格点模拟对称性破缺过程

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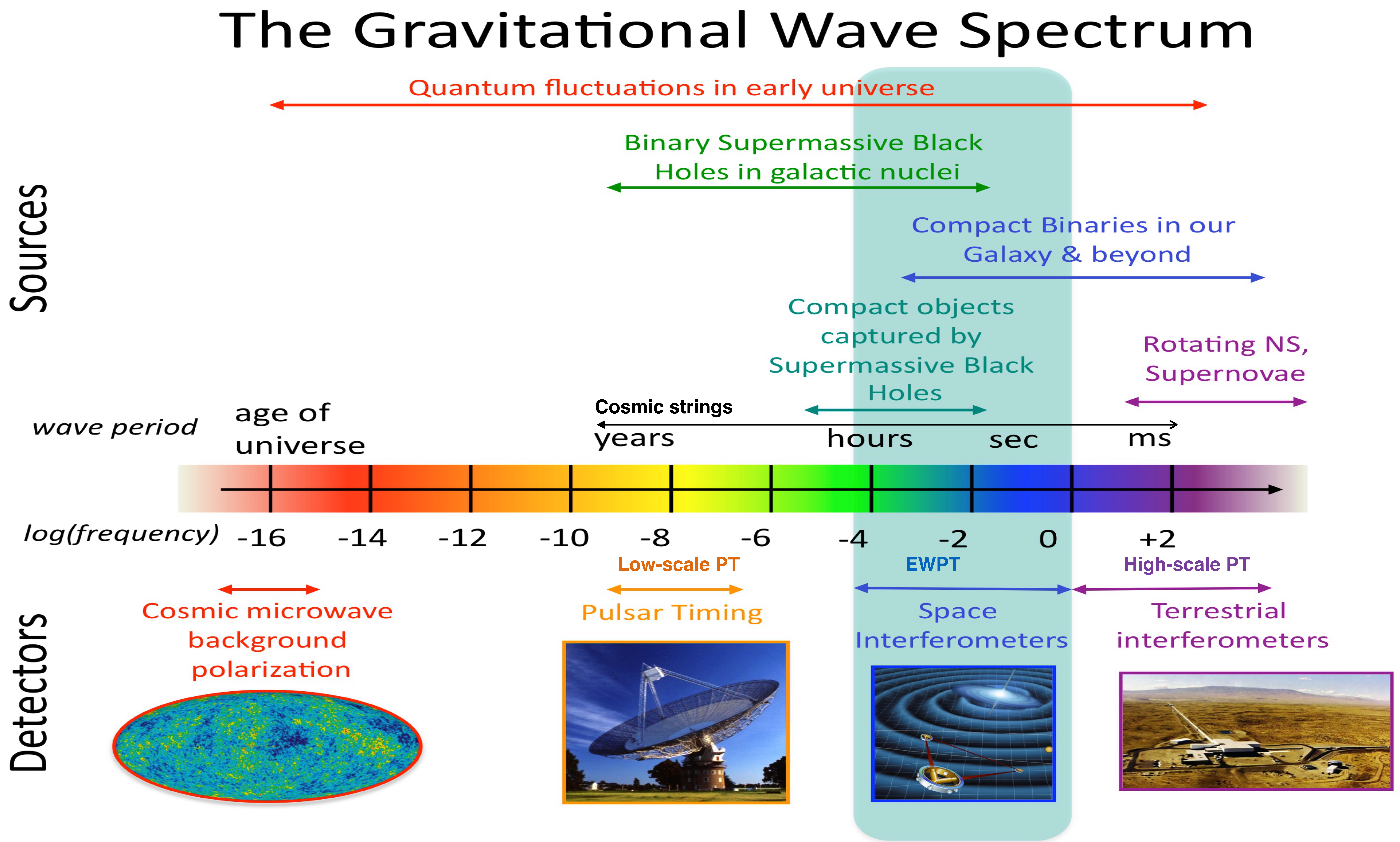
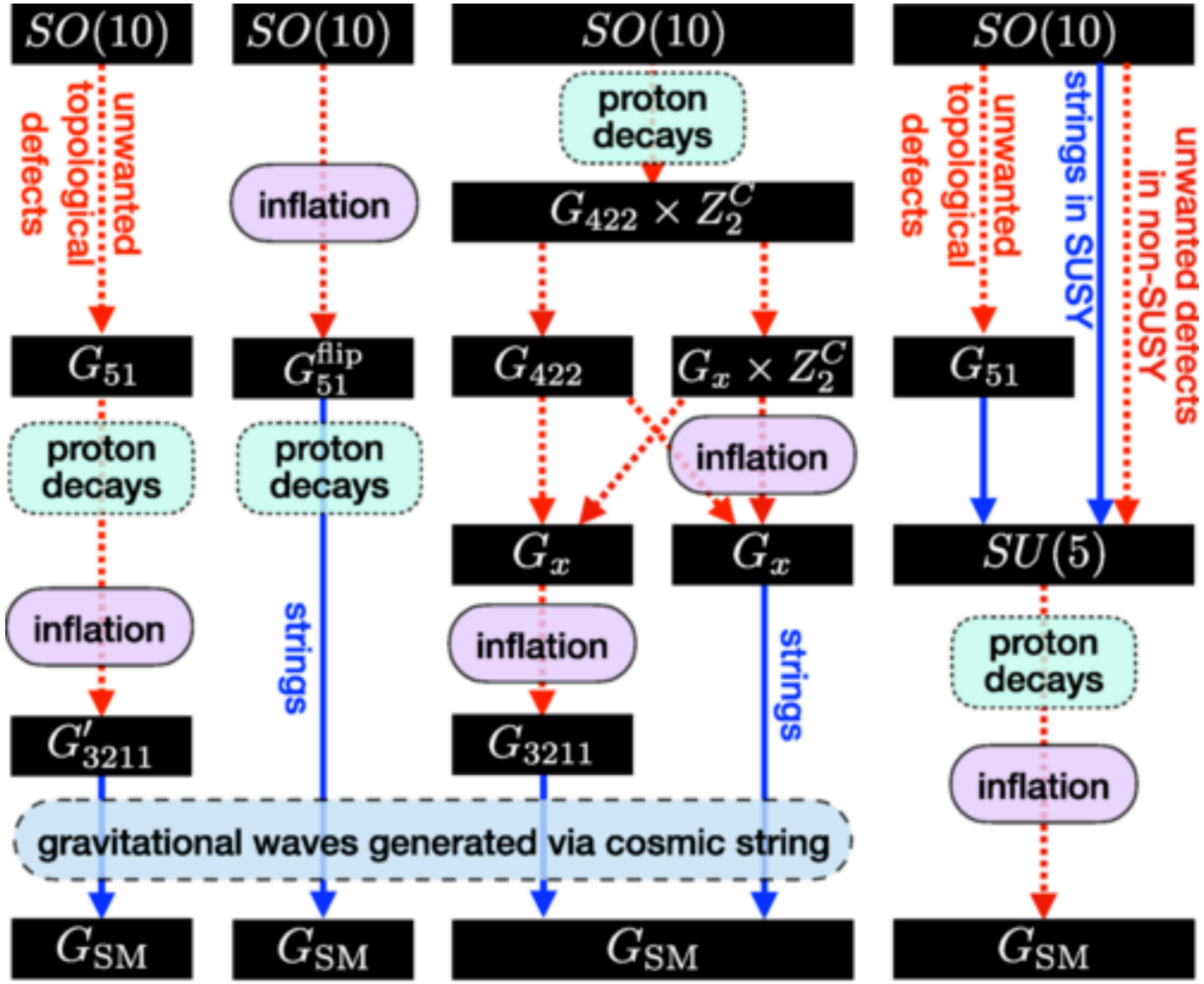
报告提纲

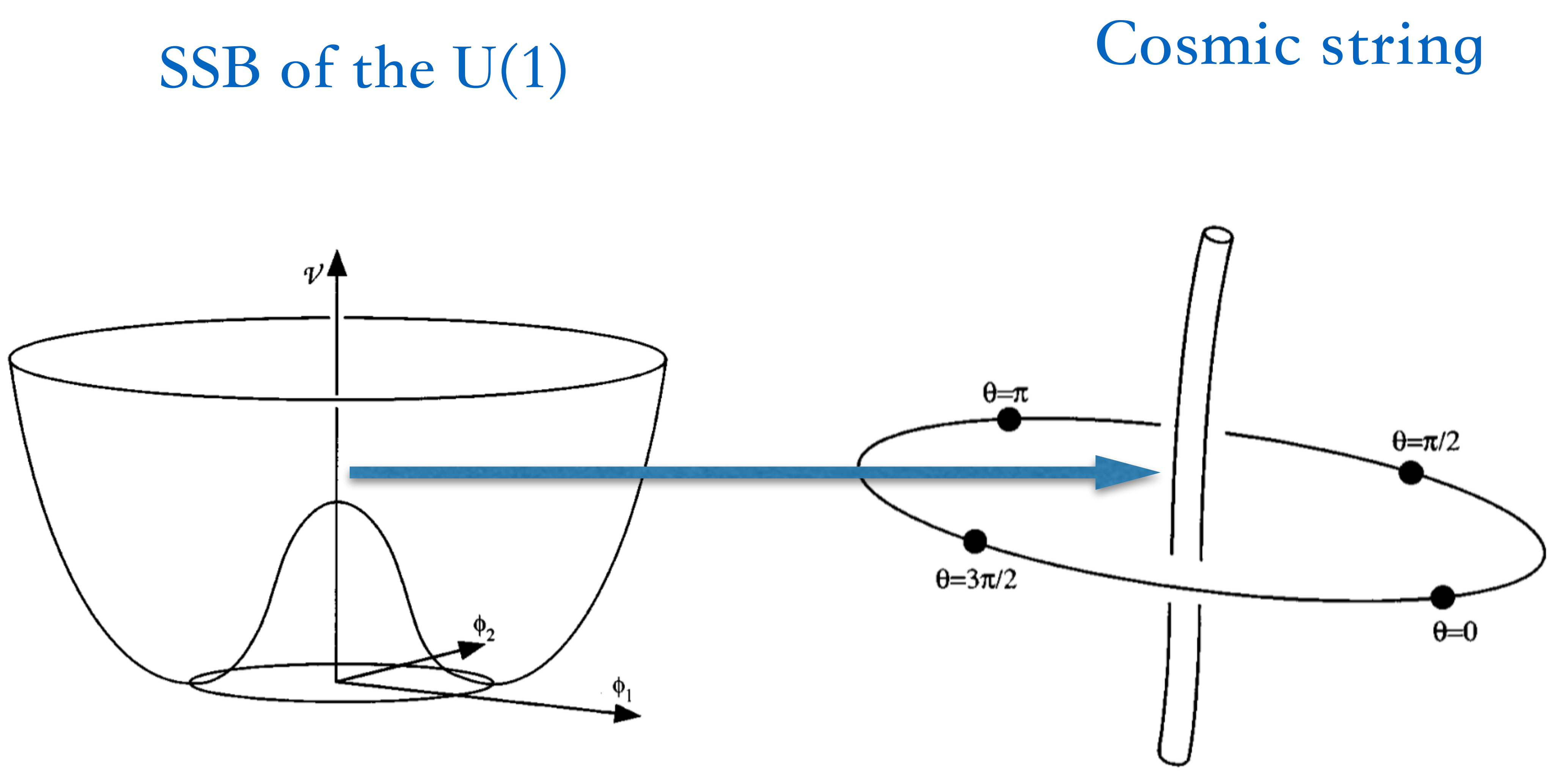
Stochastic gravitational wave background and New physics



▸ Cosmic string

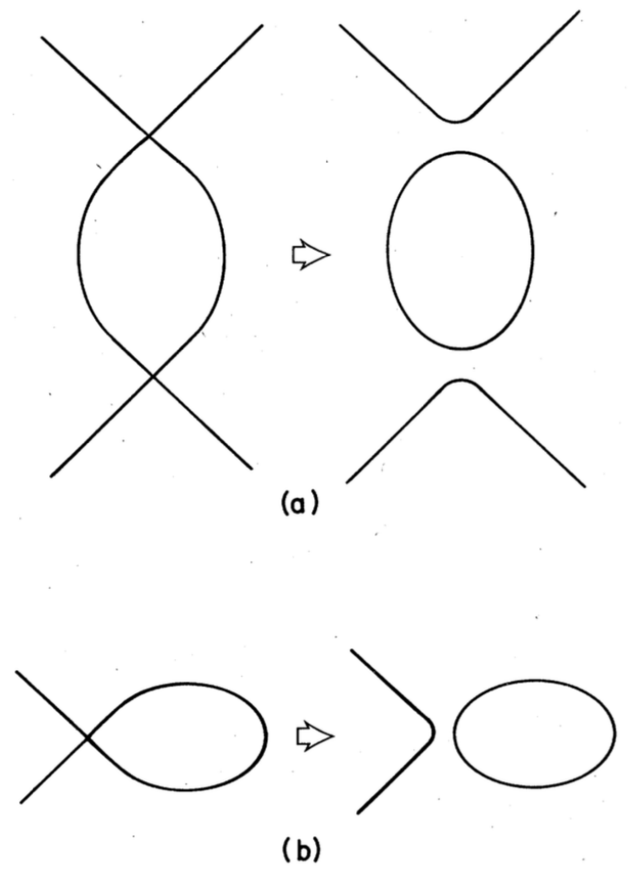


Stephen F. King, Silvia Pascoli, Jessica Turner, Ye-Ling Zhou, Phys. Rev. Lett. **126**, 021802

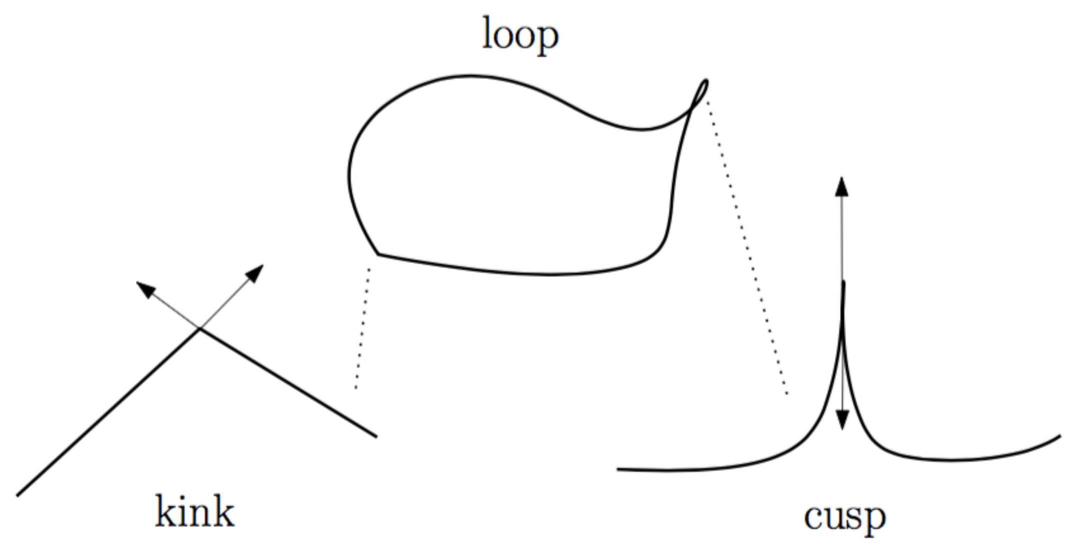


T. W. B. Kibble

Loops

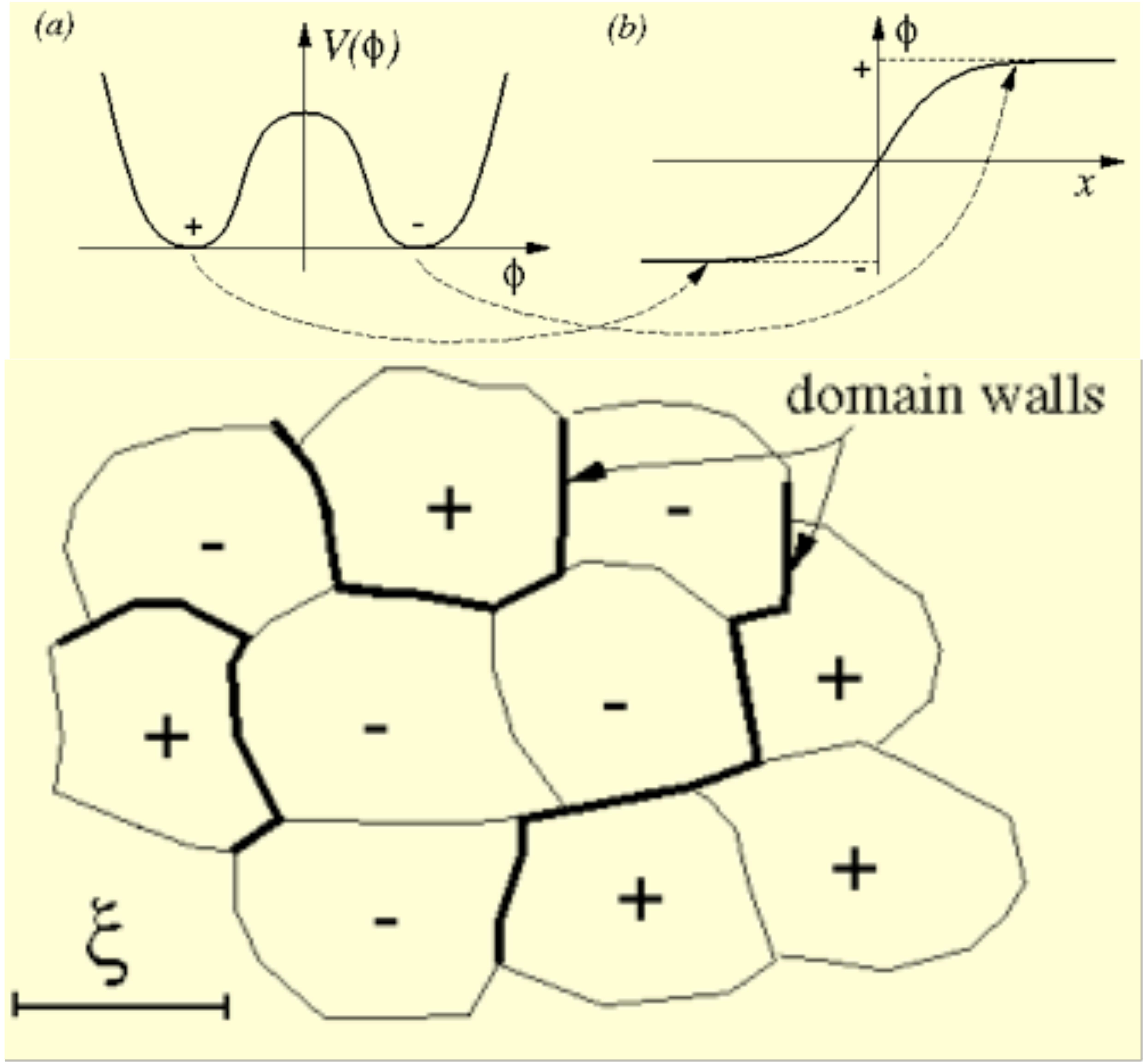


GW radiations

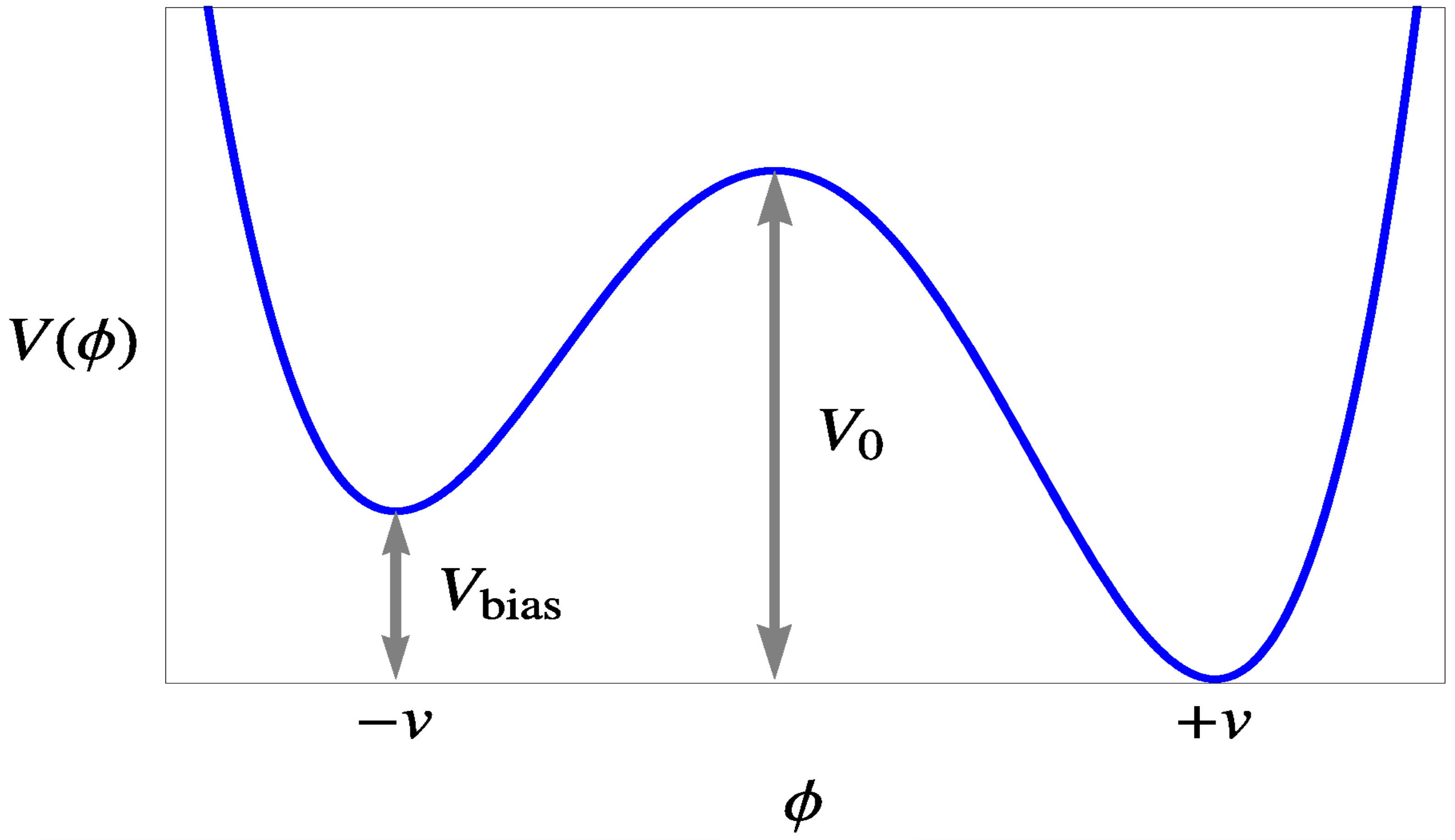


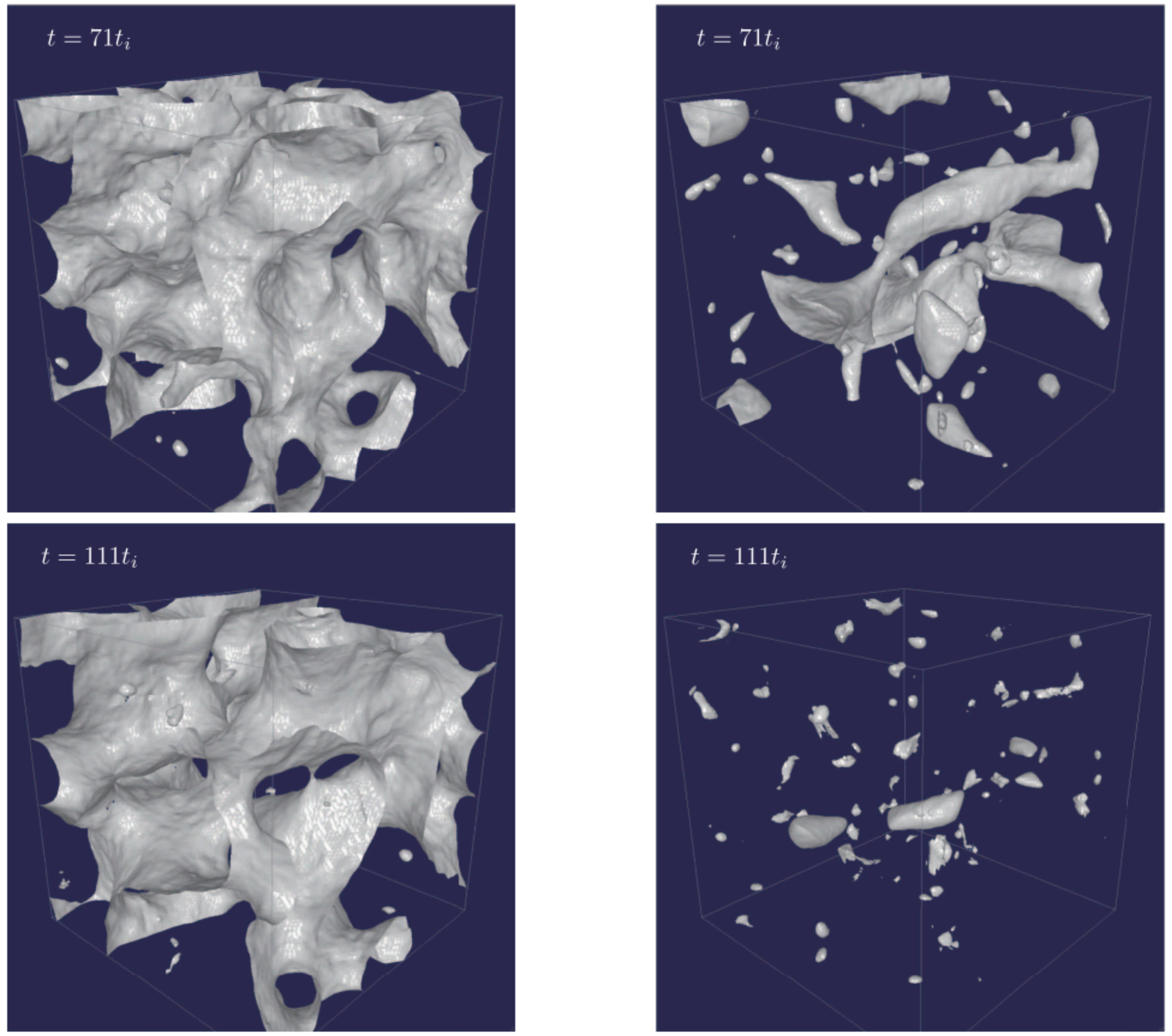
Yann Gouttenoire *et al* JCAP07(2020)032

Domain wall



Kibble mechanism





1002.1555

Cosmic string simulation: the DFSZ axion model

The action in expanding universe with spatially flat FLRW metric:

$$S = -\int d^4x\sqrt{-g}\left(g^{\mu\nu}\,\frac{1}{2}\,\partial_\mu\varphi^*\partial_\nu\varphi + V(\varphi)\right)$$

PQ complex scalar: $\varphi = \phi_1 + i\phi_2$

$$V(\varphi) = \frac{1}{4}\lambda(|\varphi|^2 - v^2)^2 + \frac{\lambda}{6}T^2|\varphi|^2 + \frac{m^2(T)v^2}{N_{DW}^2}(1-\cos(N_{DW}\theta)) - \Xi v^3(\varphi e^{-i\delta} + \text{h.c.})$$

$m^2(T)$: $\min\left[\frac{\alpha_a\Lambda^4}{f_a^2(T/\Lambda)^{6.68}}, m_a^2\right]$

θ: axion field

$\Xi v^3(\varphi e^{-i\delta} + \text{h.c.})$: bias term

PQ era, PQ symmetry broken, second order phase transition, $T_c \sim 10^9$-10^{11}GeV

→ Axion(global) strings form and enters the scaling regime

QCD era, axion acquires a non-zero mass due to QCD non-perturbative effect, $T \sim 100$MeV

→ String-domain wall hybrid networks form and eventually decay

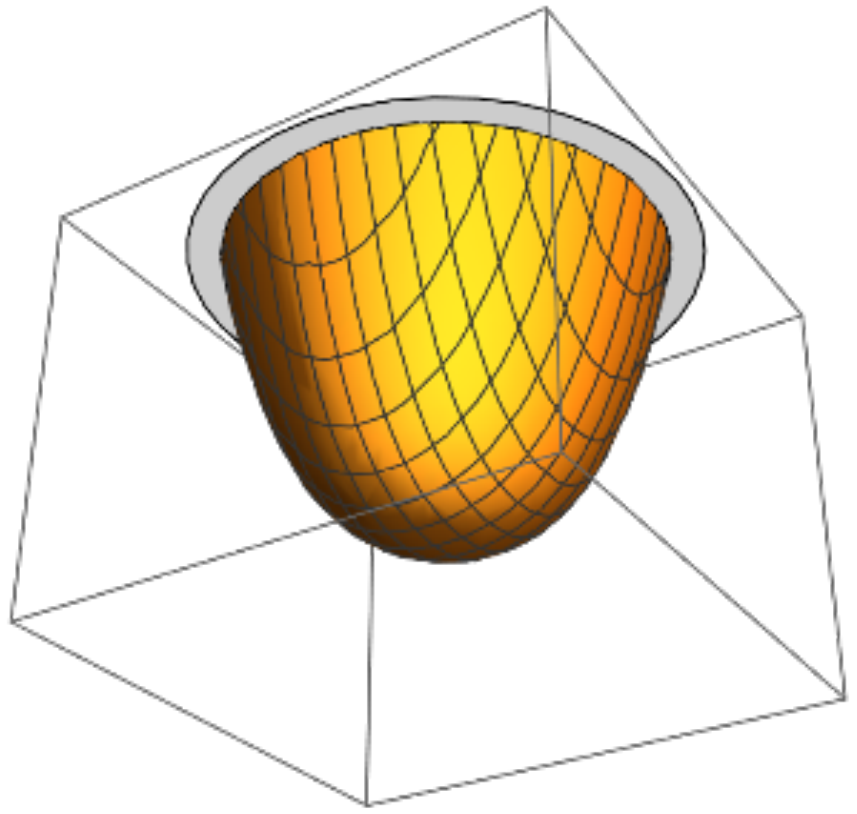
→ Gravitational waves and axion radiated by topological defects of two eras → Detection of axion dark matter

Yang Li, **Ligong Bian**, Rong-Gen Cai, Jing Shu, 2311.02011

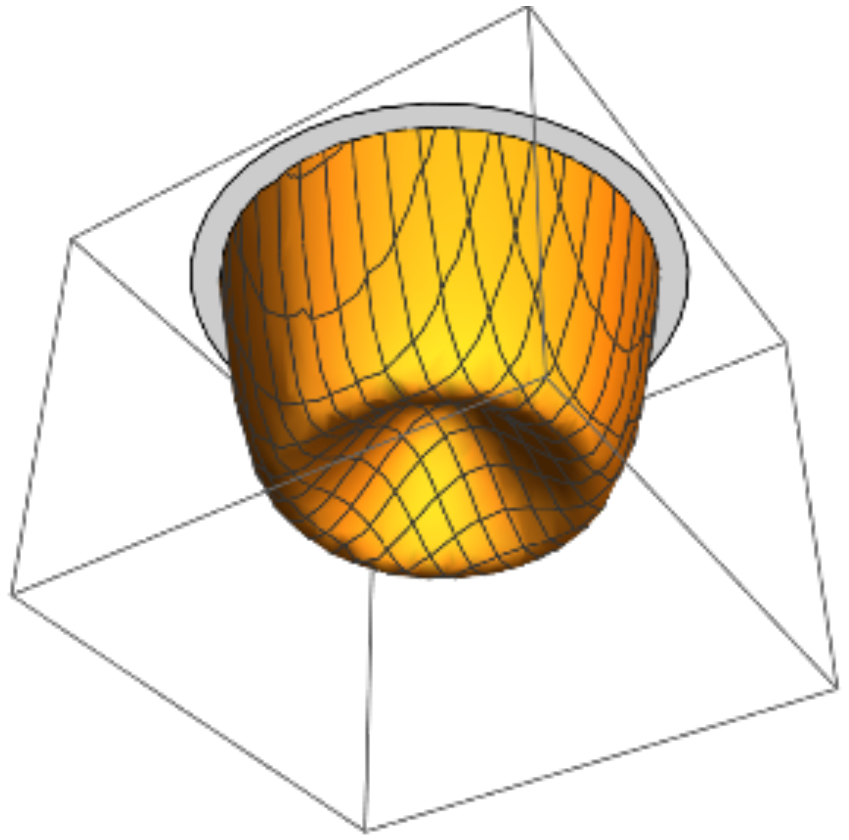
▸ Shape of the potential

PQ era:

before PQ transition



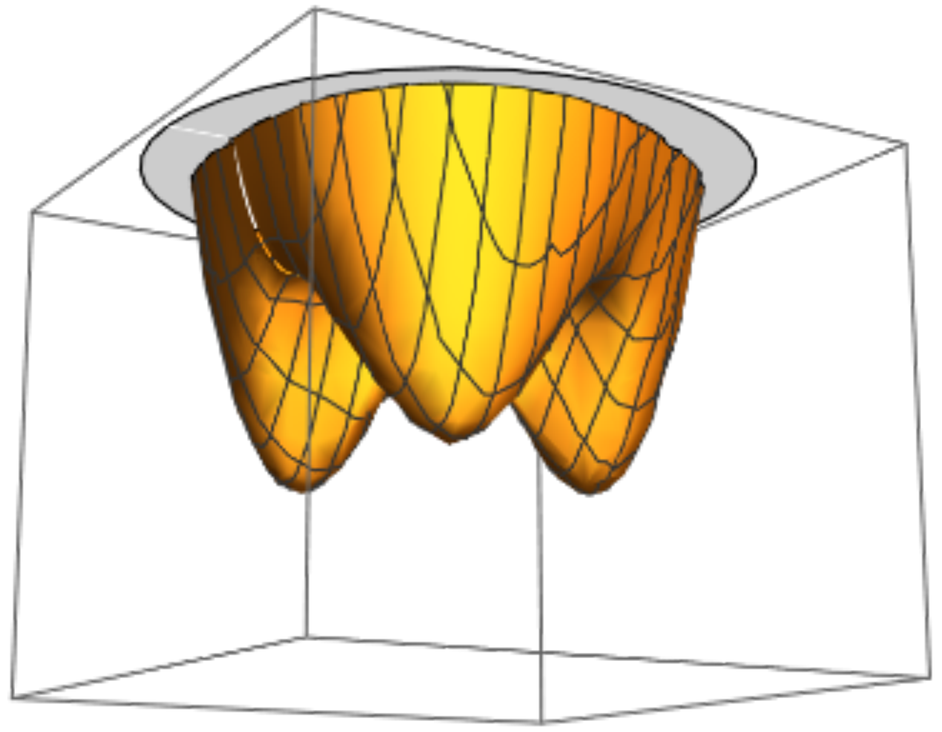
after PQ transition



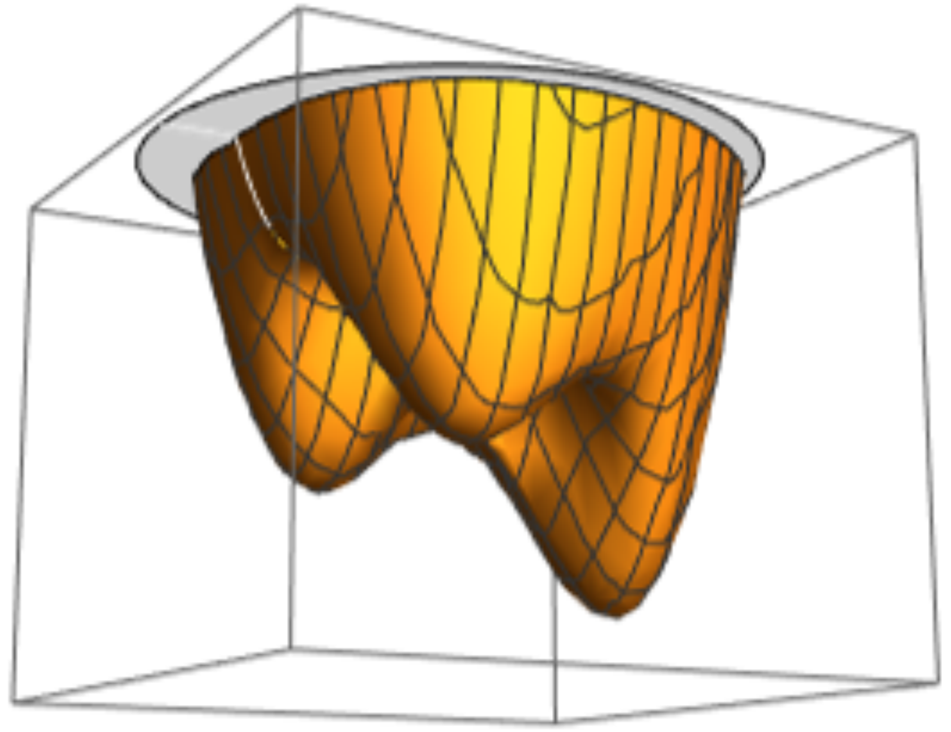
QCD era:

with nonzero axion mass

without bias term



with bias term



$N_{DW}=3$

▸ Simulation setups

Equations of motion

PQ era-the first stage

$$\begin{cases} \phi_1'' + 2\frac{a'}{a}\phi_1' - \nabla^2\phi_1 = -a^2[\lambda\phi_1(\phi_1^2 + \phi_2^2 - v^2 + \frac{1}{3}T^2) - \frac{m^2(T)v^2}{N_{\rm DW}^2}(\cos\theta\cos N_{\rm DW}\theta + N_{\rm DW}\sin\theta\sin N_{\rm DW}\theta) - 2\Xi v^3\cos\delta] \\ \phi_2'' + 2\frac{a'}{a}\phi_2' - \nabla^2\phi_2 = -a^2[\lambda\phi_2(\phi_1^2 + \phi_2^2 - v^2 + \frac{1}{3}T^2) - \frac{m^2(T)v^2}{N_{\rm DW}^2}(\sin\theta\cos N_{\rm DW}\theta - N_{\rm DW}\cos\theta\sin N_{\rm DW}\theta) - 2\Xi v^3\sin\delta] \end{cases}$$

Initial condition

thermal spectrum

$$\mathcal{P}_{\phi_1}(k) = \mathcal{P}_{\phi_2}(k) = \frac{n_k}{w_k} = \frac{1}{w_k}\frac{1}{e^{w_k/T} - 1}, \quad \mathcal{P}_{\dot\phi_1}(k) = \mathcal{P}_{\dot\phi_2}(k) = n_k w_k = \frac{w_k}{e^{w_k/T} - 1}$$

$$w_k = \sqrt{k^2/R^2 + m_{\rm eff}^2} \qquad m_{\rm eff}^2 = \lambda(T^2/3 - v^2)$$

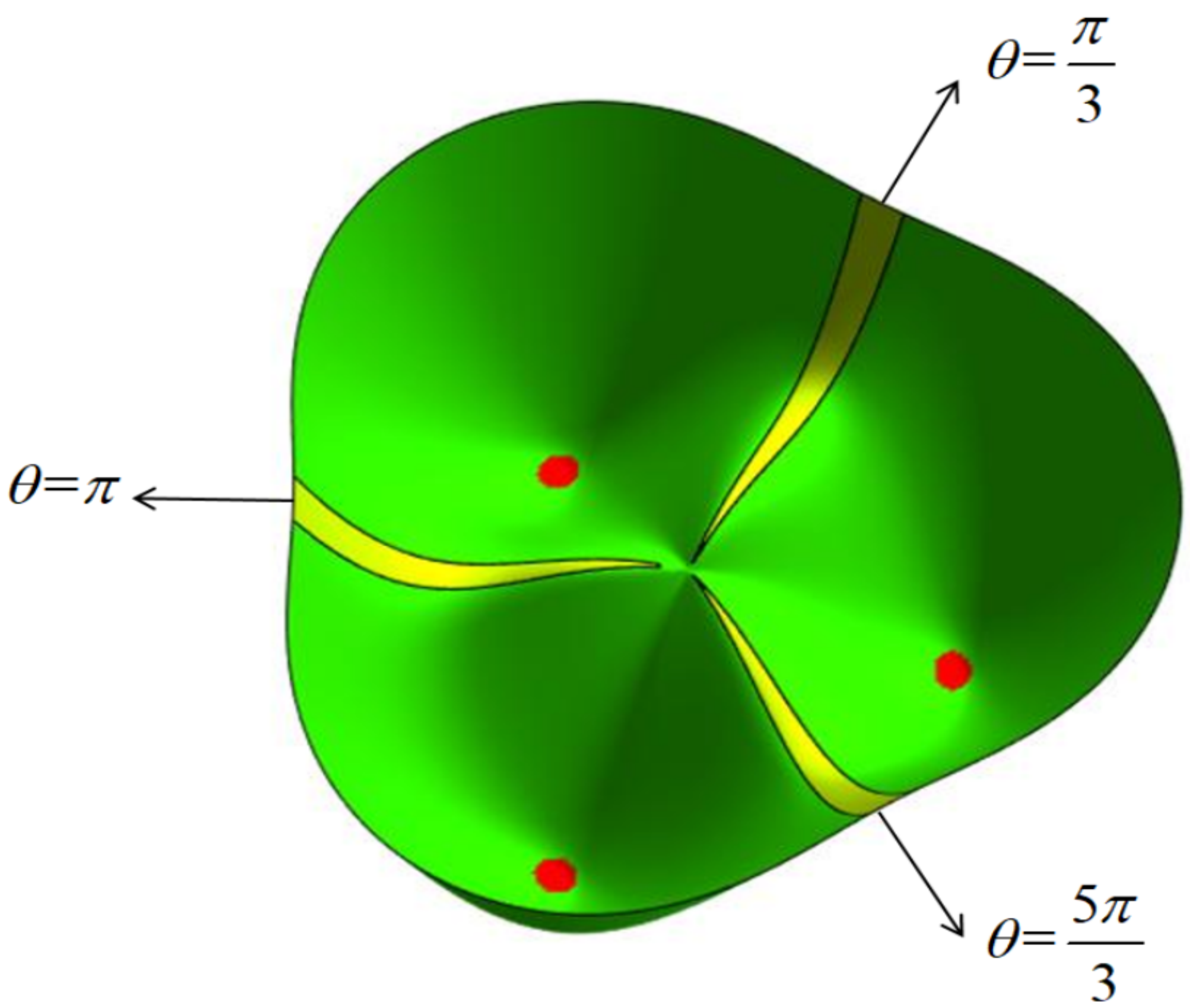
two-point correlation functions

$$\langle\phi_i(\mathbf{k})\phi_j(\mathbf{k}')\rangle = (2\pi)^3\mathcal{P}_\phi(k)\delta(\mathbf{k}-\mathbf{k}')\delta_{ij},$$
$$\langle\dot\phi_i(\mathbf{k})\dot\phi_j(\mathbf{k}')\rangle = (2\pi)^3\mathcal{P}_{\dot\phi}(k)\delta(\mathbf{k}-\mathbf{k}')\delta_{ij},$$
$$\langle\phi_i(\mathbf{k})\dot\phi_j(\mathbf{k}')\rangle = 0\,.$$

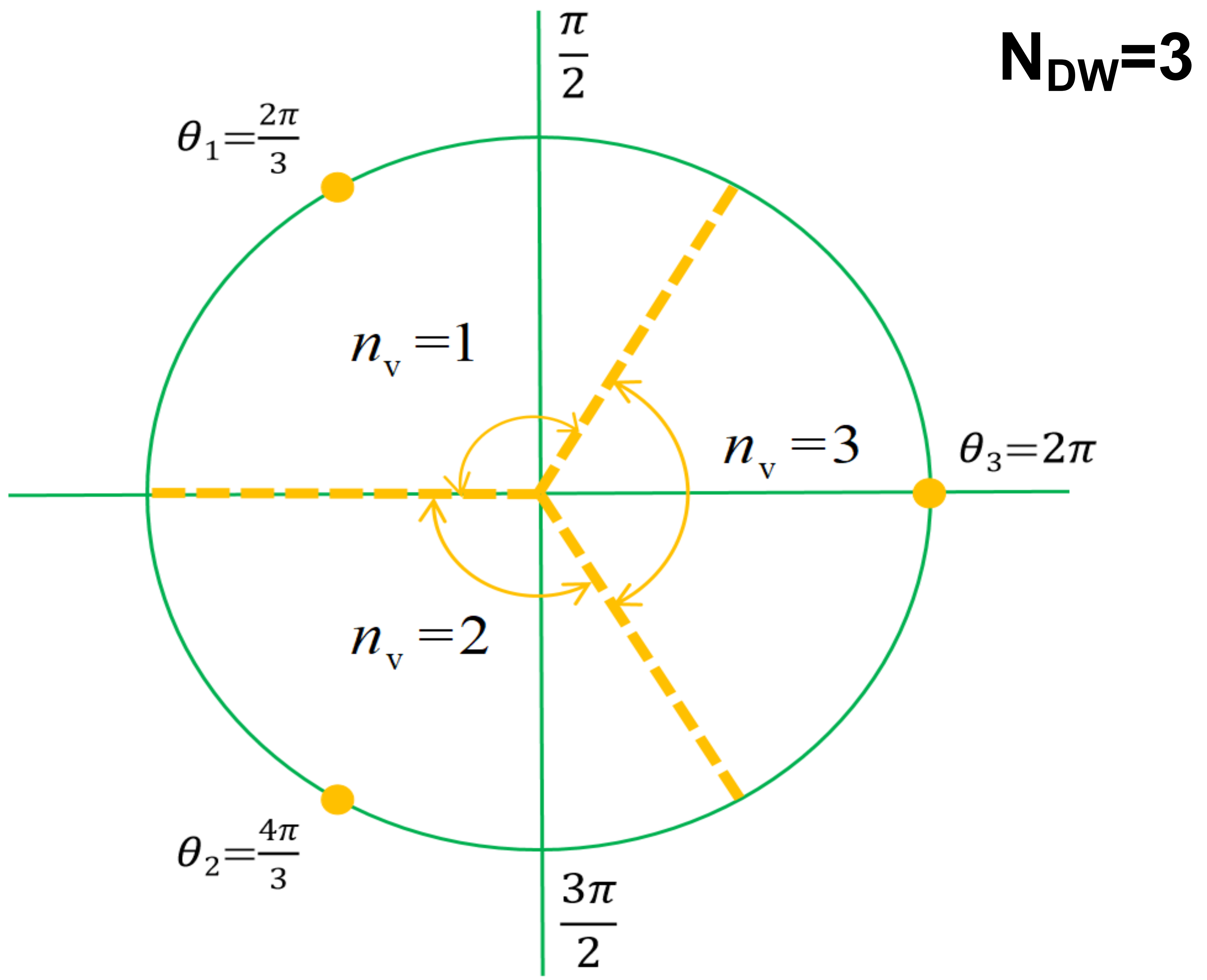
$$\langle|\phi_i(\mathbf{k})|^2\rangle = \left(\frac{N}{\delta x_{\rm phy}}\right)^3\mathcal{P}_{\phi_i}(k), \qquad \langle\phi_i(\mathbf{k})\rangle = 0,$$
$$\langle|\dot\phi_i(\mathbf{k})|^2\rangle = \left(\frac{N}{\delta x_{\rm phy}}\right)^3\mathcal{P}_{\dot\phi_i}(k), \qquad \langle\dot\phi_i(\mathbf{k})\rangle = 0,$$

▸ Domain wall's identification

Top view of the shape of potential energy



The distribution of fields in phase space



Comoving area density

$$A/V = C \sum_{\text{links}} \delta \frac{|\nabla\theta|}{|\theta_{,x}| + |\theta_{,y}| + |\theta_{,z}|}$$

θ,i (i = x, y, z) : spatial derivatives of the dimensionless axion field θ(x)

Area parameter A of DW (scaling parameter of DW)

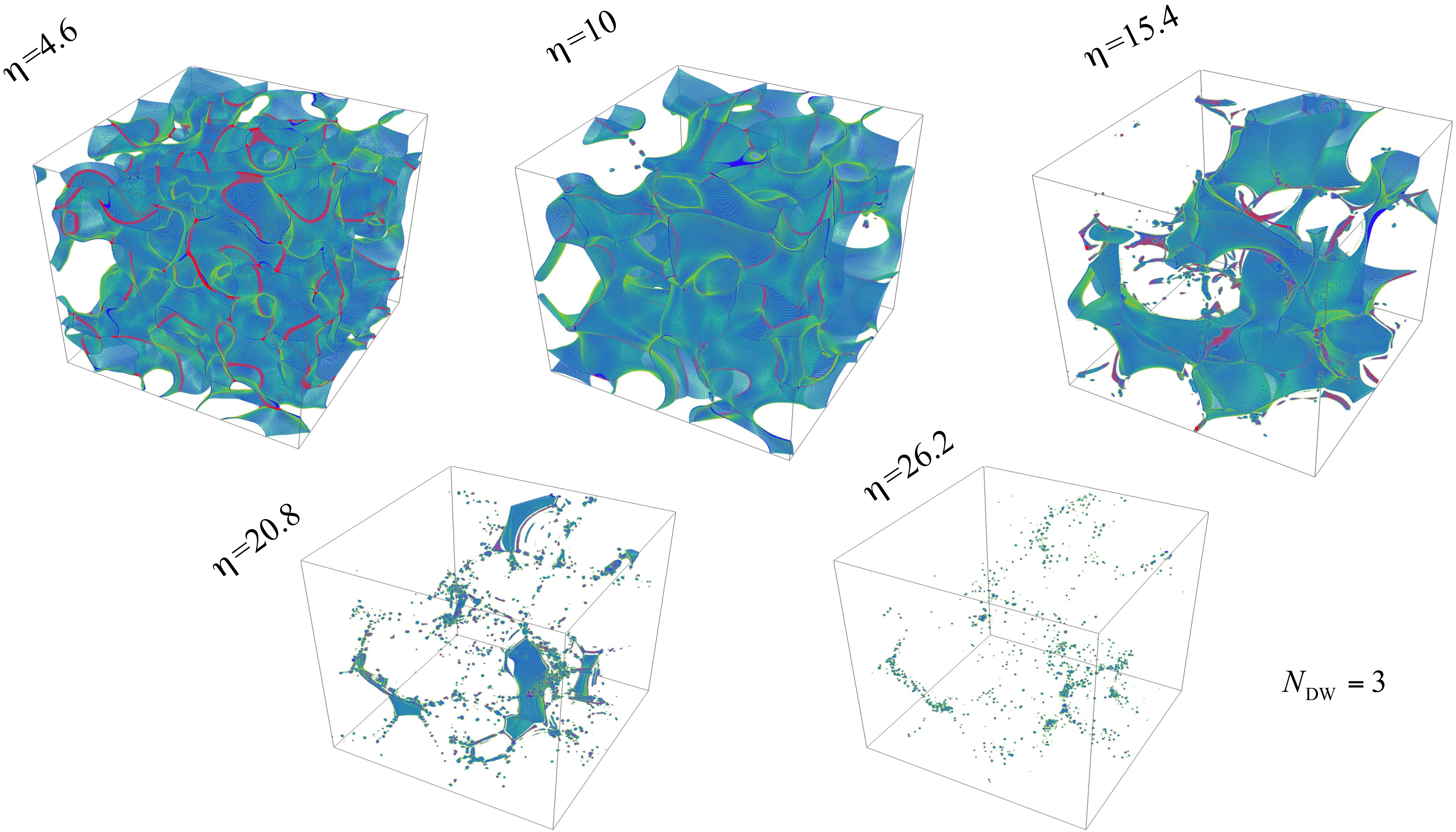
$$\xi_{\mathrm{dw}} \equiv \mathcal{A} = \frac{\rho_{\mathrm{wall}}}{\sigma_{\mathrm{wall}}} t, \quad \text{with } \rho_{\mathrm{wall}} = \frac{\sigma_{\mathrm{wall}} A}{R(t) V}$$

$$A = \Delta A \sum_{\text{links}} \delta \frac{|\nabla\theta|}{|\theta_{,x}| + |\theta_{,y}| + |\theta_{,z}|}$$

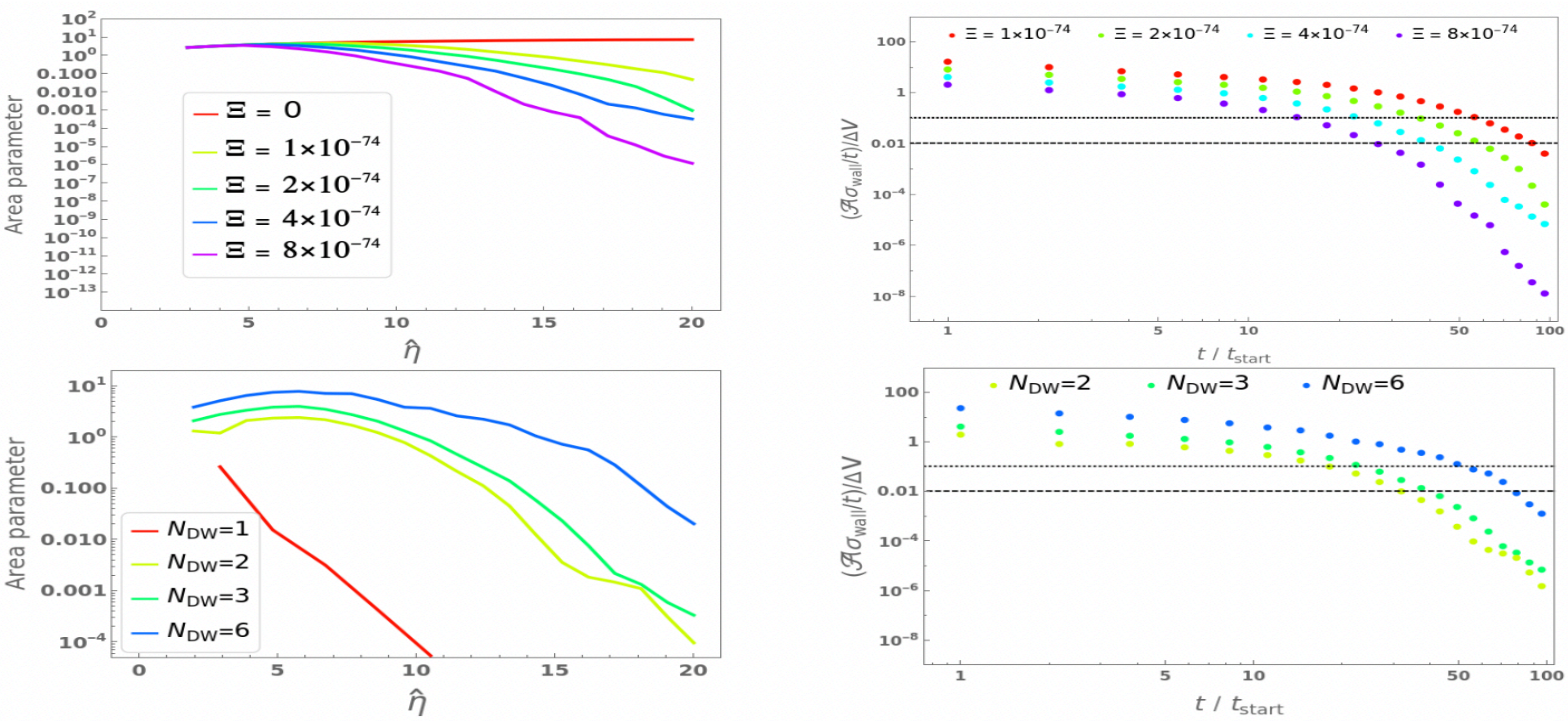
$\Delta A = (\delta x)^2$ is the comoving area of one grid surface

String-wall evolution

Axion string-domain wall hybrid network destruction with gravitational waves and axions emission



▸ Domain wall Area parameter



DW decay

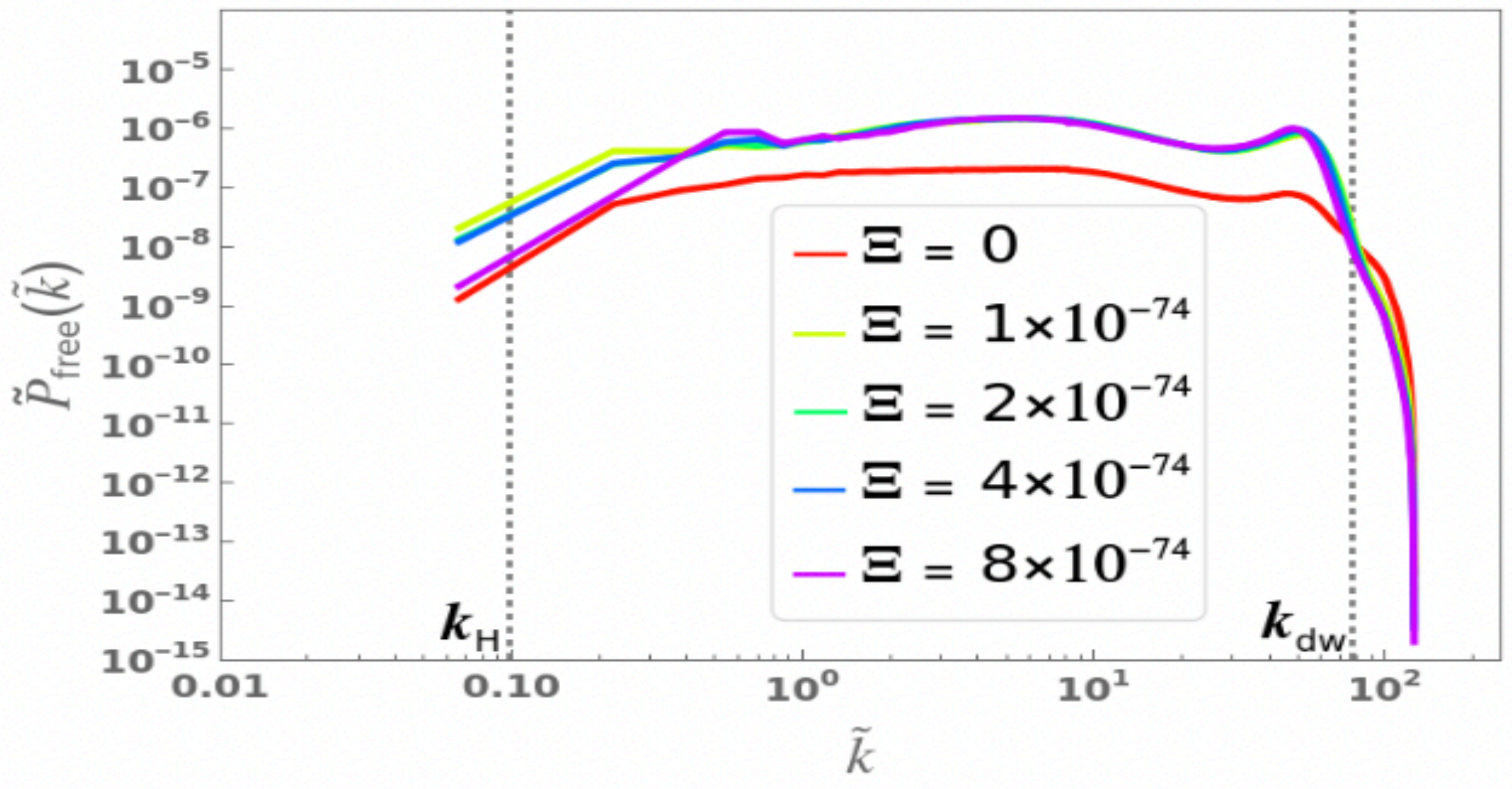
$$\Delta V = 2\Xi v^4(1-\cos(2\pi/N_{\rm DW})) \sim \rho_{\rm wall} = \mathcal{A}\sigma_{\rm wall}/t$$

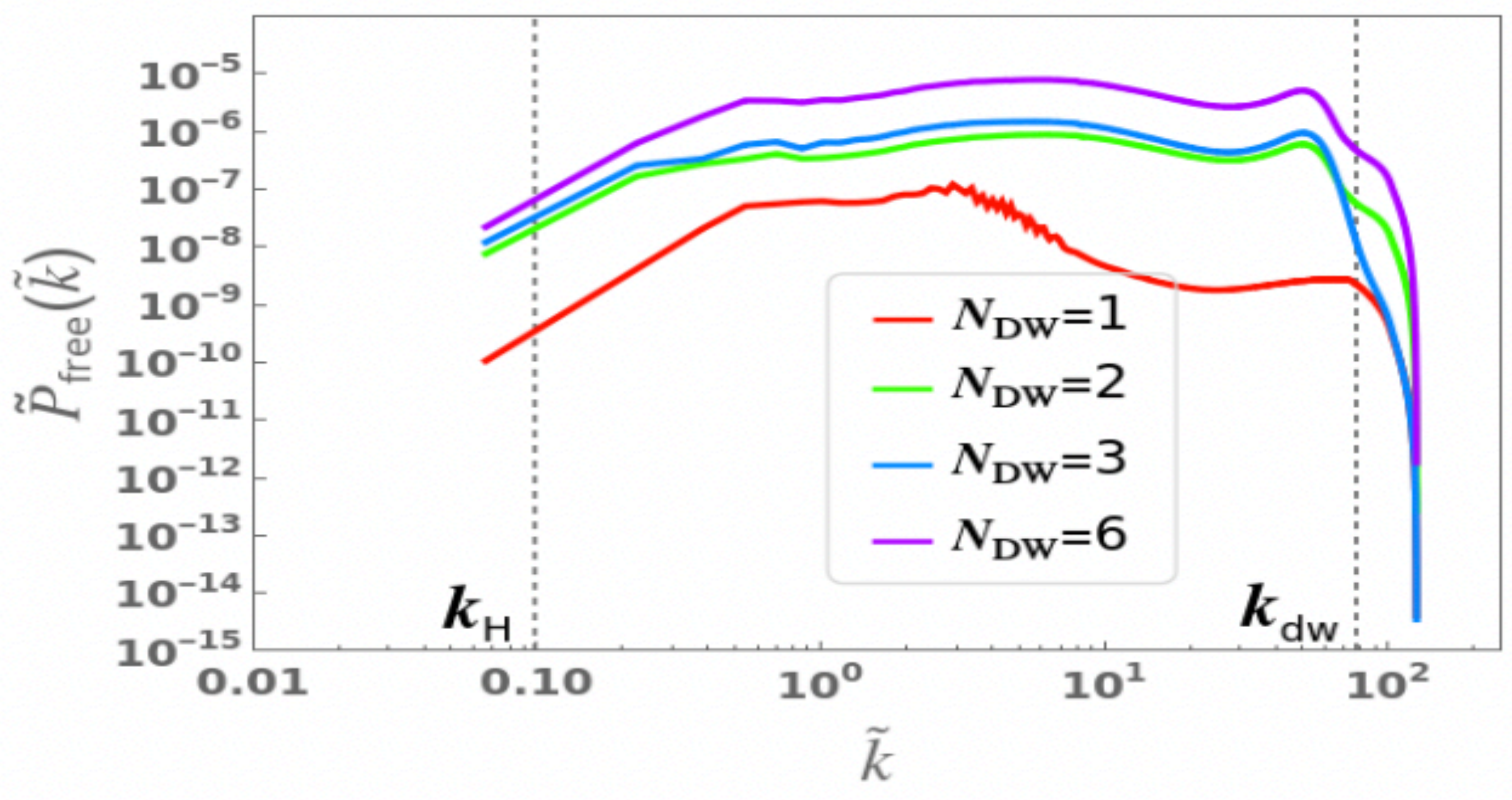
$$\Rightarrow \quad t_{\rm dec} = t_{\rm form}\left(\frac{C_d \mathcal{A}_{\rm form}\sigma_{\rm wall}}{t_{\rm form}\Xi N_{\rm DW}^4 f_a^4(1-\cos(2\pi/N_{\rm DW}))}\right)^{1/p}$$

with $p = 1 + \ln C_d / \ln(t_{dec}/t_{form})$

$C_d = \Delta V/(\mathcal{A}\sigma_{\rm wall}/t) \sim \mathcal{O}(10^2)$

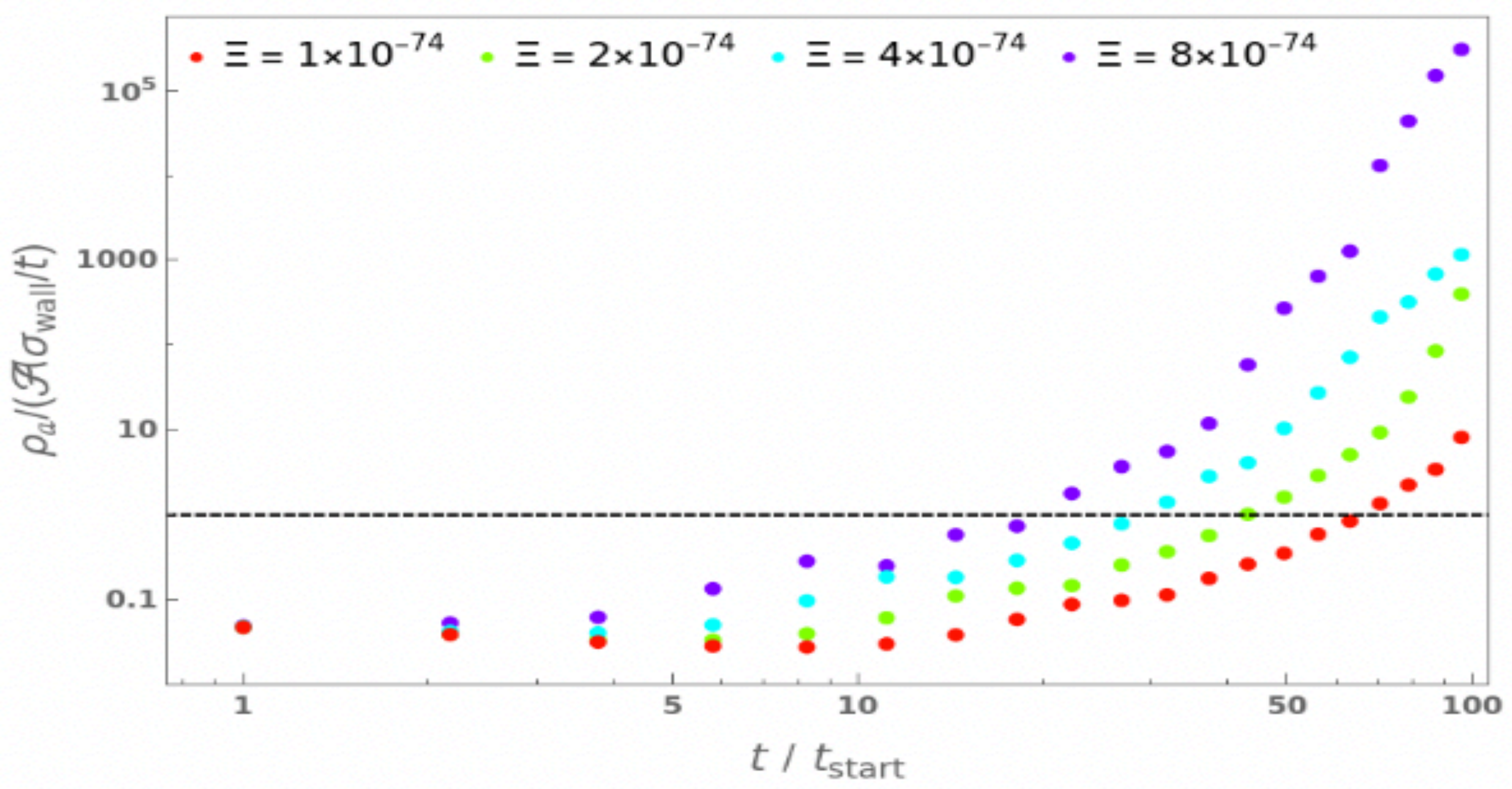
▸ Domain wall decay to free axions

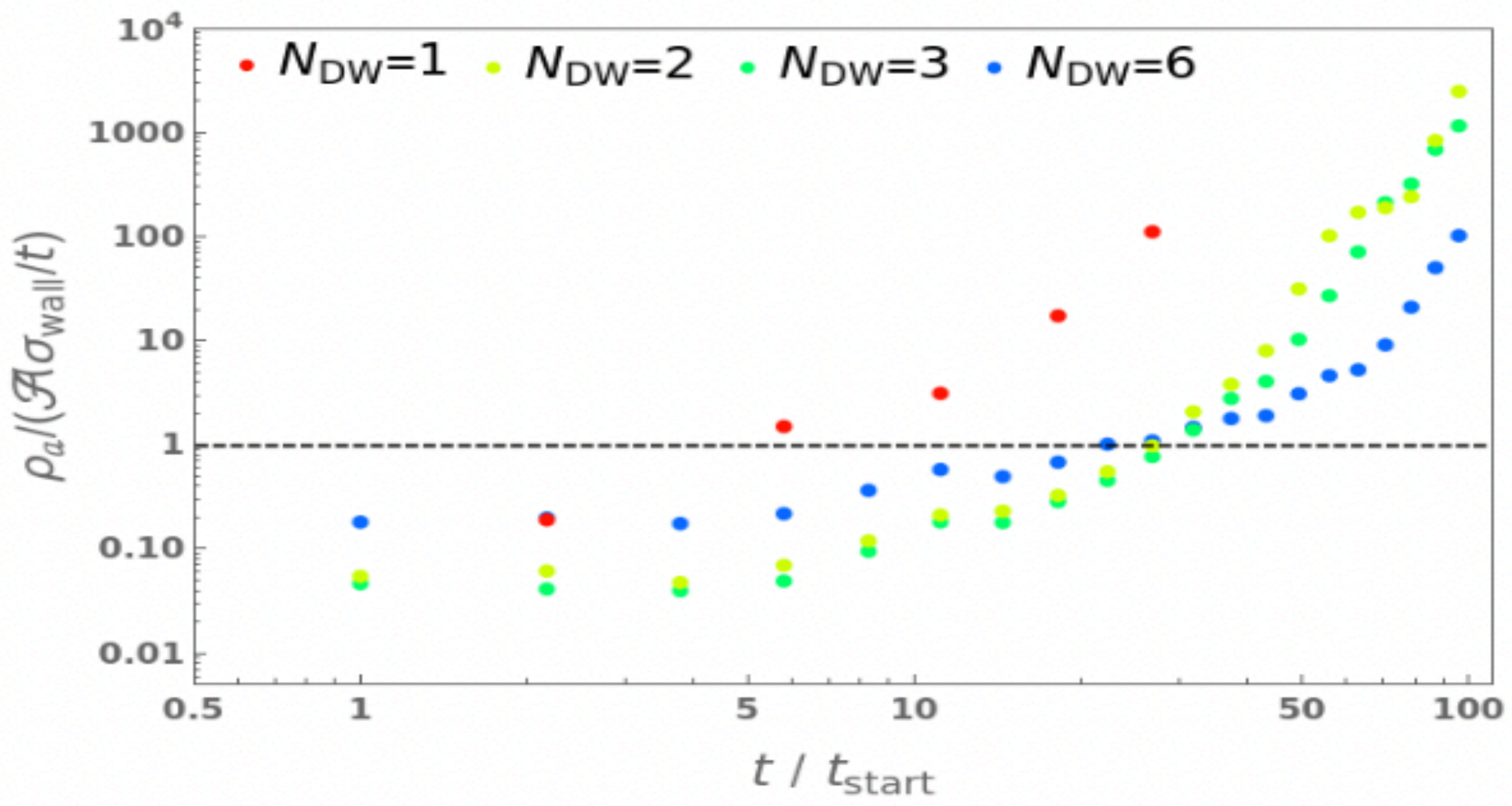




$$\frac{1}{2}\langle \dot{a}_{\rm free}(\mathbf{k})^* \dot{a}_{\rm free}(\mathbf{k}')\rangle = (2\pi)^3 \frac{2\pi^2}{k^3} P_{\rm free}(k)\delta(\mathbf{k}-\mathbf{k}') \qquad W(\mathbf{x}) = W_{\rm st}(\mathbf{x}) \times W_{\rm dw}(\mathbf{x}), \qquad \dot{a}_{\rm free}(\mathbf{x}) = W(\mathbf{x})\dot{a}(\mathbf{x}), \qquad \dot{a}(\mathbf{x}) = f_a \frac{\phi_1\dot{\phi}_2 - \dot{\phi}_1\phi_2}{\phi_1^2+\phi_2^2}$$

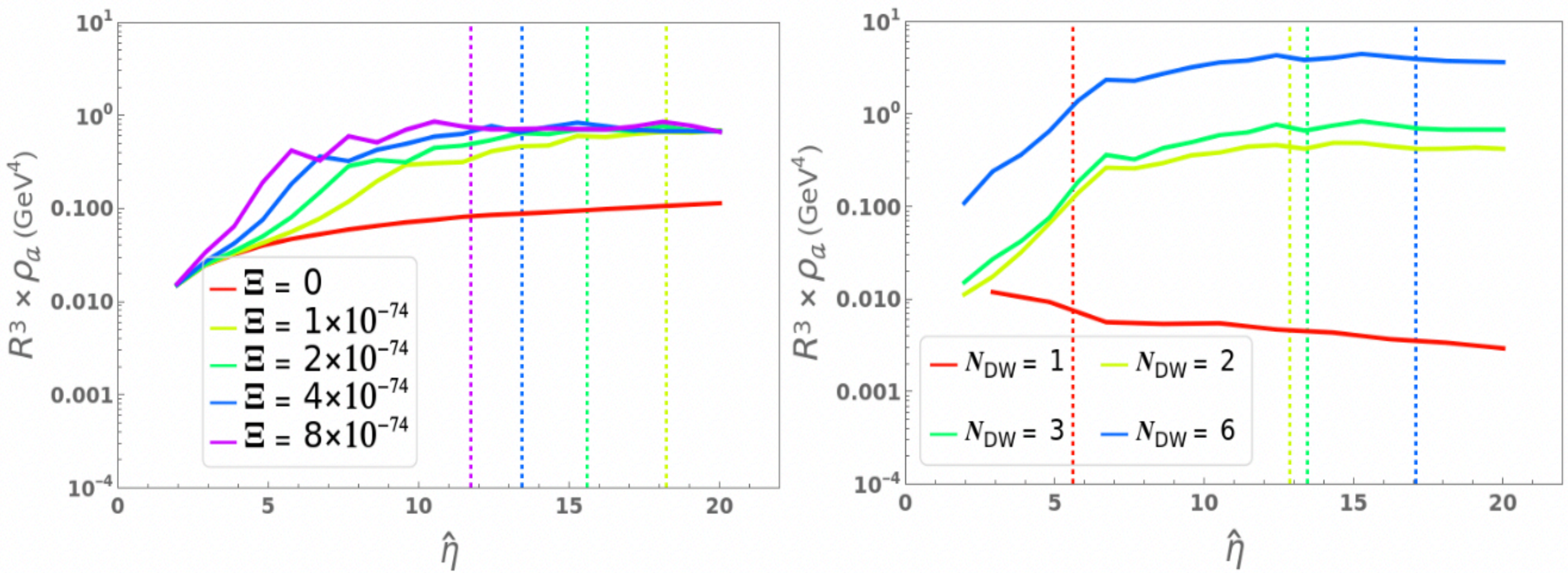
Window function Wdw(x) will take 0 near the DW core and 1 near the true vacuum





$$\rho_a \simeq 2 \times \tfrac{1}{2}\langle \dot{a}^2_{\rm free}(\mathbf{x})\rangle \qquad \rho_a \sim \mathcal{A}\sigma_{\rm wall}/t \sim 8\mathcal{A}mf_a^2/t_{\rm dec}$$

▸ Domain wall decay to free axions



$$\rho_a(t_{\rm dec}) \simeq \frac{2(2p-1) f_a^4 N_{\rm DW}^4}{(3-2p) C_d} \Xi \sin^2(\pi/N_{\rm DW})$$

Axion energy density tends to be a constant at the final moment, and the energy density of the radiated free axion is (almost) proportional to the bias term and NDW

▸ Domain wall decay to axion Dark matter

$$\Omega_{a,\mathrm{new}}(t_0)h^2 = \Omega_a^{\mathrm{DW}} h^2 + \Omega_{a,0}(t_0)h^2 + \Omega_{a,\mathrm{st}}(t_0)h^2$$

Axion DM from Wall decay：

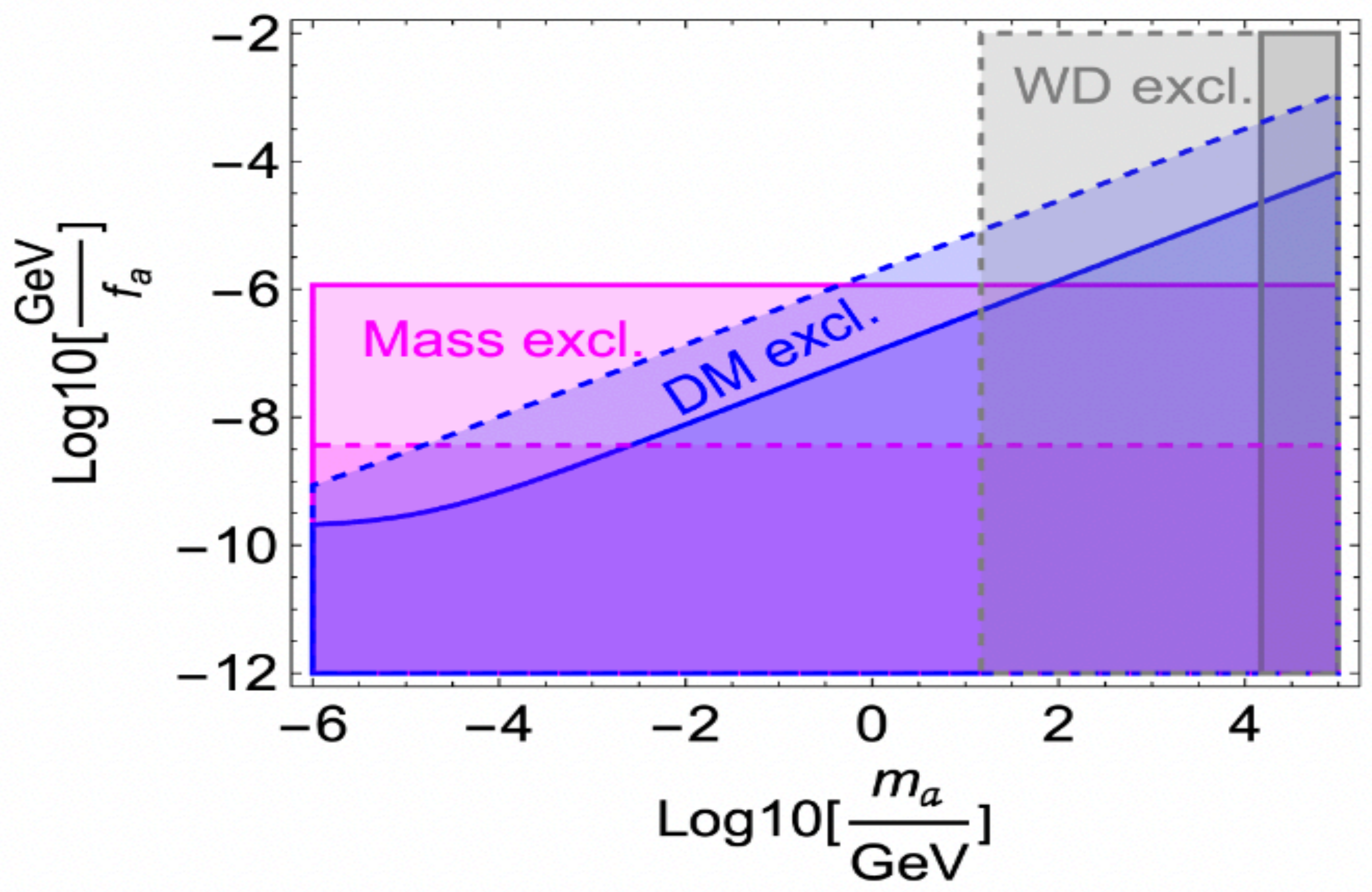
$$\Omega_a h^2 = \frac{\rho_a(t_0)}{\rho_{0,c}/h^2} = 1.024\times 10^{-19}\left(\frac{8}{3}\right)^{3/(2p)}\left(\frac{2p-1}{3-2p}\right)C_d^{3/(2p)-1}\left(\frac{m}{\mathrm{GeV}}\right)^{3/p-3/2}\left(\frac{f_a}{\mathrm{GeV}}\right)^{4-3/p}\Xi^{1-3/(2p)}\left(\frac{\csc(\pi/N_{\mathrm{DW}})}{N_{\mathrm{DW}}^2}\right)^{3/p-2}$$

Misalignment mechanism：

$$(\Omega_{a,0}(t_0)h^2 \simeq 4.63\times 10^{-3}(f_{a,\mathrm{new}}/10^{10}\mathrm{GeV})^{1.19})$$

Axion DM from pure axion string：

$$\Omega_{a,\mathrm{st}}(t_0)h^2 = 2.0\times(f_a/10^{12}\,\mathrm{GeV})^{1.19}$$



WD excl.:

$$t_{\mathrm{dec}} < t_{\mathrm{WD}} \qquad t_{\mathrm{WD}} = \frac{3}{16\pi G\sigma_{\mathrm{wall}}}$$

Mass excl.:

bias term dominating over the QCD instanton effect in the contributions to axion mass term

$$\Xi < 2\times 10^{-45} N_{\mathrm{DW}}^{-2}\left(\frac{10^{10}\mathrm{GeV}}{f_a}\right)^4$$

DM excl.

$$\Omega_a(t_0)h^2 \le 0.12$$

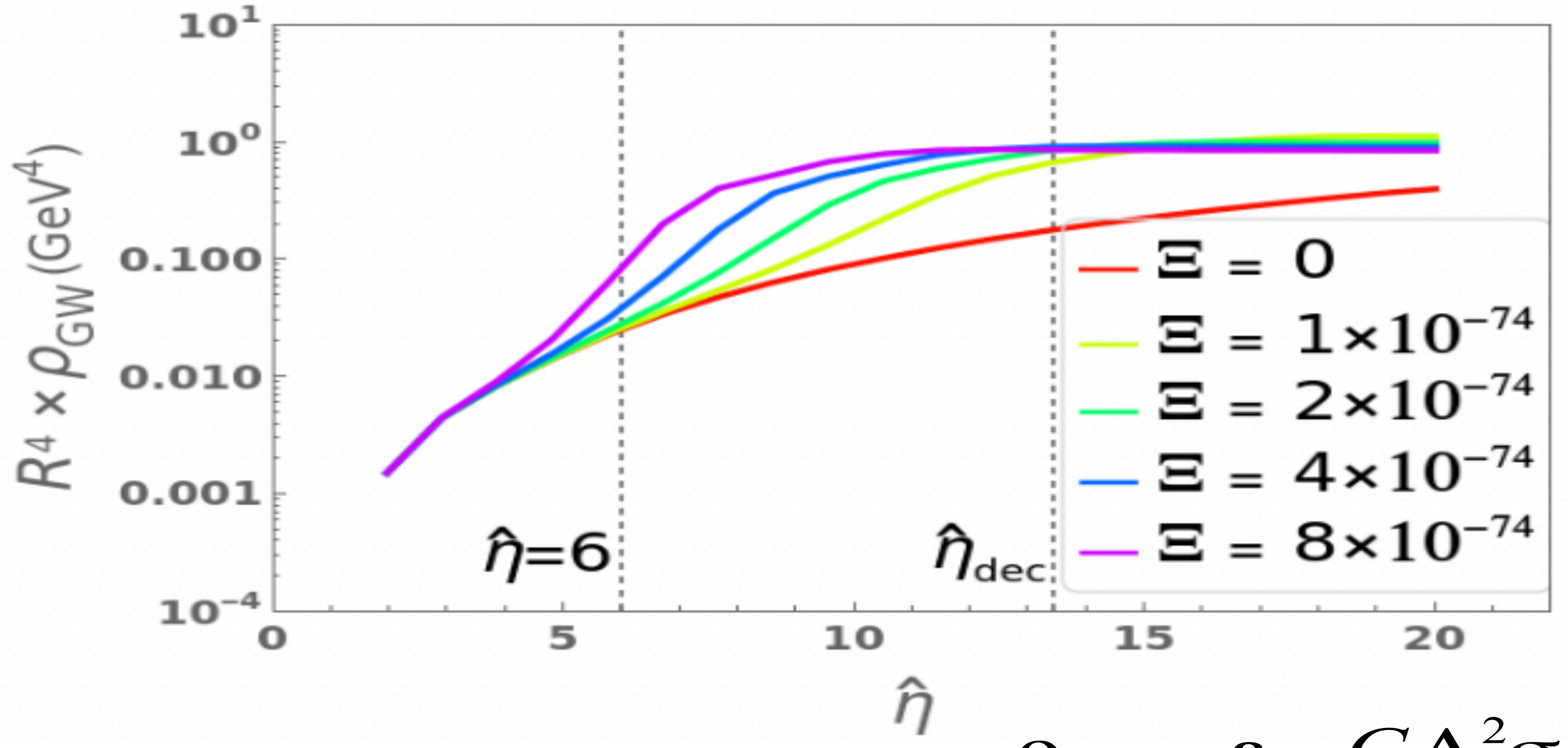
String-wall GW

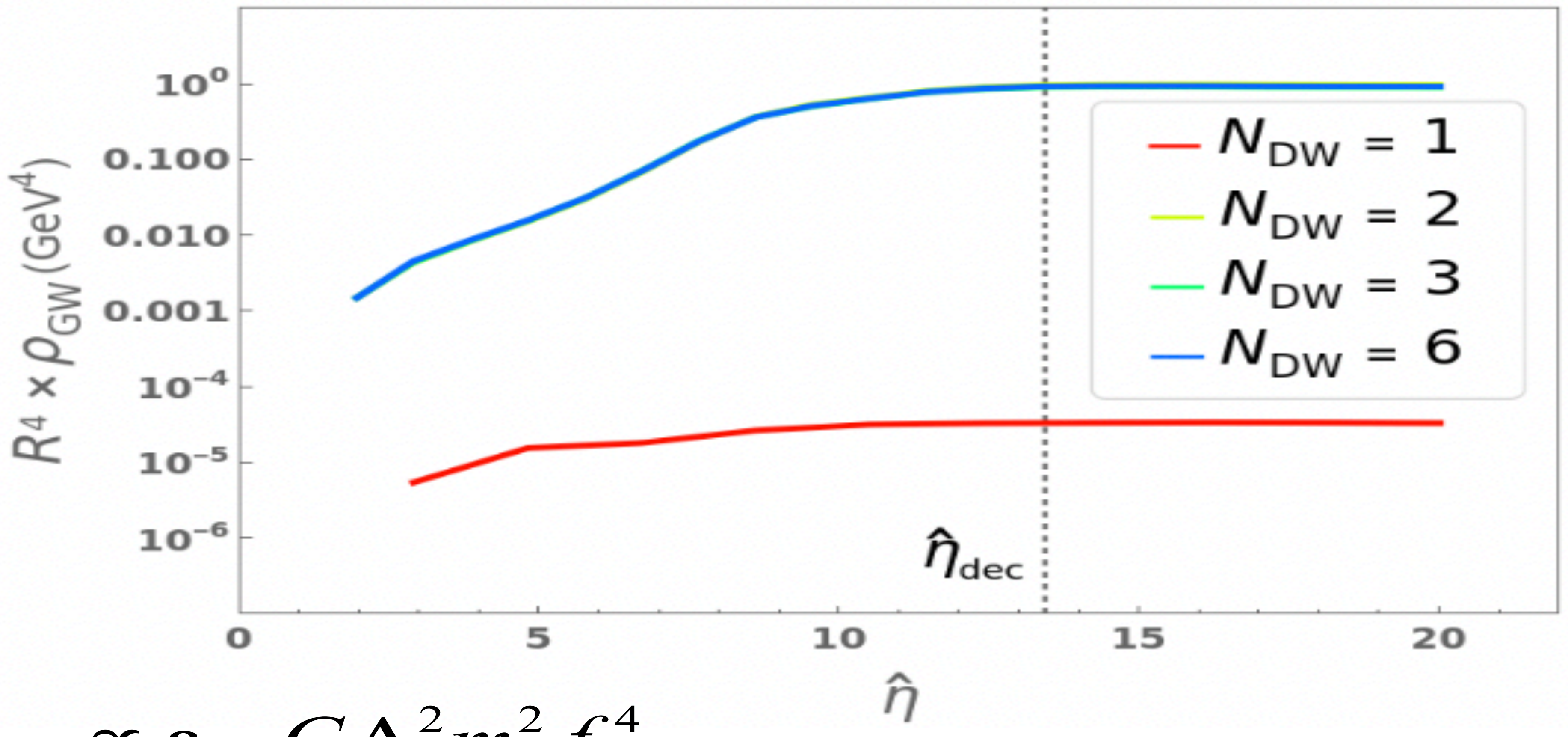
$$\ddot{h}_{ij} + 3\frac{\dot{R}}{R}\dot{h}_{ij} - \frac{\nabla^2}{R^2}h_{ij} = \frac{16\pi G}{R^2}\Pi^{TT}_{ij}$$

$$\Pi_{ij} \equiv T_{ij} - pg_{ij} = T_{ij} - \frac{\delta_{ij}}{3}\sum_l T_{ll}$$

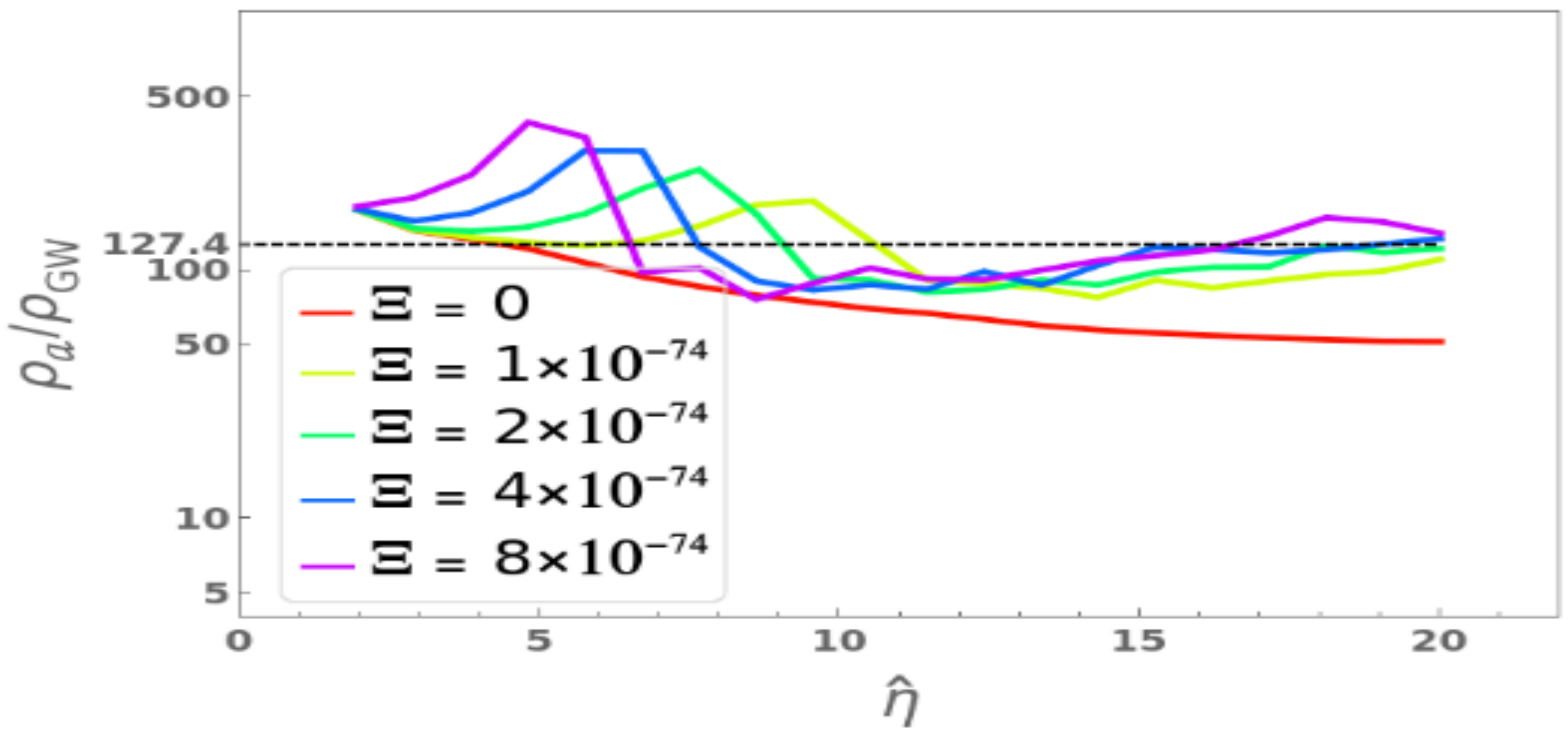
$$= (\partial_i\phi_1\partial_j\phi_1 + \partial_i\phi_2\partial_j\phi_2) - \frac{\delta_{ij}}{3}\sum_l(\partial_l\phi_1\partial_l\phi_1 + \partial_l\phi_2\partial_l\phi_2)$$

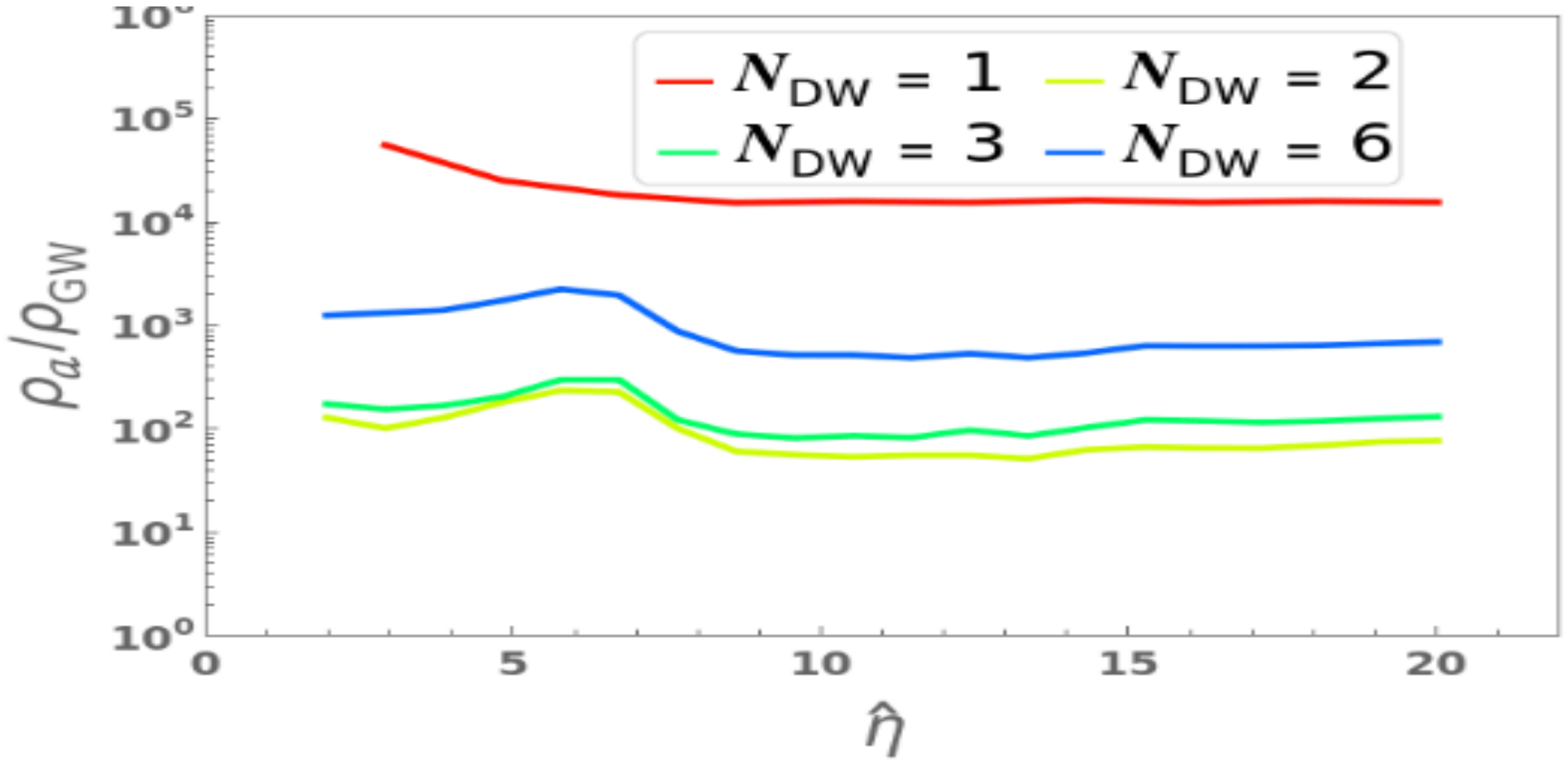
$$\rho_{\rm GW}(t) = t_{00} = \frac{1}{32\pi G}\langle\partial_\mu h^{TT}_{ij}(\mathbf{x},t)\partial_\nu h^{TT,ij}(\mathbf{x},t)\rangle|_{\mu=\nu=0} = \frac{1}{32\pi G}\langle\dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t)\rangle$$





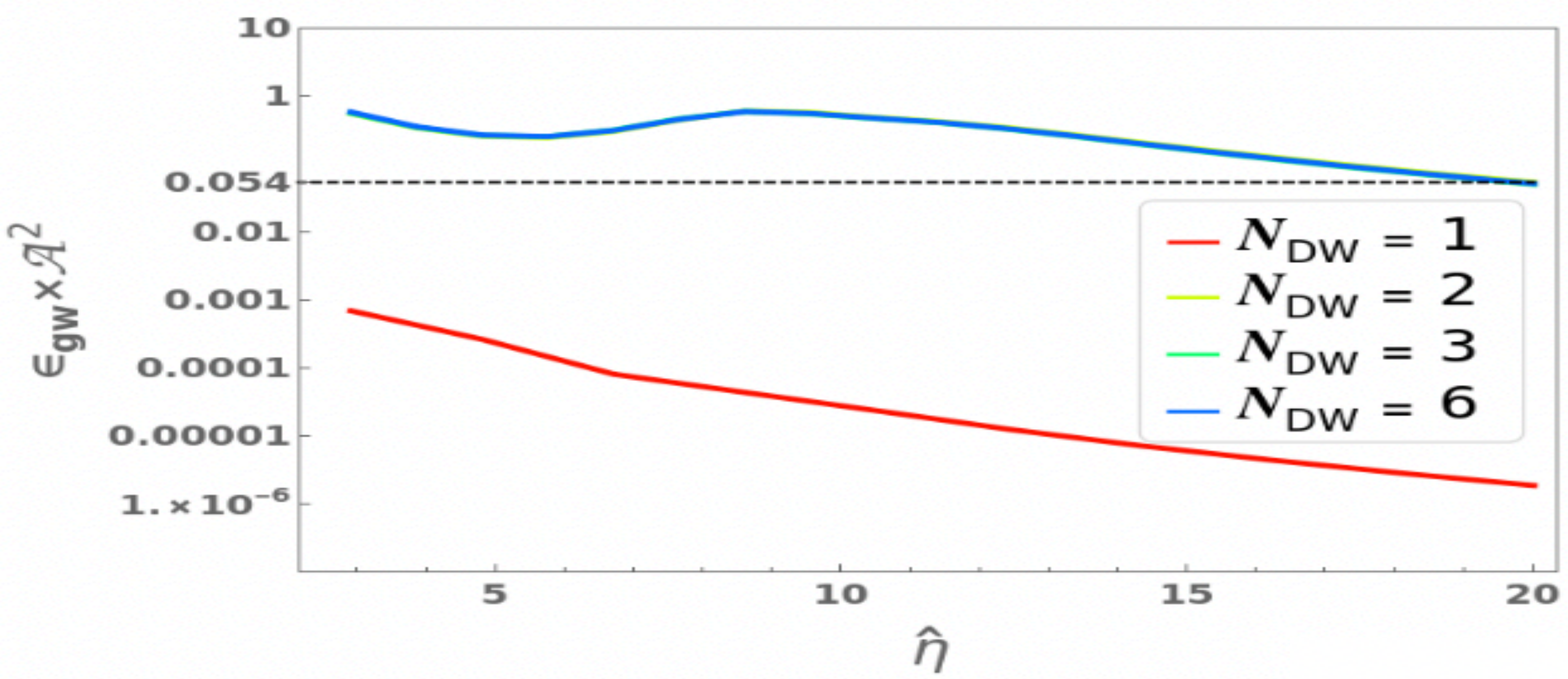
$$\rho_{\rm gw} = \varepsilon_{\rm gw} G A^2 \sigma^2_{\rm wall} \propto \varepsilon_{\rm gw} G A^2 m^2 f_a^4$$

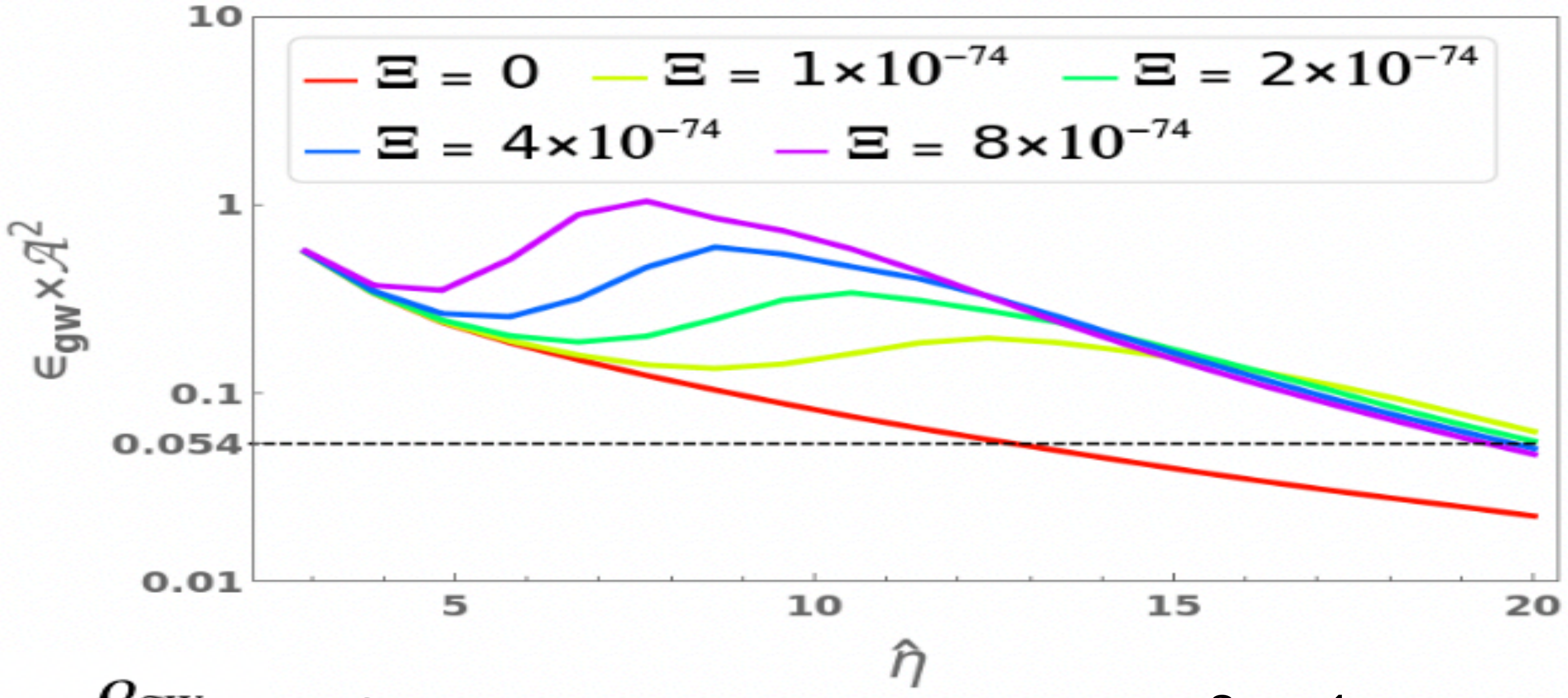




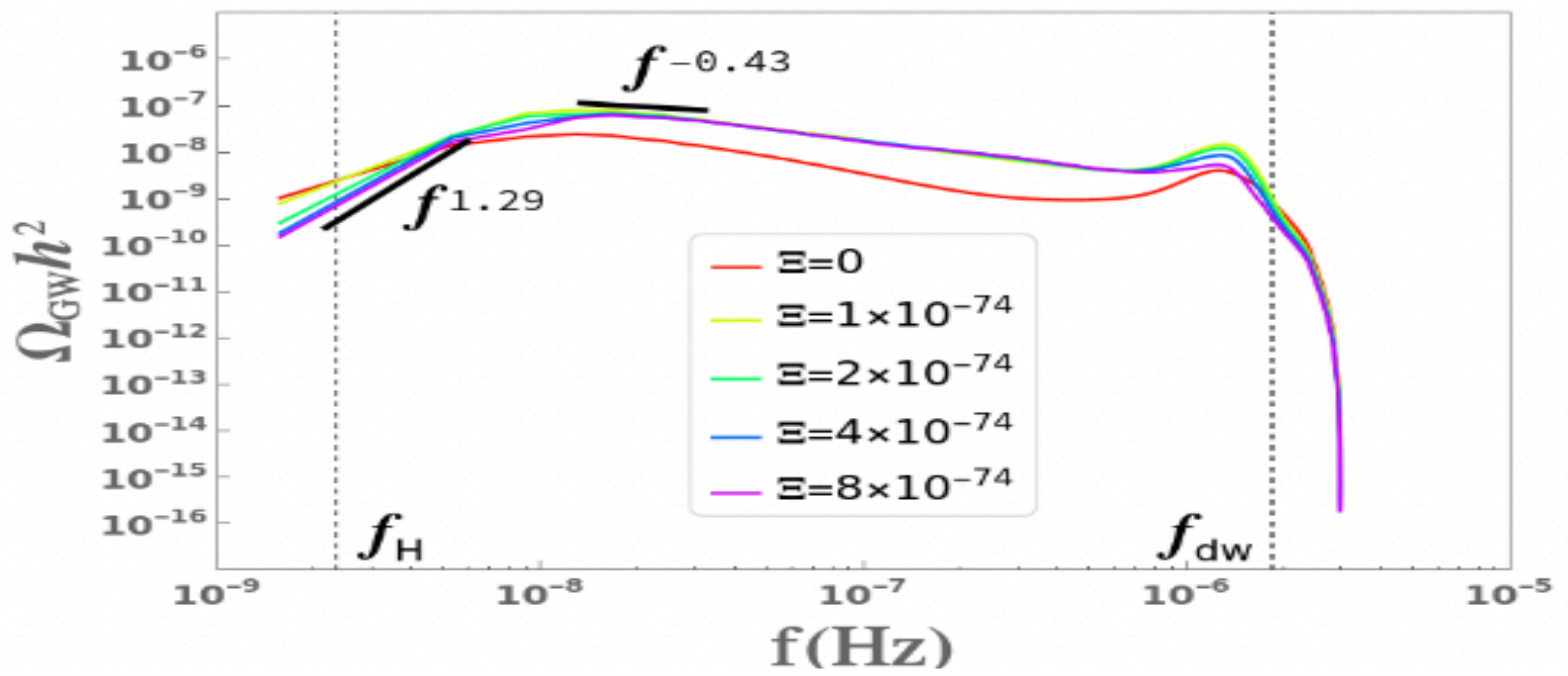
axion string-wall networks mainly decay to axion

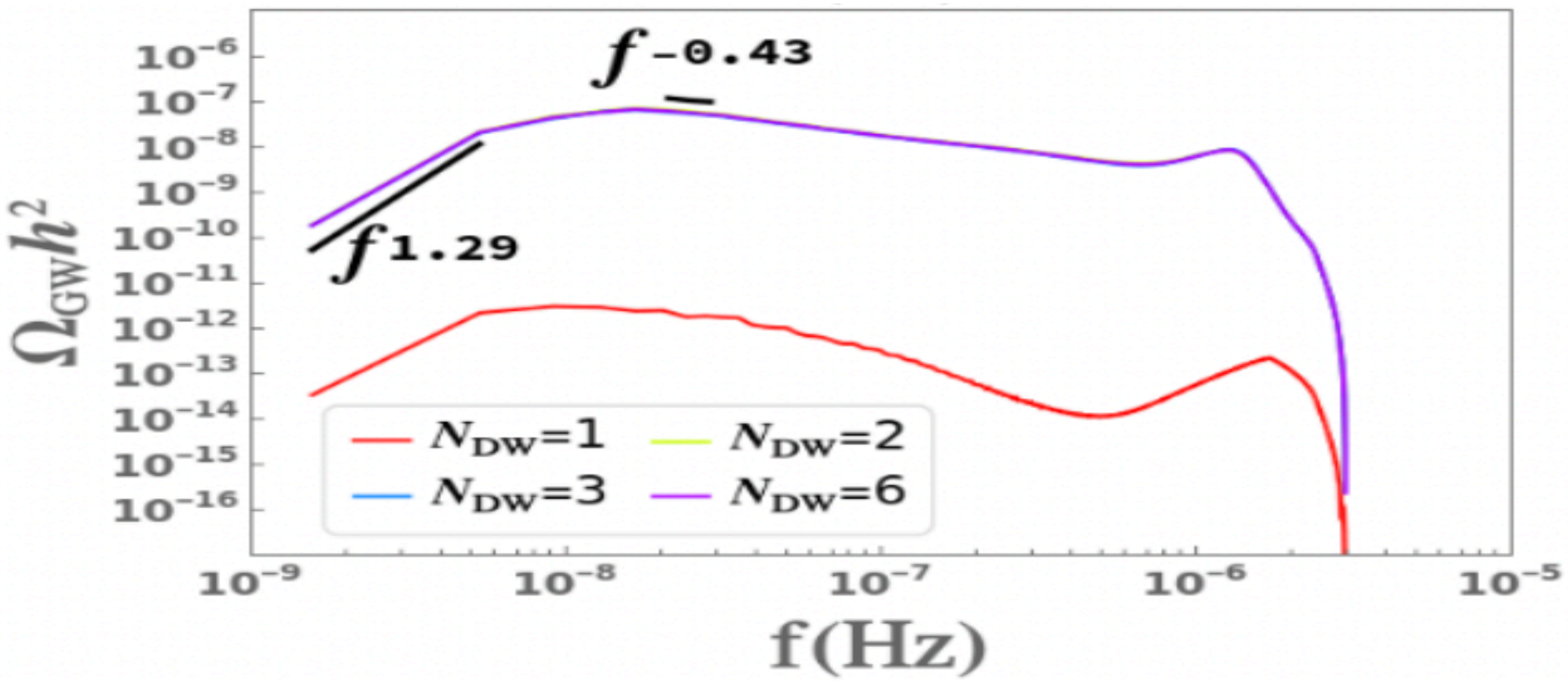
▸ String-wall GW





Efficiency parameter $\epsilon_{gw}\mathcal{A}^2 = \dfrac{\rho_{gw}}{G\sigma^2_{wall}}$ ~ Const. $\rho_{gw} \propto Gm^2 f_a^4$



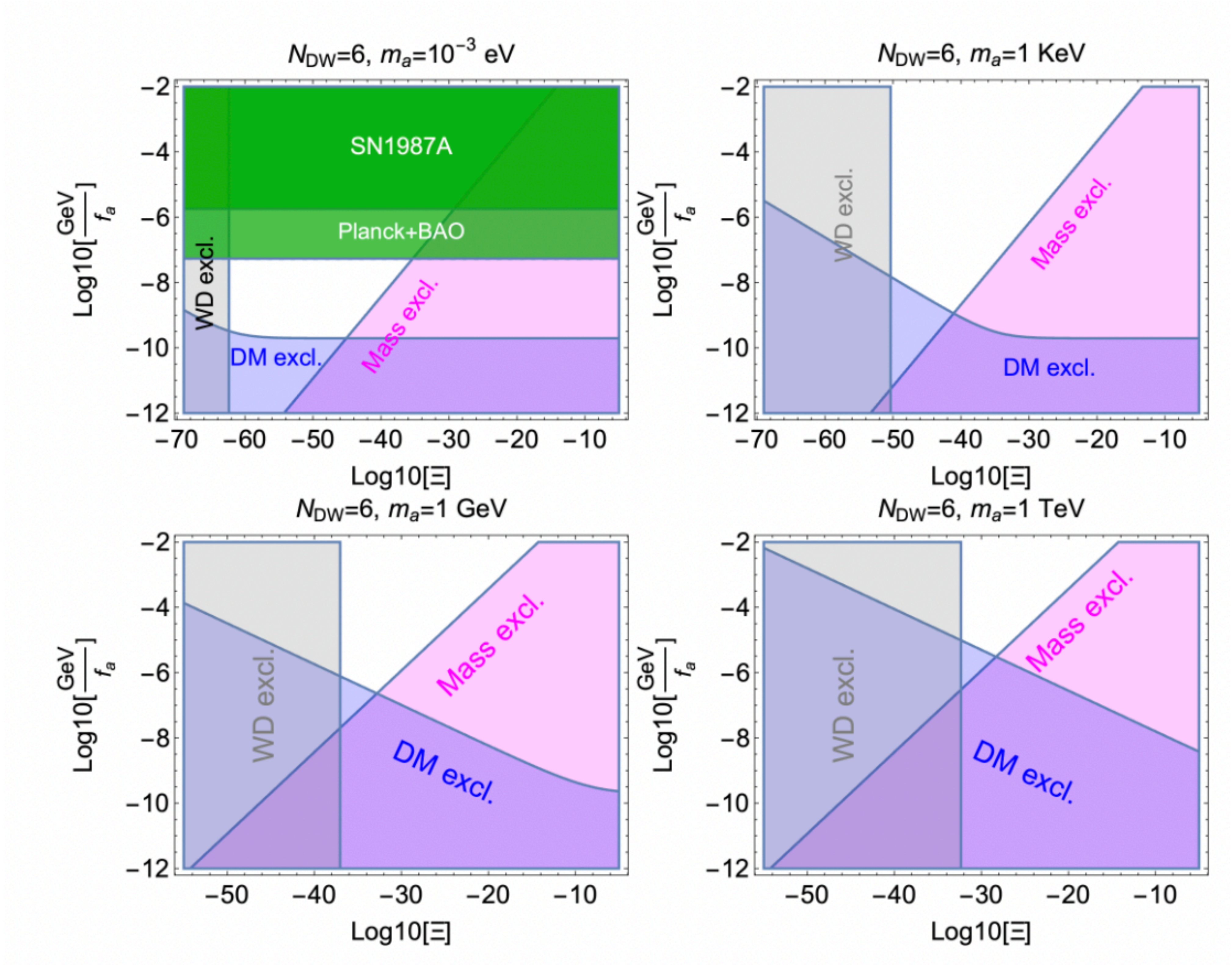


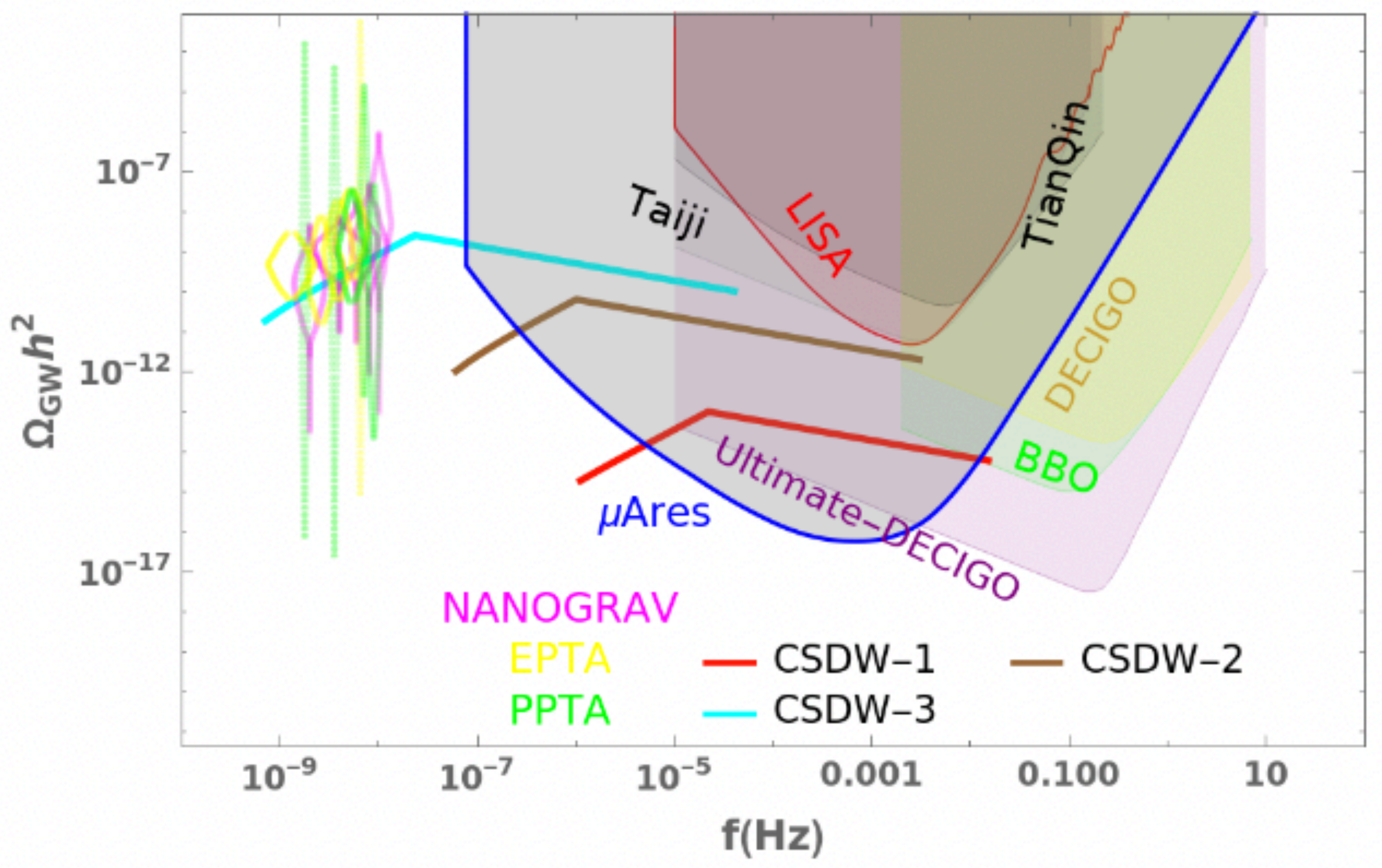
$$\Omega_{GW} \equiv \frac{1}{\rho_c}\frac{d\rho_{GW}}{d\ln k} = \frac{1}{32\pi G}\frac{1}{(2\pi)^3 V}\int d\Omega |\mathbf{k}|^3 |\dot{h}_{ij}(\mathbf{k},t)|^2$$

$$\Omega_{GW,0}h^2 = \frac{h^2}{\rho_{c,0}}\frac{d\rho_{GW,0}}{d\ln k_{p,0}} = \Omega_{rad,0}h^2\left(\frac{g_0}{g_e}\right)^{1/3}\left\{\frac{1}{\rho_{c,e}}\frac{d\rho_{GW,e}}{d\ln k_{co,e}}\right\}$$

▸ Axion DM and GW from String-wall

Our research provides the possibility of searching for axion models or constraining model parameters through GW detection experiments





Simultaneously explanation of DM and GWs with axion string-wall networks, only for N_{DW}>1 of ALP scenarios.

For N_{DW} = 1, the GW energy density appears undetectable for QCD axions and axion-like particles.

▶ Axion DM with PMF

- The action and potential are

$$S = \int d^4x\sqrt{-g}\left[-\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi) - \alpha\frac{\phi}{v}F_{\mu\nu}\tilde{F}^{\mu\nu}\right]$$

$$V(\phi) = \frac{m_a(T)^2 v^2}{N_{DW}^2}\left(1 - \cos\left(N_{DW}\frac{\phi}{v}\right)\right)$$

$$m_a(T)^2 = \frac{\alpha_a\Lambda^4}{f_a^2(T/\Lambda)^{6.68}} \quad (T > 100\ MeV,\ f_a = \frac{v}{N_{DW}})$$

$$V_{eff}(\phi) = V(\phi) + \alpha\frac{\phi}{v}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

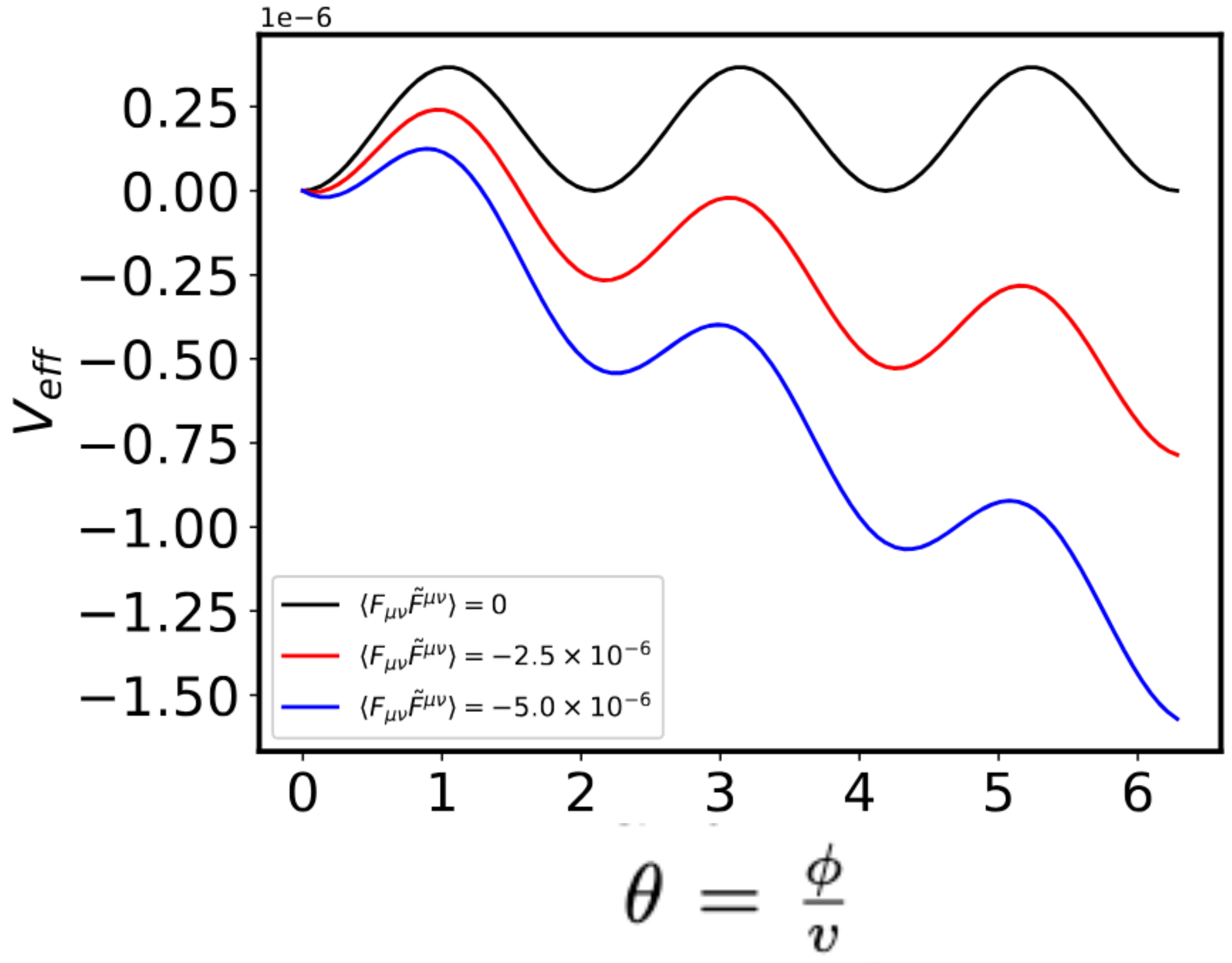
- The EOMs are

$$\phi'' = \partial_i\partial_i\phi - 2\mathcal{H}\phi' - a^2\frac{dV}{d\phi} - a^2\frac{\alpha}{4v}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$E_i' = -\epsilon_{ijk}\partial_j B_k - \frac{\alpha}{v}\phi' B_i + \frac{\alpha}{v}\epsilon_{ijk}\partial_j\phi E_k$$

$$\partial_i A_i' = -\frac{\alpha}{v}\partial_i\phi\epsilon_{ijk}\partial_j A_k$$

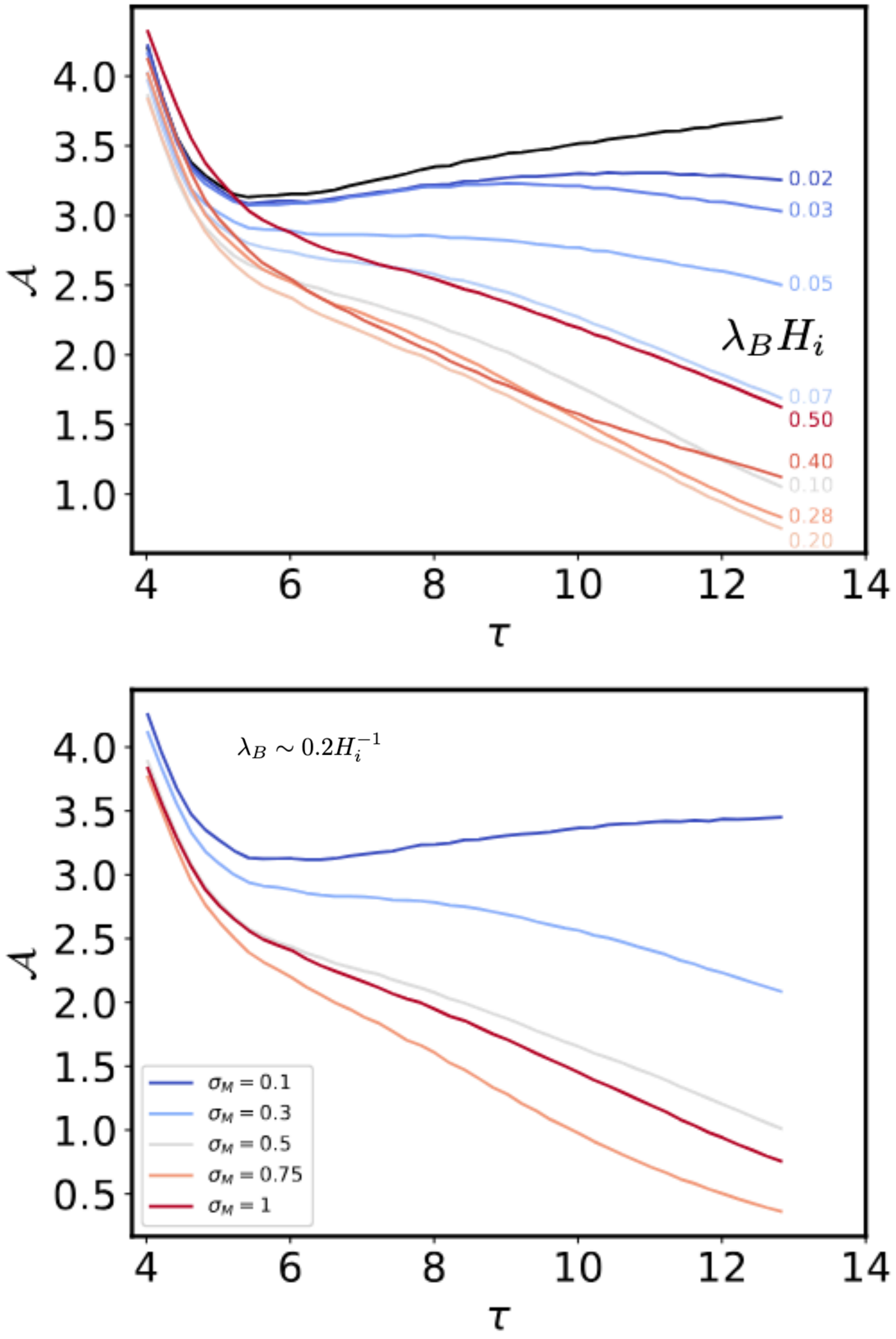
$E_i = \frac{A_i'}{a^2}$, $B_i = \frac{1}{a^2}\epsilon_{ijk}\partial_j A_k$ $\quad F_{\mu\nu}\tilde{F}^{\mu\nu} = 4\sum_i E_i B_i$ is Chern-Simons (CS)

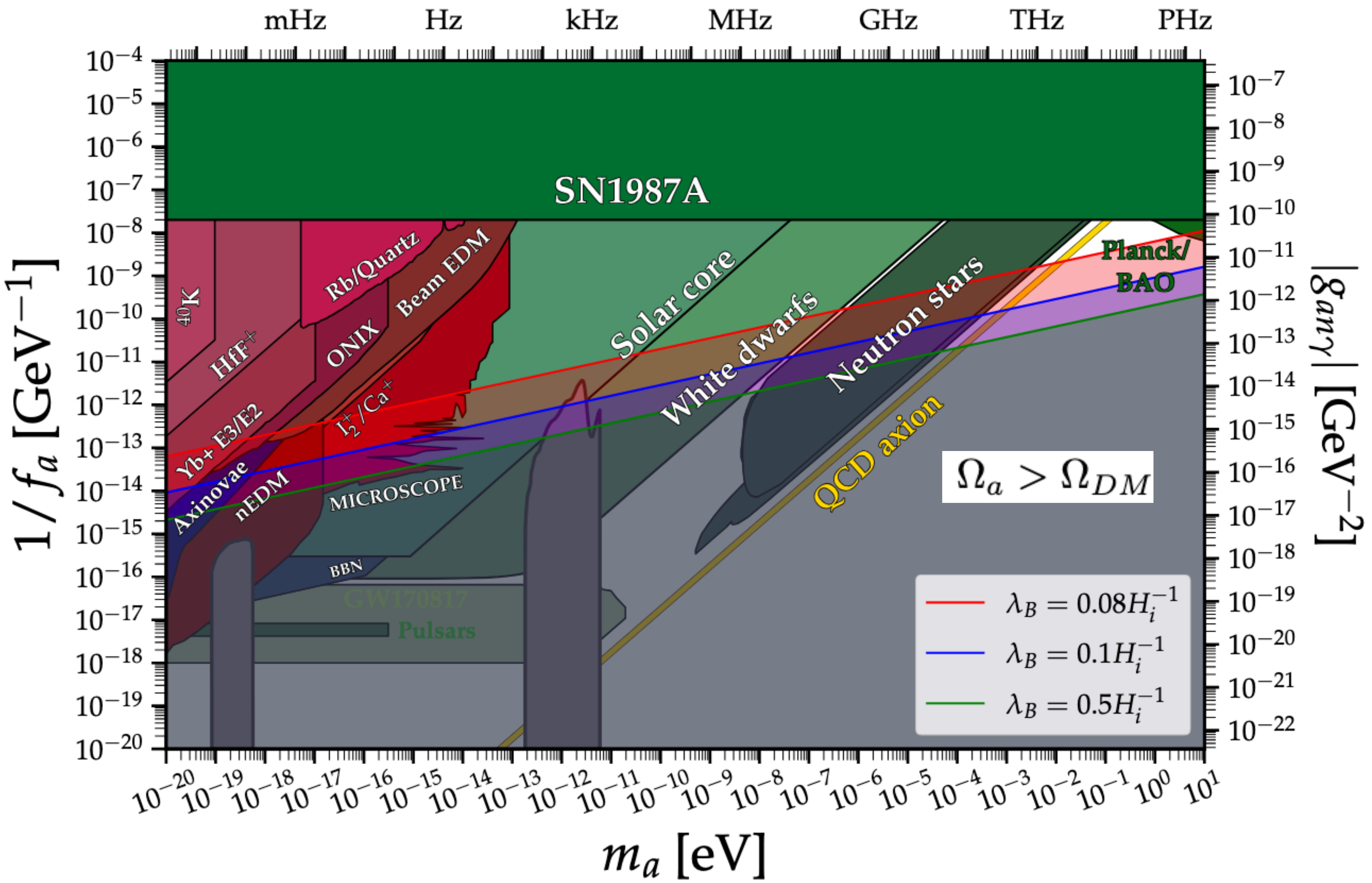


magnetic fields $\quad B_i(\vec{k}) = B_{ini}\Theta(k - k_{UV})(\delta_{ij} - \hat{k}_i\hat{k}_j - i\sigma_M\epsilon_{ijl}\hat{k}_l)g_j(\vec{k})k^n$

gj(k⃗) is Gauss random field and σ_M∈[−1, 1] determines the helicity of magnetic field. n is the spectrum index. (2409.16124)

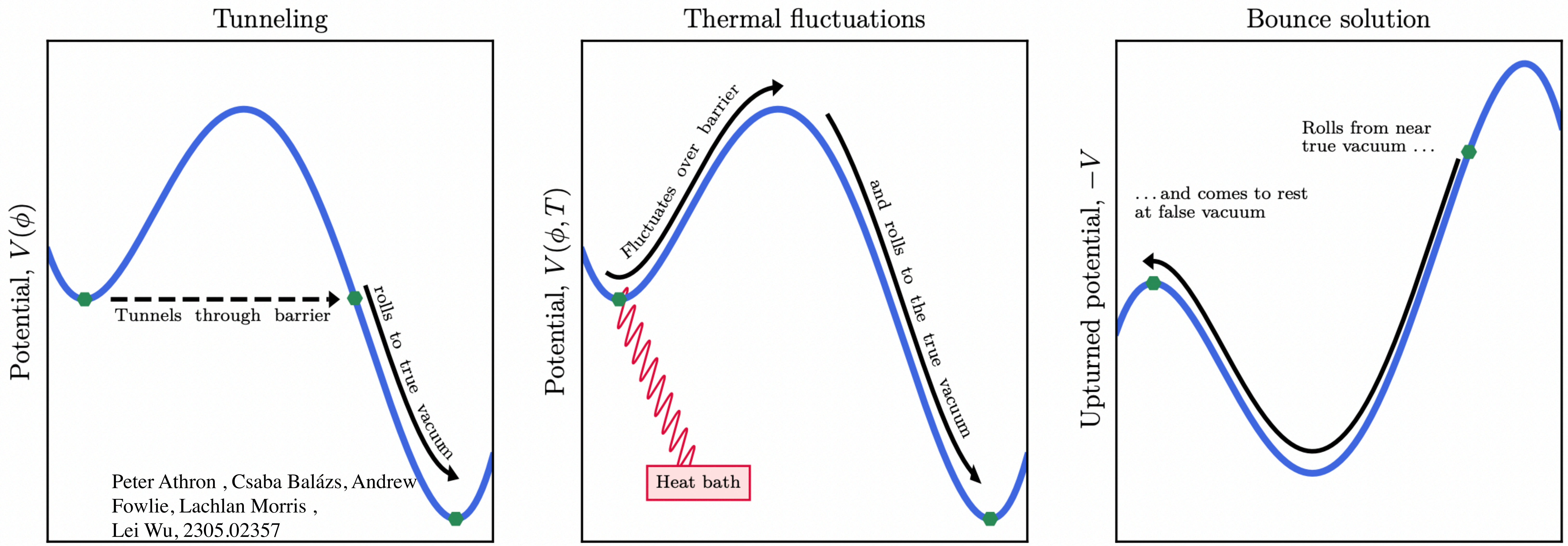
▸ Axion DM with PMF



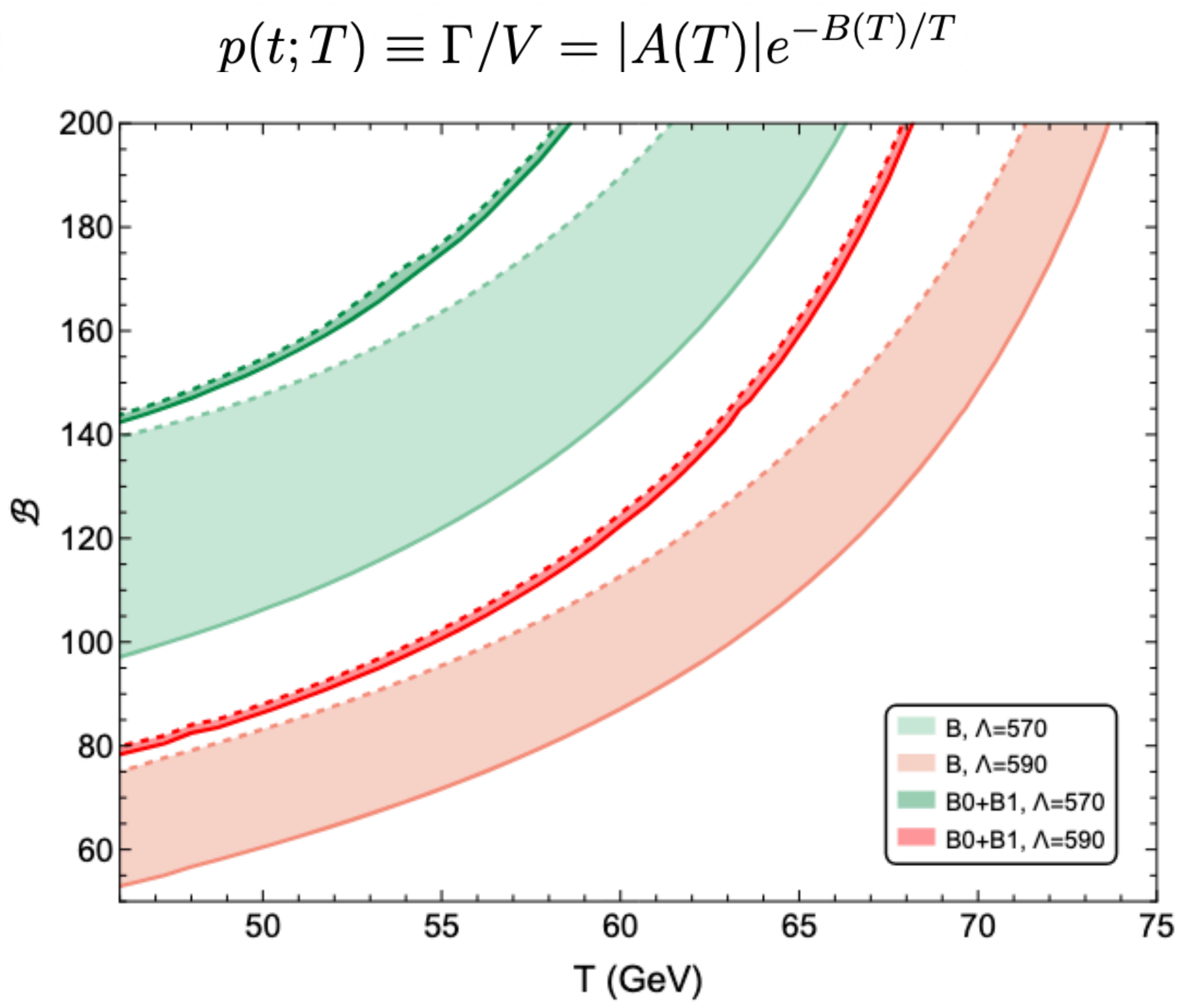


Zhao, Di, **Bian***, Shu*, 2511.22949

Vacuum decay and FOPT



Thermal EFT+MC lattice simulation



Jie Liu, Renhui Qin, Ligong Bian, e-Print: 2512.05565

Classical lattice simulation

PT parameters

Effective action $\rightarrow \beta,\ H_*$

Energy budget $\rightarrow \alpha,\ \kappa(\alpha, v_w)$

Bubble wall dynamics $\rightarrow v_w$

GW power spectrum

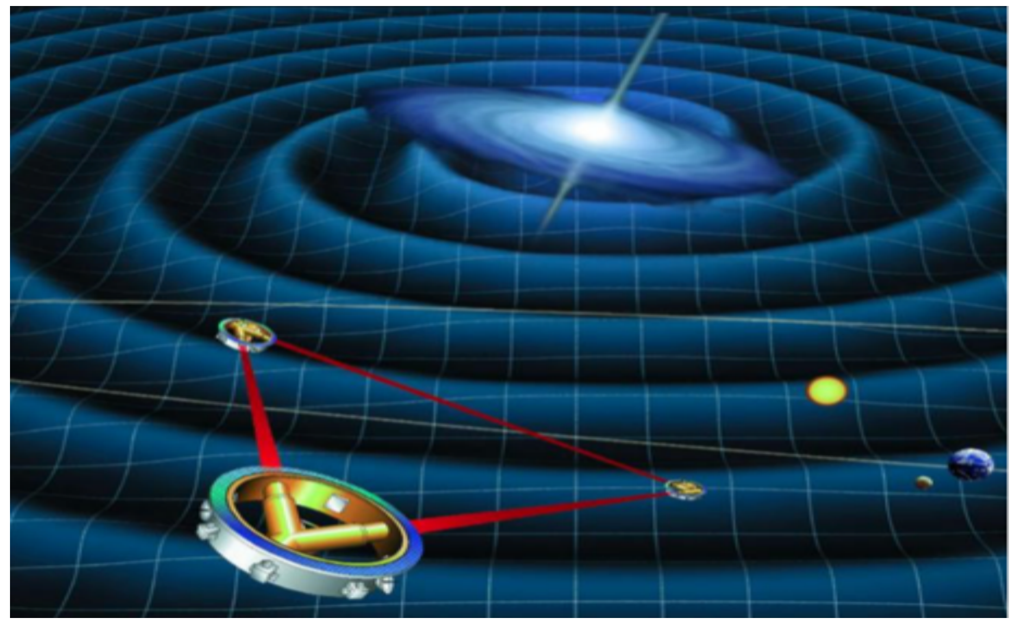
Numerical simulations $\rightarrow$

$h^2\Omega_{GW}(f; H_*, \alpha, \beta, v_w)$

LISA sensitivity

Configuration + noise level $\rightarrow$

$h^2\Omega_{sens}(f)$



Chiara Caprini *et al* JCAP03(2020)024

LISA,TianQin,Taiji,$\cdots$

Wigner function in field theory

$$P_R(T) \equiv \int_R dx |\psi(x,T)|^2 \quad \Longrightarrow \quad P_R(t) \equiv \int_R \mathcal{D}\phi |\Psi(\phi,t)|^2$$

$$W(q,p;t) \sim |\Psi(x,t)|^2 \qquad \Psi(\phi,t) = \langle \phi(x)|\Psi(t)\rangle = \int \mathcal{D}\phi_i \langle \phi|\hat{U}(t|t_0)|\phi_i\rangle\langle\phi_i(\mathbf{x})|\Psi(t_0)\rangle$$

$$W(q,p;t) = \int du e^{-\frac{i}{\hbar}pu}\left\langle q+\frac{u}{2}\right|\hat{\rho}(t)\left|q-\frac{u}{2}\right\rangle \Longrightarrow W[\phi(x),\Pi(x);t] = \int \mathcal{D}\varphi(x)\exp\left[-\frac{i}{\hbar}\int dx\Pi(x)\varphi(x)\right]\times\left\langle\phi(x)+\frac{\varphi(x)}{2}\right|\hat{\rho}(t)\left|\phi(x)-\frac{\varphi(x)}{2}\right\rangle$$

Wigner function: (ϕ, Π) phase space Quasi-probability distribution

$$\hat{\rho}(0) = \frac{1}{Z}e^{-\beta\hat{H}}$$

$$W[\phi(x),\Pi(x);0] = \frac{1}{Z}\int \mathcal{D}\varphi(x)\exp\left[-\frac{i}{\hbar}\int dx\Pi(x)\varphi(x)\right]\times\left\langle\phi(x)+\frac{\varphi(x)}{2}\right|e^{-\beta\hat{H}}\left|\phi(x)-\frac{\varphi(x)}{2}\right\rangle$$

$$\approx \frac{1}{Z}exp\left[-\beta\int dx[\frac{1}{2}\Pi^2(x)+\frac{1}{2}(\nabla\phi(x))^2+V(\phi)]\right]$$

$$Z \equiv Tr\hat{\rho} = \int \mathcal{D}\phi(x)\frac{\mathcal{D}\Pi(x)}{2\pi}W[\phi,\Pi;t]$$

$$W[\phi(x),\Pi(x);t] \gtrless 0$$

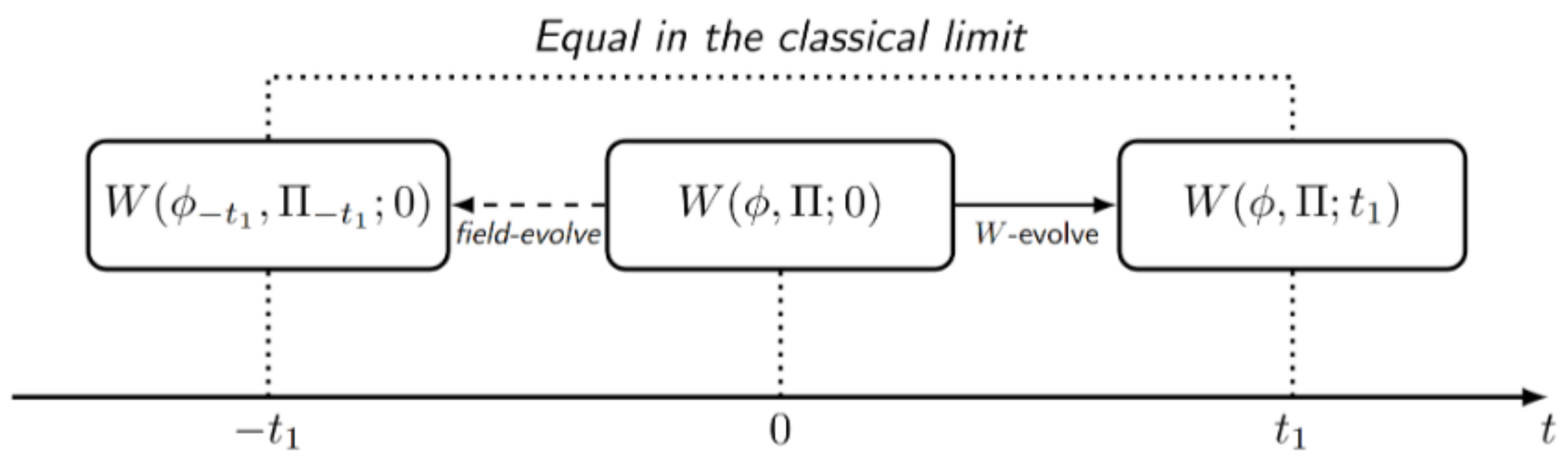
$\phi(x)$ is the real scale field

$\Pi(x) \equiv \frac{\delta\mathcal{L}}{\delta\dot{\phi}(x)}$

$\hat{\rho}(t) = |\Psi(t)\rangle\langle\Psi(t)|$

Stanisław Mrówczyński and Berndt Müller, Phys. Rev. D, 50:7542–7552, Dec 1994.

Wigner function's EOM



$$\hat{L} = \int d^3x \left(\frac{1}{2}\partial^\mu \hat{\phi}\partial_\mu \hat{\phi} - V(\hat{\phi})\right) \qquad \hat{H} = \int d^3x \hat{\mathcal{H}} = \int d^3x \left[\frac{\hat{\Pi}^2}{2} + \frac{(\nabla\hat{\phi})^2}{2} + V(\hat{\phi})\right]$$

$$i\hbar \frac{\partial}{\partial t}\hat{\rho}(t) = [\hat{H}, \hat{\rho}(t)] \qquad \frac{\partial}{\partial t} W[\phi, \pi; t] = -2H \frac{1}{i\hbar} \sin\left(\frac{i\hbar}{2}\Lambda\right) W[\phi, \pi; t]$$

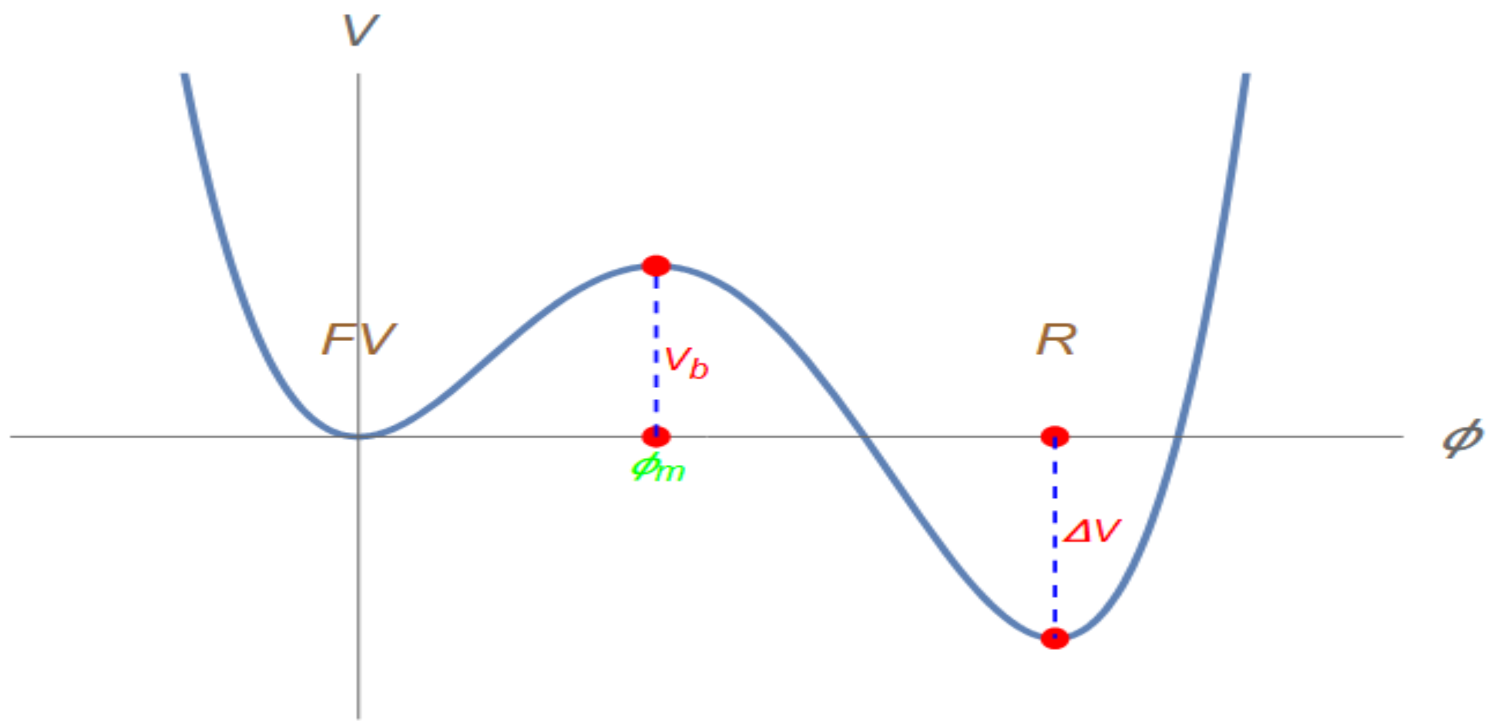
$$\Lambda = \frac{\overleftarrow{\delta}}{\delta \Pi}\frac{\overrightarrow{\delta}}{\delta \phi} - \frac{\overleftarrow{\delta}}{\delta \Phi}\frac{\overrightarrow{\delta}}{\delta \Pi} \text{ is the Poisson bracket operator}$$

Ignore $O(\hbar^2)$

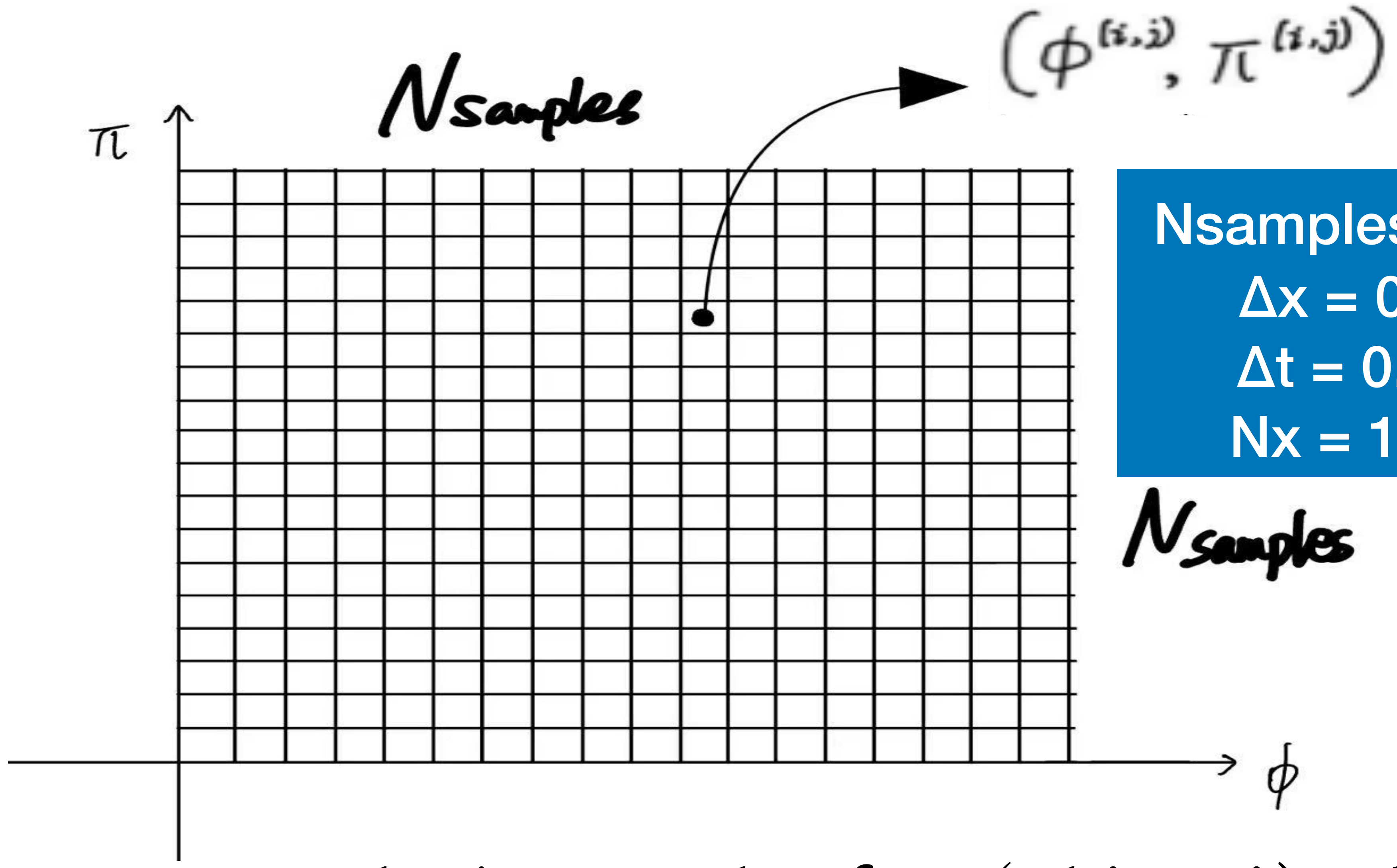
$$\left[\frac{\partial}{\partial t} + \int d^3x \left(\frac{\delta H}{\delta \Pi}\frac{\delta}{\delta \phi} - \frac{\delta H}{\delta \phi}\frac{\delta}{\delta \Pi}\right)\right] W[\phi, \Pi; t] = 0$$

$$W[\phi, \Pi; t] = W[\phi_{-t}, \Pi_{-t}; 0]$$

$$\begin{cases} \dfrac{\delta H}{\delta \Pi} = \dfrac{d\phi}{dt} = \Pi \\ -\dfrac{\delta H}{\delta \phi} = \dfrac{d\Phi}{dt} = \nabla^2 \phi - \dfrac{\delta V(\phi)}{\delta \phi} \end{cases}$$



$$P_{\rm FV}(T) \sim e^{-\Gamma T} \Longrightarrow \Gamma = -\frac{1}{P_{\rm FV}}\frac{d}{dT}P_{\rm FV}$$



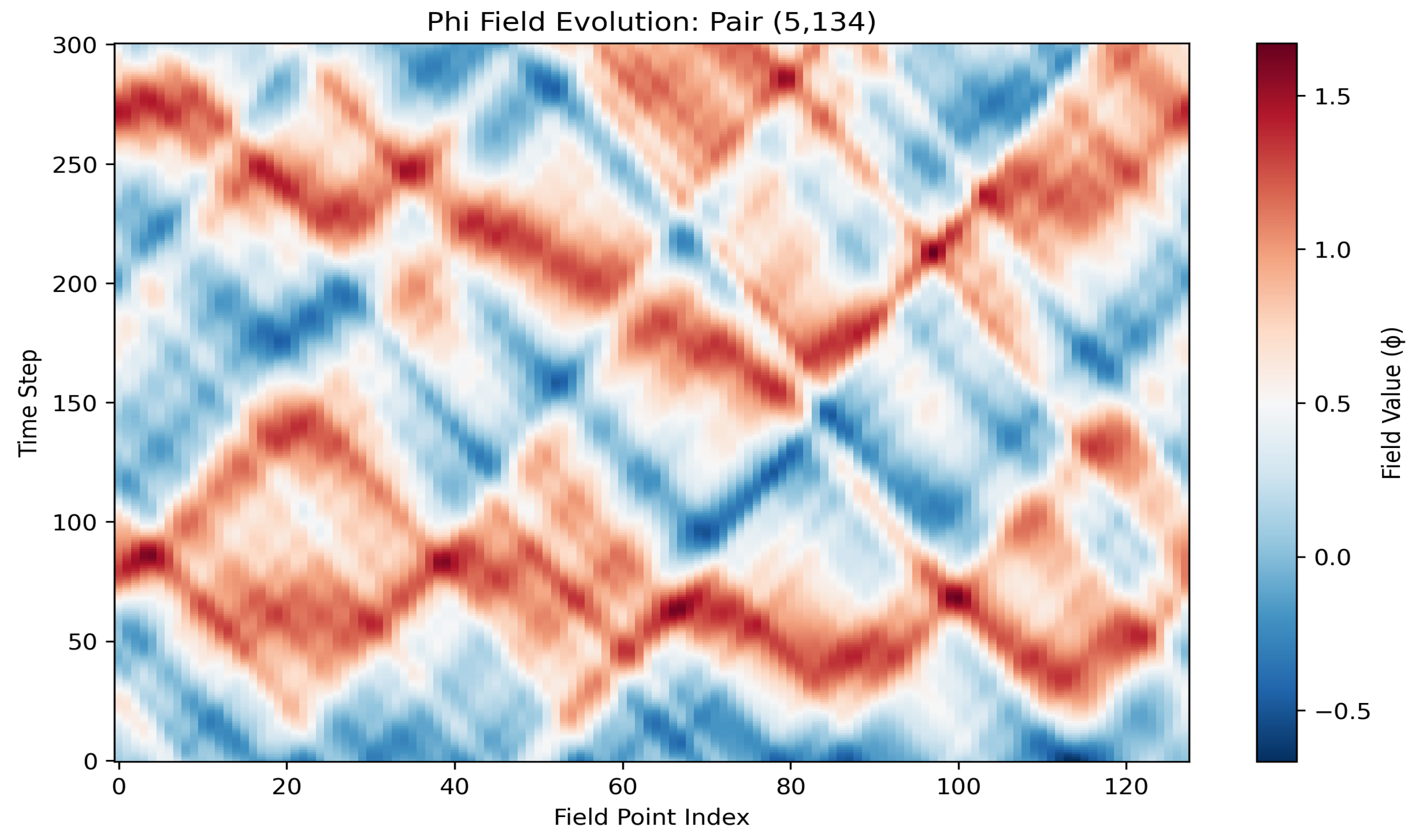
Nsamples = 10^5
Δx = 0.25
Δt = 0. 01
Nx = 1024

Nsamples*Nsamples for (phi, pi) phase space
Summation range: $i, j = 1,2,3 \dots N_{samples}$

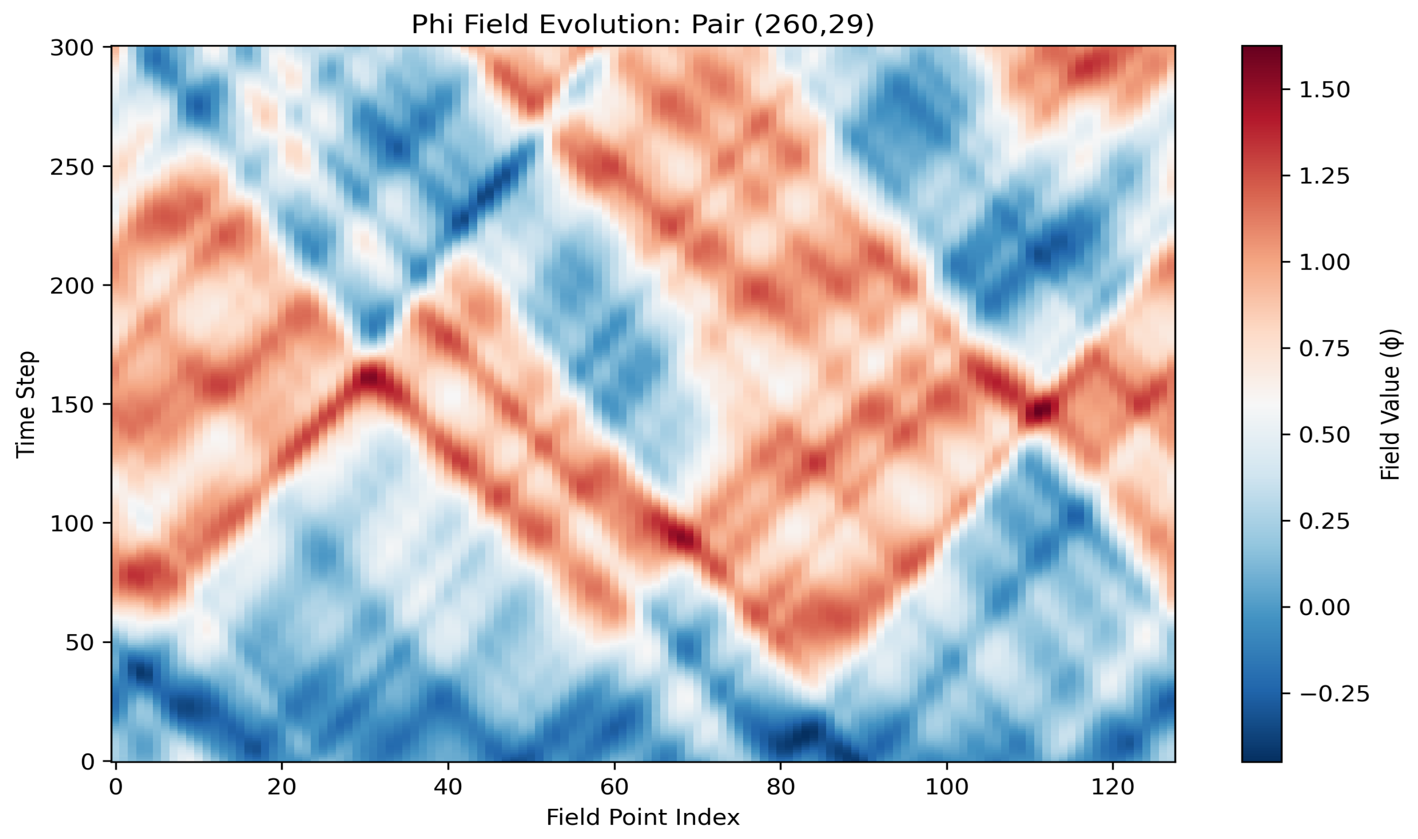
Haiyang Wang, Renhui Qin, **Ligong Bian**, arXiv: 2506.18334

Simulation results

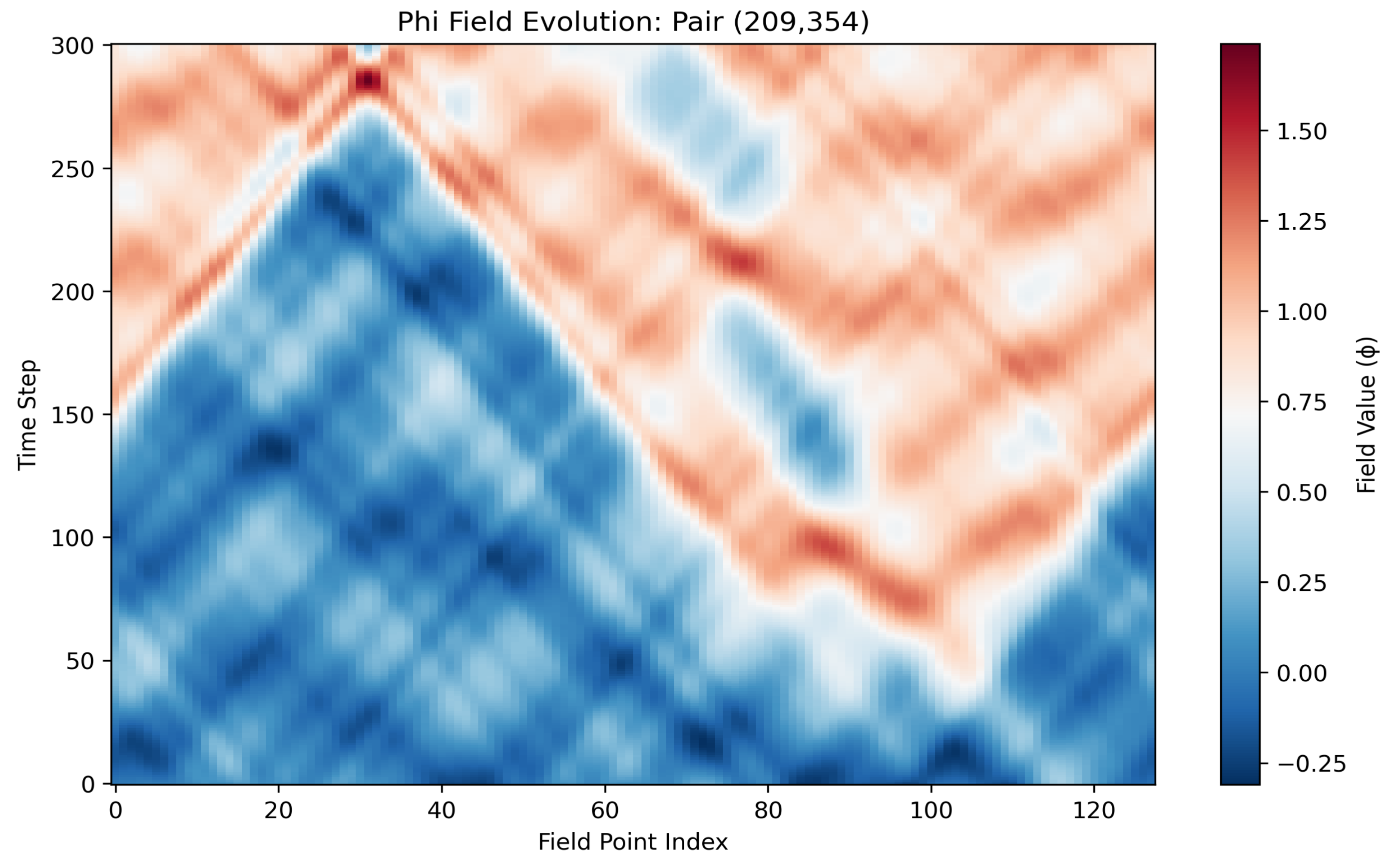
1D field



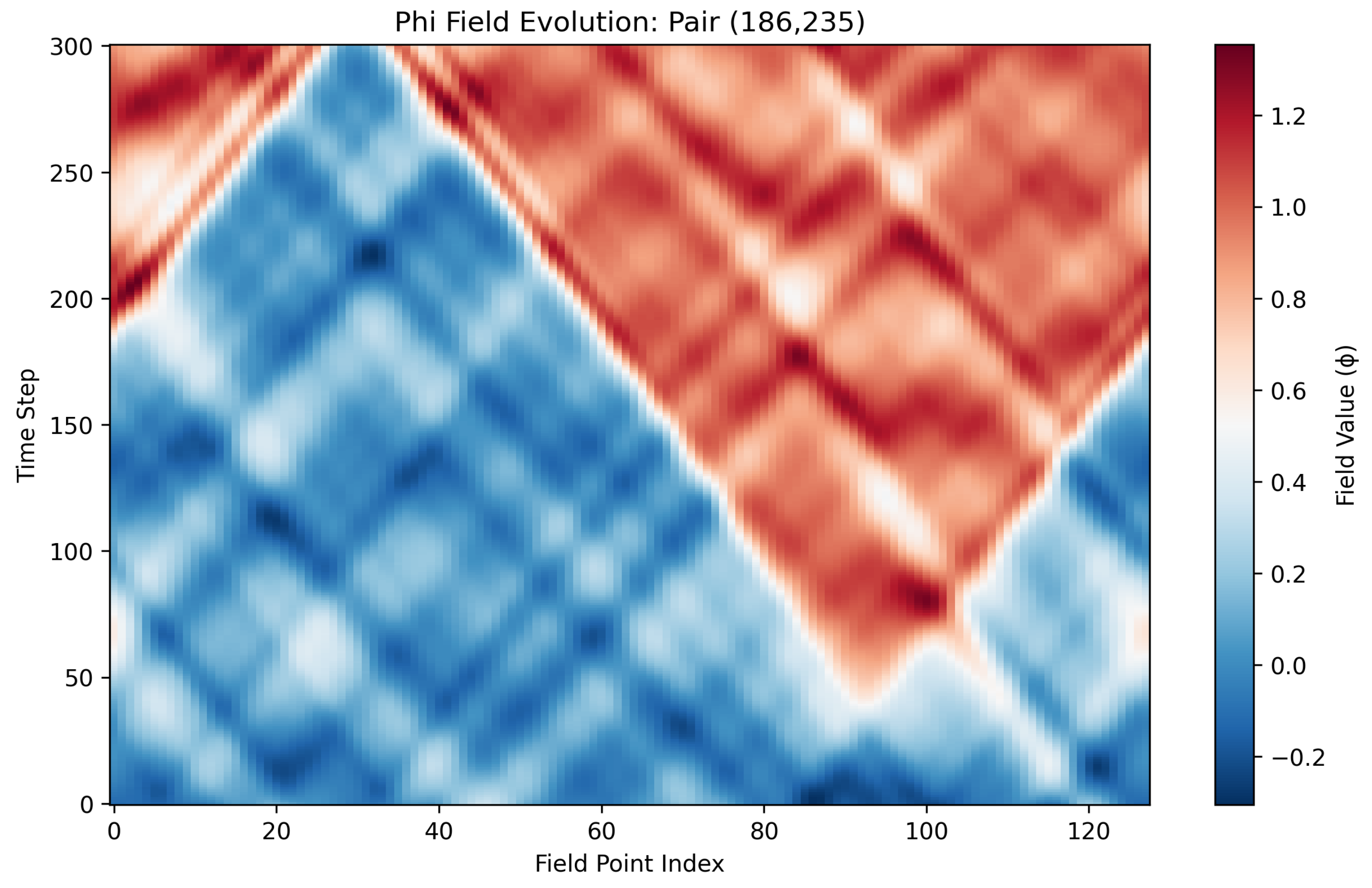
β=0.8



β=1.0

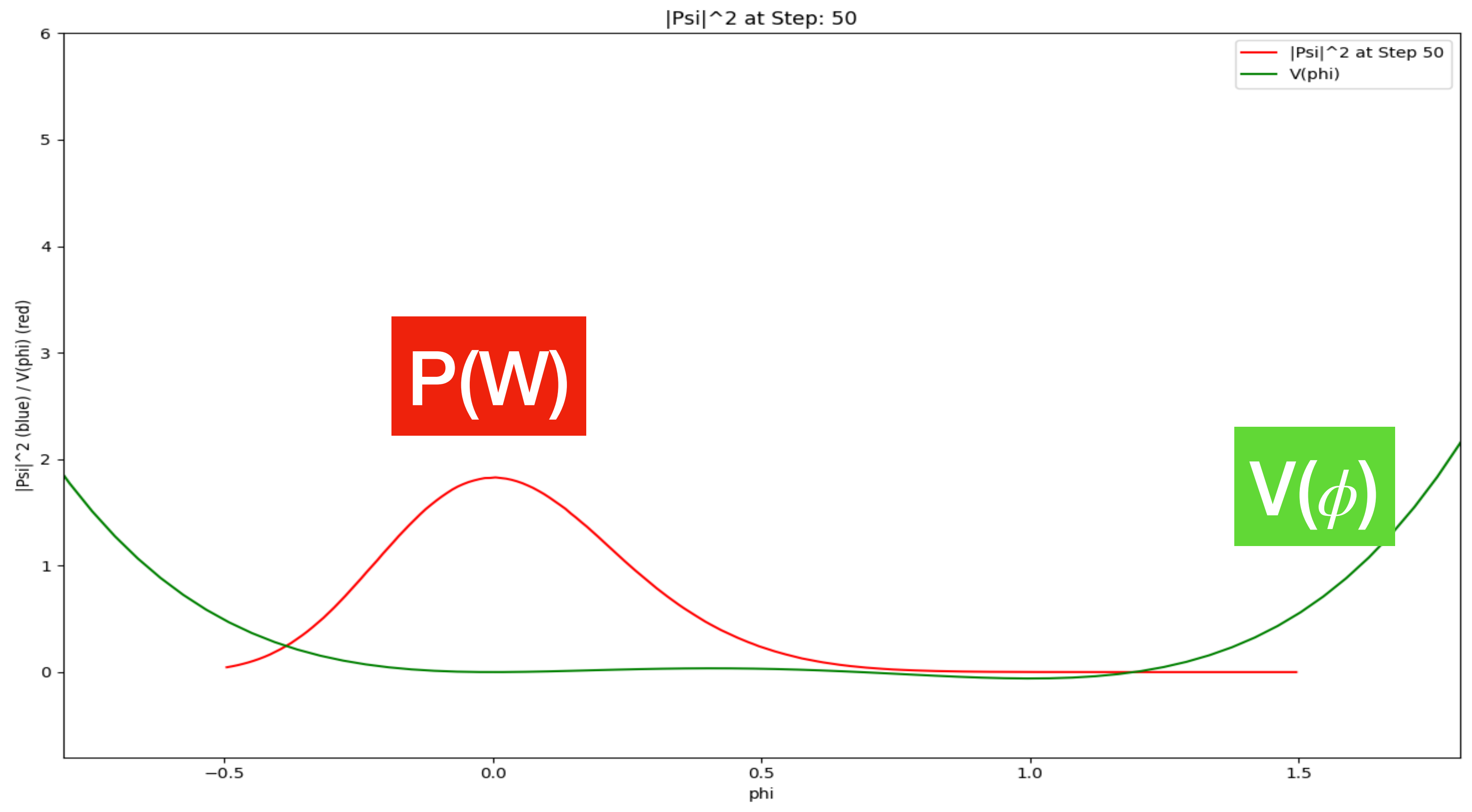


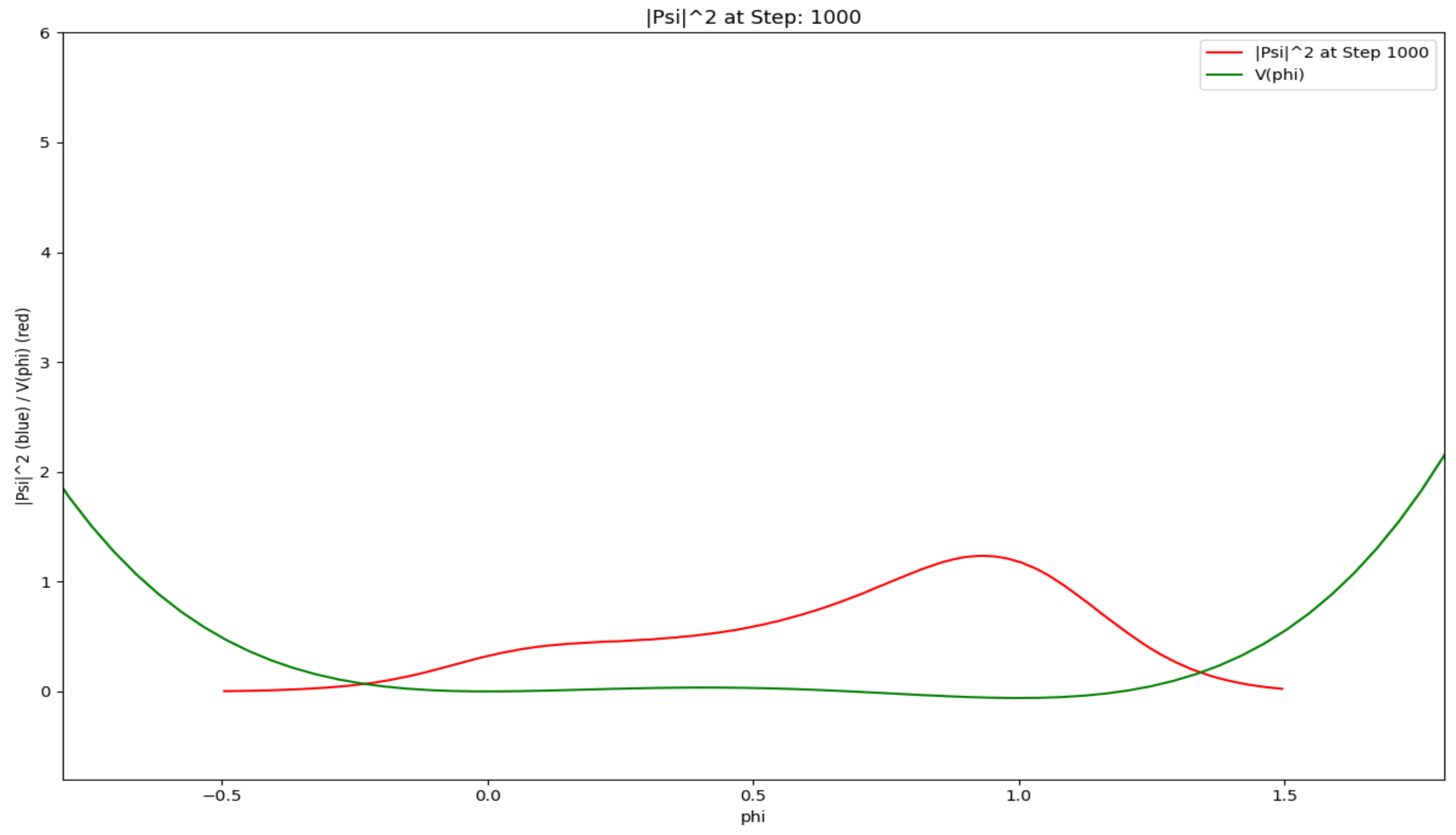
β=1.2



β=1.4

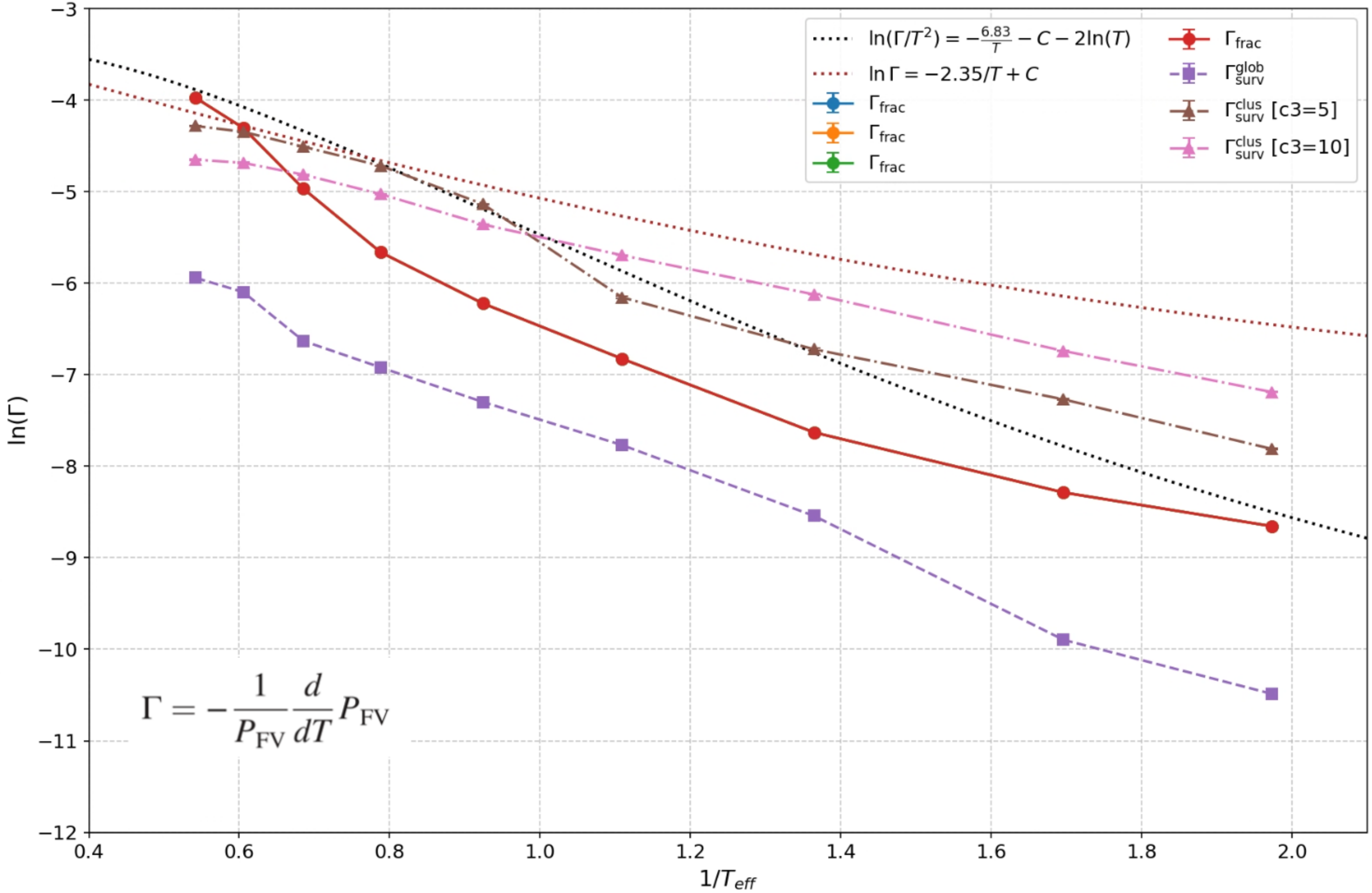
Wave function evolution





$\beta\omega^* = 1$

Simulation results



$$\Gamma = -\frac{1}{P_{\rm FV}}\frac{d}{dT}P_{\rm FV}$$

Lattice electroweak theory-classical simulation

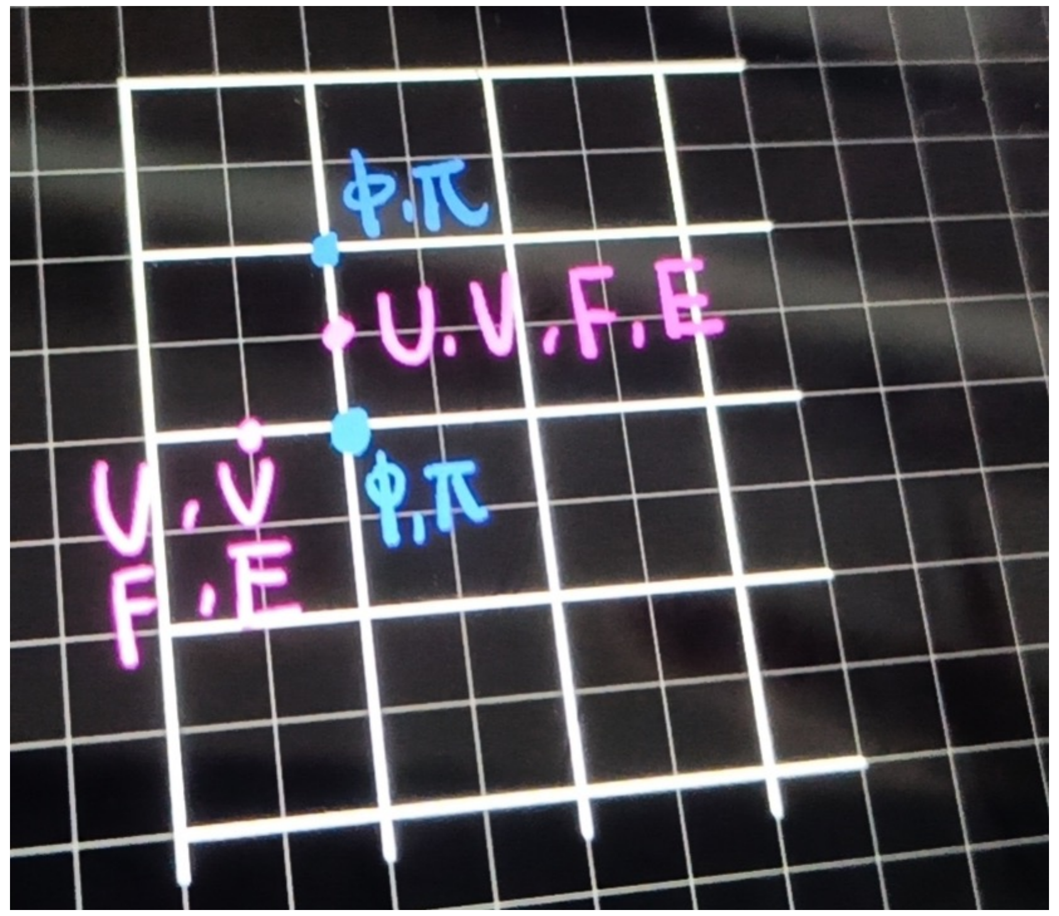
$\Phi(t, x)$: Higgs field doublet defined on sites;

$U_i(t, x)$ and $V_i(t, x)$: SU(2) and U(1) link fields, defined on the link between the neighboring sites x and x + i , $\Phi(t, x)$, $U_i(t, x)$ and $V_i(t, x)$ are defined at time steps t + Δt, t + 2Δt, . . .;

Conjugate momentum fields: $\Pi(t+\Delta t/2, x)$, $F(t+\Delta t/2, x)$ and $E(t+\Delta t/2, x)$, are defined at time steps t + Δt/2, t + 3Δt/2.

$$U_i(t,x) = \exp\left(-\frac{i}{2}g\Delta x\sigma^a W_i^a\right)$$
$$U_0(t,x) = \exp\left(-\frac{i}{2}g\Delta t\sigma^a W_0^a\right)$$
$$V_i(t,x) = \exp\left(-\frac{i}{2}g\Delta x B_i\right)$$
$$V_0(t,x) = \exp\left(-\frac{i}{2}g\Delta t B_0\right).$$

Temporal gauge
$U_0(t, x) = I_2$, $V_0(t, x) = 1$



$$D_i\Phi = \frac{1}{\Delta x}\left[U_i(t,x)V_i(t,x)\Phi(t,x+i) - \Phi(t,x)\right]$$
$$D_0\Phi = \frac{1}{\Delta t}\left[U_0(t,x)V_0(t,x)\Phi(t+\Delta t,x) - \Phi(t,x)\right].$$
$$\Phi(t+\Delta t,x) = \Phi(t,x) + \Delta t\Pi(t+\Delta t/2,x)$$
$$V_i(t+\Delta t,x) = \frac{1}{2}g'\Delta x\Delta t E_i(t+\Delta t/2,x)V_i(t,x)$$
$$U_i(t+\Delta t,x) = g\Delta x\Delta t F_i(t+\Delta t/2,x)U_i(t,x),$$

leapfrog

Di, Wang, Zhou, **Bian***, Cai*,Liu*, Phys.Rev.Lett. 126 (2021) 251102

1st order EWPT simulation-quantum tunneling

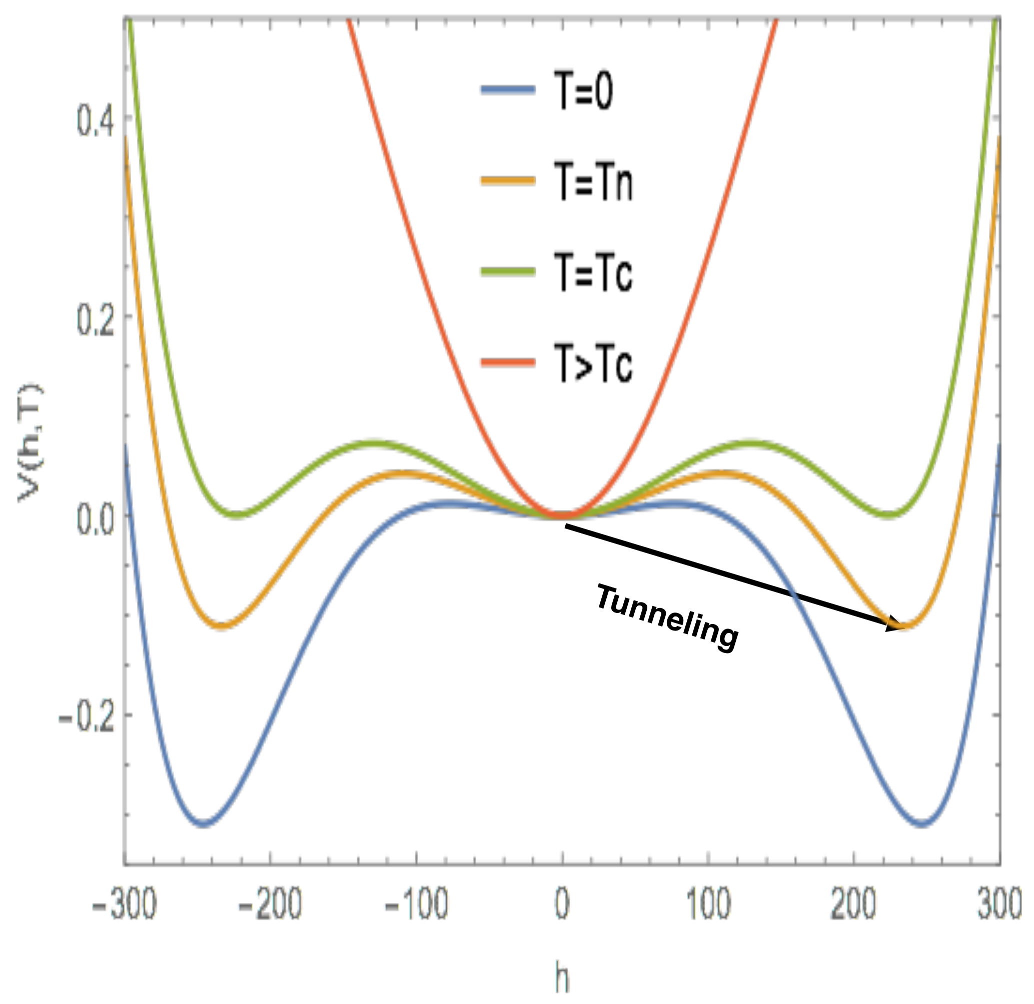
Field basis equation of motion

$$
\begin{aligned}
\partial_0^2\Phi =& D_i D_i \Phi - \frac{dV(\Phi)}{d\Phi}, \\
\partial_0^2 B_i =& -\partial_j B_{ij} + g' \, \mathrm{Im}[\Phi^\dagger D_i \Phi], \\
\partial_0^2 W_i^a =& -\partial_k W_{ik}^a - g\,\epsilon^{abc} W_k^b W_{ik}^c + g\,\mathrm{Im}[\Phi^\dagger \sigma^a D_i \Phi]\,, \\
\partial_0 \partial_j B_j - & g' \, \mathrm{Im}[\Phi^\dagger \partial_0 \Phi] = 0, \\
\partial_0 \partial_j W_j^a + & g\,\epsilon^{abc} W_j^b \partial_0 W_j^c - g\,\mathrm{Im}[\Phi^\dagger \sigma^a \partial_0 \Phi] = 0.
\end{aligned}
$$

Lattice implementation

$$
\begin{aligned}
\Pi(t+\Delta t/2, x) =& \Pi(t-\Delta t/2, x) + \Delta t\Big\{\frac{1}{\Delta x^2}\sum_i [U_i(t,x)V_i(t,x)\Phi(t,x+i) \\
& - 2\Phi(t,x) + U_i^\dagger(t,x-i)V_i^\dagger(t,x-i)\Phi(t,x-i)] - \frac{\partial U}{\partial \Phi^\dagger}\Big\} \\
\mathrm{Im}[E_k(t+\Delta t/2,x)] =& \mathrm{Im}[E_k(t-\Delta t/2,x)] + \Delta t\Big\{\frac{g'}{\Delta x}\mathrm{Im}[\Phi^\dagger(t,x+k)U_k^\dagger(t,x)V_k^\dagger(t,x)\Phi(t,x)] \\
& - \frac{2}{g'\Delta x^3}\sum_i \mathrm{Im}[V_k(t,x)V_i(t,x+k)V_k^\dagger(t,x+i)V_i^\dagger(t,x) \\
& + V_i(t,x-i)V_k(t,x)V_i^\dagger(t,x+k-i)V_k^\dagger(t,x-i)]\Big\} \\
\mathrm{Tr}[i\sigma^m F_k(t+\Delta t/2,x)] =& \mathrm{Tr}[i\sigma^m F_k(t-\Delta t/2,x)] + \Delta t\Big\{\frac{g}{\Delta x}\mathrm{Re}[\Phi^\dagger(t,x+k)U_k^\dagger(t,x)V_k^\dagger(t,x)i\sigma^m\Phi(t,x)] \\
& - \frac{1}{g\Delta x^3}\sum_i \mathrm{Tr}[i\sigma^m U_k(t,x)U_i(t,x+k)U_k^\dagger(t,x+i)U_i^\dagger(t,x) \\
& + i\sigma^m U_k(t,x)U_i^\dagger(t,x+k-i)U_k^\dagger(t,x-i)U_i(t,x-i)]\Big\},
\end{aligned}
$$

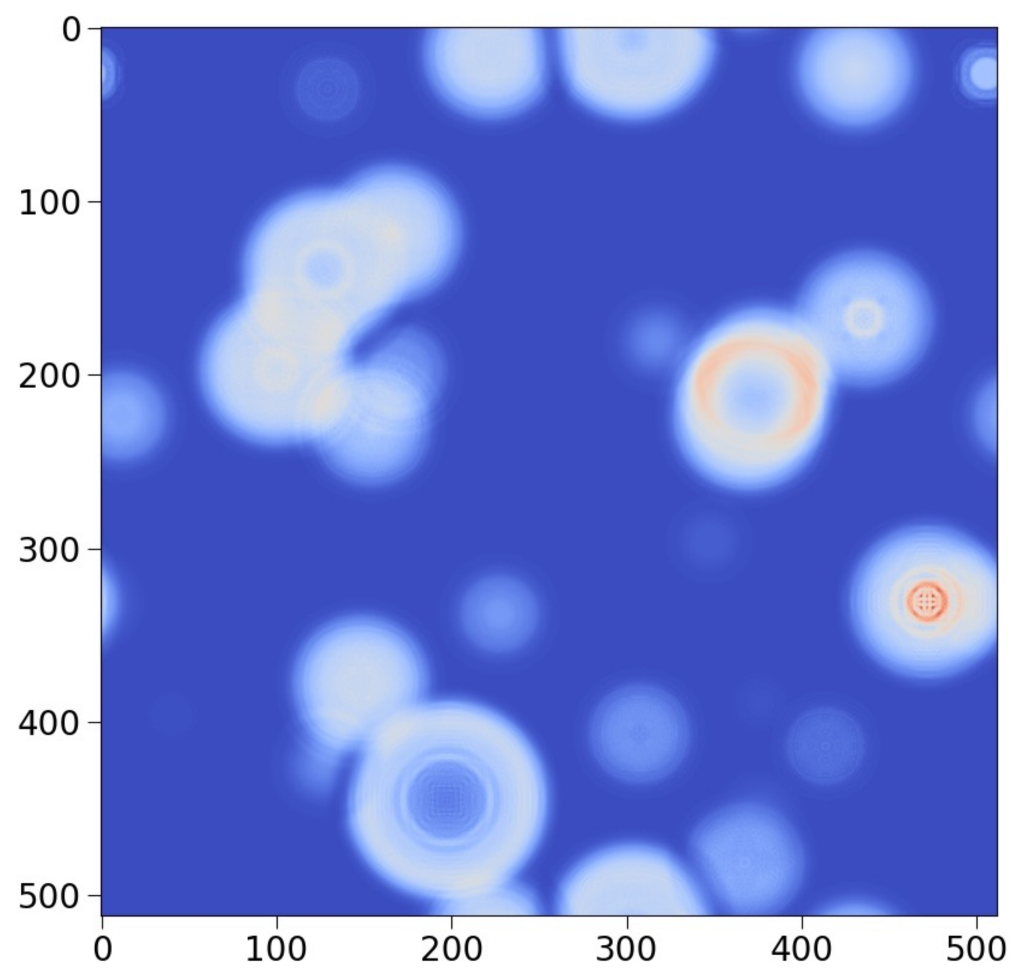
Finite-T Veff



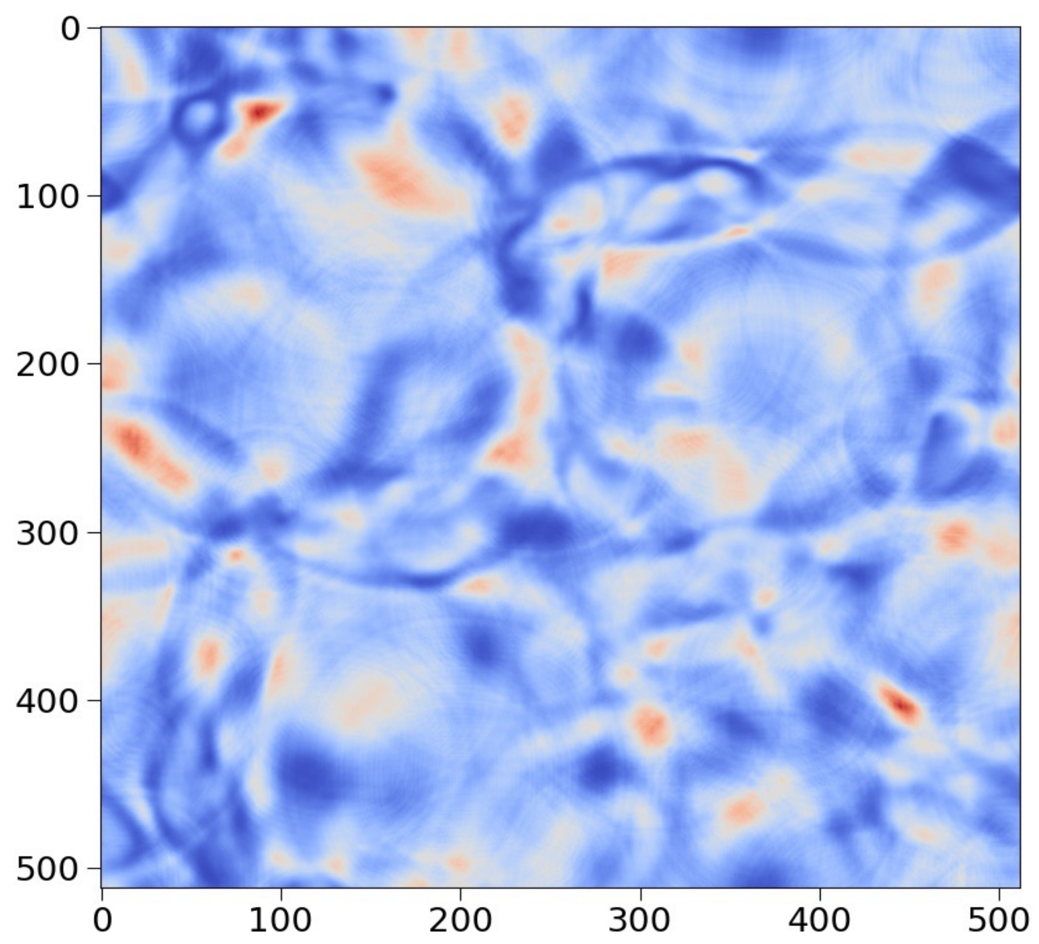
Finite-T calculation

temperature (T_*)
duration (β)
strength (α)

Nucleation



Expansion&Percolation



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1st Phase transition + Magnetic field = Matter-antimatter asymmetry ?

Higgs doublet; U(1) gauge field strength

$$\mathcal{L} = (D_\mu\Phi)^\dagger(D^\mu\Phi) - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}Y_{\mu\nu}Y^{\mu\nu} - \frac{1}{2}Y^{\rm ex}_{\mu\nu}Y^{\mu\nu} + \mathcal{V}(\Phi)$$

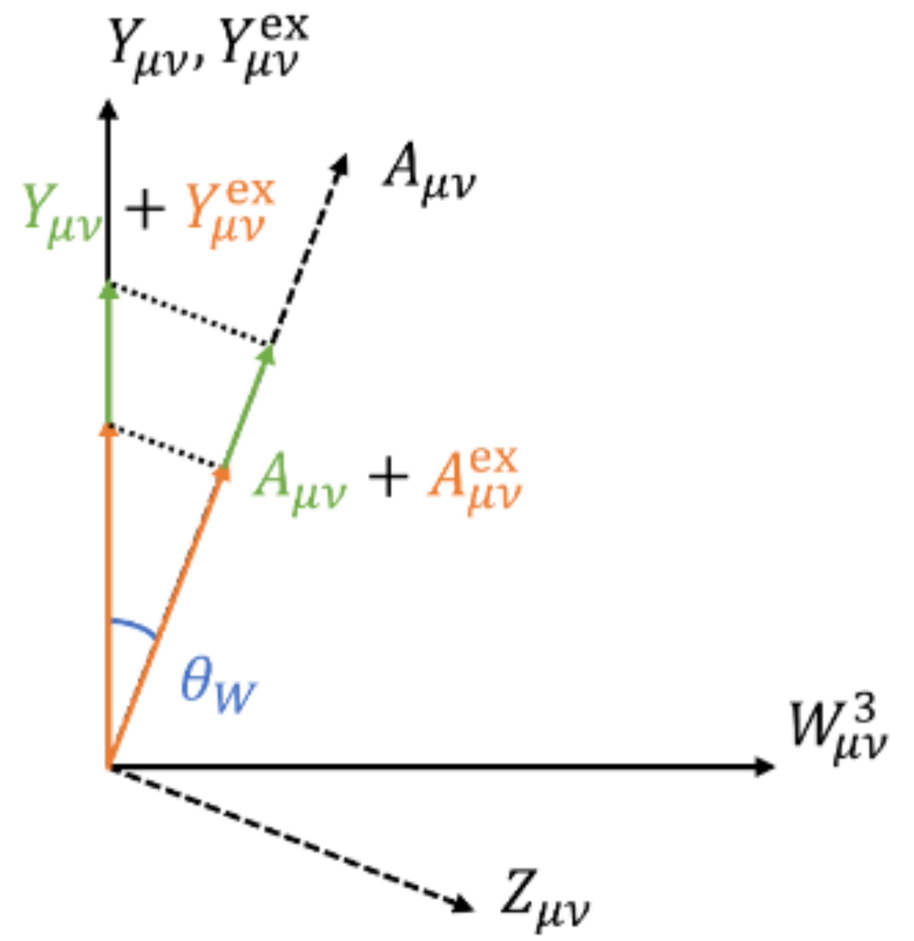
SU(2) gauge field strength; External U(1) gauge field strength. Dose not change with time

$$\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)_\mathrm{Y} \quad \Phi,\ W^a_\mu,\ Y_\mu \xrightarrow{\text{Spontaneous symmetry breaking}} \mathrm{U}(1)_\mathrm{em} \quad A_\mu,\ H,\ Z_\mu,\ W^\pm_\mu$$

$$D_\mu = \partial_\mu - \mathrm{i}g\frac{\sigma^a}{2}W^a_\mu - \mathrm{i}g'\frac{1}{2}(Y_\mu + Y^{\rm ex}_\mu)$$

is the covariant derivative.

$$\mathcal{V}(\Phi) = -\mu^2(\Phi^\dagger\Phi - \eta^2) + A(\Phi^\dagger\Phi-\eta^2)^{3/2} + \lambda(\Phi^\dagger\Phi - \eta^2)^2$$



$$\partial_\mu j^\mu_\mathrm{B} = N_\mathrm{g}\left[\frac{g^2}{16\pi^2}\mathrm{Tr}\left(W_{\mu\nu}\tilde{W}^{\mu\nu}\right) - \frac{g'^2}{32\pi^2}Y_{\mu\nu}\tilde{Y}^{\mu\nu}\right]$$

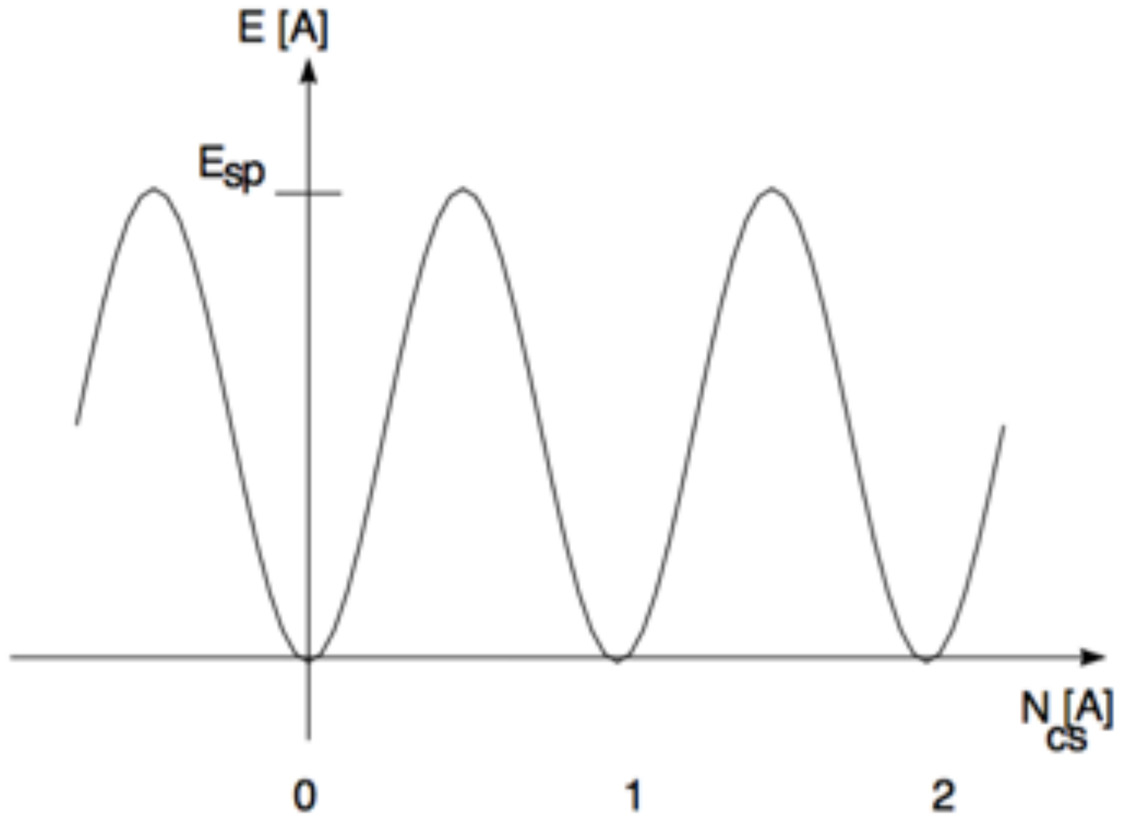
Adler, PR 177, 2426 (1969)
Bell, Jackiw, NCA 60, 47 (1969)
't Hooft, PRL 39, 8 (1976)
't Hooft, PRD 14, 3432 (1976)

$$\Delta n_\mathrm{B} = n_\mathrm{B}(t) - n_\mathrm{B}(0) = N_\mathrm{g}\frac{\Delta N_\mathrm{CS}(t)}{V} = N_\mathrm{g}\frac{N_\mathrm{CS}(t) - N_\mathrm{CS}(0)}{V}$$

Number of families of fermions $N_\mathrm{g} = 3$

Chern-Simons Number density:

$$\frac{N_\mathrm{CS}(t)}{V} = \frac{1}{V}\frac{1}{32\pi^2}\varepsilon^{ijk}\int \mathrm{d}^3x\left[-g'^2(Y_i + Y^{\rm ex}_i)(Y_{jk} + Y^{\rm ex}_{jk}) + g^2\left(W^a_iW^a_{jk} - \frac{g}{3}\varepsilon^{abc}W^a_iW^b_jW^c_k\right)\right]$$



Helicity density:

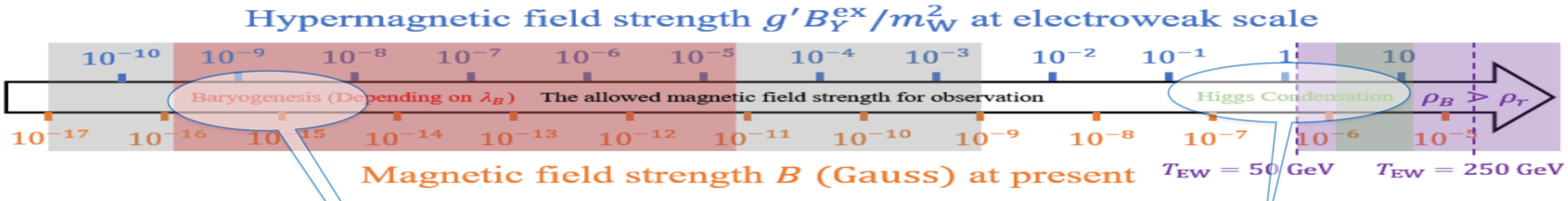
$$h_Y = \frac{H_Y}{V} = \frac{1}{V}\varepsilon^{ijk}\int \mathrm{d}x^3\,(Y_i + Y^{\rm ex}_i)(Y_{jk} + Y^{\rm ex}_{jk}) = \frac{1}{V}\int \mathrm{d}x^3\,(\boldsymbol{Y} + \boldsymbol{Y}^{\rm ex})\cdot(\boldsymbol{B}_Y + \boldsymbol{B}^{\rm ex}_Y)$$

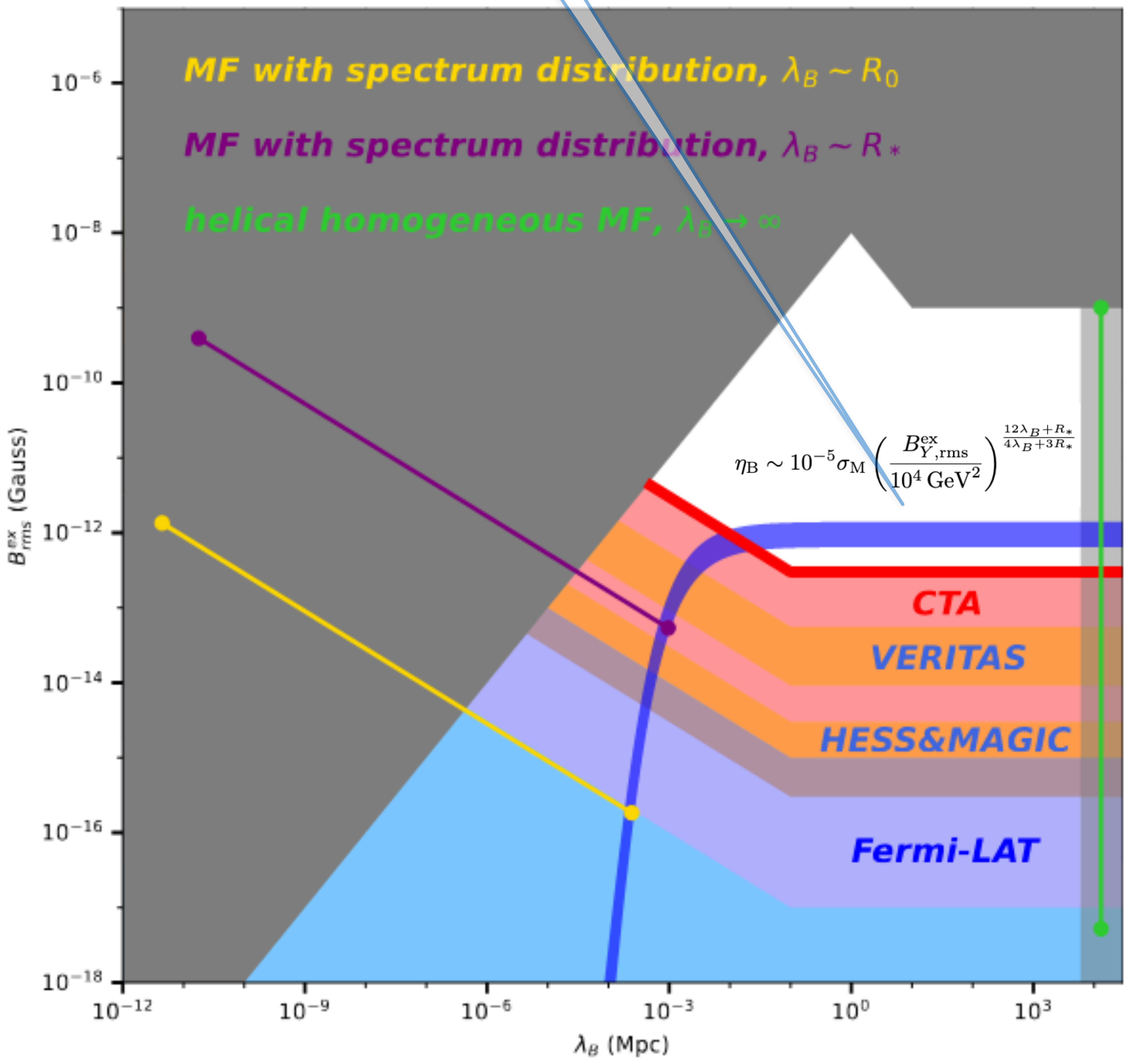
Net Baryon density:

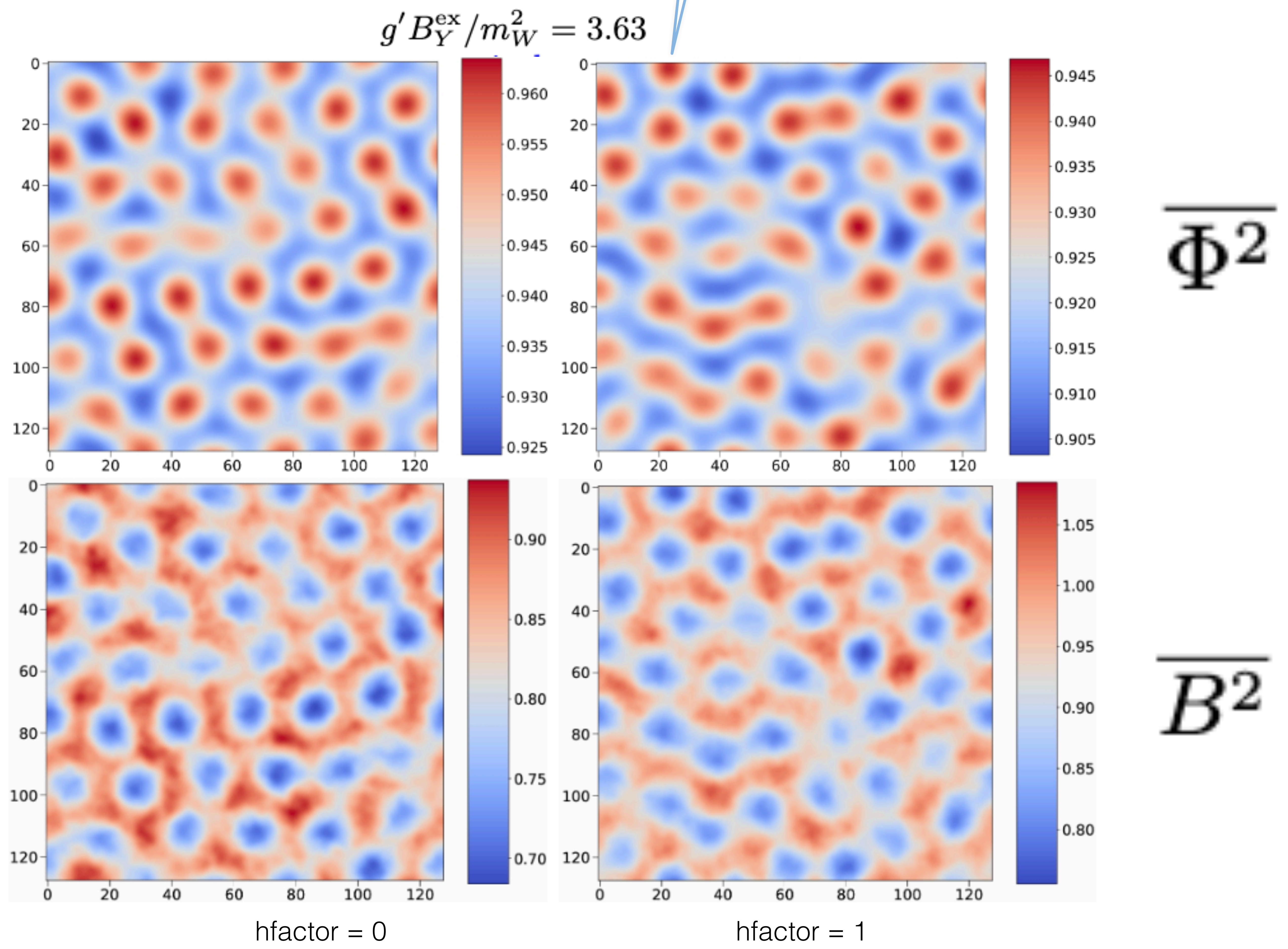
$$\eta_\mathrm{B}(t) = \frac{n_\mathrm{B}}{s} = \frac{45N_\mathrm{g}}{2\pi^2 g_{*S}T^3}\frac{\Delta N_\mathrm{CS}(t)}{V}$$

Di, **Bian***, Cai*, 2409.16124, 2508.07416

Bubble collision and PMF







Di, **Bian***, Cai*,2409.16124, 2508.07416

❖ Summary

- GW of ALP particles in the DFSZ case can be probed by GW detectors
- GW of Axion/ALP dark matter scenario in KSVZ cannot be probed
- Direct measurement of the vacuum decay rate at finite temperature
- Bubble dynamics prefer PMF and GW production from EW lattice

❖ Future

- Dark matter and PT
- Topological defects: Magnetic monopoles, cosmic strings, domain walls
- Magnetic fields, baryogenesis, and Axion dynamics

Thanks

谢谢！

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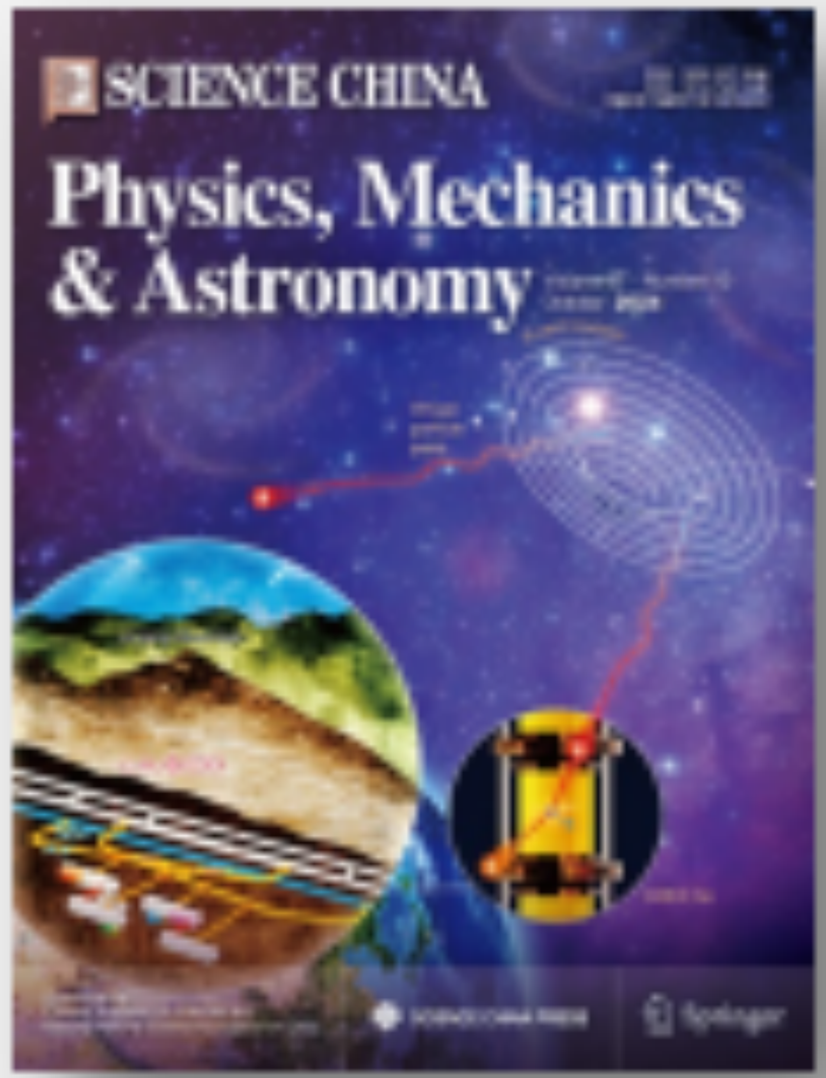

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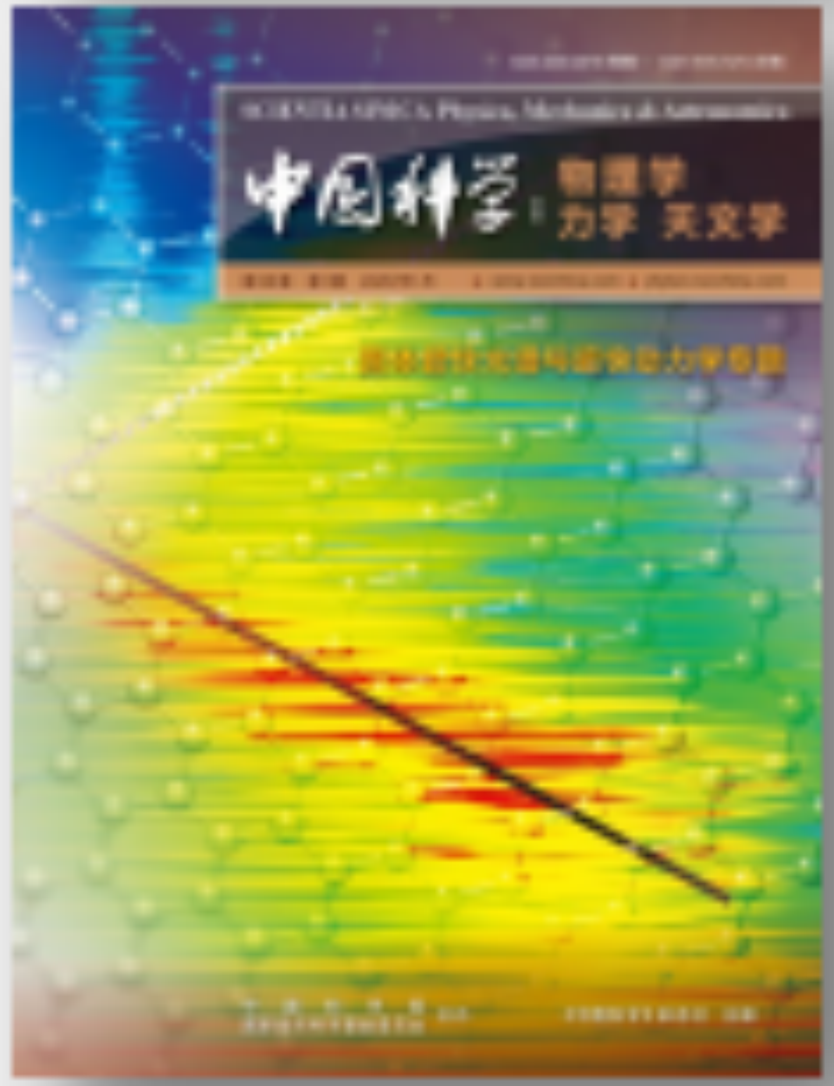

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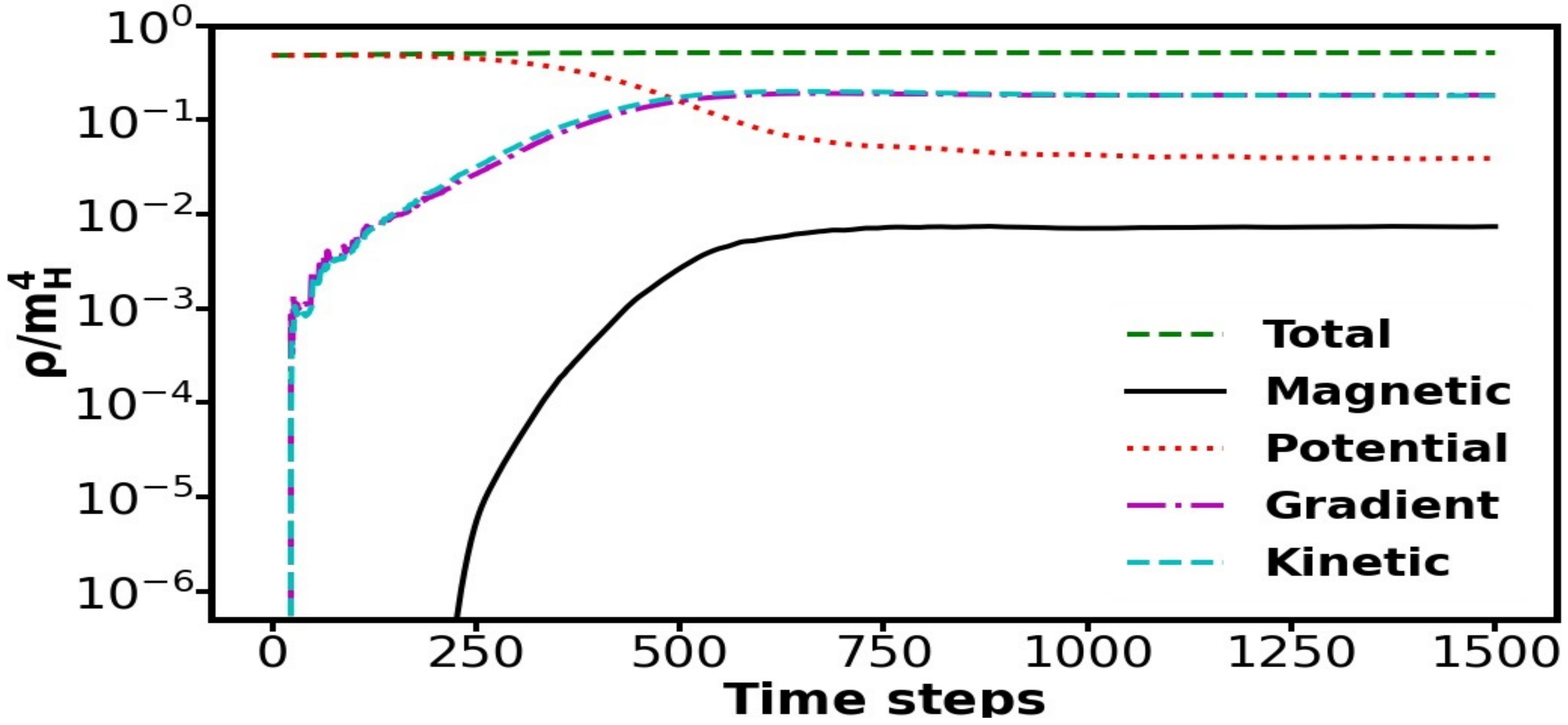
Bubble collision and GWs generation

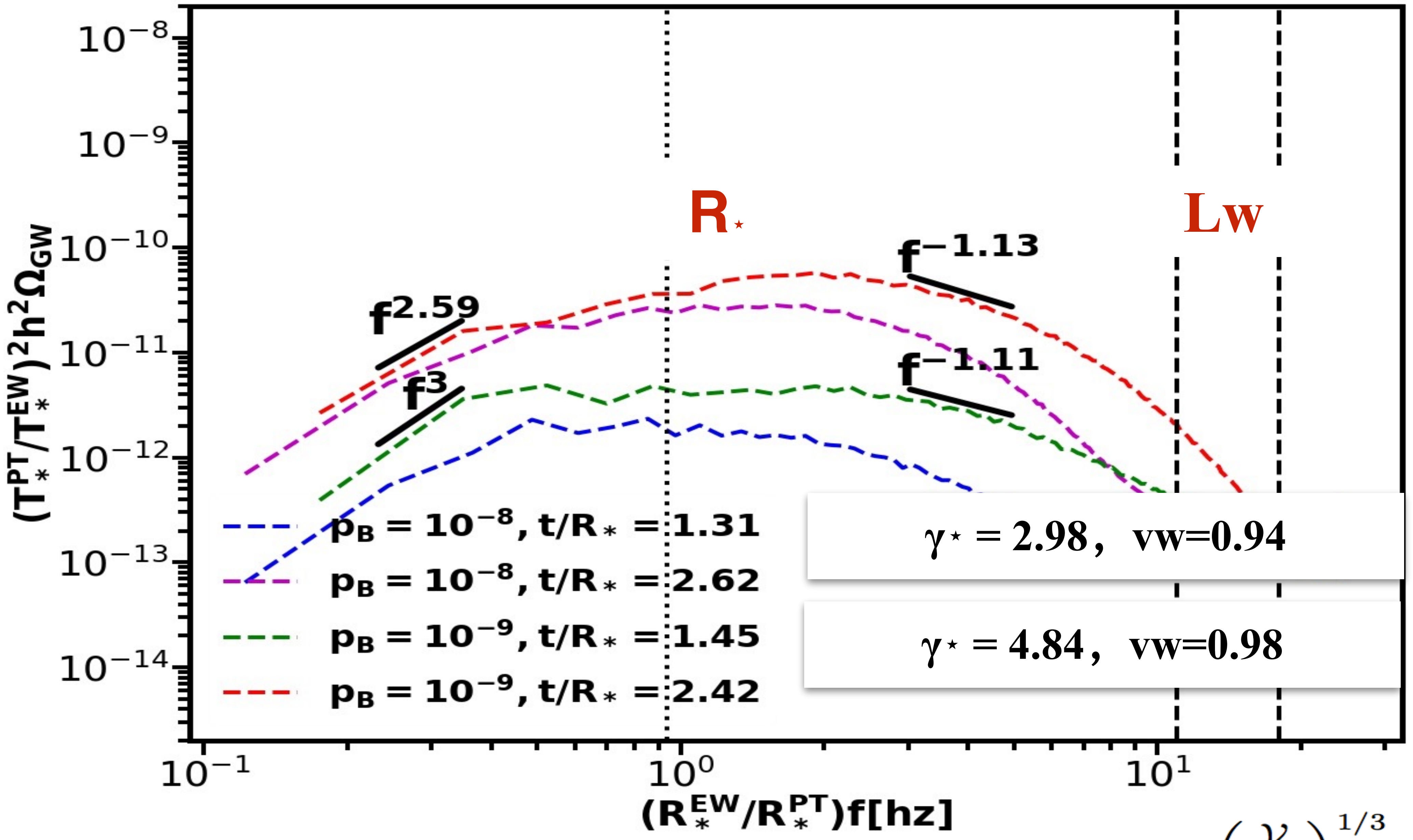
$$\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{\mathrm{TT}}$$

$$T_{\mu\nu} = \partial_\mu \Phi^\dagger \partial_\nu \Phi - g_{\mu\nu}\frac{1}{2}\mathrm{Re}[(\partial_i \Phi^\dagger \partial^i \Phi)^2]$$

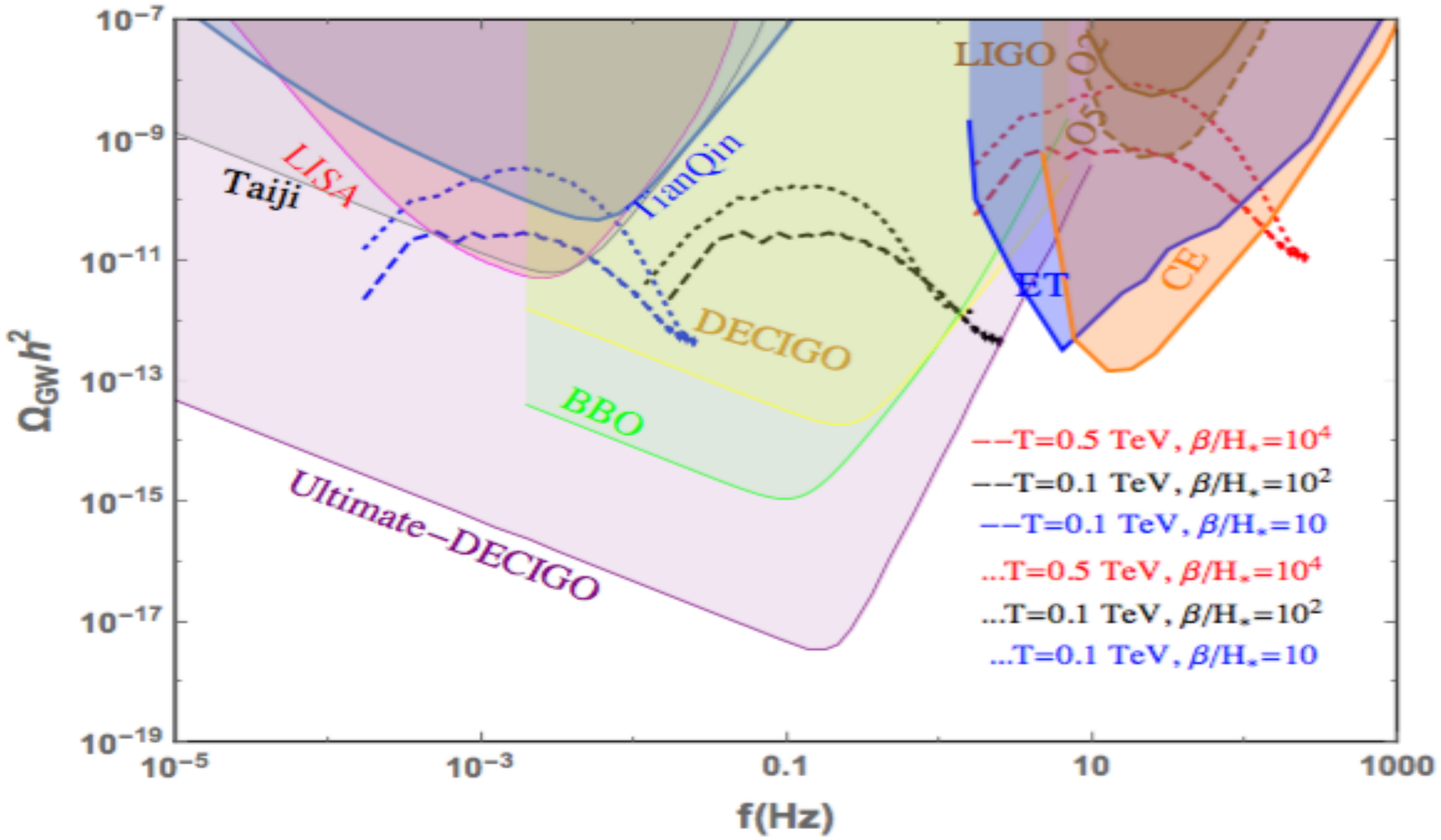
$$\langle \dot{h}_{ij}^{TT}(\mathbf{k},t)\dot{h}_{ij}^{TT}(\mathbf{k}',t)\rangle = P_{\dot{h}}(\mathbf{k},t)(2\pi)^3\delta(\mathbf{k}+\mathbf{k}')$$

$$\frac{d\Omega_{\mathrm{gw}}}{d\ln(k)} = \frac{1}{32\pi G\rho_c}\frac{k^3}{2\pi^2}P_{\dot{h}}(\mathbf{k},t)$$





$\gamma_* = R_* /(2Rc)$ $\quad R_* = \left(\frac{\mathcal{V}}{N_b}\right)^{1/3}$



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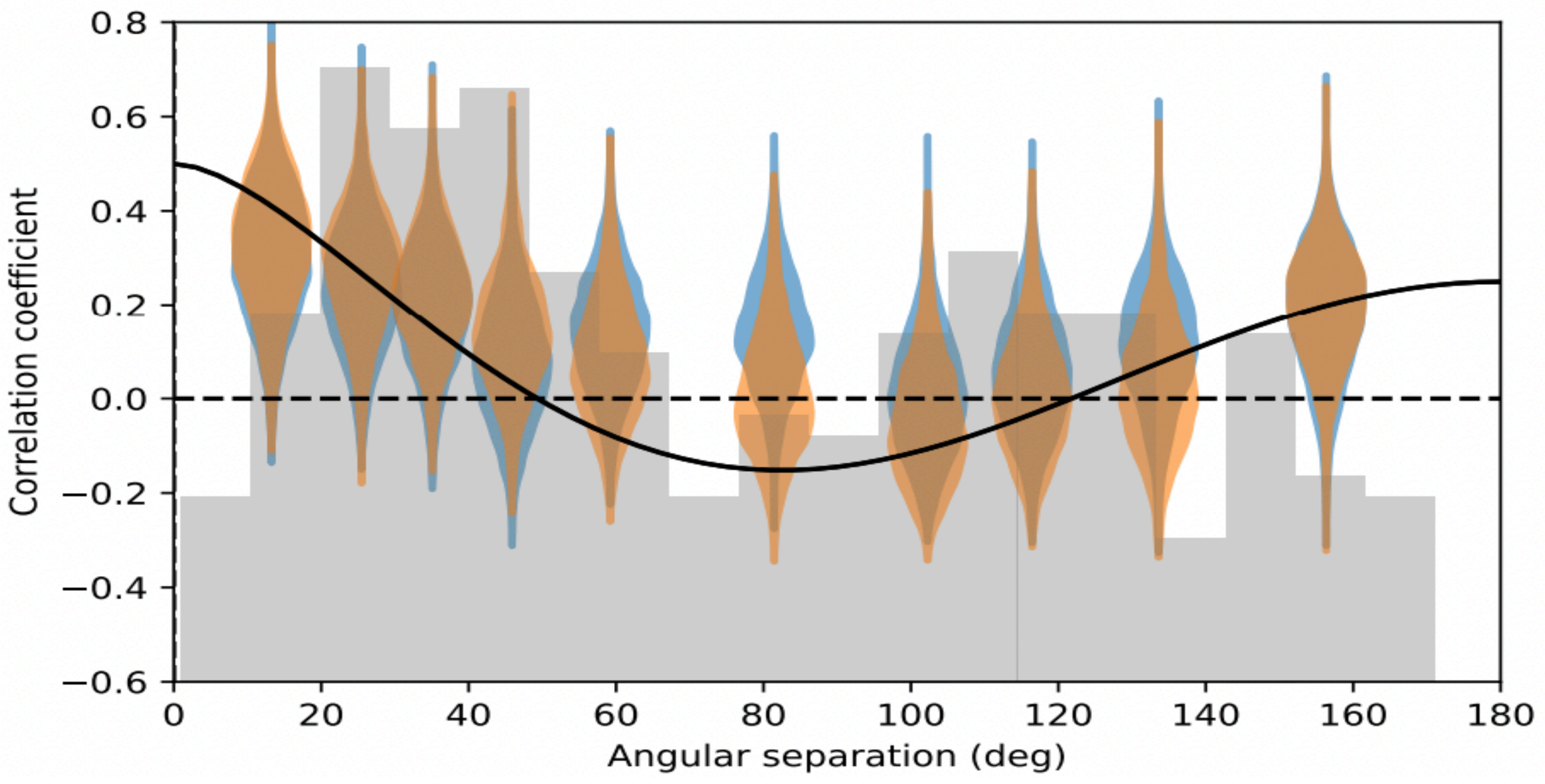
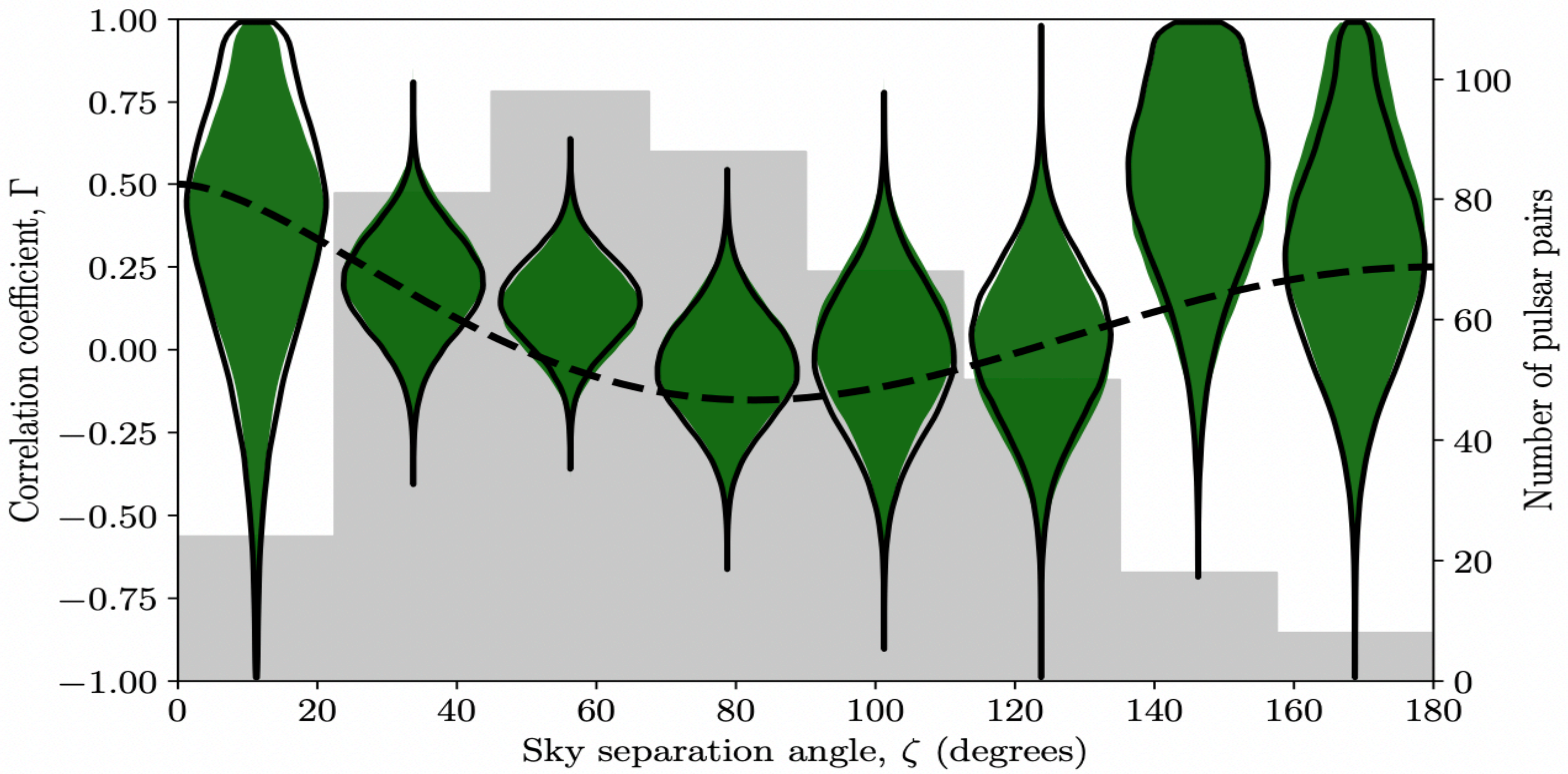
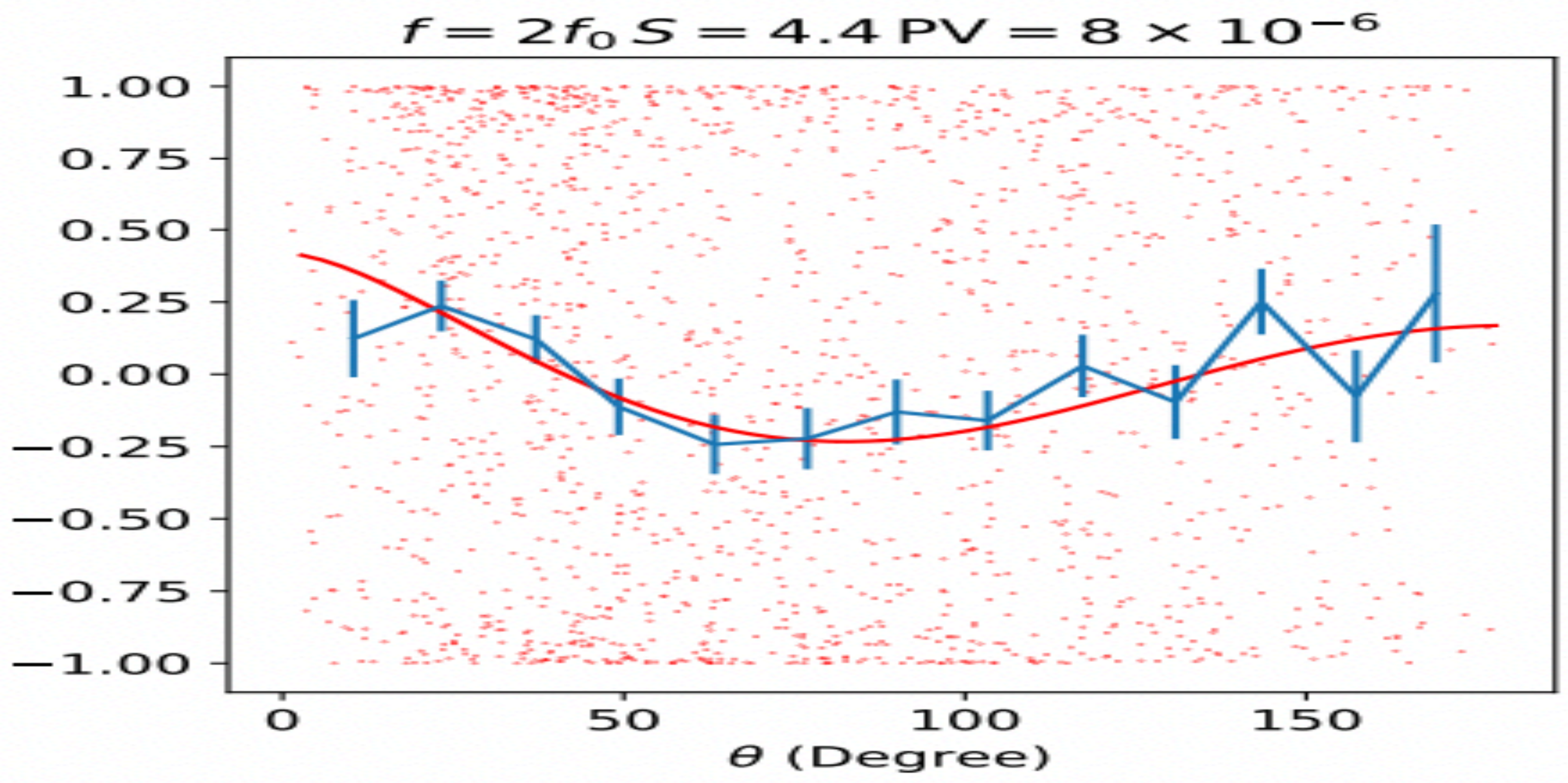
New dataset from PTAs



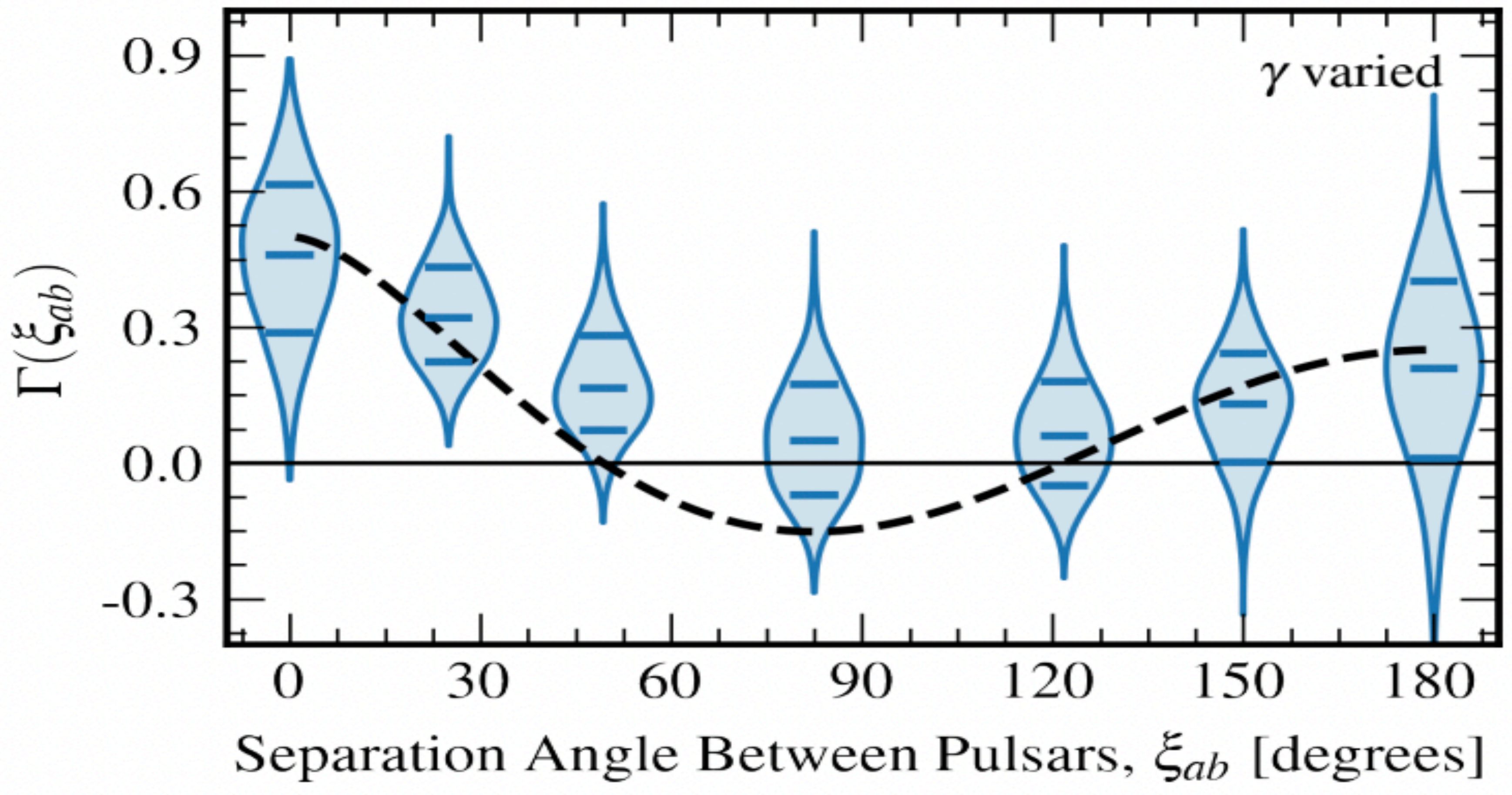
EPTA,2306.16214



PPTA,2306.16215

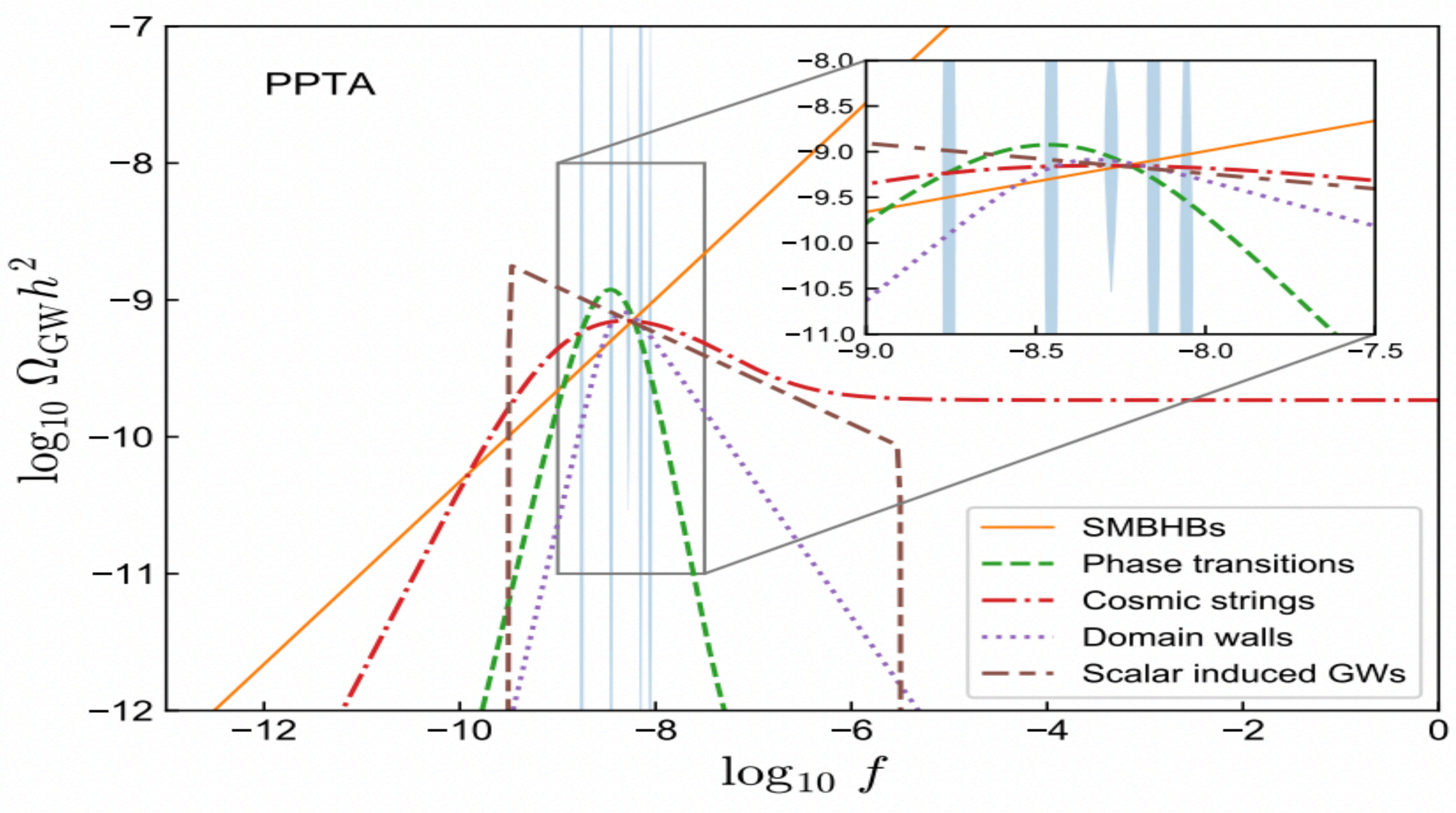


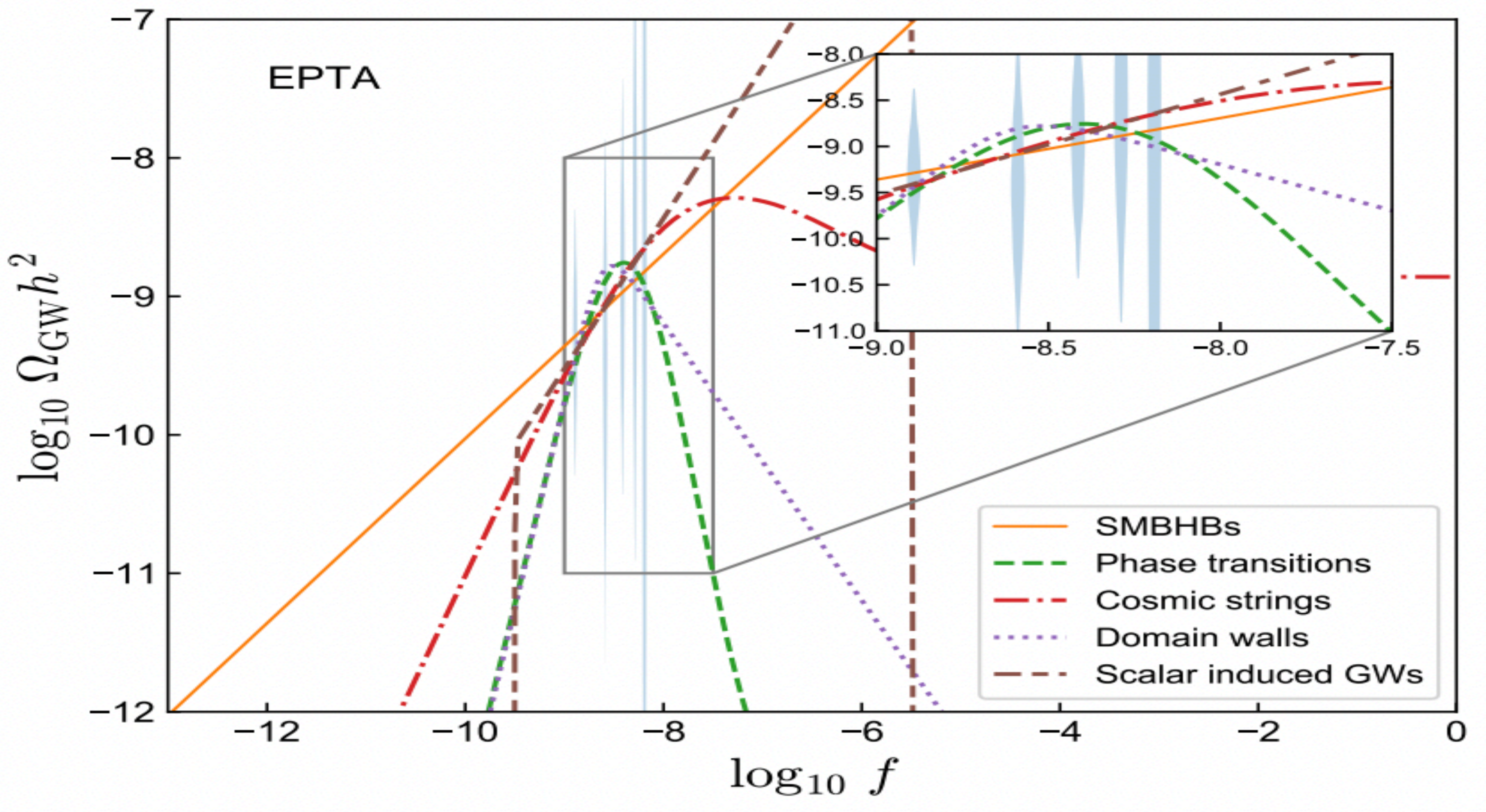
CPTA ,2306.16216

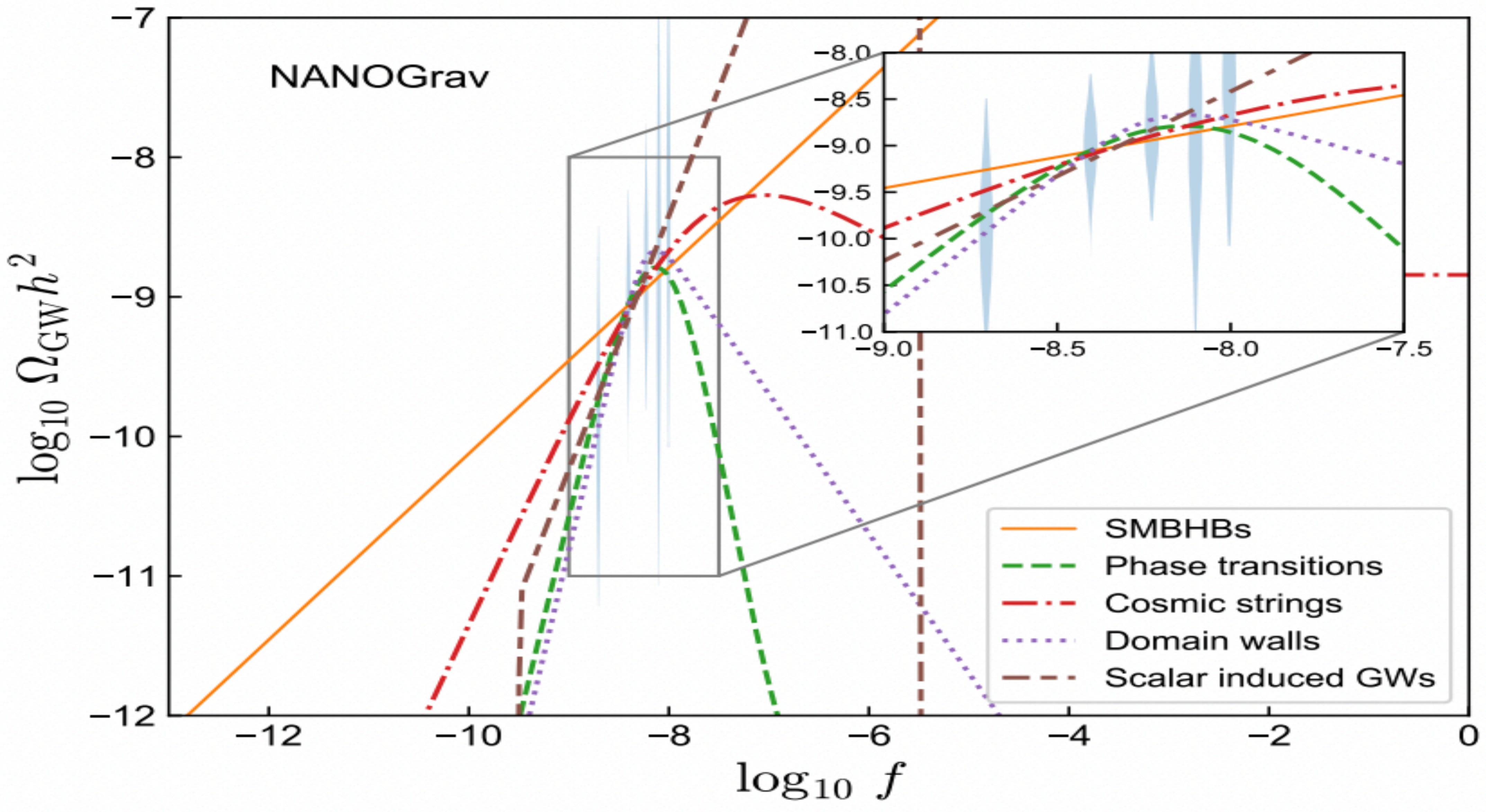


NANOGrav,2306.16213

Gravitational wave sources for Pulsar Timing Arrays



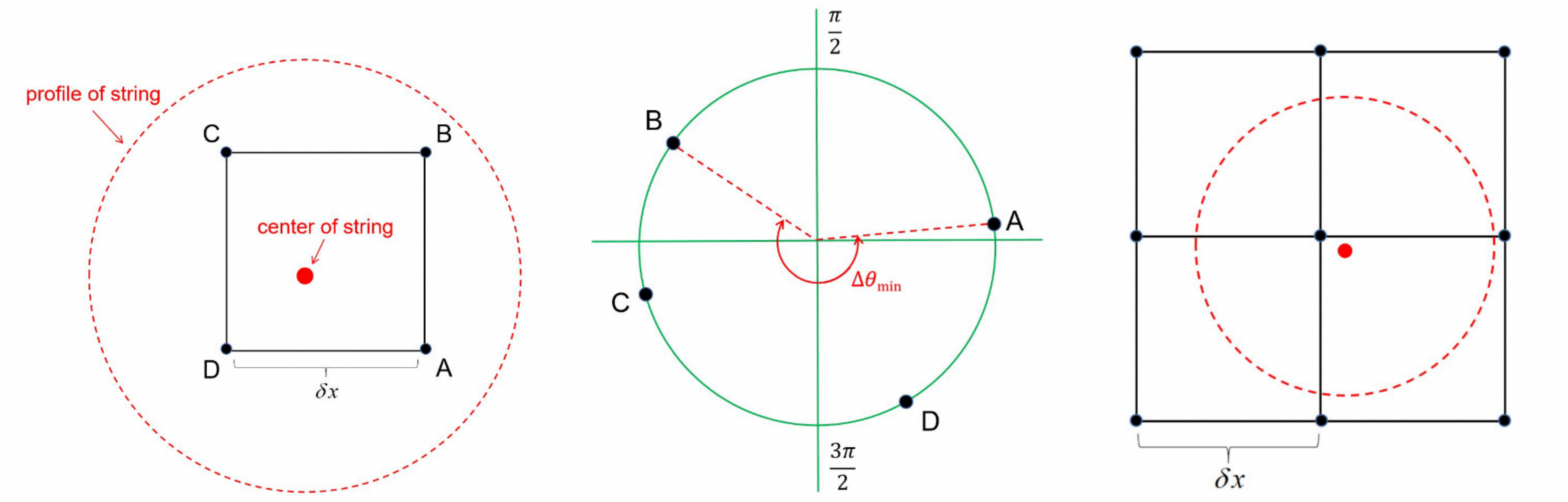




Bian,Ge,Shu,Wang,Yang,Zong,2307.02376

▸ Identification of Cosmic string

String penetrates the square loop if the minimum phase range which contains the four points is greater than π and the phase changes continuously



For a specific square loop, assuming that the minimum phase at four points is θmin

(1) $\theta_{\min} < \pi$.
(2) There exists at least one phase at another point minus $\theta_{\min}$ is greater than π.
(3) There exists at least one phase at another point minus $\theta_{\min}$ is smaller than π.
(4) Denote the phase closest to π in all phases greater than π as θ_a, and denote the phase closest to π in all phases smaller than π as θ_b, it is required to meet $\theta_a - \theta_b < \pi$.
(5) Calculate the difference between the phases at each of two adjacent points in a counterclockwise direction, the multiplication of the four differences is required to be negative.

Bubble dynamics of FOPT

From QFT

$$p(t;T) \equiv \Gamma/V = |A(T)|e^{-B(T)/T}$$

Tunneling

$$A(T) = T\left(\frac{S_3[\bar\phi(T)]}{2\pi T}\right)^{\frac{3}{2}}\left(\frac{\det'[-\nabla^2 + V''(\bar\phi)]}{\det[-\nabla^2 + V''(\phi_f)]}\right)^{-\frac{1}{2}}$$

Fluctuation

$$A(T) = \frac{\sqrt{-\lambda_-}}{2\pi}\left(\frac{S_3[\bar\phi(T)]}{2\pi T}\right)^{\frac{3}{2}}\left(\frac{\det'[-\nabla^2 + V''(\bar\phi)]}{\det[-\nabla^2 + V''(\phi_f)]}\right)^{-\frac{1}{2}}$$

$$= \frac{1}{2\pi}\left(\frac{S_3[\bar\phi(T)]}{2\pi T}\right)^{\frac{3}{2}}\left(\frac{\det^+[-\nabla^2 + V''(\bar\phi)]}{\det[-\nabla^2 + V''(\phi_f)]}\right)^{-\frac{1}{2}},$$

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False vacuum fraction considering bubbles expansion

Bubble Volume

$$h(t) = \exp\left[-\int_{t_c}^{t} \mathrm{d}t' \, \frac{4\pi}{3} v_\mathrm{w}^3 (t-t')^3 \frac{\Gamma(t')}{\mathcal{V}}\right]$$

$$\frac{\Gamma}{\mathcal{V}} = \frac{\Gamma_f}{\mathcal{V}} e^{\beta(t-t_f)}$$

$$\beta \equiv \frac{\mathrm{d}}{\mathrm{d}t}\log\left(\frac{\Gamma(t)}{\mathcal{V}}\right)\bigg|_{t=t_f}$$

$$-\log h(t) \simeq \int^{t} \mathrm{d}t' \frac{4\pi}{3} v_\mathrm{w}^3 (t-t')^3 \frac{\Gamma_f}{\mathcal{V}} e^{\beta(t'-t_f)}$$

$$= \frac{4\pi}{3} v_\mathrm{w}^3 \frac{\Gamma_f}{\mathcal{V}} \frac{3!}{\beta^4} e^{\beta(t-t_f)}.$$

h(tf)=1/e

$$8\pi \frac{v_\mathrm{w}^3}{\beta^4}\frac{\Gamma_f}{\mathcal{V}} = 1$$

$$h(t) = \exp\left[-e^{\beta(t-t_f)}\right]$$

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