



NNLO精度的共线碎裂函数 (Collinear Fragmentation Functions at NNLO)

沈晓民

中国科学院 近代物理研究所

Based on: Jun Gao, **XS**, Hongxi Xing, Yuxiang Zhao, Bin Zhou (NPC Collaboration)

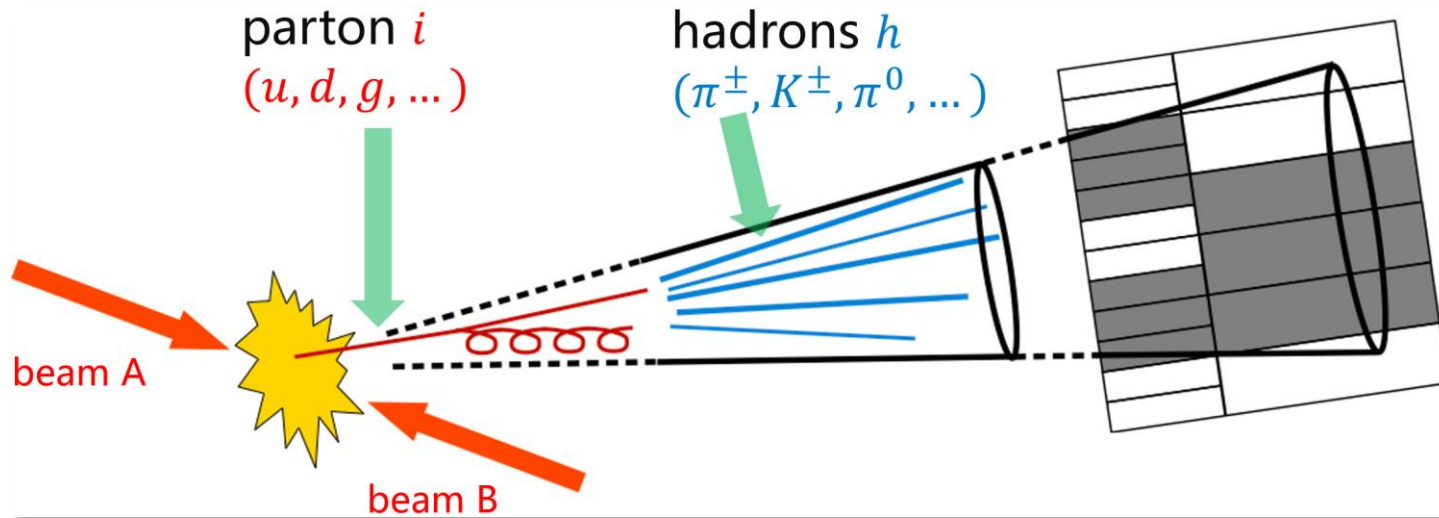
PRL 135, 041902, 2025

第十八届粒子物理、核物理和宇宙学交叉学科前沿问题研讨会
2026-04-12 桂林

Outline

- Introduction to Fragmentation Functions (FFs, 碎裂函数)
- Determination of collinear FFs
- NPC analysis of **NNLO FFs** + constraints on PDFs

Fragmentation Functions (碎裂函数) in parton model



FF = number density of finding

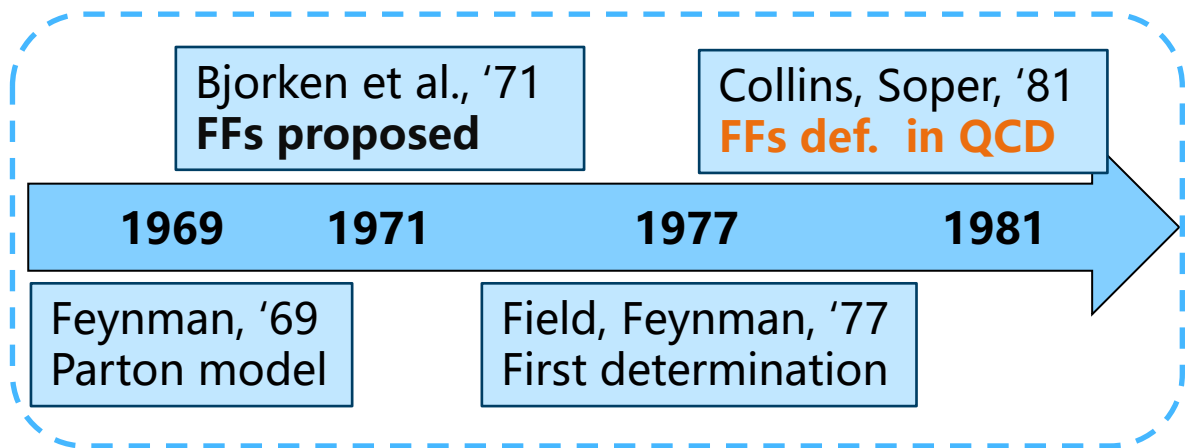
- a specific hadron h
- with momentum fraction z "in" parton i

$$D_{h/i} \left(z = \frac{p_h^+}{p_i^+} \right) \longleftrightarrow f_{i/h} \left(x = \frac{p_i^+}{p_h^+} \right)$$

碎裂函数

部分子分布函数

Fragmentation Functions (FFs) in QCD



$$D_{h/q}(z) = \frac{z}{4} \sum_X \int \frac{d\xi^+}{2\pi} e^{iP_h^- \xi^+ / z} \text{Tr} \left[\langle 0 | \mathcal{W}(\infty^+, \xi^+) \psi_q(\xi^+, 0^-, \vec{0}_T) | P_h, S_h; X \rangle \right. \\ \left. \times \langle P_h, S_h; X | \bar{\psi}_q(0^+, 0^-, \vec{0}_T) \mathcal{W}(0^+, \infty^+) | 0 \rangle \gamma^- \right]$$

$$D(z, Q_0)$$

$$D(z, Q)$$

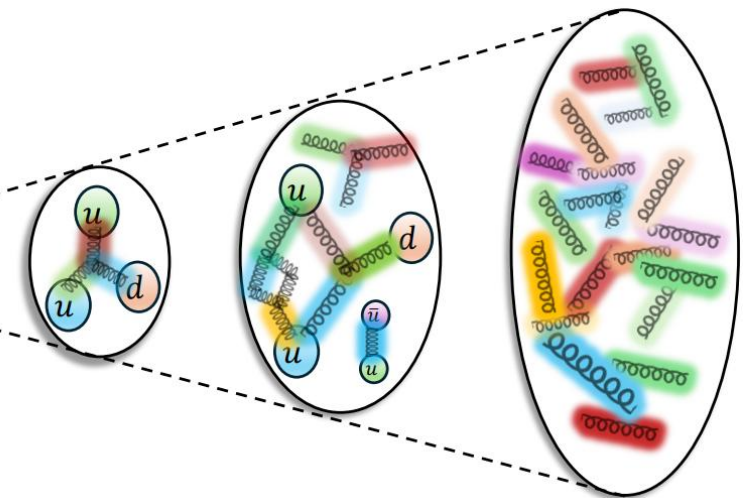
$$\frac{dD_{h/i}(z, Q)}{d \ln^2 Q} = P_{ji}(y) \otimes D_{h/j}\left(\frac{z}{y}, Q\right)$$

timelike DGLAP evolution

PDFs

Hadron

Parton distribution function describes the probability of finding a quark or gluon

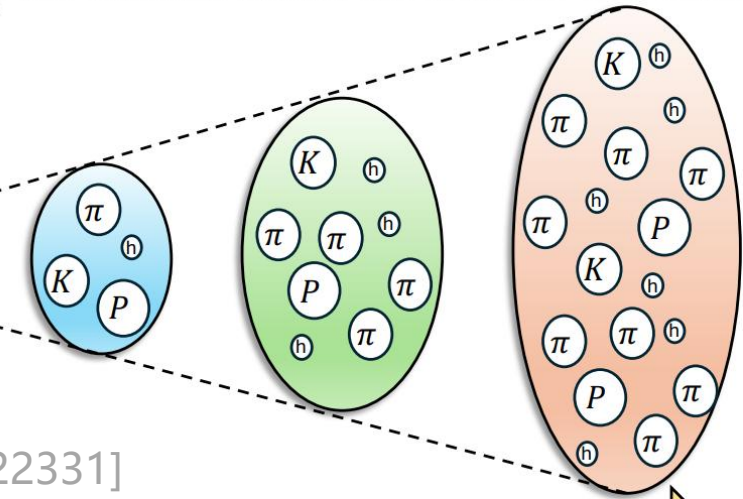


Crossing Symmetry $\left\{ \begin{array}{l} e^- + h \rightarrow e^- + X \\ e^- + e^+ \rightarrow h + X \end{array} \right\}$

FFs

Parton

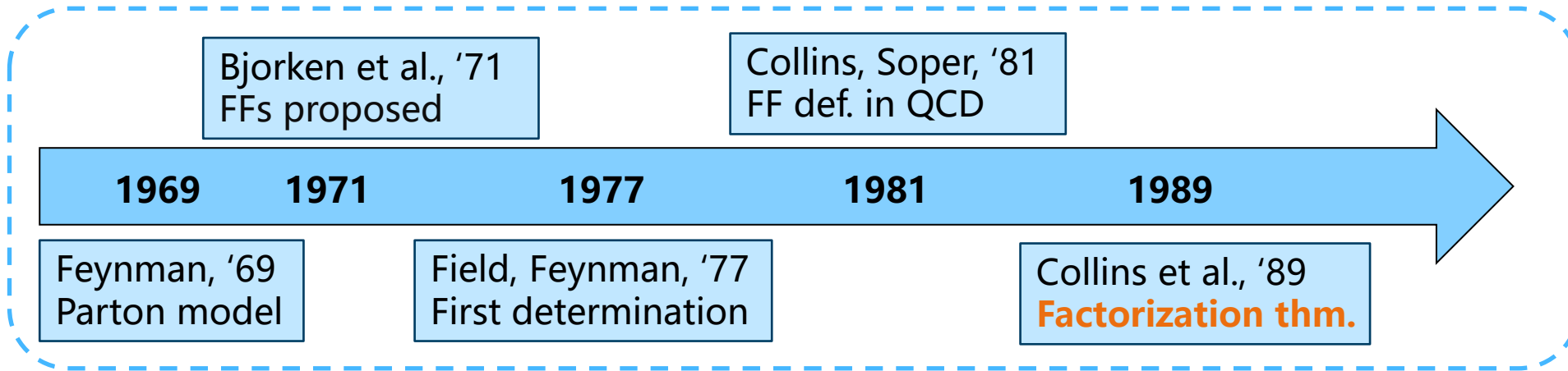
Fragmentation function describes the probability of producing a specific hadron.



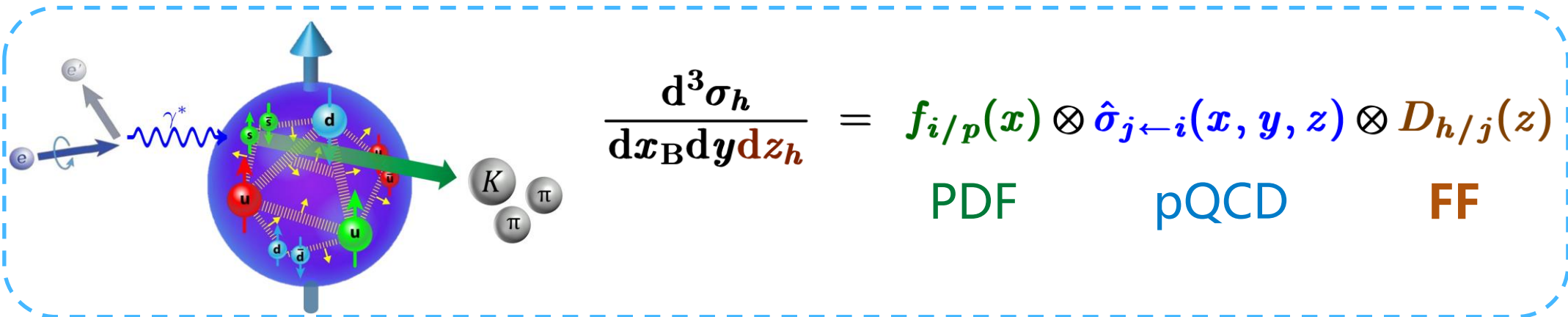
[Figure from 2410.22331]



FFs are key ingredients of QCD factorization framework



➤ Semi-Inclusive DIS (SIDIS) : $e + N \rightarrow e + \mathbf{h} + X$



➤ $e^+e^- \rightarrow h + X$ (SIA) $\Rightarrow \sigma = \text{pQCD} \otimes \mathbf{FF}$

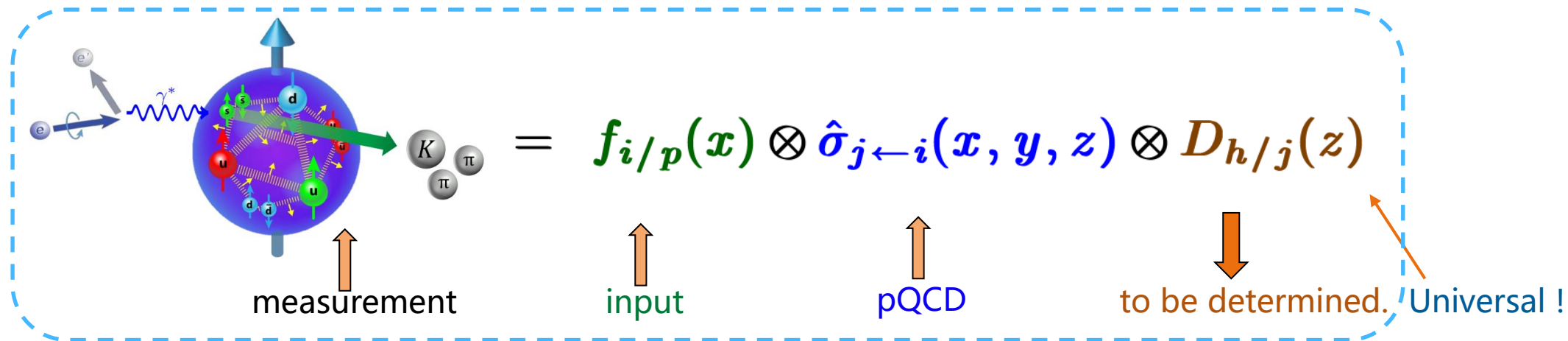
➤ $pp \rightarrow h + X \Rightarrow \sigma = \mathbf{PDF} \otimes \mathbf{PDF} \otimes \text{pQCD} \otimes \mathbf{FF}$



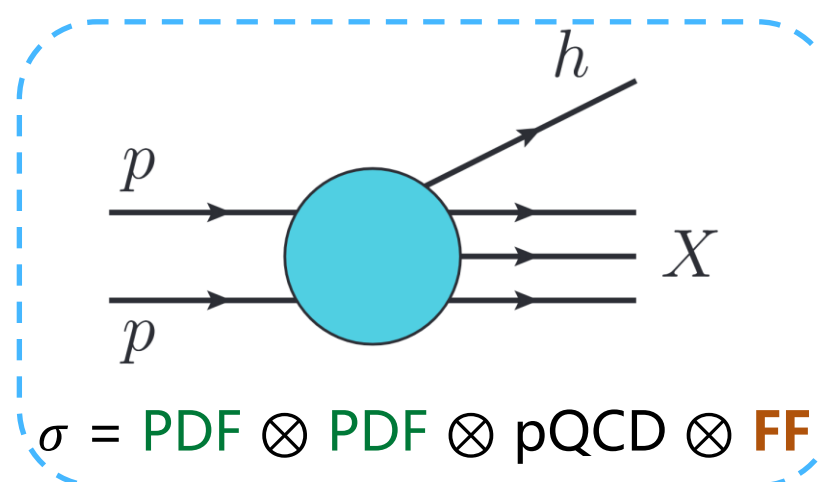
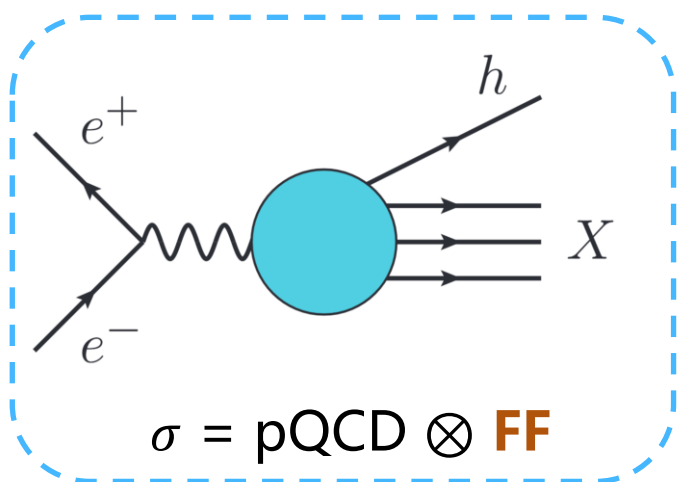
Determination of collinear FFs

Determination of FFs: global data fits

❖ e p collision:



❖ ee and pp



Global analyses of collinear FFs to light hadrons

collaboration	NNFF	JAM	DSS+	BDSSV	MAP	NPC
SIA (ee)	✓	✓	✓	✓	✓	✓
SIDIS (ep)	✗	✓	✓	✓	✓	✓
pp incl. hadron	✗	✗	✓	✗	✗	✓
hadron in jet	✗	✗	✗	✗	✗	✓
FFs (charged h)	π^\pm, K^\pm, p	π^\pm, K^\pm	π^\pm, K^\pm, p	π^\pm	π^\pm, K^\pm	π^\pm, K^\pm, p
FFs (neutral h)			η			K^0, η, Λ
pQCD order	NNLO	NLO	NLO	appr. NNLO	appr. NNLO	NLO

➤ **NLO** analyses from **Non-perturbative Physics Collaboration (NPC)**

- NPC23 FFs to light **charged** hadrons:

Gao, Liu, **XS**, Xing, Zhao, *PRL* 132, 261903, '24

Gao, Liu, **XS**, Xing, Zhao, *PRD* 110, 114019, '24 (Editors' suggestion)

- NPC23 FFs to light **neutral** hadrons:

Gao, Liu, Li, **XS**, Xing, Zhao, Zhou, *PRD* 112, 054045, '25

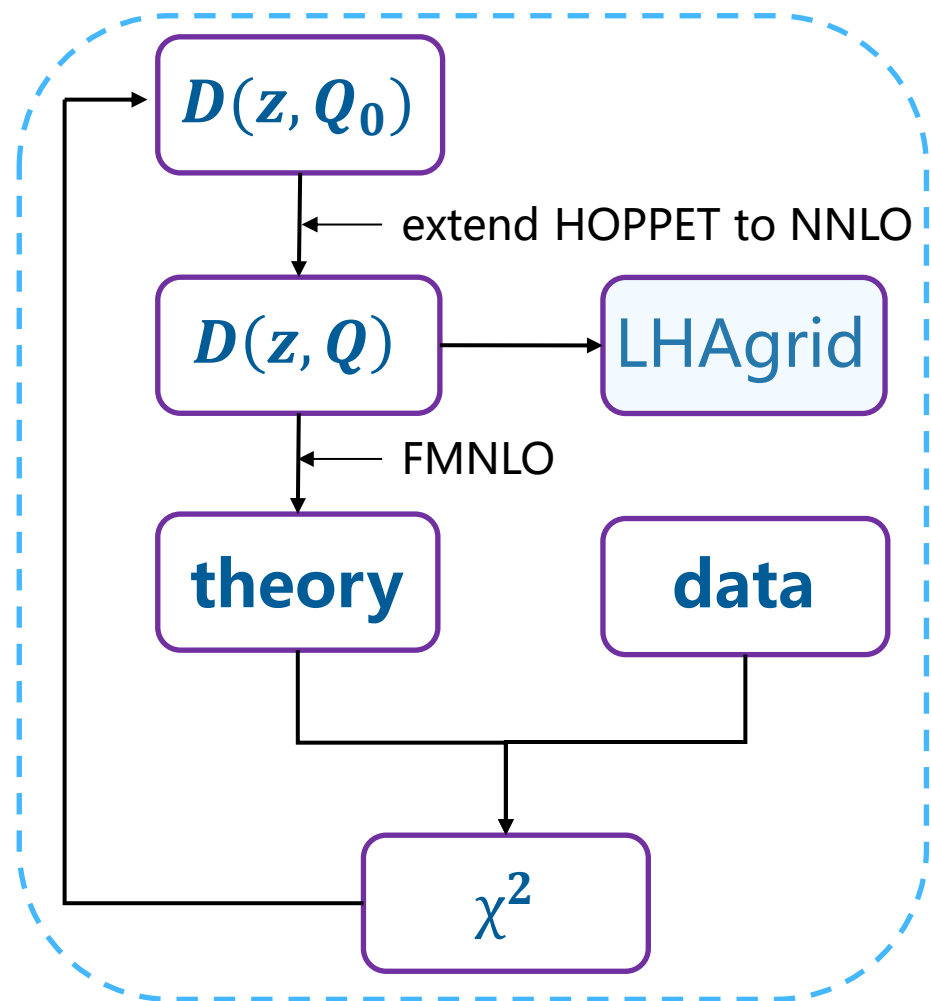


NPC analysis of FFs at NNLO

[Gao, **XS**, Xing, Zhao, Zhou, *PRL* 135, 041902, 2025]

collaboration	NNFF	JAM	DSS+	BDSSV	MAP	NPC	NPC
SIA (ee)	✓	✓	✓	✓	✓	✓	✓
SIDIS (ep)	✗	✓	✓	✓	✓	✓	✓
pp incl. hadron	✗	✗	✓	✗	✗	✓	✗
hadron in jet	✗	✗	✗	✗	✗	✓	✗
FFs	π^\pm, K^\pm, p	π^\pm, K^\pm	π^\pm, K^\pm, p, h^\pm η	π^\pm	π^\pm, K^\pm	π^\pm, K^\pm, p, h^\pm K^0, η, Λ	π^\pm, K^\pm
pQCD order	NNLO	NLO	NLO	appr. NNLO	appr. NNLO	NLO	NNLO

The framework



- FFs at starting scale $Q_0 = 1.4 \text{ GeV}$

$$zD_i^h(z, Q_0) = z^{\alpha_i^h} (1-z)^{\beta_i^h} \exp\left(\sum_{n=0}^m a_{i,n}^h z^{n/2}\right)$$

+ charge/isospin symmetries

- FFs at arbitrary energy scale Q

3-loop timelike DGLAP evolution

[Mitov, Moch, Vogt, Almasy]

[Chen, Yang, Zhu, Zhu, '20]

- SIA/SIDIS coefficient functions at NNLO

[Bonino+, '24], [Goyal+, '24]

The **first** global FF fit (ee+SIDIS) at full NNLO accuracy

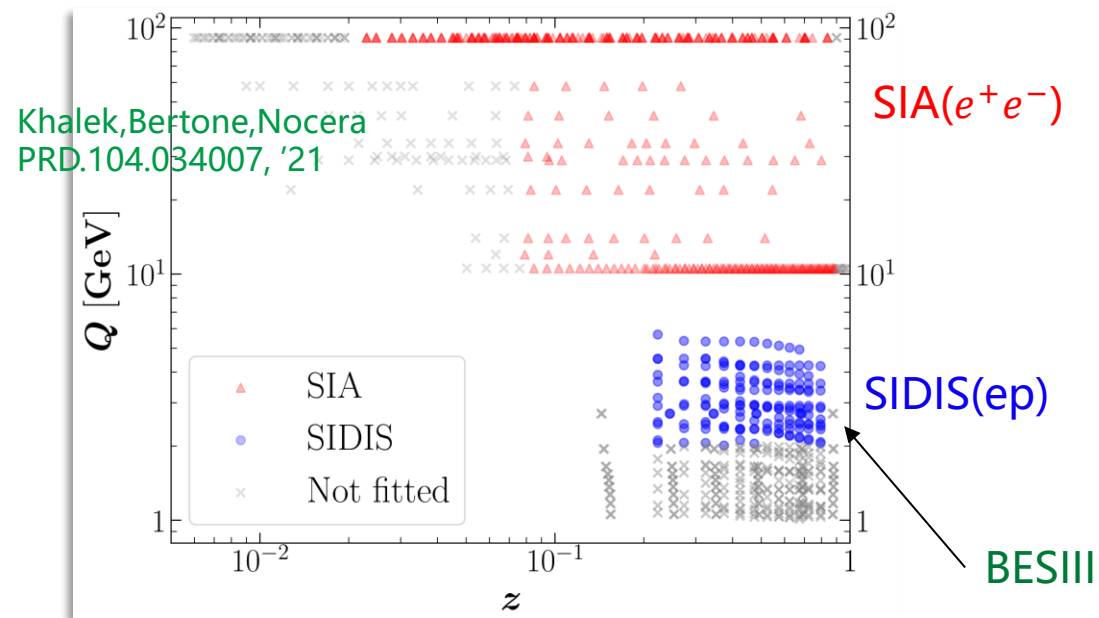
The datasets

- SIA(e^+e^-) data used in the fit:

exp.	\sqrt{s}/GeV	lum. (n_Z)	year	final states	hadrons
DELPHI	189	157.7 pb ⁻¹	2002	inc. had.	π^\pm, K^\pm
OPAL	m_Z	780 000	1994	$Z \rightarrow q\bar{q}$	π^\pm, K^\pm
ALEPH	m_Z	520 000	1995	$Z \rightarrow q\bar{q}$	π^\pm, K^\pm
DELPHI	m_Z	1 400 000	1998	$Z \rightarrow q\bar{q}$	π^\pm, K^\pm
				$Z \rightarrow b\bar{b}$	π^\pm, K^\pm
SLD	m_Z	400 000	2004	$Z \rightarrow q\bar{q}$	π^\pm, K^\pm
				$Z \rightarrow b\bar{b}$	π^\pm, K^\pm
				$Z \rightarrow c\bar{c}$	π^\pm, K^\pm
TASSO	44	34 pb ⁻¹	1989	inc. had.	π^\pm, π^0
TASSO	34	77 pb ⁻¹	1989	inc. had.	π^\pm, K^\pm
TPC/2 γ	29	70 pb ⁻¹	1988	inc. had.	π^\pm, K^\pm
Belle	10.52	68 fb ⁻¹	2013	inc. had.	π^\pm, K^\pm
BaBar	10.54	0.91 fb ⁻¹	2013	inc. had.	π^\pm, K^\pm
BESIII	2.0-3.671	253 pb ⁻¹	2025	inc. had.	π^\pm, K^\pm

[BESIII+NPC, PRL135, 151901, 2025]

- **separated** kinematic region of e^+e^- and ep data before BESIII measurement



- This work uses both ee and ep data at $Q \sim 3 \text{ GeV}$

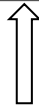
The first test on universality of FFs at $Q \sim 3 \text{ GeV}$ (using ee + ep)



The results

- Our FFs describe both ee and ep data well
- test of leading-twist factorization

$E_{h,min}$ [GeV]	BESIII		COMPASS		B-factories		HE-SIA		global		
	N_{pt}	χ^2/N_{pt}	N_{pt}	χ^2/N_{pt}	N_{pt}	χ^2/N_{pt}	N_{pt}	χ^2/N_{pt}	N_{pt}	χ^2	χ^2/N_{pt}
0.5	242	1.26	358	1.65	233	1.06	426	1.19	1259	1650.2	1.31
0.6	212	1.21	290	1.59	228	0.92	423	0.97	1153	1338.8	1.16
0.7	182	1.11	214	1.47	223	0.61	413	0.84	1032	997.2	0.97
0.8	152	0.98	142	1.30	218	0.53	407	0.82	919	781.8	0.85
0.9	122	1.05	94	1.29	213	0.52	407	0.80	836	687.1	0.82
1.0	98	1.14	54	0.97	209	0.49	403	0.80	764	587.2	0.77

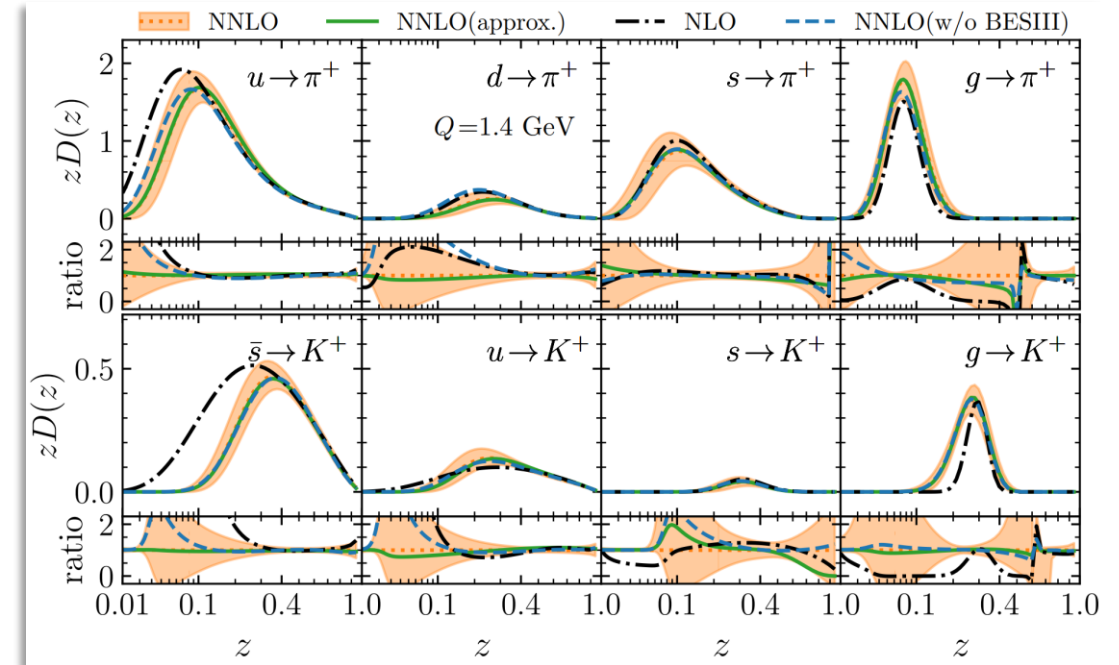


energy cut of the identified hadron

- $\chi^2_{NNLO} < \chi^2_{NLO}$.

Previous studies: $\chi^2_{appr.NNLO} > \chi^2_{NLO}$.

[Gao, XS, Xing, Zhao, Zhou, *PRL* 135, 041902, 2025]



LHAg grids of our FFs have been submitted to the LHAPDF repository.

<https://www.lhapdf.org/pdfsets.html>

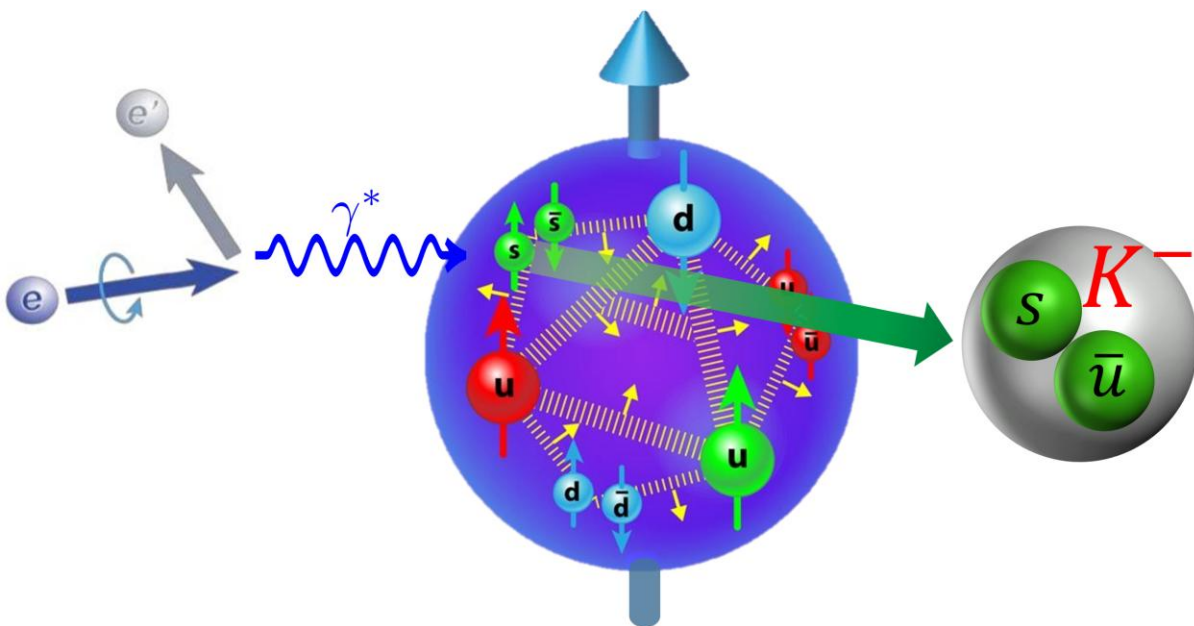


Application: constraining proton PDFs at NNLO

SIDIS may also constrain PDFs:

$$\frac{d^3\sigma_h}{dx_B dy dz_h} = f_{i/p}(x) \otimes \hat{\sigma}_{j \leftarrow i}(x, y, z) \otimes D_{h/j}(z)$$

unpolarized PDF
FF



➤ LO xsec of SIDIS off an isoscalar target (COMPASS)

$$\frac{d\sigma^{K^+}}{dx dy dz} - \frac{d\sigma^{K^-}}{dx dy dz}$$

$$\sim 2 \left(u_v(x) + d_v(x) \right) \left(D_u^{K^+}(z) - D_{\bar{u}}^{K^+}(z) \right)$$

$$+ \left(s(x) - \bar{s}(x) \right) \left(D_s^{K^+}(z) - D_{\bar{s}}^{K^+}(z) \right) + \dots$$

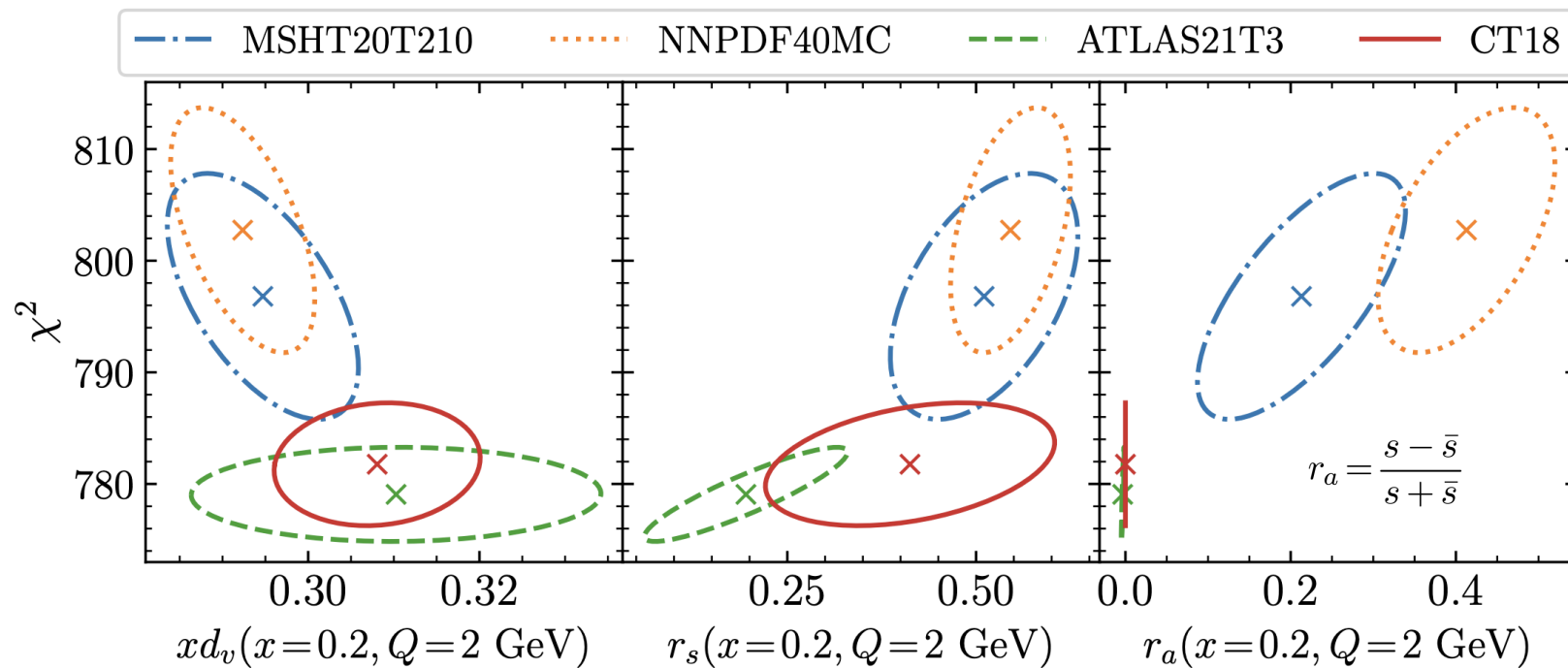
PDF
FF

is sensitive to **strangeness asymmetry**

$$r_a = \frac{s - \bar{s}}{s + \bar{s}}$$

Application: constraining proton PDFs at NNLO

➤ Correlation between χ^2 and PDFs



➤ We generate **modified PDFs** which reflect the impact of SIDIS data

- Reweighting of the [NNPDF4.0](#) PDF set
- Profiling of the [MSHT20](#) PDF set



Summary and outlook

- FFs are key inputs for calculations of hadron production rate from first principles.
- We present the **first** global (SIA+SIDIS) FFs determination at full **NNLO**.
- find good agreement with low-Q data from both **SIDIS and e+e-** annihilation.
- find a preference for a reduced asymmetry in the strange (anti-)quark PDFs.

collaboration	NPC	NPC
SIA (ee)	✓	✓
SIDIS (ep)	✓	✓
pp incl. hadron	✓	✗
hadron in jet	✓	✗
FFs	π^\pm, K^\pm, p K^0, η, Λ	π^\pm, K^\pm
pQCD order	NLO	NNLO

FF sets from NPC available from <https://www.lhapdf.org/pdfsets.html>

NLO charged hadron:

Gao, Liu, **XS**, Xing, Zhao, *PRL* 132, 261903, 2024

Gao, Liu, **XS**, Xing, Zhao, *PRD* 110, 114019, 2024

NLO neutral hadron:

Gao, Liu, Li, **XS**, Xing, Zhao, Zhou, *PRD* 112, 054045, 2025

NNLO:

Gao, **XS**, Xing, Zhao, Zhou, *PRL* 135, 041902, 2025

Thank you!



NPC collaboration gathering on July 19th 2025



in neighborhood of Huizhou city (host of EICc)

Jun Gao, ChongYang Liu, Meng Yang Li, **XiaoMin Shen**, **HongXi Xing**, **YuXiang Zhao**, Bin Zhou, YiYu Zhou
Shanghai JiaoTong Univ., South China Normal Univ., Institute of Modern Physics, CAS

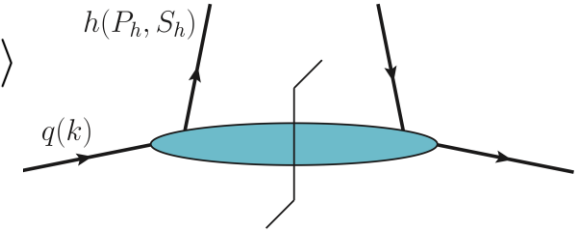
NPC=Non-perturbative Physics Collaboration

Slide from Jun Gao

Determination of FFs

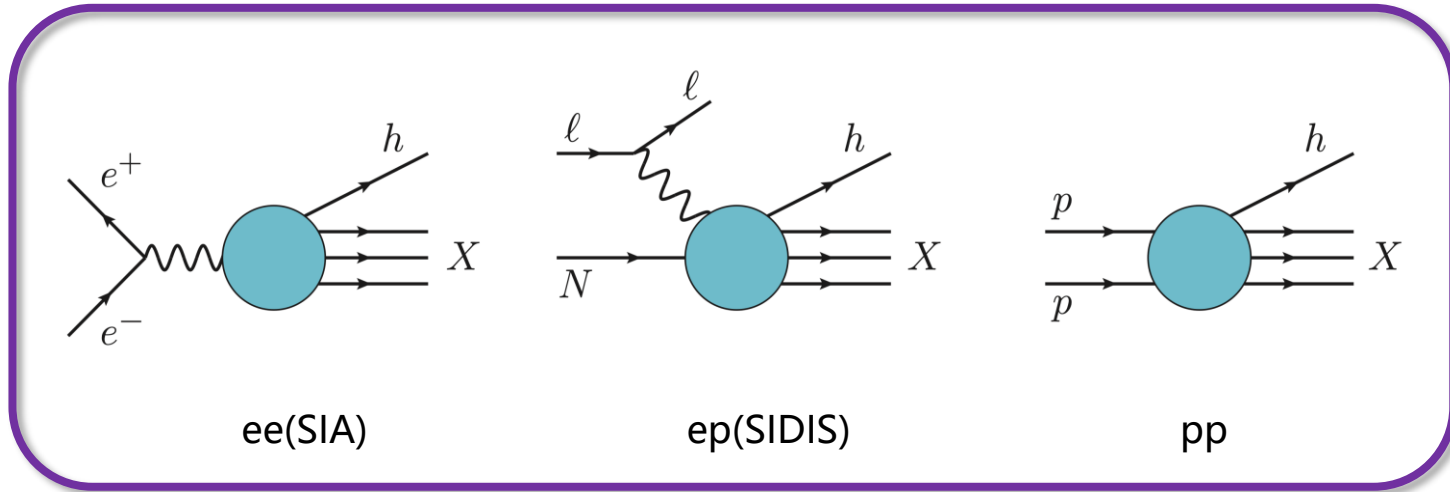
- ❖ No lattice QCD calculation yet

$$D_{h/q}(z) = \frac{z}{4} \sum_X \int \frac{d\xi^+}{2\pi} e^{iP_h^- \xi^+ / z} \text{Tr} \left[\langle 0 | \mathcal{W}(\infty^+, \xi^+) \psi_q(\xi^+, 0^-, \vec{0}_T) | P_h, S_h; X \rangle \right. \\ \left. \times \langle P_h, S_h; X | \bar{\psi}_q(0^+, 0^-, \vec{0}_T) \mathcal{W}(0^+, \infty^+) | 0 \rangle \gamma^- \right]$$

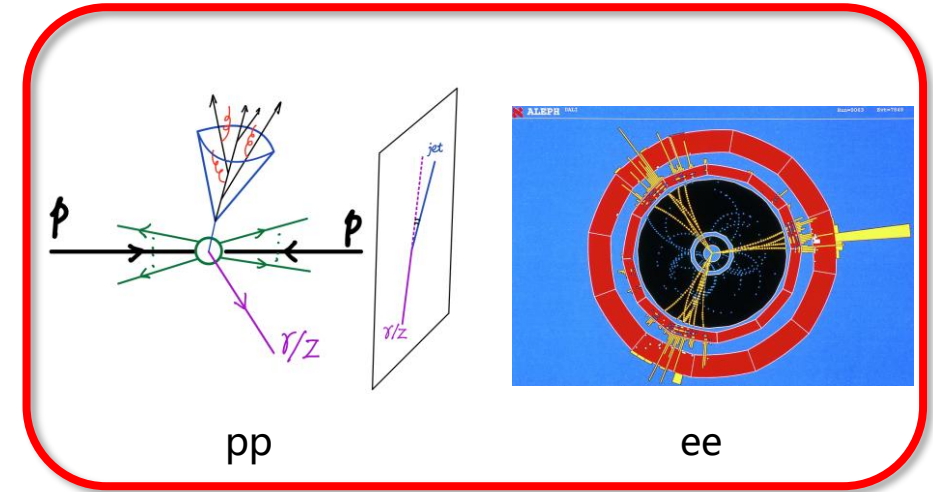


- ❖ Using quantum computers [Li, Xing, Zhang, 2406.05683]
[Galvez-Viruet et al. 2510.18869]
- ❖ In simpler theory: 't Hooft model [Jia, Mo, Xiong, 2310.17640]

NPC FFs analyses incorporate various types of data



single-inclusive hadron production



hadron-in-jet measurements

- Hadron-in-jet data provides **direct probe of z dependence**

$$\frac{p_{T,h}}{p_{T,j}} \xrightarrow{\text{LO}} z$$

- All theoretical predictions calculated with **FMNLO**.

[Liu, **XS**, Zhou, Gao, 2305.14620 (JHEP)]

Application: constraining proton PDFs at NNLO

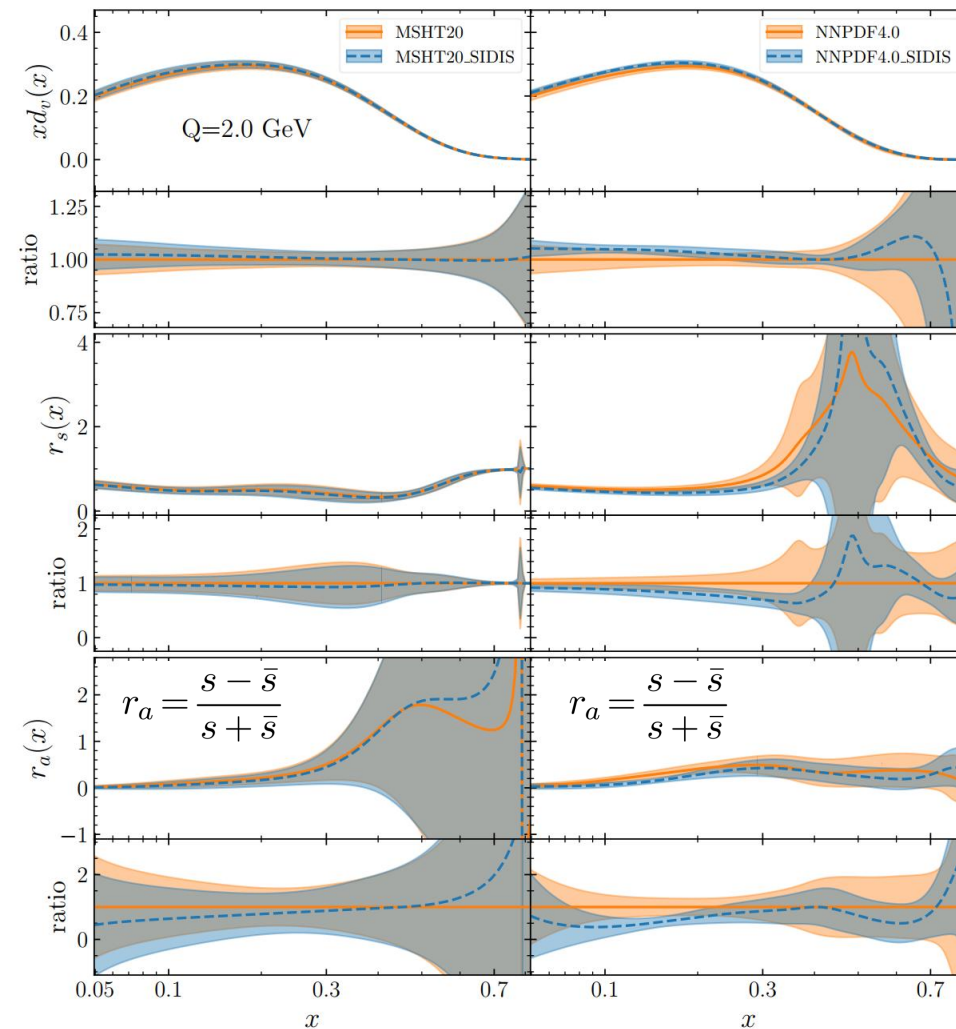
[Gao, **XS**, Xing, Zhao, Zhou, *PRL* **135**, 041902, 2025]

PDF sets before and after reweighting/profiling

	$x d_v(x = 0.2, Q = 2\text{GeV})$	$r_s(x = 0.2, Q = 2\text{GeV})$	$r_a(x = 0.2, Q = 2\text{GeV})$
NNPDF4.0	0.2924 ± 0.0084	0.547 ± 0.079	0.408 ± 0.107
NNPDF4.0(reweighting)	0.3021 ± 0.0069	0.438 ± 0.066	0.281 ± 0.086
MSHT20	0.295 ± 0.011	0.511 ± 0.124	0.213 ± 0.126
MSHT20(profiling)	0.298 ± 0.011	0.481 ± 0.121	0.167 ± 0.136

PDF values d_v, r_s, r_a at 68% C.L. before and after reweighting (profiling) for NNPDF4.0 (MSHT20)

$$d_v \equiv d - \bar{d}, \quad r_s \equiv \frac{s + \bar{s}}{\bar{u} + \bar{d}}, \quad r_a \equiv \frac{s - \bar{s}}{s + \bar{s}}$$



Backup

NNPDF40MC

