

Cosmological history and direct detection of QCD axion

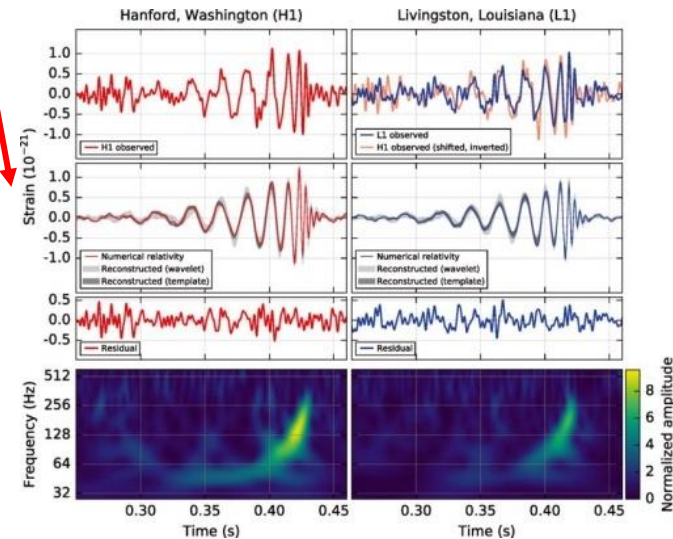
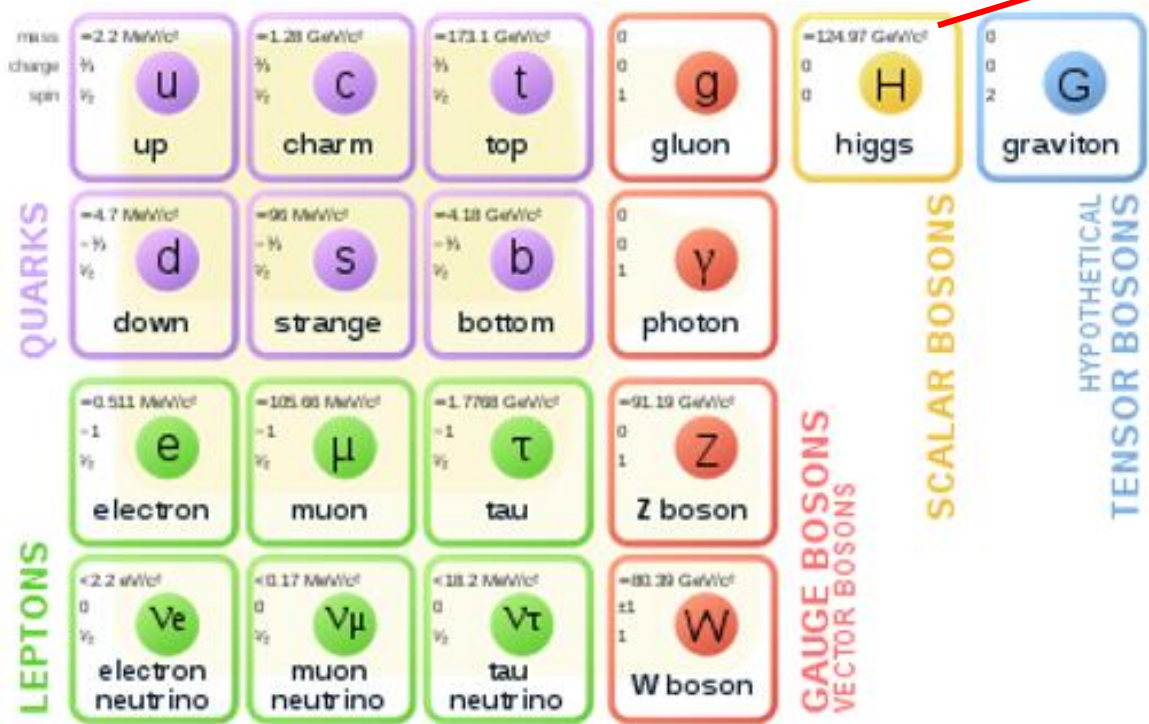
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e-Print: 2506.19918 [hep-ph]

e-Print: 2511.14851 [hep-ph]

Current Status of Particle Physics:



+ anything else?

Left-over problems:

- The identity of dark matter
- Gauge hierarchy problem
- Strong CP problem
- The identity of inflaton field
- Baryogenesis
- Cosmological constant
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Left-over problems:

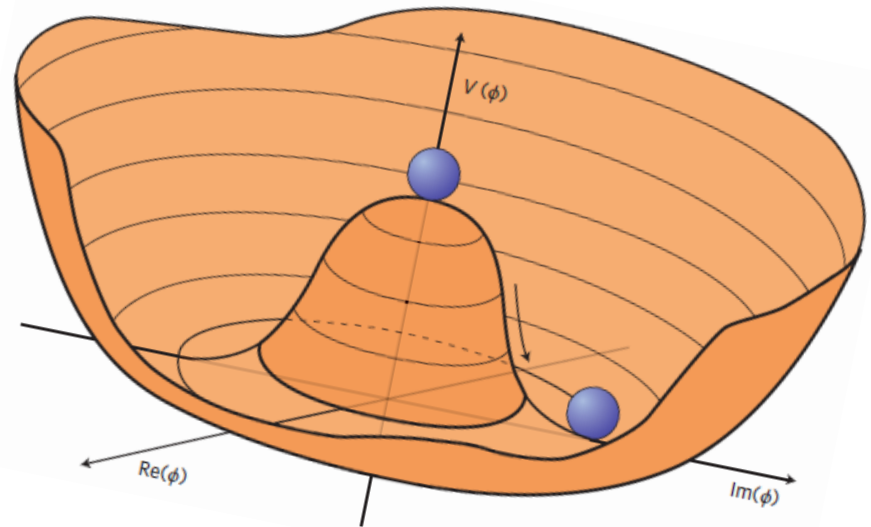
- The identity of dark matter → misalignment mechanism
 - Gauge hierarchy problem → relaxion
 - Strong CP problem → QCD axion
 - The identity of inflaton field
 - Baryogenesis
 - Cosmological constant
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 -
- can be related to axion

Theory motivations:

Strong CP-problem:

$$\underbrace{(\theta - \arg \det M_q)}_{\bar{\theta} < 10^{-10}} \frac{\alpha_s}{8\pi} G\tilde{G}$$

Introduce axion field:



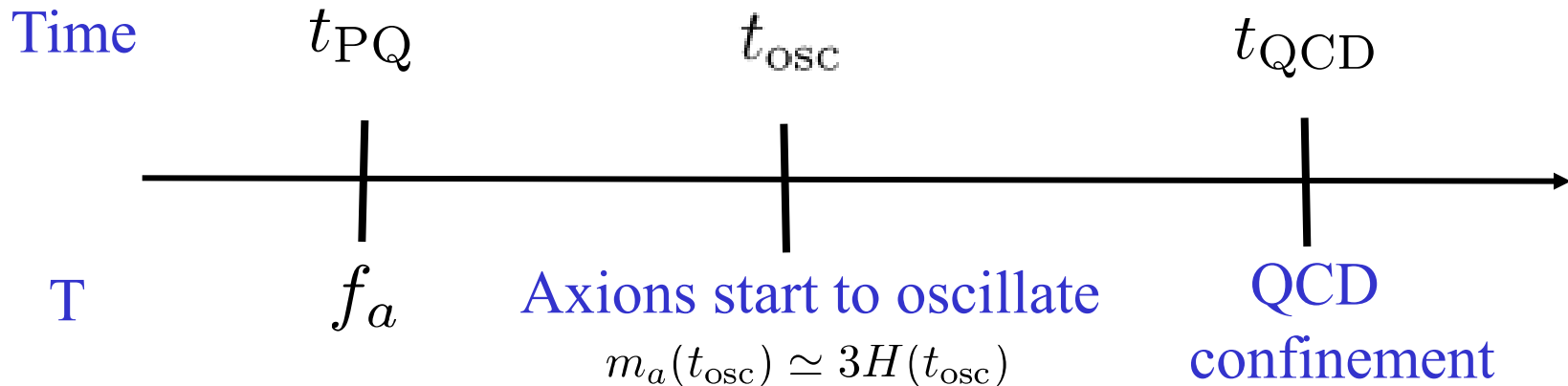
Couplings:

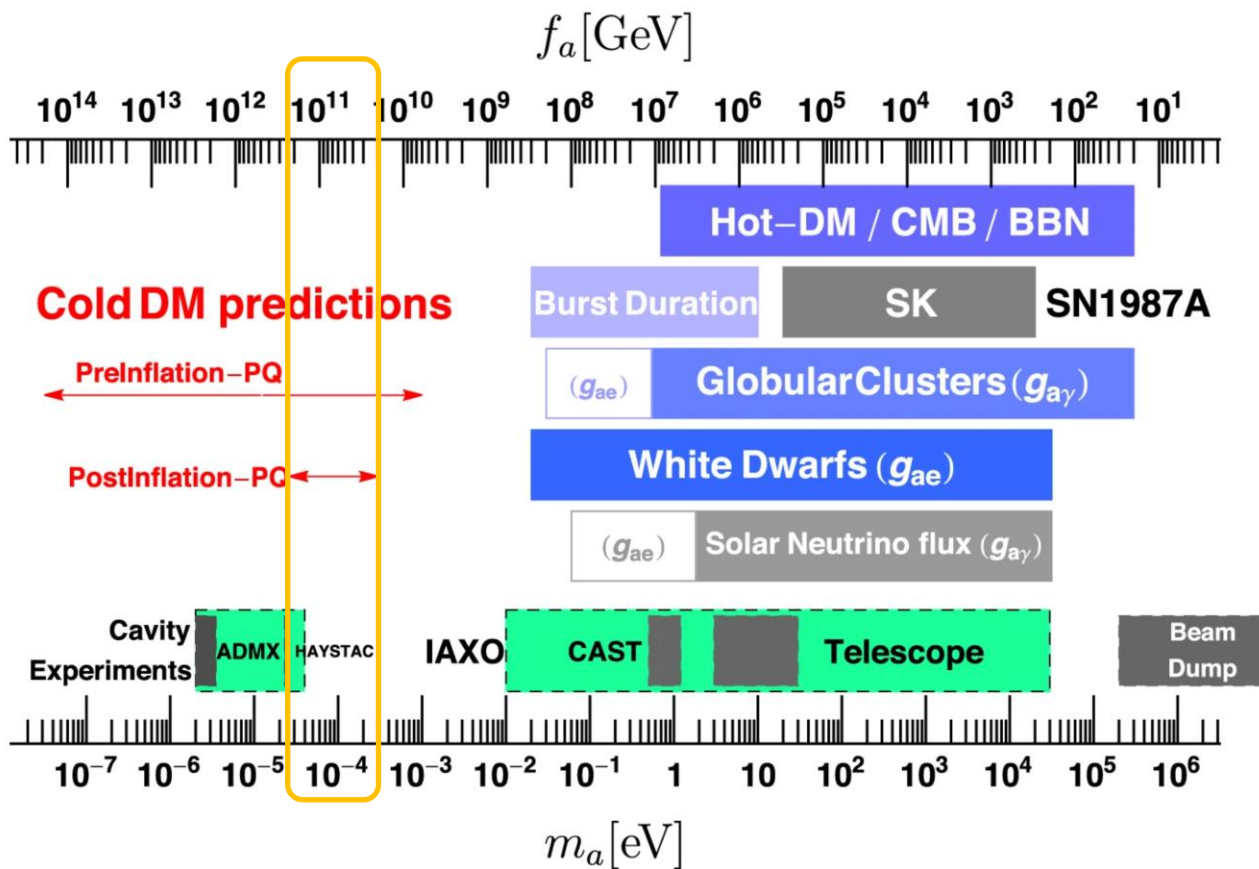
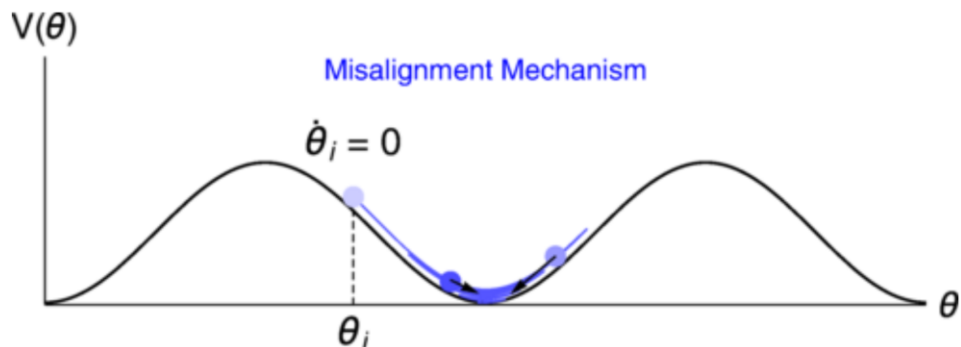
axion-gluon-gluon
axion-photon-photon
axion-fermion-fermion

Standard QCD axion thermal history

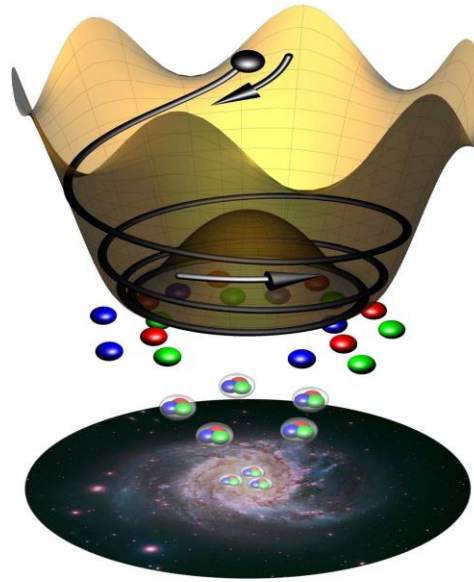
$$V(\theta_a, T) \equiv V(\theta_a) \times h(T) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u + m_d)^2} \sin^2 \left(\frac{\theta_a}{2} \right)} \times h(T)$$

$$h(T) = \begin{cases} 1 & T \leq T_{\text{QCD}} , \\ \left(\frac{T_{\text{QCD}}}{T} \right)^8 & T > T_{\text{QCD}} . \end{cases}$$

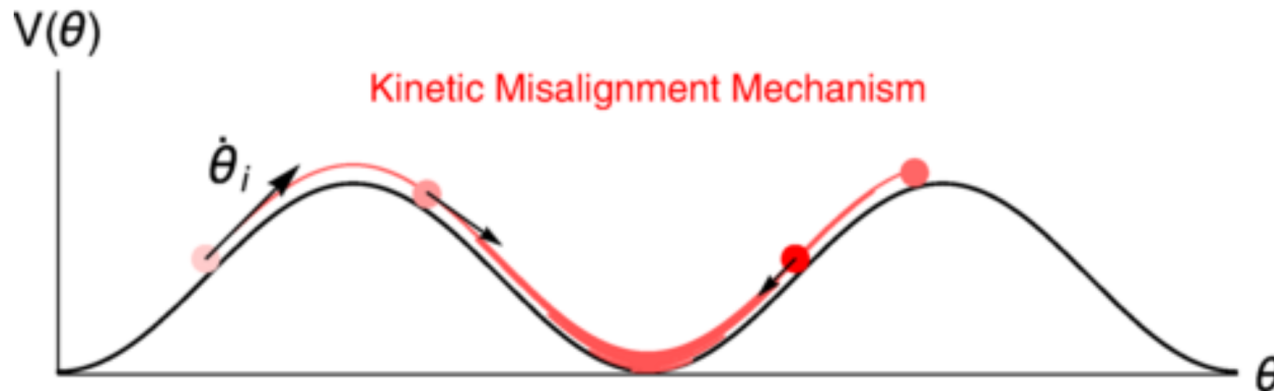




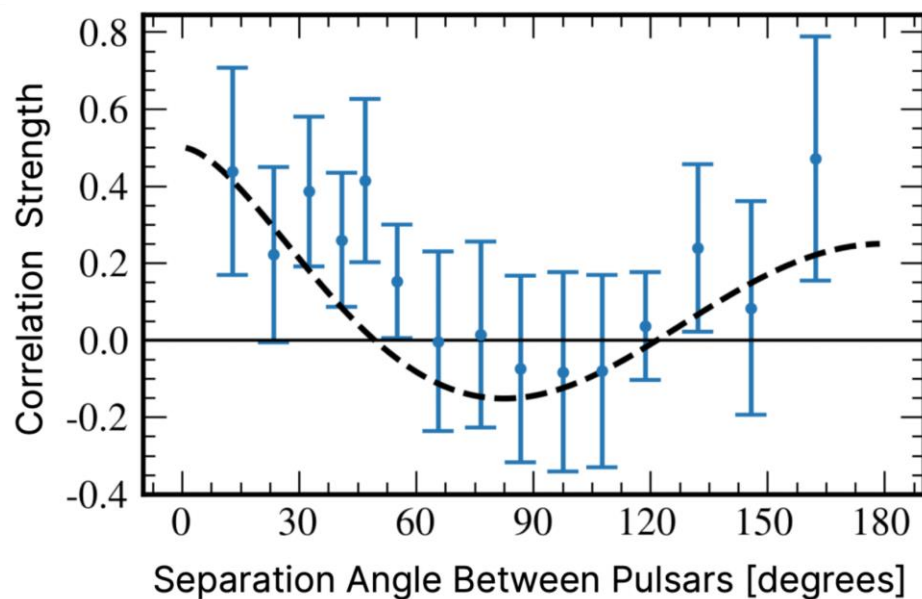
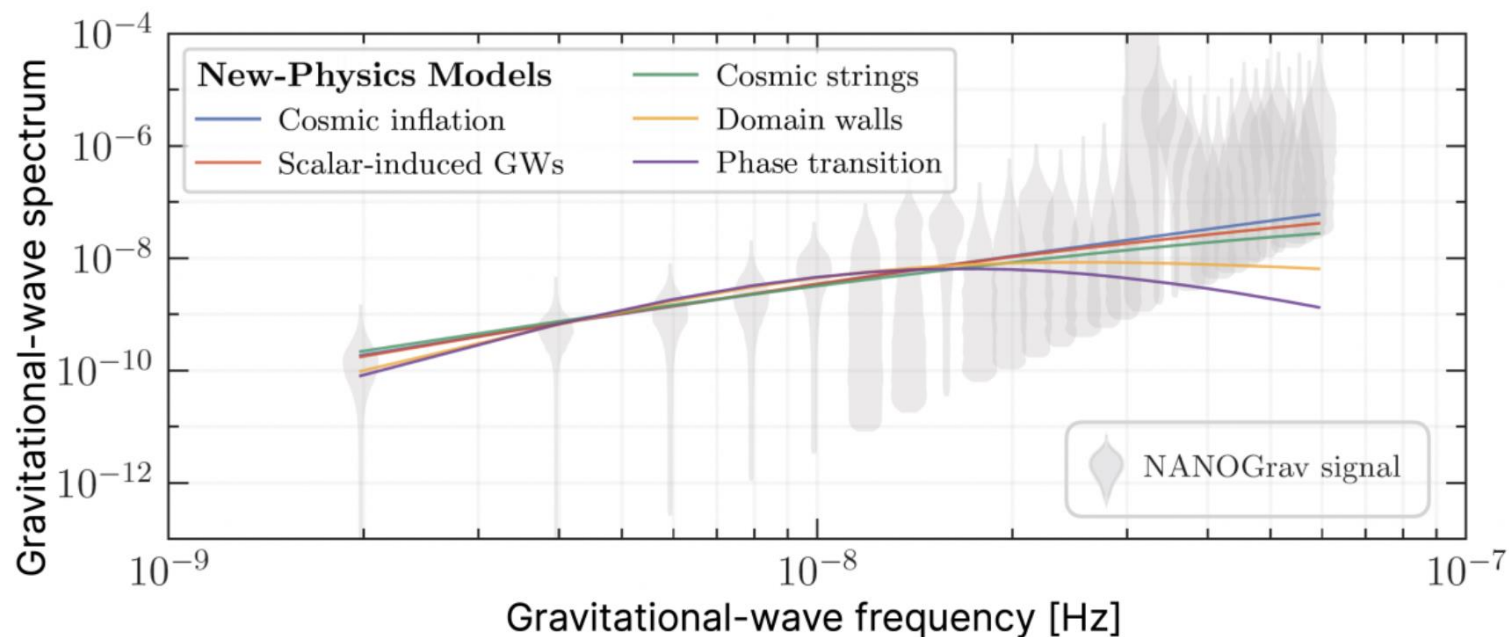
Kinetic Misalignment

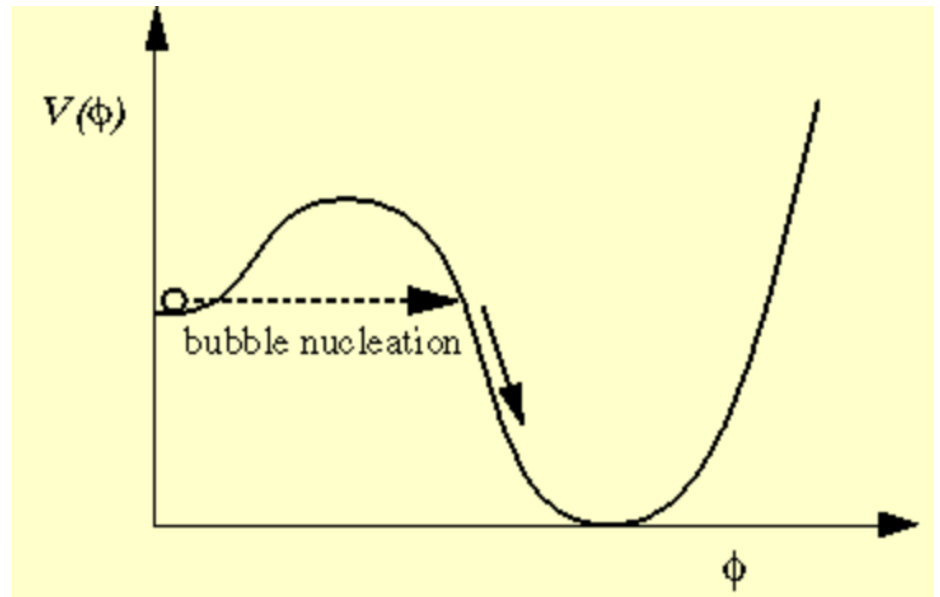
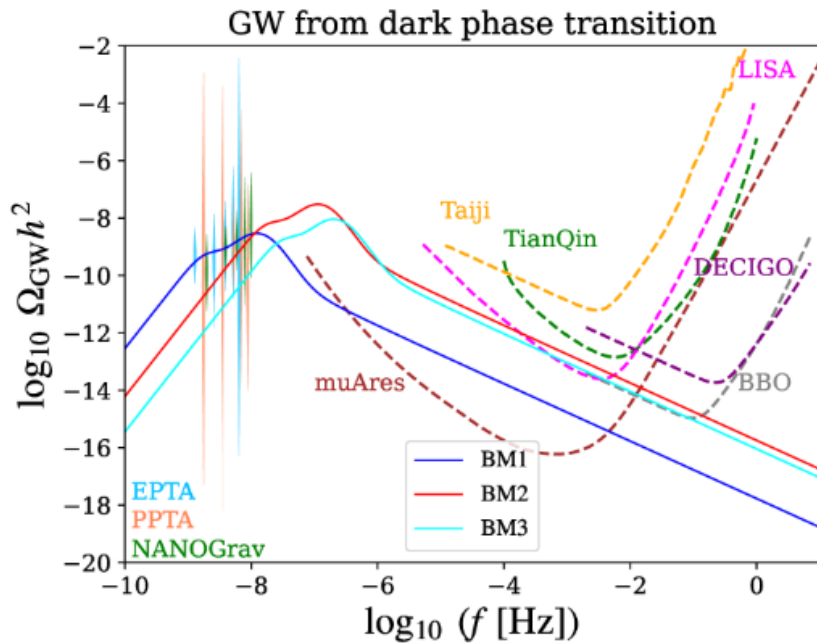


The kination era may change the prediction on the axion relic abundance.



First Order Phase Transition





Strong FOPT @ O(QCD confinement scale).

Reheat the Universe significantly due to large latent heat release.

Modify the thermal history and may lead to interesting phenomena.



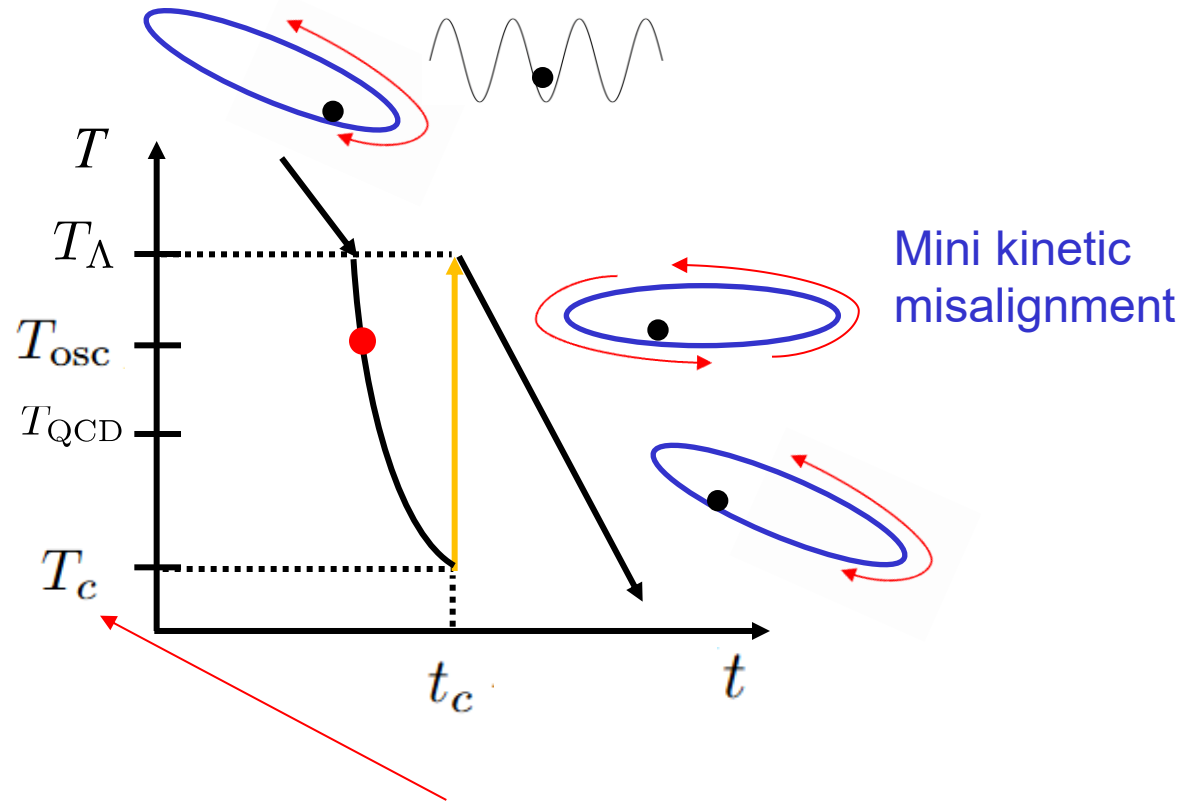
Reset axion DM relic abundance
Domain Wall production



May overclose the universe

May lead to interesting features of GW at higher frequency.

QCD axion history with the FOPT




FOPT may happen at
O(10 MeV-GeV) regime.

Mini Kinetic Misalignment

Based on the WKB approximation, the ratio of the axion number density to the entropy density is conserved.

The axion field amplitude at the time of phase transition:

$$\theta_{a,\text{PT}} = \theta_a(t = t_{\text{PT}}) \simeq \theta_{a,i} \left(\frac{m_a(T_{\text{osc}})}{m_a(T_{\text{PT}})} \right)^{1/2} \left(\frac{T_{\text{PT}}}{T_{\text{osc}}} \right)^{3/2} \left(\frac{g_{*S}(T_{\text{PT}})}{g_{*S}(T_{\text{osc}})} \right)^{1/2} \boxed{\cos \phi_{\text{PT}}} .$$


a random phase

$$K(\boxed{T_\Lambda}) = \dot{\theta}_{a,\text{PT}}^2 f_a^2 / 2 .$$

$$V_{\text{max}}(\boxed{T_\Lambda}) = 2 f_a^2 m_a(T_\Lambda)^2$$

Mini kinetic misalignment occurs if

$$K(T_\Lambda) \gg V_{\text{max}}(T_\Lambda)$$

QCD Axion evolution after FOPT

The reheating time scale (t_{reh}) after FOPT is model dependent.

If $t_{reh} > H_{PT}^{-1}$, the universe is reheated uniformly.

The evolution of the axion field remains the same spatially.

$$\ddot{\theta}_a + 3H(t)\dot{\theta}_a + m_a^2(t) \sin \theta_a = 0$$

When $K(T_\Lambda) \gg V_{\max}(T_\Lambda)$

$$\theta_a(t) = \theta_a(t_{PT}) + \frac{\dot{\theta}_{a,PT}}{H_\Lambda} \left[1 - \left(\frac{t_{PT}}{t} \right)^{1/2} \right]$$

When $K(T_{\text{trap}}) \simeq V_{\max}(T_{\text{trap}})$, axion is trapped.

$$T_{\text{trap}}/T_\Lambda = (K(T_\Lambda)/V_{\max}(T_\Lambda))^{-1/14}$$

6 from kinetic energy + 8 from potential raise

Resetting Misalignment

A reset of the misalignment is easily achieved.

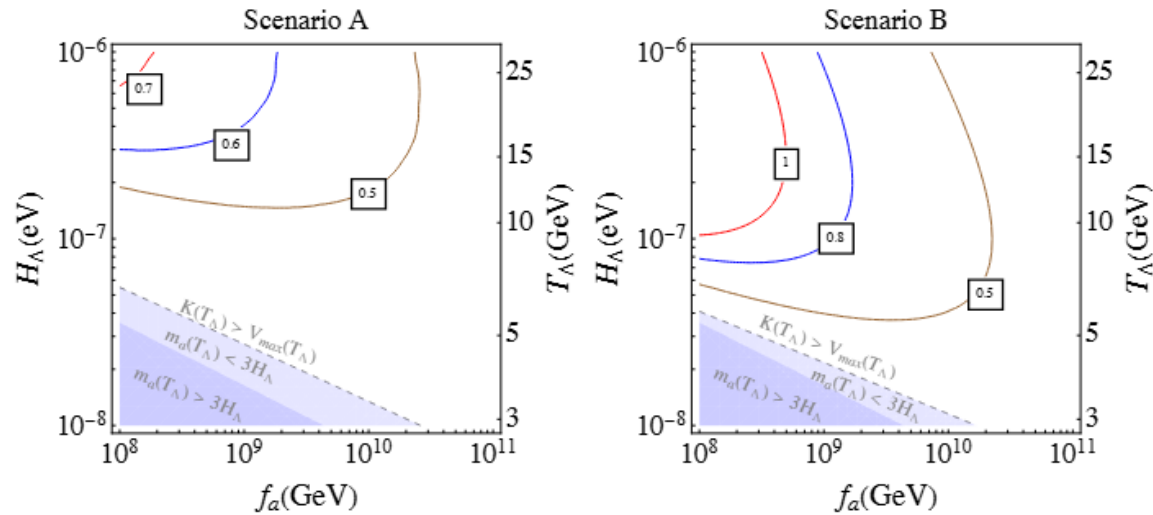
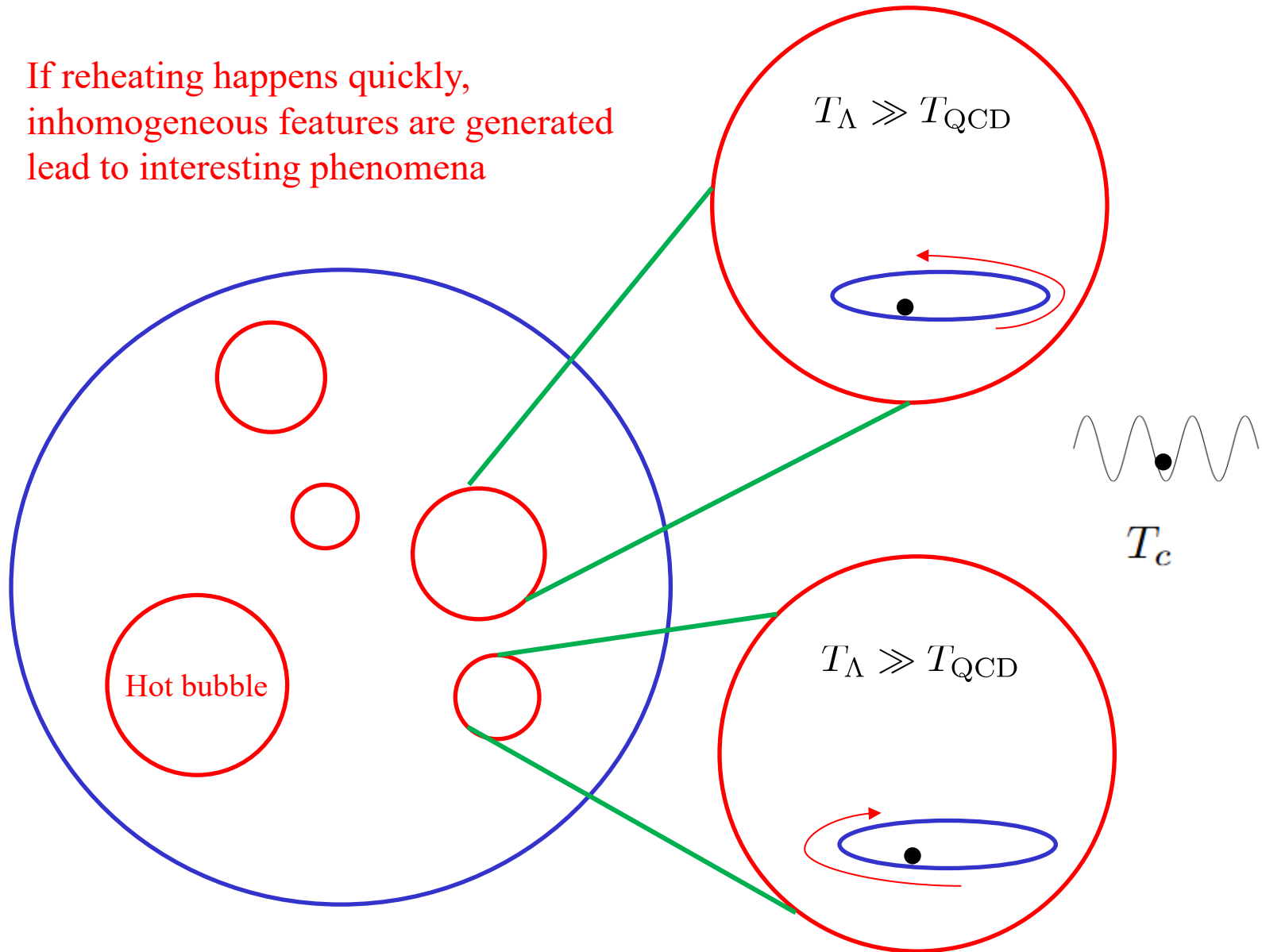


FIG. 2. The contours of the reset θ_a value. Here we choose $\theta_{a,i} = 0.2$ and $\sin \phi_{\text{PT}} = 1/\sqrt{2}$. We also show the parameter space where the kinetic energy is unimportant, and the axion field oscillates as matter or is frozen after reheating.

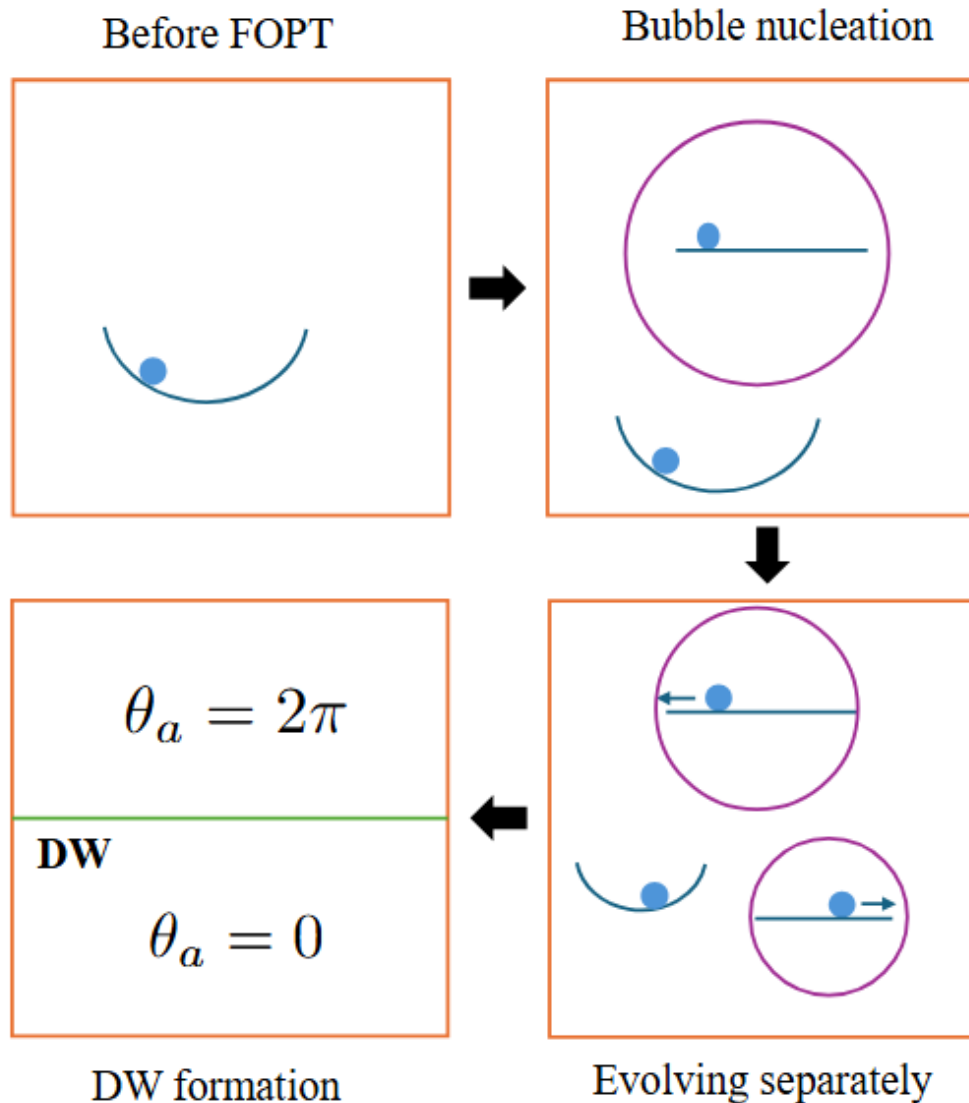
Interesting implication to QCD axion relic abundance!

Inhomogeneity From Bubble Emergence

If reheating happens quickly,
inhomogeneous features are generated
lead to interesting phenomena

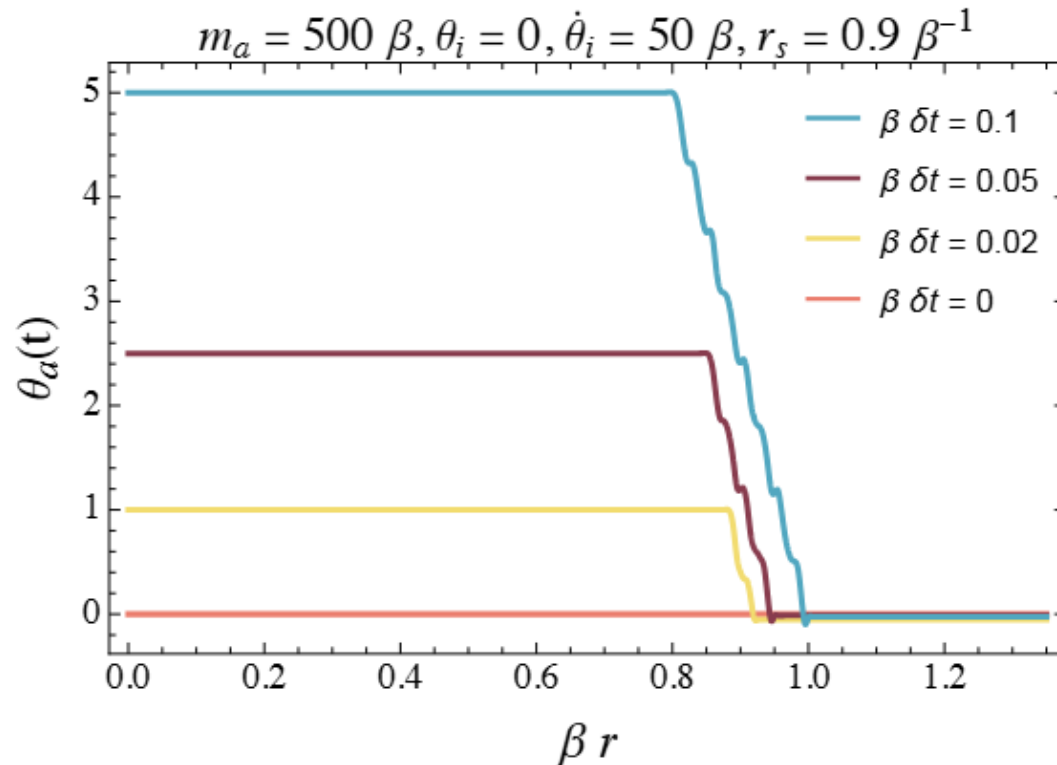


Inhomogeneity From Bubble Emergence



Inhomogeneity From Bubble Emergence

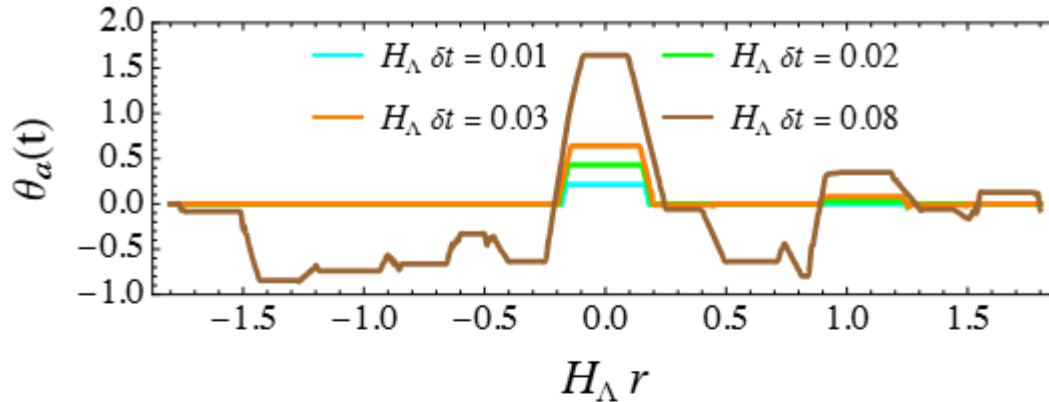
$$\ddot{\theta}_a + 3H\dot{\theta}_a - \frac{\nabla^2}{a(t)^2}\theta_a + m_a^2(T) \sin \theta_a = 0$$



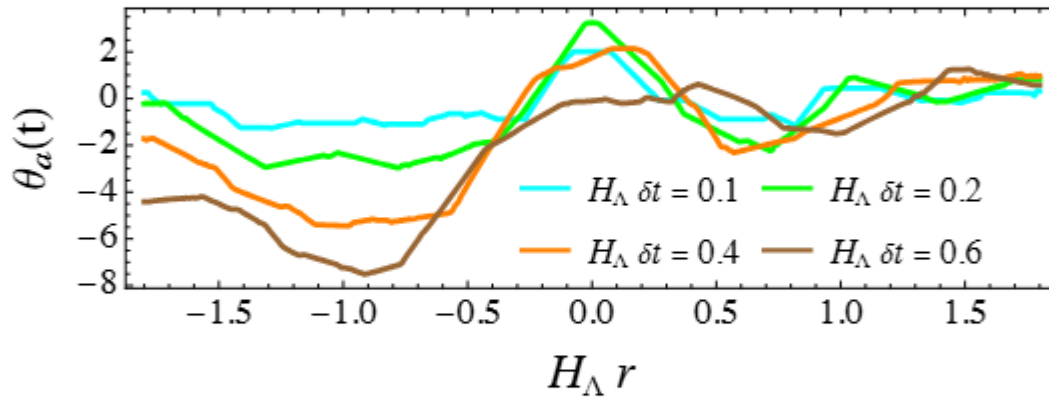
Reheating happens when the bubble is nearly to merge with other bubbles.

$$t_{reh} \sim (\beta H)^{-1}$$

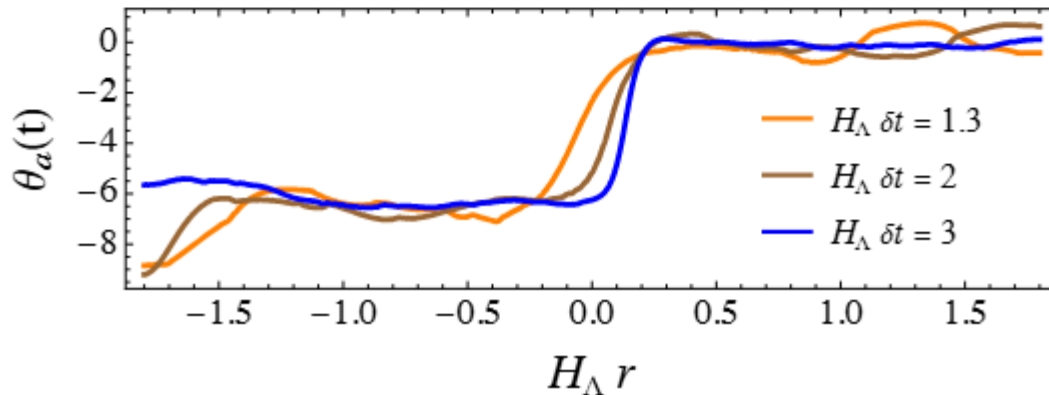
Domain Wall Formation



$m_a \sim 0$




m_a
non-negligible



trapped,
domain wall
formation

Domain Wall Formation

Condition for domain wall formation:

$$\frac{\dot{\theta}_{a,\text{PT}}}{\beta} \left(\frac{T_{\text{trap}}}{T_{\Lambda}} \right) > \pi$$


redshift factor

There is cosmic string produced, thus no boundary for DWs

Finite-sized enclosed domain walls:

Axion radiation from closed DWs.

Change in axion relic abundance.

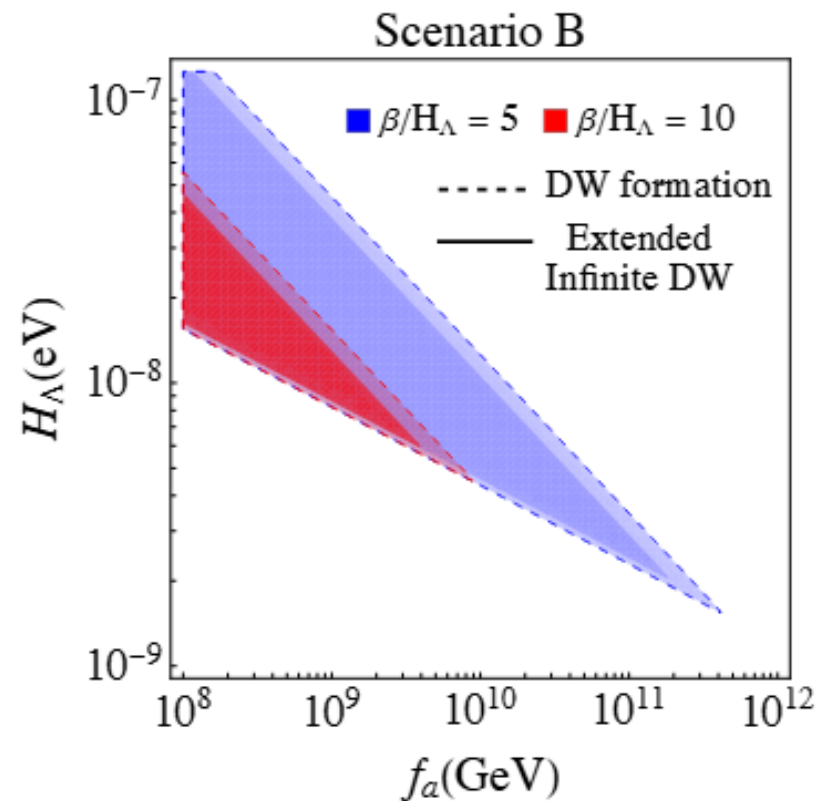
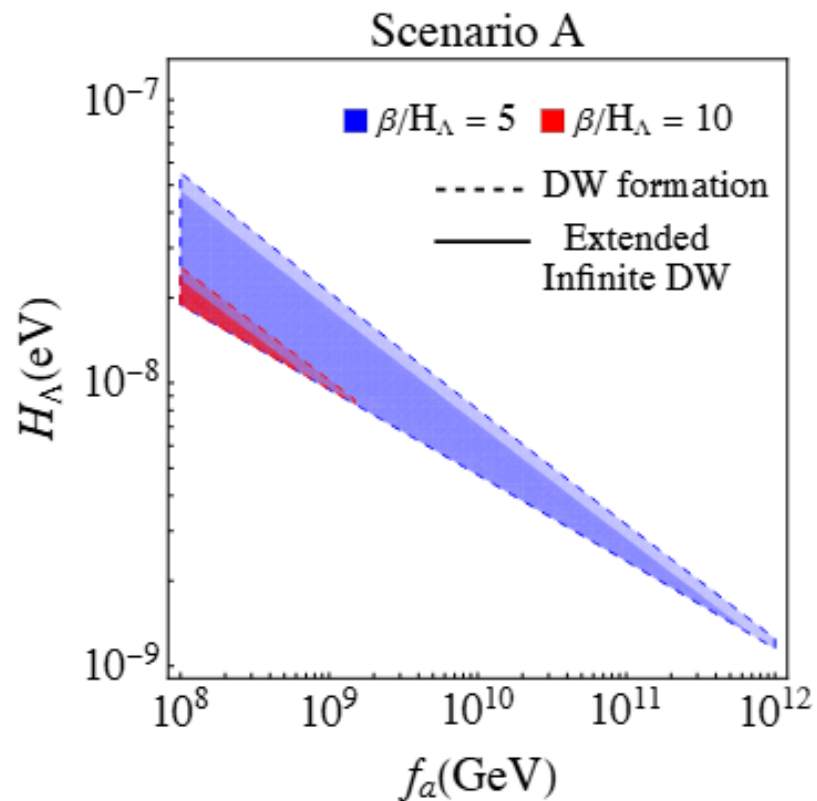
Infinitely extended domain walls:

Percolation theorem: 31%

Scaling regime: $\rho_w \sim \sigma/t$

$m_{a,0} f_a^2 \sim \Lambda_{\text{QCD}}^2 f_a \gg 1 \text{ MeV}^3$ overclose the universe

Domain Wall Formation

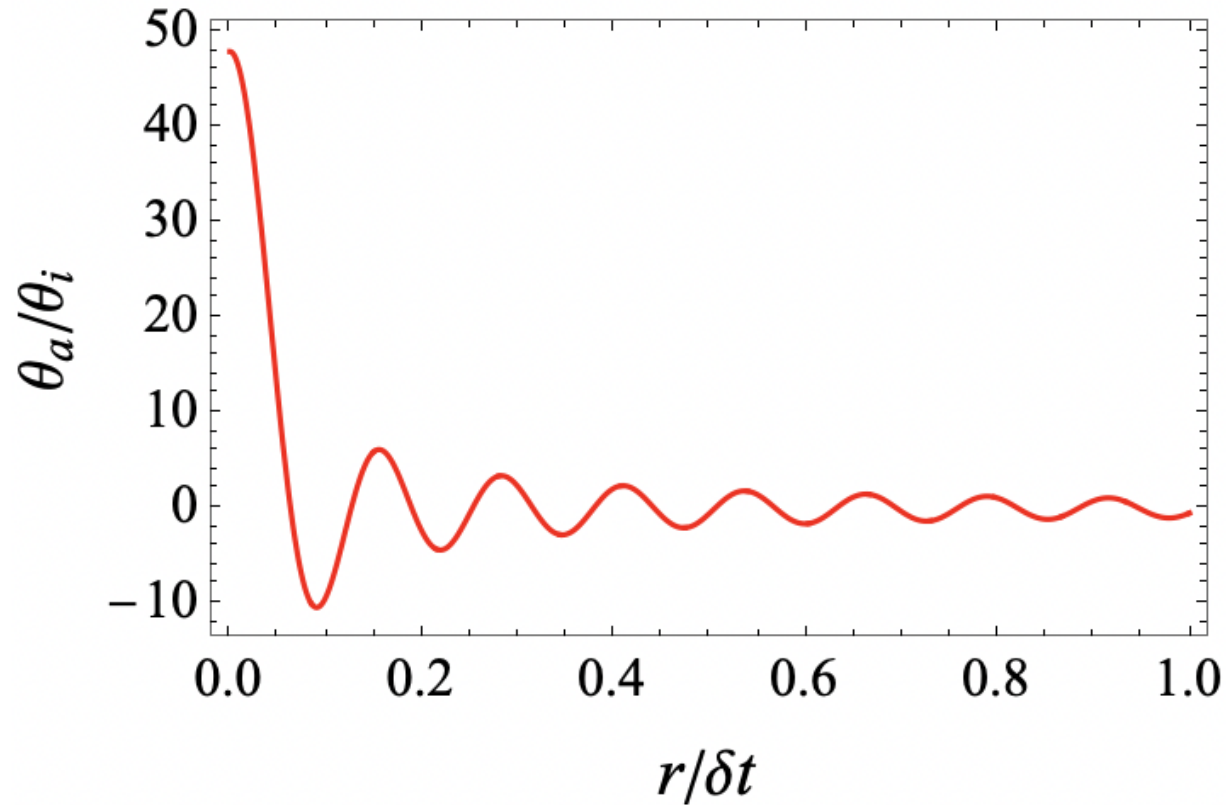


$$\theta_{a,i} = 2.$$

Domain Wall Formation

Instantaneous reheating: $t_{reh} \ll (\beta H)^{-1}$

$$m_a \delta t = 100, \phi_s = 0$$



Only the center gains large field value.
Small enclosed bubbles are produced.

Conclusion 1

FOPT at QCD scale is hinted by NANOGrav.

The existence of QCD axion is very motivated.

⇒ Combining these two aspects leads to interesting cosmology!

Mini kinetic misalignment is a natural prediction.

Axion relic abundance is reset.

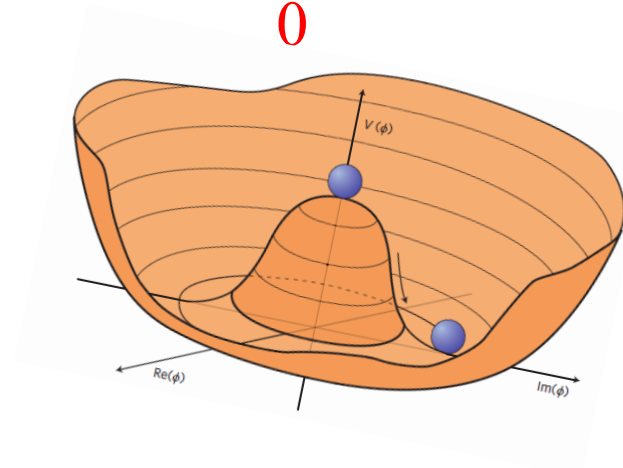
Domain wall formation, without cosmic strings, is expected.

⇒ Enclosed DWs decay away and change relic abundance.

Infinitely extended DWs easily overclose the universe.

QCD axion direct detection

$$\underbrace{(\theta - \arg \det M_q)}_0 \frac{\alpha_s}{8\pi} G\tilde{G}$$



The introduction of axion sets the average value of $\bar{\theta}$ to zero.

Axion DM leads to a time dependent $\bar{\theta}$.

$$a(t, \vec{x}) \approx \frac{\sqrt{2\rho_{\text{DM,local}}}}{m_a} \sin(\omega_a t - \vec{p} \cdot \vec{x})$$

Chiral Lagrangian

Axion affects the nucleon and mesons through Chiral Lagrangian.

lead to pion-axion mixing

$$\begin{aligned} u &\rightarrow e^{i\phi_u} u \\ d &\rightarrow e^{i\phi_d} d, \\ \phi_u + \phi_d &= \theta \end{aligned}$$

$$(\theta - \arg \det M_q) \frac{\alpha_s}{8\pi} G\tilde{G}$$

$$\mathcal{L} = -\frac{1}{4}f_\pi^2 \text{Tr}[\partial_\mu U \partial^\mu U^\dagger] + B_0 \text{Tr}[(MU_0)U + (MU_0)^\dagger U^\dagger]$$

$$-c_1 \bar{N}((MU_0)P_L + (MU_0)^\dagger P_R)N \Rightarrow \text{change nucleon mass}$$

$$-c_2 \bar{N}(U^\dagger (MU_0)^\dagger U^\dagger P_L + U(MU_0)U P_R)N$$

change nucleon-pion interactions

$$c + \frac{m_u m_d \sin \theta}{f_\pi [m_u^2 + m_d^2 + 2m_u m_d \cos \theta]^{1/2}} \quad \pi \quad c + \frac{m_u m_d \sin \theta}{f_\pi [m_u^2 + m_d^2 + 2m_u m_d \cos \theta]^{1/2}}$$

Simple example

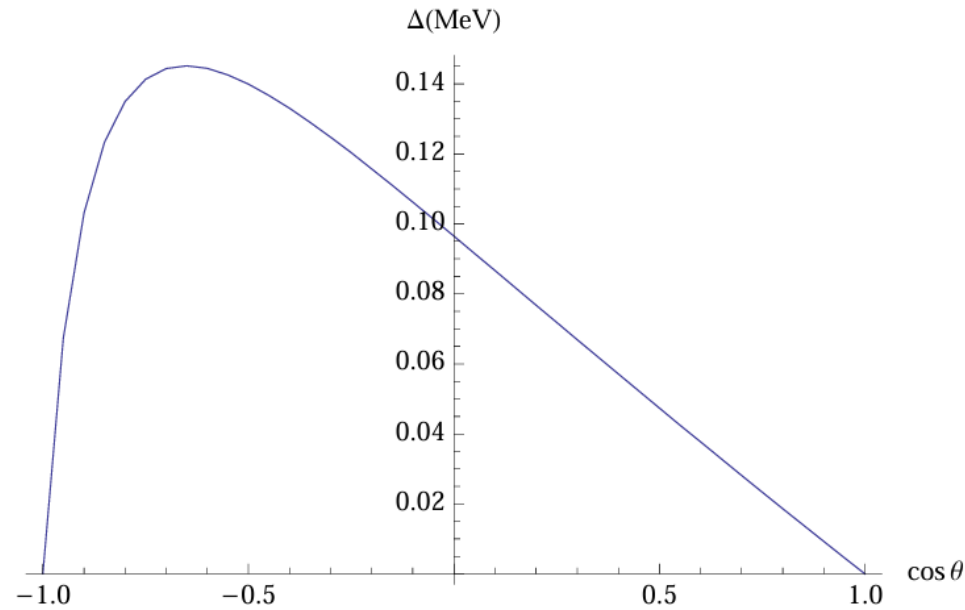
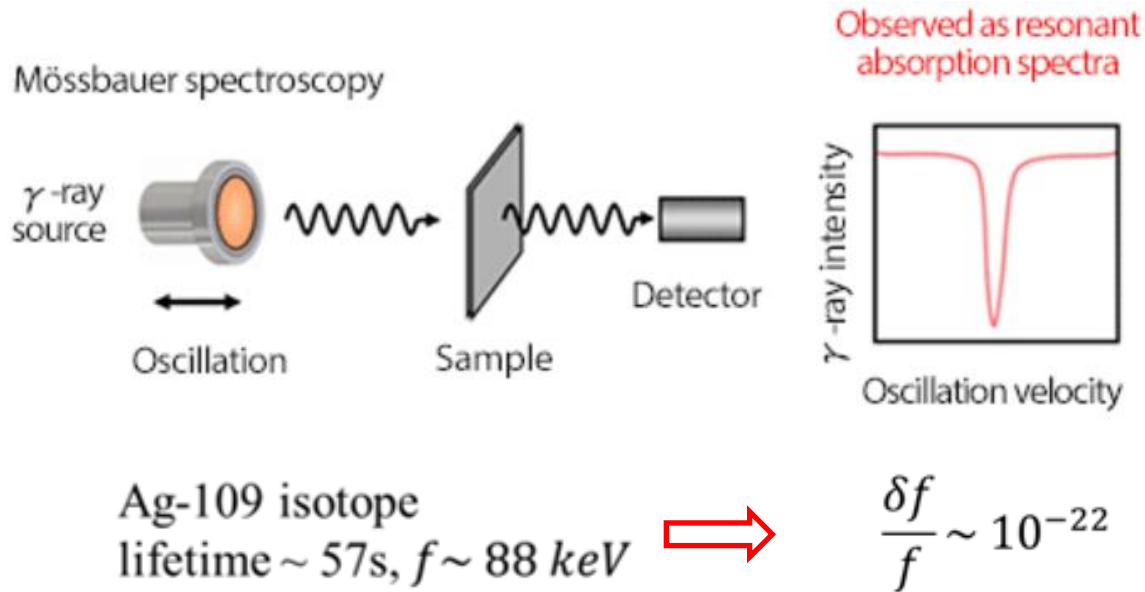


Figure 2: *Shift in the deuteron binding energy as a function of $\cos \theta$*

Mössbauer Effect



It is very challenging to measure the natural linewidth experimentally.

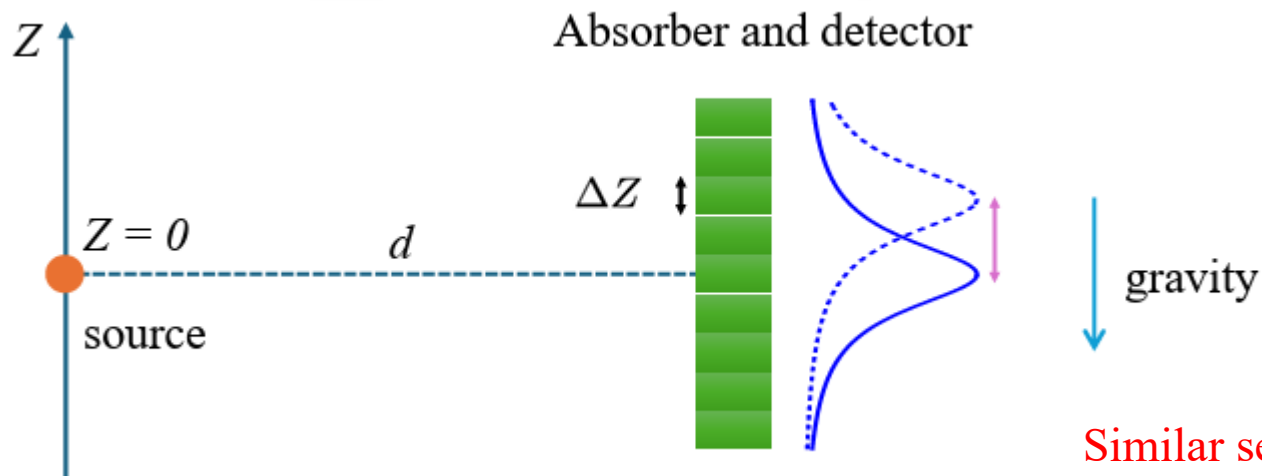
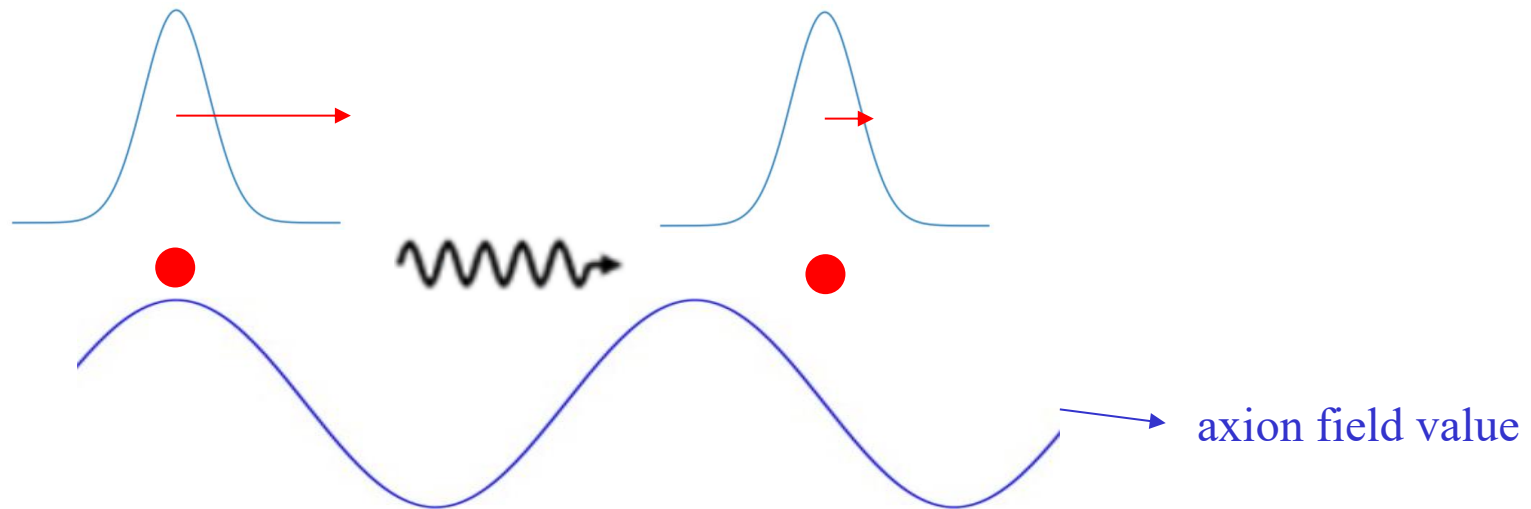
So far, the best measurement has achieved 7 times of the nature width.

$$\frac{\delta f}{f} = g \Delta Z$$

\downarrow
 μm

If axion leads to the change of nucleon-pion coupling, it will change the energy splits.

Experiment Setup



$$\delta E_{\text{bind}}(d) = \Delta E_{\text{bind}}(t + d, \vec{d}) - \Delta E_{\text{bind}}(t, 0)$$

Similar setup has been proposed to search for gravitational wave.
Sci. Bull. 69, 2795 (2024)

Correction and Cancellation

assume related corrections
common factor
(1 + $\epsilon_{\alpha S} \theta^2$)

| 能量项 | 1/2 ⁻ (MeV) | 7/2 ⁺ (MeV) | $\Delta E = E_{7/2^+} - E_{1/2^-}$ |
|--------------------|------------------------|------------------------|------------------------------------|
| $E_{\alpha S}$ | -17611.06 | -17502.45 | +108.61 |
| $E_{\alpha V}$ | 13056.42 | 12985.91 | -70.51 |
| $E_{\alpha TV}$ | 19.46 | 19.17 | -0.29 |
| $E_{\beta S}$ | 2536.06 | 2508.91 | -27.15 |
| $E_{\gamma S}$ | -869.23 | -856.75 | +12.48 |
| $E_{\gamma V}$ | -100.45 | -99.36 | +1.09 |
| $E_{\delta S}$ | 46.32 | 44.55 | -1.77 |
| $E_{\delta V}$ | 196.86 | 189.64 | -7.22 |
| $E_{\delta TV}$ | 2.20 | 2.13 | -0.07 |
| E_{kin} | 1476.73 | 1461.69 | -15.04 |
| E_{cou} | 333.96 | 334.49 | +0.53 |
| E_{pair} | -9.03 | -10.49 | -1.46 |
| E_{Total} | -921.76 | -922.57 | -0.81 |

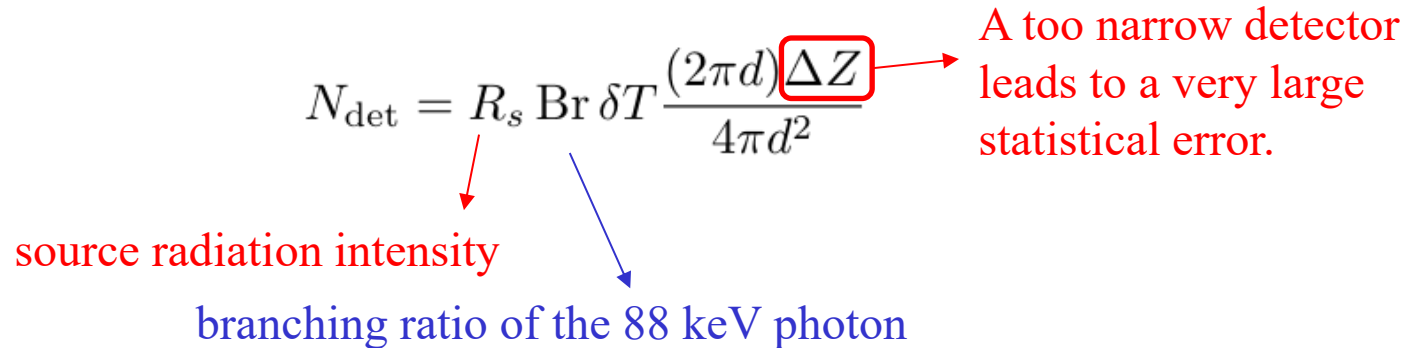
assume unrelated
corrections

uncommon factor
(1 + $\epsilon_i \theta^2$)

$$\delta E \sim (1 + \epsilon_{\alpha S} \theta^2) 100 \text{ MeV}$$

Systematic and Statistical Uncertainties

$$N_{\text{det}} = R_s \text{ Br } \delta T \frac{(2\pi d) \Delta Z}{4\pi d^2}$$



source radiation intensity

branching ratio of the 88 keV photon

A too narrow detector leads to a very large statistical error.

A competition:

the systematic uncertainty

worse with a larger detector width

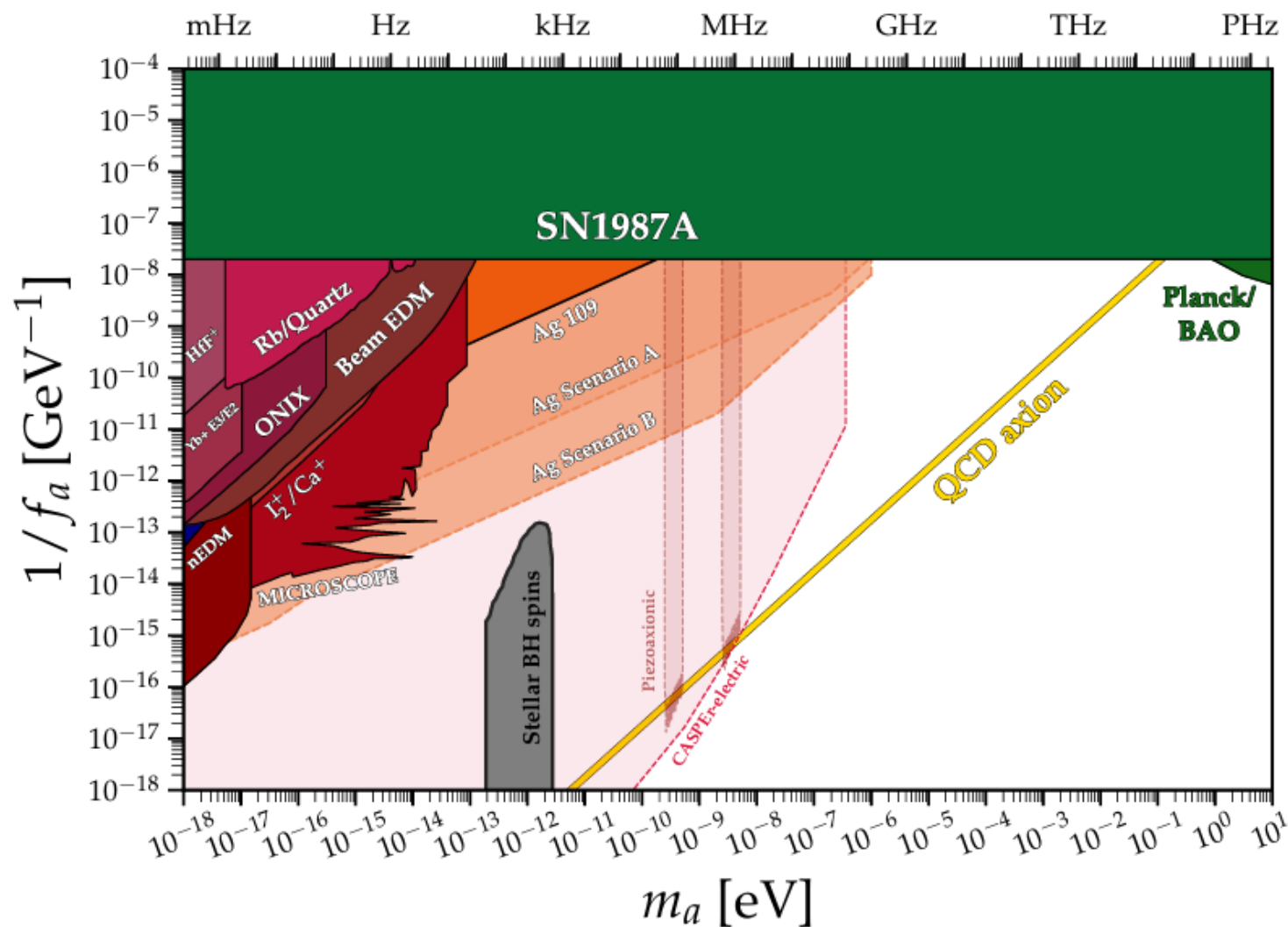
the statistical uncertainty

better with a larger detector width

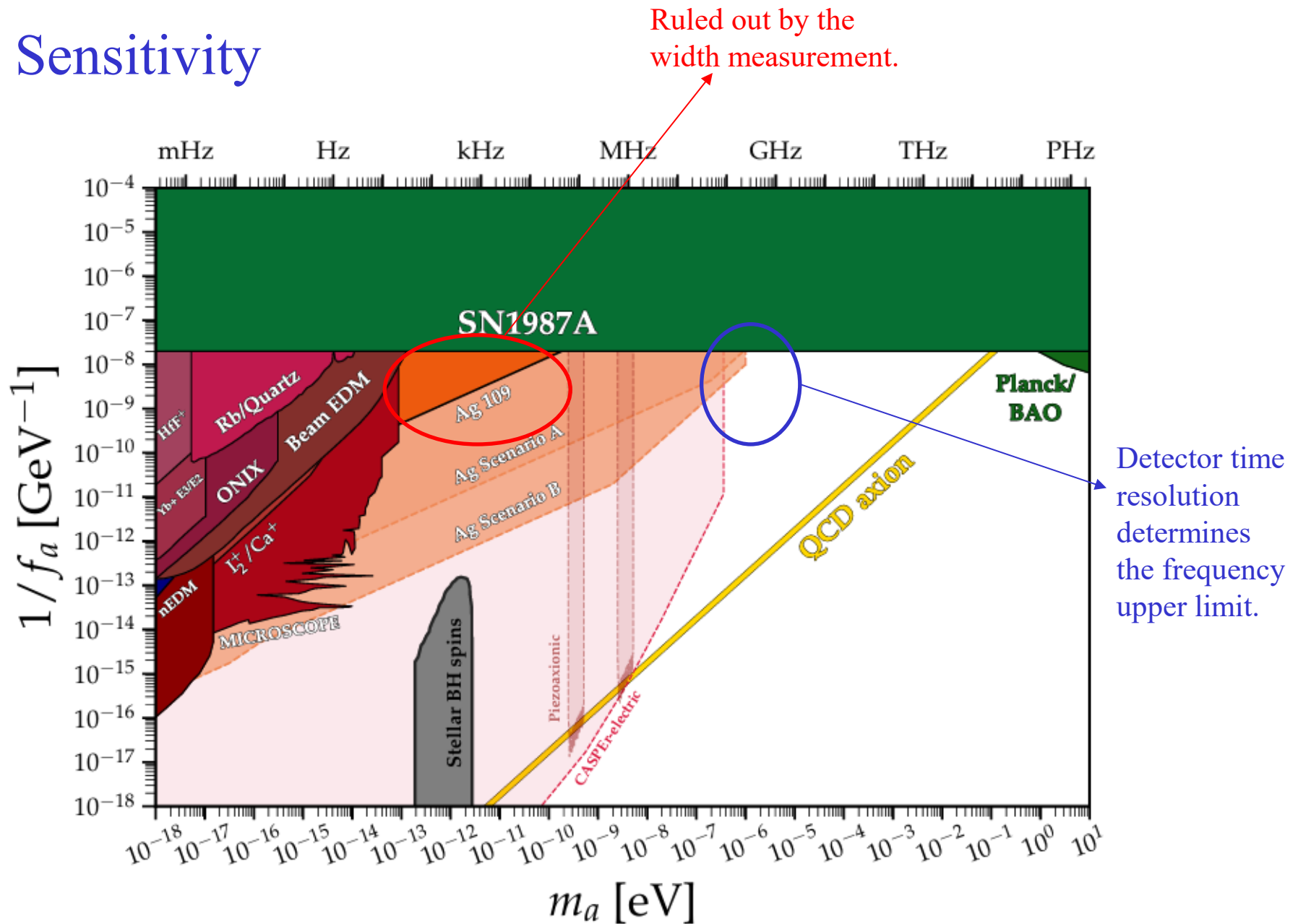
A smart data analysis strategy is introduced to balance these two competing aspects.

Sensitivity

| | $g(g_{\oplus})$ | d(m) | ΔZ | ϵf_S | $m_{a,\text{max}}(\text{eV})$ | $R_s(\text{Ci})$ |
|---|-----------------|------|-----------------|----------------|-------------------------------|------------------|
| A | 1 | 1 | $10\mu\text{m}$ | 0.04 | 10^{-6} | 1 |
| B | 10^{-4} | 100 | 1 dm | 0.04 | 10^{-6} | 10 |



Sensitivity



Conclusion 2

QCD axion dark matter leads to time-dependent nuclear binding energy.

This can be tested using Mossbauer Spectroscopy.

Ag-109 has an intrinsic width as 10^{-22} , perfect for this purpose.

Current width measurement has already imposed interesting constraints.

A large parameter space can be probed in the future.