

Thermal production of gravitational waves in the early universe

Xun-Jie Xu

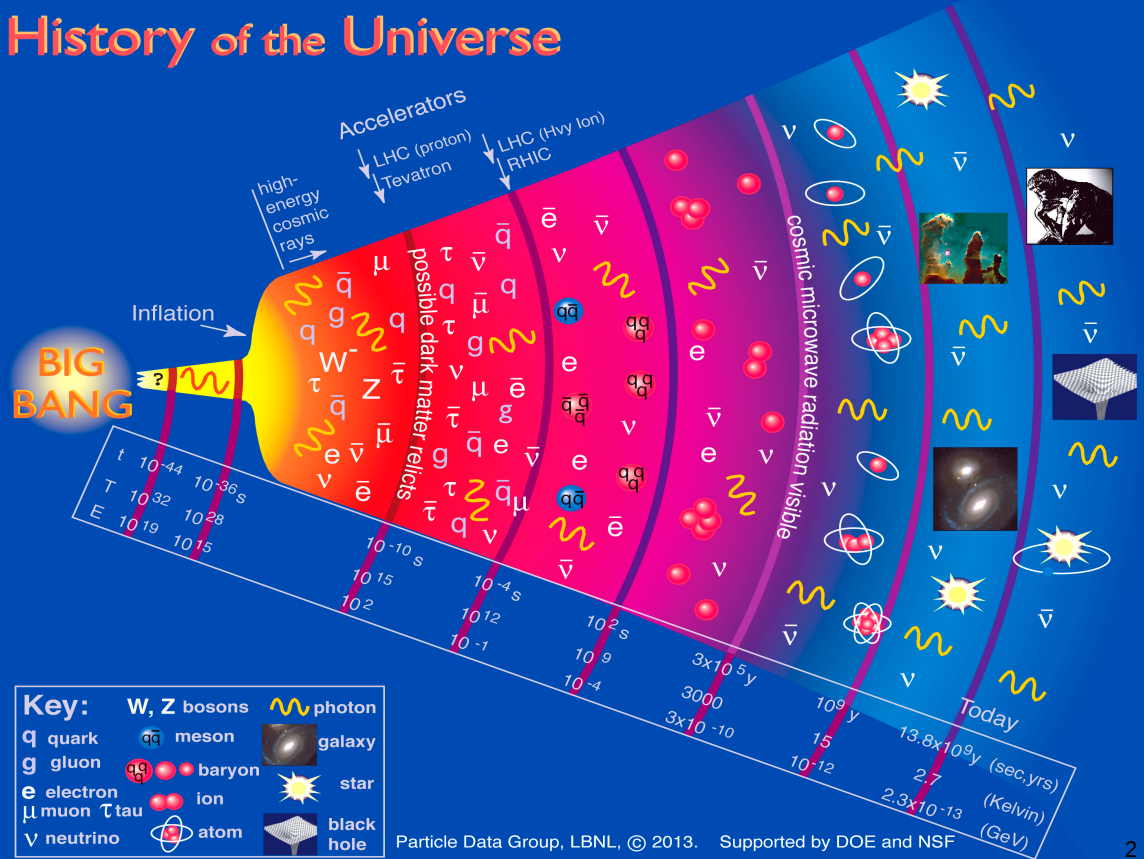
Institute of High Energy Physics (IHEP)
Chinese Academy of Sciences (CAS)



<https://xunjiexu.github.io/>

Based on 2503.10756 and 2505.08868 (JHEP $\times$ 2)

History of the Universe

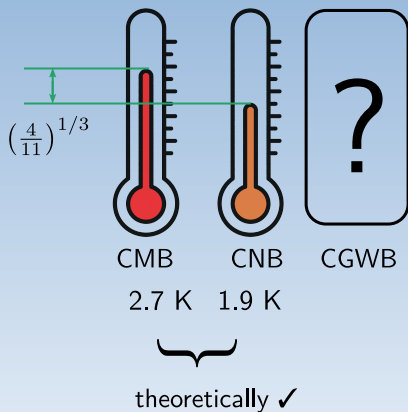


The Cosmic Gravitational Wave Background (CGWB)

the lightest species: $\gamma \rightarrow \text{CMB}$, $\nu \rightarrow \text{CNB}$, graviton/GW $\rightarrow \text{CGWB}$

observation ✓
 $\Rightarrow$ modern cosm.

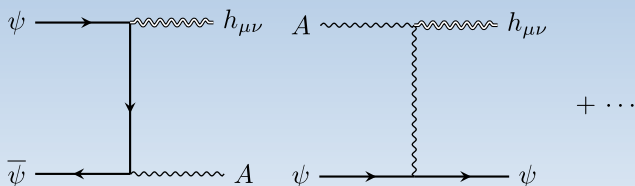
predicted, not detected yet



because
 known interactions
 known thermal history

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} M_P^2 \mathcal{R} + \mathcal{L}_{\text{SM}} + \dots \right)$$

gravitons,
 with known interactions



hot SM $\xrightarrow[\text{high E}]{\text{scattering}}$ $h_{\mu\nu}$ $\xrightarrow{\text{red shift}}$ CGWB

Just like CMB/CNB, CGWB $\in$ thermal relics

Main diff: ~~equilibrium~~

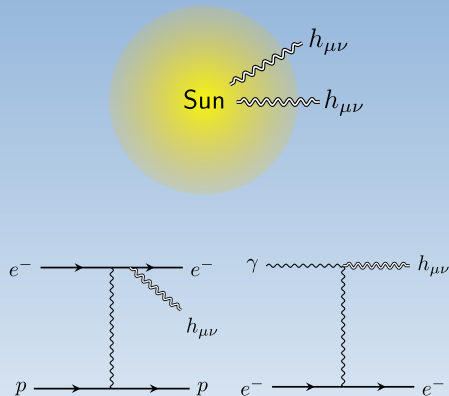
abundance $\Leftarrow$ thermal production

Infrared Photons and Gravitons*

STEVEN WEINBERG†

Department of Physics, University of California, Berkeley, California

(Received 1 June 1965)



Recently recalculated by
Camilo Garcia-Cely, Andreas Ringwald, PRL
[2407.18297]

The solar gravitational radiation power is then

$$P_{\odot} \simeq 6 \times 10^{14} \text{ erg/sec.} = 6 \times 10^7 \text{ Watt} \quad (4.24)$$

Although this is not much more power than used by the city of Berkeley, it nevertheless compares favorably with the gravitational radiation from previously considered classical sources like planetary motion. A planet of mass m moving in a circular orbit of radius R around a star of mass M emits gravitational radiation with power

$$P = (32/5) G c^{-5} m^2 R^4 (GM/R^3)^3. \quad (4.25)$$

For the Jupiter-Sun system this is 7.6×10^{11} erg/sec. Venus and the Earth radiate comparable amounts, and the other planets considerably less, so the thermal gravitational radiation (4.14) from the Sun appears to be the dominant source of gravitational radiation from the solar system. A binary star like Sirius A and B radiates more classically—in this case Eq. (4.25) gives 8×10^{14} erg/sec—but it also radiates more thermally. Thus thermal collisions possibly may provide the most important source of gravitational radiation in the universe. I have no idea what it is good for.

CGWB from SM: from LLO (leading-log order) to complete LO

Gravitational wave background from Standard Model physics: Complete leading order #1

J. Ghiglieri (SUBATECH, Nantes), G. Jackson (U. Bern, AEC), M. Laine (U. Bern, AEC), Y. Zhu (Munich, Tech. U.) (Apr 23, 2020)

Published in: *JHEP* 07 (2020) 092 • e-Print: [2004.11392](#) [hep-ph]

pdf DOI cite claim reference search 89 citations

Gravitational wave background from Standard Model physics: Qualitative features #2

J. Ghiglieri (U. Bern, AEC), M. Laine (U. Bern, AEC) (Apr 10, 2015)

Published in: *JCAP* 07 (2015) 022 • e-Print: [1504.02569](#) [hep-ph]

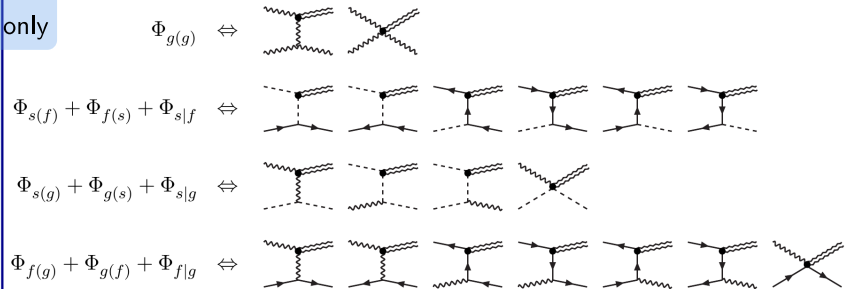
pdf DOI cite claim reference search 95 citations

LO

LLO

Not an easy task
... even t-channel only

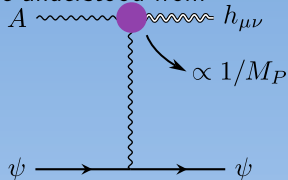
IR divergence
Debye screening
...



How does CGWB look like? — similar to CMB

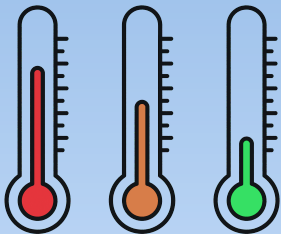
Height:

... can be understood from



f_{peak} :

... colder than CMB, due to g_*

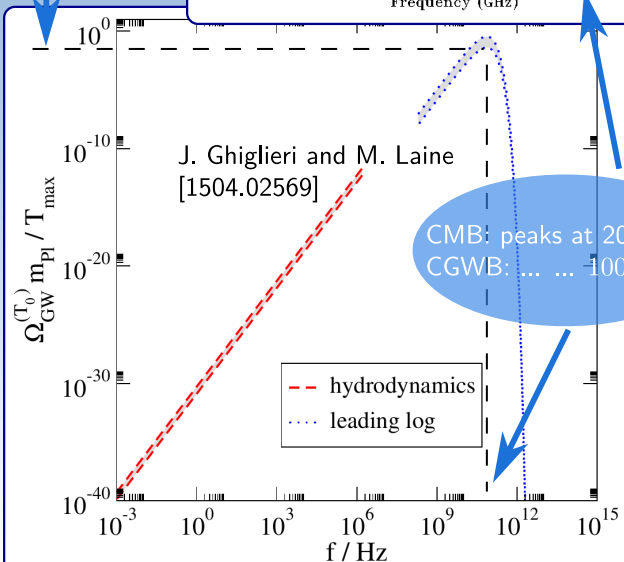
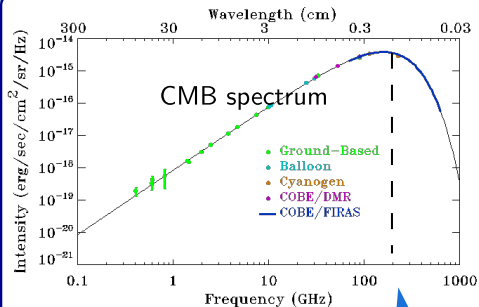


CMB

CNB

CGWB

$$\text{Height} \propto \frac{T}{M_P}$$



J. Ghiglieri and M. Laine
[1504.02569]

CMB: peaks at 200 GHz
CGWB: 100 GHz

Gravitational Waves as a Big Bang Thermometer

#1

Andreas Ringwald (DESY), Jan Schütte-Engel (Hamburg U. and Illinois U., Urbana), Carlos Tamarit (Munich, Tech. U.) (Nov 9, 2020)

Published in: JCAP 03 (2021) 054 • e-Print: [2011.04731](https://arxiv.org/abs/2011.04731) [hep-ph]

pdf

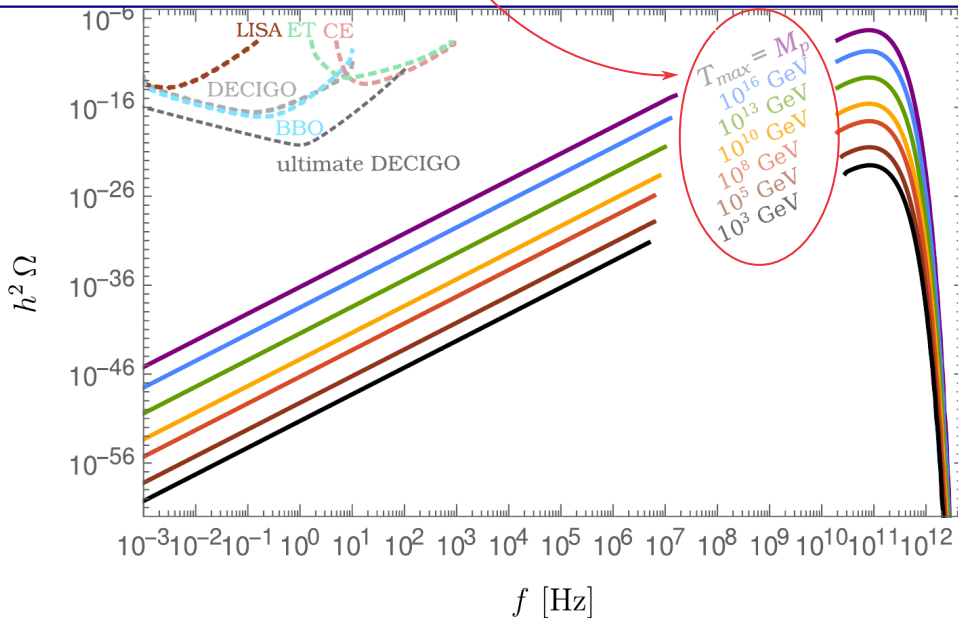
DOI

cite

claim

reference search

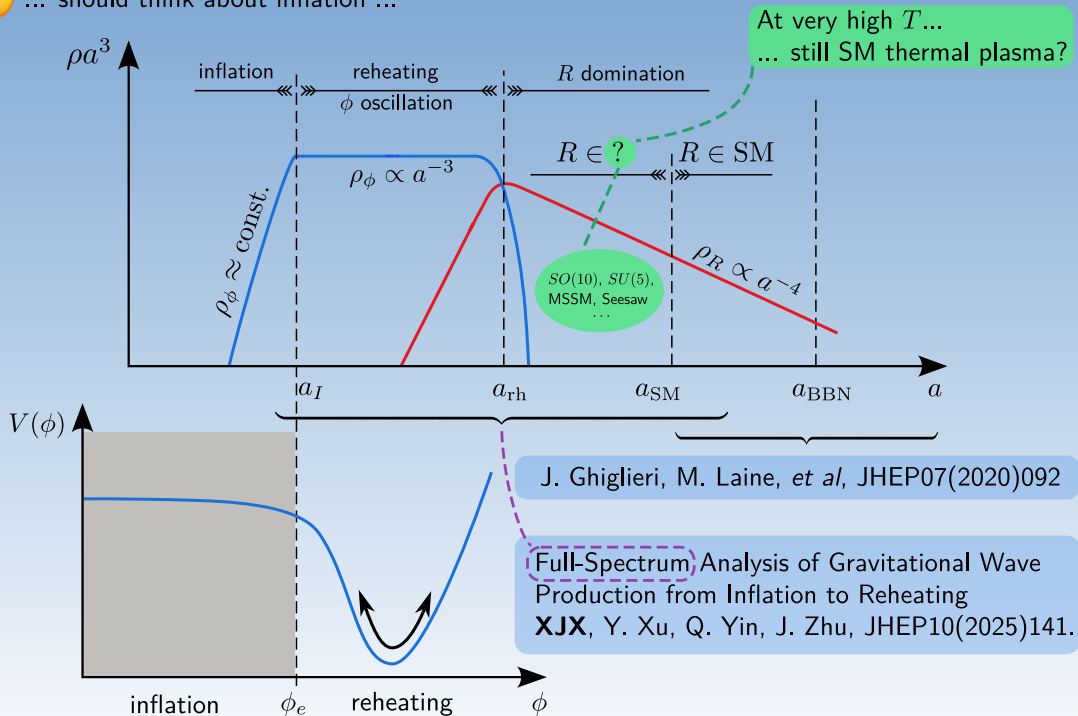
115 citations

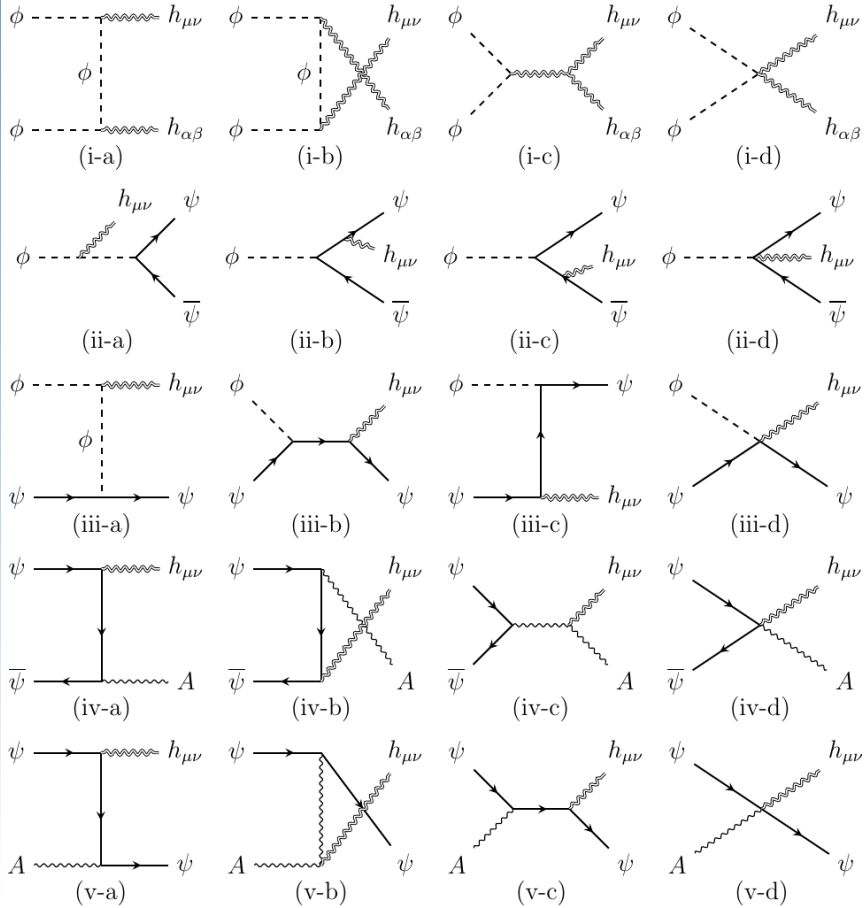


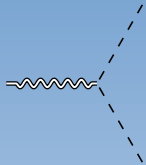
How high has T ever reached?



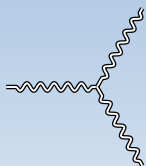
... should think about inflation ...







$$= -\frac{i}{M_P} [p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - \eta^{\mu\nu} (p_1 \cdot p_2 - m_\phi^2)]$$



$$=$$

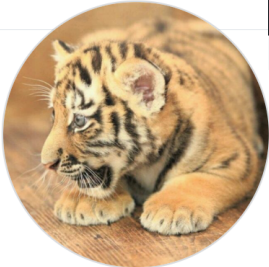
(* Created with the Wolfram Language : www.wolfram.com *)

```
(2*I)*K*FV[ph1, e]*FV[ph1, f]*MT[a, d]*MT[b, c] +
((3*I)/2)*K*FV[ph1, f]*FV[ph2, e]*MT[a, d]*MT[b, c] +
((3*I)/2)*K*FV[ph1, e]*FV[ph2, f]*MT[a, d]*MT[b, c] +
(2*I)*K*FV[ph2, e]*FV[ph2, f]*MT[a, d]*MT[b, c] +
I*K*FV[ph1, f]*FV[ph3, e]*MT[a, d]*MT[b, c] +
I*K*FV[ph2, f]*FV[ph3, e]*MT[a, d]*MT[b, c] +
I*K*FV[ph1, e]*FV[ph3, f]*MT[a, d]*MT[b, c] +
I*K*FV[ph2, e]*FV[ph3, f]*MT[a, d]*MT[b, c] +
I*K*FV[ph1, e]*FV[ph3, f]*MT[a, d]*MT[b, c] -
I*K*FV[ph1, d]*FV[ph1, f]*MT[a, e]*MT[b, c] -
I*K*FV[ph1, f]*FV[ph2, d]*MT[a, e]*MT[b, c] -
(I/2)*K*FV[ph1, d]*FV[ph2, e]*MT[a, f]*MT[b, c] -
I*K*FV[ph2, d]*FV[ph2, f]*MT[a, e]*MT[b, c] -
(I/2)*K*FV[ph1, f]*FV[ph3, d]*MT[a, e]*MT[b, c] -
(I/2)*K*FV[ph2, f]*FV[ph3, d]*MT[a, e]*MT[b, c] -
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(I/2)*K*FV[ph1, d]*FV[ph2, e]*MT[a, f]*MT[b, c] -
I*K*FV[ph2, d]*FV[ph2, e]*MT[a, f]*MT[b, c] -
(I/2)*K*FV[ph1, e]*FV[ph3, d]*MT[a, f]*MT[b, c] -
(I/2)*K*FV[ph2, e]*FV[ph3, d]*MT[a, f]*MT[b, c]
```

First 24 terms
495 terms in total
So, don't compute it manually!

A few words about tools:

- FeynGrav [2406.14872] is useful, but ...
- Anna Tokareva's lecture [2412.08690] and github (github.com/tokareva90) very useful!
- We rebuilt the tool to automatically derive Feynman rules and calculate diagrams



Quan-feng WU (吴泉锋)
Fenyutanchan

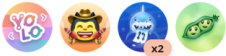
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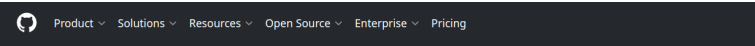
Theoretical Physics PostDoc.

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TPD, IHEP, CAS
Beijing 100049, China
15:47 - same time

Achievements





Fenyutanchan / **amplitude-for-inflaton-reheaton-graviton** Public

README MIT license

Amplitude Calculation of Inflaton, Reheaton, and Graviton

Main Note

See [note](#) and [arXiv:2503.10756](#) (on [INSPIRE-HEP](#)).

Requirement

- [Wolfram Mathematica](#)
- [xAct](#)
- [FeynCalc](#)

Repository Structure

- [Mathematica notebooks](#): The directory of the reproducible Mathematica notebooks.
- [note](#): The L^AT_EX source directory of the main note.

License

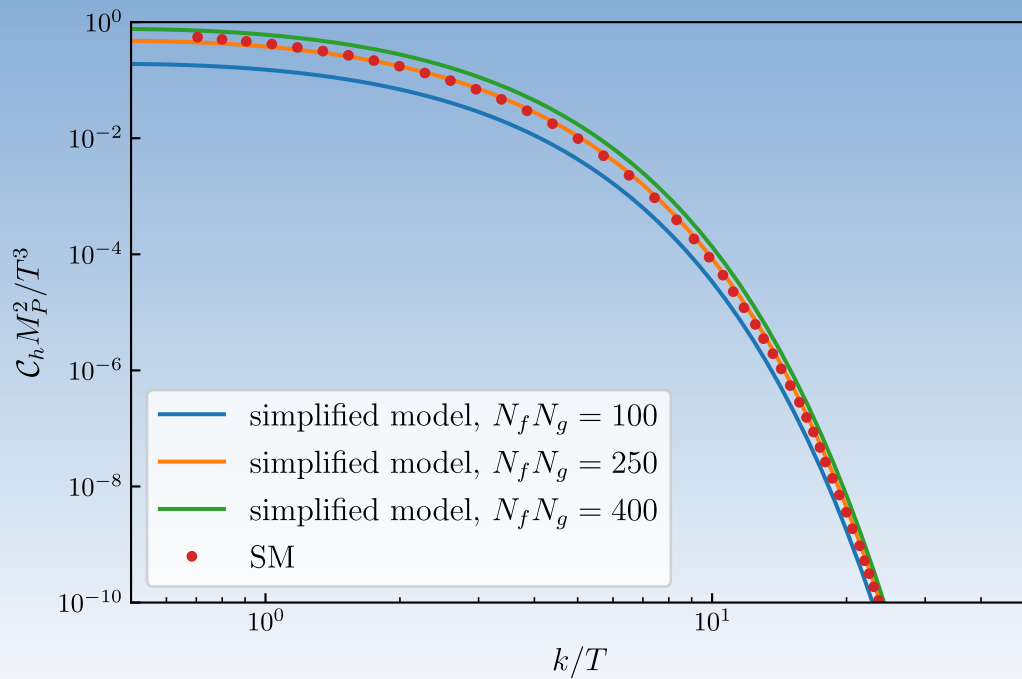
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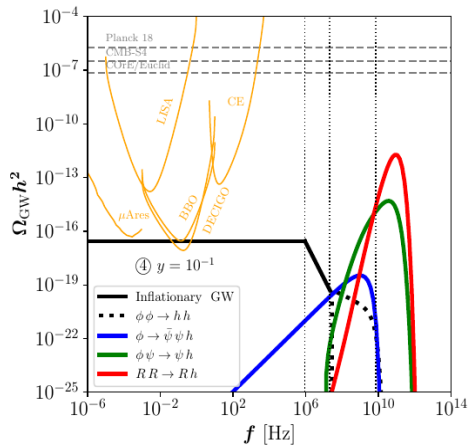
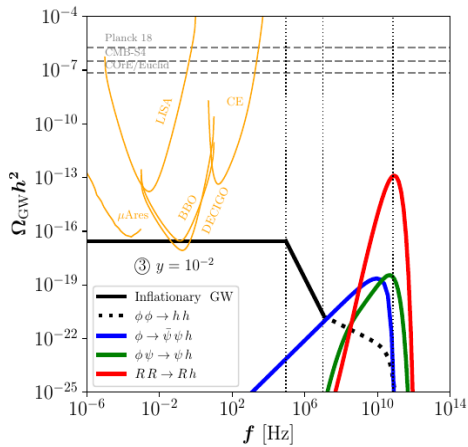
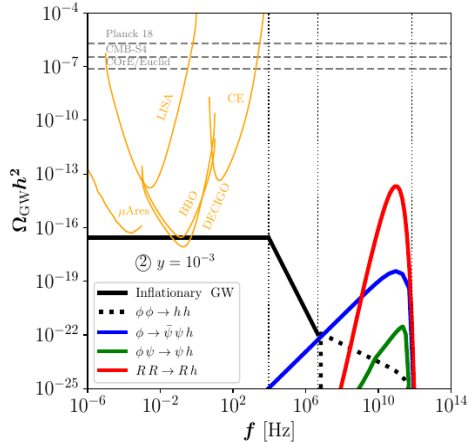
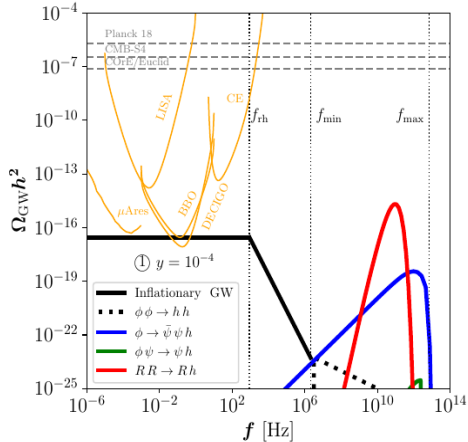
Citation

Please cite the paper of [arXiv:2503.10756](#) (on [INSPIRE-HEP](#)) if you use this code in your research:

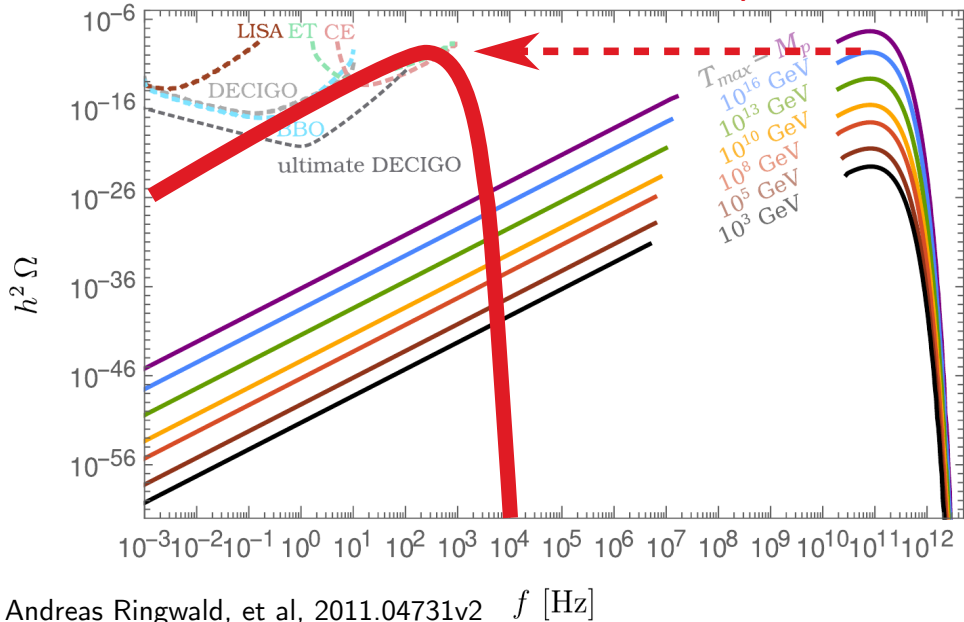
mature software/packages

<https://github.com/Fenyutanchan/amplitude-for-inflaton-reheaton-graviton>



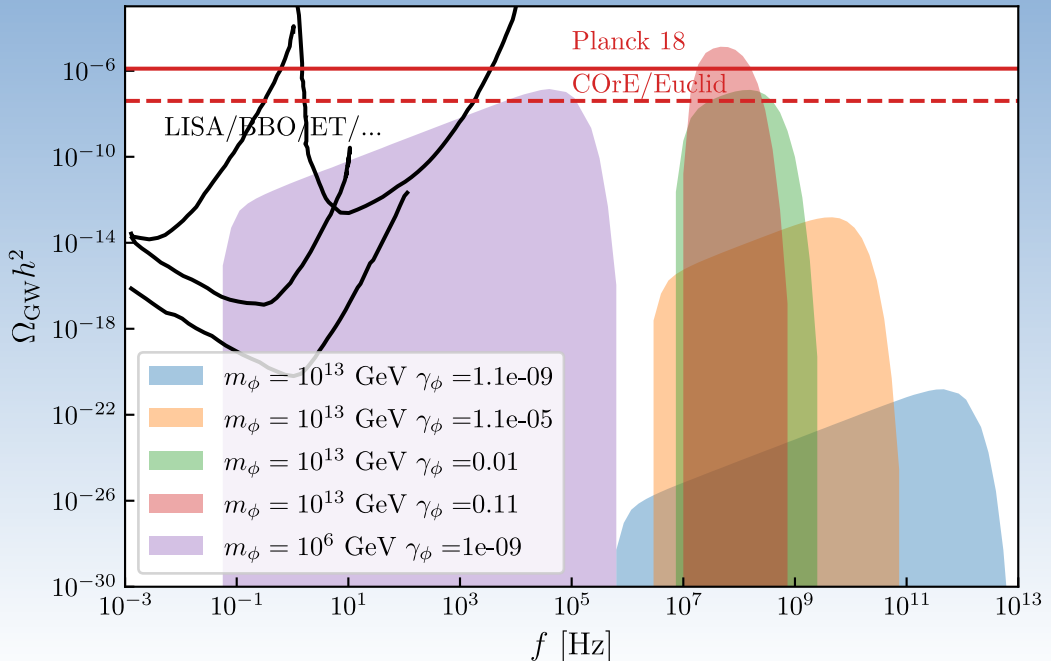


Can we shift CGWB to lower frequencies?



Yes, if inflatons are light but dense ...

Nicolás Bernal, Quan-feng Wu, XJX, Yong Xu,
Pre-thermalized Gravitational Waves, [2503.10756].



Why it works?

Relations between GW properties and thermal plasma properties:

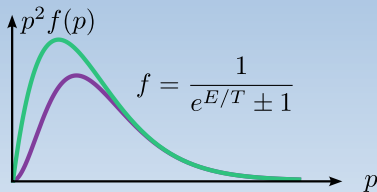
Hot (high $\langle E \rangle$)	Cold (low $\langle E \rangle$)	
Dense (high n)	Rarefied (low n)	

f_{peak}
height } of Ω_{GW}

Want high Ω_{GW} at lower f_{peak} ? Take "Cold+Dense"!

Is it possible to increase n without increasing T ?

Yes, bosons only, fermions always $f \leq 1$
(Pauli blocking)



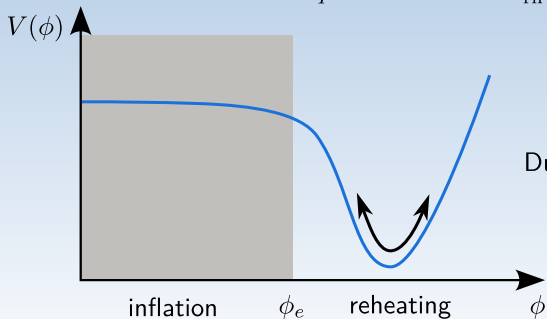
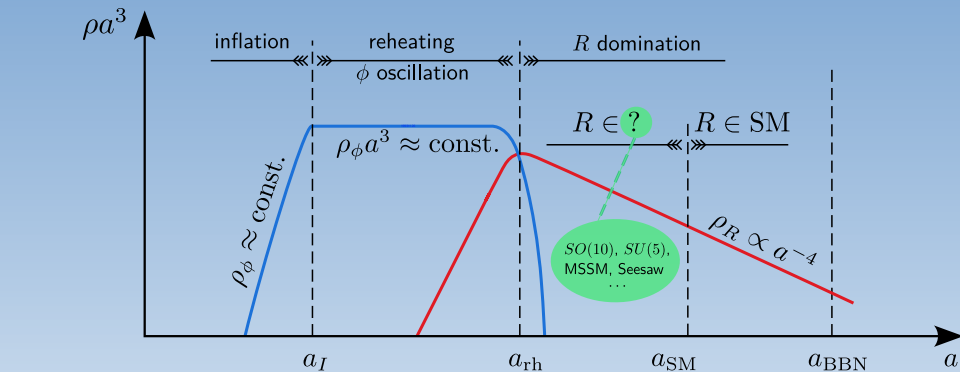
$$n = \frac{\zeta(3)}{\pi^2} T^3 \times \begin{cases} 1 & (\text{BE}) \\ 3/4 & (\text{FD}) \end{cases}$$



- typically, $f \lesssim 1$.
- Therefore, $n \sim 0.1 T^3$
- Can we have $n = 1000 T^3$?

... cold but dense ...

How?

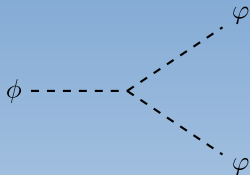


During reheating, inflaton itself is cold & dense.

Can radiation (R) inherit this feature?

Consider ...

inflaton (ϕ) decay to bosonic radiation (φ , massless)



$$\langle p_\varphi \rangle = \frac{1}{2} m_\phi$$

$$n_\varphi = 2n_\phi$$

Inflaton is cold ($p_\phi = 0$), and can be dense ($n_\phi \gg \frac{1}{\pi^2} m_\phi^3$).

The produced radiation would inherit “cold & dense” from it:

$$n_\varphi \gg \frac{2}{\pi^2} m_\phi^3 \gg \frac{1}{\pi^2} \langle p_\varphi \rangle^3$$

very small $m_\phi \rightarrow$
e.g. 10^{10} GeV

$$n_\varphi = 10000000000 T^3$$



So far, so good ...

However,

$$n_\varphi/T^3 \gg 1 \xrightarrow[\text{kinetic equilibrium}]{\text{chemical equilibrium}} ???$$

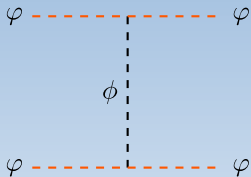
However, $n_\varphi/T^3 \gg 1$ $\xrightarrow[\text{kinetic equilibrium}]{\text{chemical equilibrium}}$ $n_\varphi/T^3 = 0.1$



Can φ reach KEQ and CEQ?

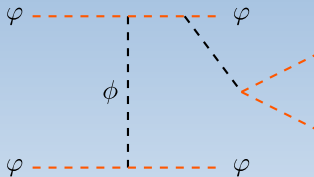
Leading order

KEQ $\Leftarrow$ sufficient collisions
to exchange momentum
CEQ $\Leftarrow$ sufficient N_φ collisions
to change particle number



Yes & No, depending on ...

We choose coupling $\rightarrow$ No KEQ



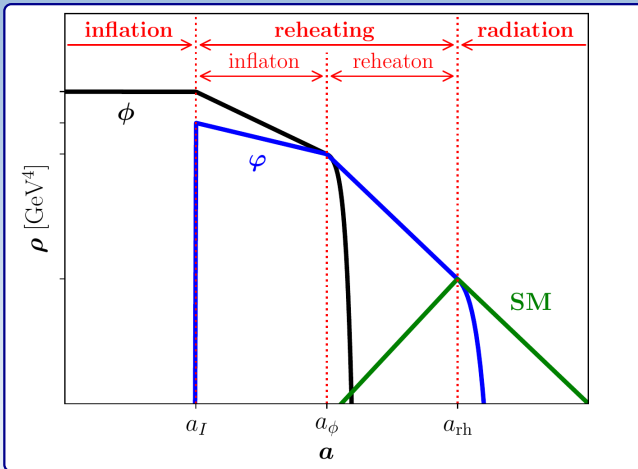
No! Too much suppressed

Adding reheaton φ to the game

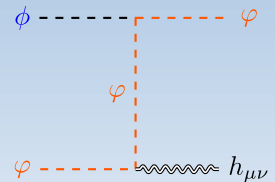
Reheaton (φ) $\left\{ \begin{array}{l} \text{to connect } \phi \text{ and SM} \\ \text{to inherit "cold\&dense" from } \phi \end{array} \right.$

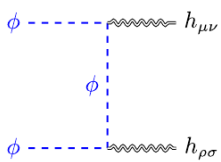
Requirement:

- interacts weakly
- behaves as radiation

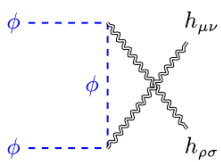


One of the dominant GW production channels

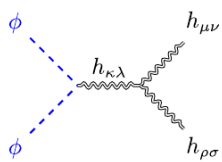




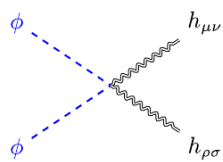
II-A



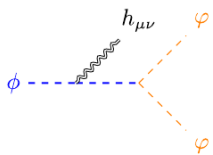
II-B



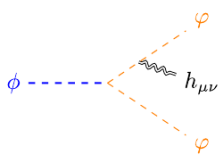
II-C



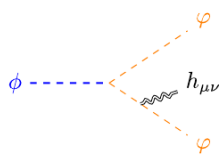
II-D



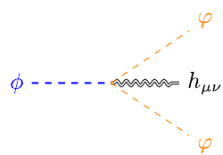
III-A



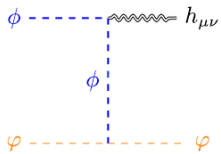
III-B



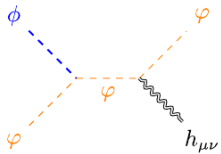
III-C



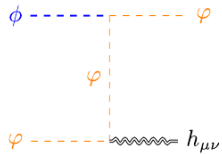
III-D



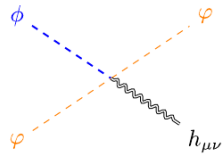
IV-A



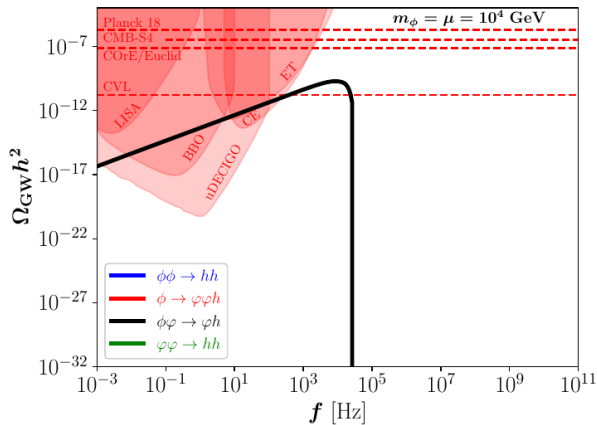
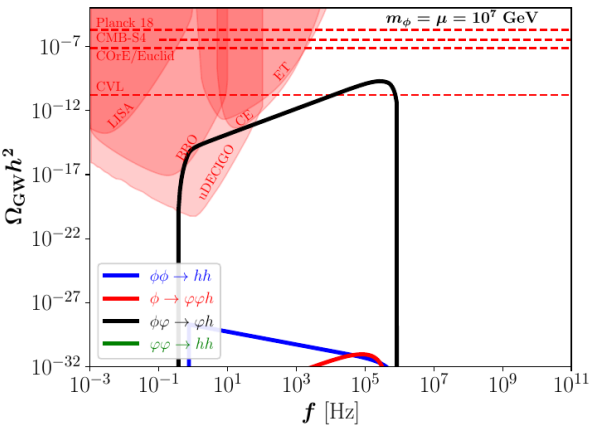
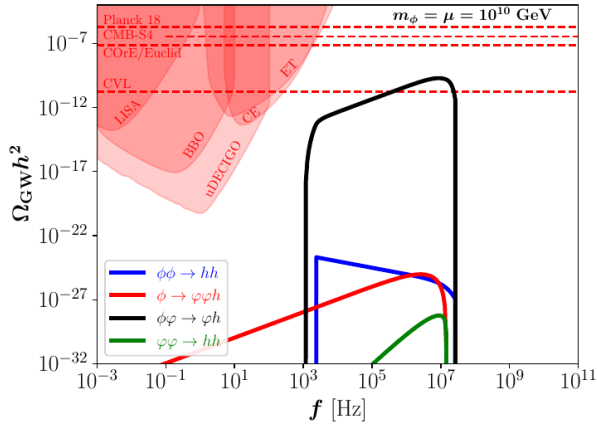
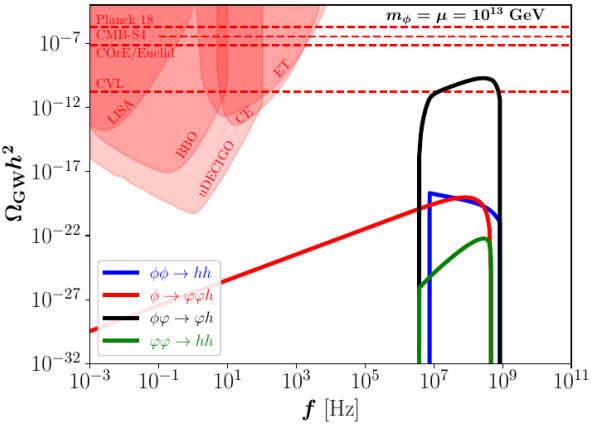
IV-B



IV-C

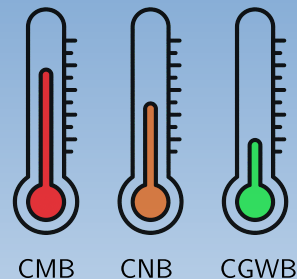


IV-D

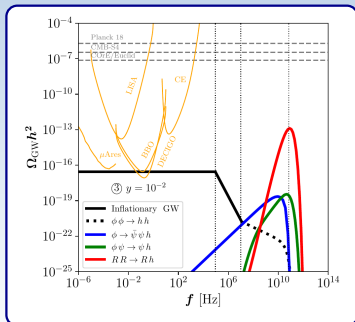


Summary

- CGWB = Cosmic Gravitational Wave Background
 - guaranteed background
 - the only assumption: the Universe was hot and dense
 - peaks at ~ 100 GHz
- Taking inflation into account
 - finite $T_{\max}$
 - transition from inflation $\rightarrow$ reheating $\rightarrow$ SM

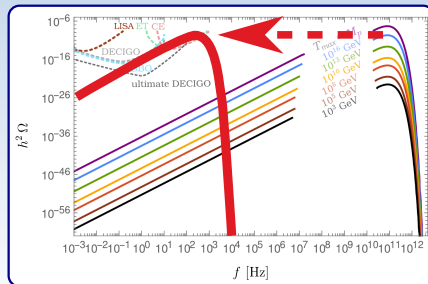


Full-spectrum analysis



- If ... “cold&dense” ...

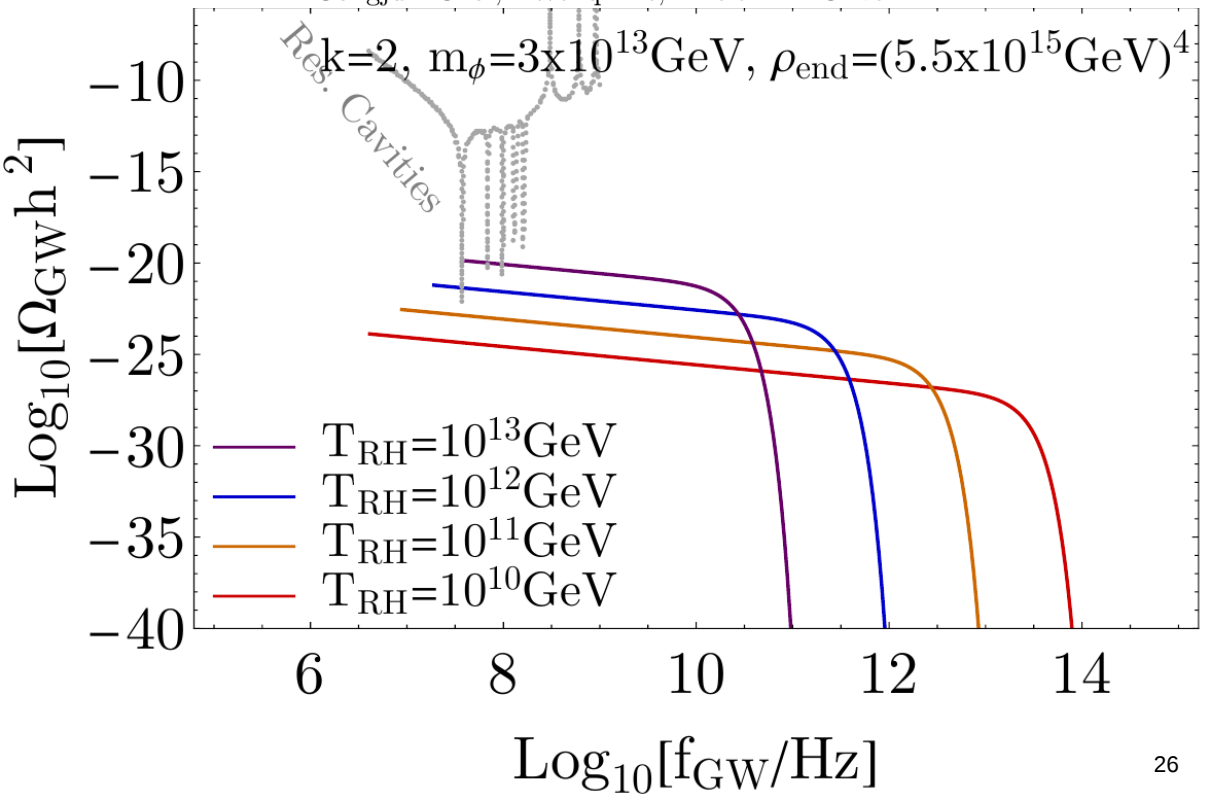
– motivated/achieved by reheaton



Backup

Minimal Production of Prompt Gravitational Waves during Reheating

Gongjun Choi,^{*} Wenqi Ke,[†] Keith A. Olive[‡] 2402.04310



A slide taken from Carlos Tamarit's talk.

<https://indico.cern.ch/event/1074510/contributions/4519382/>

How to detect high-freq gravitational waves?

Direct searches

Interferometers [Nishizawa et al, Akutsu et al, Chou et al]

Cavity experiments measuring **rotation of polarization** [Cruise & Ingley]

Experiments measuring **resonant spin precession** of electrons [Ito et al]

Inverse Gertsenshtein effect: Partial conversion of gravitational waves into electromagnetic radiation

Can use **axion experiments** featuring photon-axion conversion!
[Cruise, Ellji et al]

Indirect searches

Gertsenshtein effect can modify the **CMB** spectrum! [Domcke & Cely]