第十八届粒子物理、核物理和宇宙学交叉学科前沿问题研讨会

Preliminarily

Neural Network Emulators for Self-Interacting Dark Matter Halo Evolution

项树诚

中国科学院紫金山天文台(PMO)

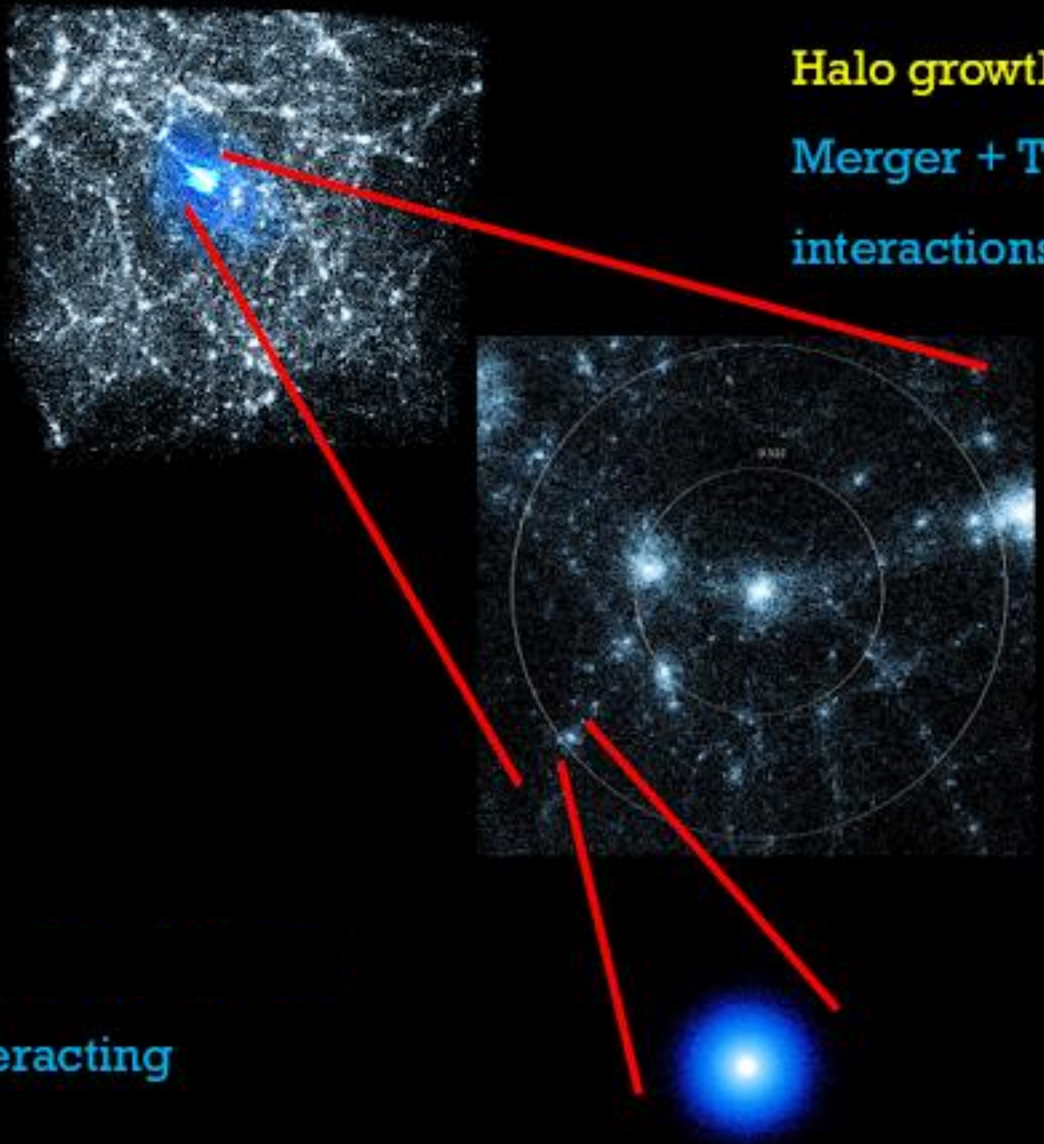
2026/04/11 桂林

In collaboration with Daneng Yang ...

Structure formation is hierarchical

Large scales

Cold Dark Matter



Halo growth

Merger + Tidal interactions

Small scales

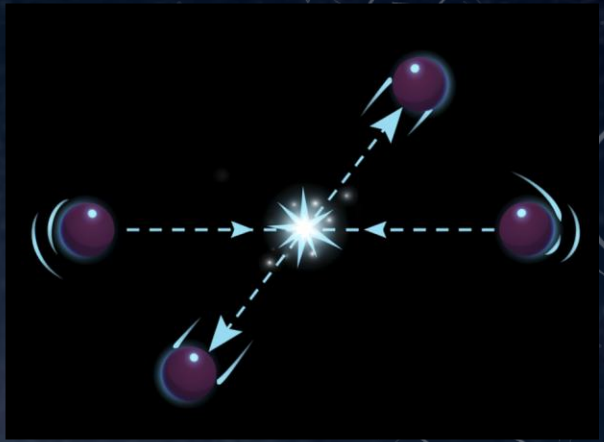
Virialize

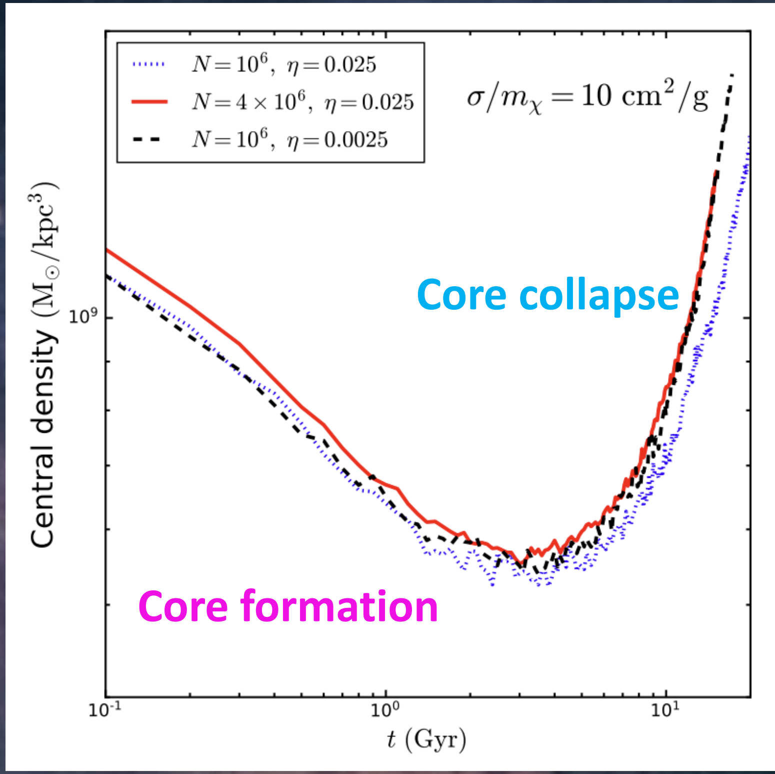
+ Could be self-interacting

- Self-Interacting Dark Matter (SIDM)
- A universal kernel for SIDM halo evolution
- Model architecture and performance

Recent small scale challenges	LCDM	SIDM
Dwarf clustering	✗	✓
Strong lensing perturber	✗	✓
GGSL	✗	**Two-component**
Diverse rotation curves	🤞	✓
Little red dots	✗	✓
BHB mergers	✗ (?)	✓
Stellar stream perturbers	(?)	✓

Self-Interacting Dark Matter (SIDM) could address several small-scale challenges





Hou & Yang et al. 2025 JCAP

Elastic scatterings lead to heat transport in **Halos,** which drives

- Core formation
- Core collapse

Spergel & Steinhardt 2000,
Tulin and Yu 2017 (Review) etc.

An analytic model for SIDM halo evolution on normalized evolution time

Heat transport by SIDM (κ) can be absorbed into evolution time: **t -> t σ**

$$\frac{\partial}{\partial r}\left(r^2 \boxed{\kappa m}\frac{\partial v^2}{\partial r}\right) = r^2 \rho v^2 \frac{D}{\boxed{Dt}} \ln\frac{v^3}{\rho}$$

SIDM particles in a halo are like fluid, instead

κ (heat conductivity) ∝ # of scatterings ∝ **σ (cross section)**

$\tilde{t} \equiv t/t_c$ **∝ t σ**

$$\rho_{\rm SIDM}(r) = \frac{\rho_s}{\frac{\left(r^\beta + \boxed{r_c}^\beta\right)^{1/\boxed{\beta}}}{\boxed{r_s}}\left(1+\frac{r}{r_s}\right)^2}$$

A 3 parameter parametric model:

Yang+2305.16176 JCAP

The SIDM halo density profile is fully specified given **3 parameters: Vmax(τ), Rmax(τ), τ**

Incorporate accretion history

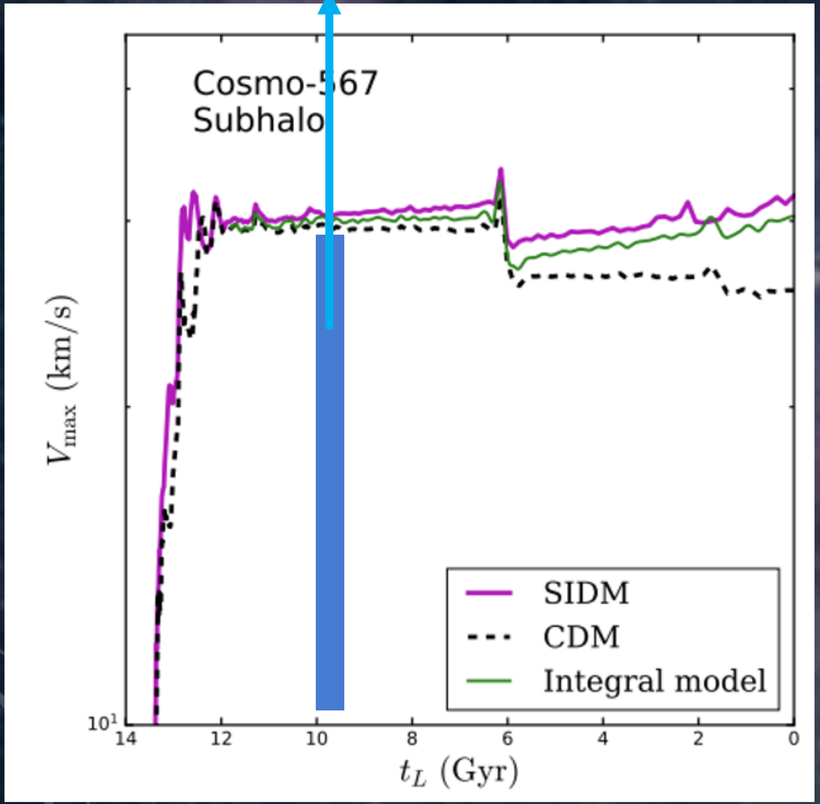
Sum over many isolated increments in the gravothermal phase: Δτ=(Δt)/tc

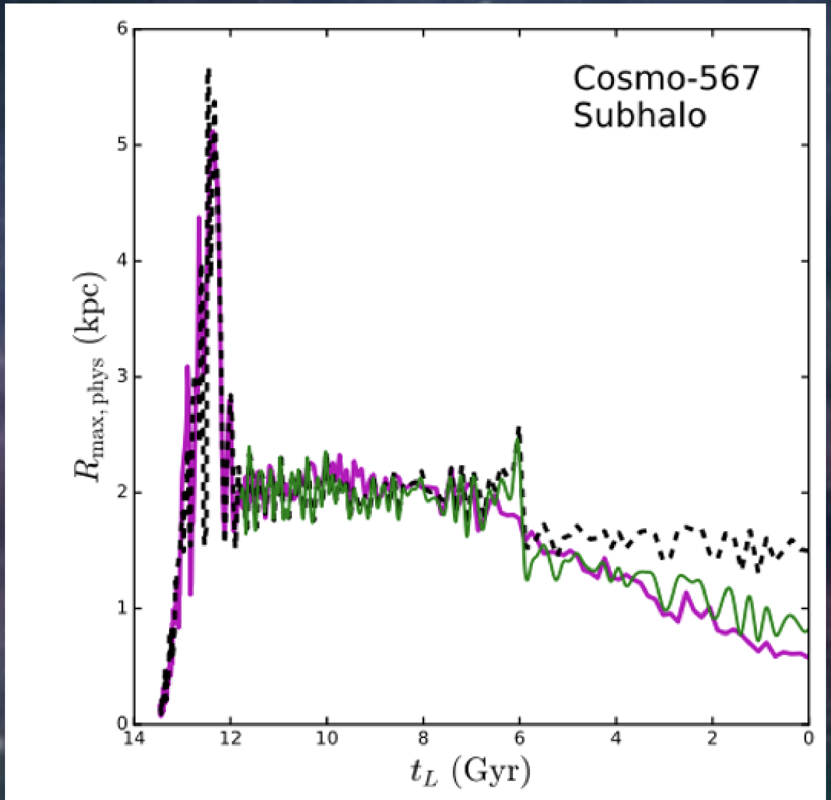
Continuum limit

Analytic kernel

$$V_{\max}^{\rm SIDM}(t) = V_{\max}^{\rm CDM}(t) + \int_{t_f}^{t} \frac{dt'}{t_c(t')} \boxed{\frac{dV_{\max,\rm Model}(\tilde{t}')}{d\tilde{t}'}},$$

$$r_{\max}^{\rm SIDM}(t) = r_{\max}^{\rm CDM}(t) + \int_{t_f}^{t} \frac{dt'}{t_c(t')} \boxed{\frac{dr_{\max,\rm Model}(\tilde{t}')}{d\tilde{t}'}},$$

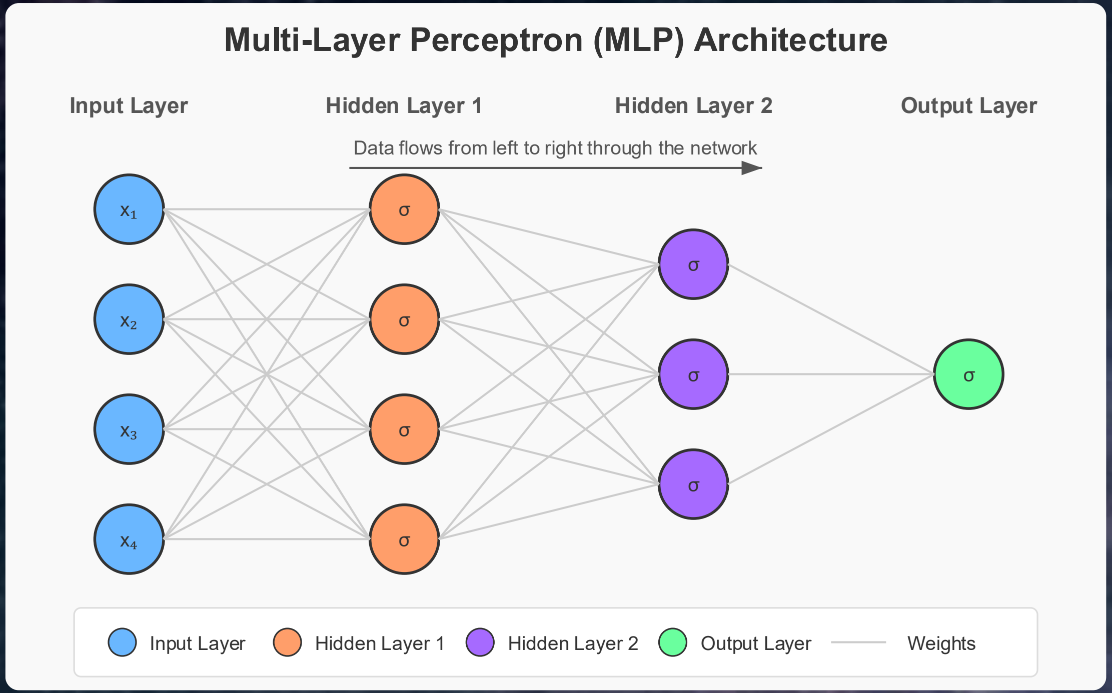




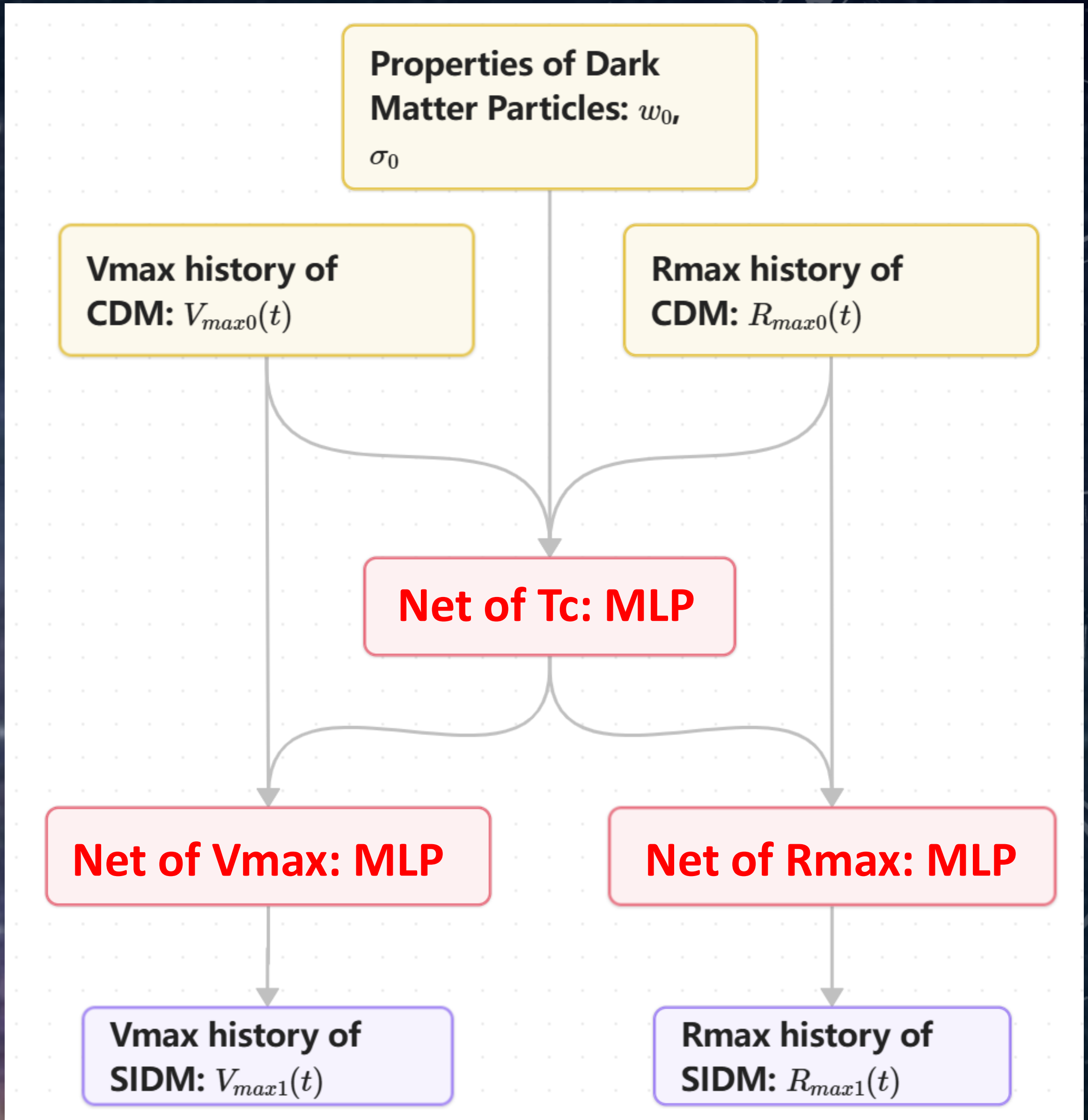
Merger/Stripping induced relaxation may change τ

Merger/Stripping induced relaxation may limit Δτ

Model Architecture



MLP: a feedforward neural network that transforms input data into output predictions through multiple fully connected layers with nonlinear activations.



Yang et al. 2024 ApJ

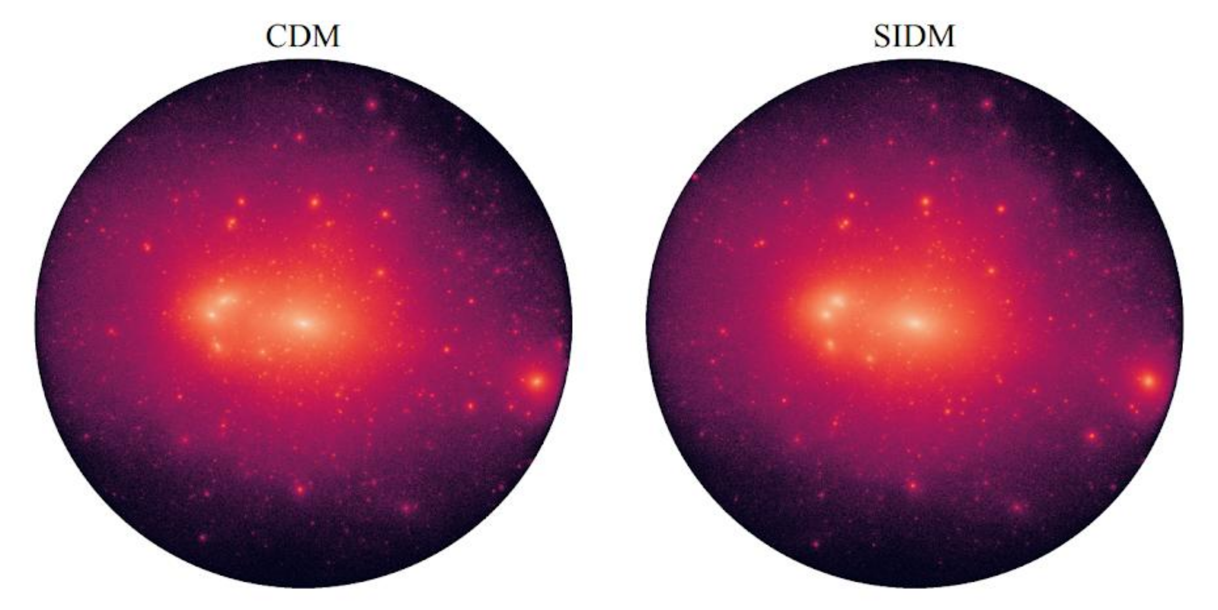
Training & Dataset

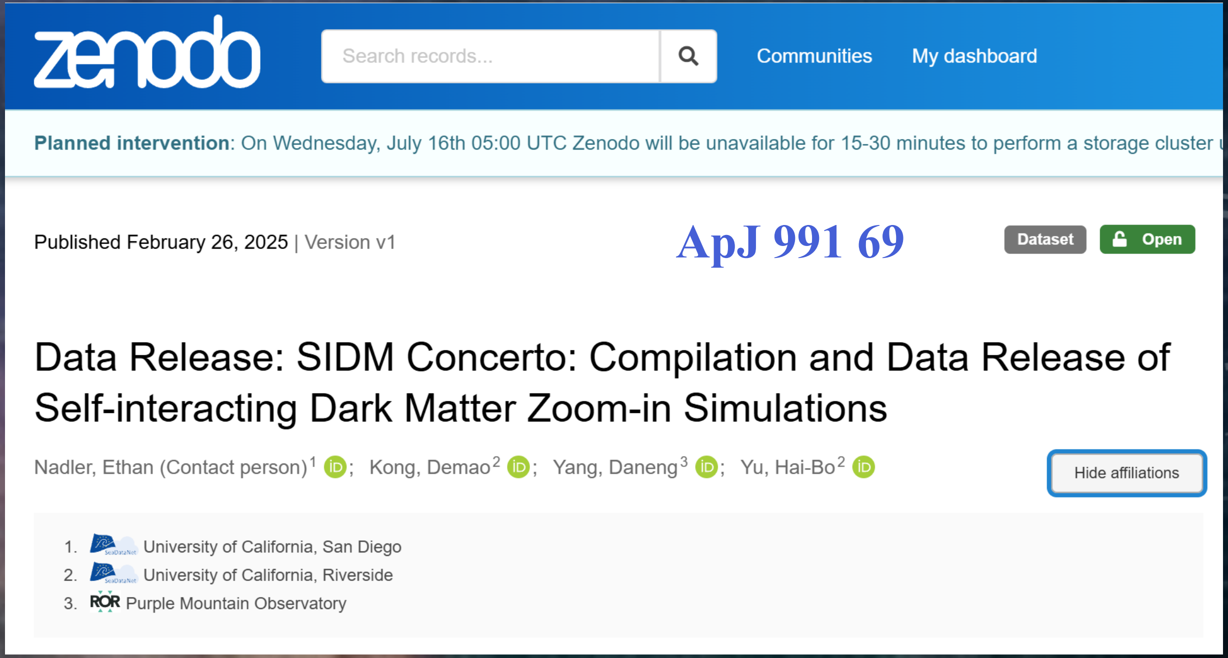
Pre-train with parametric model predictions (integral approach)

- One model currently
- Will expand by considering various SIDM models

Fine tune with simulation data

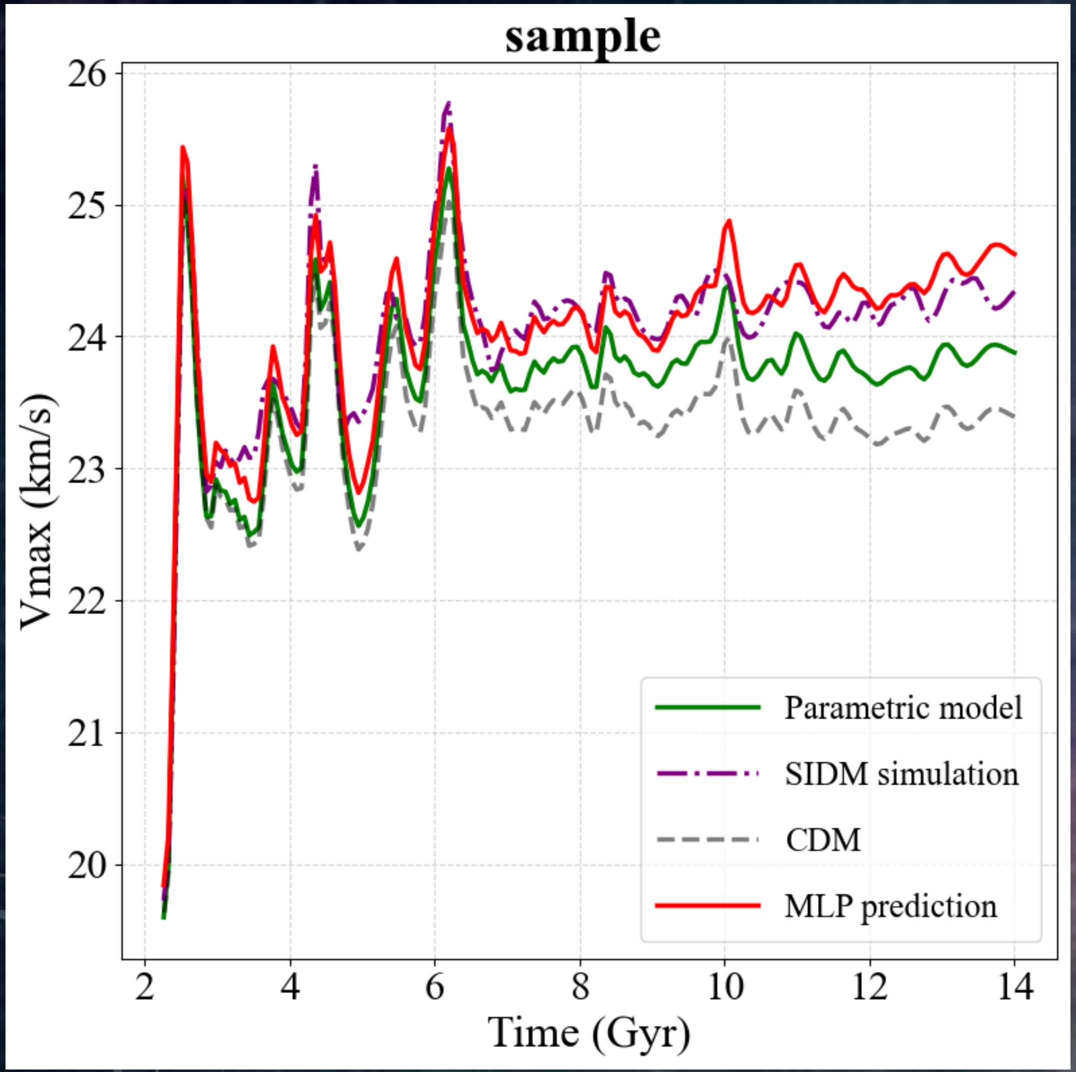
- Milky-Way zoom-in simulations from Yang et al. 2024 ApJ (H416 from **SIDM-Concerto)**
- **~1800 halos matched between CDM and SIDM simulations**

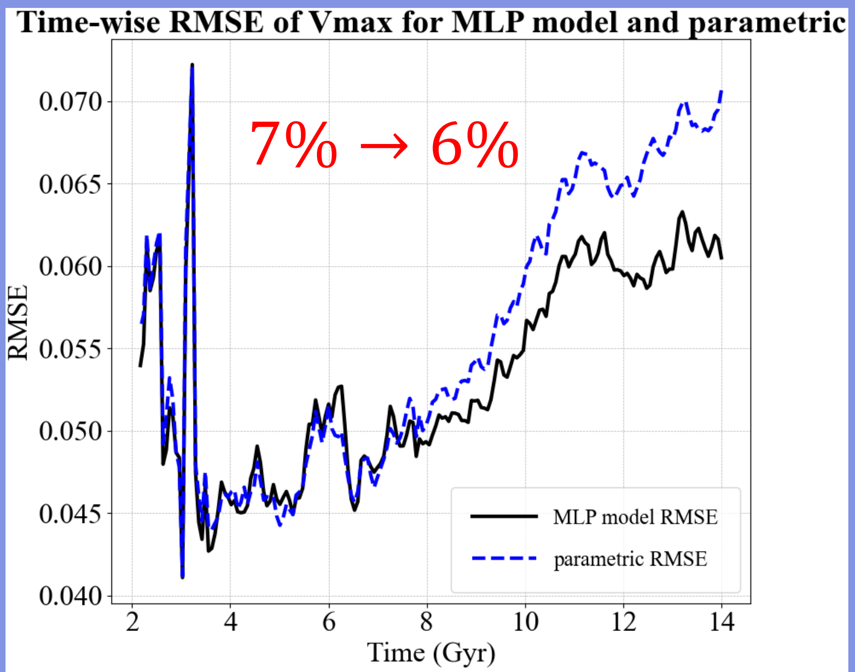


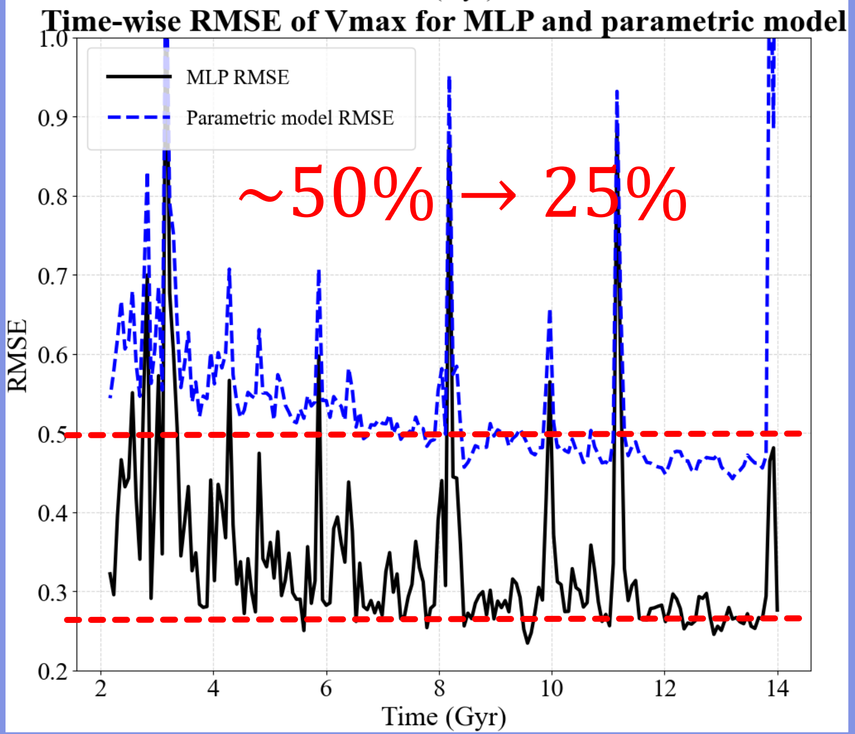


Model Performance: MLP

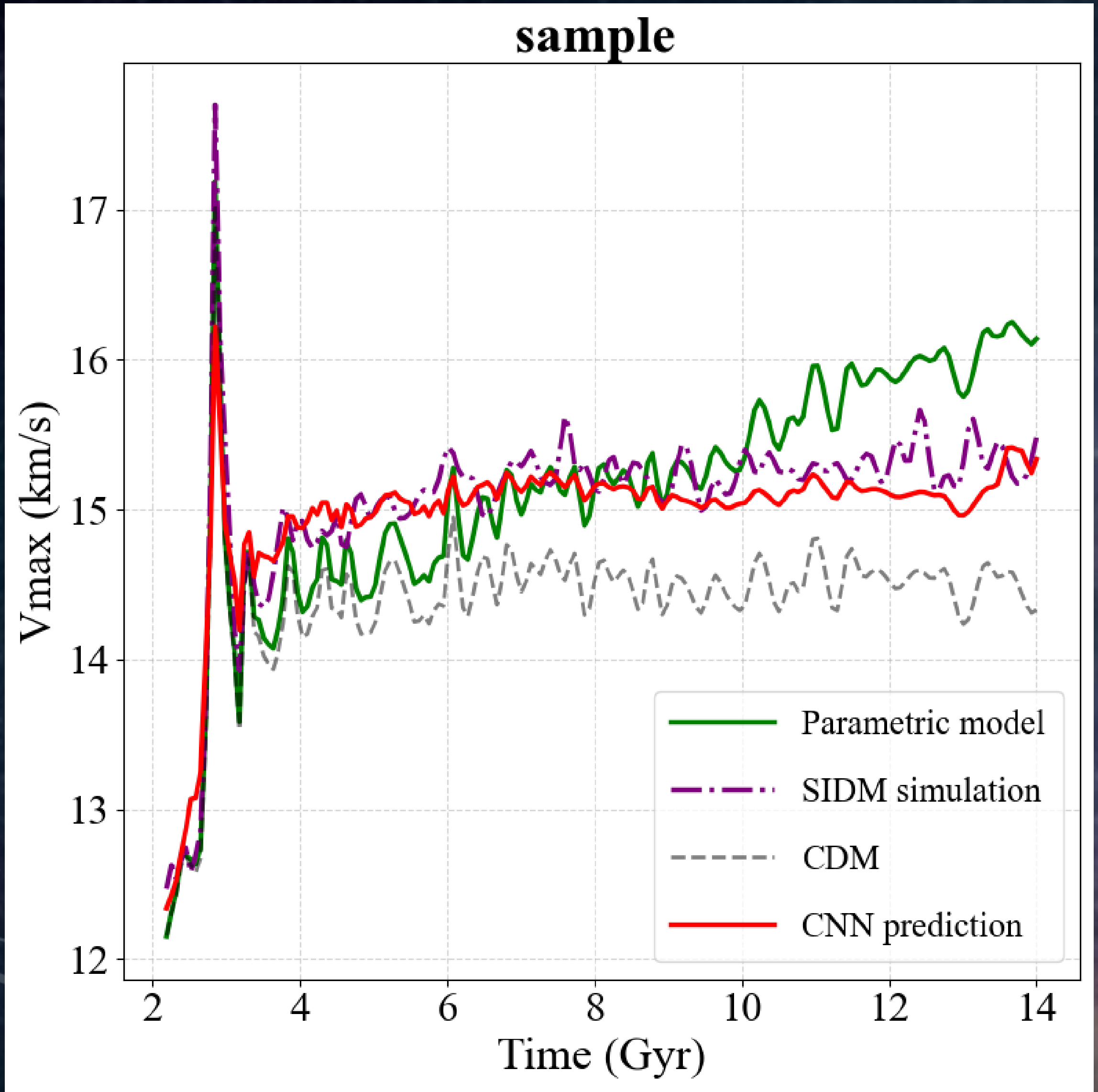
RMSE: Root Mean Square Error

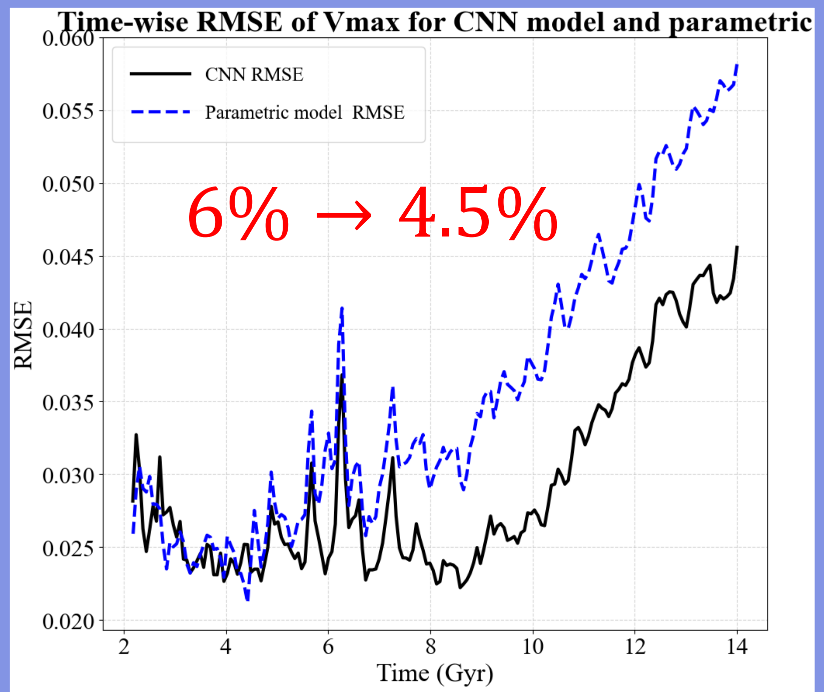


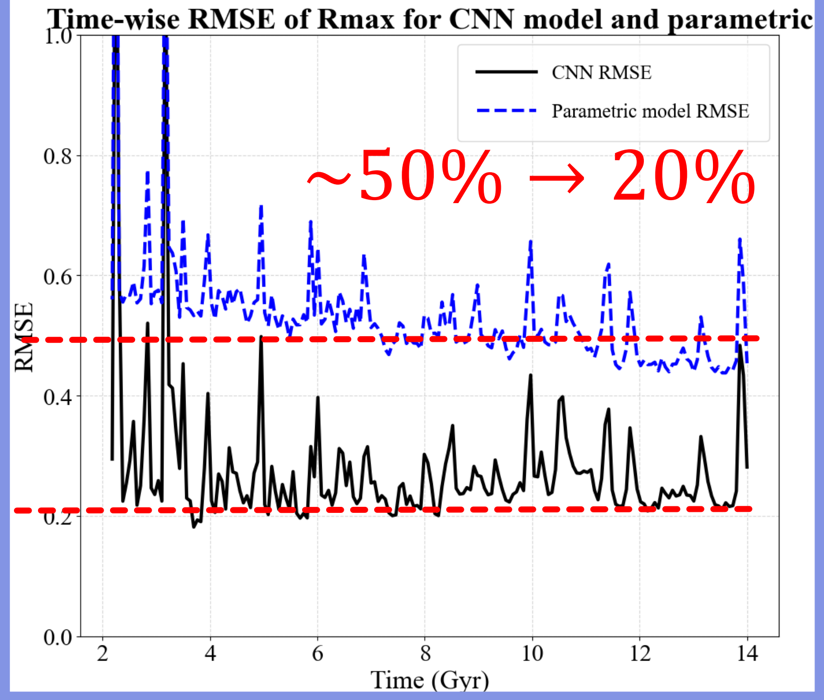




MODEL PERFORMANCE: CNN







An Inference Agent for SIDM halos?

See Daneng's Talk for background

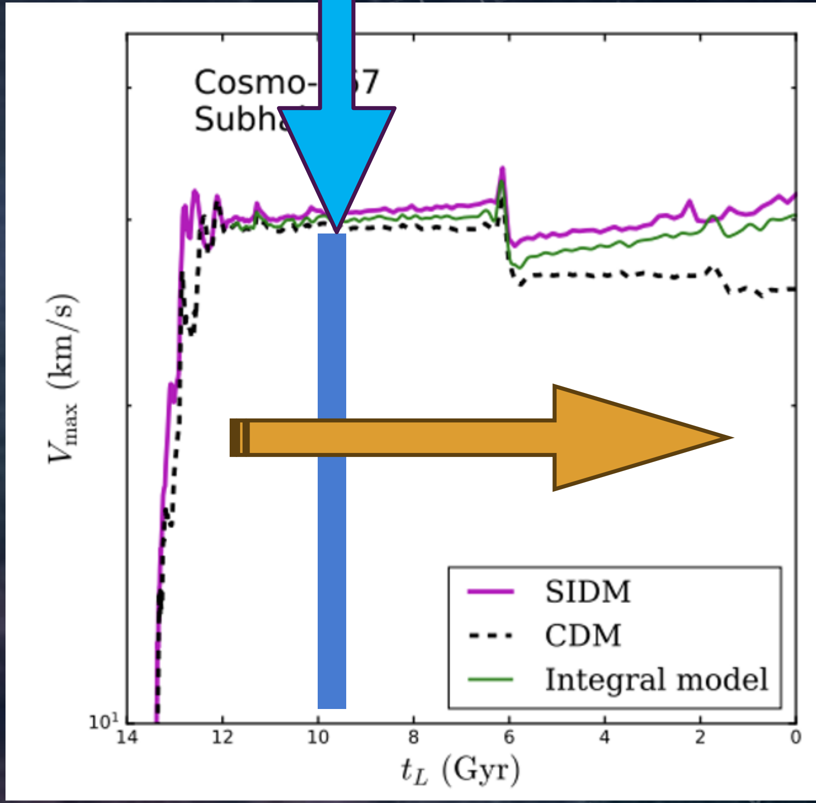
$$\mathbf{y}_k \equiv (V_{\max}, R_{\max})_k$$

$$\Delta \mathbf{y}_k = \Delta \mathbf{y}_k^{\mathrm{tr}} + \Delta\tau_k \mathbf{b}_{\mathrm{SIDM}}(s_k;\theta) + \boldsymbol{\epsilon}_k$$

(Non-linear) Transport

Residual Drift

Residual Scatter



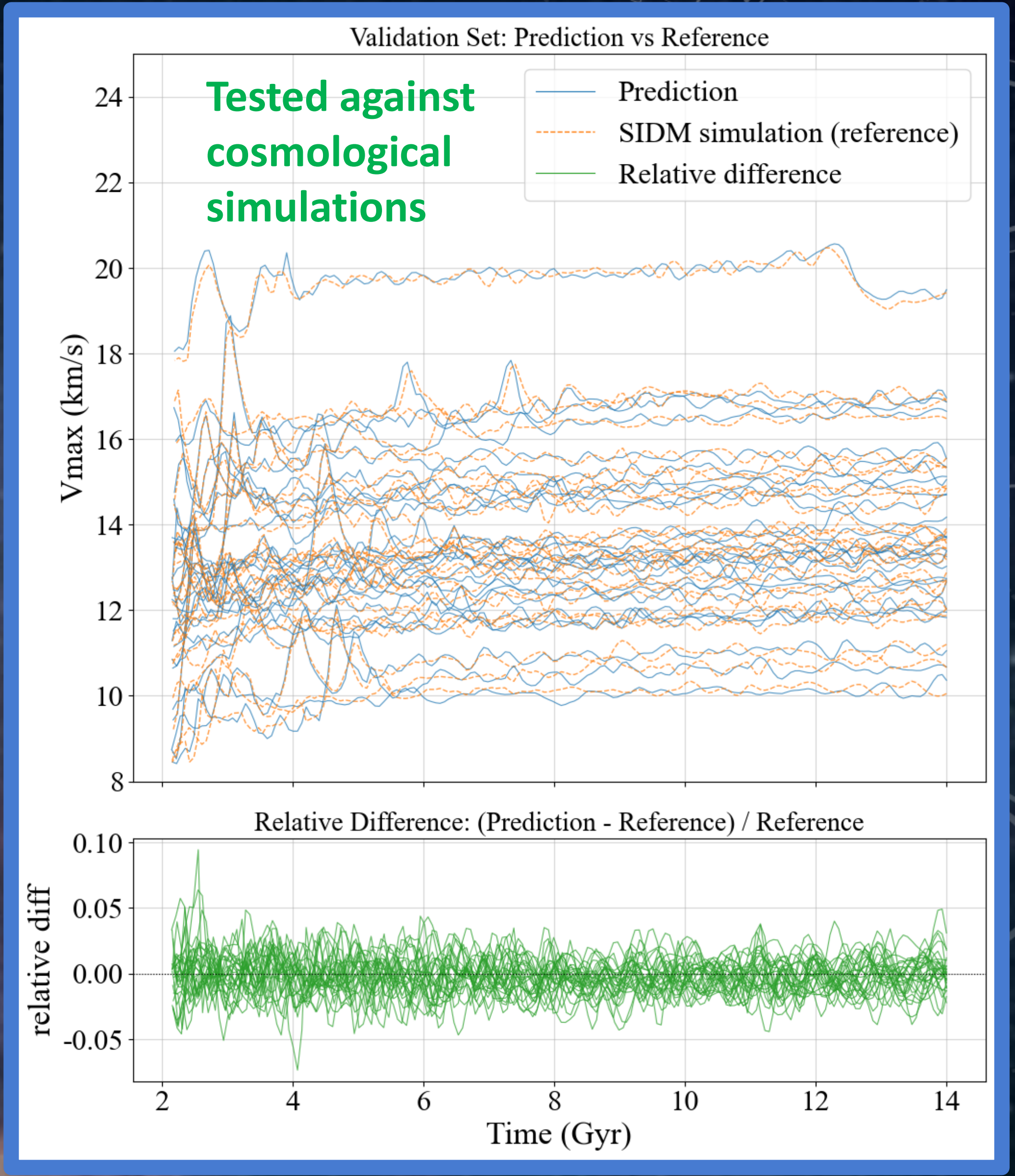
- A universal kernel approximates SIDM effects on (Vmax,Rmax) per time step
- Removing the time conditioning in bSIDM mirrors the universality

arXiv:26XX.XXXX

Summary

- Parametric SIDM model is fast but limited by fixed functional form
- We train neural networks to learn ΛCDM → SIDM mapping directly
- Two approaches: full-history mapping and time-universal incremental model
- Neural networks outperform parametric model (≥50% improvement) with greater flexibility

Thanks for your attention!



BACK UP

$$V_{\max}(t) = V_{\max,\mathrm{CDM}}(t_f) + \int_{t_f}^{t} dt' \frac{dV_{\max,\mathrm{CDM}}(t')}{dt'} + \int_{t_f}^{t} \frac{dt'}{t_c(t')} \frac{dV_{\max,\mathrm{Model}}(\tau')}{d\tau'}$$

$$R_{\max}(t) = R_{\max,\mathrm{CDM}}(t_f) + \int_{t_f}^{t} dt' \frac{dR_{\max,\mathrm{CDM}}(t')}{dt'} + \int_{t_f}^{t} \frac{dt'}{t_c(t')} \frac{dR_{\max,\mathrm{Model}}(\tau')}{d\tau'},$$

$$\frac{1}{V_{\max,\mathrm{CDM}}(t)} \frac{dV_{\max,\mathrm{Model}}(\tau)}{d\tau} = 0.1777 - 13.20\tau^2 + 66.62\tau^3 - 94.34\tau^4 + 63.54\tau^6 - 21.93\tau^8$$

$$\frac{1}{R_{\max,\mathrm{CDM}}(t)} \frac{dR_{\max,\mathrm{Model}}(\tau)}{d\tau} = 0.007623 - 1.440\tau + 1.013\tau^2 - 0.5502\tau^3,$$

$$\frac{\rho_s}{\rho_{s,0}} = 2.033 + 0.7381\tau + 7.264\tau^5 - 12.73\tau^7 + 9.915\tau^9 + (1 - 2.033)(\ln 0.001)^{-1} \ln(\tau + 0.001)\,,$$
$$\frac{r_s}{r_{s,0}} = 0.7178 - 0.1026\tau + 0.2474\tau^2 - 0.4079\tau^3 + (1 - 0.7178)(\ln 0.001)^{-1} \ln(\tau + 0.001)\,,$$
$$\frac{r_c}{r_{s,0}} = 2.555\sqrt{\tau} - 3.632\tau + 2.131\tau^2 - 1.415\tau^3 + 0.4683\tau^4, \tag{2.3}$$

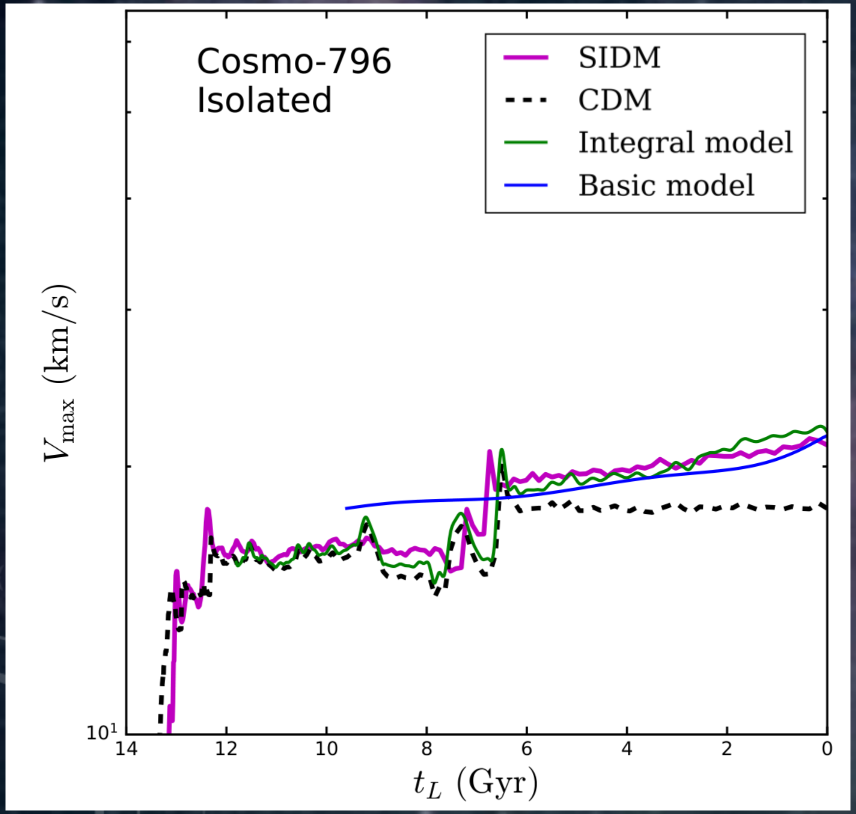
$$r_s = R_{max}/2.1626$$
$$\rho_s = \left(V_{max}/(1.648 r_s)\right)^2/G$$

PARAMETRIC MODEL

D. Yang, et al., 2024
https://github.com/DanengYang/parametricSIDM

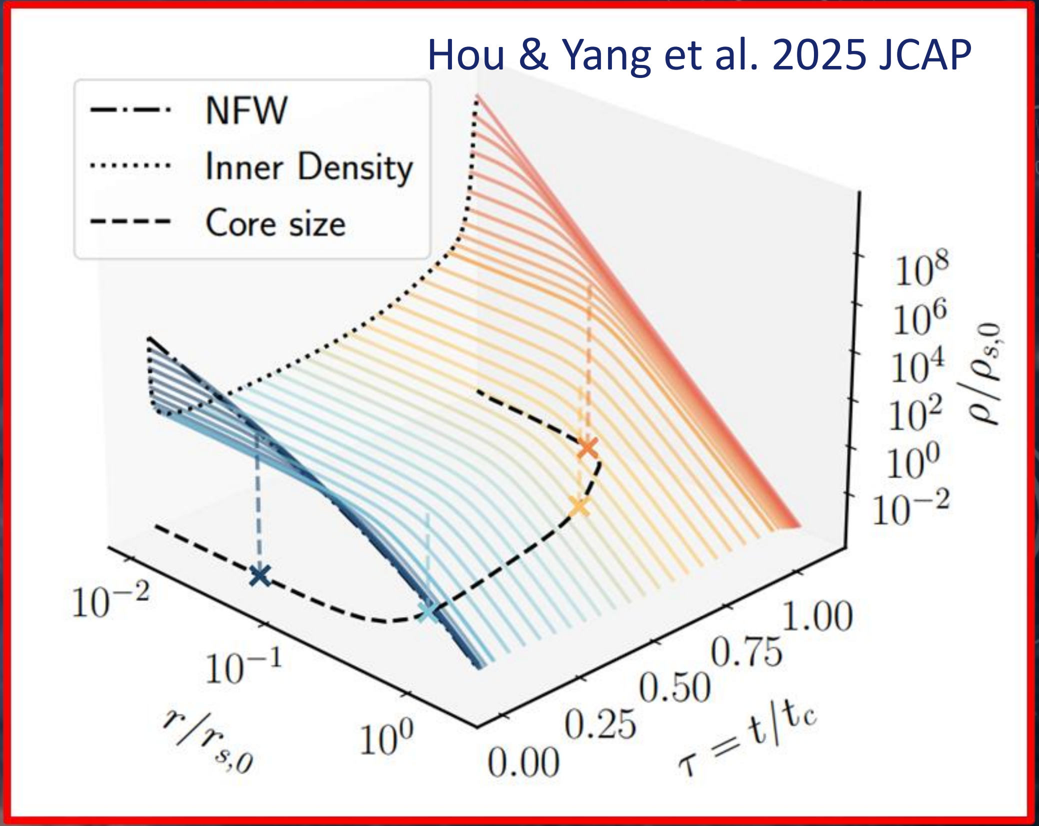
High-resolution SIDM N-Body simulations： expensive !

Analytic predictions?



Predict the evolution of a halo under SIDM given its evolution in CDM based on a few analytic equations

gravothermal evolution look universal !

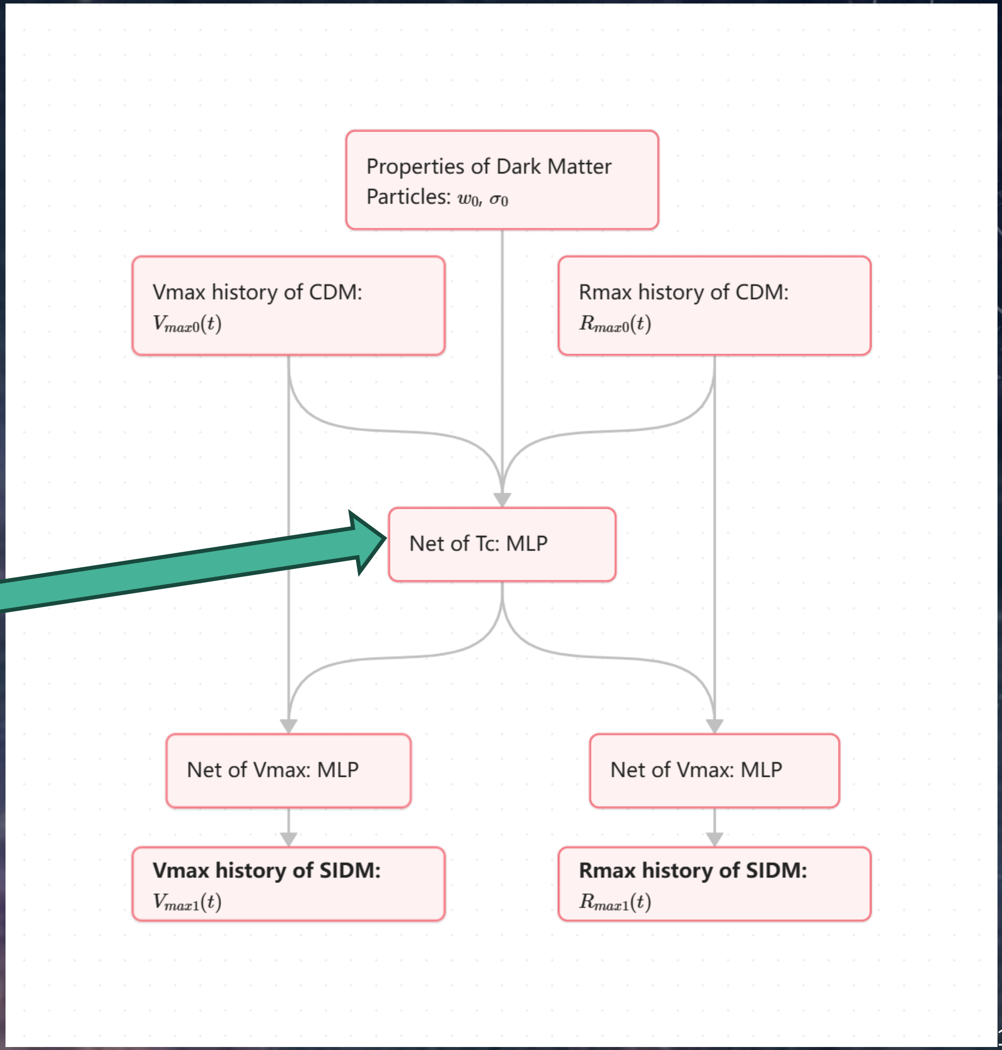


Analytic kernel: the calibrated universal solution

FULL-SEQUENCE: MLP

To use neural networks instead of formulas of parametric model？
: Firstly similar to integral approach

Intermediate value

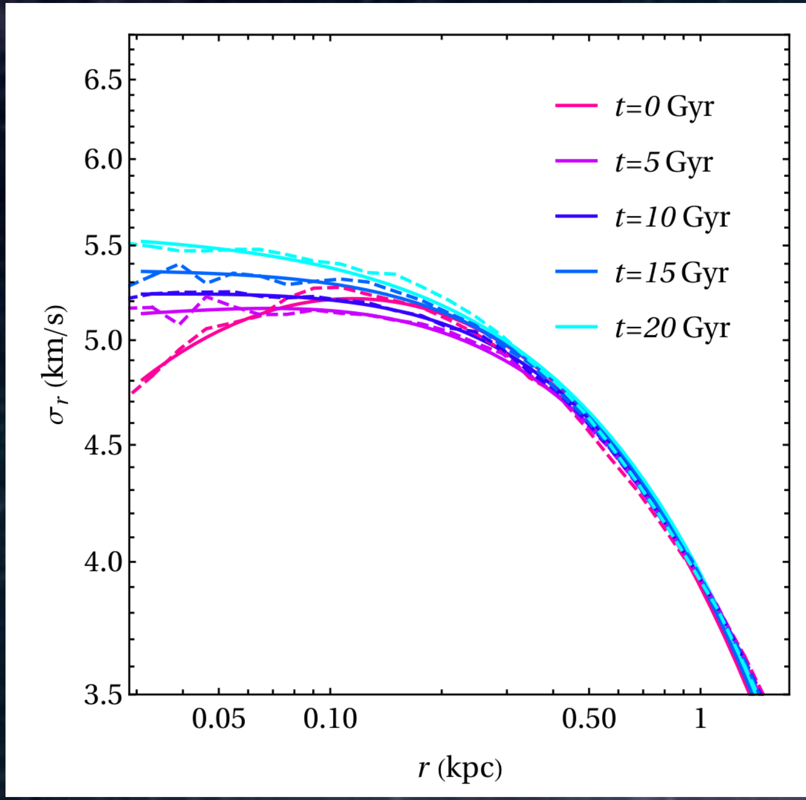


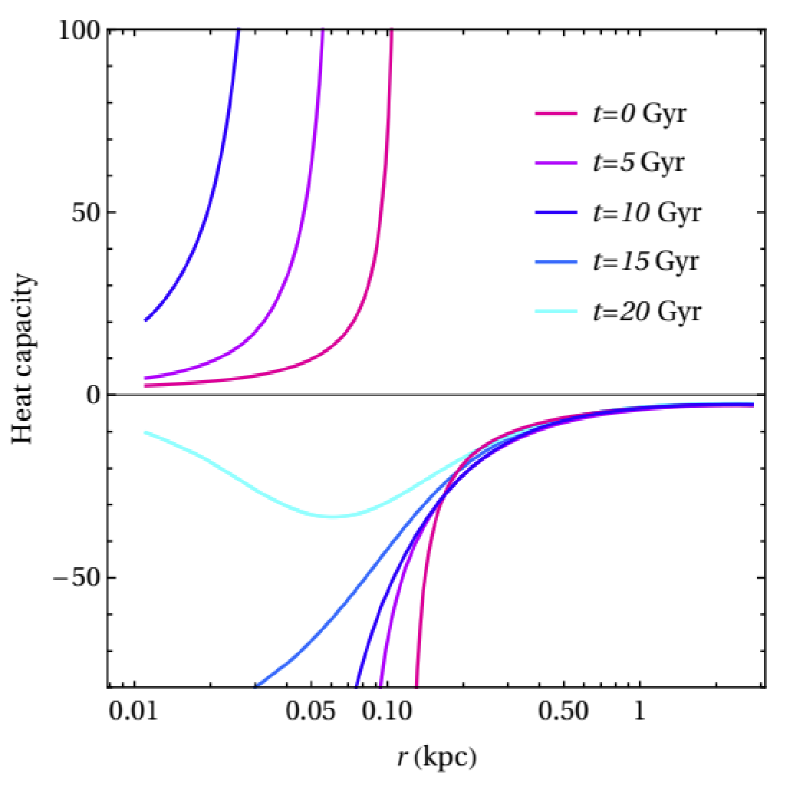
$$F[\mathrm{Vmax0}(t), \mathrm{tc}(t)] \rightarrow \mathrm{Vmax1}(t)$$

$$G[\mathrm{Rmax0}(t), \mathrm{tc}(t)] \rightarrow \mathrm{Rmax1}(t)$$

Evolution of SIDM halos

A thermodynamics picture





Core formation
- heat flux + capacity => core formation

Core shrink (heat flux small)
+ heat flux + capacity => quasi-stable core

Core collapse
+ heat flux - capacity => core collapse

Yang & Yu 2022 JCAP

STEP-BY-STEP INFERENCE

State parameters of the SIDM halo at time T_{i-1}:
$V_{max}(T_{i-1}), R_{max}(T_{i-1})$

State parameters of the CDM halo at time T_{i-1} and T_{i-1}

Particle parameters

（Relevant state values from previous times）

Neural Network

State parameters of the SIDM halo at time T_i:
$V_{max}(T_i), R_{max}(T_i)$