

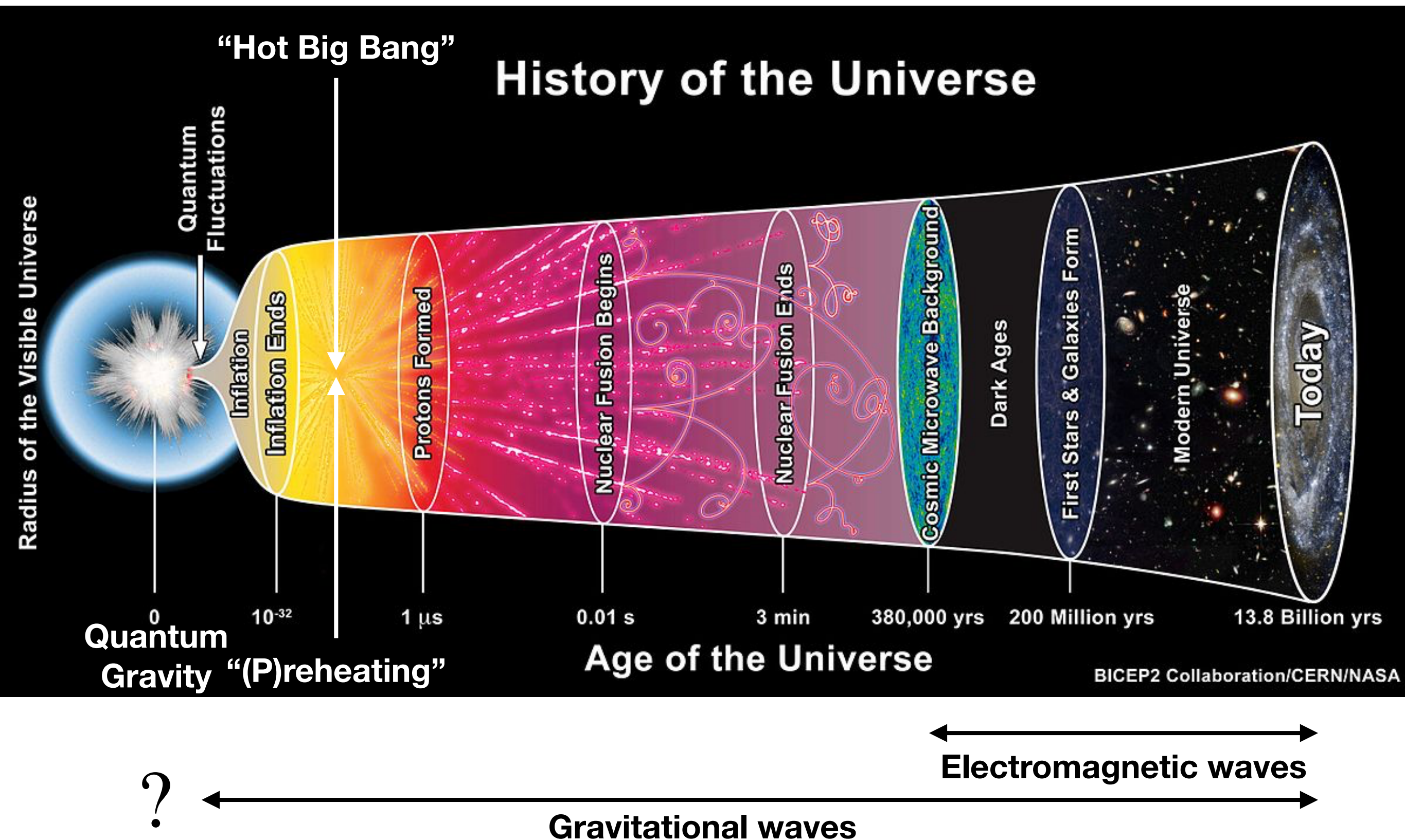
Gravitational waves from early Universe and late Universe

Shao-Jiang Wang

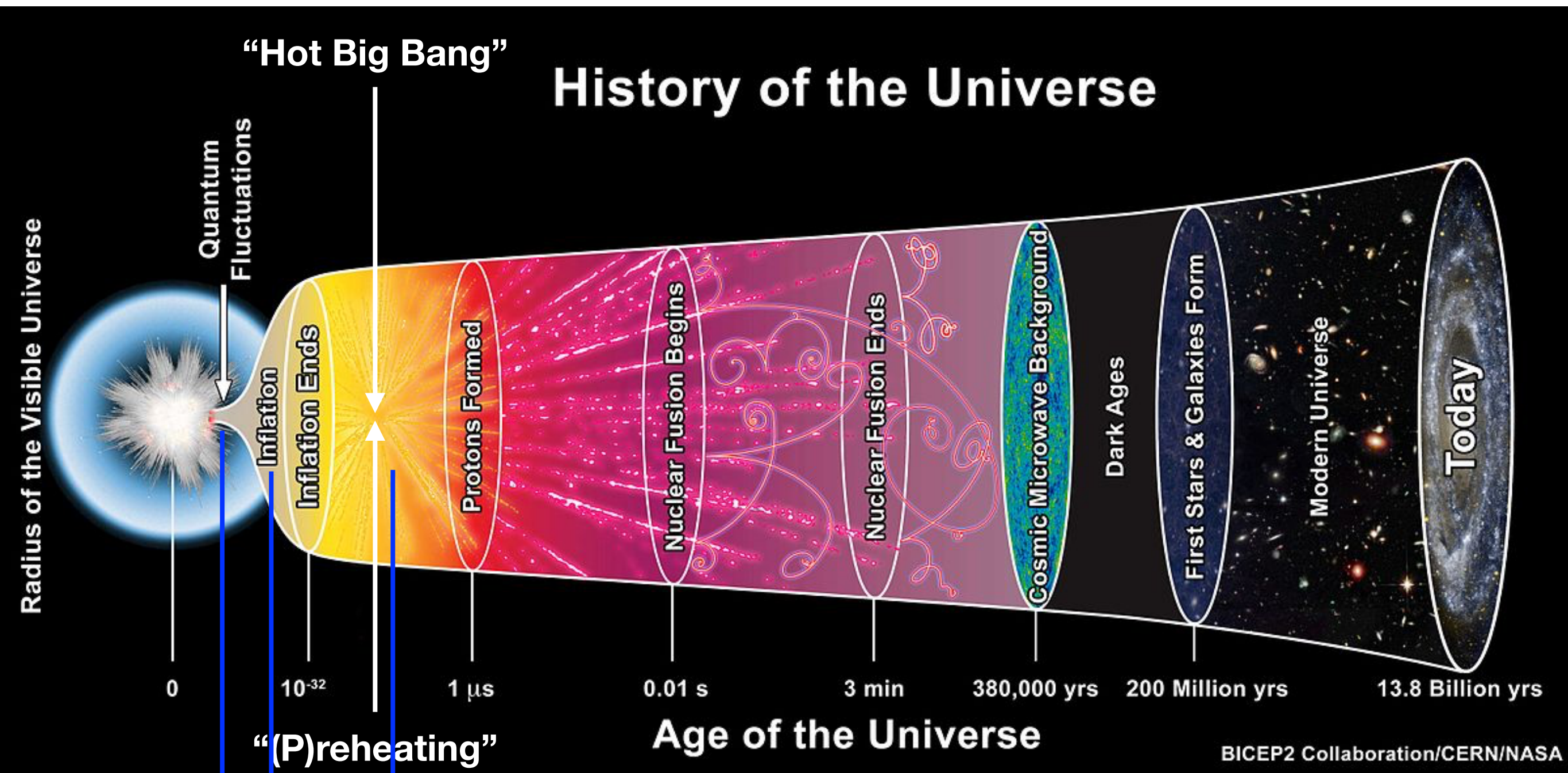
**Institute of Theoretical Physics
Chinese Academy of Sciences**

**Workshop on Gravitational Waves, Cosmology, and Fundamental Physics
2025-11-01 @ HIAS-UCAS @ 10:00-10:30**

Cosmic history



Inflation era

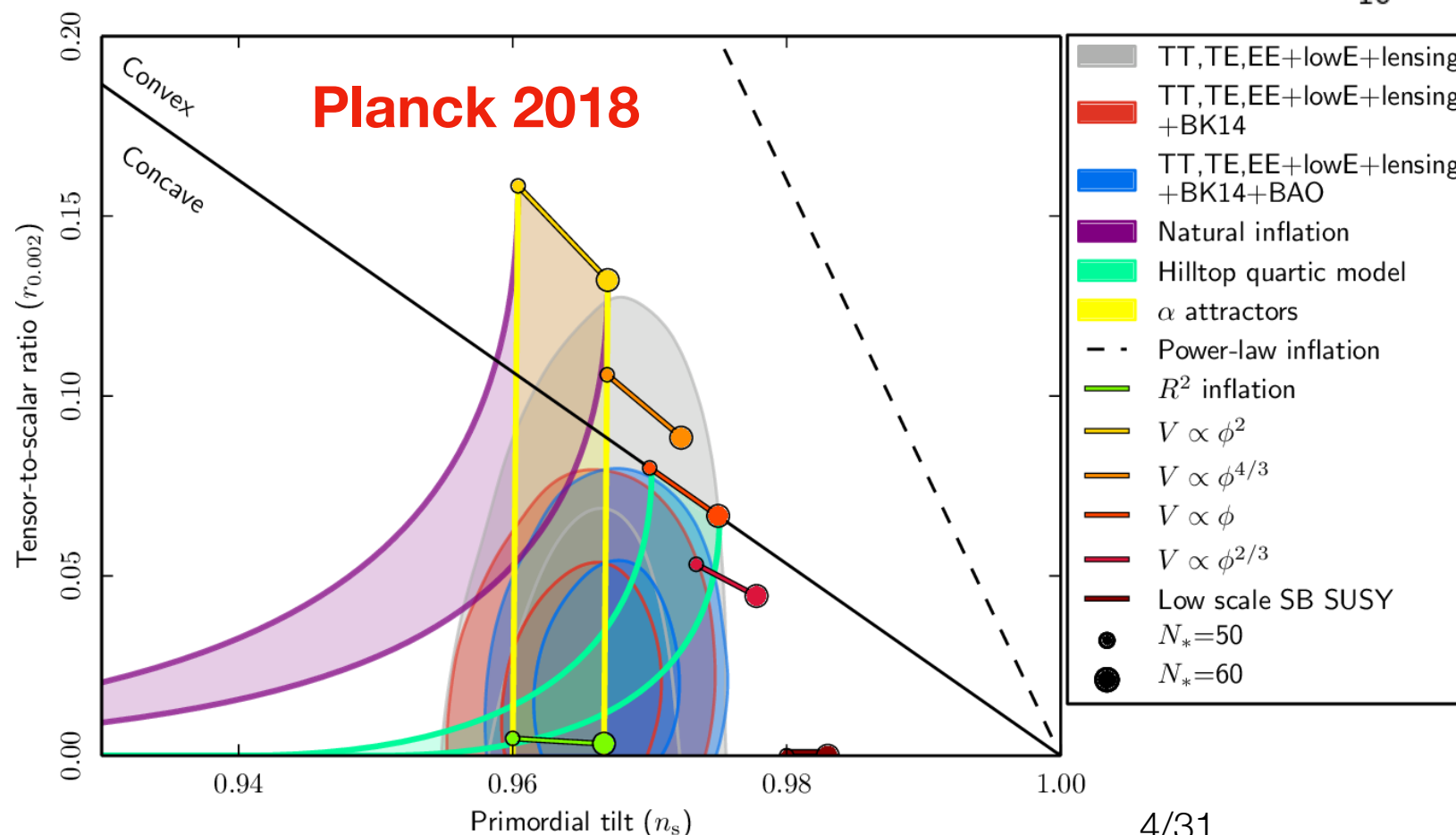
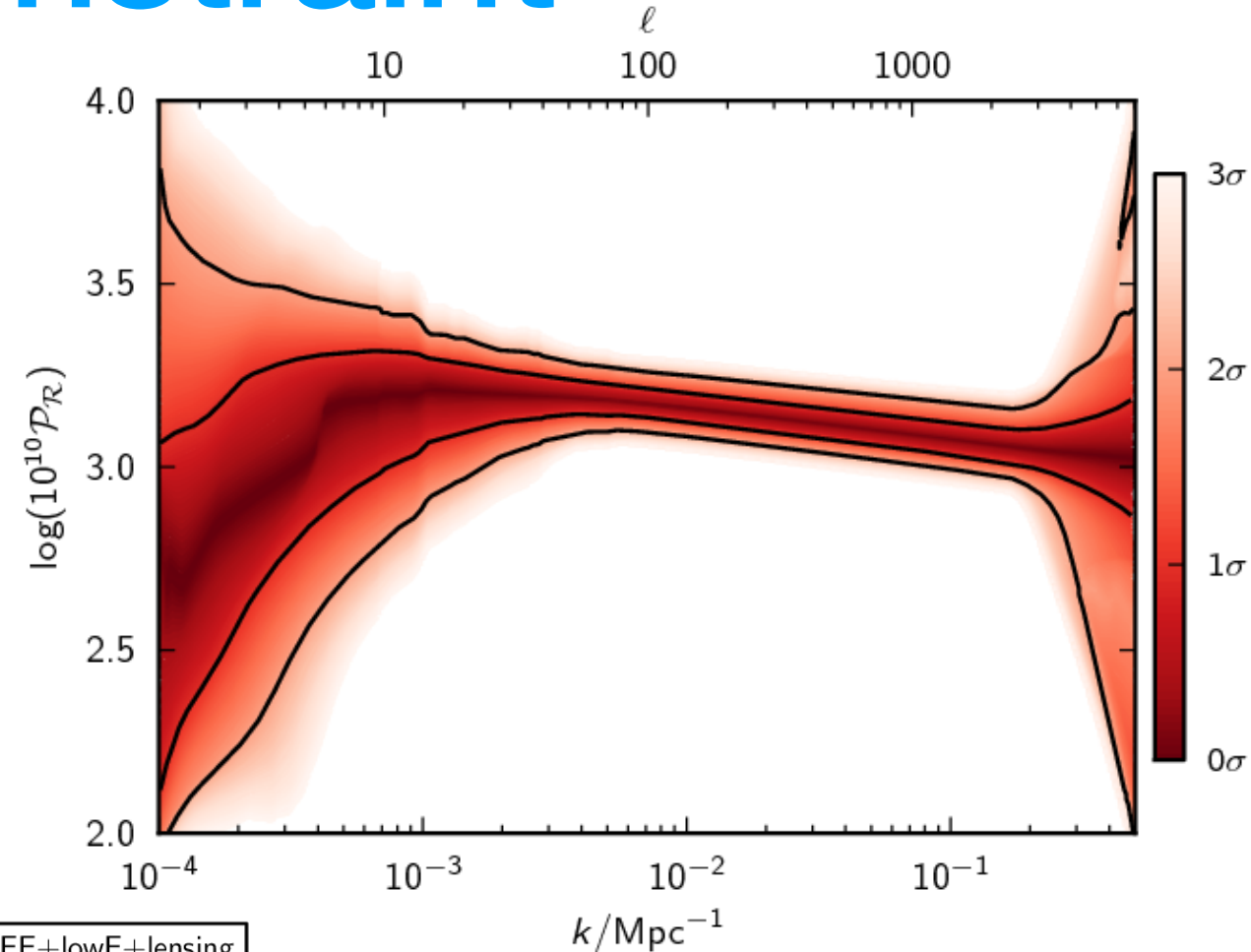


Primordial GWs

Induced GWs

Planck constraint

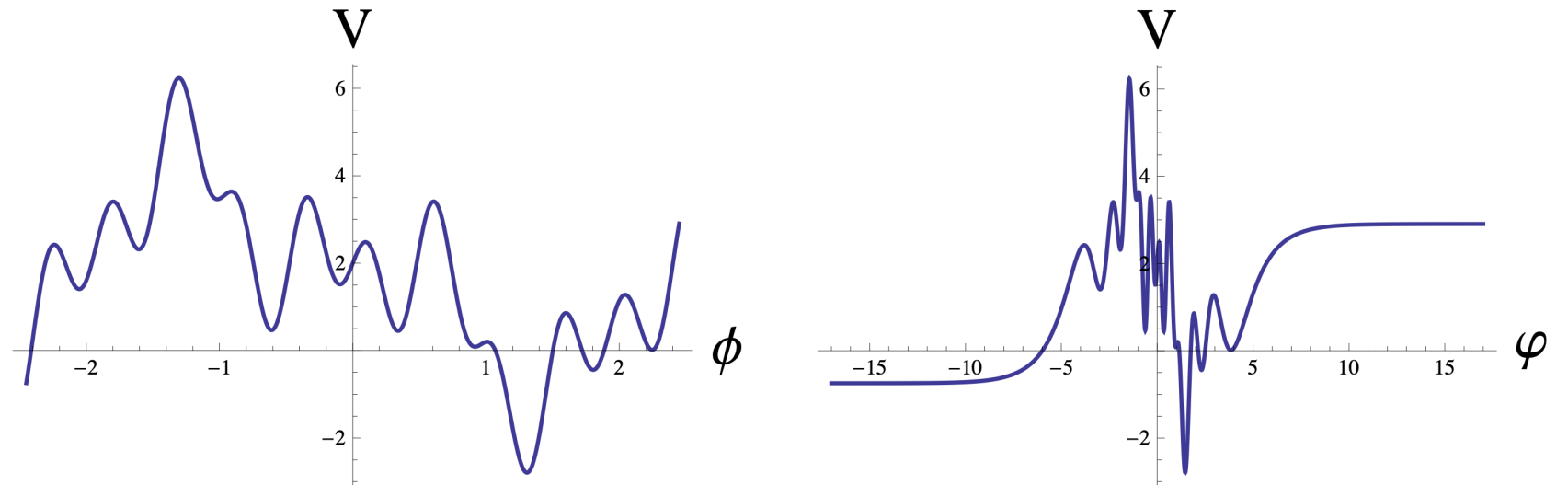
Adiabatic Gaussian
random perturbations
with a nearly-scale-
invariant spectrum &
negligible primordial
gravitational waves



A single-field
slow-roll inflation
with an
exponentially-flat
potential

“Attractive” model

Exponentially-flat
Starobinsky inflation
Induced inflation
Higgs inflation
=
Attractor models

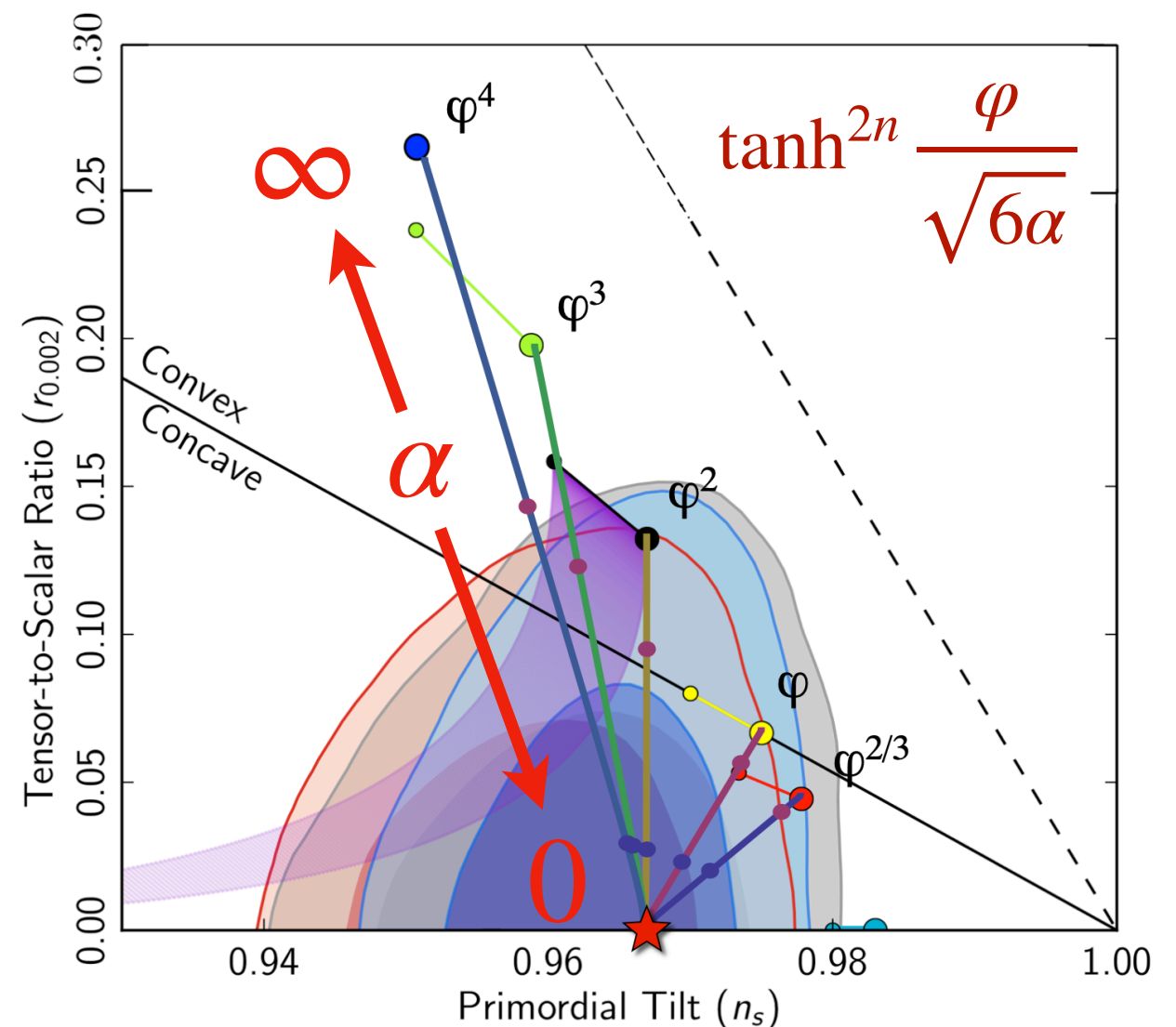


M. Galante, R. Kallosh, A. Linde, and D. Roest,
“Unity of Cosmological Inflation Attractors,”
Phys. Rev. Lett. 114 no. 14, (2015) 141302,
arXiv:1412.3797

$$\mathcal{L}_E = \sqrt{-g} \left[\frac{1}{2} R - \frac{\alpha}{(1 - \phi^2/6)^2} \frac{(\partial\phi)^2}{2} - \alpha f^2 (\phi/\sqrt{6}) \right]$$

$$\frac{\phi}{\sqrt{6}} = \frac{1 - \rho}{1 + \rho} : \quad K_E = \frac{3\alpha}{2} \frac{1}{\rho^2}, \quad V_E = \alpha f^2 \left(\frac{1 - \rho}{1 + \rho} \right)$$

$$\phi = -\sqrt{\frac{3}{2}} \alpha \ln \rho : \quad K_E = 1, \quad V_E = \alpha f^2 \left(\tanh \frac{\phi}{\sqrt{6}\alpha} \right)$$



ACT constraint



Photo Credit: Jon Ward

V

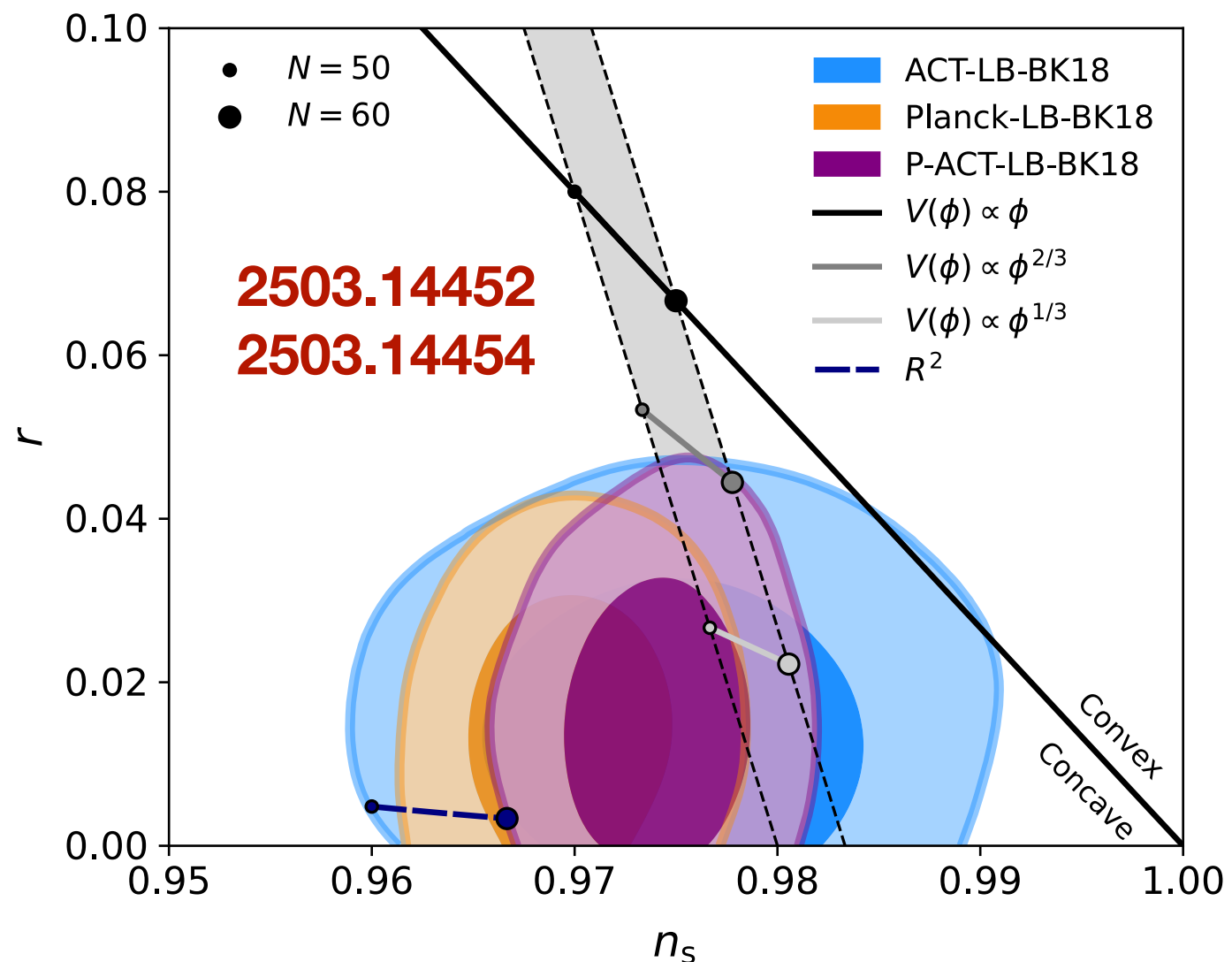
ACT: polynomially-flat

$$V_{\text{pol}}(\phi) = V_0 \left(1 - \left(\sqrt{\frac{2}{3\alpha}} \phi \right)^{-k} + \dots \right)$$

Planck: exponentially-flat

$$V_{\text{exp}}(\phi) = V_0 \left(1 - e^{-\sqrt{\frac{2}{3\alpha}} \phi} + \dots \right)$$

ϕ



2505.13646 Renata Kallosh and Andrei Linde
 “On the Present Status of Inflationary
 Cosmology” *Gen. Rel. Grav.* 57 (2025) 10, 135

Renata Kallosh, Andrei Linde, and Diederik
 Roest “Atacama Cosmology Telescope,
 South Pole Telescope, and Chaotic Inflation”
Phys. Rev. Lett. 135 (2025) 16, 161001

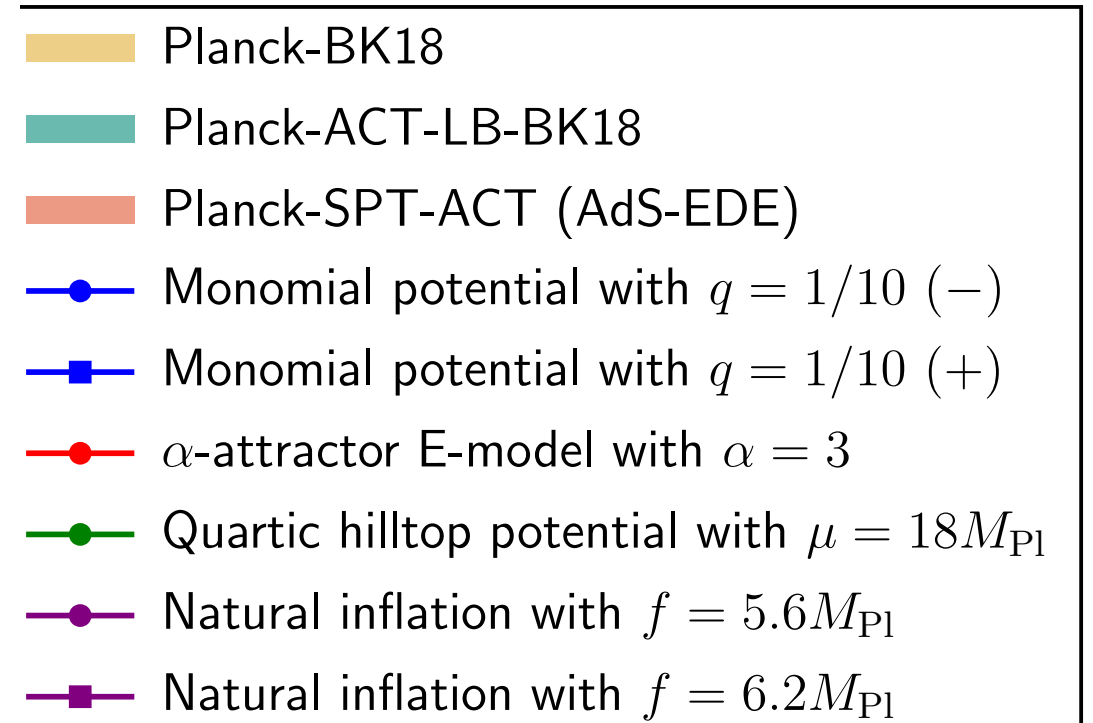
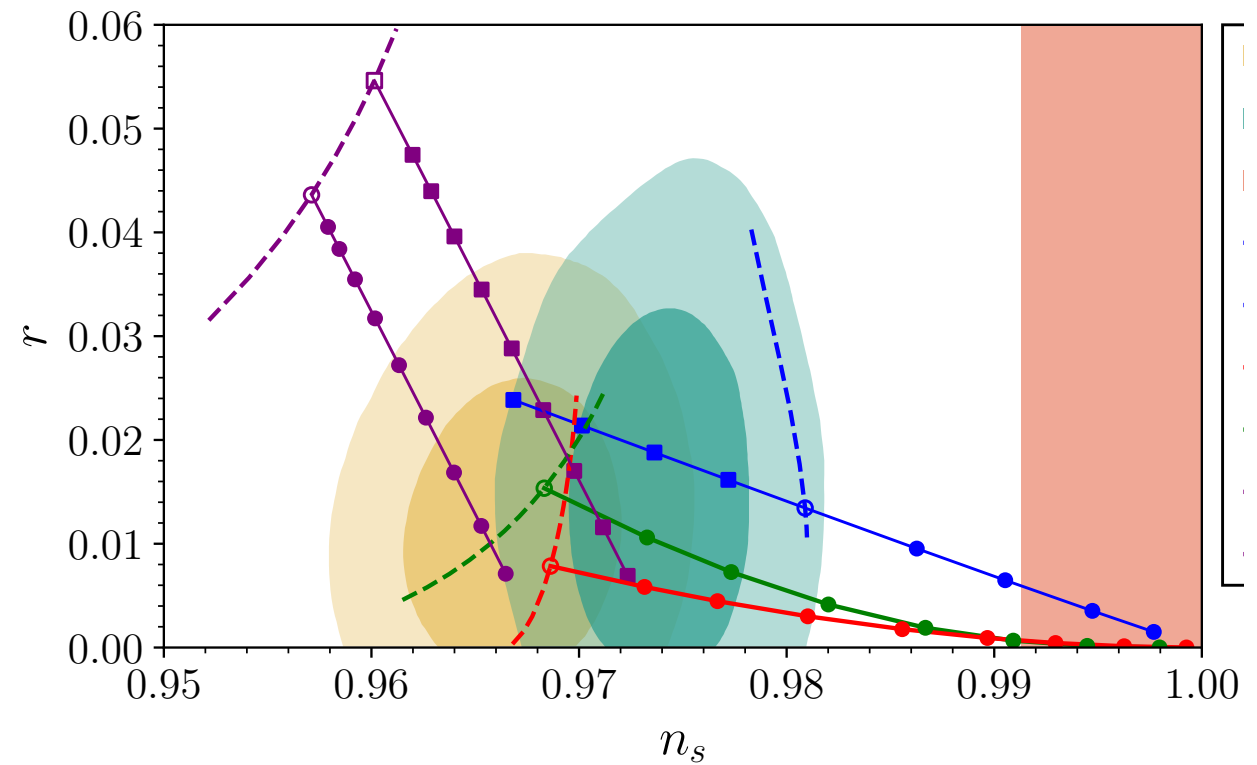
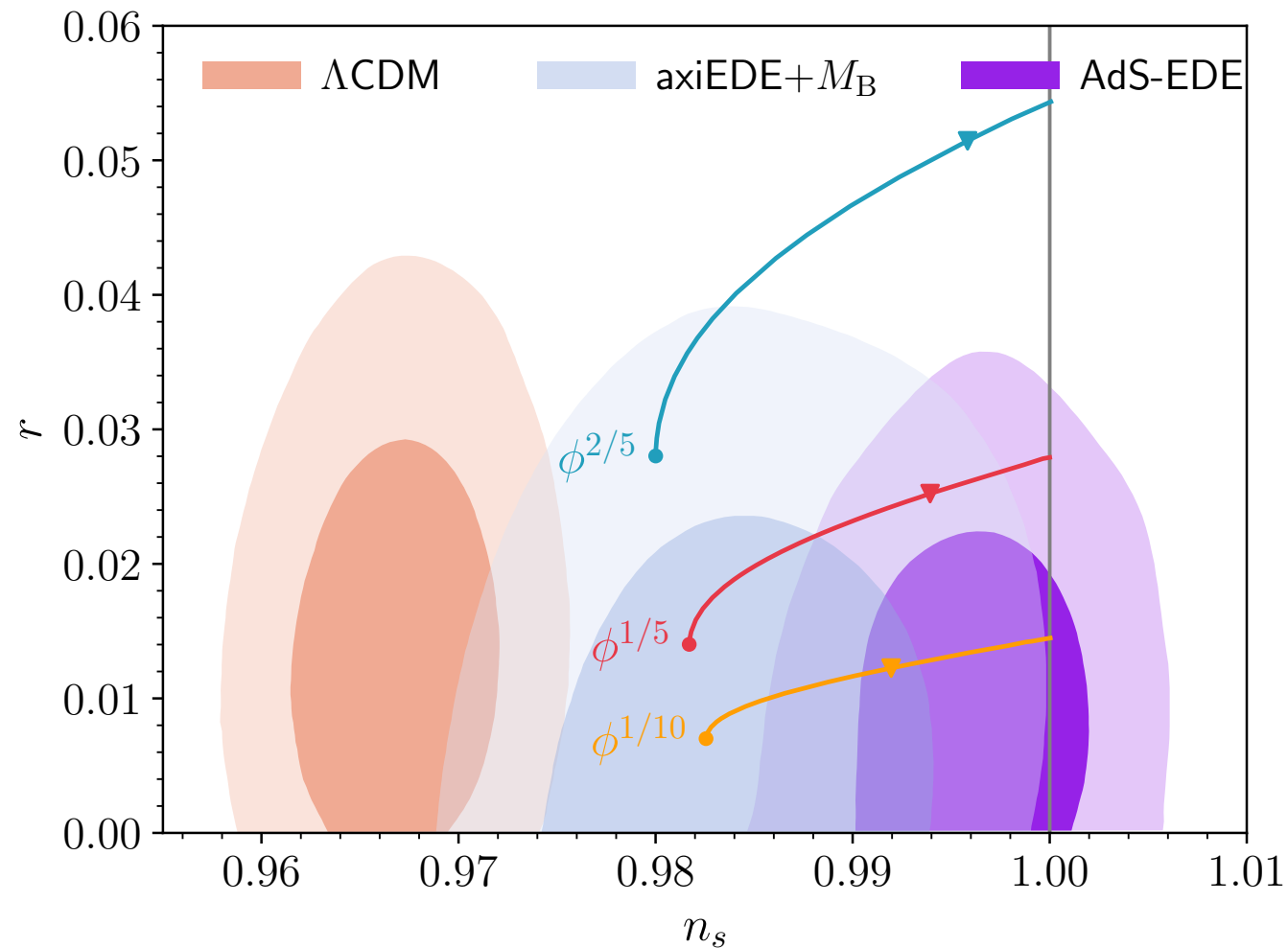
Model in ACTion

2310.12932 Chengjie Fu, SJW "Reconciling early dark energy with a Harrison-Zeldovich spectrum"
Phys. Rev. D 109 (2024) 4, L041304

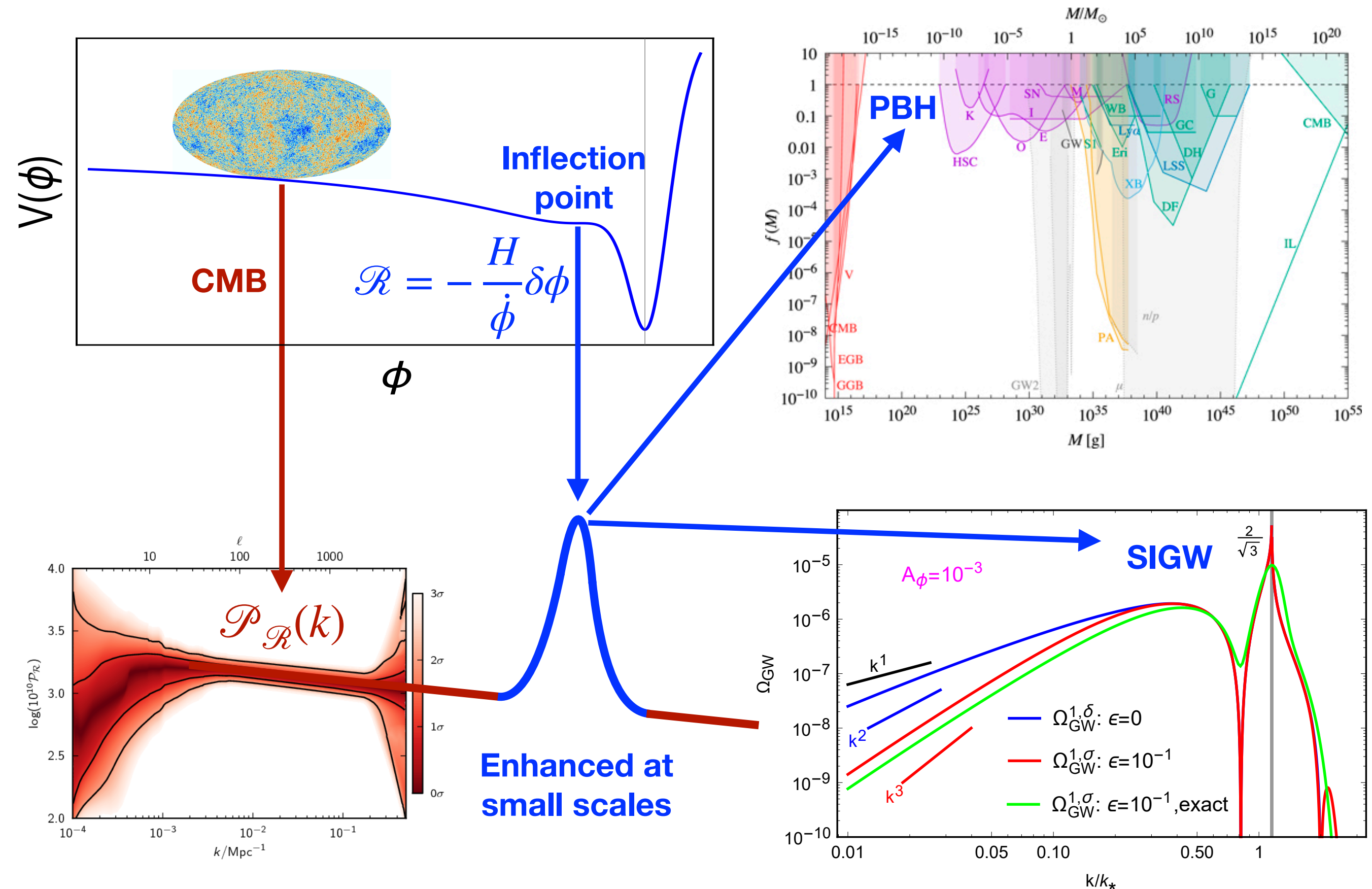
2510.24682 Chengjie Fu, Di Lu, SJW "The Harrison-Zeldovich attractor: From Planck to ACT"

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} (g^{\mu\nu} - \xi G^{\mu\nu}) \nabla_\mu \phi \nabla_\nu \phi - V \right]$$

$$V(\phi) = \lambda M_{\text{Pl}}^{4-p} \phi^p \quad \xi < 0$$

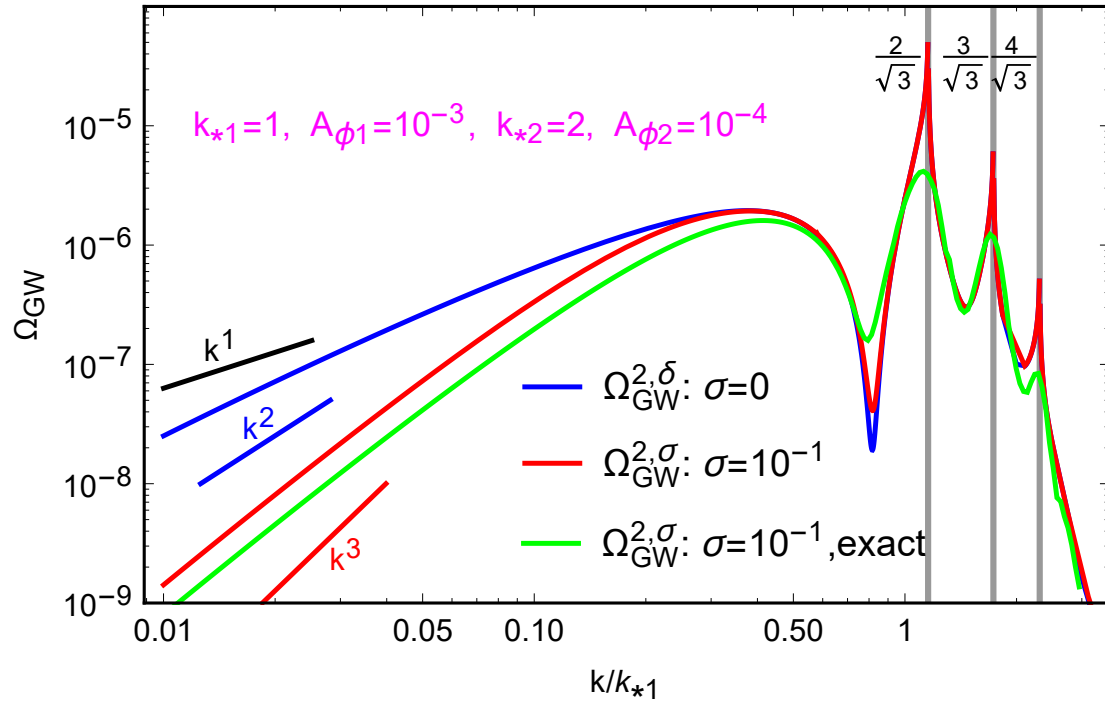


Induced GWs



Resonant multi-peaks

1901.10152 Cai, Pi, **SJW**, Yang *JCAP* 05 (2019) 013



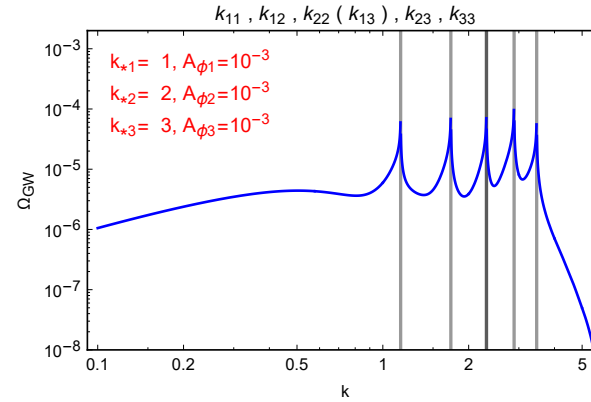
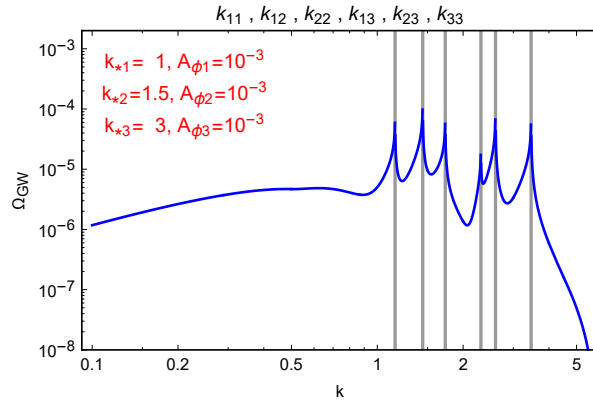
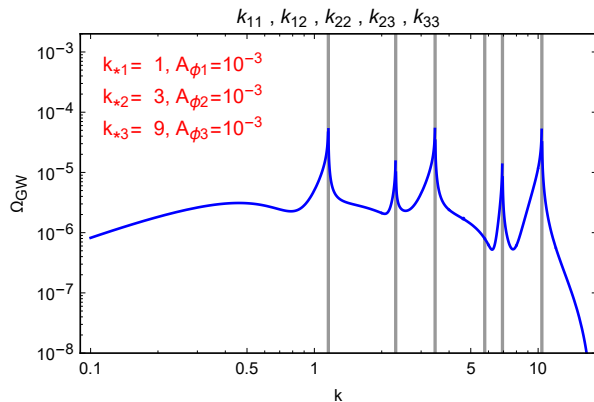
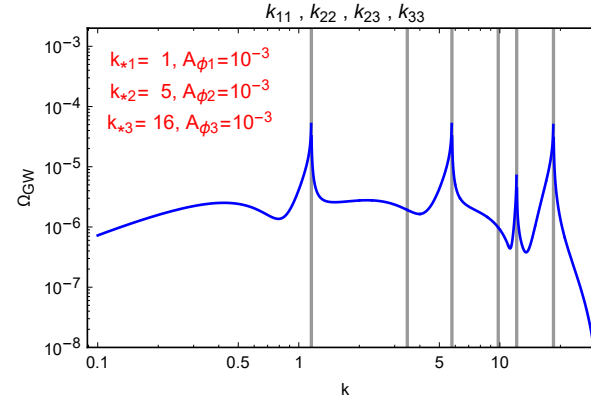
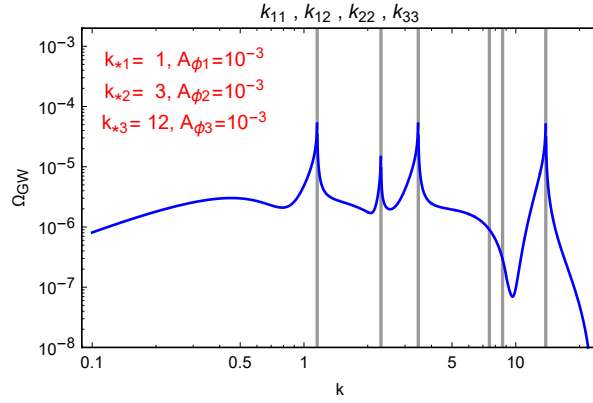
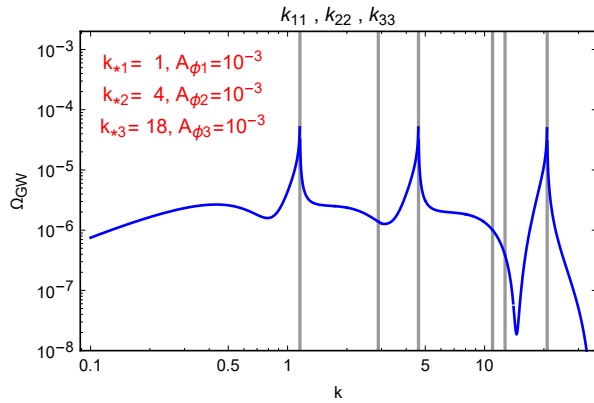
$$\delta - \text{peak} : \tilde{P}_\phi(k) = \sum_{i=1}^n A_{\phi i} \delta(\ln \frac{k}{k_{*i}});$$

$$\Omega_{\text{GW}}^{n,\delta}(k) = \frac{1}{24} \sum_{i,j=1}^n A_{\phi i} A_{\phi j} \frac{k_{*i} k_{*j}}{k^2} \mathcal{T} \left(\frac{k_{*i}}{k}, \frac{k_{*j}}{k} \right) \mathcal{F}^\delta(k; k_{*i}, k_{*j})$$

$$\sigma - \text{peak} : \tilde{P}_\phi(k) = \sum_{i=1}^n \frac{A_{\phi i} k_{*i}}{\sqrt{2\pi}\sigma} e^{-\frac{(k - k_{*i})^2}{2\sigma^2}};$$

$$\Omega_{\text{GW}}^{n,\sigma}(k) = \frac{1}{24} \sum_{i,j=1}^n A_{\phi i} A_{\phi j} \frac{k_{*i} k_{*j}}{k^2} \mathcal{T} \left(\frac{k_{*i}}{k}, \frac{k_{*j}}{k} \right) \mathcal{F}^\sigma(k; k_{*i}, k_{*j})$$

Resonant amplification peaked at $k_{ij} = \frac{1}{\sqrt{3}}(k_{*i} + k_{*j})$



2406.05034

Zeng, Cai, SJW

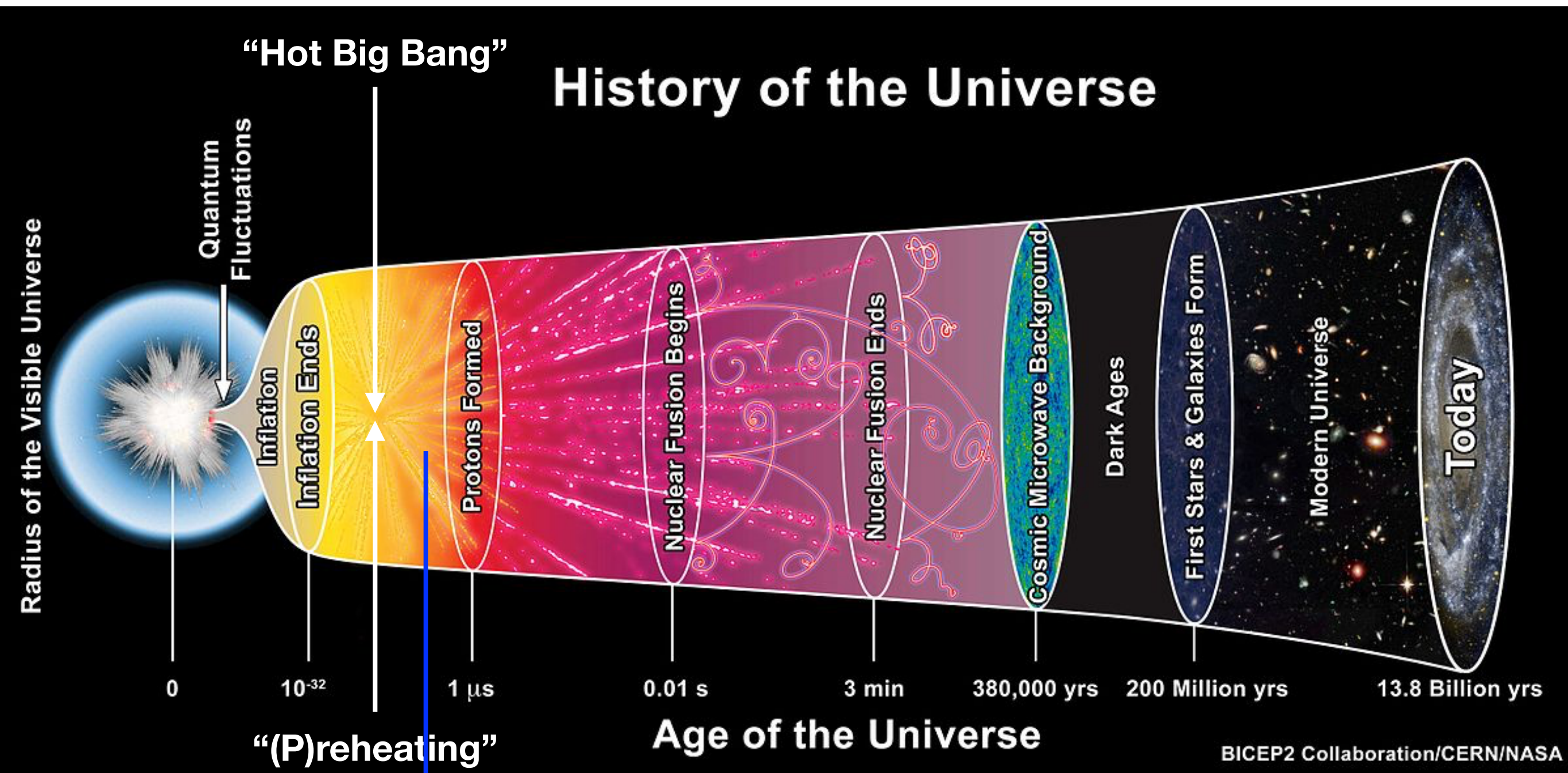
Multiple peaks in gravitational waves induced from primordial curvature perturbations with non-Gaussianity

JCAP 10 (2024) 045

2508.10812 Zeng, Ning, Cai, SJW,
Scalar-induced gravitational waves with non-Gaussianity up to all orders

2510.02106 Zeng isocurvature

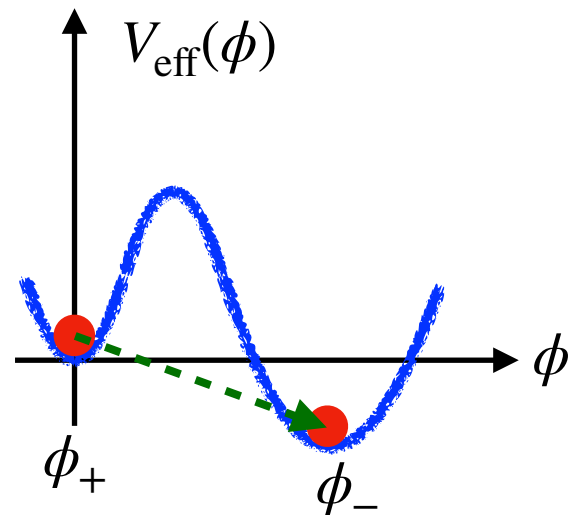
Radiation era



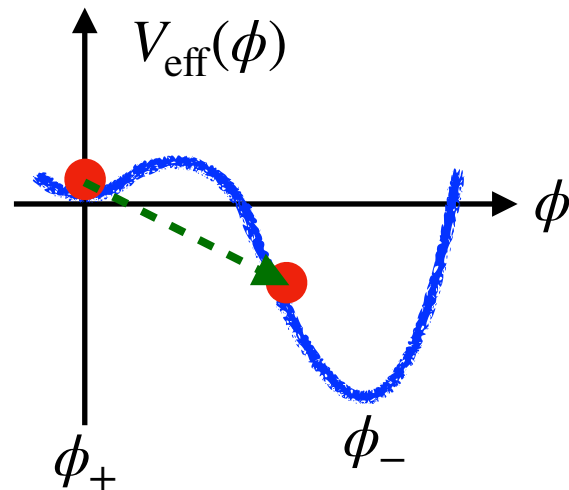
GWs from FOPT

GWs from FOPTs

Thermal barrier



Vacuum barrier

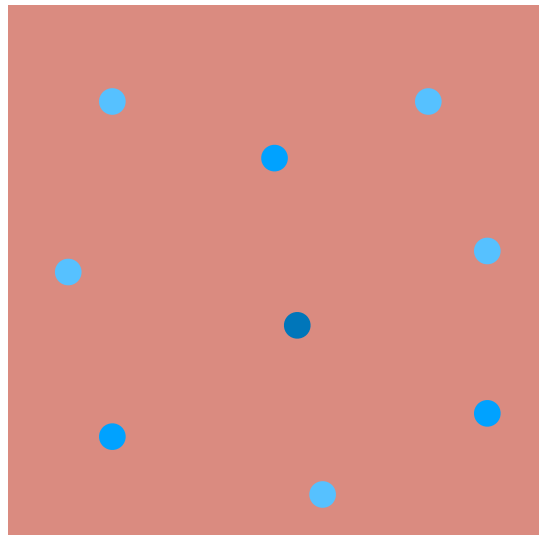


Exponential nucleation

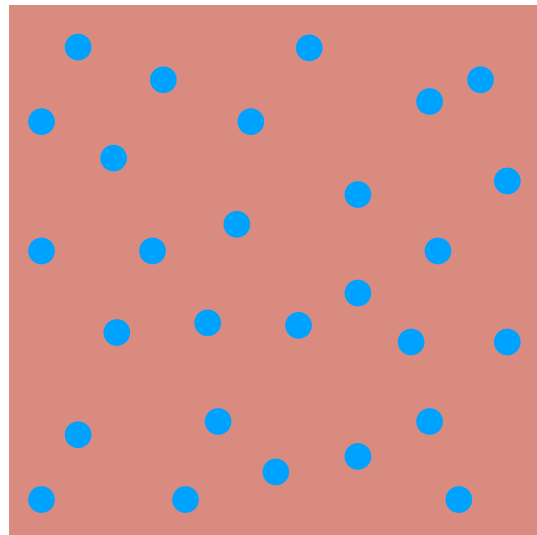
$$\Gamma(t) = \Gamma_0 e^{\beta_1(t-t_f)}$$

Simultaneous nucleation

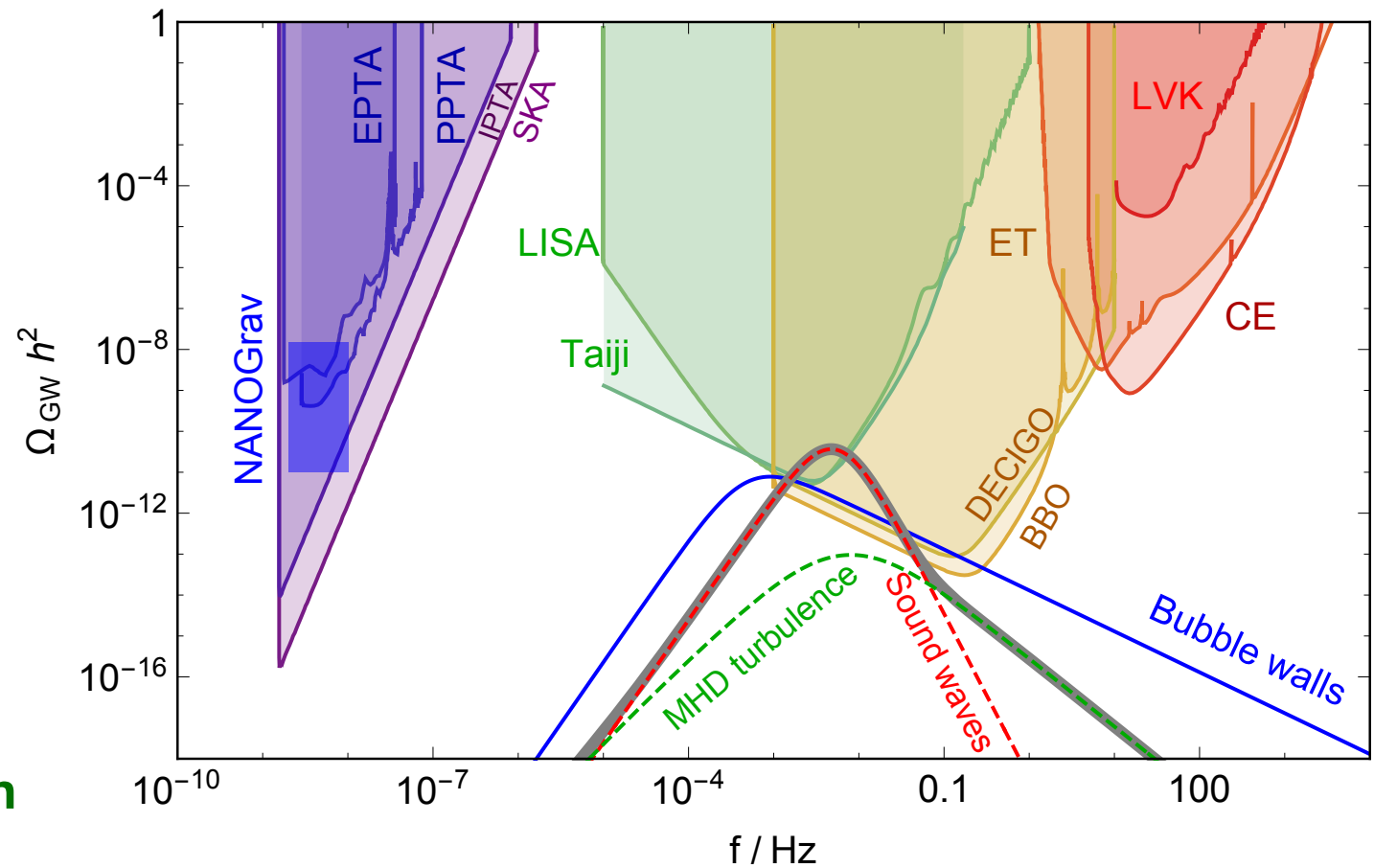
$$\Gamma(t) = \Gamma_0 e^{-\frac{1}{2}\beta_2^2(t-t_m)^2}$$



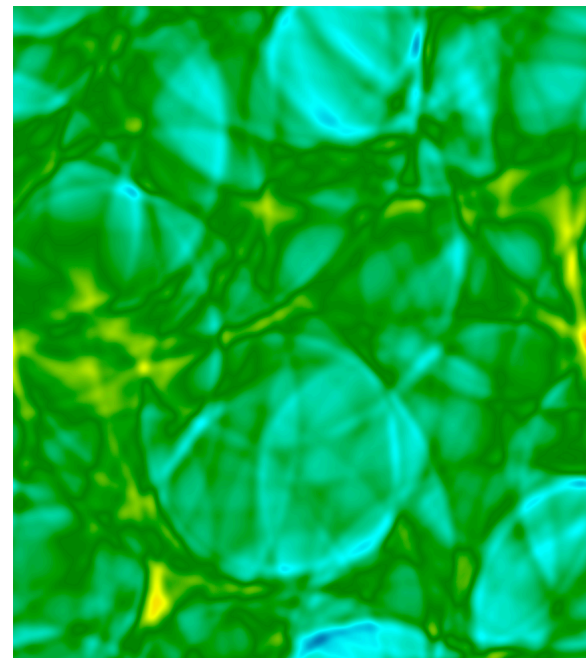
Fluid motions



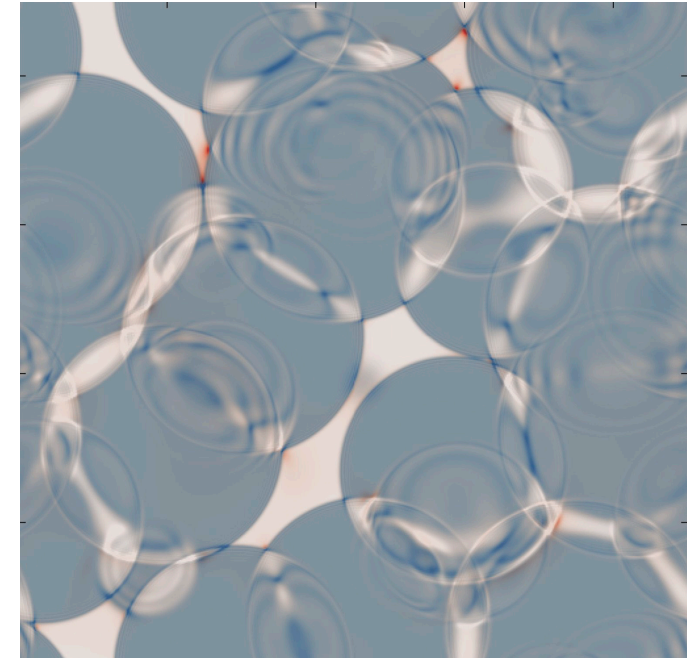
Wall collisions



Fluid motions

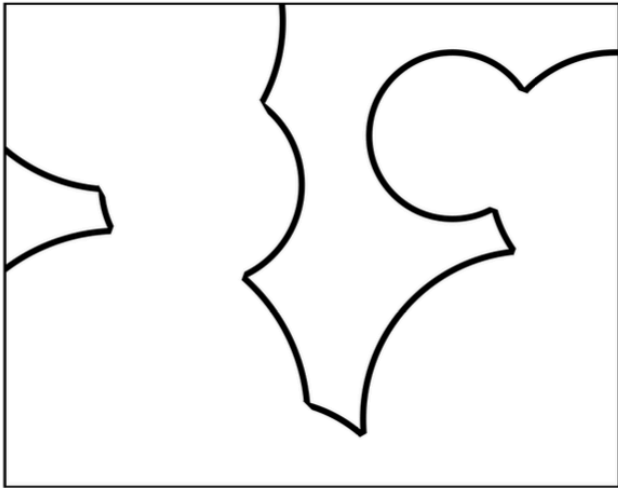


Wall collisions

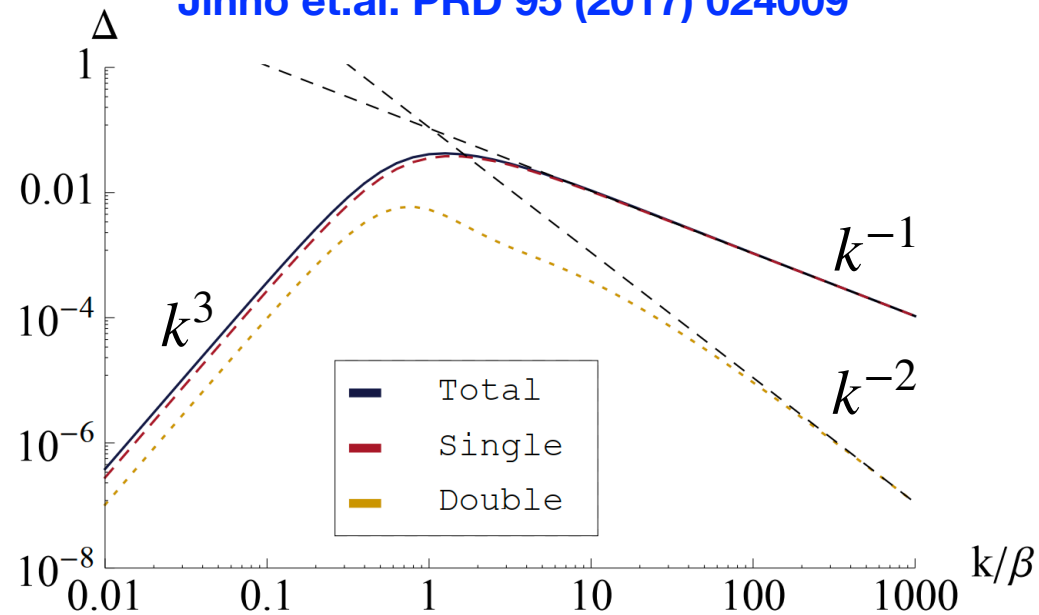


GWs from wall collisions

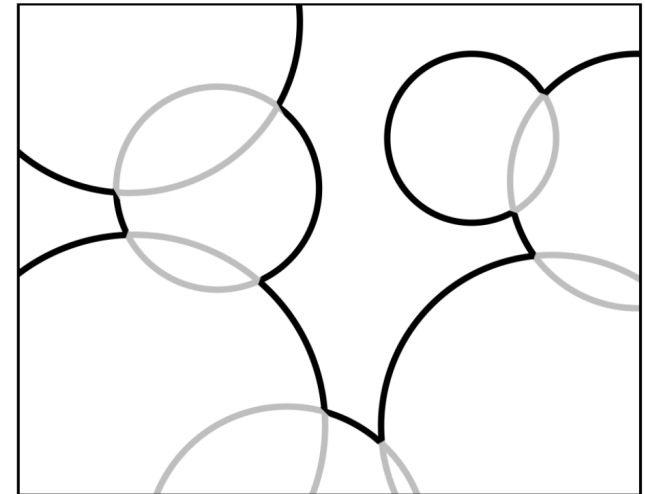
Envelope approximation



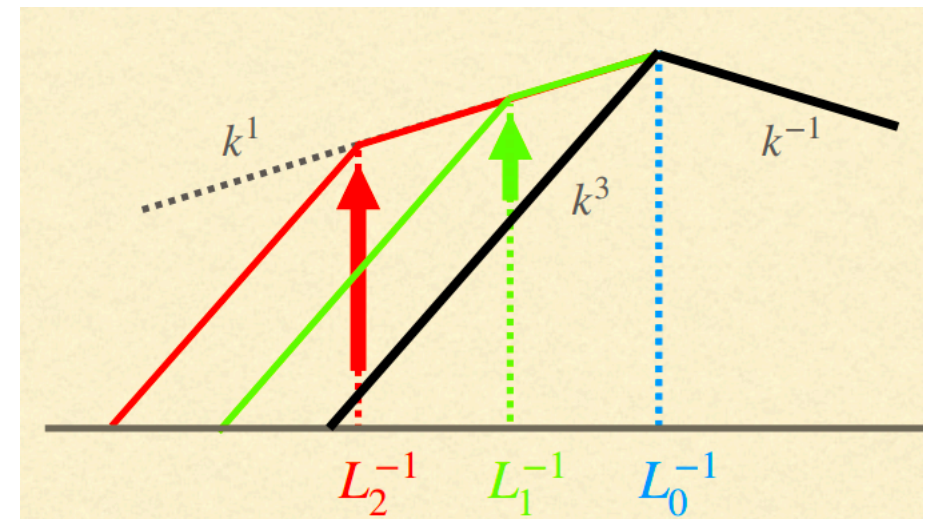
Jinno et.al. PRD 95 (2017) 024009



Beyond envelope approximation



Jinno et.al. JCAP 01 (2019) 060



Analytic template of wall collisions

$$h^2 \Omega_{\text{env}} = 1.67 \times 10^{-5} \left(\frac{100}{g_{\text{dof}}} \right)^{\frac{1}{3}} \left(\frac{H_*}{\beta} \right)^2 \left(\frac{\kappa_\phi \alpha}{1 + \alpha} \right)^2 \frac{0.48 v_w^3}{1 + 5.3 v_w^2 + 5 v_w^4} S_{\text{env}}(f)$$

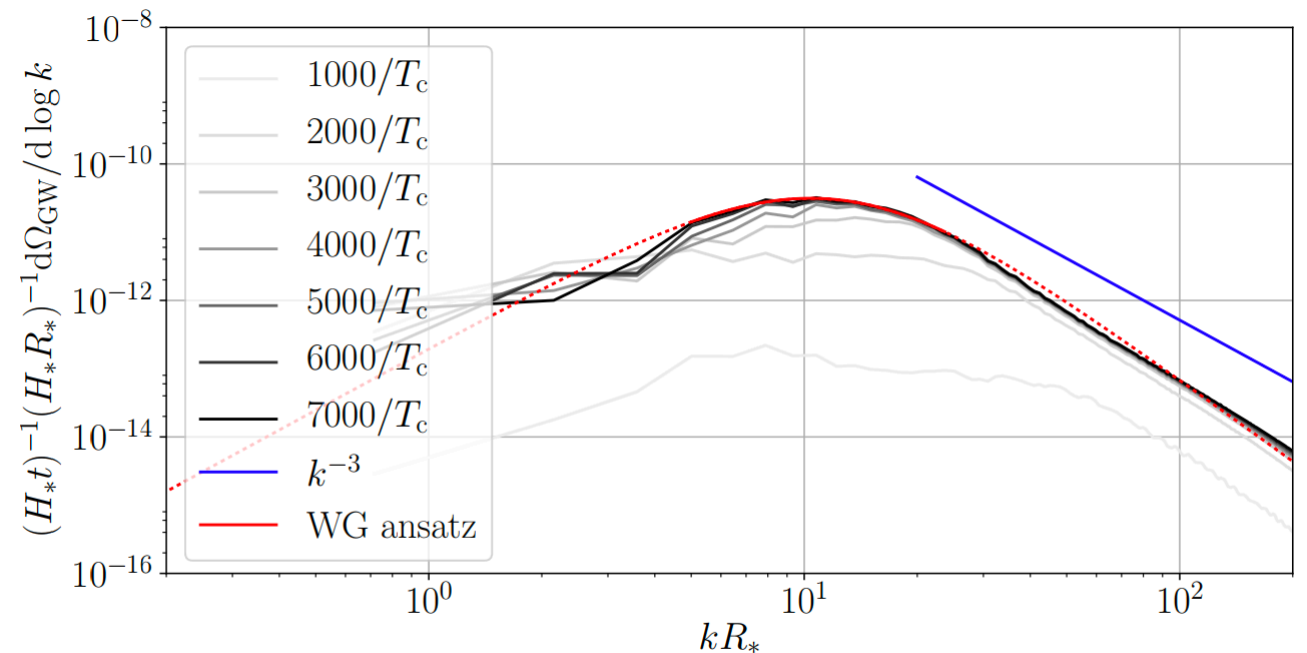
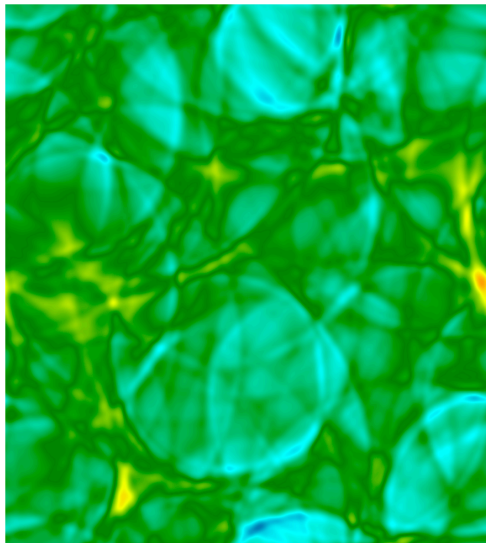
$$\frac{f_{\text{env}}}{\text{Hz}} = 1.65 \times 10^{-5} \left(\frac{g_{\text{dof}}}{100} \right)^{\frac{1}{6}} \left(\frac{T_*}{100 \text{ GeV}} \right) \frac{0.35(\beta/H_*)}{1 + 0.069 v_w + 0.69 v_w^4}$$

$$S_{\text{env}}(f) = \left[c_l \left(\frac{f}{f_{\text{env}}} \right)^{-3} + (1 - c_l - c_h) \left(\frac{f}{f_{\text{env}}} \right)^{-1} + c_h \left(\frac{f}{f_{\text{env}}} \right) \right]^{-1}$$

GWs from fluid motions

Numerical simulations

Sound waves Helsinki-Sussex: 2013, 2015, 2017, 2019



Scalar-plasma EoMs: friction ansatz $\frac{\partial p_{\delta f}}{\partial \phi} \propto u^\mu \partial_\mu \phi$

$$\nabla_\mu \nabla^\mu \phi - \frac{\partial V_{\text{eff}}}{\partial \phi} = - \frac{\partial p_{\delta f}}{\partial \phi}$$

$$\nabla_\mu T_f^{\mu\nu} + \nabla^\nu \phi \frac{\partial V_T}{\partial \phi} = \nabla^\nu \phi \frac{\partial p_{\delta f}}{\partial \phi}$$

$$E = \gamma \epsilon : \quad \dot{E} + \partial_i (E v^i) + p[\dot{\gamma} + \partial_i (\gamma v^i)] - \frac{\partial V}{\partial \phi} \gamma (\dot{\phi} + v^i \partial_i \phi) = \eta \gamma (\dot{\phi} + v^i \partial_i \phi)$$

$$P_i = \gamma (\epsilon + p) U_i : \quad \dot{P}_i + \partial_j (P_i v^j) + \partial_i p + \frac{\partial V}{\partial \phi} \partial_i \phi = - \eta \gamma (\dot{\phi} + v^j \partial_j \phi) \partial_i \phi$$

$$-\ddot{\phi} + \nabla^2 \phi - \frac{\partial V}{\partial \phi} = \eta \gamma (\dot{\phi} + v^i \partial_i \phi)$$

$$h^2 \Omega_{\text{sw}} = 2.65 \times 10^{-6} \left(\frac{100}{g_{\text{dof}}} \right)^{\frac{1}{3}} \left(\frac{H_*}{\beta} \right) \left(\frac{\kappa_{\text{sw}} \alpha}{1 + \alpha} \right)^2 v_w \frac{7^{7/2} (f/f_{\text{sw}})^3}{(4 + 3(f/f_{\text{sw}})^2)^{7/2}} \Upsilon$$

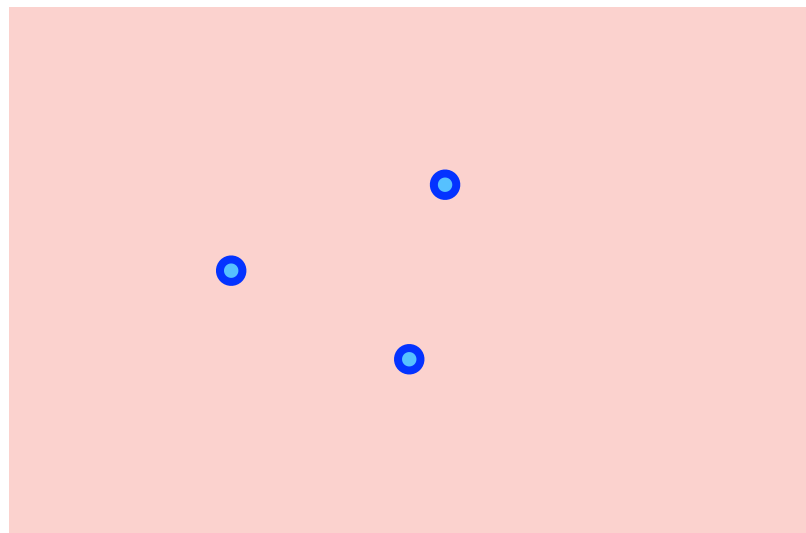
$$\frac{f_{\text{sw}}}{\text{Hz}} = 1.9 \times 10^{-5} \left(\frac{g_{\text{dof}}}{100} \right)^{\frac{1}{6}} \left(\frac{T_*}{100 \text{ GeV}} \right) \left(\frac{1}{v_w} \right) \left(\frac{\beta}{H_*} \right)$$

$$\Upsilon \equiv 1 - (1 + 2\tau_{\text{sw}} H_*)^{-1/2}$$

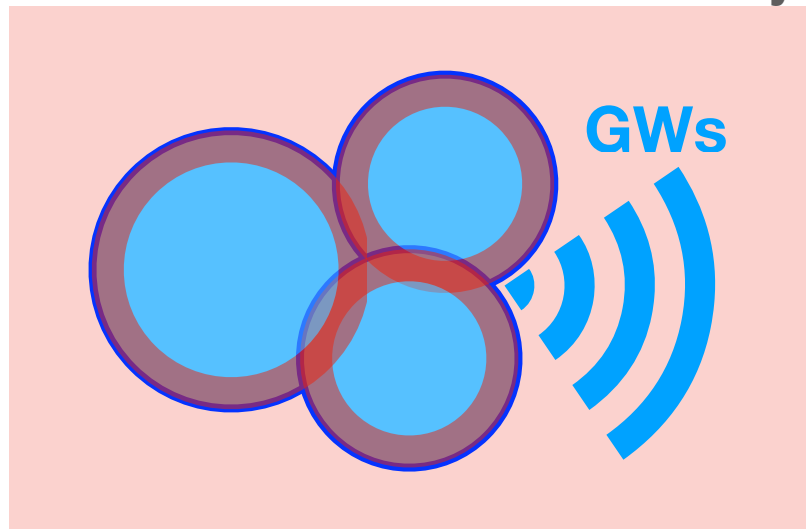
$$\tau_{\text{sw}} H_* \approx (8\pi)^{1/3} v_w / (\beta/H_*) / \bar{U}_f$$

$$\bar{U}_f^2 = 3\kappa_{\text{sw}} \alpha / [4(1 + \alpha)] \quad \text{Huai-Ke Guo et al. 20}$$

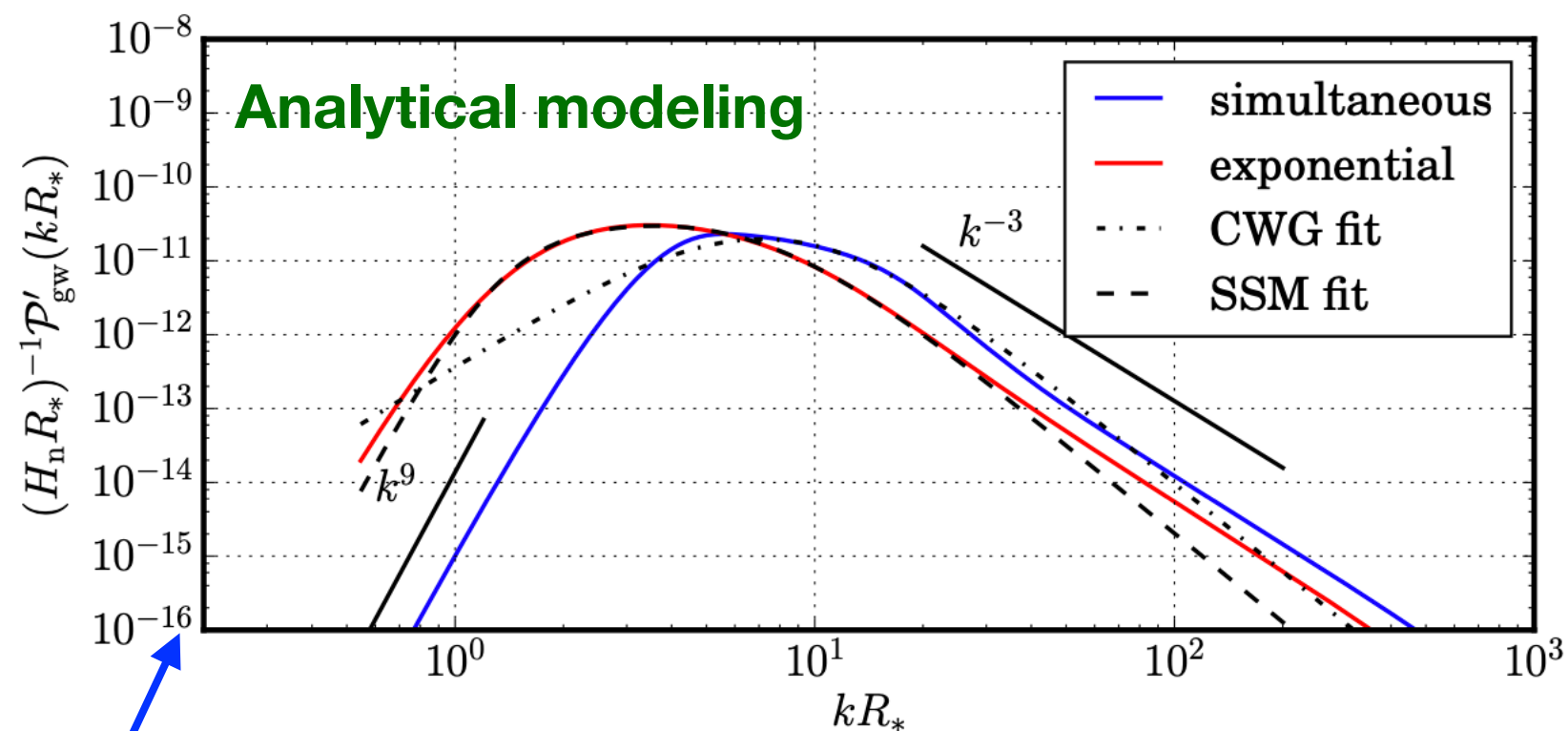
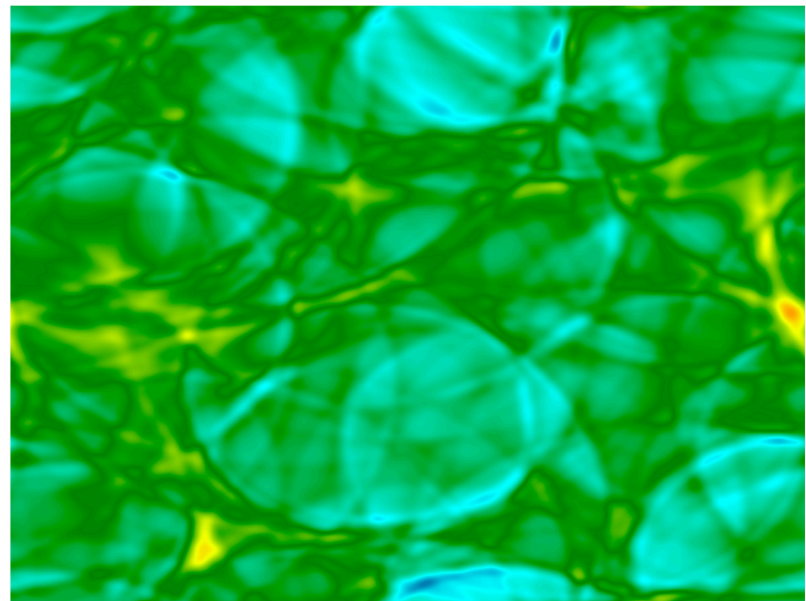
GWs from sound-shell superpositions



Sound-shell forced-collisions at early time

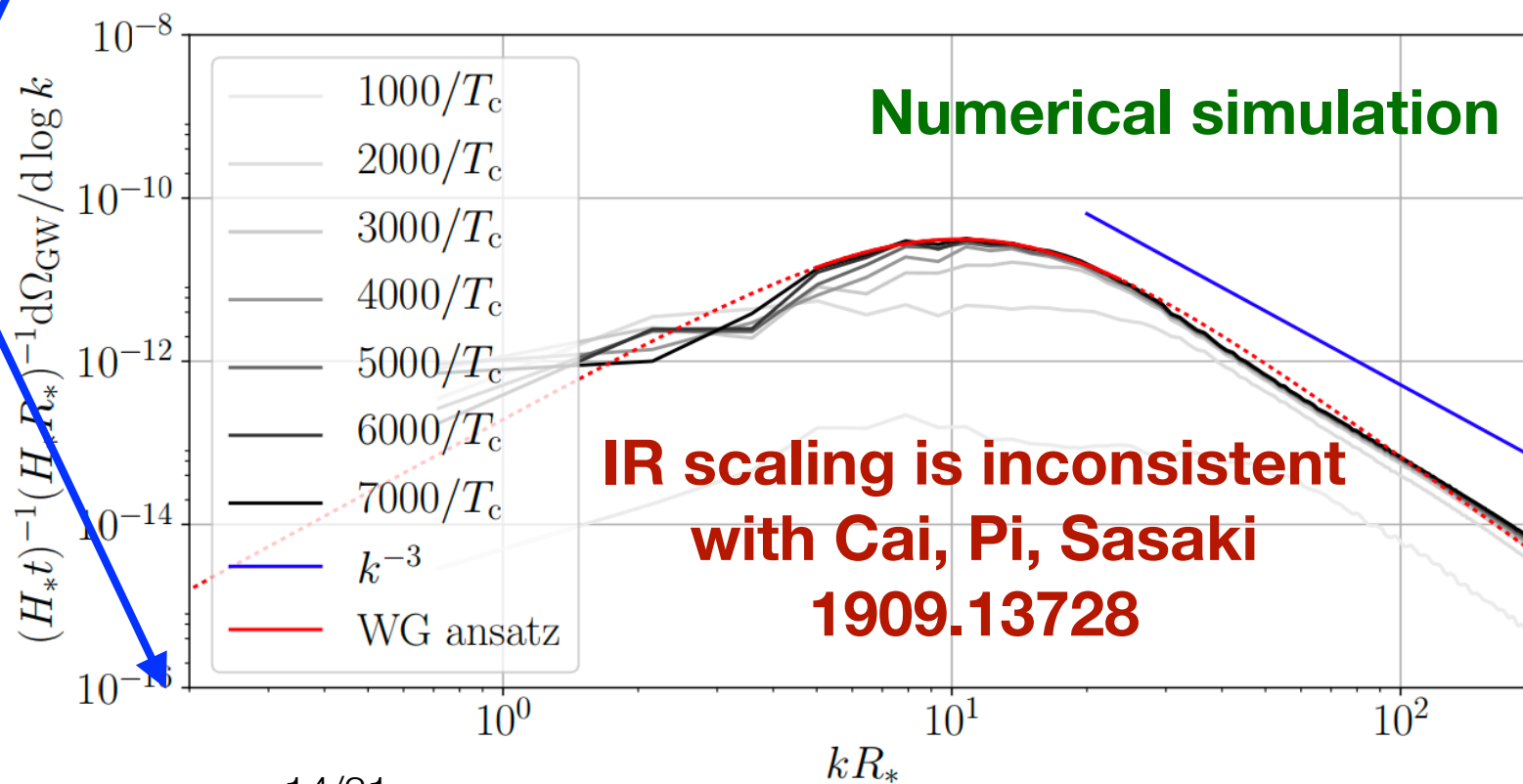


Sound-shell free-collisions at late time

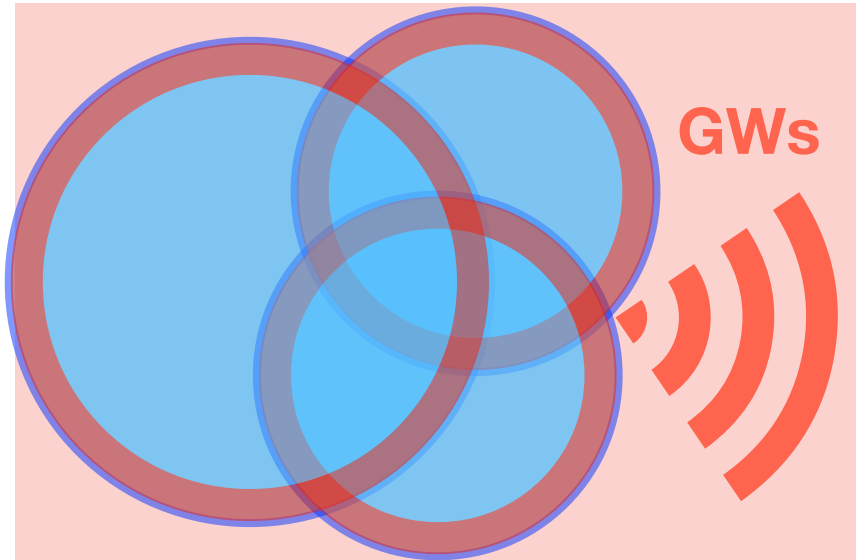


Sound shell model Hindmarsh 18 PRL, Hindmarsh & Hijazi 19

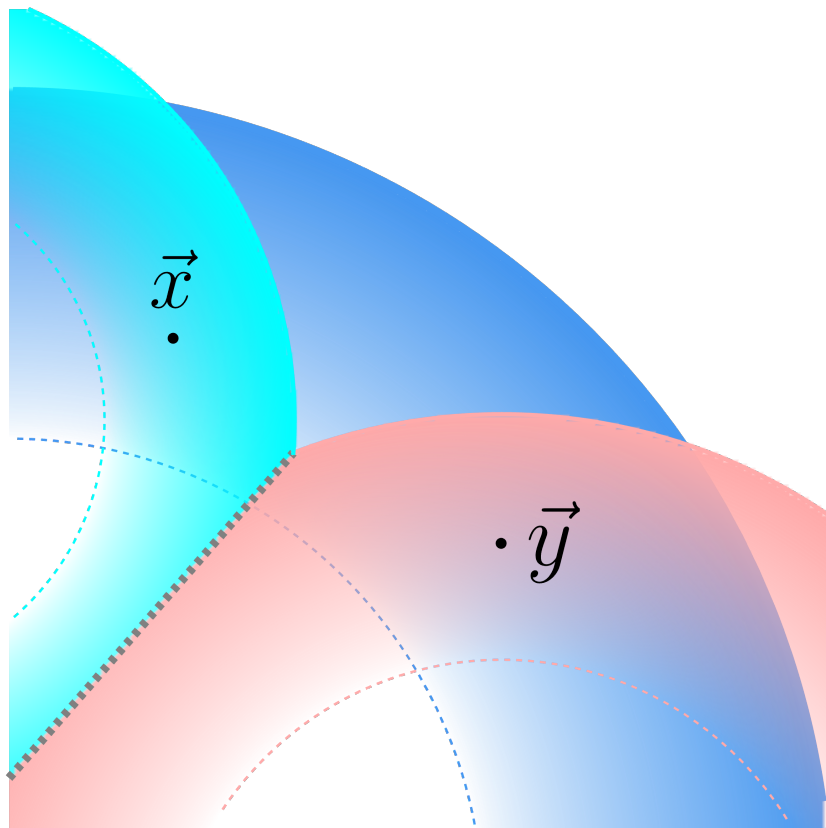
(a) Weak, $v_w = 0.92$



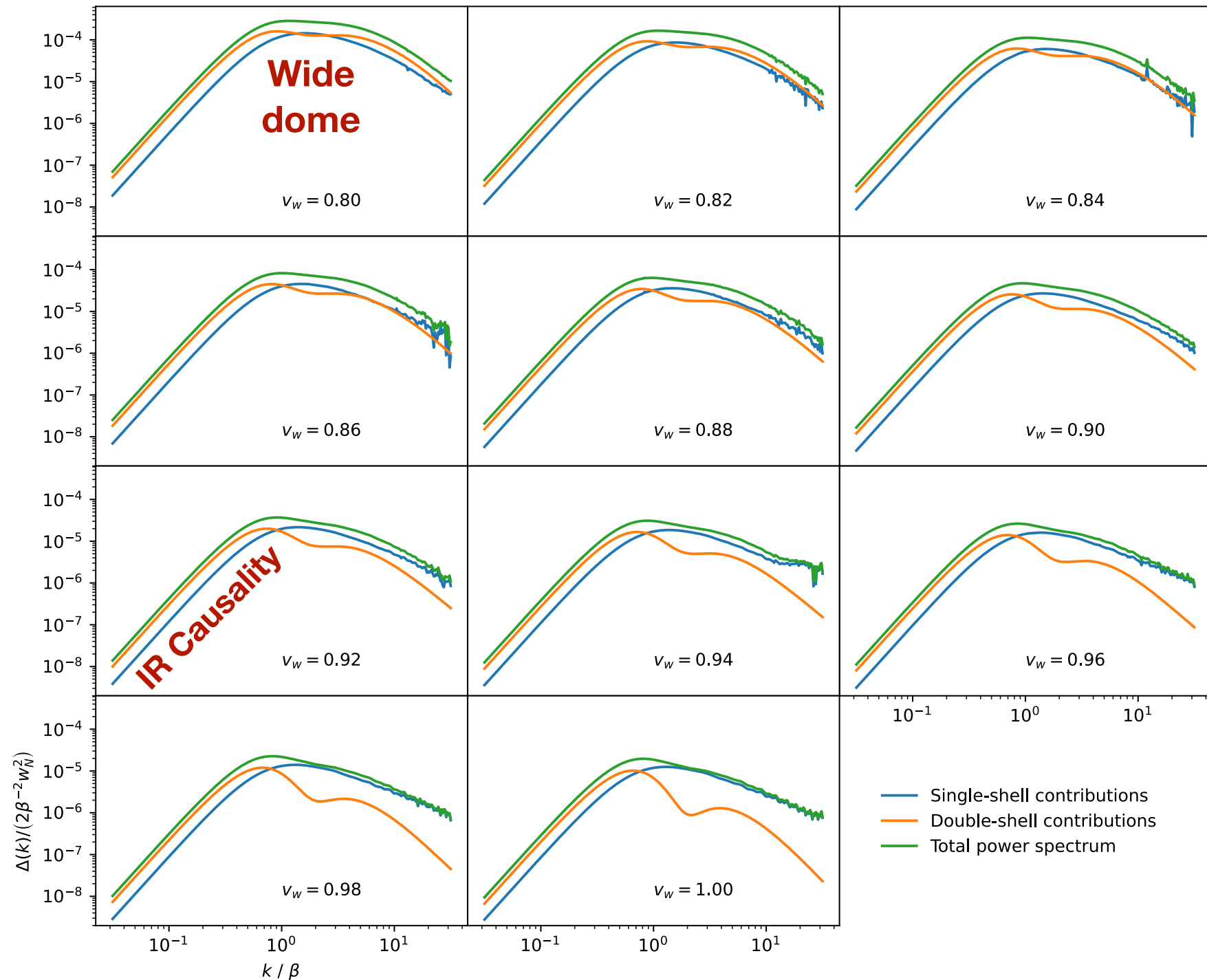
GWs from sound-shell envelopes



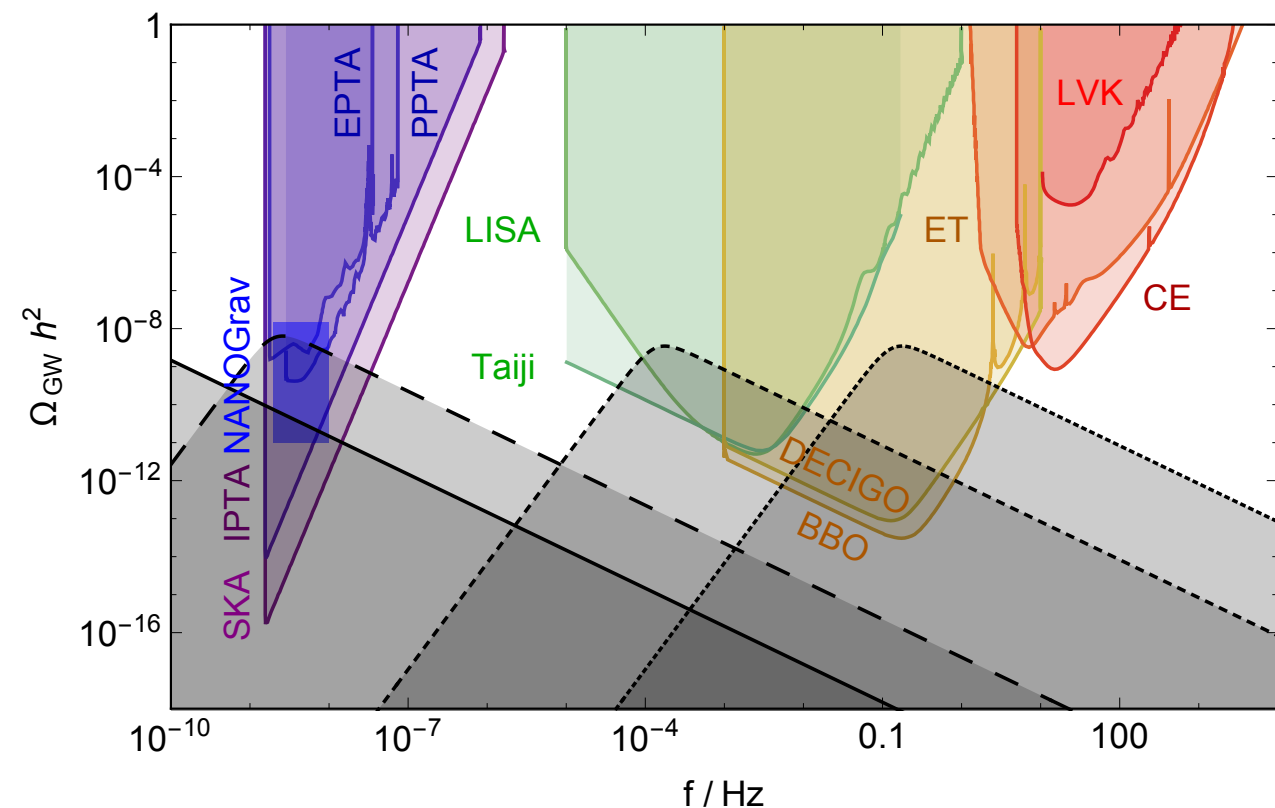
SGWBs from sound-shell envelopes before the full percolation of bubble walls



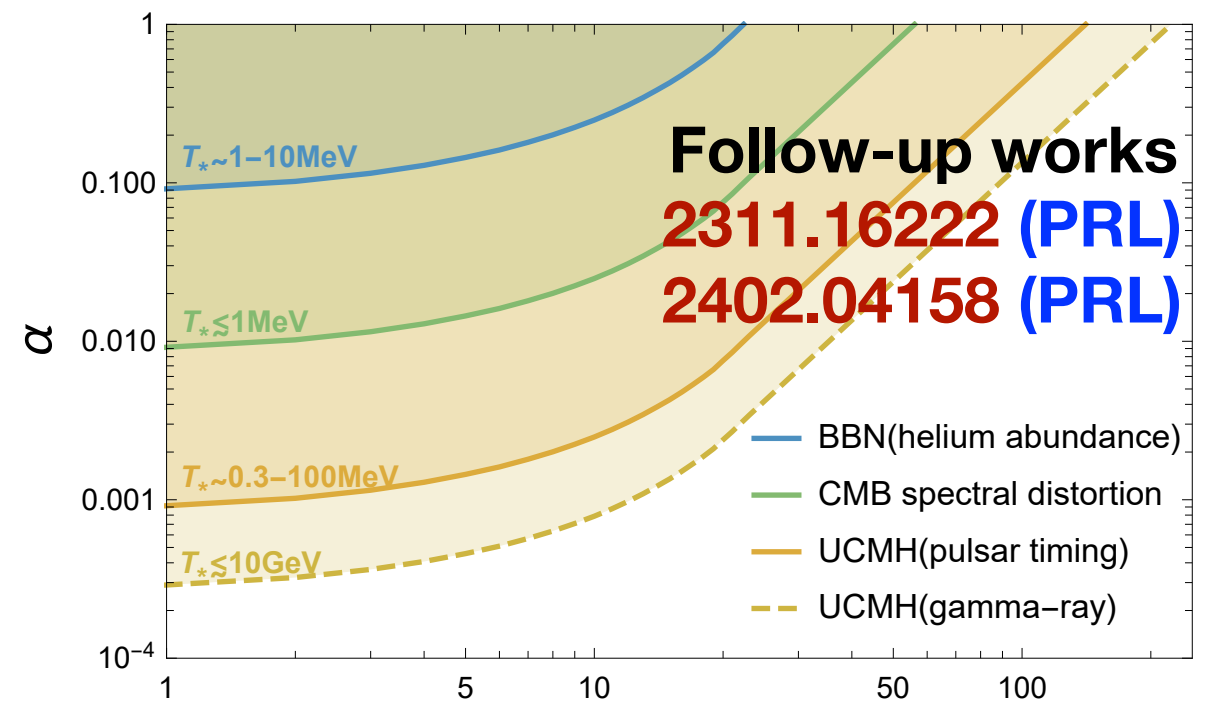
2305.00074 Rong-Gen Cai, SJW, Zi-Yan Yuwen
“Hydrodynamic sound shell model”
 Phys. Rev. D 108 (2023) 2, L021502



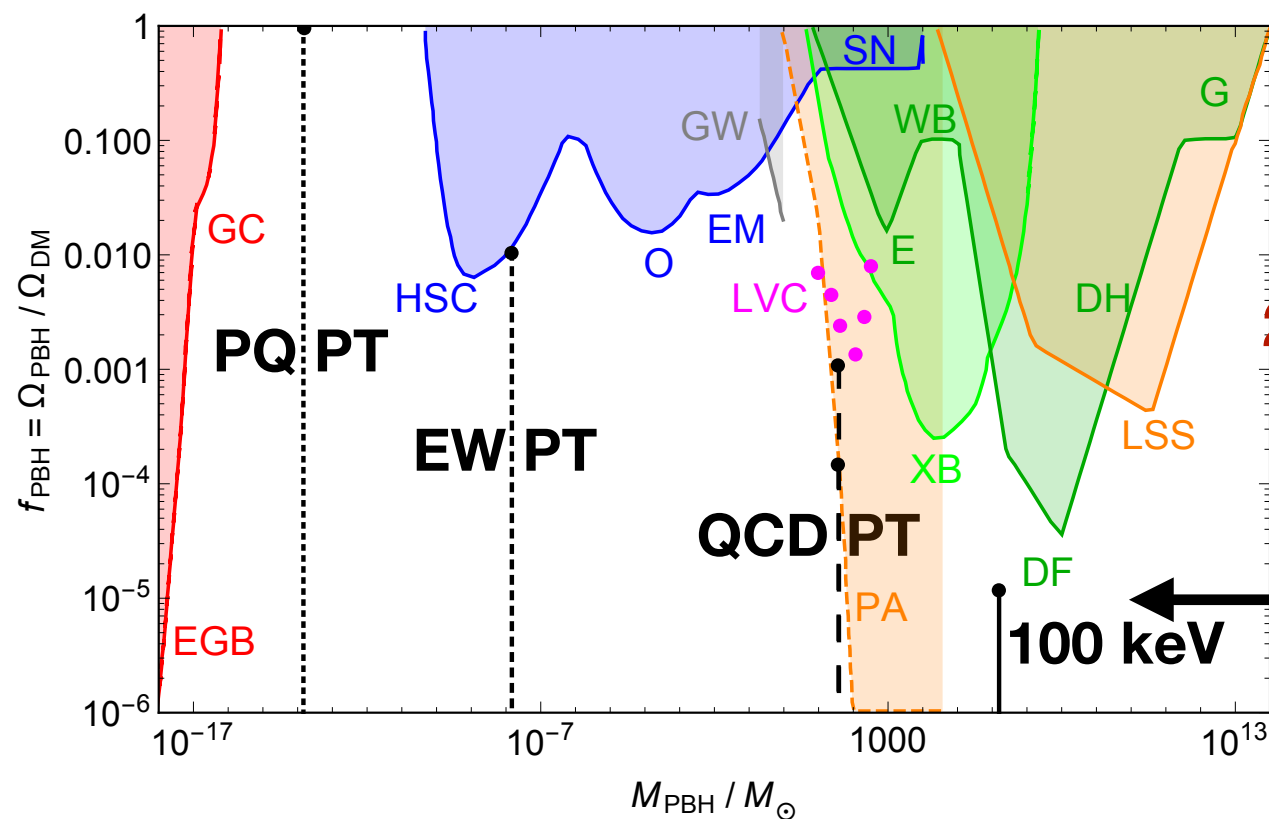
PBH/Curvature perturbations from FOPT



2208.14086 Liu, Bian, Cai, Guo, Wang (PRL)



2106.05637 Liu, Bian, Cai, Guo, Wang (PRD Letter)

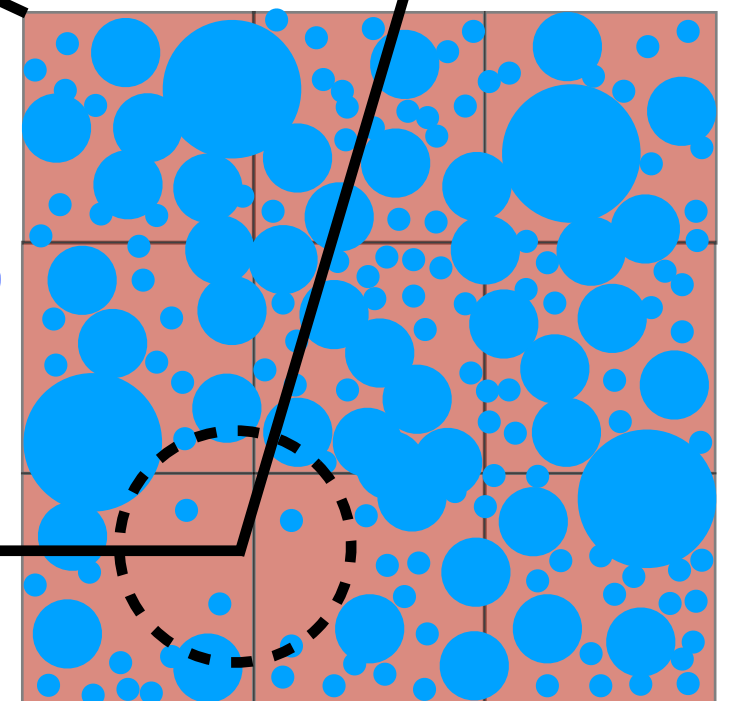


GWs

Curvature perturbations

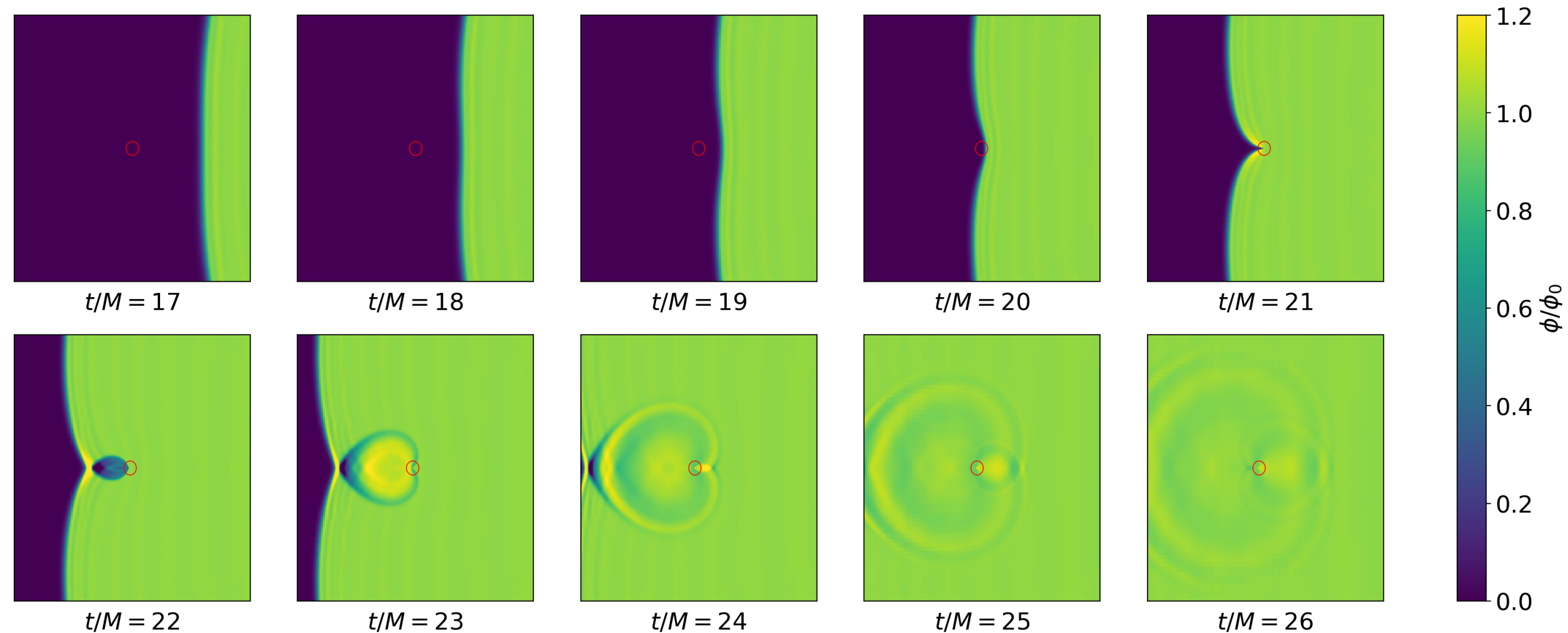
Follow-up work
2307.04239 (PRL)

PBHs



2404.06506 Cai, Hao, Wang (SCPMA)

Bubble-PBH collisions

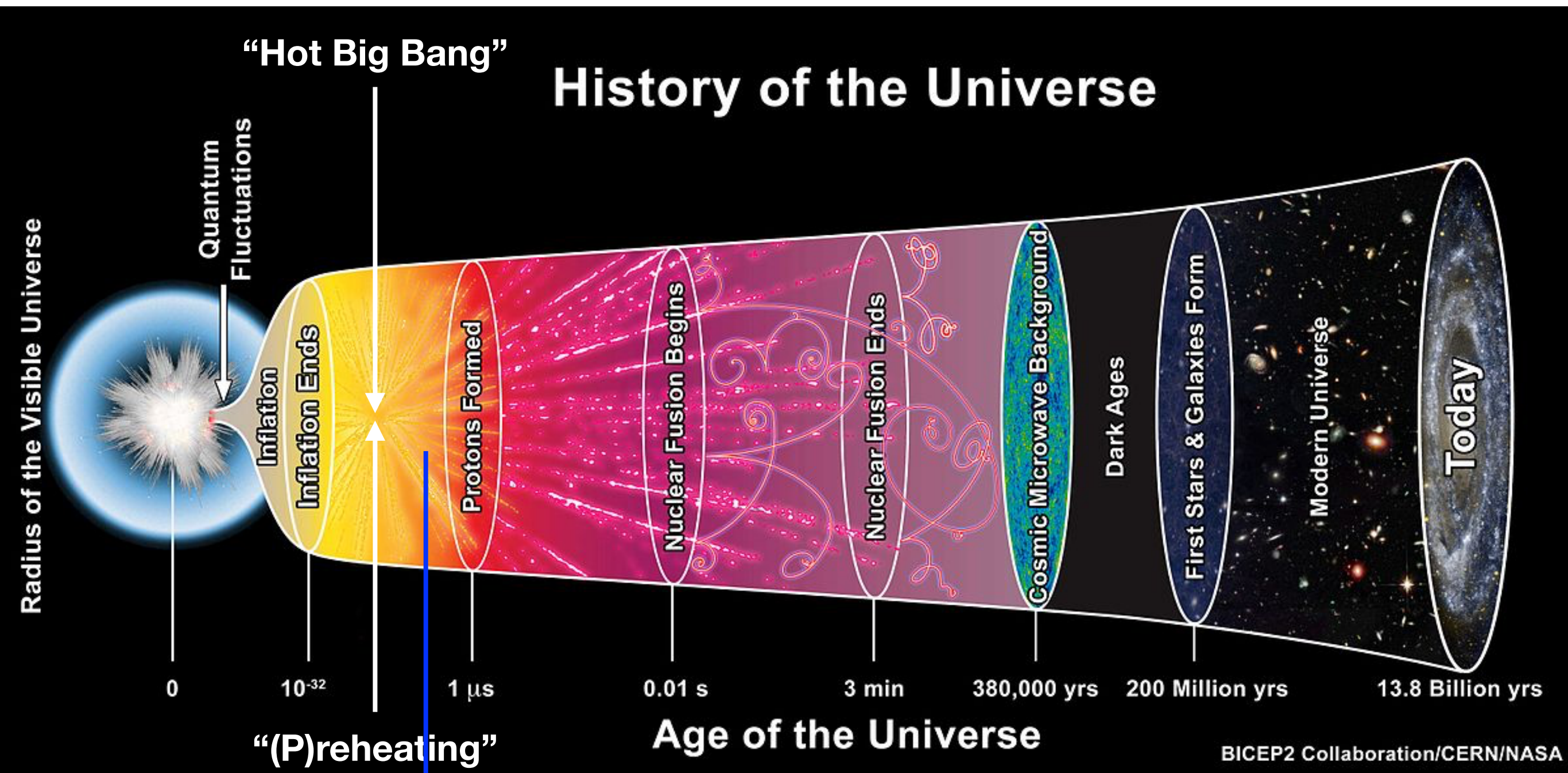


Full numerical relativity with **GRChombo**

- Phenomenologies** {
1. PBH mass growth $\Rightarrow$ small but important for late/multi FOPT
 2. PBH velocity recoil $\Rightarrow$ enhance PBH binary formation rate
 3. Gravitational waves $\Rightarrow$ high-frequency GWs but difficult

2406.05838 Yuwen, Joana, **SJW**, Cai *Phys.Rev.Res.* 7 (2025) 2, 023180

Radiation era



GWs from gravitational collapse

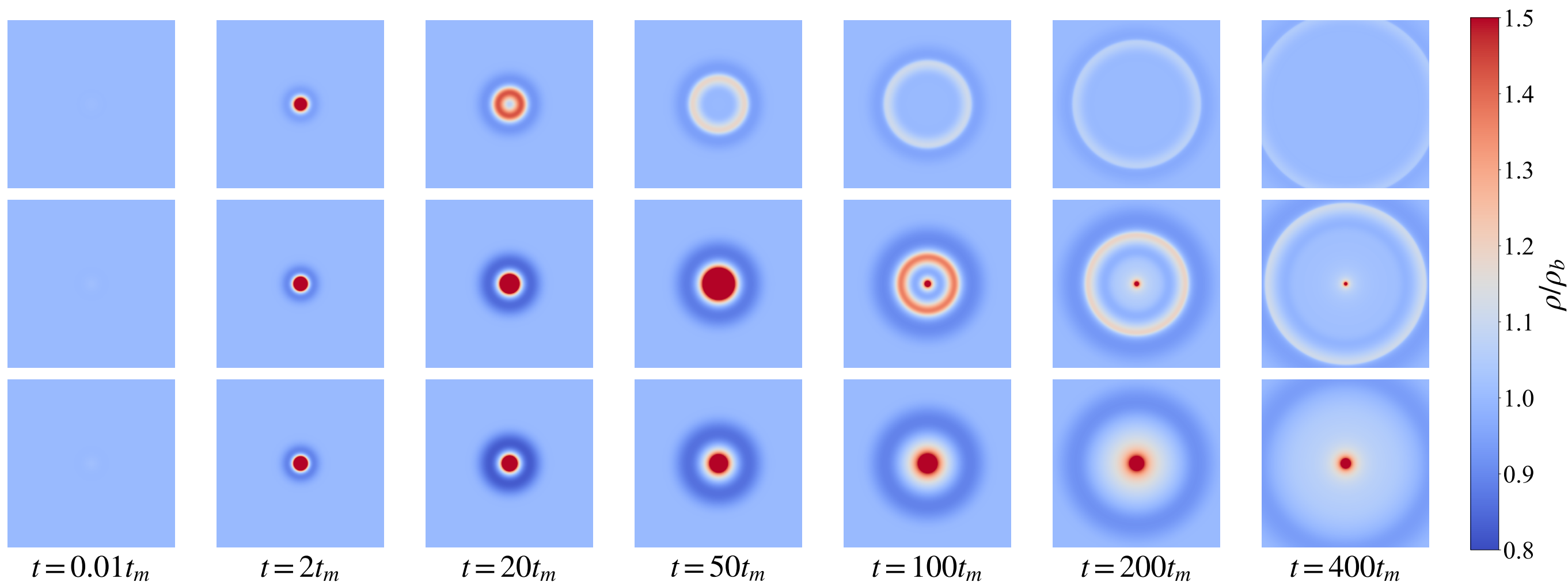
GWs from gravitational collapses

2504.11275 Zeng, Ning, Yuwen, SJW, Cai **Relic gravitational waves from primordial gravitational collapses**

2504.12243 Ning, Zeng, Yuwen, SJW, Cai **Sound waves from primordial black hole formations**

FOPT bubble expansion → sound waves → gravitational waves

Gravitational collapse → sound waves → gravitational waves



GWs from gravitational collapses

2504.11275 Zeng, Ning, Yuwen, SJW, Cai **Relic gravitational waves from primordial gravitational collapses**

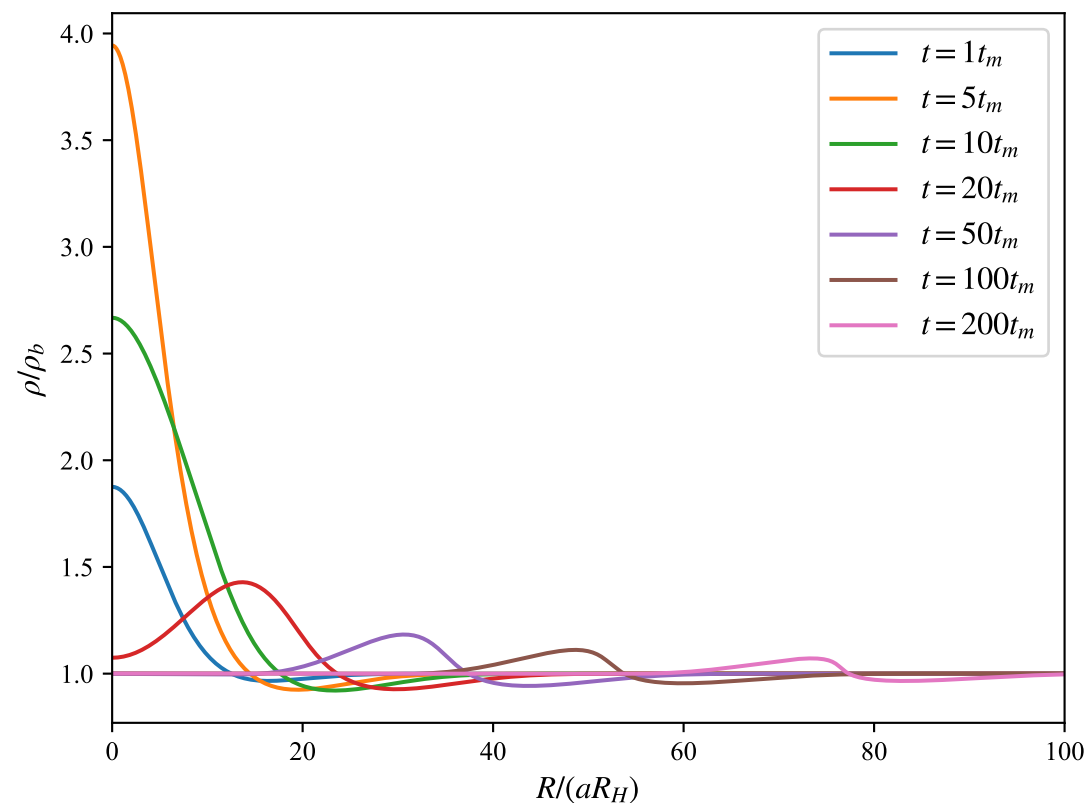
2504.12243 Ning, Zeng, Yuwen, SJW, Cai **Sound waves from primordial black hole formations**

FOPT bubble expansion $\rightarrow$ sound waves $\rightarrow$ gravitational waves

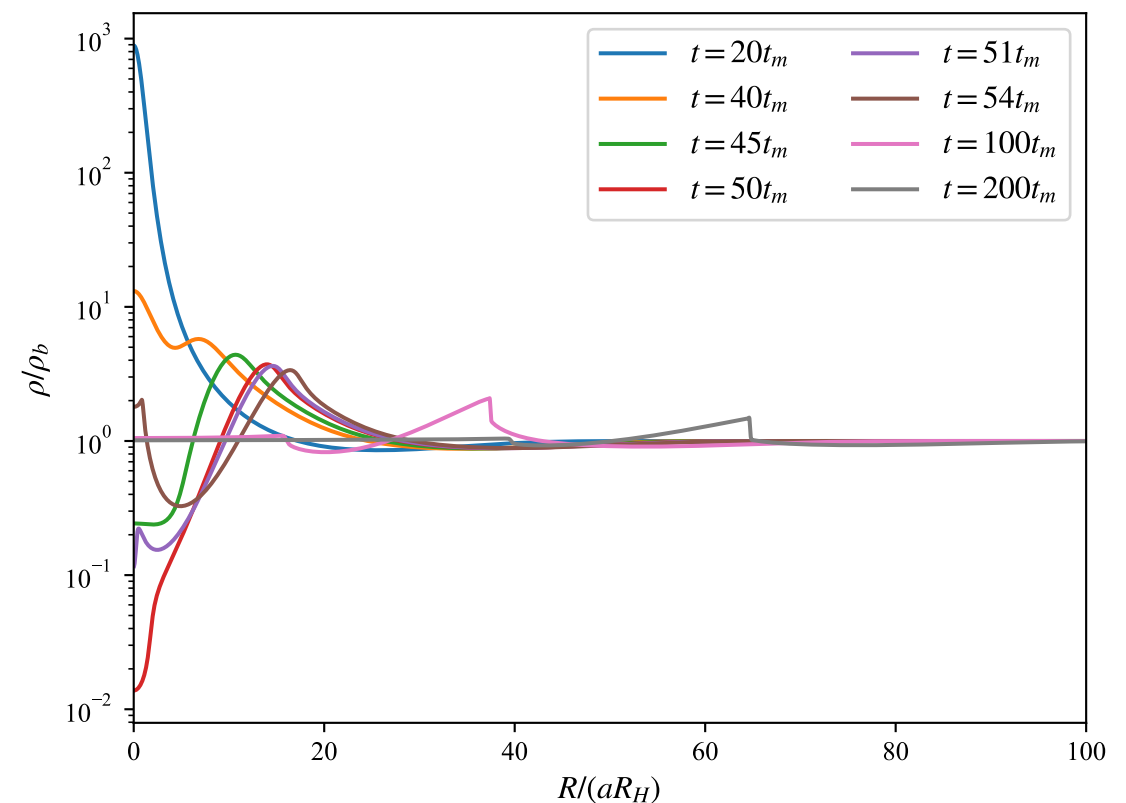
Gravitational collapse $\rightarrow$ sound waves $\rightarrow$ gravitational waves

Subcritical case: no PBH is formed

$$\delta_m = 0.3 \ll \delta_c$$



$$\delta_m = 0.495 \lesssim \delta_c$$



GWs from gravitational collapses

2504.11275 Zeng, Ning, Yuwen, SJW, Cai **Relic gravitational waves from primordial gravitational collapses**

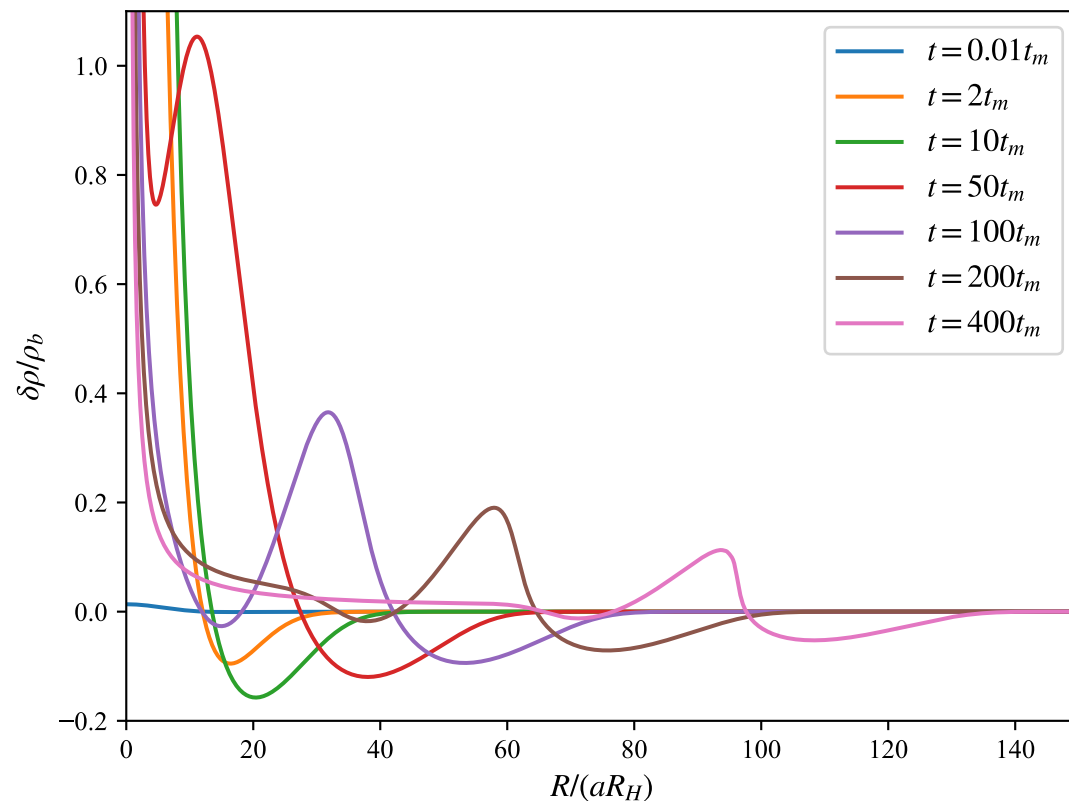
2504.12243 Ning, Zeng, Yuwen, SJW, Cai **Sound waves from primordial black hole formations**

FOPT bubble expansion → sound waves → gravitational waves

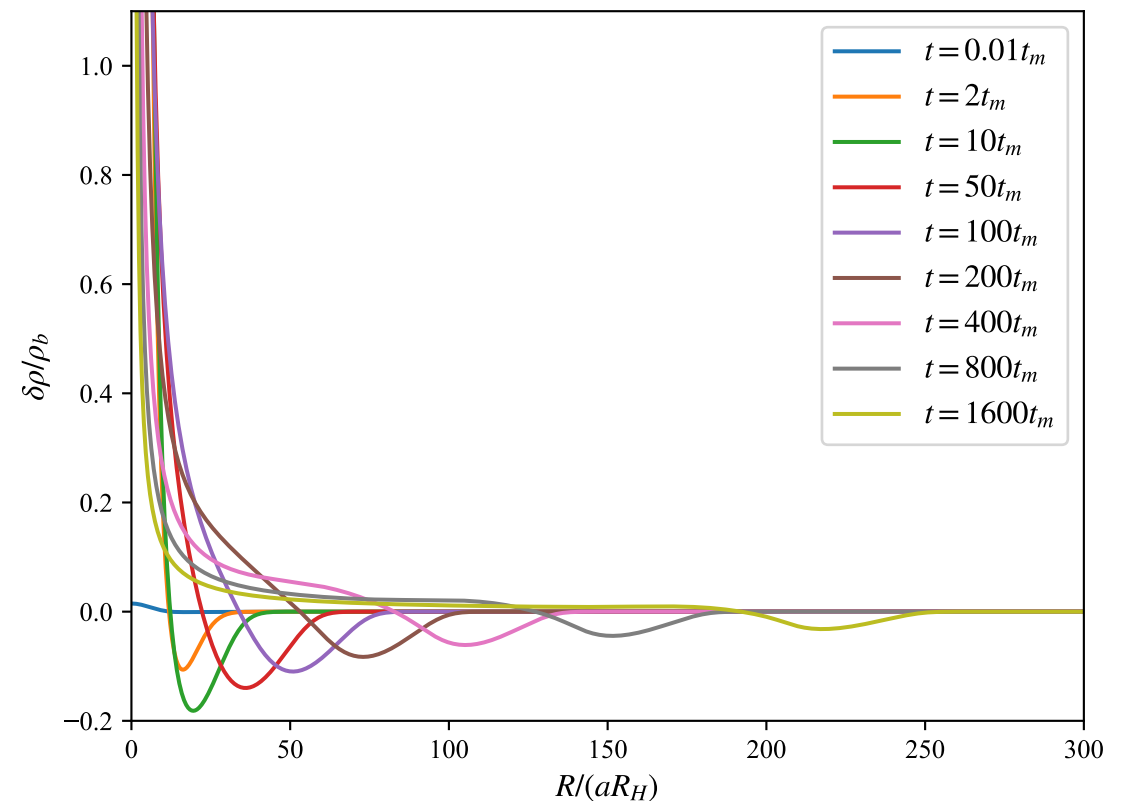
Gravitational collapse → sound waves → gravitational waves

Supercritical case: PBH is formed

$$\delta_m = 0.5 \gtrsim \delta_c$$



$$\delta_m = 0.55 > \delta_c$$



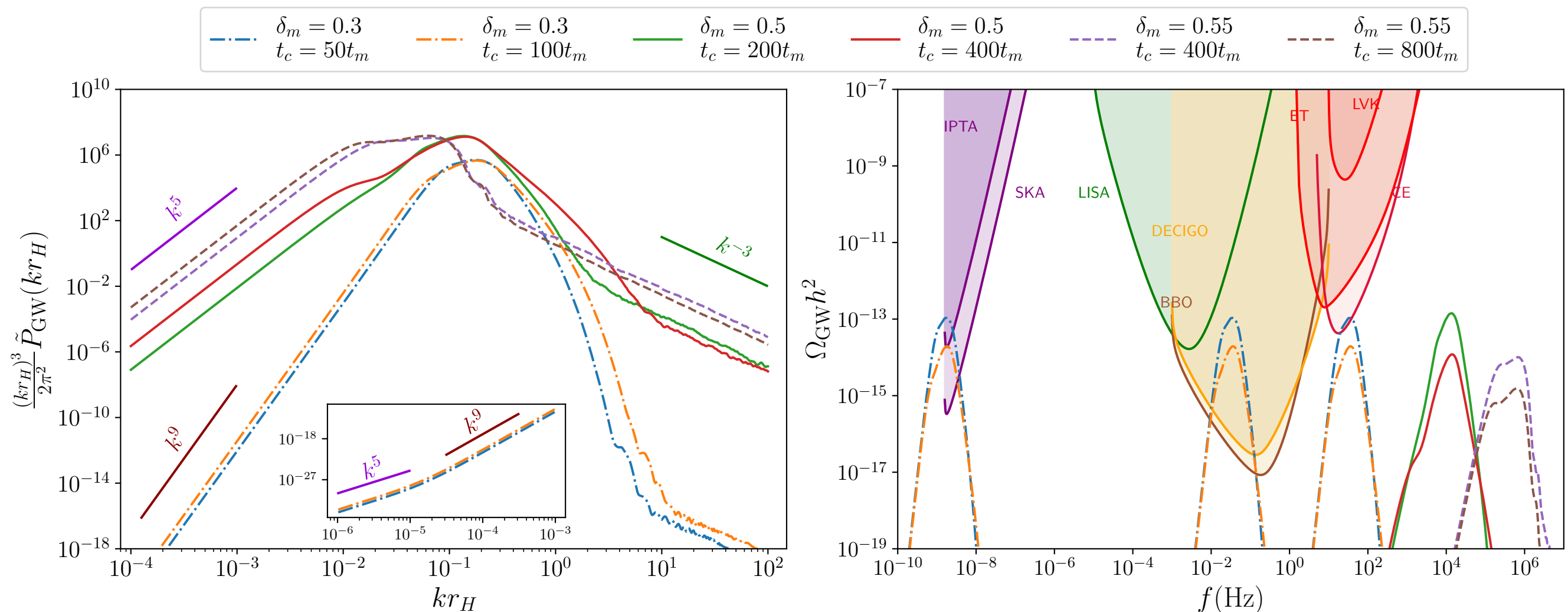
GWs from gravitational collapses

2504.11275 Zeng, Ning, Yuwen, SJW, Cai **Relic gravitational waves from primordial gravitational collapses**

2504.12243 Ning, Zeng, Yuwen, SJW, Cai **Sound waves from primordial black hole formations**

FOPT bubble expansion → sound waves → gravitational waves

Gravitational collapse → sound waves → gravitational waves



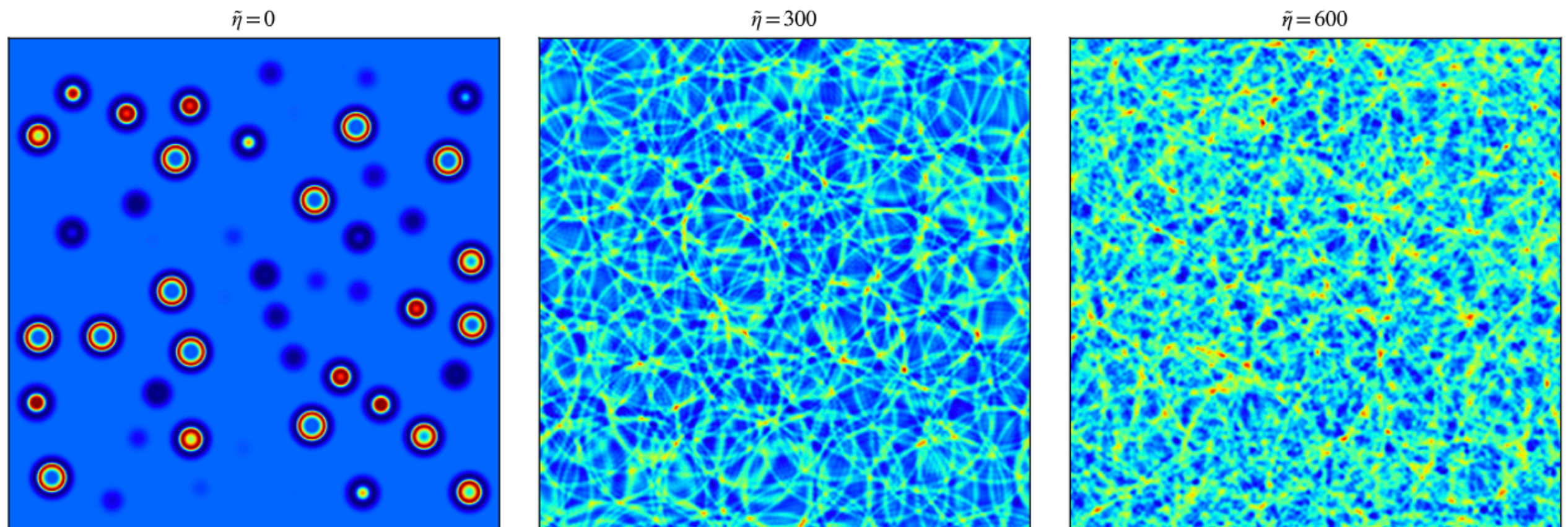
GWs from gravitational collapses

2504.11275 Zeng, Ning, Yuwen, SJW, Cai **Relic gravitational waves from primordial gravitational collapses**

2504.12243 Ning, Zeng, Yuwen, SJW, Cai **Sound waves from primordial black hole formations**

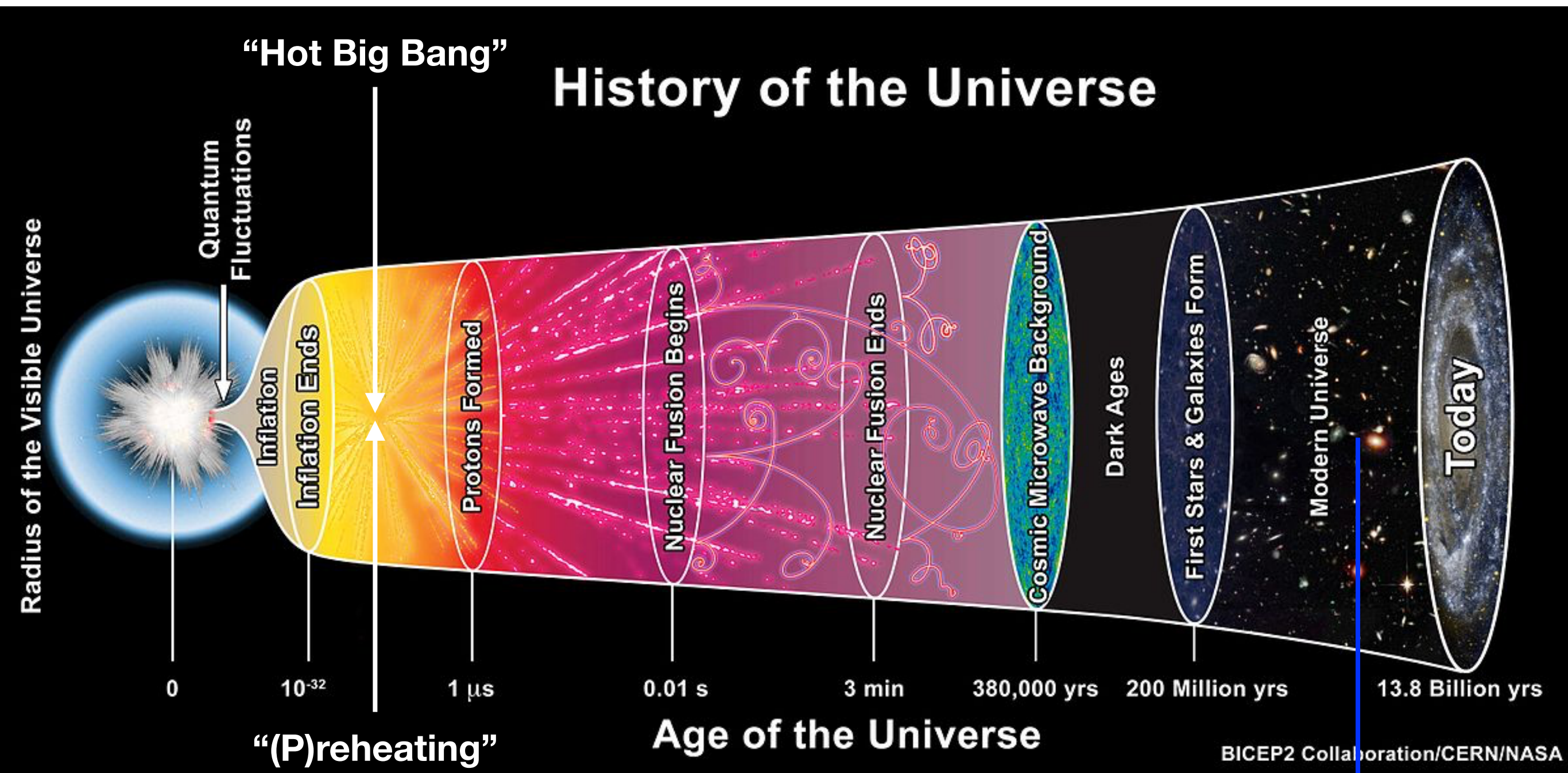
FOPT bubble expansion $\rightarrow$ sound waves $\rightarrow$ gravitational waves

Gravitational collapse $\rightarrow$ sound waves $\rightarrow$ gravitational waves



To appear soon

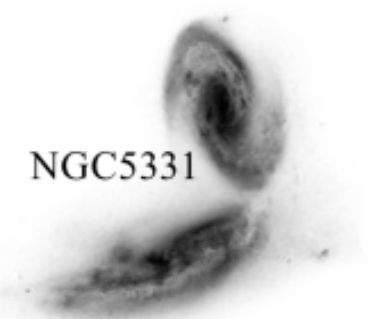
Matter era



GWs from inspiraling system

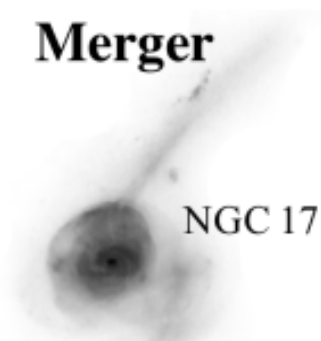
SMBH binary

Galaxy Merger



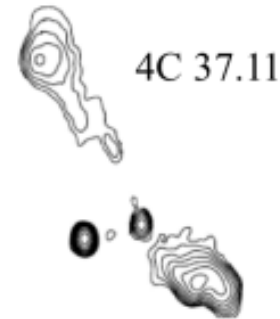
Dynamical friction drives massive objects to central positions

Stellar Core Merger



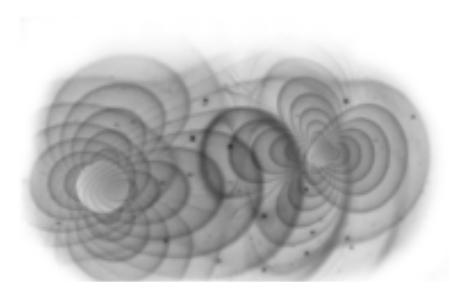
Dynamical friction less efficient as SMBHs form a binary.

Binary Formation



Stellar and gas interactions may dominate binary inspiral?

Continuous GWs



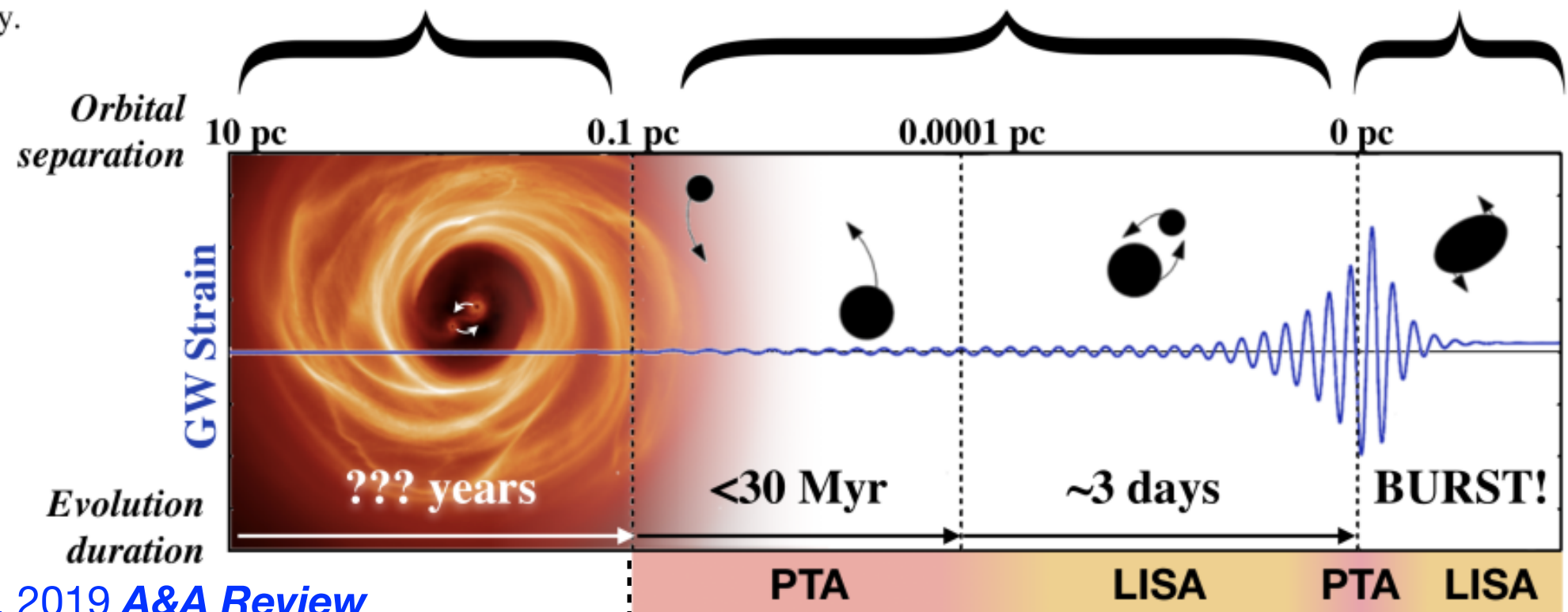
Gravitational radiation provides efficient inspiral. Circumbinary disk may track shrinking orbit.

Coalescence, Memory & Recoil



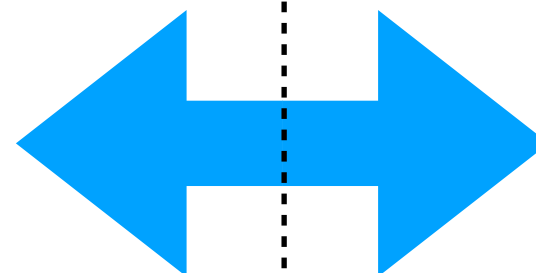
Post-coalescence system may experience gravitational recoil.

The Lifecycle of Binary Supermassive Black Holes



Sara Burke-Spolaor et al. 2019 *A&A Review*

Last-parsec problem?



Circularization from GWs

SMBH binary with eccentricity

Model : Siyuan Chen et al. 2017 *MNRAS*

function of the form

$$h_{c,\text{fit}}(f) = a_0 \bar{f}^{a_1} e^{-a_2 \bar{f}} + b_0 \bar{f}^{b_1} e^{-b_2 \bar{f}} + c_0 \bar{f}^{-c_1} e^{-c_2/\bar{f}} \quad (15)$$

where a_i, b_i, c_i are constants to be determined by the fit and $\bar{f} = f/(10^{-8}\text{Hz})$. We find that setting

$$\begin{array}{lll} a_0 = 7.27 \times 10^{-14} & a_1 = 0.254 & a_2 = 0.807 \\ b_0 = 1.853 \times 10^{-12} & b_1 = 1.77 & b_2 = 3.7 \\ c_0 = 1.12 \times 10^{-13} & c_1 = 0.676 & c_2 = 0.6 \end{array}$$

reproduces the spectrum within a maximum error of **1.5%** in log-amplitude (i.e. **3.5%** in amplitude), as shown in figure 3. It also

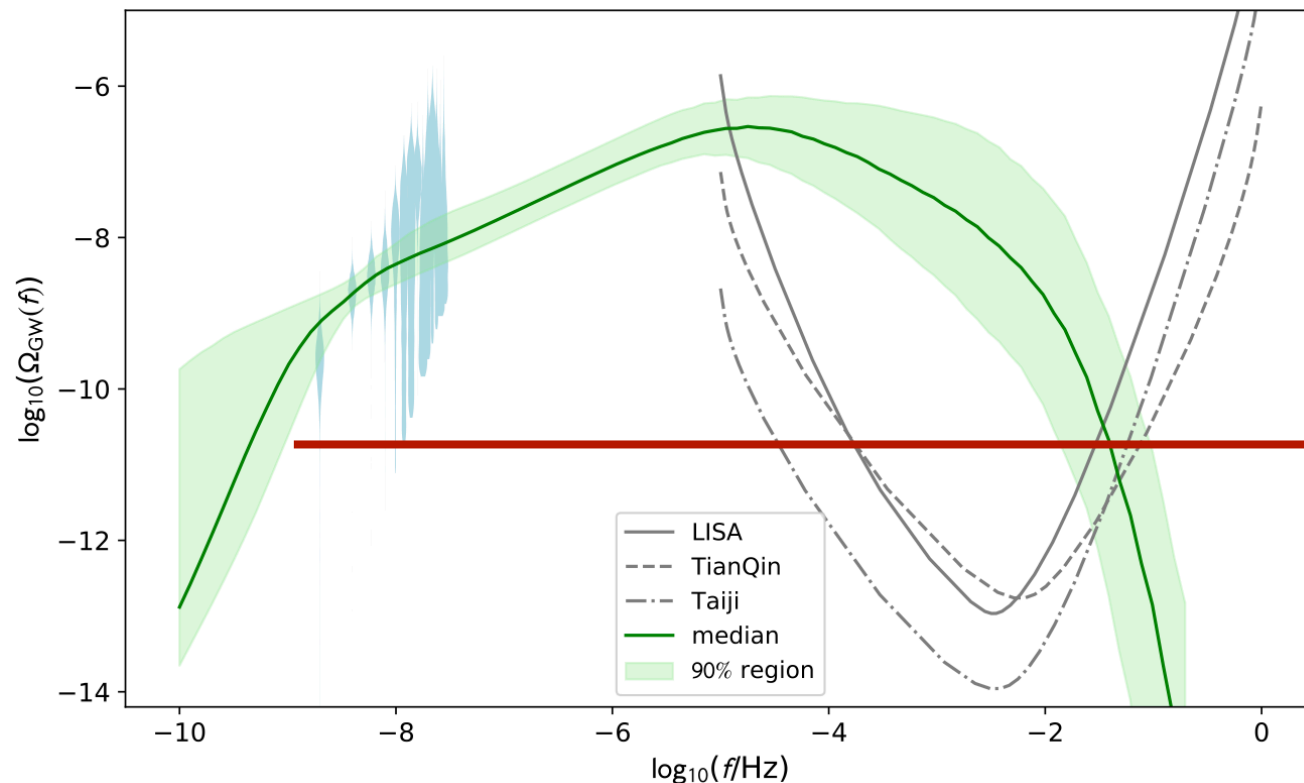


Figure 1. characteristic amplitude spectrum for different eccentricities calculated with $n = 12500$ harmonics computed with no lower limit on f_n .

Data : Bi, Wu, Chen, Huang 2023 *SCPMA*

NANOGrav 15yr data prefers a large turn-over eccentricity when GWs begin to dominate the SMBHB evolution, but why?

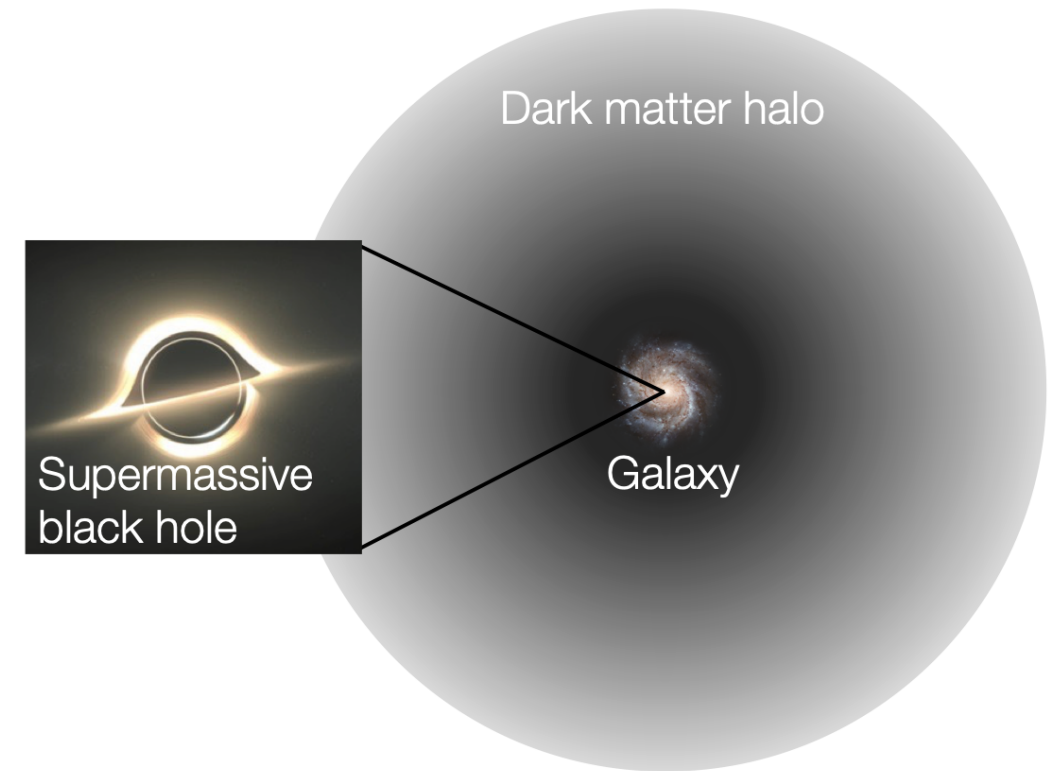
SMBH with DM spike

NFW profile

$$\rho_{\text{NFW}}(r) = \frac{\rho_s}{\frac{r}{r_s} \left(1 + \frac{r}{r_s}\right)^2}$$

Inner cusp

$$\rho_{\text{halo}}(r \ll r_0) = \rho_0 \left(\frac{r_0}{r}\right)^{\gamma_0} \begin{cases} \rho_0 = \rho_s \\ r_0 = r_s \\ \gamma_0 = 1 \end{cases}$$



Adiabatic growth (Gondolo & Silk 1999 *PRL*)

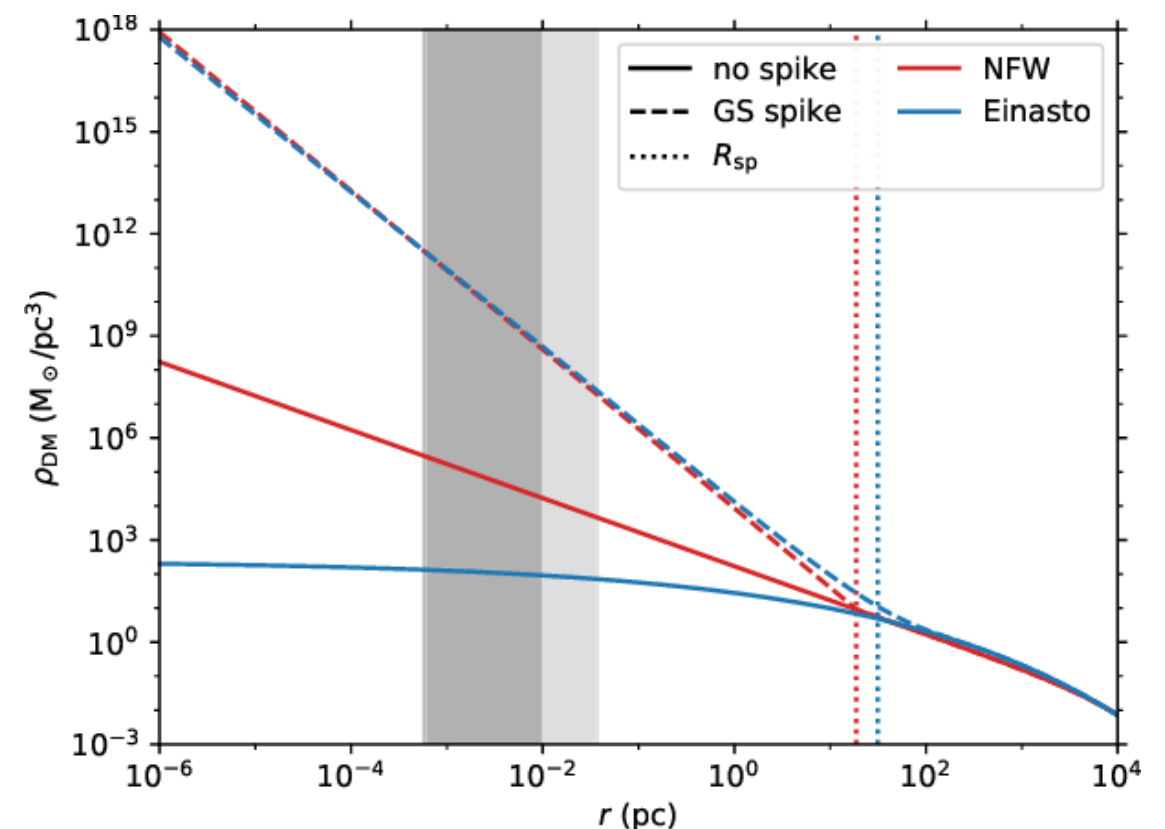
$$\rho_{\text{spike}}(r_{\text{ISCO}} < r < R_{\text{sp}}) = \rho_{\text{sp}} \left(\frac{R_{\text{sp}}}{r}\right)^{\gamma_{\text{sp}}}$$

$$R_{\text{sp}} \propto r_0 \left(\frac{M_{\text{BH}}}{\rho_0 r_0^3}\right)^{\frac{1}{3-\gamma_0}} \quad \gamma_{\text{sp}} \equiv \frac{9-2\gamma_0}{4-\gamma_0}$$

Dynamical friction (Chandrasekhar 1943 *ApJ*)

$$F_i = \frac{4\pi G^2 m_i^2 \rho_{\text{spike}}(r_i) \ln \Lambda}{v_i^2}$$

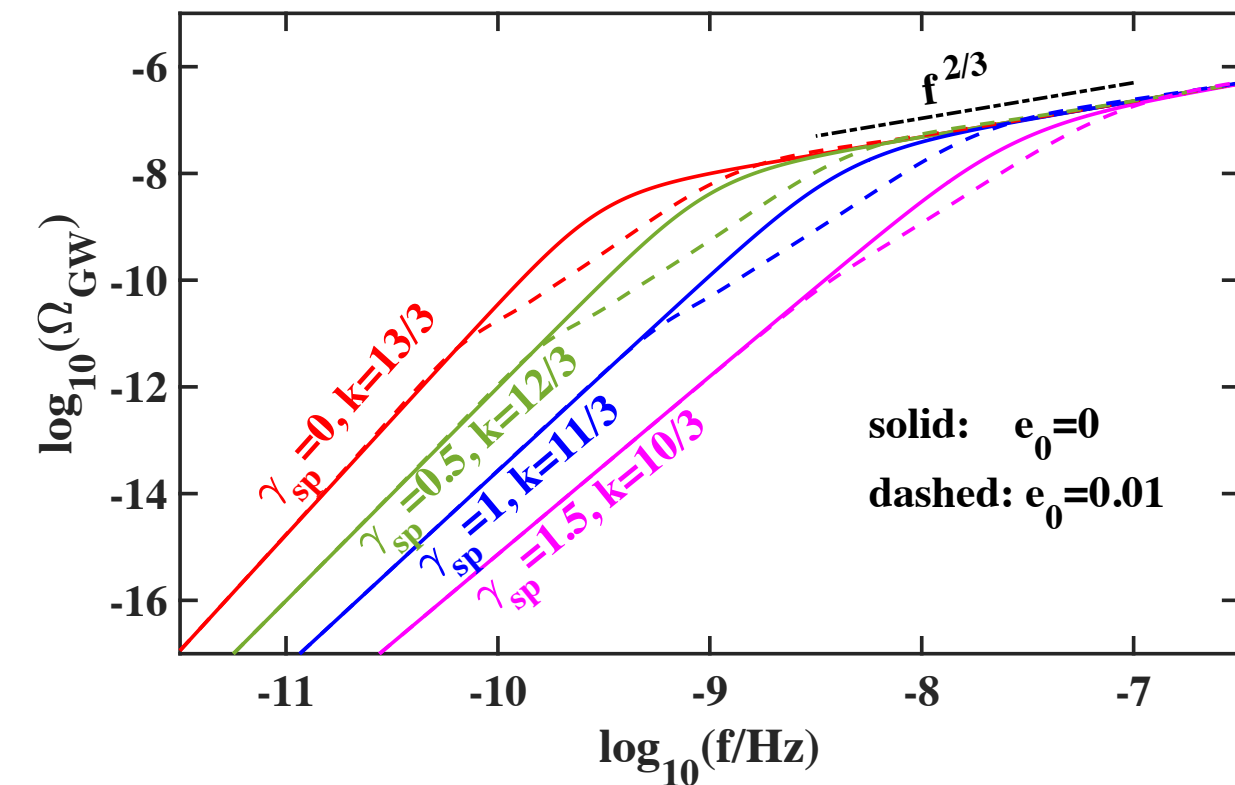
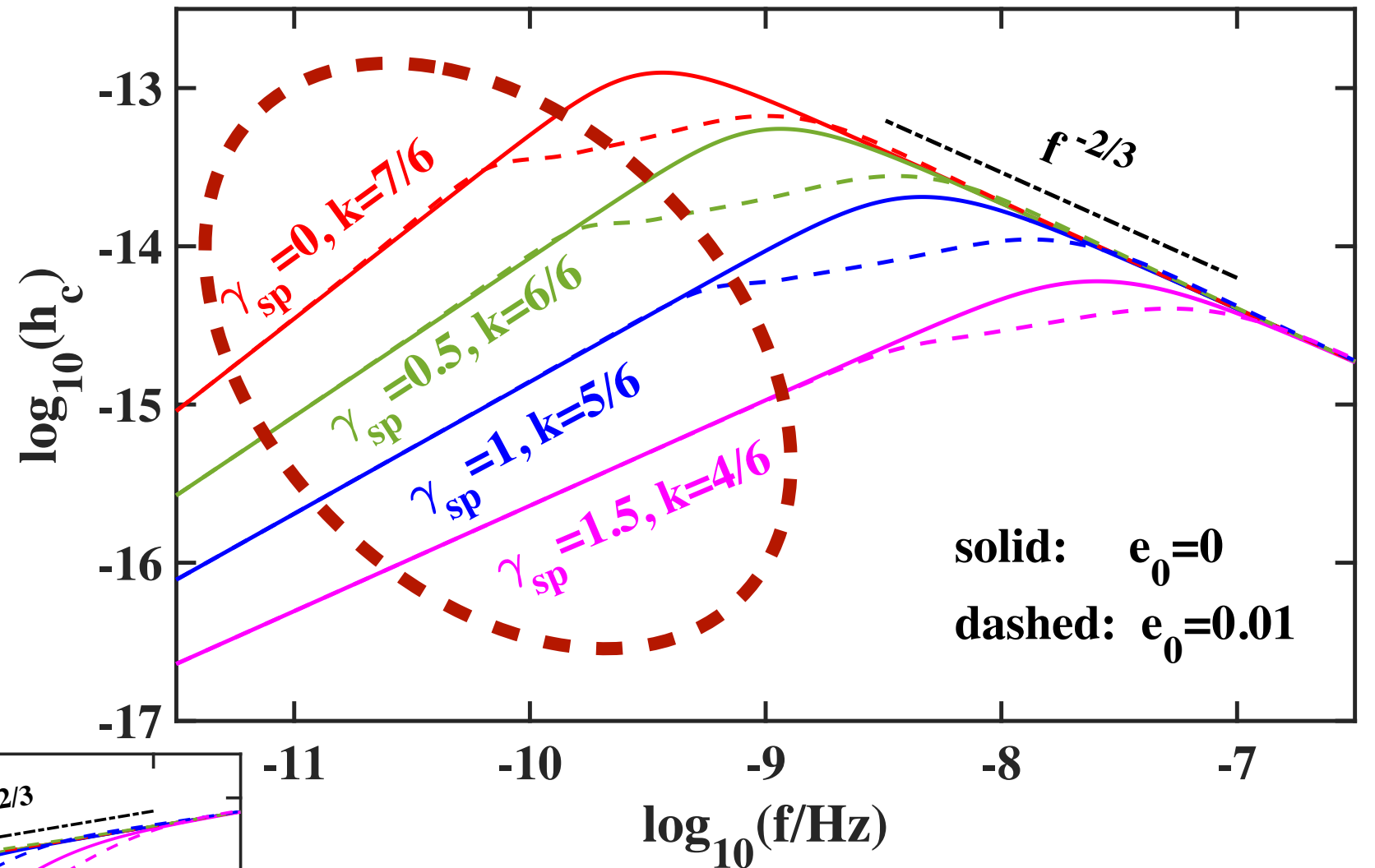
SMBH Shen, Yuan, Jiang, Tsai, Yuan, Fan 2023 *MNRAS*



Eccentric SMBHBs with DM spike

Direct measurement
for DM spike index at
very low frequencies

$$h_c \sim f^{7/6 - \gamma_{\text{sp}}/3}$$

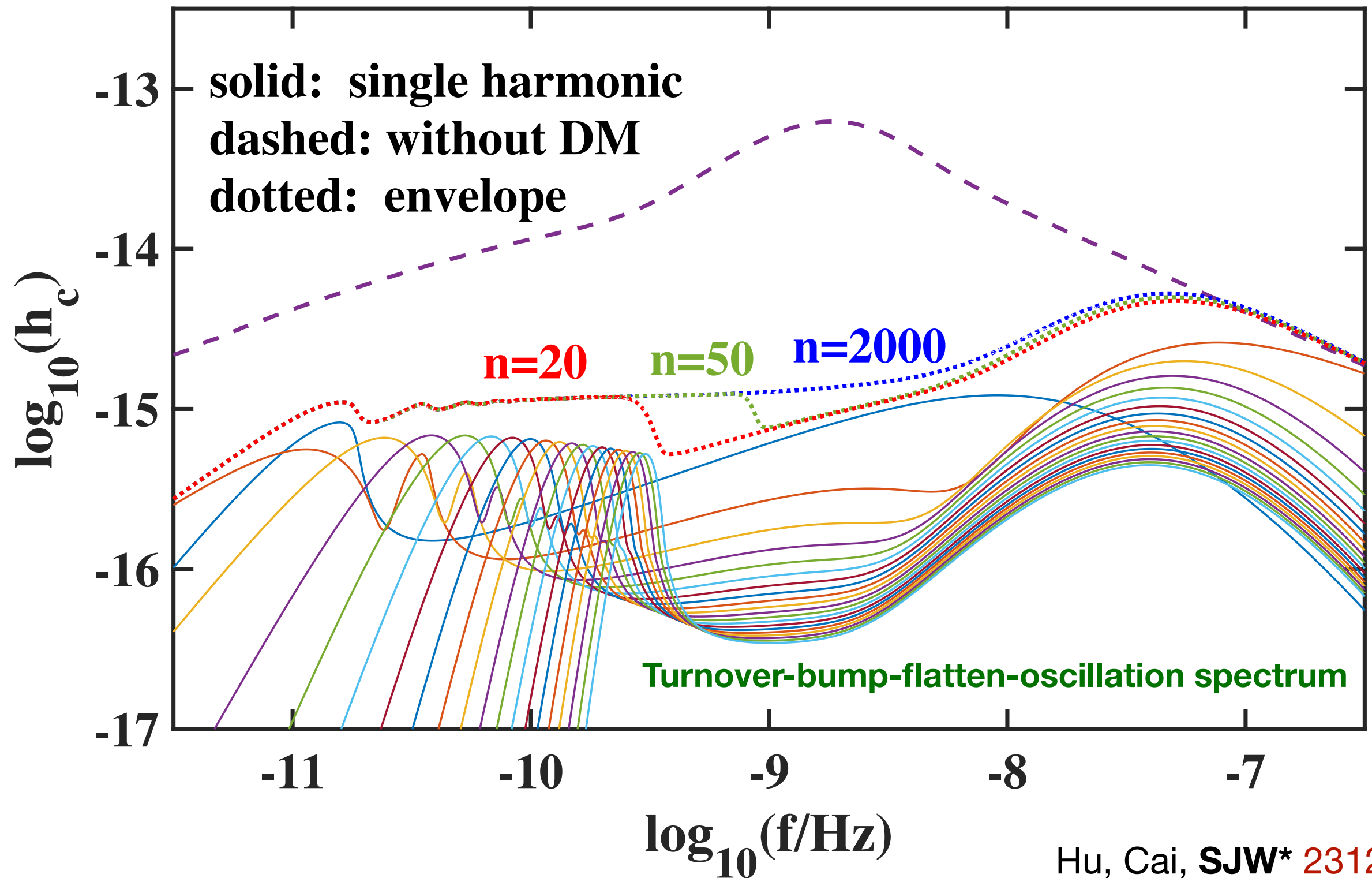


$$\Omega_{\text{GW}}(f) \equiv \frac{1}{\rho_c} \frac{d\rho_{\text{GW}}}{d \ln f} = \frac{\pi}{4} \frac{f^2 h_c^2(f)}{G \rho_c} \sim f^{13/3 - 2\gamma_{\text{sp}}/3}$$

Hu, Cai, **SJW*** 2312.14041

Eccentric SMBHBs with DM spike

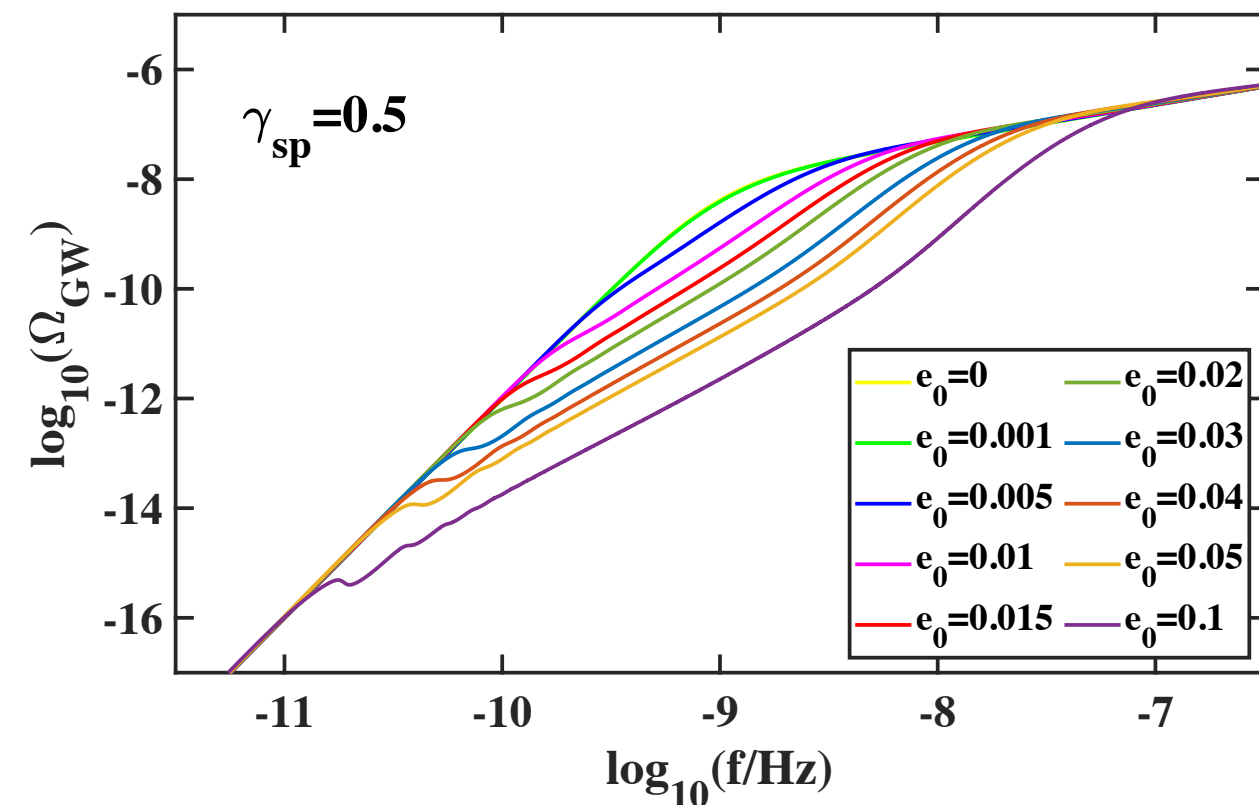
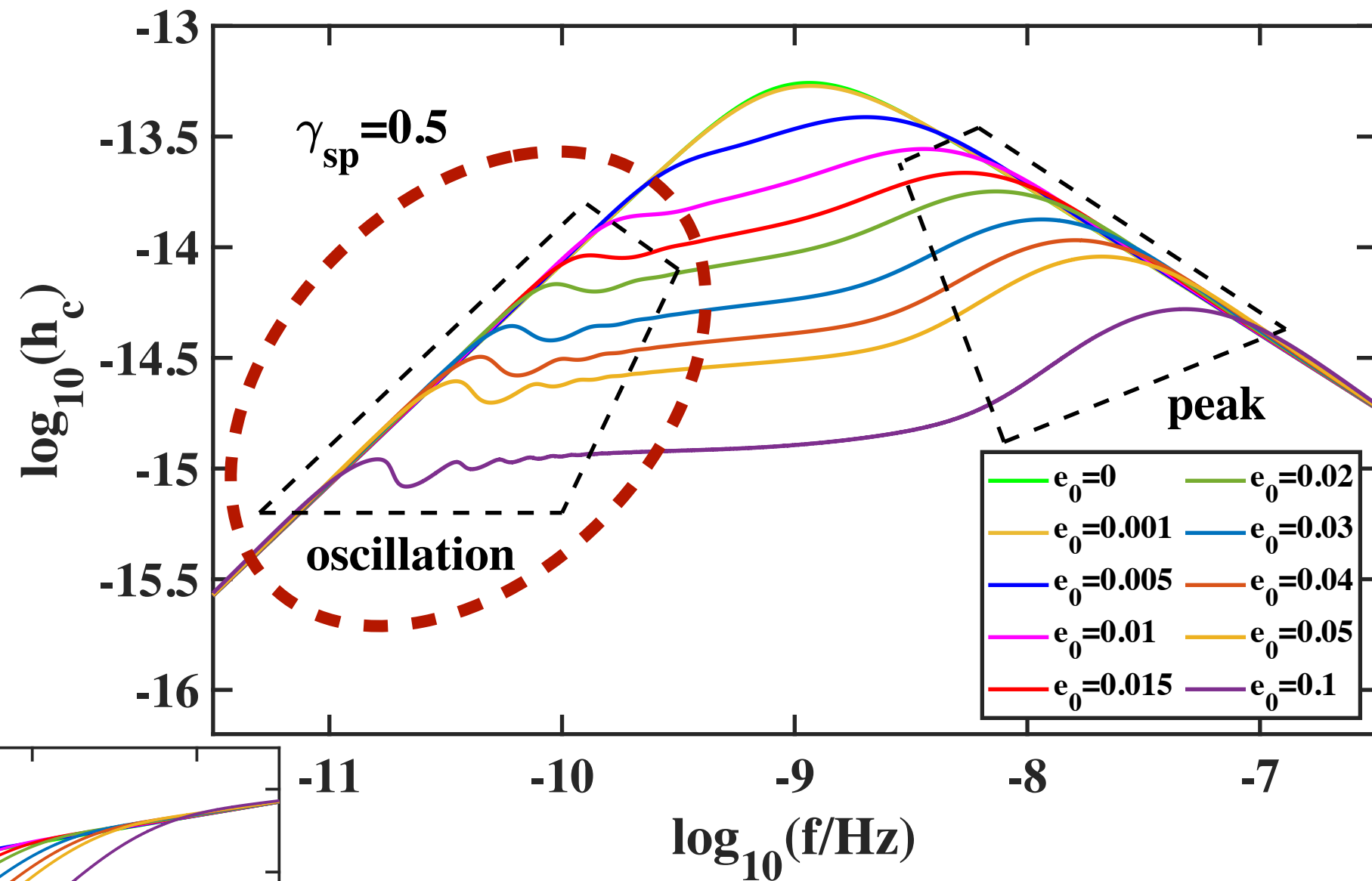
We have found an oscillation feature for DM spike as the envelope from summing harmonics



Hu, Cai, **SJW*** 2312.14041

Eccentric SMBHBs with DM spike

Smoking gun of DM
spike around SMBH
for longer PTA
observational
duration
More FAST network?



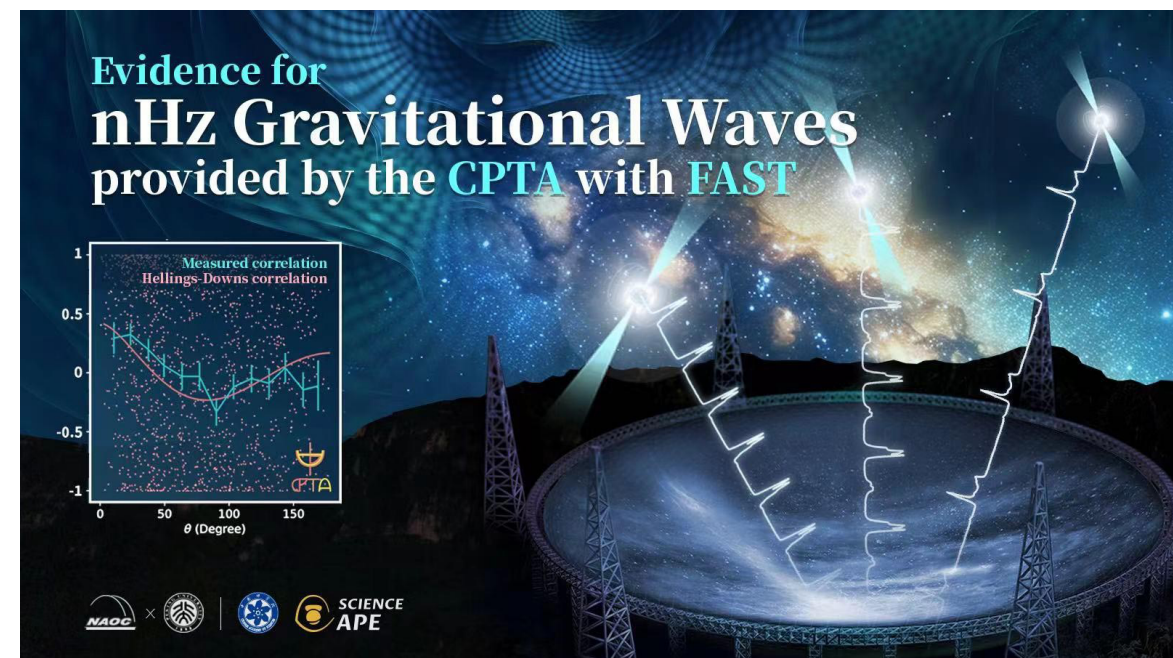
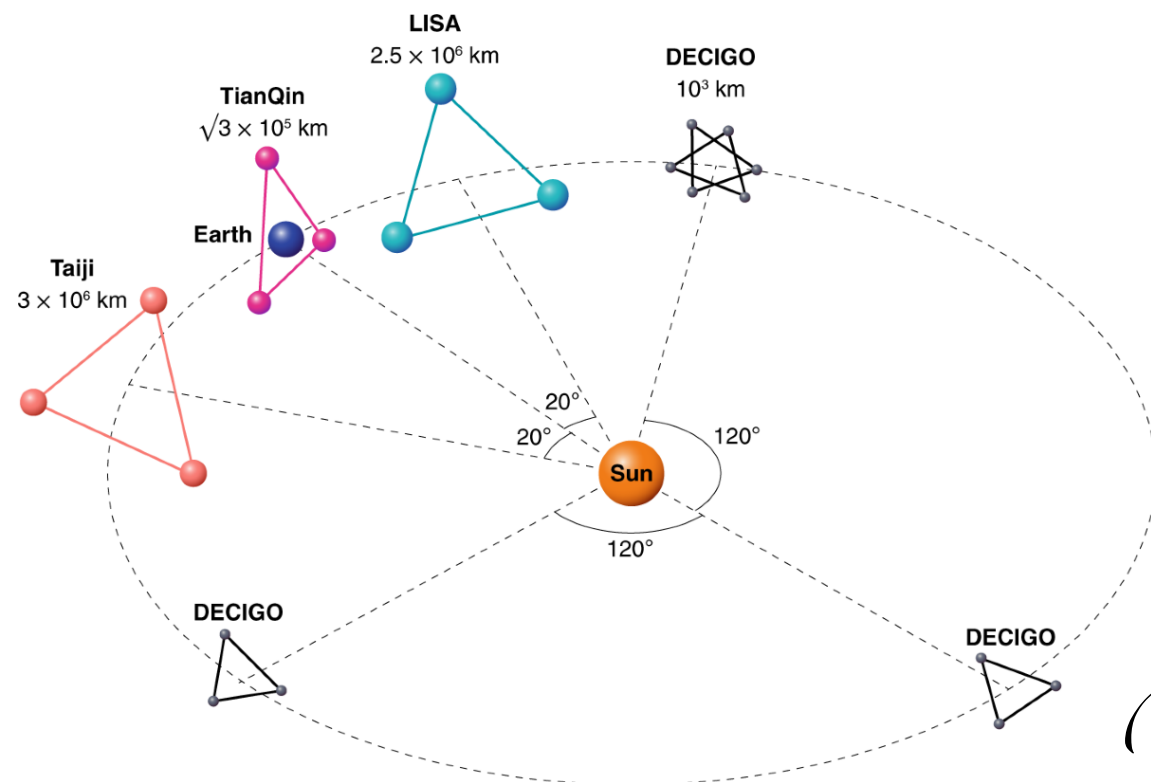
Pure gravitational probe into DM
if it is only subjected to gravity

Hu, Cai, **SJW*** 2312.14041

Conclusions and discussions

Take-home message

1. Primordial GWs : we might need a new paradigm shift for inflationary model-building
2. Induced GWs : we might eventually need numerical simulation to precision prediction
3. GW from FOPT : we might need full numerical simulation of bubble, PBH, perturbations
4. GW from gravitational collapse: GWs from general perturbations are more general than we think
5. GW from long-term inspiraling system : great probe with DM brake and DM flip effects



Thank you

b-EMRI with DM spike

