

# $f_0(980)$ 结构的能标敏感性研究

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# Overview

I: The controversy of  $f_0(980)$  structure

II:  $2\pi$ DAs and  $H_{14}$  decays

i: From  $\pi$ -LCDAs to  $2\pi$ DAs

ii: Energy dependent structure of  $f_0(980)/[\pi\pi]_S$  in  $B_s/D_s$  decays

III: Summary and Prospect

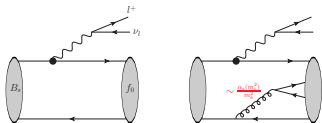
# The controversy of $f_0(980)$ structure

- Experimental identification is particularly challenging (large width)
- From the quantum theory, it is a superposition of all possible Fock states

$$|f_0(980)\rangle, \quad |[\pi\pi]_S\rangle = \psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \psi_{q\bar{q}q\bar{q}}|q\bar{q}q\bar{q}\rangle + \dots$$

- **Hadron spectroscopy** provides clear evidence for the complex config
  - conventional resonant with possible gluonball component  
[J.D. Weinstein and N. Isgur, PRL 48 (1982) 659]
  - exotic multiparticle state (compact tetraquark state, molecular state)  
[Y.R. Liu, H.X. Chen, W. Chen, X. Liu and S.L. Zhu, PPNP 107 (2019) 237-320]  
[F.K. Guo, C. Hanhart, U.G. Meissner, Q. Wang, Q. Zhao and B.S. Zou, RMP 90 (2018) 015004]
- **The underlying partonic dynamics** can not be extracted directly from spectral analysis, even though it reveals these exotic configurations
- **Semileptonic  $B, D$  decays** are powerful probe of the underlying structure
- **LCDAs, scale-dependent functions with a strict twist expansion**

- **Semileptonic  $B, D$  decays** are powerful probe of the underlying structure
- **color transparency mechanism in  $B_{(s)} \rightarrow f_0 l^+ \nu_l$  decays**



† high Fock states' contribution is doubly suppressed by  $\alpha_s$  and  $\mathcal{O}(1/Q^2)$

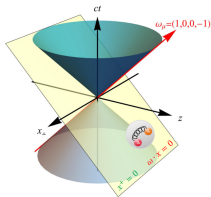
- **the mechanism fails in  $D_s \rightarrow f_0 l^+ \nu_l$  decays**

- **the cascade decay analyses of  $D_s \rightarrow (f_0 \rightarrow \pi\pi) e \nu$  under  $q\bar{q}$  ansatz can explain data**
- **$D_s \rightarrow f_0$  FFs & Flatté resonant model & fine tuning of the mixing angle**  
 [SC and S. L. Zhang, EPJC 84 (2024) 379]  
 [D.D. Hu, X.G. Wu, H.J. Tian, T. Zhong, and H.B. Fu, PRD 112 (2025) 056023]
  - the scale dependence have revealed in  $D_s \rightarrow f_0$  form factor  $\Downarrow$
  - the seemingly agreement in  $D_{14}$  decays is attributed to the cascade framework
  - the Flatté parameterization disrupts the assessment of color transparency  $\Uparrow$
- LCDAs, scale-dependent functions with a strict twist expansion

- From  $\pi$ -LCDAs to  $2\pi$ DAs

[SC, PRD 112 (2025) L111301]

- Energy dependent structure of  $f_0(980)/[\pi\pi]_S$  in  $B_s/D_s$  decays [SC, L.Y. Dai, J.M. Shen and S.L. Zhang, PRD 113 (2026) L031901]



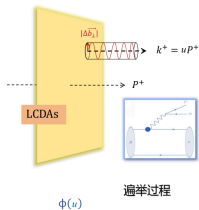
- The Lorentz and gauge invariant ME

$$\langle 0 | \bar{u}(x) \gamma_\mu \gamma_5 d(-x) | \pi^-(p) \rangle = f_\pi \int_0^1 du e^{i(2u-1)p \cdot x} \left[ i p_\mu \left( \phi(u, \mu) + \frac{x_\perp^2}{4} \phi_1^4(u, \mu) \right) - \dots \right]$$

$$\langle 0 | \bar{u}(x) \gamma_\mu \gamma_5 d(-x) | \rho^-(p) \rangle = f_\rho m_\rho \int_0^1 du e^{i(2u-1)p \cdot x} \left[ p_\mu \frac{\epsilon(\lambda) \cdot x}{p \cdot x} \left( \phi_{\parallel}(u, \mu) - \phi_{\perp}^3 \right) - \dots \right]$$

$$\langle 0 | \bar{q}(x) \gamma_\mu \gamma_5 q(-x) | f_0(p) \rangle = p_\mu \int_0^1 du e^{i(2u-1)p \cdot x} [\phi(u, \mu) + \dots]$$

- LCDAs are dimensionless functions of  $u$  and renormalization scale  $\mu$
- the probability amplitudes to **find the meson** in a state with minimal number of constituents and have small transversal separation of order  $1/\mu$
- longitudinal  $\otimes$  transversal dofs
- TMD ( $\mu$  dependence) by *RGE*
- LMD by collinear subgroup of conformal group  $\Rightarrow$  collinear twist  $\Rightarrow$  the Gegenbauer polynomials



- LCDAs of pion achieved **great success** in describing large  $Q^2$  processes.

△ the establishment and development of the pQCD factorization  $F_\pi(Q^2)$

|  |       |   |
|--|-------|---|
| $\int_0^1 du_i \phi(u_1, \tilde{Q}_1) T_H(u_i, Q) \phi(u_2, \tilde{Q}_2)$  | 1980s | [Lepage & Brodsky 1980, Efremov & Radyushkin 1980]  |
| $\int_b^{\bar{b}} du_1 \int_{a/u_1}^{1-a/u_1} \phi(u_1, k_1\tau) \bar{T}_H(u_i, Q) \phi(u_2, k_2\tau)$                                 | 1990s | [Huang, Shen & Kroll 1991, Huang, Shen & Ma 1994]   |
| $\psi(u_1, \mu_{r_1}) T_H(u_i, b_i, Q) S_i(u_i) e^{-S(u_i, b_i, Q)} \psi(u_2, \mu_{r_2})$  | 2000s | [Botts & Sterman 1989, Li & Sterman 1992, Li 1999]  |
| $\sum_{t_i} \psi^{t_1}(u_1, \mu_{r_1}) T_H^{t_i, LO+NLO}(u_i, b_i, Q) S_i(u_i) e^{-S(u_i, b, Q)} \psi^{t_2}(u_2, \mu_{r_2})$           | 2010s | [Li, Shen, Wang & Zou 2011, SC, Fan & Xiao 2014]  |
| $\sum_{t_i} \psi^{t_1}(u_1, b_1, \mu_{r_1}) T_H^{t_i, LO+NLO}(u_i, b_i, Q) S_i(u_i) e^{-S(u_i, b, Q)} \psi^{t_2}(u_2, b_2, \mu_{r_2})$ | 2020s | [Chai & SC 2025]<br>[Chen <sup>2</sup> , Feng & Jia 2024], [Ji, Shi, Wang <sup>3</sup> & Yu 2025] |

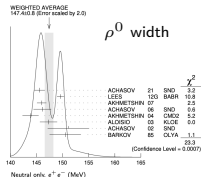
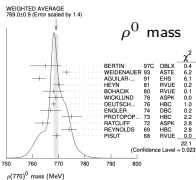
- LCDAs of proton serves as the fundamental input to explain  $ep$  scattering  
[Chen<sup>2</sup>, Feng, Hu, Jia 2025], [Huang, Shi, Wang, Zhao 2025], [Yu, SC, Han, Li, Yu 2025]
- The non-perturbative input for HFP theoretical studies that determines the precision and accuracy predicted the CPVs in the  $B \rightarrow \pi\pi, K\pi$  decays and et.al.,
- ...

# The limitations of the single particle picture

- $V, S$  meson LCDAs reveal certain limitations in the precision testing era

△ the probability amplitudes to **find the light meson** in a state with . . .

△ the  $\pi\pi$  mass in  $[0.554, 0.996]$  GeV is selected to identify candidates for the  $\rho(770)$



- A second-best approach frequently employed in phenomenology is the **cascade decay framework**

$$\mathcal{M}(B^0 \rightarrow \pi^0 \pi^- l^+ \nu_l) = \mathcal{M}(B^0 \rightarrow \rho^- l^+ \nu_l) \text{BW}(s) \mathcal{M}(\rho \rightarrow \pi\pi)$$

- **model dependence, large uncertainties in  $f_0$  involved processes**
- How to accurately describe the **width effects** of unstable intermediate particles, the contributions and **interference effects** of different partial waves, and the **QCD backgrounds** from non-resonant states

## $2\pi$ DAs provide a most general description of $\pi\pi$ spectral

- Chiral-even LC expansion with gauge factor  $[x, 0]$  [Polyakov 1999, Diehl 1998]

$$\langle \pi^a(k_1) \pi^b(k_2) | \bar{q}_f(zn) \gamma_\mu \tau q_f(0) | 0 \rangle = \kappa_{ab} k_\mu \int du e^{iuz(k \cdot n)} \Phi_{\parallel}^{ab, ff'}(u, \zeta, k^2, \mu)$$

$$\langle 0 | \bar{q}(x) \gamma_\mu \gamma_5 q(-x) | f_0(p) \rangle = p_\mu \int_0^1 du e^{i(2u-1)p \cdot x} \phi(\mu, u)$$

- $2\pi$ DAs is decomposed in terms of  $C_n^{3/2}(2u-1)$  and  $C_\ell^{1/2}(2\zeta-1)$

$$\Phi^{J=1}(u, \zeta, k^2, \mu) = 6u(1-u) \sum_{n=0, \text{even}}^{\infty} \sum_{l=1, \text{odd}}^{n+1} B_{n\ell}^{J=1}(k^2, \mu) C_n^{3/2}(2u-1) C_\ell^{1/2}(2\zeta-1)$$

$$\Phi^{J=0}(u, \zeta, k^2, \mu) = 6u(1-u) \sum_{n=1, \text{odd}}^{\infty} \sum_{l=0, \text{even}}^{n+1} B_{n\ell}^{J=0}(k^2, \mu) C_n^{3/2}(2u-1) C_\ell^{1/2}(2\zeta-1)$$

- Evolution from  $4m_\pi^2$  to large  $k^2$**  via the Watson theorem of  $\pi\pi$  scattering amplitudes

$$B_{n\ell}^J(k^2) = B_{n\ell}^J(0) \text{Exp} \left[ \sum_{m=1}^{N-1} \frac{k^{2m}}{m!} \frac{d^m}{dk^{2m}} \ln B_{n\ell}^J(0) + \frac{k^{2N}}{\pi} \int_{4m_\pi^2}^{\infty} ds \frac{\delta_\ell^J(s)}{s^N(s-k^2-i0)} \right]$$

$\triangle$   $2\pi$ DAs in a wide range of energies is given by  $\delta_\ell^J$  and a few subtraction constants

# 2πDAs

- The subtraction constants of  $B_{n\ell}(k^2)$  at low  $k^2$  (around the threshold)

| (nl) | $B_{n\ell}^{\parallel}(0)$ | $c_1^{\parallel,(nl)}$ | $\frac{d}{dk^2} \ln B_{n\ell}^{\parallel}(0)$ | $B_{n\ell}^{\perp}(0)$    | $c_1^{\perp,(nl)}$ | $\frac{d}{dk^2} \ln B_{n\ell}^{\perp}(0)$ |
|------|----------------------------|------------------------|---|---------------------------|--------------------|---|
| (01) | 1                          | 0                      | 1.46 $\rightarrow$ 1.80                       | 1                         | 0                  | 0.68 $\rightarrow$ 0.60                   |
| (21) | -0.113 $\rightarrow$ 0.218 | -0.340                 | 0.481   | 0.113 $\rightarrow$ 0.185 | -0.538             | -0.153                                    |
| (23) | 0.147 $\rightarrow$ -0.038 | 0                      | 0.368   | 0.113 $\rightarrow$ 0.185 | 0                  | 0.153                                     |
| (10) | -0.556                     | -                      | 0.413   | -                         | -                  | -   |
| (12) | 0.556                      | -                      | 0.413   | -                         | -                  | -   |

△ firstly studied in the effective low-energy theory based on instanton vacuum [Polyakov 1999]

△ updated with the kinematical constraints and the new  $a_2^{\pi}, a_2^{\rho}$  [SC 2019, 2023]

- 2πDAs were introduced at leading twist [Polyakov 1999, Diehl 1998]

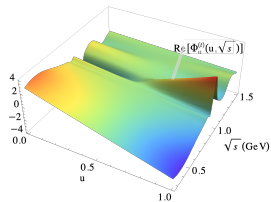
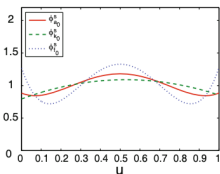
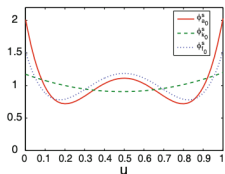
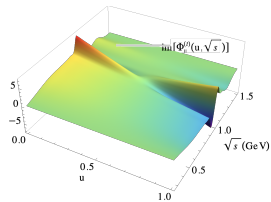
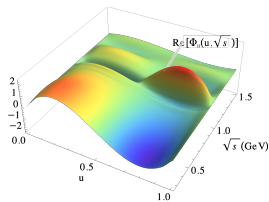
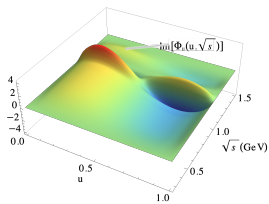
$$\langle \pi^a(k_1) \pi^b(k_2) | \bar{q}_f(zn) \gamma_{\mu} \tau q_f(0) | 0 \rangle = \kappa_{ab} k_{\mu} \int dx e^{iuz(k \cdot n)} \Phi_{\parallel}^{ab,ff'}(u, \zeta, k^2)$$

- improved to twist-three level recently [SC 2025]

$$\langle \pi(k_1) \pi(k_2) | \bar{q}(0) q(x) | 0 \rangle = \int du e^{i\bar{u}k \cdot x} \frac{ik^2(k \cdot x)}{2f_{2\pi}^{\perp}} \Phi_{\parallel}^{(s)},$$

$$\langle \pi(k_1) \pi(k_2) | \bar{q}(0) \sigma^{\mu\nu} q(x) | 0 \rangle = -\frac{i}{f_{2\pi}^{\perp}} \int du e^{i\bar{u}k \cdot x} \left[ \frac{k_{\mu} \bar{k}_{\nu} - k_{\nu} \bar{k}_{\mu}}{2\zeta - 1} \Phi_{\perp} - k^2 \frac{k_{\mu} x_{\nu} - k_{\nu} x_{\mu}}{k \cdot x} \Phi_{\parallel}^{(t)} \right].$$

# $2\pi$ DAs of isospin scalar two-pion system



[Han, et.al., Eur. Phys. J. A (2013) 49: 78]

[Lü, et.al., Phys. Rev. D 75. 056001(2007)]

- asymmetry of the twist-3  $2\pi$ DAs to the parton momentum fraction  $u$
- symmetric for  $f_0$  obtained under the single particle approximation
- where QCD sum rules dictate that the asymmetric component vanishes

## $H \rightarrow \pi\pi$ form factors in $H_{I4}$ decays

- **QCD dynamics of  $H_{I4}$  decays is incorporated in  $H \rightarrow \pi\pi$  form factors** instead of the  $H \rightarrow f_0$  ffs followed by  $f_0 \rightarrow \pi\pi$  in the cascade decay

$$\begin{aligned}
 i\langle \pi^+(k_1)\pi^-(k_2) | \bar{s}\gamma_\nu(1 - \gamma_5)c | D_s(p) \rangle = & F_\perp(q^2, k^2, \zeta) \frac{2}{\sqrt{k^2}\sqrt{\lambda_B}} i\epsilon_{\nu\alpha\beta\gamma} q^\alpha k^\beta \bar{k}^\gamma \\
 & + F_t(q^2, k^2, \zeta) \frac{q_\nu}{\sqrt{q^2}} + F_0(q^2, k^2, \zeta) \frac{2\sqrt{q^2}}{\sqrt{\lambda_B}} \left( k_\nu - \frac{k \cdot q}{q^2} q_\nu \right) \\
 & + F_\parallel(q^2, k^2, \zeta) \frac{1}{\sqrt{k^2}} \left( \bar{k}_\nu - \frac{4(q \cdot k)(q \cdot \bar{k})}{\lambda_B} k_\nu + \frac{4k^2(q \cdot \bar{k})}{\lambda_B} q_\nu \right)
 \end{aligned}$$

- $\lambda = \lambda(m_B^2, k^2, q^2)$  is the Källén function,  $q \cdot k = (m_B^2 - q^2 - k^2)/2$ ,  
 $q \cdot \bar{k} = \sqrt{\lambda}\beta_\pi(k^2) \cos \theta_\pi/2 = \sqrt{\lambda}(2\zeta - 1)$ ,  $\beta_\pi(k^2) = \sqrt{1 - 4m_\pi^2/k^2}$

- **QCDF** (QCD factorization) in the large two-pion mass

• [P. Böer, T. Feldmann and D. van Dyk, JHEP 02 (2017) 133]  $T_I \propto F_{B \rightarrow \pi} \otimes \phi_\pi$ ,

- **SU(3) flavor symmetry/breaking** with the intermediate resonance

• [R.M. Wang, Y.G. Xu, J.H. Sheng, X.D. Cheng and et.al., 2301.00090, PRD 112 (2025) 033002]

# $H \rightarrow \pi\pi$ form factors

- **LQCD** (Lattice QCD) in the  $\rho$  resonance region with a simple BW model
  - [L. Leskovec and et.al, PRL 134 (2025) 161901, **Editors' Suggestion**]
- **HChPT** (Heavy-meson Chiral Perturbative Theory) in the large  $q^2$  by taking dispersive methods in terms of Omnés functions
  - [X.-W. Kang, B. Kubis, C. Hanhart, and U.-G. Meißner, PRD 89 (2014) 053015]

in the full phase-space by a novel parameterization with unitarity

  - [F. Herren, B. Kubis and R. van Tonder, PRD 112 (2025) 014037, **Editors' Suggestion**]
- **LCSRs** (Light-cone sum rules) in the small and intermediate  $q^2$ 
  - [SC, A. Khodjamirian and J. Virto, JHEP 05 (2017) 157]  **$B$ -meson LCSRs**, [S. Descotes-Genon, A. Khodjamirian, J. Virto and K.K. Vos, JHEP 12 (2019) 083, 06 (2023) 034]  **$B \rightarrow K\pi$**
  - [C. Hambrock and A. Khodjamirian, NPB 905 (2016) 379-390]  **$2\pi$ DAs LCSRs of  $F_{\parallel, \perp}$**
  - [SC, A. Khodjamirian and J. Virto, PRD(R) 96 (2017) 051901] **timelike-helicity FF  $F_t$  and  $F_0$**
  - [SC, PRD 99 (2019) 053005]  **$2\pi$ DAs updates and  $B \rightarrow [\pi\pi]_{S,P}$  FFs**
  - [SC and J.M Shen, EPJC 6 (2020) 554, SC and S.L Zhang, EPJC 84 (2024) 379] **Pheno**
  - [SC, PRD 112 (2025) L111301] first study of **twist-three  $2\pi$ DAs and  $|V_{ub}|$  extraction**
  - [SC, L.Y. Dai, J.M. Shen and S.L. Zhang, PRD 113 (2026) L031901]  **$D_s \rightarrow [\pi\pi]_S e\nu$  this talk**

# $D_s \rightarrow [\pi\pi]_S e^+ \nu_e$ decay

- $D_s \rightarrow f_0 e^+ \nu$  [CLEO '09],  $D_{(s)} \rightarrow a_0 e^+ \nu$  [BESIII '18, '21],  $D^+ \rightarrow f_0/\sigma e^+ \nu$  [BESIII '19]
- $D_s \rightarrow f_0 (\rightarrow \pi^0 \pi^0, K_S K_S) e^+ \nu$  [BESIII 22],  $D_s \rightarrow f_0 (\rightarrow \pi^+ \pi^-) e^+ \nu$  [BESIII 23]

$$\mathcal{B}(D_s \rightarrow f_0 (\rightarrow \pi^0 \pi^0) e^+ \nu) = (7.9 \pm 1.4 \pm 0.3) \times 10^{-4}$$

$$\mathcal{B}(D_s \rightarrow f_0 (\rightarrow \pi^+ \pi^-) e^+ \nu) = (17.2 \pm 1.3 \pm 1.0) \times 10^{-4}$$

$$f_+^0(0) |V_{cs}| = 0.504 \pm 0.017 \pm 0.035$$

- single particle (narrow width limit)  $D_s \rightarrow f_0 e^+ \nu$

$$\frac{d\Gamma(D_s^+ \rightarrow f_0 l^+ \nu)}{dq^2} = \frac{G_F^2 |V_{cs}|^2 \lambda^{3/2}(m_{D_s}^2, m_{f_0}^2, q^2)}{192\pi^3 m_{D_s}^3} |f_+(q^2)|^2, \quad D_s \rightarrow f_0 \text{ FF}$$

- improvement with the width effect by resonant models  $D_s \rightarrow [f_0 \rightarrow] \pi\pi e^+ \nu$

$$\frac{d\Gamma(D_s^+ \rightarrow [\pi\pi]_S l^+ \nu)}{dsdq^2} = \frac{1}{\pi} \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_{D_s}^3} |f_+(q^2)|^2 \frac{\lambda^{3/2}(m_{D_s}^2, s, q^2) g_1 \beta_\pi(s)}{|m_S^2 - s + i(g_1 \beta_\pi(s) + g_2 \beta_K(s))|^2}, \quad \text{BESIII}$$

- calculate directly the signal channel  $D_s \rightarrow [\pi\pi]_S e^+ \nu$

$$\frac{d^2\Gamma(D_s^+ \rightarrow [\pi\pi]_S l^+ \nu)}{dsdq^2} = \frac{G_F^2 |V_{cs}|^2 \beta_{\pi\pi}(s) \sqrt{\lambda_{D_s} q^2}}{3(4\pi)^5 m_{D_s}^3} \sum_{\ell=0}^{\infty} |F_0^{(\ell)}(q^2, s)|^2, \quad D_s \rightarrow \pi\pi \text{ FF}$$

# $D_s \rightarrow f_0$ FFs and cascade decay $D_s \rightarrow (f_0 \rightarrow) \pi\pi e^+ \nu$

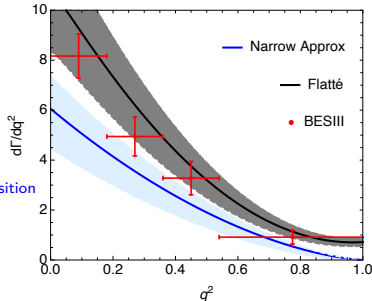
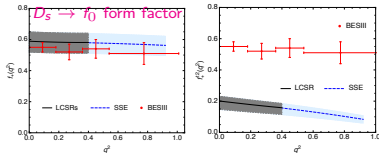
- $\{M^2, s_0\} = \{5.0 \pm 0.5, 6.0 \pm 0.5\} \text{GeV}^2$

| this work       | 3pSRs(07) | LFQM(09) | CLFD/DR(08) | LCSRs(10)       |
|-----------------|-----------|----------|-------------|-----------------|
| $0.63 \pm 0.04$ | 0.96      | 0.87     | 0.86/0.90   | $0.30 \pm 0.03$ |

- the BESIII result in the  $\pi^+\pi^-$  system  $f_+(0) = 0.518 \pm 0.018 \pm 0.036$  [BESIII 23]

different input of the decay constant  $\tilde{f}_{f_0} = 335 \text{ MeV}$ , much larger than 180 MeV in LCSRs(10)  
we add the first gegenbauer expansion terms in the LCDAs, up-to-date parameters

$\bar{s}s - \bar{n}n$  mixing scenario of  $f_0$  with  $\theta = 20^\circ \pm 10^\circ$



- Twist-3 LCDAs give dominate contribution in  $D_s \rightarrow f_0$  transition

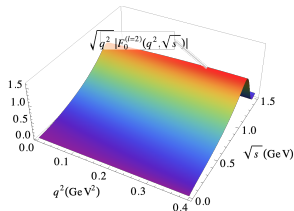
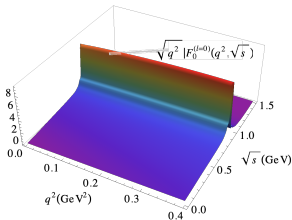
- the uncertainty estimation is conservative
- without NLO correction
- we need a model independent calculation
- for the QCD understanding
- and the future partial-wave measurement

Differential decay width of  $D_s^+ \rightarrow (f_0 \rightarrow) [\pi\pi]_S e^+ \nu_e$

# $D_s \rightarrow [\pi\pi]_S$ FFs and $D_s \rightarrow [\pi\pi]_S e^+ \nu$ decay

- The LCSRs  $\ell'$ -wave  $D_s \rightarrow [\pi\pi]_S$  form factors ( $\ell' = \text{even} \ \& \ \ell' \leq n + 1$ )

$$\sqrt{q^2} F_0^{(\ell')} (q^2, k^2) = \frac{m_c(m_c + m_s) \sqrt{q^2} \sqrt{\lambda_{D_s}}}{m_{D_s}^2 f_{D_s}} \sum_{n=1, \text{odd}}^{\infty} \frac{\beta_{\pi}(k^2)}{\sqrt{2\ell' + 1}} J_n^0(q^2, k^2, M^2, s_0) B_{n\ell', \parallel}^{l'=0}(k^2) I_{\ell\ell'}$$

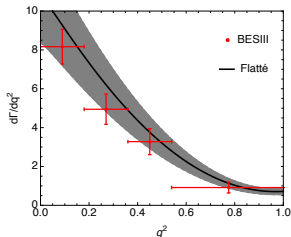
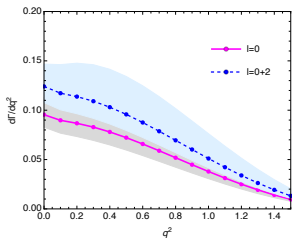


- Twist-2 and twist-3 contributions to  $D_s \rightarrow \pi\pi, f_0$  form factors at  $q^2 = 0$   
under the  $q\bar{q}$  approximation

| Form Factors  | Twist-2  | Twist-3   | Total   |
|---|--|---|---|
| $\sqrt{q^2} F_0^{(l=0)}(0) = \sqrt{q^2} F_t^{(l=0)}(0)$ | $0.20_{-0.02}^{+0.02} - i0.24_{-0.02}^{+0.02}$ | $-0.41_{-0.05}^{+0.04} + i0.51_{-0.04}^{+0.02}$ | $-0.21_{-0.01}^{+0.02} + i0.27_{-0.02}^{+0.03}$ |
| $\sqrt{q^2} F_0^{(l=2)}(0) = \sqrt{q^2} F_t^{(l=2)}(0)$ | $0.27_{-0.02}^{+0.03} + i0.21_{-0.01}^{+0.02}$ | $-0.55_{-0.03}^{+0.02} - i0.41_{-0.04}^{+0.05}$ | $-0.28_{-0.02}^{+0.02} - i0.20_{-0.01}^{+0.02}$ |
| $f_+(0) = f_0(0)$                                       | $0.20_{-0.05}^{+0.03}$                         | $0.41_{-0.06}^{+0.04}$                          | $0.61_{-0.07}^{+0.05}$                          |

# $D_s \rightarrow [\pi\pi]_S$ FFs and $D_s \rightarrow [\pi\pi]_S e^+ \nu$ decay

- The differential decay width on momentum transfers



| $D_s \rightarrow [\pi\pi]_S e^+ \nu_e$ | $D_s \rightarrow [f_0 \rightarrow \pi\pi] e^+ \nu_e$ [23] | Data [25]      |
|--|---|----------------|
| $0.81^{+0.34}_{-0.14}$                 | $18.8^{+4.5}_{-3.8}$                                      | $17.2 \pm 1.6$ |

- non- $q\bar{q}$  Fock states are the dominate component of  $[\pi\pi]_S$  in charm decays
- in consistent with the assessment of color transparency
- much different in  $B$  decays non-asymptotic leading twist dominated [SC 2025]
- go further to multi-particle DiPion LCDAs in CHARM ( $q\bar{q}g, q\bar{q}q\bar{q})$

## Summary and Prospect

- $2\pi$ DAs provide a general description of the  $\pi\pi$  mass spectrum
- $2\pi$ DAs are a crucial input for the study of  $H_{I4}$  decays
- $2\pi$ DAs have been studied at the three-twist level, explaining the multi-particle picture of scalar mesons
- wishlists related to the (future) colliders
  - possible anomalies in the FCNC processes of the  $D \rightarrow \pi\pi I^+ I^-$  decay
  - the processes  $e^+e^-$  annihilation  $\gamma^* \rightarrow \pi\pi\gamma$  and  $\gamma^*\gamma \rightarrow \pi\pi$

Thank you for your patience.