Wess-Zumino-Witten Interactions of Axions with Three-Flavor

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2406.11948 (Phys.Rev.Lett. 134, 081803 (2025)),
2505.24822 (JHEP 02 (2026) 117)

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The axion effective Lagrangian at quark-level

- A more detailed effective Lagrangian

$$\mathscr{L}_{\text{eff},0} = \bar{q}_0(iD_\mu\gamma^\mu - \mathbf{m}_{q,0})q_0 + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2$$
$$+ g_{ag,0}\frac{a}{f_a}G\tilde{G} + g_{a\gamma,0}\frac{a}{f_a}F\tilde{F} + \frac{\partial_\mu a}{f_a}\left(\bar{q}_L\mathbf{k}_{L,0}\gamma^\mu q_L + \bar{q}_R\mathbf{k}_{R,0}\gamma^\mu q_R + \ldots\right)$$

Bauer et al, PRL 127 (2021), 081803

- Quark mass $\mathbf{m}_{q,0}$ diagonal and real

- Coupling to both left/right fermions $\mathbf{k}_{L,0}$ and $\mathbf{k}_{R,0}$

The axion-dependent chiral rotation

- Use an axion-dependent chiral rotation to eliminate $aG\tilde{G}$ term

Bauer et al, PRL 127 (2021), 081803

$$q_0(x) = \exp\left[-i(\boldsymbol{\delta}_{q,0} + \boldsymbol{\kappa}_{q,0}\gamma_5)c_{gg}\frac{a(x)}{f_a}\right] q(x) \qquad \mathrm{Tr}(\kappa_{q,0}) = 1$$

- New effective Lagrangian

$$\mathscr{L}_{\text{eff}} = \bar{q}(iD_\mu\gamma^\mu - \mathbf{m}_q(a))q + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2$$
$$+ g_{a\gamma}\frac{a}{f_a}F\tilde{F} + \frac{\partial_\mu a}{f_a}\left(\bar{q}_L\mathbf{k}_L(a)\gamma^\mu q_L + \bar{q}_R\mathbf{k}_R(a)\gamma^\mu q_R + \dots\right)$$

The axion-dependent chiral rotation

- Define the chiral rotations (2-flavor for simplicity)

$$\boldsymbol{\theta}_L \equiv \boldsymbol{\delta}_{q,0} - \boldsymbol{\kappa}_{q,0} \qquad U_L \equiv \exp\left[-i\boldsymbol{\theta}_L a/f_a\right]$$

$$\boldsymbol{\theta}_R \equiv \boldsymbol{\delta}_{q,0} + \boldsymbol{\kappa}_{q,0} \qquad U_R \equiv \exp\left[-i\boldsymbol{\theta}_R a/f_a\right]$$

- The relations between parameters

$$\mathbf{m}_q(a) = U_L^\dagger \mathbf{m}_0 U_R \to \begin{pmatrix} m_{u,0} e^{-2i\kappa_{u,0} c_{gg} \frac{a}{f_a}} & 0 \\ 0 & m_{d,0} e^{-2i\kappa_{d,0} c_{gg} \frac{a}{f_a}} \end{pmatrix}$$

$$\mathbf{k}_L(a) = U_L^\dagger [\mathbf{k}_{L,0} + c_{gg}\boldsymbol{\theta}_{L,0}] U_L \to \mathbf{k}_{L,0} + c_{gg}\boldsymbol{\theta}_{L,0}$$

$$\mathbf{k}_R(a) = U_R^\dagger [\mathbf{k}_{R,0} + c_{gg}\boldsymbol{\theta}_{R,0}] U_R \to \mathbf{k}_{R,0} + c_{gg}\boldsymbol{\theta}_{R,0}$$

Anomalous axion contribution

$$g_{a\gamma} = g_{a\gamma_0} - 2N_c c_{gg} \mathrm{Tr}\left[\mathbf{Q}^2 \boldsymbol{\kappa}_{q,0}\right]$$

The consistent ChPT axion Lagrangian at meson level

$$\mathscr{L}_{\text{eff}} = \bar{q}(iD_\mu\gamma^\mu - \mathbf{m}_q(a))q + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2$$

$$+ g_{a\gamma}\frac{a}{f_a}F\tilde{F} + \frac{\partial_\mu a}{f_a}\left(\bar{q}_L \mathbf{k}_L(a)\gamma^\mu q_L + \bar{q}_R \mathbf{k}_R(a)\gamma^\mu q_R + \ldots\right)$$

- ChPT Lagrangian matching

$$U = \exp[(\sqrt{2}i/f_\pi)\pi^a\boldsymbol{\tau}^a]$$

$$\mathscr{L}_{\chi\text{PT}} = \frac{f_\pi^2}{8}\left[(D^\mu U)(D_\mu U)^\dagger\right] + \frac{f_\pi^2}{4}B_0\text{Tr}\left[\mathbf{m}_q(a)U^\dagger + h.c.\right] + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2 + g_{a\gamma}\frac{a}{f_a}F\tilde{F}$$

- The axion derivative coupling

Bauer et al, PRL 127 (2021), 081803

$$D^\mu U \to D^\mu U - i\frac{\partial^\mu a}{f_a}\left(\mathbf{k}_L U - U\mathbf{k}_R\right)$$

The importance of consistency

- The physical results should be independent of auxiliary parameters

$$q_0(x) = \exp\left[-i(\boldsymbol{\delta}_{q,0} + \boldsymbol{\kappa}_{q,0}\gamma_5)c_{gg}\frac{a(x)}{f_a}\right] q(x)$$

- The most important channel BR($K \to \pi a$) is off by a factor of 37 for 35 years

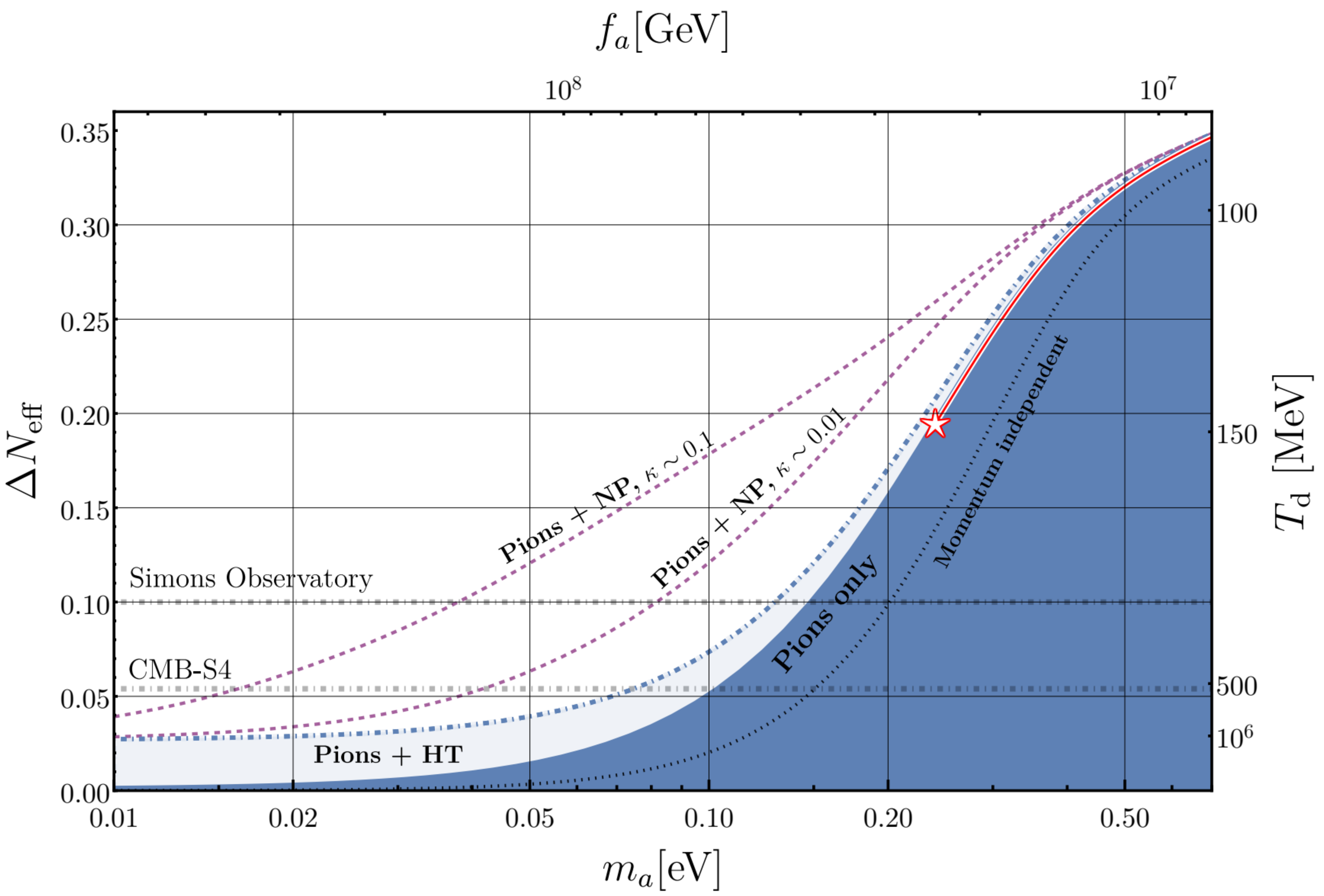
H. Georgi, D. B. Kaplan and L. Randall, Phys. Lett. B 169, 73-78 (1986)

- Model-independent expression for $K \to \pi a$ and $\pi^- \to e^- \bar{\nu}_e a$ have been obtained for all axion couplings, only in 2021

Bauer et al, PRL 127 (2021), 081803

Why accurate interactions are important?

- Prediction for thermal axion and its near future test by CMB observation
  - Thermal axion production (high T): $q\bar{q} \to ga, qg \to qa$
    Ferreira, Notari PRL 120 (2018)191301
  - QCD phase transition:
    D'Eramo, Hajkarim, Yun PRL 128 (2022)152001
  - Improved axion-pion scattering production: $\pi\pi \leftrightarrow \pi a$
    Notari, Rompineve, Villladoro PRL 131 (2023)011004



QCD axion: $m_a < 0.24$ eV

Wess-Zumino-Witten Interactions in QCD

- WZW terms can describe anomalies in QCD, ensuring gauge invariance and completing chiral L

- Low-energy dynamics of mesons:
  e.g. multiple mesons and photons interactions
  $\pi_0/\eta/\eta' \to \gamma\gamma, \eta' \to 4\pi, \gamma^* \to 3\pi, 5\pi$

- Axion should be involved in WZW interactions systematically, not only in $a - \gamma - \gamma$ interactions

  - Raised by Harvey Hill Hill in [PRL 99 (2007) 261601], but not solved in previous study

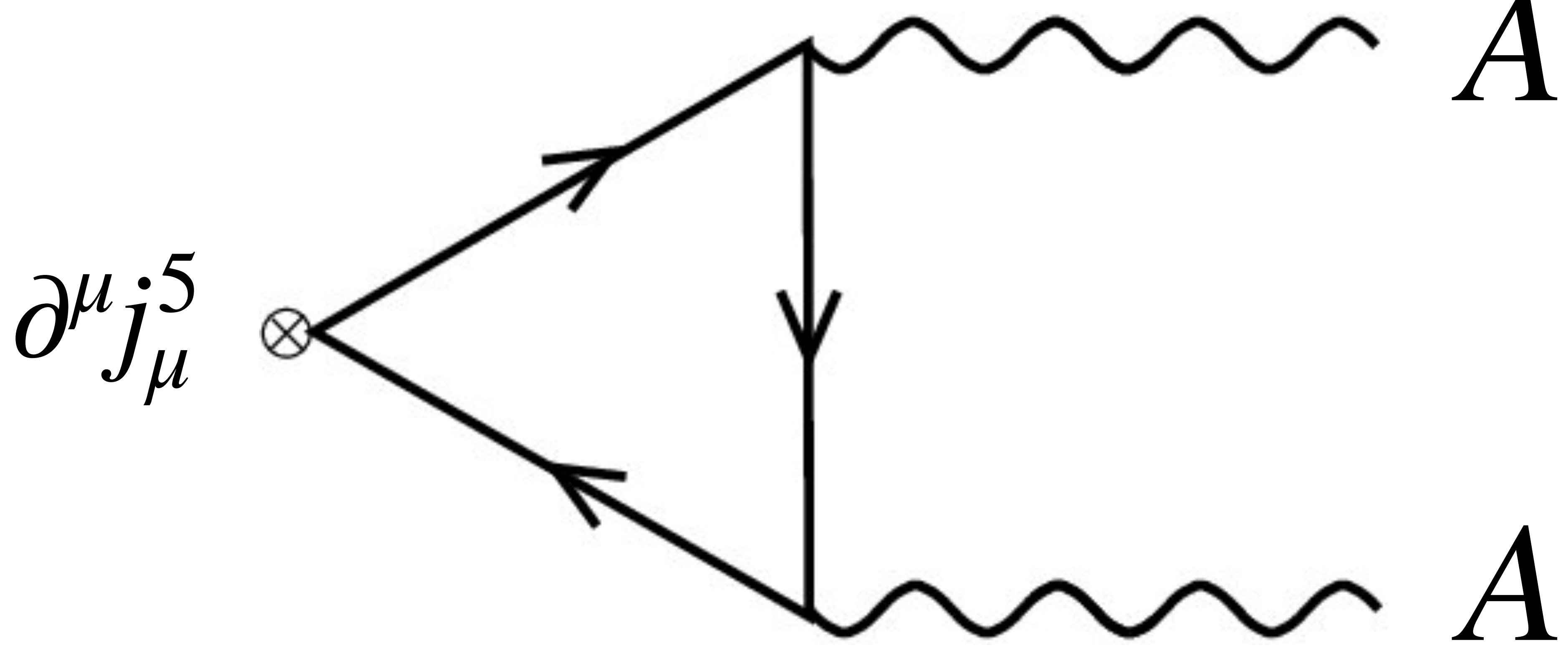
Challenges in axion-meson interactions

- 1. Physics should not depend on the choice of chiral basis
- 2. How to systematically include axion interactions into WZW terms
- 3. Global symmetry (e.g. PQ, $U(1)_B$) will induce mixed anomaly to be dealt with

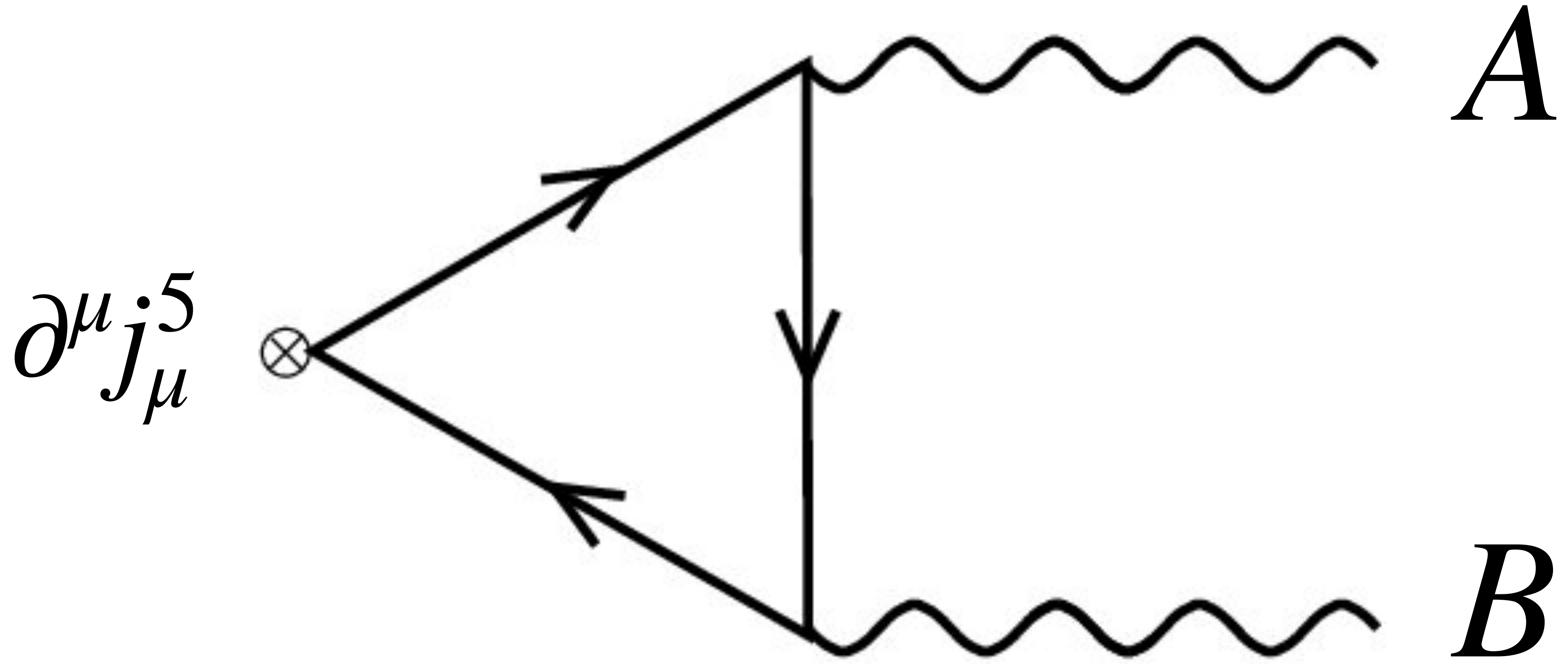
Global currents and background vector fields

- Background fields can couple to currents of $\mathscr{L}_{\chi\mathrm{PT}}$
  - Baryon currents $U(1)_B$ in neutron star, ω meson
  - Z boson vector in neutrino dense environment
- SM gauge invariance needs counter terms

$$\partial^\mu j^5_\mu = \frac{1}{48\pi^2}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$$



$$\delta\partial^\mu j^5_\mu \propto \epsilon_{\mu\nu\rho\sigma}A^{\mu\nu}B^{\rho\sigma}$$



WZW counter terms for global symmetry

J. A. Harvey, C. T. Hill, and R. J. Hill,
PRL 99 (2007) 261601,
PRD 77(2008) 085017

- Generic WZW interactions with counter terms

  - Vector fields in 1-form: $\mathscr{A}_{L/R} \equiv \mathbb{A}_{L/R} + \mathbb{B}_{L/R}$
    Similar to Hidden Local Symmetry

$$\mathscr{L}^{\text{full}}_{\text{WZW}}(U, \mathscr{A}_{L/R}) = \mathscr{L}_{\text{WZW}}(U, \mathscr{A}_L, \mathscr{A}_R) + \mathscr{L}_c(\mathbb{A}_{L/R}, \mathbb{B}_{L/R})$$

- Counter terms ensures SM invariance

$$\Gamma_c = -2\mathscr{C}\int Tr\Big[(\mathbb{A}_L d\mathbb{A}_L + d\mathbb{A}_L\mathbb{A}_L)\mathbb{B}_L + \frac{1}{2}\mathbb{A}_L(\mathbb{B}_L d\mathbb{B}_L + d\mathbb{B}_L\mathbb{B}_L) - \frac{3}{2}i\mathbb{A}_L^3\mathbb{B}_L - \frac{3}{4}i\mathbb{A}_L\mathbb{B}_L\mathbb{A}_L\mathbb{B}_L - \frac{i}{2}\mathbb{A}_L\mathbb{B}_L^3\Big] - (L \leftrightarrow R)$$

- Suitable for chiral gauge fields and background fields

Axion treatment in three flavor

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \bar{q} i \not{D} q - (\bar{q}_L \boldsymbol{m}_q q_R + {\rm H.c.}) + \frac{1}{2}\partial_\mu a\, \partial^\mu a - \frac{m_a^2}{2} a^2$$
$$+ \frac{\partial^\mu a}{f_a}(\bar{q}_L \gamma^\mu \boldsymbol{k}_L q_L + \bar{q}_R \gamma^\mu \boldsymbol{k}_R q_R) + c_{gg} \frac{\alpha_s}{4\pi} \frac{a}{f_a} G_{\mu\nu} \widetilde{G}^{\mu\nu} + \frac{a}{f_a} \sum_{\mathcal{A}_{1,2}} c_{\mathcal{A}_1 \mathcal{A}_2} F_{\mathcal{A}_1 \mu\nu} \widetilde{F}^{\mu\nu}_{\mathcal{A}_2} + \mathcal{L}_c\,,$$

- $D_\mu = \partial_\mu - ig(A_L P_L + A_R P_R)$
- Hints from quark-level L: $D_\mu \to D_\mu + i\frac{\partial_\mu a}{f_a}\left(\mathbf{k}_L P_L + \mathbf{k}_R P_R\right)$
- Hints from ChPT L: $D^\mu U \to D^\mu U - i\frac{\partial^\mu a}{f_a}\left(\mathbf{k}_L U - U\mathbf{k}_R\right)$

Pseudoscalars in three flavor

- U matrix for mesons in three flavor

$$U = \exp\left[(\sqrt{2}i/f_\pi)\pi^a \boldsymbol{t}^a\right] \equiv \exp\left[(\sqrt{2}i/f_\pi)\boldsymbol{\Phi}\right]$$

$$\boldsymbol{\Phi} = \begin{pmatrix} \pi_0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi_0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\,\overline{K}^0 & -\frac{2}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 \end{pmatrix}$$

- η' mass from instanton effect $\bar{\theta}\frac{\alpha_s}{8\pi}G_{\mu\nu}\widetilde{G}^{\mu\nu} \to -\frac{\tau}{2}\left(-i\log\det U - \bar{\theta}\right)^2$

Axionized Lagrangian:

$$\mathcal{L}_{\chi\mathrm{PT}} \supset -\frac{\tau}{2}\left(-i\log\det U - 2c_{gg}\frac{a}{f_a}\right)^2 = -\frac{m_0^2}{2}\left(\eta_0 - \frac{c_{gg}}{\sqrt{3}}\frac{f_\pi}{f_a}a\right)^2$$

Axion treatment in three flavor

- Vector fields in 1-form: $\mathscr{A}_{L/R} \equiv \mathbb{A}_{L/R} + \mathbb{B}_{L/R}$
  Similar to Hidden Local Symmetry

$$\mathbb{A}_L = \frac{e}{s_w} W^i \boldsymbol{T_i} + \frac{e}{c_w} W^0 \mathbf{Y}_Q, \quad \mathbb{A}_R = \frac{e}{c_w} W^0 \mathbf{Y}_q$$

- Axion 1-form field can be added into background fields: $\mathbb{B}_{L/R} \to \mathbb{B}_{L/R} + \mathbf{k}_{L/R,0} \dfrac{da}{f_a}$

- 3-flavor ChPT with SM gauge bosons and background fields

$$\mathbb{B}_V \equiv \mathbb{B}_L + \mathbb{B}_R = g \begin{pmatrix} \rho_0 + \omega & \sqrt{2}\rho^+ & \sqrt{2}K^*_+ \\ \sqrt{2}\rho^- & -\rho_0 + \omega & \sqrt{2}K^*_0 \\ \sqrt{2}K^*_- & \sqrt{2}\,\overline{K}^*_0 & \sqrt{2}\phi \end{pmatrix} + (\boldsymbol{k}_L + \boldsymbol{k}_R)\frac{da}{f_a},$$

$$\mathbb{B}_A \equiv \mathbb{B}_L - \mathbb{B}_R = g \begin{pmatrix} a_1 + f_1 & \sqrt{2}a^+ & \sqrt{2}K^*_{A+} \\ \sqrt{2}a^- & -a_1 + f_1 & \sqrt{2}K^*_{A0} \\ \sqrt{2}K^*_{A-} & \sqrt{2}\,\overline{K}^*_{A0} & \sqrt{2}f_s \end{pmatrix} + (\boldsymbol{k}_L - \boldsymbol{k}_R)\frac{da}{f_a}$$

The consistent full axion Lagrangian at low-energy

- ChPT: $D^\mu = \partial^\mu - i \sum_{\mathcal{A}} (\mathcal{A}_L^\mu P_L + \mathcal{A}_R^\mu P_R)$

$$\mathcal{L}_{\chi\mathrm{PT}} = \frac{f_\pi^2}{8} \mathrm{Tr}\left[(D^\mu U)(D_\mu U)^\dagger\right] + \frac{f_\pi^2}{4} B_0 \,\mathrm{Tr}\left[\boldsymbol{m}_q U^\dagger + \mathrm{H.c.}\right] - \frac{m_0^2}{2}\left(\eta_0 - \frac{c_{gg}}{\sqrt{3}}\frac{f_\pi}{f_a} a\right)^2$$
$$+ \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_a^2}{2} a^2 + \frac{a}{f_a} \sum_{\mathcal{A}_{1,2}} c_{\mathcal{A}_1 \mathcal{A}_2} F_{\mathcal{A}_1 \mu\nu} \widetilde{F}^{\mu\nu}_{\mathcal{A}_2} ,$$

- Full WZW: $\mathscr{L}^{\mathrm{full}}_{\mathrm{WZW}}(U, \mathscr{A}_{L/R}) = \mathscr{L}_{\mathrm{WZW}}(U, \mathscr{A}_L, \mathscr{A}_R) + \mathscr{L}_c(\mathbb{A}_{L/R}, \mathbb{B}_{L/R})$

- Full $\mathscr{L}$: $\mathscr{L}^{\mathrm{full}}_{\mathrm{axion}} \equiv \left[\mathscr{L}_{\chi\mathrm{PT}} + \mathscr{L}^{\mathrm{full}}_{\mathrm{WZW}}\right]\left(U, \mathbf{m}_q(a), \mathscr{A}_{L/R} + \mathbf{k}_{L/R}(a)\, da/f_a\right)$

Matching between $\mathscr{L}_{\rm eff}$ and $\mathscr{L}^{\rm full}_{\rm axion}$

$$\mathscr{L}_{\rm eff}(q, \mathscr{A}_{L/R} | \boldsymbol{m}_q, \boldsymbol{k}_{L/R}, c_{gg}) \supset c_{gg} \frac{\alpha_s}{4\pi}\frac{a}{f_a} G\tilde{G}$$

$$q \to \exp\left[-i\left(\boldsymbol{\delta}_q + \boldsymbol{\kappa}_q \gamma_5\right) \frac{c_{gg} a}{f_a}\right] q$$

$$\left\{\mathscr{L}_{\rm eff}(q, \mathscr{A}_{L/R} | \boldsymbol{m}'_q, \boldsymbol{k}'_{L/R}, c'_{gg}) + \delta\mathscr{L}^{\rm ano}_a\right\} \supset c_{gg}[1 - {\rm Tr}(\boldsymbol{\kappa}_q)] \frac{\alpha_s}{4\pi}\frac{a}{f_a} G\tilde{G}$$

matching

matching

$$\left[\mathscr{L}_{\chi{\rm PT}} + \mathscr{L}^{\rm full}_{\rm WZW}\right](U, \mathscr{A}_{L/R} | \boldsymbol{m}_q, \boldsymbol{k}_{L/R}, c_{gg}) \supset -\frac{\tau}{2}\left(-i \log[\det U] - 2 c_{gg} a/f_a\right)^2$$

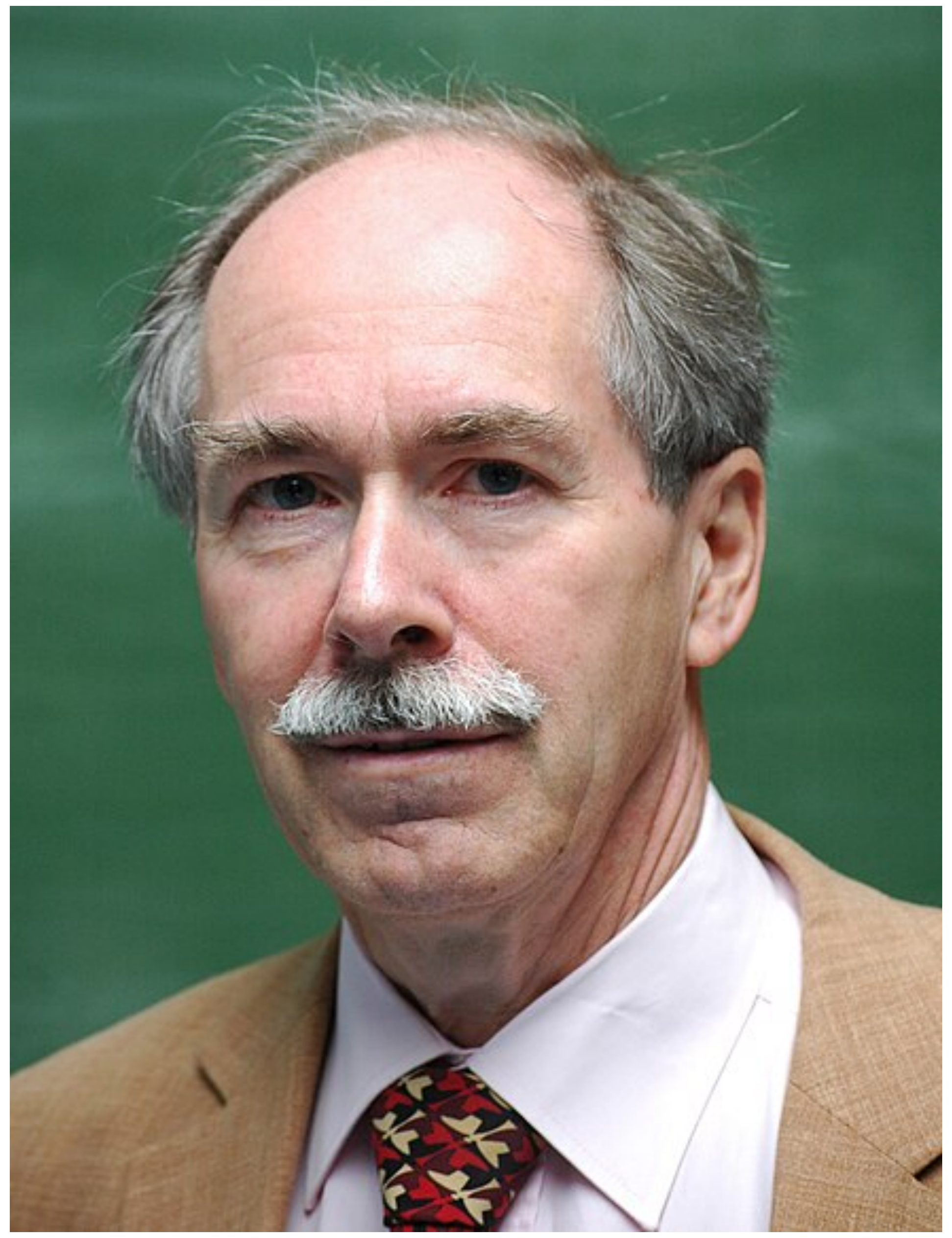
$$U \to U_L U U_R^\dagger$$

$$\left\{\left[\mathscr{L}_{\chi{\rm PT}} + \mathscr{L}^{\rm full}_{\rm WZW}\right](U, \mathscr{A}_{L/R} | \boldsymbol{m}'_q, \boldsymbol{k}'_{L/R}, c'_{gg}) + \delta\mathscr{L}^{\rm ano}_{\rm WZW}\right\} \supset -\frac{\tau}{2}\left(-i \log[\det U] - 2 c_{gg}[1 - {\rm Tr}(\boldsymbol{\kappa}_q)] a/f_a\right)^2$$

- Important: consistency for any κ_q rotation

Gerard 't Hooft UV-IR anomaly matching condition

- The anomalies of global symmetries must match between the ultraviolet (UV) and infrared (IR) descriptions of a QFT



Gerard 't Hooft

$$\mathscr{L}_{\rm eff}\left(q, \mathscr{A}_{L/R} \,\middle|\, \boldsymbol{m}_q, \boldsymbol{k}_{L/R}, c_{gg}\right) \supset c_{gg}\frac{\alpha_s}{4\pi}\frac{a}{f_a}G\tilde{G}$$

$$q \to \exp\left[-i\left(\boldsymbol{\delta}_q + \boldsymbol{\kappa}_q\gamma_5\right)\frac{c_{gg}a}{f_a}\right]q$$

$$\left\{\mathscr{L}_{\rm eff}\left(q, \mathscr{A}_{L/R} \,\middle|\, \boldsymbol{m}'_q, \boldsymbol{k}'_{L/R}, c'_{gg}\right) + \delta\mathscr{L}_a^{\rm ano}\right\} \supset c_{gg}\left[1 - {\rm Tr}(\boldsymbol{\kappa}_q)\right]\frac{\alpha_s}{4\pi}\frac{a}{f_a}G\tilde{G}$$

matching

matching

$$\left[\mathscr{L}_{\chi {\rm PT}} + \mathscr{L}_{\rm WZW}^{\rm full}\right]\left(U, \mathscr{A}_{L/R} \,\middle|\, \boldsymbol{m}_q, \boldsymbol{k}_{L/R}, c_{gg}\right) \supset -\frac{\tau}{2}\left(-i\log[\det U] - 2c_{gg}a/f_a\right)^2$$

$$U \to U_L U U_R^\dagger$$

$$\left\{\left[\mathscr{L}_{\chi {\rm PT}} + \mathscr{L}_{\rm WZW}^{\rm full}\right]\left(U, \mathscr{A}_{L/R} \,\middle|\, \boldsymbol{m}'_q, \boldsymbol{k}'_{L/R}, c'_{gg}\right) + \delta\mathscr{L}_{\rm WZW}^{\rm ano}\right\} \supset -\frac{\tau}{2}\left(-i\log[\det U] - 2c_{gg}\left[1 - {\rm Tr}(\boldsymbol{\kappa}_q)\right]a/f_a\right)^2$$

Anomaly Matching

$$\delta\mathscr{L}_a^{\rm ano} = -\delta\left[\mathscr{L}_{\rm WZW} + \mathscr{L}_c\right](\boldsymbol{\theta}_L, \boldsymbol{\theta}_R) = \delta\mathscr{L}_{\rm WZW}^{\rm ano}$$

Utilities 1: calculating decay width

$$\mathscr{L}_{\text{eff},0} = \bar{q}_0(iD_\mu\gamma^\mu - \mathbf{m}_{q,0})q_0 + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2$$
$$+ g_{ag,0}\frac{a}{f_a}G\tilde{G} + g_{a\gamma,0}\frac{a}{f_a}F\tilde{F} + \frac{\partial_\mu a}{f_a}\left(\bar{q}_L\mathbf{k}_{L,0}\gamma^\mu q_L + \bar{q}_R\mathbf{k}_{R,0}\gamma^\mu q_R + \ldots\right)$$

https://github.com/nun3366/Axion-WZW-3

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WZW Interactions of Axions

What is it?

In this work, we implement the routines to derive the relevant axion WZW interactions with the ground-state (axial-)vector mesons and electroweak gauge bosons and calculate the relevant hadronic decay widths derived in 2406.11948 and 2505.24822. We also provide the routines to plot the hadronic axion decay widths under different parameter schemes of the user's choice.

These notebooks are based on the open-source Mathematica notebooks written by Maksym Ovchynnikov and Andrii Zaporozhchenko introduced in 2310.03524 and 2501.04525.

Dependencies

To run the routines, one must first install FeynCalc and xAct-xTerior.

Repository structure

The main routines are implemented in `main.nb`, with the basis modules defined in the notebooks stored in the folder `notebooks`. In addition to the axion/ALP mass m_a and axion decay constant f_a, the user should specify seven other parameters: the axion couplings to gluons c_{gg} and left/right-handed u/d/s-quarks $c_{L/R}^{u/d/s}$. The major contribution of this work, the details of auxiliary parameter cancellation in the effective axion WZW couplings, is implemented in `WZW_axion_interactions.nb`.

Summary

- A full chiral axion Lagrangian for axion and pseudoscalar/vector mesons
- $\mathscr{L}^{\text{full}}_{\text{axion}} \equiv \left[\mathscr{L}_{\chi\text{PT}} + \mathscr{L}^{\text{full}}_{\text{WZW}}\right]\left(U, \mathbf{m}_q(a), \mathscr{A}_{L/R} + \mathbf{k}_{L/R}(a)da/f_a\right)$
  - 1. Wess-Zumino-Witten counter term is included for gauge invariance
  - 2. t'Hooft UV-IR anomaly matching is achieved
  - 3. Consistent physical amplitudes without auxiliary rotation parameters
- Three light quarks scheme: consistent treatment with η'
  - 4. Demonstrates two ways of resolving $aG\tilde{G}$ are consistent. Work for any ch-rotation.
- Important for Axion/ALP searches via mesons.
- All calculation machinery provided as Mathematica code in GitHub.

Thank you!