

Beyond general relativity: gravitational waves in non-minimally coupled theories

**Workshop on Gravitational Waves,
Cosmology, and Fundamental Physics**

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Brown University &
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Cosmology, and Fundamental Physics

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Halloween, 2025



Collaborators



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Based on:

2510.25895 S. Alexander, T. Daniel, T.M.,
T.M., S. Alexander, 2401.14452 (PRD),
S. Alexander, T. Daniel, L. Jenks, 2403.09373 (PRD)

Some motivation



Image credit: Astronomy.com

- We **only** know dark matter couples to gravity
- DM density increases around black holes
- BH/BH mergers are thus prime settings for probing DM physics
- And of course... strong field gravity: Beyond GR?

What can we learn about DM + beyond GR from GW observations?

Many efforts on this front

‘Probing Interacting Dark Sector with the next generation of gravitational-wave detectors,’ Califano et. al., 2410.06152

‘Quasinormal modes of Schwarzschild-like black hole surrounded by the pseudo-isothermal dark matter halo,’ Liu et. al., 2409.20333

* ‘Probing stochastic ultralight dark matter with space based gravitational wave interferometers,’ Y. H. Yao, Y. Tong 2412.21137

‘Detectability of dark matter density distribution via gravitational waves from binary black holes in the galactic center,’ Li, Guo, Cao, Zhang, 2506.19327

... to name just a few

‘Probing dark-matter effects with gravitational waves using the parameterized post-Einsteinian framework,’ Wilcox et. al., 2409.10846

‘Compact Binary Merger Rate with Modified Gravity in Dark-Matter Spikes,’ Fakhry et. al., 2408.11995

* ‘Probing gravitational dark matter with high frequency gravitational waves,’ Y. Xu, 2412.21137

Roadmap for the talk

- Model independent beyond GR parameterization
- Example models in simplified scenarios
- Discussion



Capturing model independent effects beyond GR

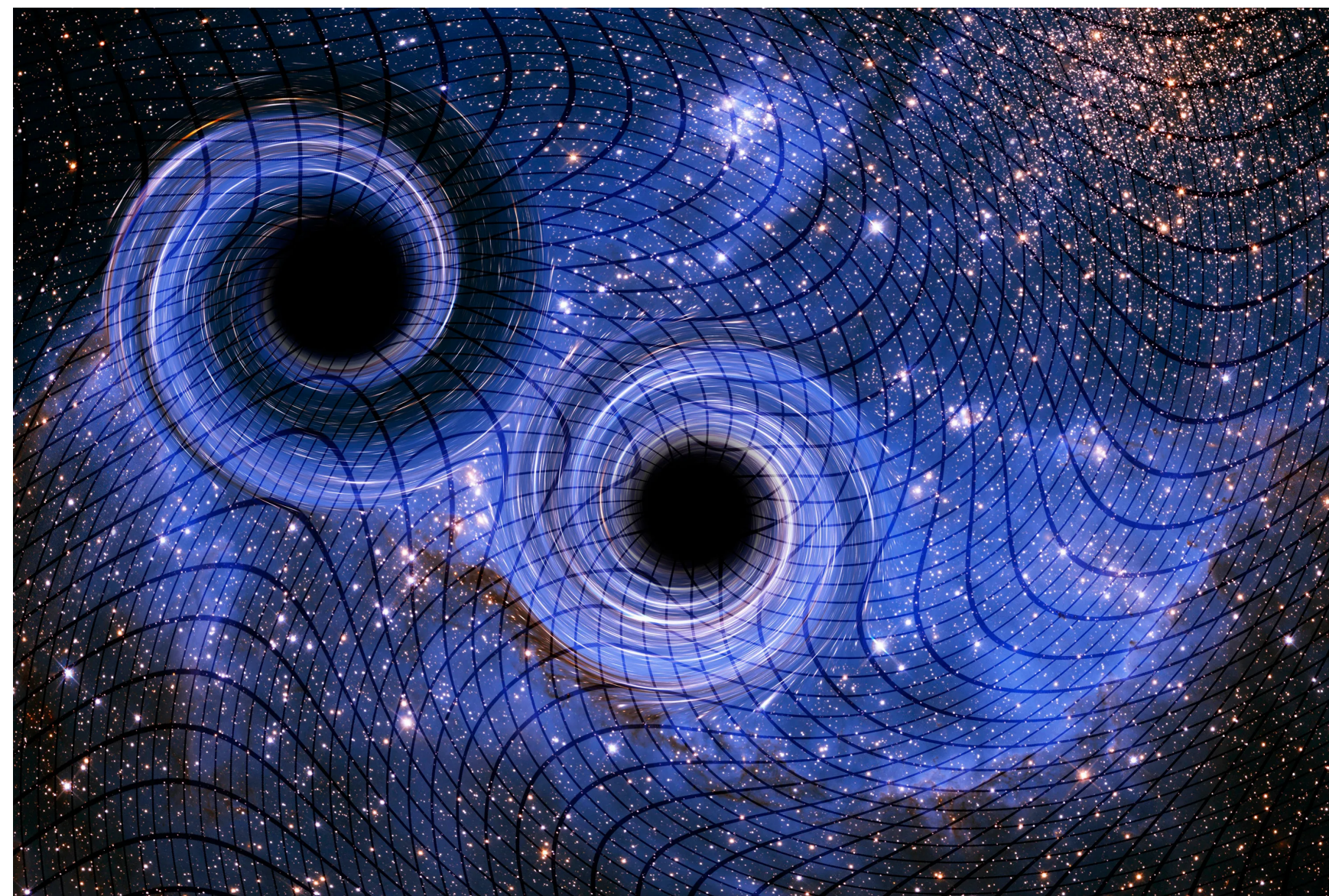


Image credit: MIT Tech. Review

Capturing model independent effects beyond GR

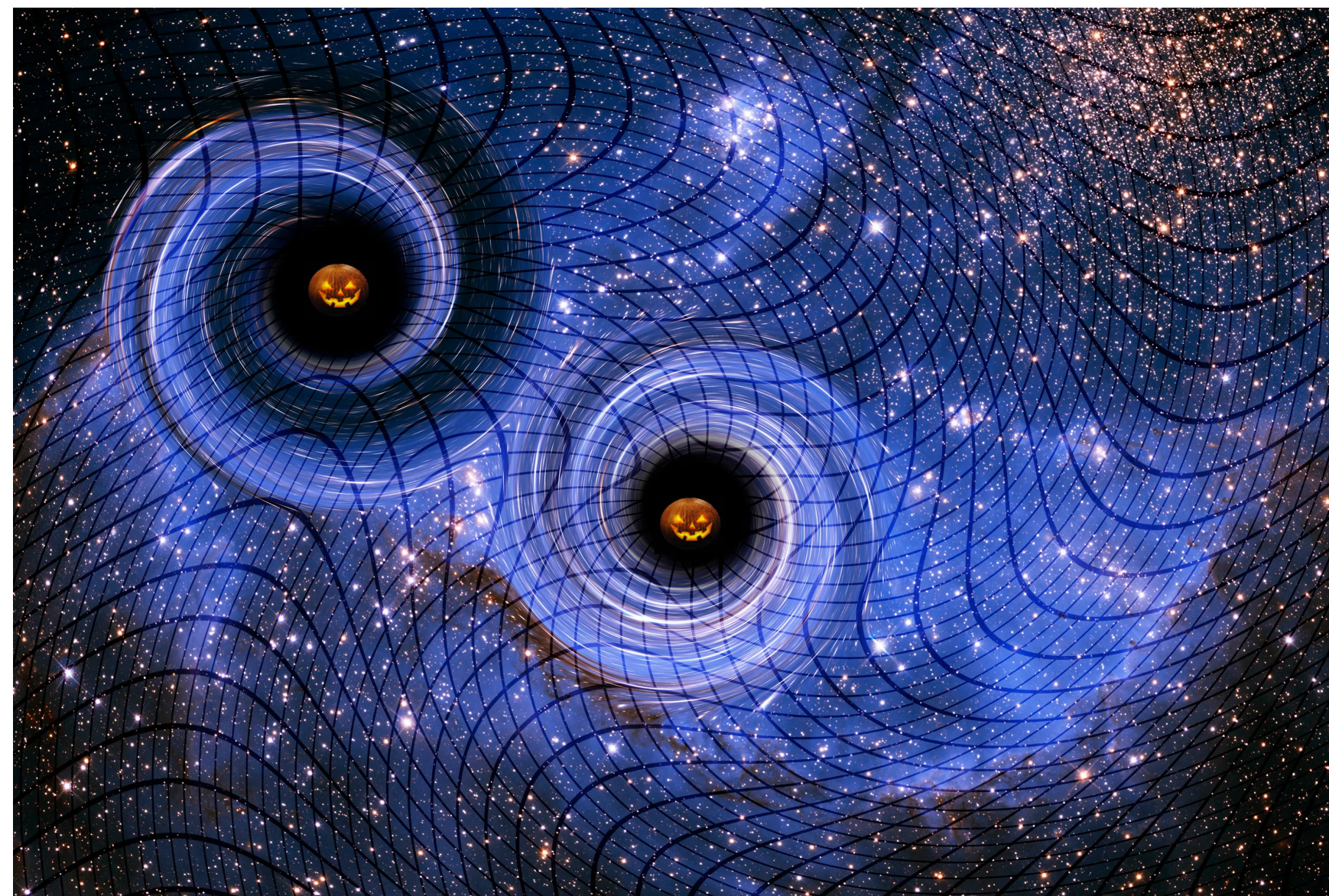


Image credit: MIT Tech. Review

Parameterizing EOM

S. Alexander, T. Daniel, T.M.,
2510.25895

$$h_{R,L} = \frac{1}{\sqrt{2}}(h_+ \pm i h_\times)$$

$$h''_{R,L} + \left\{ 2\mathcal{H} + C_O^{(1)} + C_E^{(1)} \right\} h'_{R,L} + k^2 \left\{ 1 + C_O^{(0)} + C_E^{(0)} \right\} h_{R,L} = 0,$$

Amplitude effects

$$\begin{cases} C_O^{(1)} = \lambda_{R,L} \sum_{n=0} k^{2n+1} \left[\frac{\alpha_{2n+1}}{(\Lambda_O a)^{2n+1}} \mathcal{H} + \frac{\beta_{2n+1}}{(\Lambda_O a)^{2n}} + \frac{\mu_{2n+1} \mathcal{H}^2 + \rho_{2n+1} \mathcal{H}'}{(\Lambda_O a)^{2n+2}} \right] \\ C_E^{(1)} = \sum_{n=0} k^{2n} \left[\frac{\alpha_{2n}}{(\Lambda_E a)^{2n}} \mathcal{H} + \frac{\beta_{2n}}{(\Lambda_E a)^{2n-1}} + \frac{\mu_{2n} \mathcal{H}^2 + \rho_{2n} \mathcal{H}'}{(\Lambda_E a)^{2n+1}} \right] \end{cases}$$

Well behaved in low energy ($k \rightarrow 0$) and Minkowski ($\mathcal{H} \rightarrow 0$) limits

EFT cutoffs Λ_E, Λ_O

Phase effects

$$\begin{cases} C_O^{(0)} = \lambda_{R,L} \sum_{m=0} k^{2m-1} \left[\frac{\gamma_{2m}}{(\Lambda_O a)^{2m}} \mathcal{H} + \frac{1}{(\Lambda_O a)^{2m-1}} \left(\delta_{2m} + \frac{\nu_{2m} \mathcal{H}^2 + \sigma_{2m} \mathcal{H}'}{k^2} \right) \right] \\ C_E^{(0)} = \sum_{m=0} k^{2m} \left[\frac{\gamma_{2m+1}}{(\Lambda_E a)^{2m+1}} \mathcal{H} + \frac{1}{(\Lambda_E a)^{2m}} \left(\delta_{2m+1} + \frac{\nu_{2m+1} \mathcal{H}^2 + \sigma_{2m+1} \mathcal{H}'}{k^2} \right) \right] \end{cases}$$

$\mathcal{H} = \frac{a'}{a}$

$\lambda_{R,L} = \pm 1$ (Birefringence!)

Model dependent $\{\alpha, \beta, \delta, \gamma, \mu, \nu, \rho, \sigma\}$, depend on time

General solution...

..to $h''_{R,L} + \left\{ 2\mathcal{H} + C_O^{(1)} + C_E^{(1)} \right\} h'_{R,L} + k^2 \left\{ 1 + C_O^{(0)} + C_E^{(0)} \right\} h_{R,L} = 0$ in terms of redshift is

$$h_{R,L} = \bar{h}_{R,L} \times \exp \left\{ (\lambda_{R,L})^{2n} \left[\frac{[k(1+z)]^{2n}}{2} \left(\frac{\alpha_{2n_0}}{\Lambda_E^{2n}} z_{2n} + \frac{\beta_{2n_0}}{\Lambda_E^{2n-1}} D_{2n+1} + \frac{\mu_{2n_0}}{\Lambda_E^{2n+1}} \int H dz + \frac{\rho_{2n_0}}{\Lambda_E^{2n+1}} \int (1+z)^{-1} H_z dz \right) \right. \right. \\ \left. \left. + \lambda_{R,L} \frac{[k(1+z)]^{2n+1}}{2} \left(\frac{\alpha_{2n_0+1}}{\Lambda_O^{2n+1}} z_{2n+1} + \frac{\beta_{2n_0+1}}{\Lambda_O^{2n}} D_{2n+2} + \frac{\mu_{2n_0+1}}{\Lambda_O^{2n+2}} \int H dz + \frac{\rho_{2n_0+1}}{\Lambda_O^{2n+2}} \int (1+z)^{-1} H_z dz \right) \right] \right\} \\ \times \exp \left\{ i(\lambda_{R,L})^{2m+1} \left[\frac{[k(1+z)]^{2m}}{2} \left(\frac{\gamma_{2m_0}}{\Lambda_O^{2m}} z_{2m} + \frac{\delta_{2m_0}}{\Lambda_O^{2m-1}} D_{2m+1} + \frac{\nu_{2m_0}}{\Lambda_O^{2m-1} k^2} \int H dz + \frac{\sigma_{2m_0}}{\Lambda_O^{2m-1} k^2} \int (1+z)^{-1} H_z dz \right) \right. \right. \\ \left. \left. + \lambda_{R,L} \frac{[k(1+z)]^{2m+1}}{2} \left(\frac{\gamma_{2m_0+1}}{\Lambda_E^{2m+1}} z_{2m+1} + \frac{\delta_{2m_0+1}}{\Lambda_E^{2m}} D_{2m+2} + \frac{\nu_{2m_0+1}}{\Lambda_E^{2m} k^2} \int H dz + \frac{\sigma_{2m_0+1}}{\Lambda_E^{2m} k^2} \int (1+z)^{-1} H_z dz \right) \right] \right\}$$

Models of interest have very few terms in these sums

$$D_\alpha = (1+z)^{1-\alpha} \int \frac{(1+z)^{\alpha-2}}{H(z)} dz \\ z_\alpha = (1+z)^{-\alpha} \int \frac{dz}{(1+z)^{1-\alpha}}$$

Effective distance and redshift:

- Sum over m, n implied
- ‘Zero’ subscripts on $\{\alpha, \beta, \delta, \gamma, \mu, \nu, \rho, \sigma\}$ denote slowly varying

Dispersion and propagation speed...

... are also modified:

- $$\omega_{R,L}^2 = k^2 \left\{ 1 + (\lambda_{R,L})^{2m+1} k^{2m-1} \left[\frac{\gamma_{2m}}{(\Lambda_O a)^{2m}} \mathcal{H} + \frac{1}{(\Lambda_O a)^{2m-1}} \left(\delta_{2m} + \frac{\nu_{2m} \mathcal{H}^2 + \sigma_{2m} \mathcal{H}'}{k^2} \right) \right] + (\lambda_{R,L})^{2m+2} k^{2m} \left[\frac{\gamma_{2m+1}}{(\Lambda_E a)^{2m+1}} \mathcal{H} + \frac{1}{(\Lambda_E a)^{2m}} \left(\delta_{2m+1} + \frac{\nu_{2m+1} \mathcal{H}^2 + \sigma_{2m+1} \mathcal{H}'}{k^2} \right) \right] \right\},$$
- $$v_g^{R,L} = 1 + (\lambda_{R,L})^{2m+1} k^{2m-1} \left\{ m \left[\frac{\gamma_{2m}}{(\Lambda_O a)^{2m}} \mathcal{H} + \frac{1}{(\Lambda_O a)^{2m-1}} \left(\delta_{2m} + \frac{\nu_{2m} \mathcal{H}^2 + \sigma_{2m} \mathcal{H}'}{k^2} \right) \right] + \lambda_{R,L} k \left(m + \frac{1}{2} \right) \left[\frac{\gamma_{2m+1}}{(\Lambda_E a)^{2m+1}} \mathcal{H} + \frac{1}{(\Lambda_E a)^{2m}} \left(\delta_{2m+1} + \frac{\nu_{2m+1} \mathcal{H}^2 + \sigma_{2m+1} \mathcal{H}'}{k^2} \right) \right] \right\}$$
- $$-(\lambda_{R,L})^{2m+1} k^{2m-1} \left\{ \frac{\nu_{2m} \mathcal{H}^2 + \sigma_{2m} \mathcal{H}'}{(\Lambda_O a)^{2m-1} k^2} + \lambda_{R,L} k \left[\frac{\nu_{2m+1} \mathcal{H}^2 + \sigma_{2m+1} \mathcal{H}'}{(\Lambda_E a)^{2m} k^2} \right] \right\},$$
- $$v_p^{R,L} = 1 + \frac{(\lambda_{R,L})^{2m+1} k^{2m-1}}{2} \left\{ \frac{\gamma_{2m}}{(\Lambda_O a)^{2m}} \mathcal{H} + \frac{1}{(\Lambda_O a)^{2m-1}} \left(\delta_{2m} + \frac{\nu_{2m} \mathcal{H}^2 + \sigma_{2m} \mathcal{H}'}{k^2} \right) + \lambda_{R,L} k \left[\frac{\gamma_{2m+1}}{(\Lambda_E a)^{2m+1}} \mathcal{H} + \frac{1}{(\Lambda_E a)^{2m}} \left(\delta_{2m+1} + \frac{\nu_{2m+1} \mathcal{H}^2 + \sigma_{2m+1} \mathcal{H}'}{k^2} \right) \right] \right\}.$$

Group velocity constrained by GW170817

Why non-minimal couplings?

Novel way to transfer energy in/out of gravitational systems, with applications:

- Probing non-standard dark matter interactions via GWs (this talk)
- Unique gravitational lensing signature (Zhang 2510.05575)
- Cosmic acceleration (e.g. Horndeski theory)
- Leptogenesis (Alexander, Peskin, Sheikh-Jabbari 0403069)
- Higgs inflation
- Final parsec problem (work in progress)

On to some models

Types of theories I have in mind

Non-minimal coupled (dark) matter Ψ , minimally coupled SM matter Ψ_0 ,

$$S = \int d^4x \sqrt{-g} \left(\frac{m_p^2}{2} R + \mathcal{L}_\Psi - \mathcal{L}_{int}(g, \Psi) + \mathcal{L}_{\Psi_0} \right)$$

Where generically,

$$\mathcal{L}_{int} = \tilde{R}^{\mu\nu\rho\sigma} \sum_{m=1} \frac{\tilde{\xi}_m}{(\Lambda_O)^{\tilde{\xi}_m}} \mathcal{O}_{\mu\nu\rho\sigma}^m(g, \Psi) + R^{\mu\nu\rho\sigma} \sum_{n=1} \frac{\xi_n}{(\Lambda_E)^{\xi_n}} \mathcal{E}_{\mu\nu\rho\sigma}^n(g, \Psi)$$

Loosely speaking, $\text{Tr}(\tilde{R}\mathcal{O})$ is parity odd, $\text{Tr}(R\mathcal{E})$ is even, and $\tilde{R} = \star R$

Specific examples

- Kalb-Ramond dark matter (dim 4 interactions):

$$\mathcal{L}_B = \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} - V(B), \quad H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$$

$$\mathcal{L}_{int} = \tilde{\xi}_1 \tilde{R}^{\mu\nu\rho\sigma} B_{\mu\nu} B_{\rho\sigma} + \xi_1 R^{\mu\nu\rho\sigma} B_{\mu\nu} B_{\rho\sigma} + \xi_2 R^{\mu\rho} B_{\mu\lambda} B^\lambda_\rho + \xi_3 R B_{\mu\nu} B^{\mu\nu} + \xi_4 R B_{\mu\nu} \tilde{B}^{\mu\nu}$$

- Axion-dilaton-CS-GB (dim 5 interactions):

$$\mathcal{L}_{\phi,\varphi} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\partial\varphi)^2, \quad \mathcal{L}_{int} = -\frac{\xi\phi}{\Lambda_E} \mathcal{G} - \frac{\tilde{\xi}\varphi}{\Lambda_O} R\tilde{R}$$

- Dark photon (dim 6 interactions):

$$\mathcal{L}_{em} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(A)$$

$$\mathcal{L}_{int} = \frac{\tilde{\xi}_1}{\Lambda_O^2} \tilde{R}^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + \frac{1}{\Lambda_E^2} \left(\xi_1 R^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + \xi_2 R^{\mu\rho} F_{\mu\lambda} F^\lambda_\rho + \xi_3 R F_{\mu\nu} F^{\mu\nu} + \xi_4 R F_{\mu\nu} \tilde{F}^{\mu\nu} \right)$$

Simplified field configurations

- Kalb-Ramond dark matter:

$$B_{\mu\nu}^{(0)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -a^2 V_e & 0 & 0 \\ a^2 V_e & 0 & 0 & 0 \\ 0 & 0 & 0 & a^2 V_m \\ 0 & 0 & -a^2 V_m & 0 \end{pmatrix} \quad V_e, V_m \in \mathbb{R}$$

- Axion-dilaton-CS-GB:

$$\varphi'' \approx \phi'' \approx 0, \quad \varphi', \phi' \lesssim 1$$

- Dark photon:

$$F_{\mu\nu} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -aB_y \\ 0 & 0 & 0 & aB_x \\ 0 & aB_y & -aB_x & 0 \end{pmatrix} \quad B_x, B_y \in \mathbb{R}$$

Effect on GW propagation

$$h''_{R,L} + (2\mathcal{H} + C)h'_{R,L} + k^2(1 + D)h_{R,L} = 0$$

Amplitude effects

Phase effects

- Kalb-Ramond dark matter:

$$C = \frac{d}{d\eta} P_0(\xi_i, \tilde{V}_e, \tilde{V}_m),$$

$$D = P_1(\tilde{V}_{e,m}, \xi_{1,2}) + \lambda_{R,L} P_2(\tilde{V}_{e,m}, \tilde{\xi}_1, \xi_1) \frac{\mathcal{H}}{k} + \frac{1}{k^2} \left[P_3(\tilde{V}_{e,m}, \xi_i) \mathcal{H}' + P_4(\tilde{V}_{e,m}, \xi_i) \mathcal{H}^2 \right]$$

- Axion-dilaton-CS-GB:

$$C = 16\lambda_{R,L} k \frac{\phi'}{\Lambda_O a} \mathcal{H} - \frac{16\phi'}{\Lambda_E a} \mathcal{H}^2,$$

$$D = \frac{4\phi'}{\Lambda_E a} \mathcal{H} + \lambda_{R,L} k \frac{16\phi'}{\Lambda_O a} \frac{\mathcal{H}^2}{k^2}$$

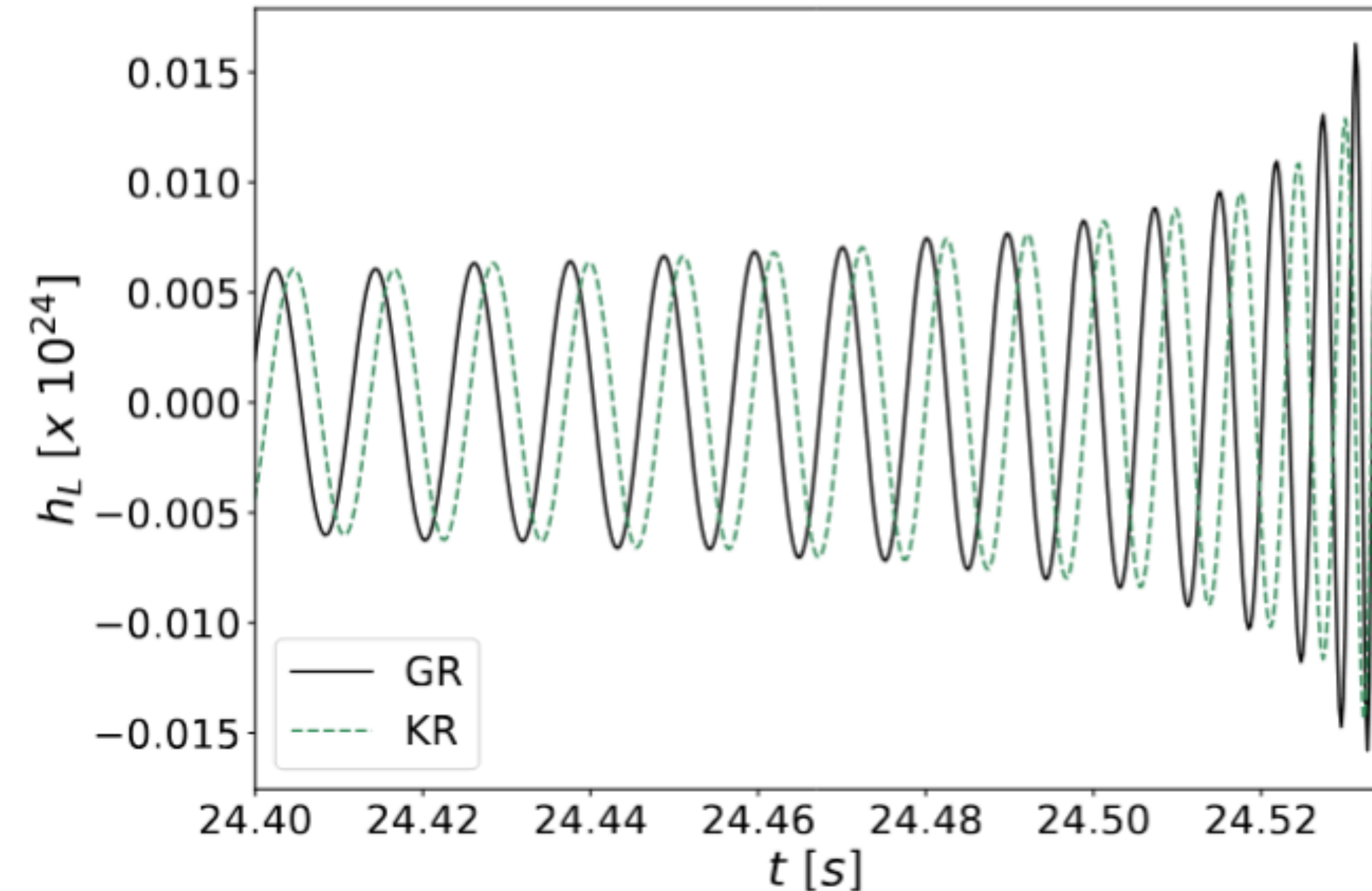
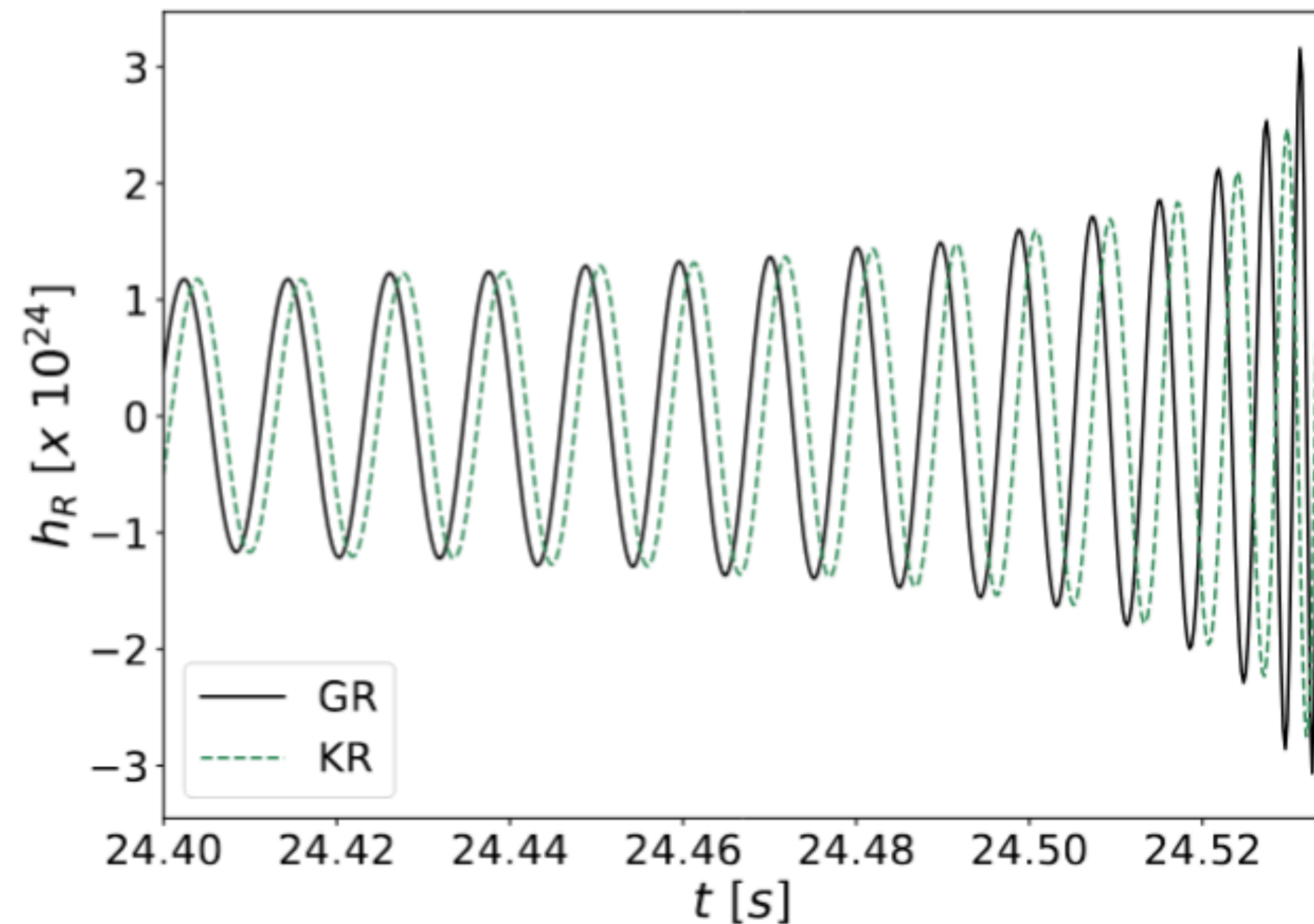
- Dark photon:

$$C = \frac{d}{d\eta} \bar{P}_0(\xi_i) |B|^2$$

$$|B|^2 = B_x^2 + B_y^2$$

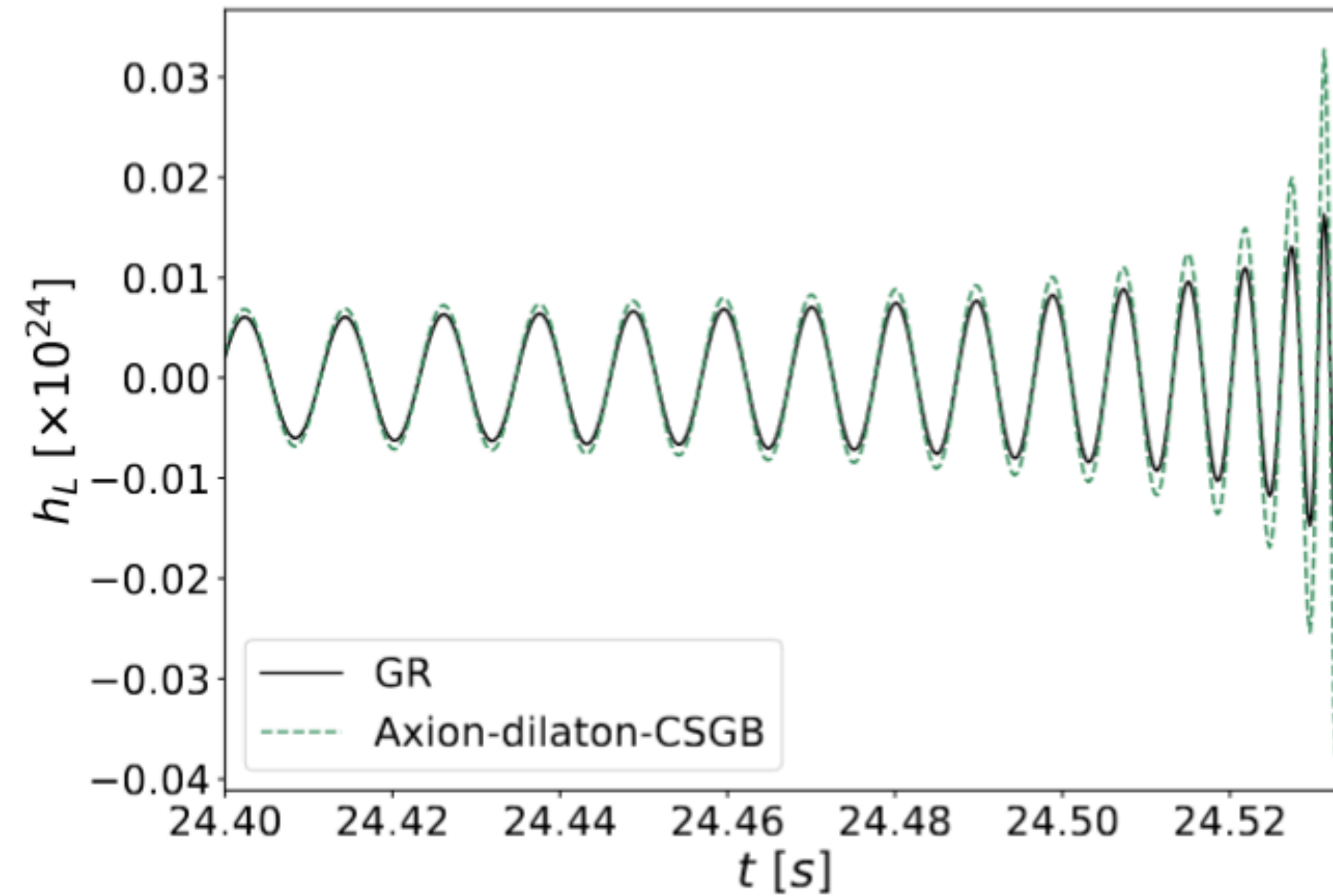
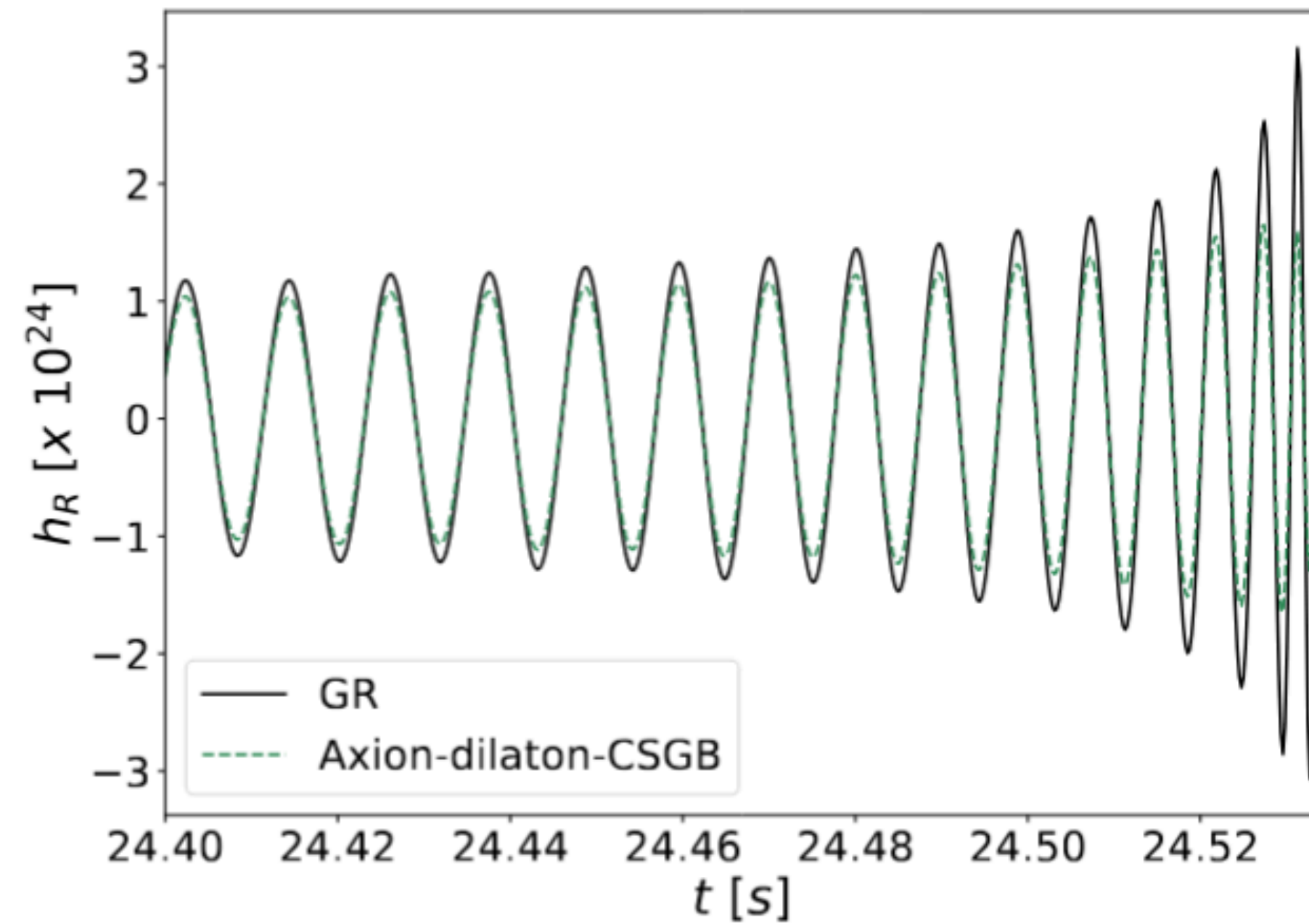
$$D = \frac{|B|^2}{(a\Lambda_E)^2} \bar{P}_1(\xi_i) - 2\lambda_{R,L} \frac{|B|^2}{(a\Lambda_O)^2} \tilde{\xi}_1 \frac{\mathcal{H}}{k} + \frac{|B|^2}{(a\Lambda_E k)^2} \left[\bar{P}_2(\xi_i) \mathcal{H}' + \bar{P}_3(\xi_i) \mathcal{H}^2 \right]$$

Kalb-Ramond effect on waveform: $h = h_{GR} \times \tilde{h}$



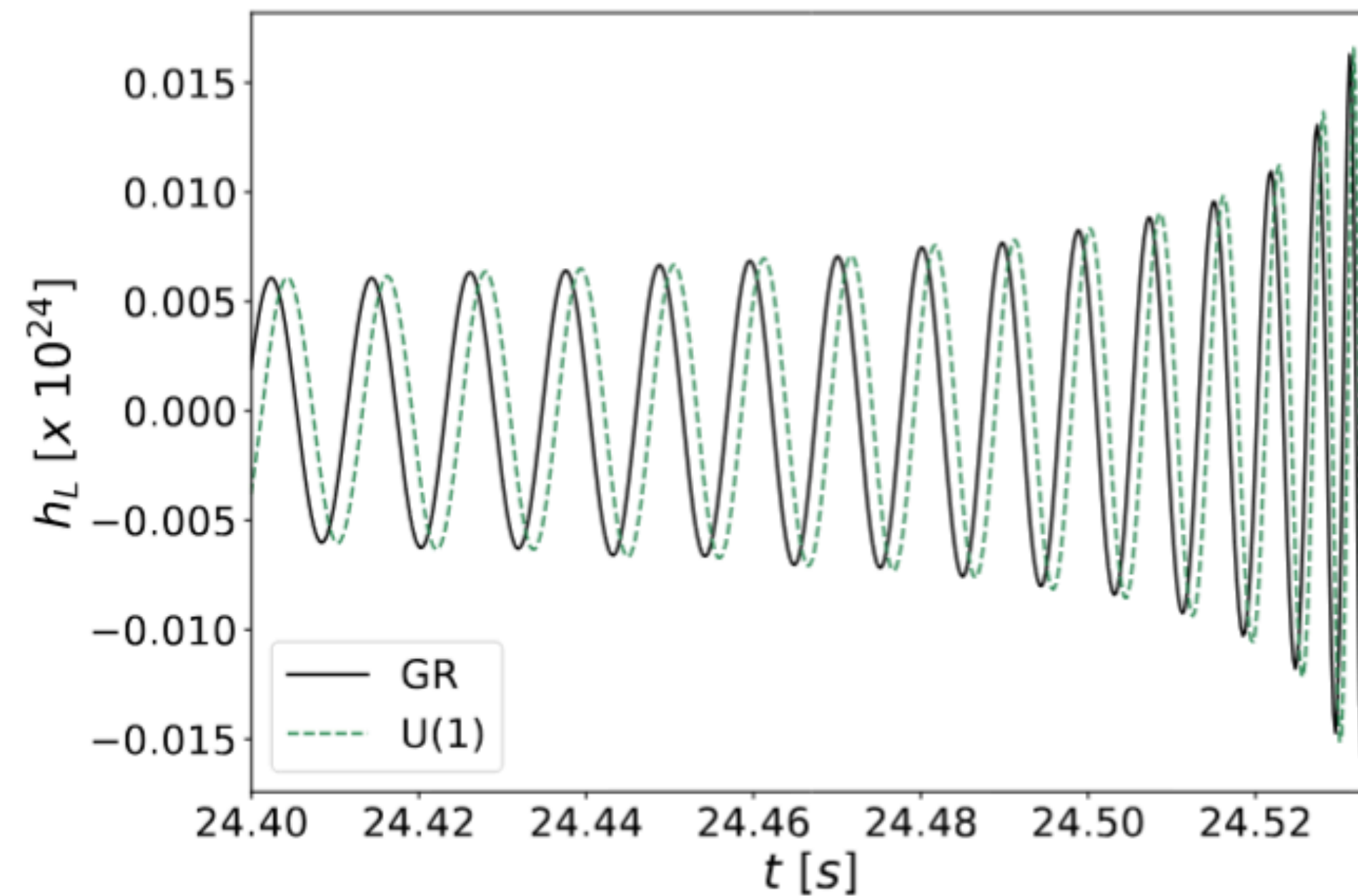
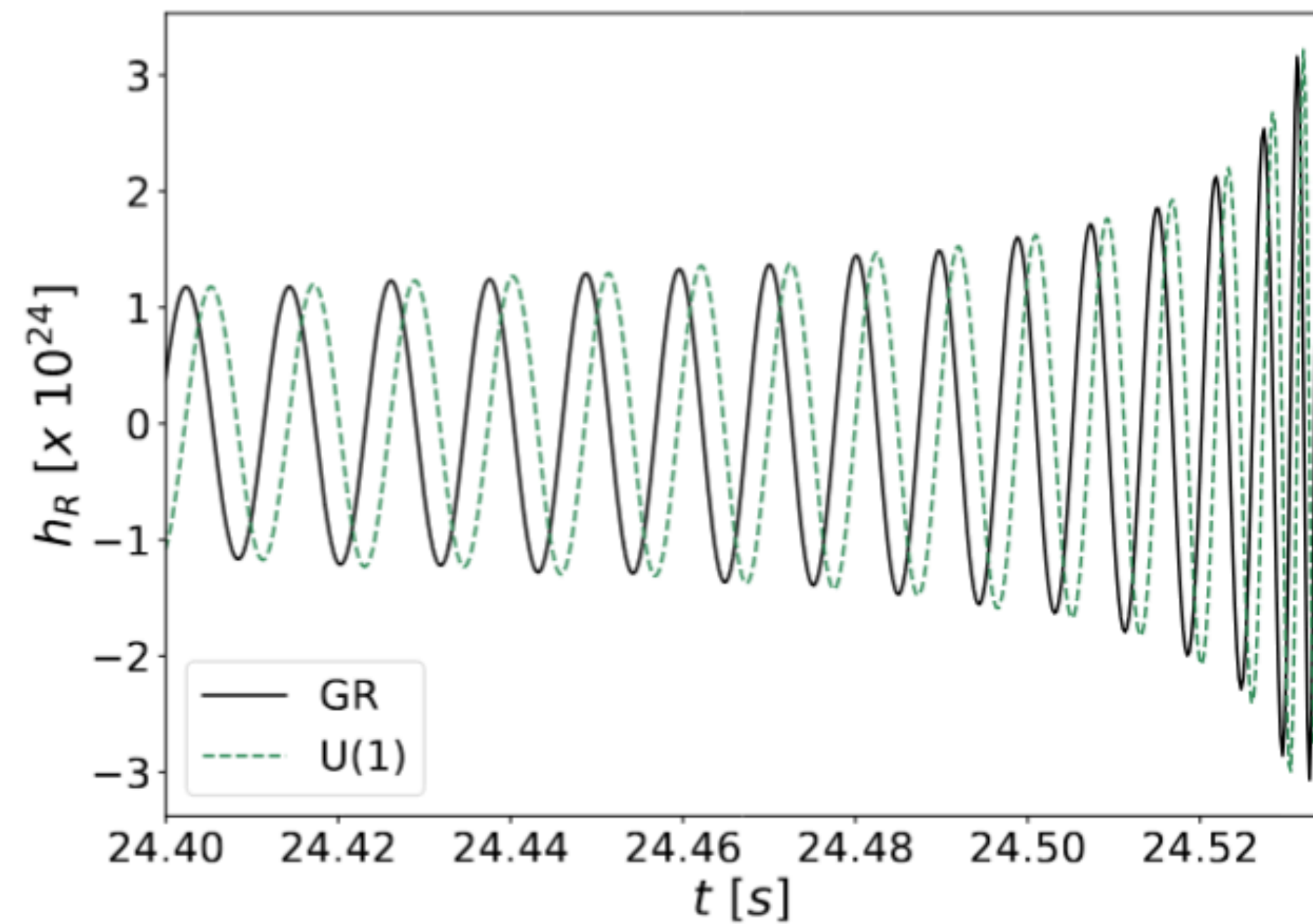
Parameters: $m_1 = 20M_\odot$, $m_2 = 18M_\odot$, $\tilde{V}_e \sim \tilde{V}_m \sim O(1)$, chosen artificially large for illustrative purposes. The relative amplitude differences come from the sim being close to face on

dCS + GB effect on waveform: $h = h_{GR} \times \tilde{h}$



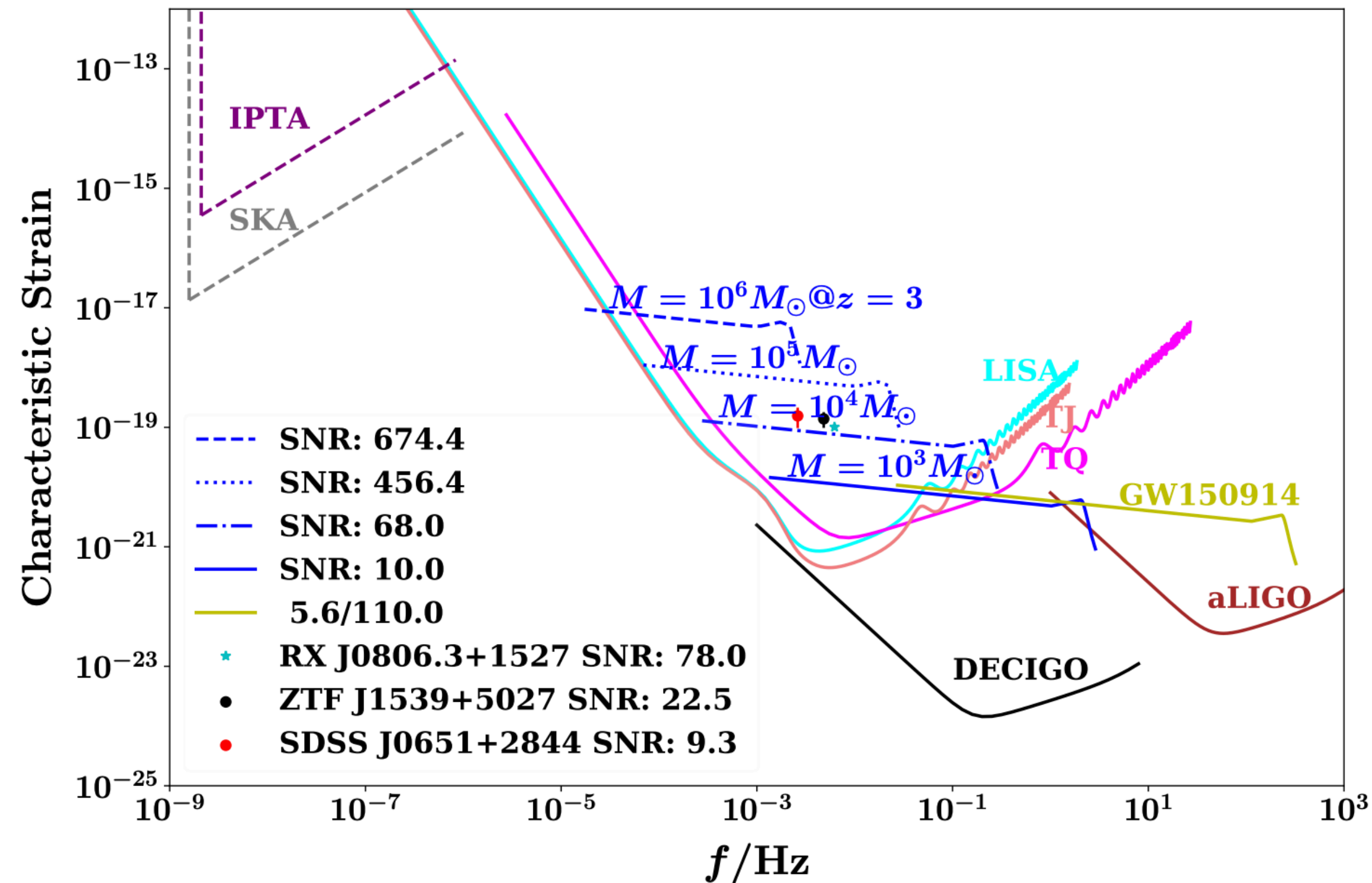
$$m_1 = 20M_{\odot}, m_2 = 18M_{\odot}, \varphi' \sim \phi' \sim O(1)$$

Dark photon effect on waveform: $h = h_{GR} \times \tilde{h}$

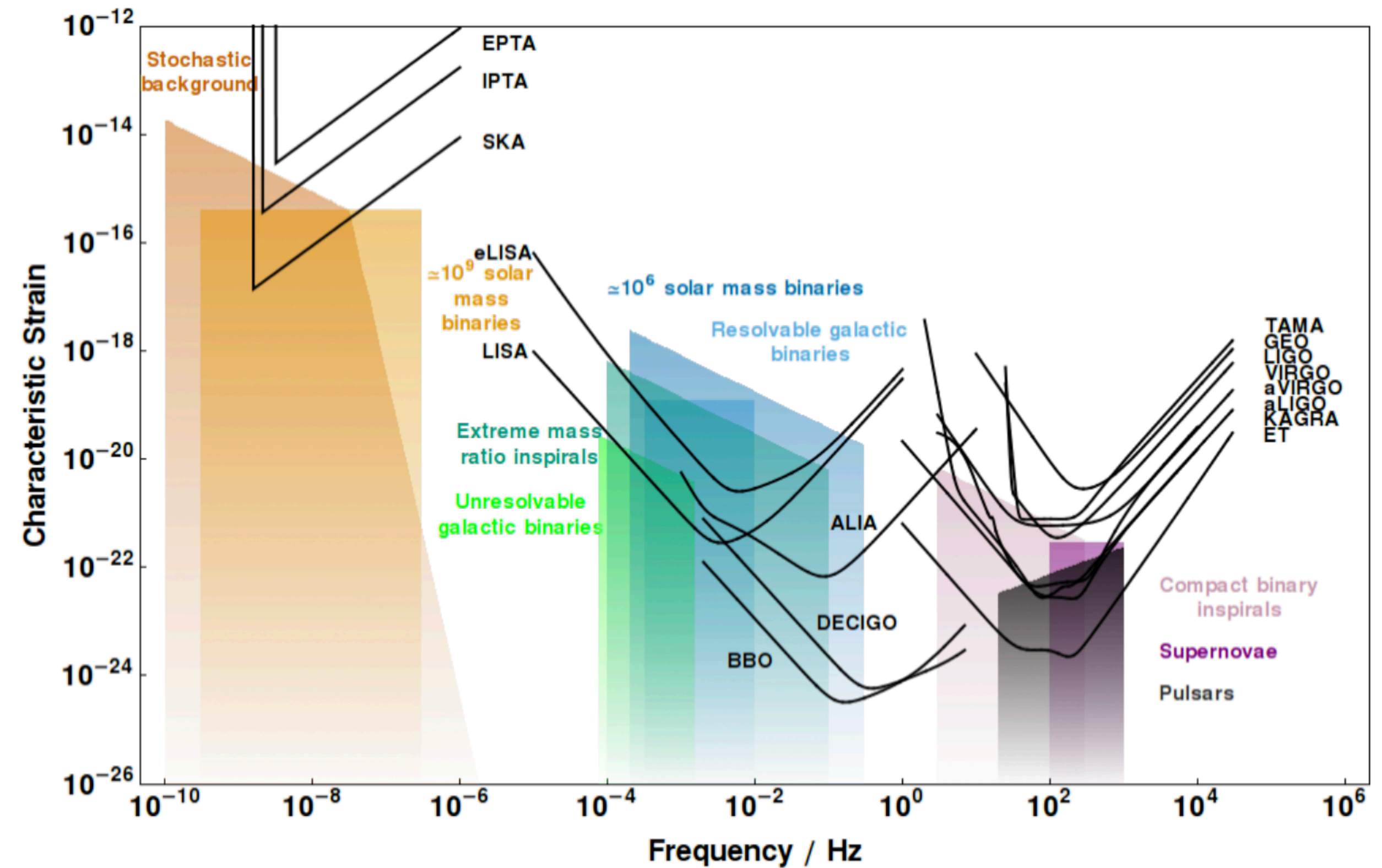


$$m_1 = 20M_{\odot}, m_2 = 18M_{\odot}, B_x \sim B_y \sim O(1)$$

What is/will be online



From Luo et. al., 2109.07442



From Moore et. al., 1408.0740

Lots of promise to learn a lot about the validity of these EFTs!

Recap & Discussion

- Non-minimal couplings to DM modify phase and amplitude of GWs, departing from GR
- Nature of the modification is model dependent and depends on GW wavenumber
- We illustrated the effects with 3 toy models in simplified configs.

Much to do:

- Realistic field configurations
- Map constraints from Zhu et. al. 2507.09705
- Early universe effects
- $\vdots$

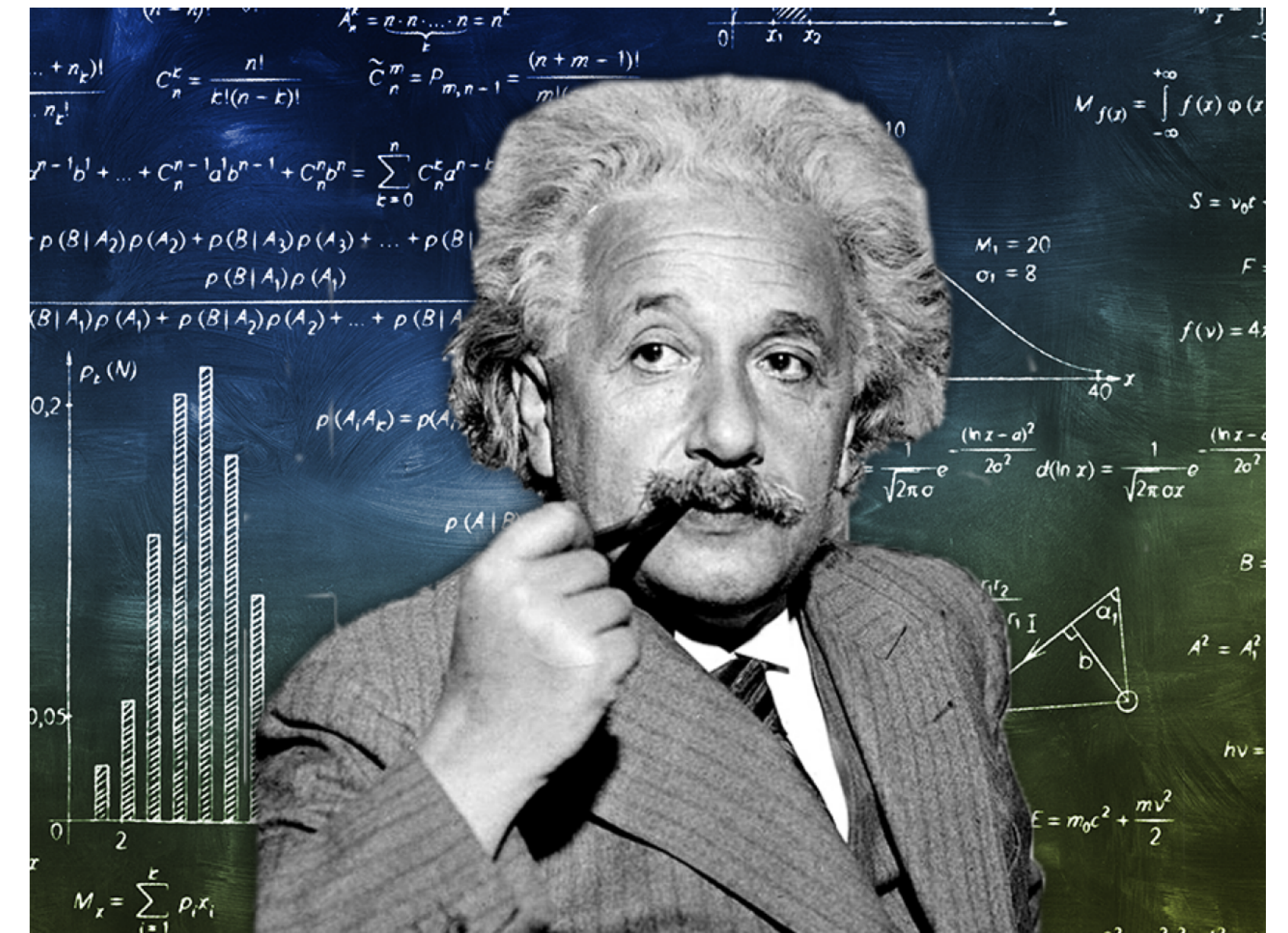


Image credit: Discovery

Thanks for your attention!