

Gravitational Waves from symmetry breaking

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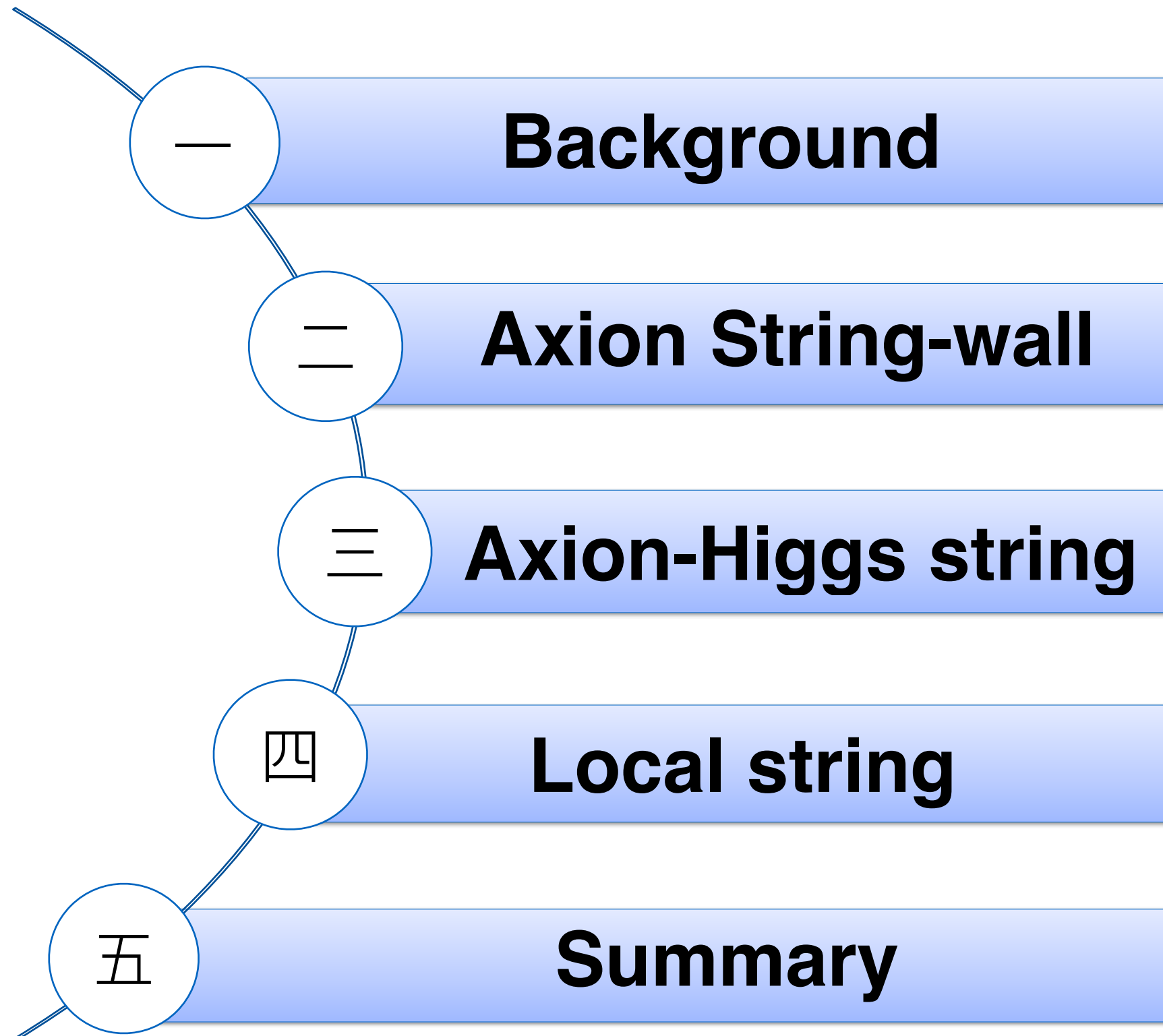
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Workshop on Gravitational Waves, Cosmology and Fundamental Physics

2025/10/30-2025/11/3

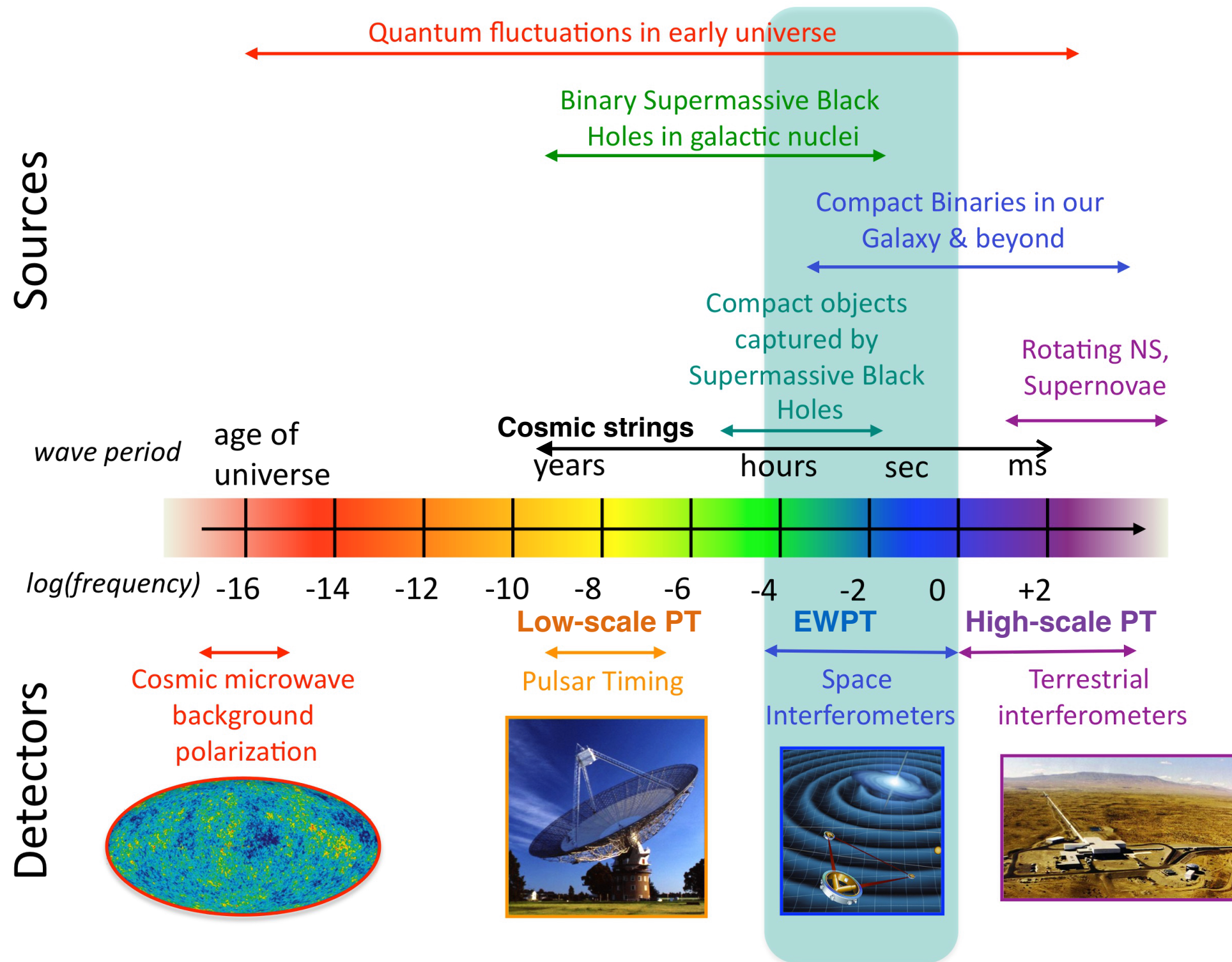
国科大杭州高等研究院

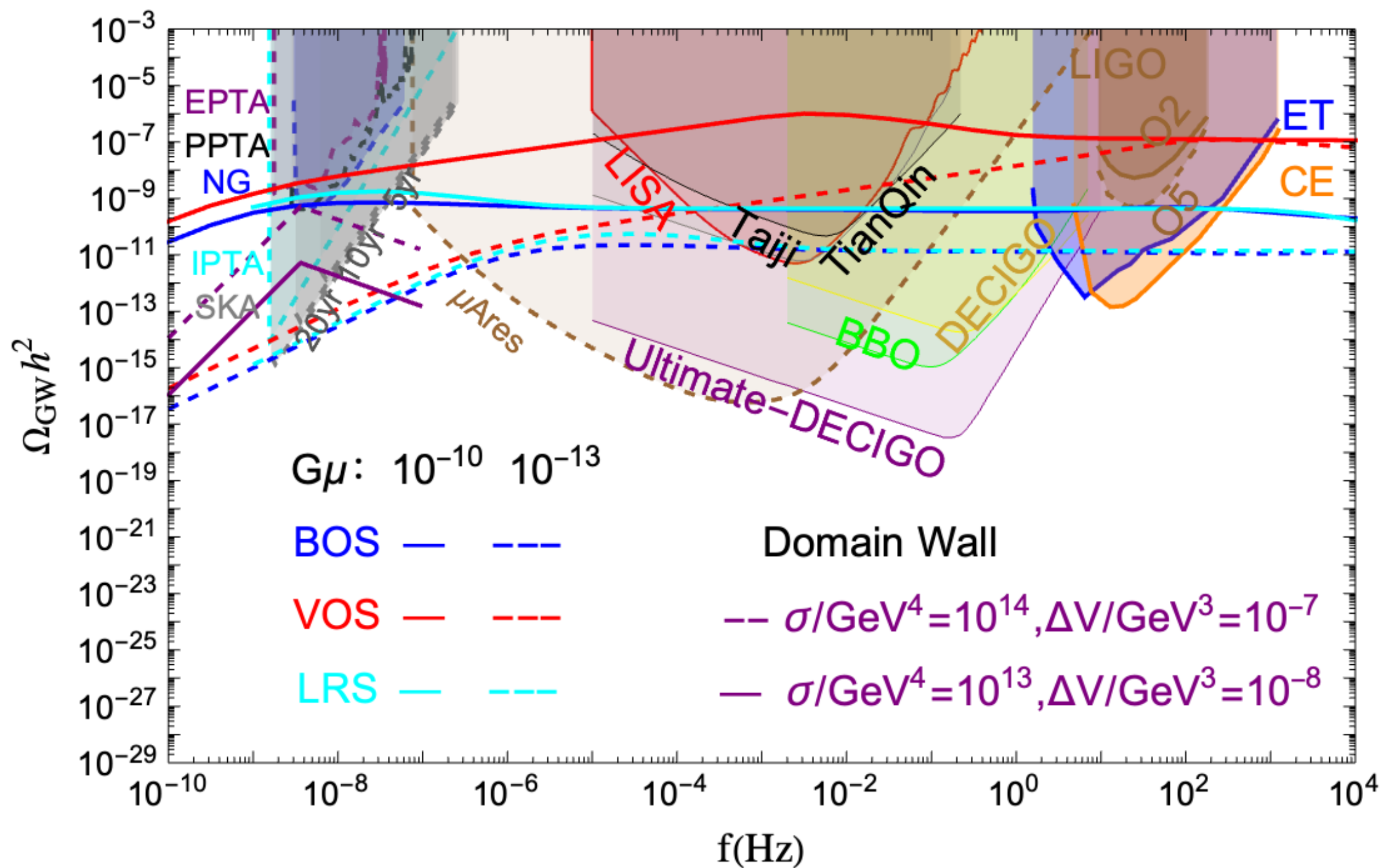
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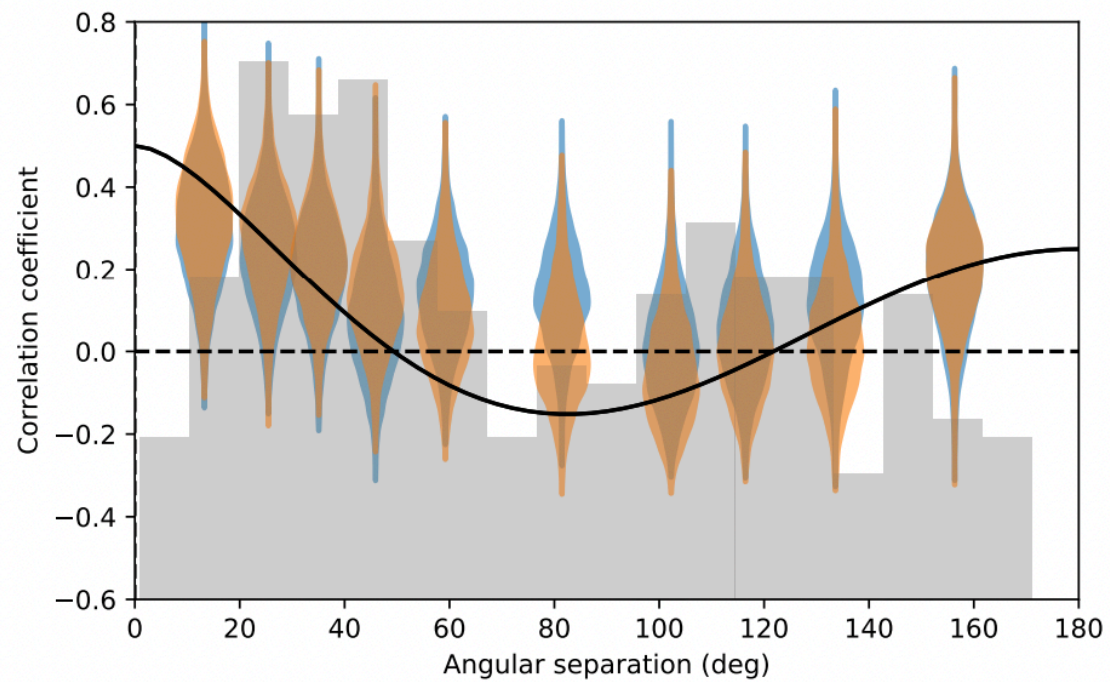
► Stochastic gravitational wave background and New physics

The Gravitational Wave Spectrum

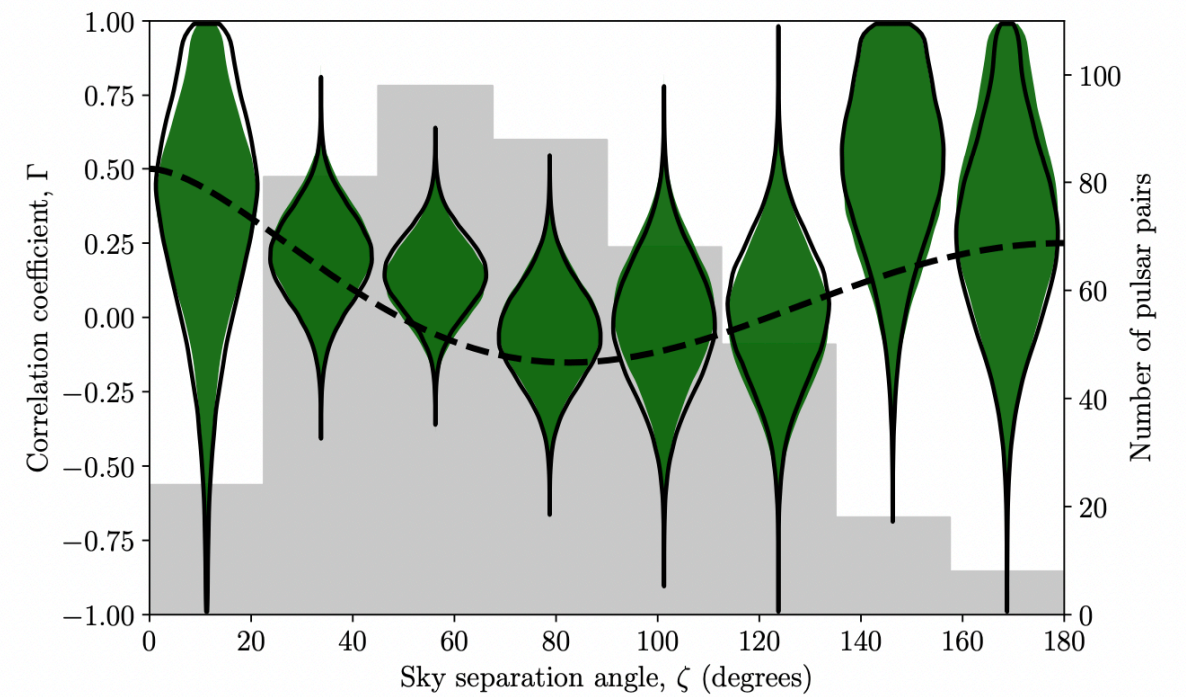




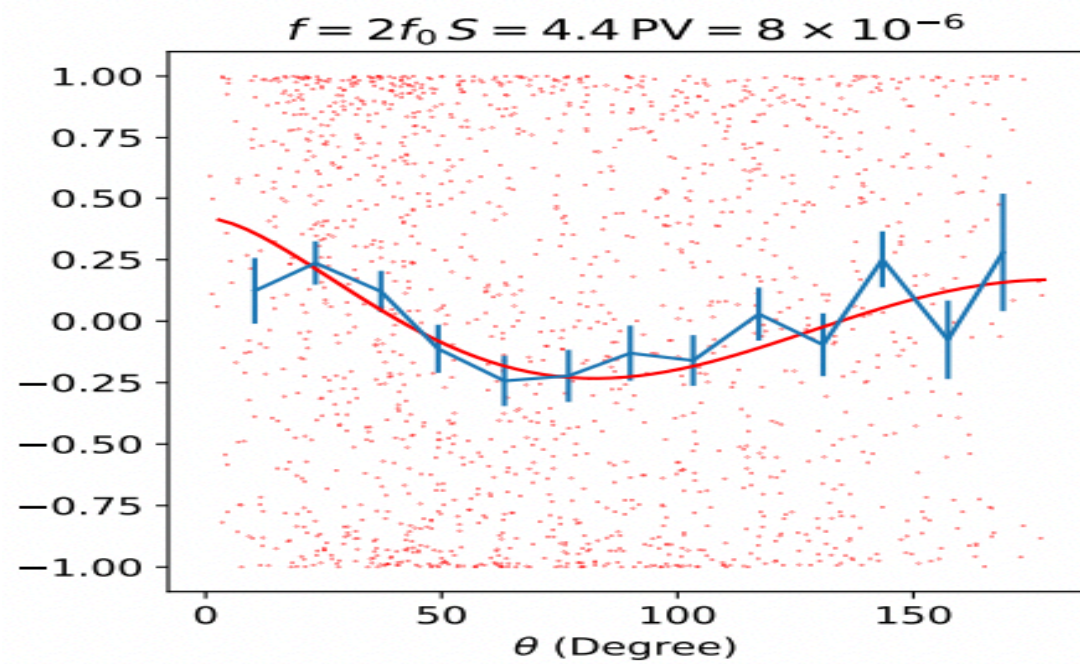
New dataset from PTAs



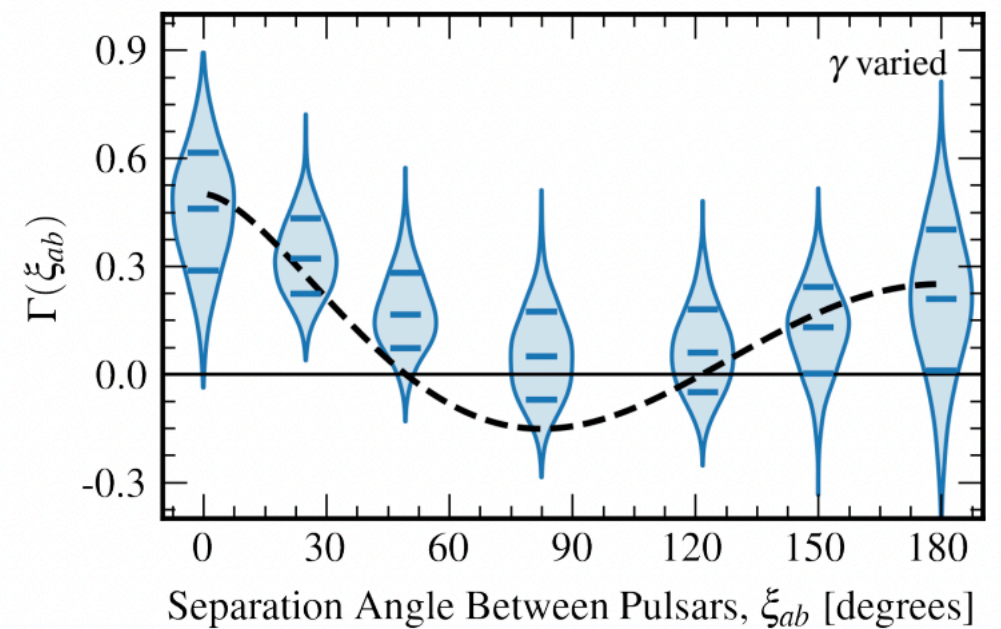
EPTA,2306.16214



PPTA,2306.16215

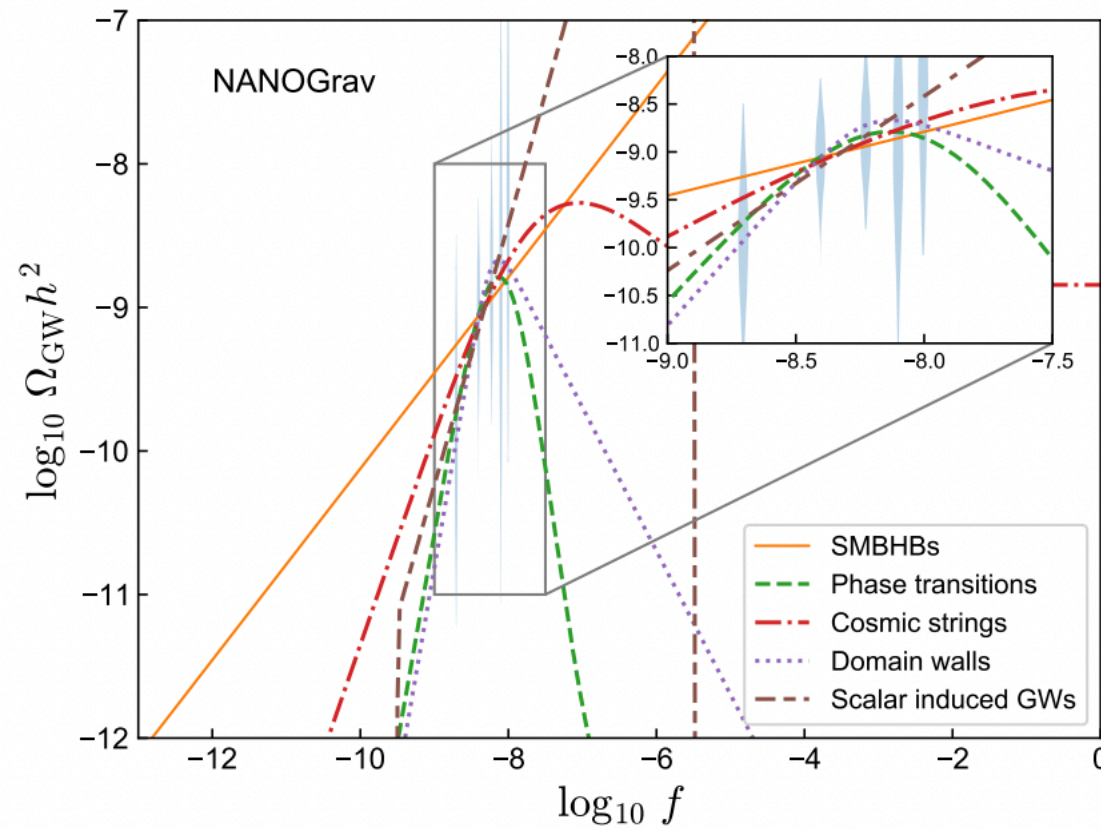
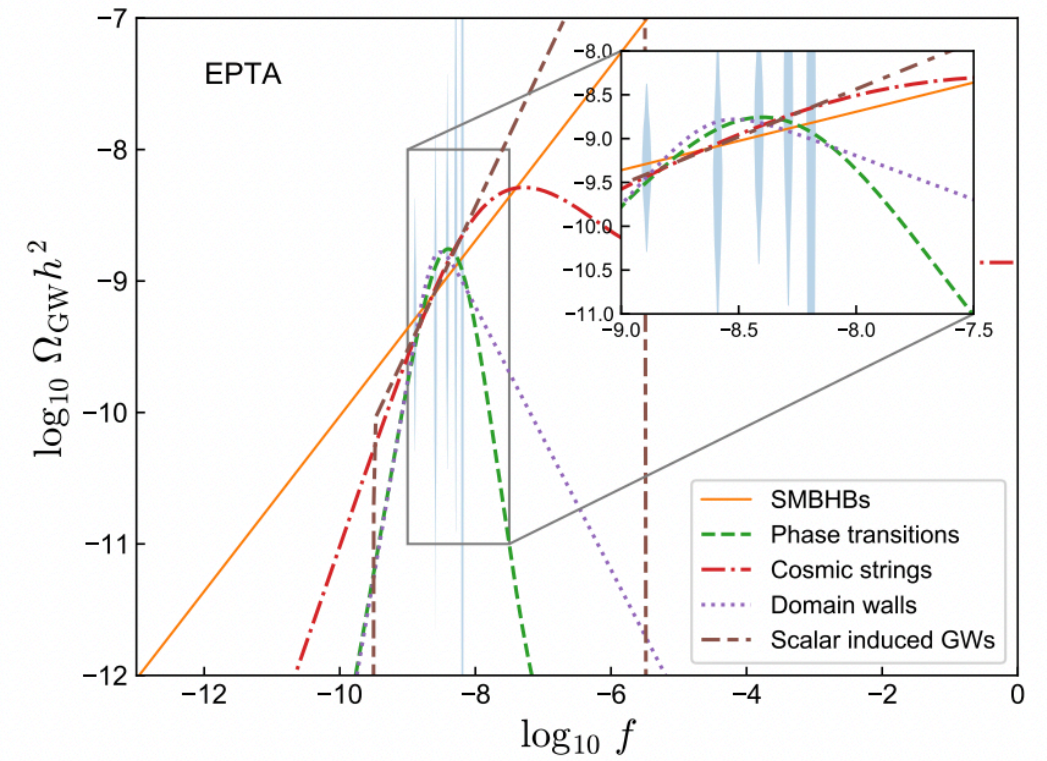
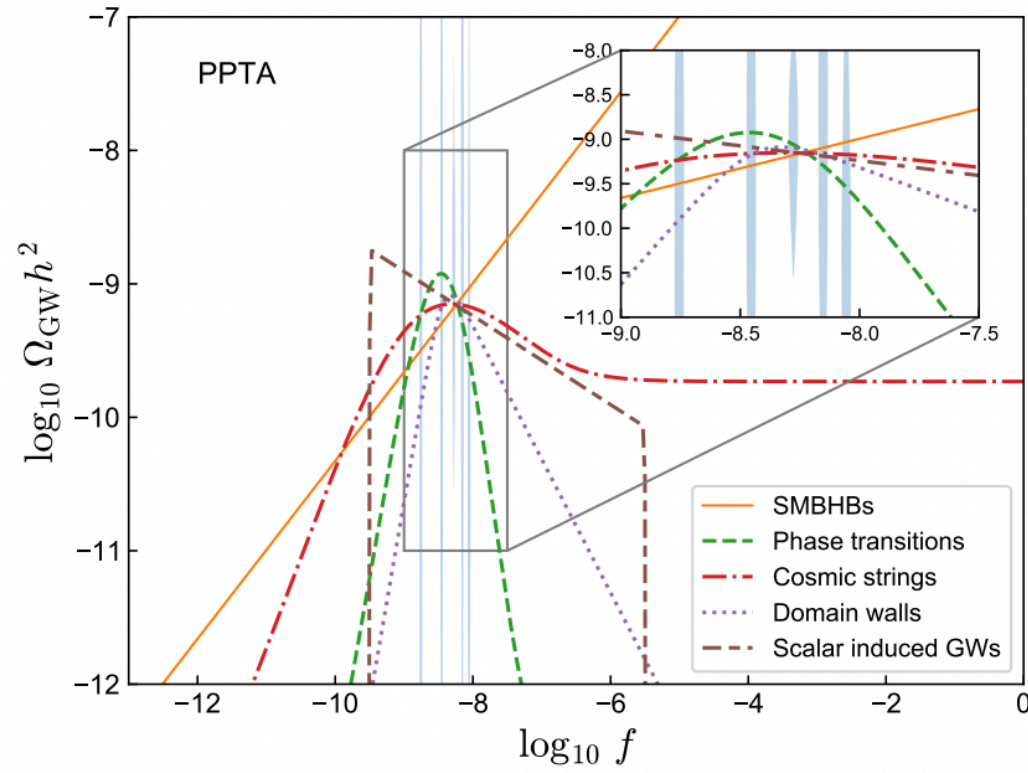


CPTA ,2306.16216



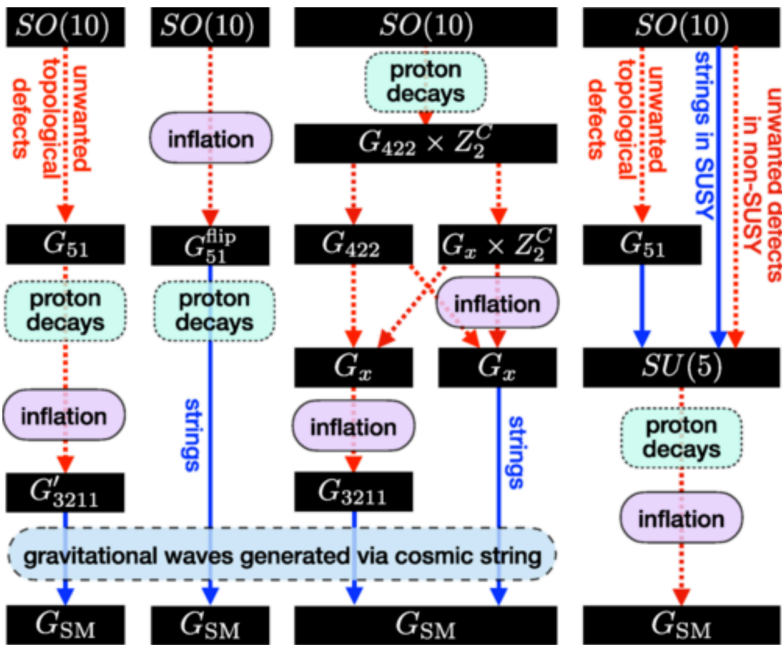
NANOGrav,2306.16213

Gravitational wave sources for Pulsar Timing Arrays



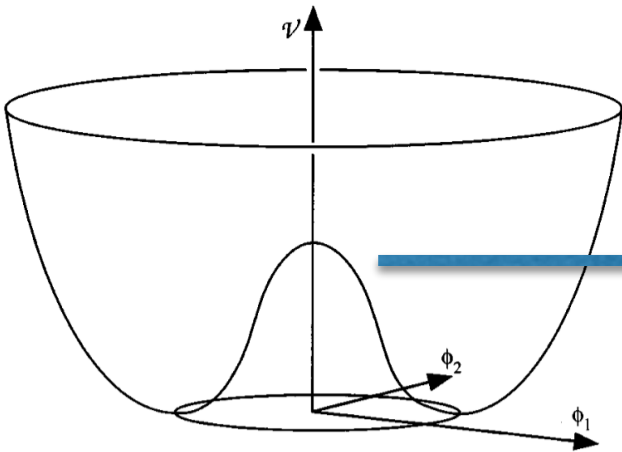
Cosmic string

GUTs

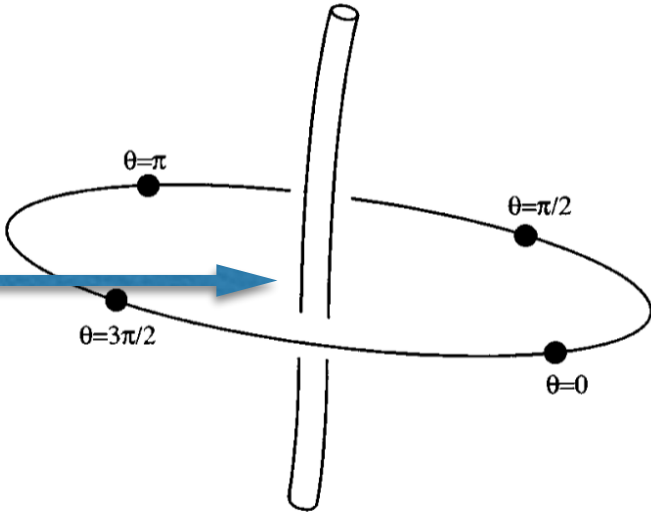


Stephen F. King, Silvia Pascoli, Jessica Turner, Ye-Ling Zhou,
 Phys. Rev. Lett. **126**, 021802

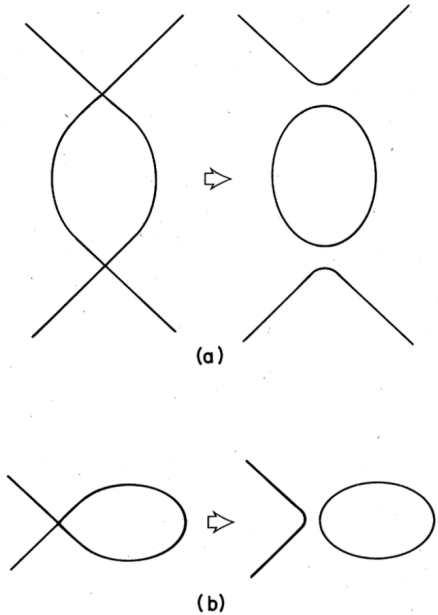
SSB of the U(1)



Cosmic string

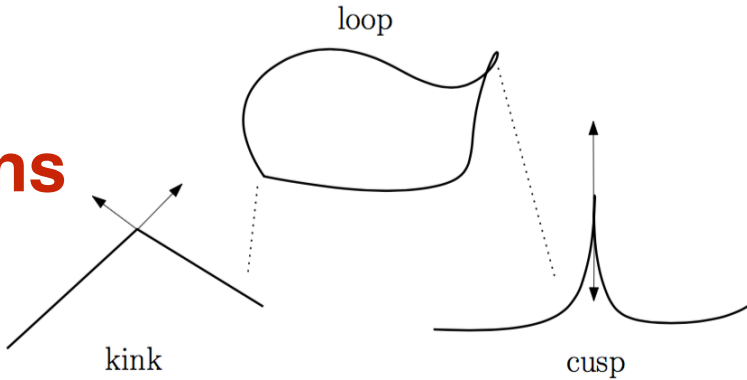


T. W. B. Kibble



Loops

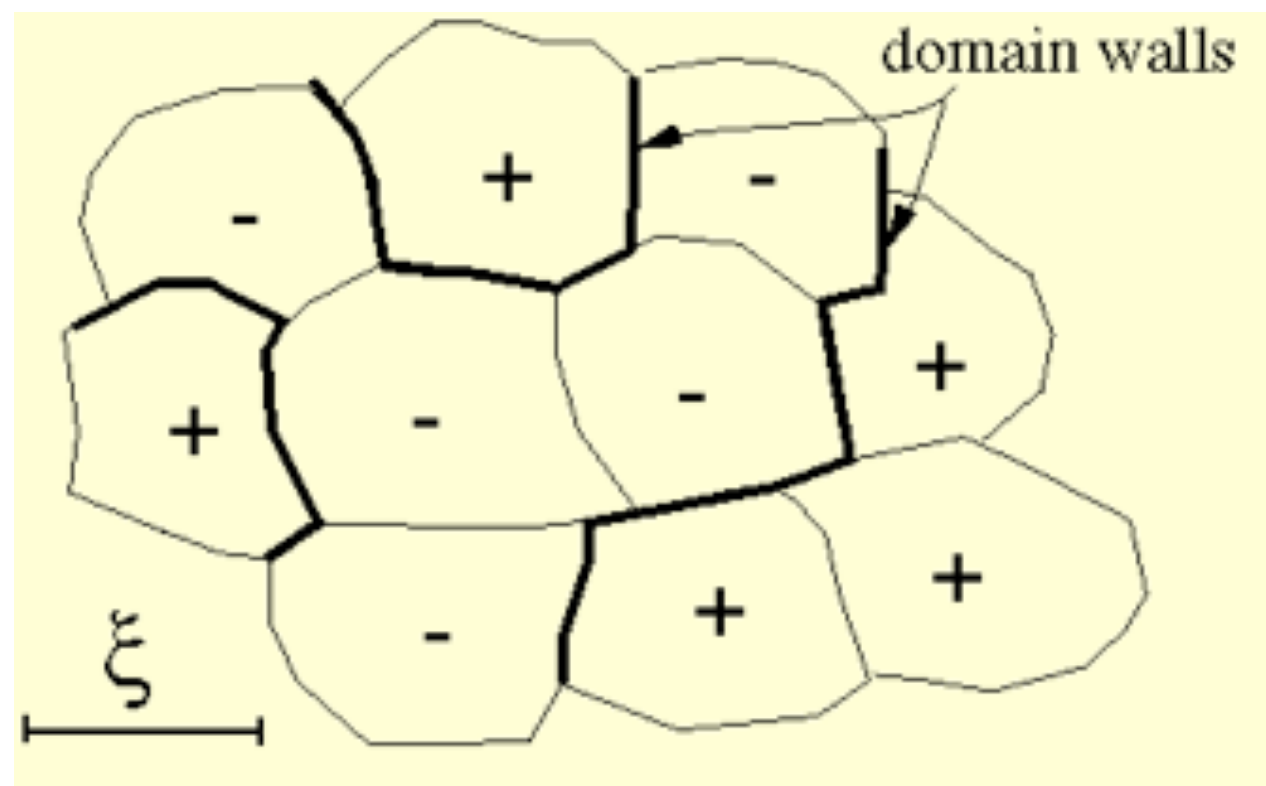
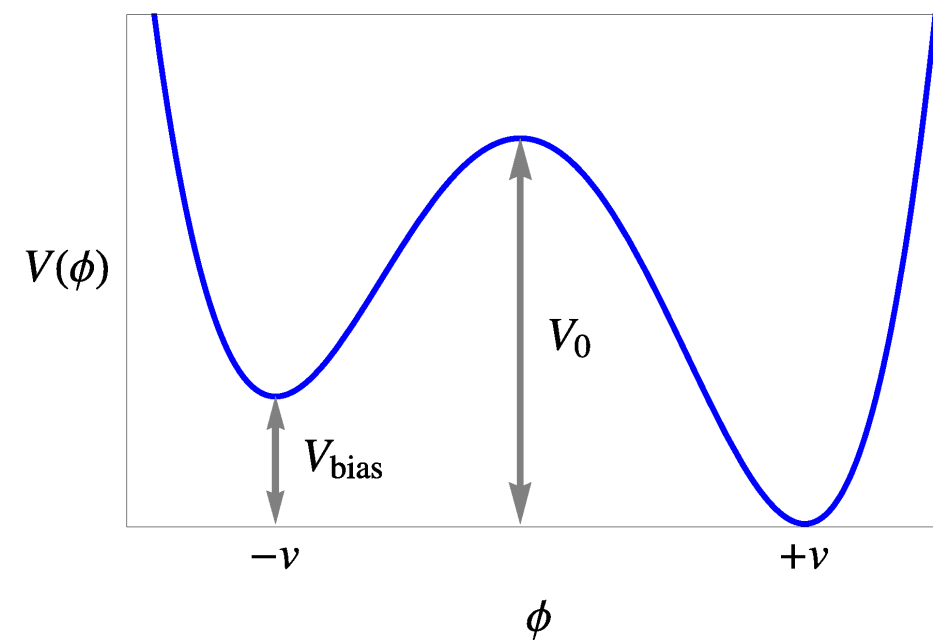
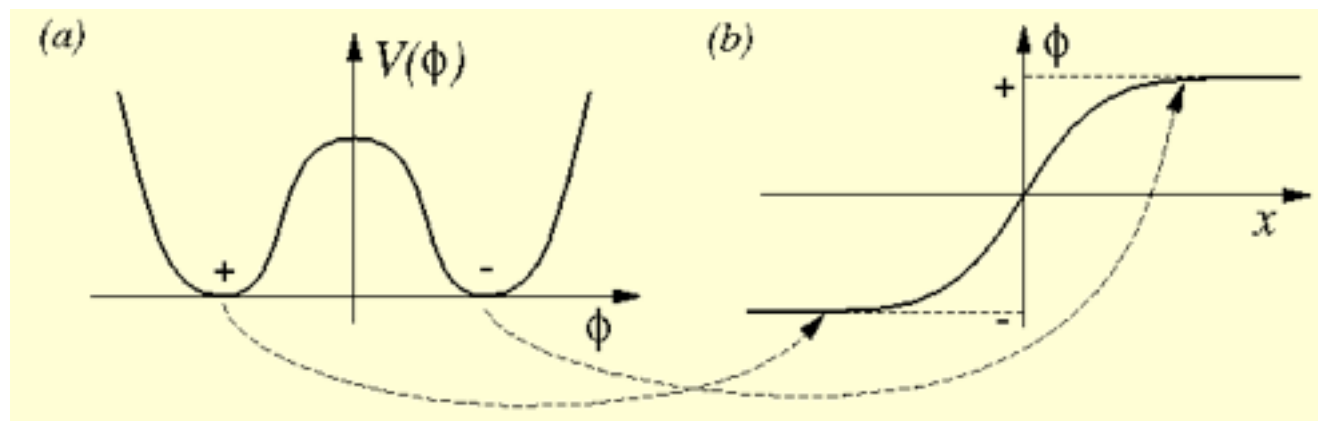
GW radiations



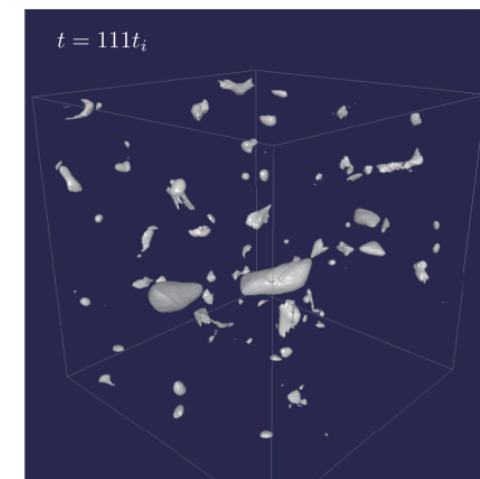
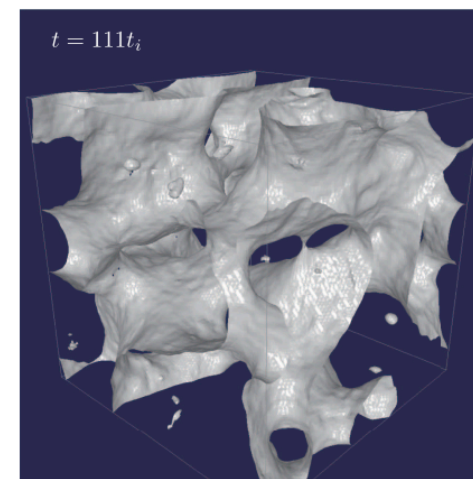
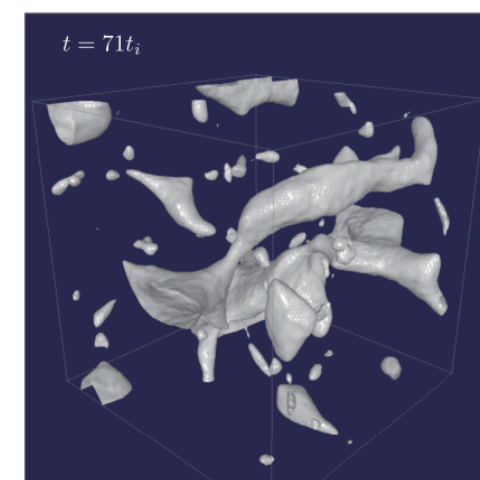
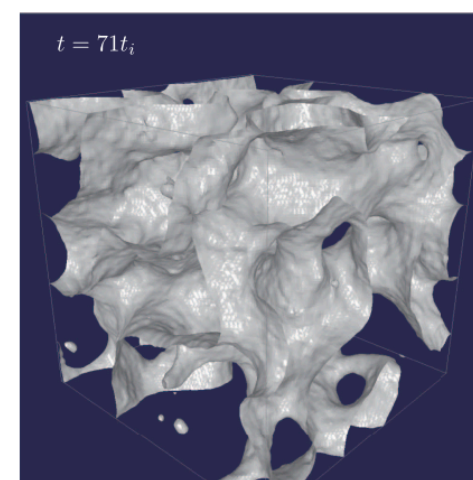
Phys.Rev.D 30 (1984) 2036

Yann Gouttenoire *et al* JCAP07(2020)032

Domain wall



Kibble mechanism



► Cosmic string simulation: the DFSZ axion model

The action in expanding universe with spatially flat FLRW metric:

$$S = -\int d^4x \sqrt{-g} \left(g^{\mu\nu} \frac{1}{2} \partial_\mu \varphi^* \partial_\nu \varphi + V(\varphi) \right)$$

PQ complex scalar: $\varphi = \phi_1 + i\phi_2$

$$V(\varphi) = \underbrace{\frac{1}{4} \lambda (|\varphi|^2 - v^2)^2 + \frac{\lambda}{6} T^2 |\varphi|^2}_{\text{axion field}} + \underbrace{\frac{m^2(T)v^2}{N_{DW}^2} (1 - \cos(N_{DW}\theta)) - \Xi v^3 (\varphi e^{-i\delta} + \text{h.c.})}_{\text{bias term}}$$

$\min \left[\frac{\alpha_a \Lambda^4}{f_a^2 (T/\Lambda)^{6.68}}, m_a^2 \right]$

PQ era, PQ symmetry broken, second order phase transition, $T_c \sim 10^9\text{-}10^{11}\text{GeV}$

QCD era, axion acquires a non-zero mass due to QCD non-perturbative effect, $T \sim 100\text{MeV}$

Axion(global) strings form and enters the scaling regime

String-domain wall hybrid networks form and eventually decay

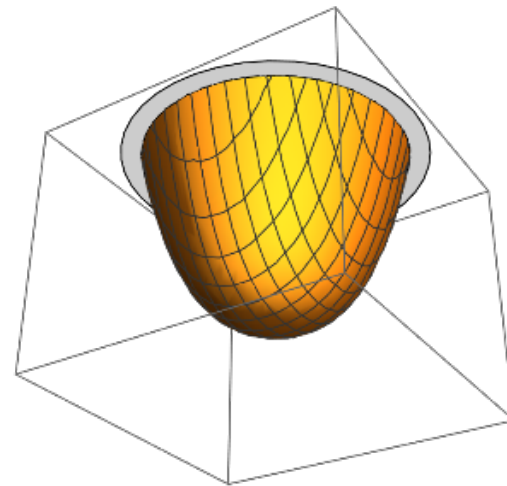
Gravitational waves and axion radiated by topological defects of two eras

→ Detection of axion dark matter

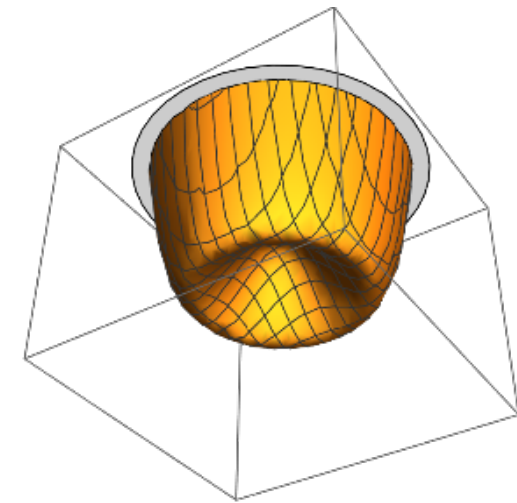
► Shape of the potential

**PQ
era:**

before PQ
transition

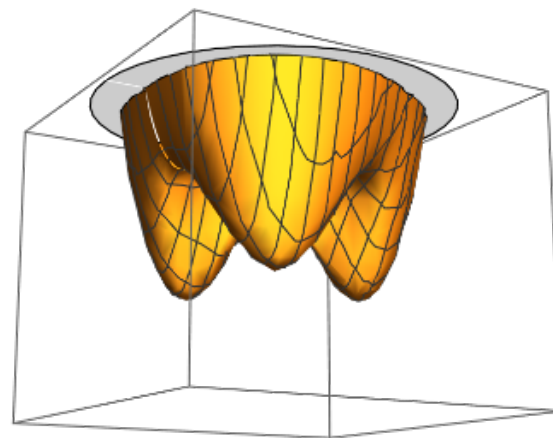


after PQ
transition

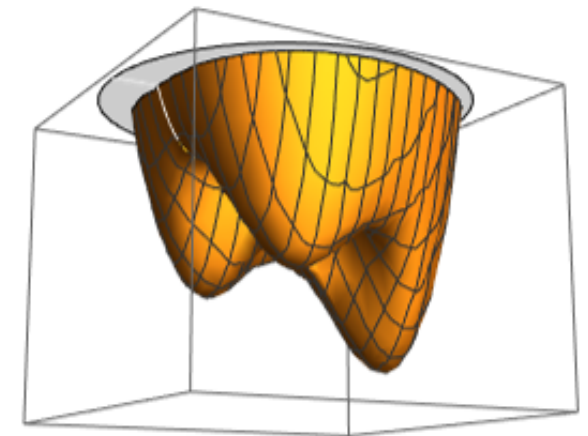


**QCD
era:**

with nonzero axion mass
without bias term



with bias
term



► Simulation setups

Equations of motion

PQ era-the first stage

$$\left\{ \begin{array}{l} \phi_1'' + 2\frac{a'}{a}\phi_1' - \nabla^2\phi_1 = -a^2\left[\lambda\phi_1(\phi_1^2 + \phi_2^2 - v^2 + \frac{1}{3}T^2) - \frac{m^2(T)v^2}{N_{\text{DW}}^2}(\cos\theta \cos N_{\text{DW}}\theta + N_{\text{DW}} \sin\theta \sin N_{\text{DW}}\theta) - 2\Xi v^3 \cos\delta\right] \\ \phi_2'' + 2\frac{a'}{a}\phi_2' - \nabla^2\phi_2 = -a^2\left[\lambda\phi_2(\phi_1^2 + \phi_2^2 - v^2 + \frac{1}{3}T^2) - \frac{m^2(T)v^2}{N_{\text{DW}}^2}(\sin\theta \cos N_{\text{DW}}\theta - N_{\text{DW}} \cos\theta \sin N_{\text{DW}}\theta) - 2\Xi v^3 \sin\delta\right] \end{array} \right.$$

Initial condition

thermal spectrum

$$\mathcal{P}_{\phi_1}(k) = \mathcal{P}_{\phi_2}(k) = \frac{n_k}{w_k} = \frac{1}{w_k} \frac{1}{e^{w_k/T} - 1}, \quad \mathcal{P}_{\dot{\phi}_1}(k) = \mathcal{P}_{\dot{\phi}_2}(k) = n_k w_k = \frac{w_k}{e^{w_k/T} - 1}$$

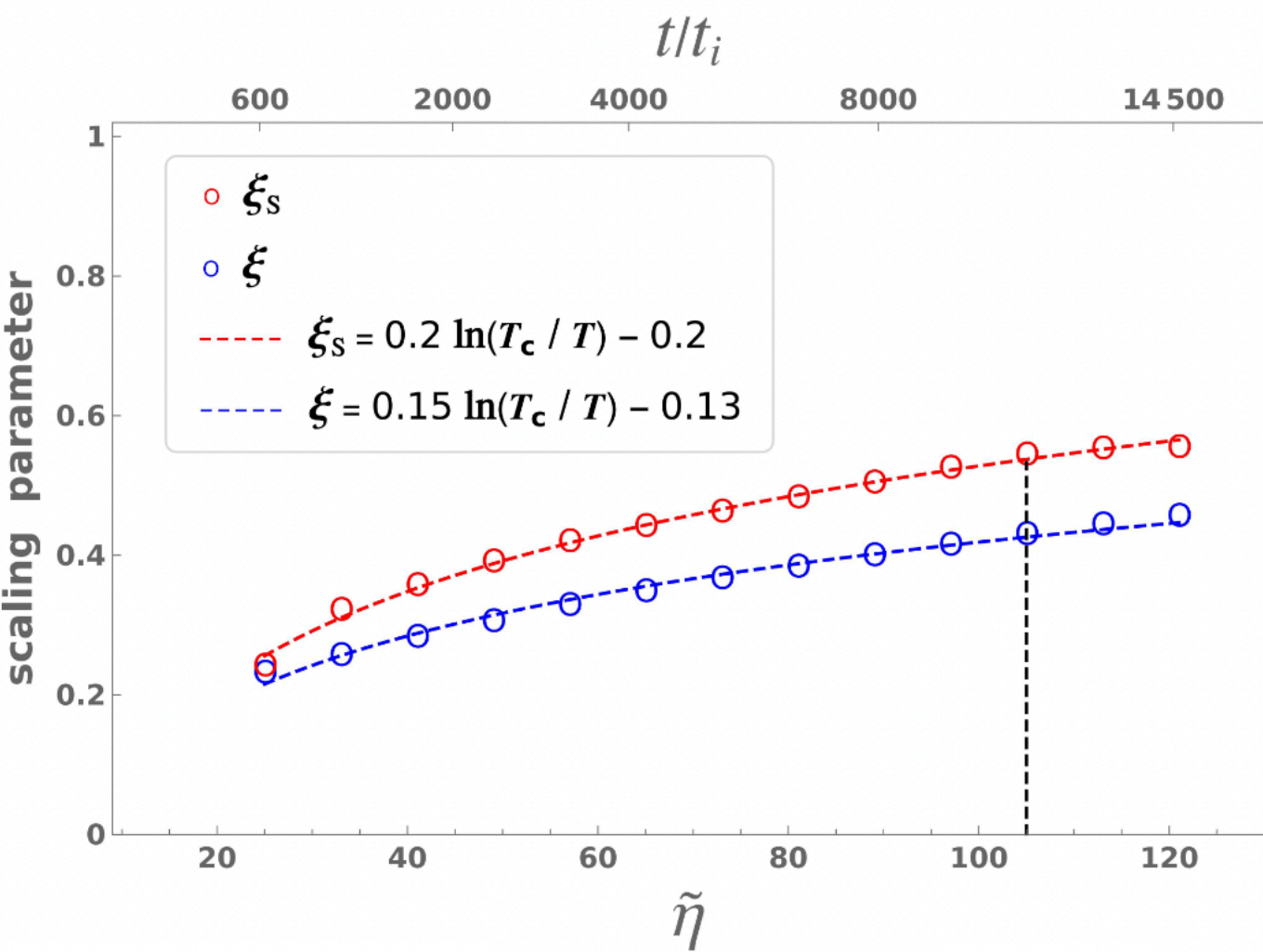
$$w_k = \sqrt{k^2/R^2 + m_{\text{eff}}^2} \quad m_{\text{eff}}^2 = \lambda(T^2/3 - v^2)$$

two-point correlation functions

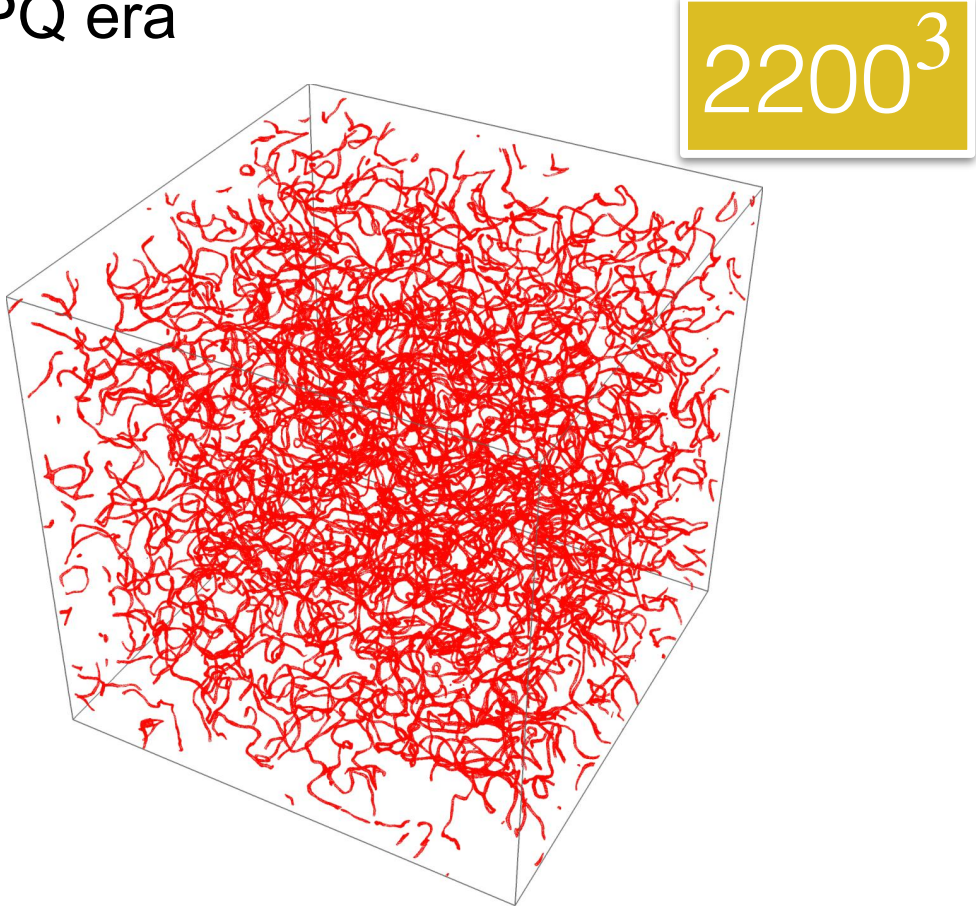
$$\begin{aligned} \langle \phi_i(\mathbf{k}) \phi_j(\mathbf{k}') \rangle &= (2\pi)^3 \mathcal{P}_\phi(k) \delta(\mathbf{k} - \mathbf{k}') \delta_{ij}, \\ \langle \dot{\phi}_i(\mathbf{k}) \dot{\phi}_j(\mathbf{k}') \rangle &= (2\pi)^3 \mathcal{P}_{\dot{\phi}}(k) \delta(\mathbf{k} - \mathbf{k}') \delta_{ij}, \\ \langle \phi_i(\mathbf{k}) \dot{\phi}_j(\mathbf{k}') \rangle &= 0. \end{aligned}$$

$$\begin{aligned} \langle |\phi_i(\mathbf{k})|^2 \rangle &= \left(\frac{N}{\delta x_{\text{phy}}} \right)^3 \mathcal{P}_{\phi_i}(k), & \langle \phi_i(\mathbf{k}) \rangle &= 0, \\ \langle |\dot{\phi}_i(\mathbf{k})|^2 \rangle &= \left(\frac{N}{\delta x_{\text{phy}}} \right)^3 \mathcal{P}_{\dot{\phi}_i}(k), & \langle \dot{\phi}_i(\mathbf{k}) \rangle &= 0, \end{aligned}$$

► Scaling parameter



String network at the final moment of PQ era



$$\xi = \frac{\rho_{st} t^2}{\mu_{st}}, \quad \text{with } \rho_{st} = \frac{\mu_{st} l}{R^2 V} \quad \longrightarrow \quad \xi = \frac{lt^2}{R^2 V}$$

$$\xi_s = \frac{l_{phy} t^2}{V_{phy}} = \frac{t^2}{L_m^2} = \frac{1}{\kappa^2} \frac{t^2}{(t - t_0)^2} \rightarrow \frac{1}{\kappa^2}$$

tension: $\mu_{st} \simeq \pi v^2 \ln(t/\delta_{st})$
 width: $\delta_{st} = 1/\sqrt{\lambda(v^2 - \frac{1}{3}T^2)}$

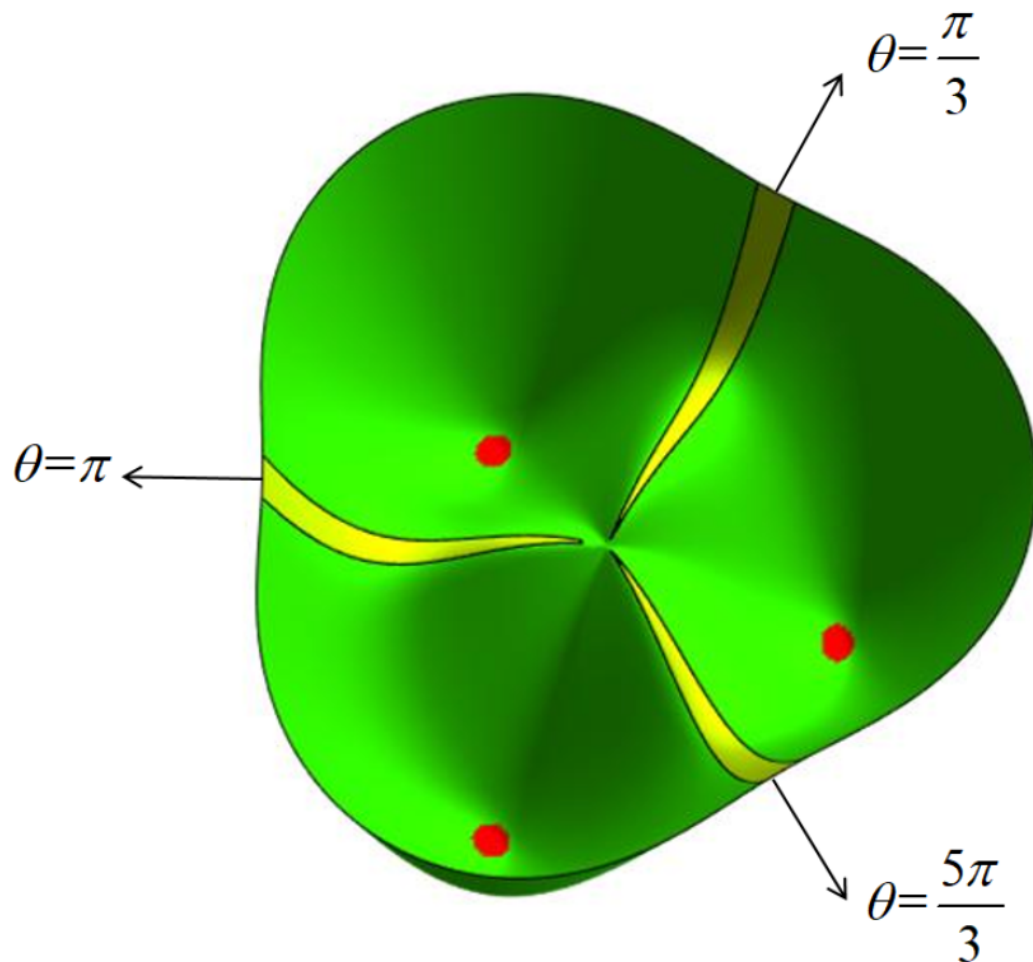
l : comoving string length
 V : comoving volume

Mean string separation $L_m = \sqrt{V_{phy}/l_{phy}}$
 Physical string length $l_{phy} = (2/3)n_c(R\delta x)$
 nc: square loops

In the scaling regime, L_m increases linearly with t , and the scaling parameter tends to be a constant. The same logarithmic increase behavior, as 1809.0924, 1906.00967, 1806.0467, 1806.05566

► Domain wall's identification

Top view of the shape of potential energy



Comoving area density

$$A/V = C \sum_{\text{links}} \delta \frac{|\nabla \theta|}{|\theta_{,x}| + |\theta_{,y}| + |\theta_{,z}|}$$

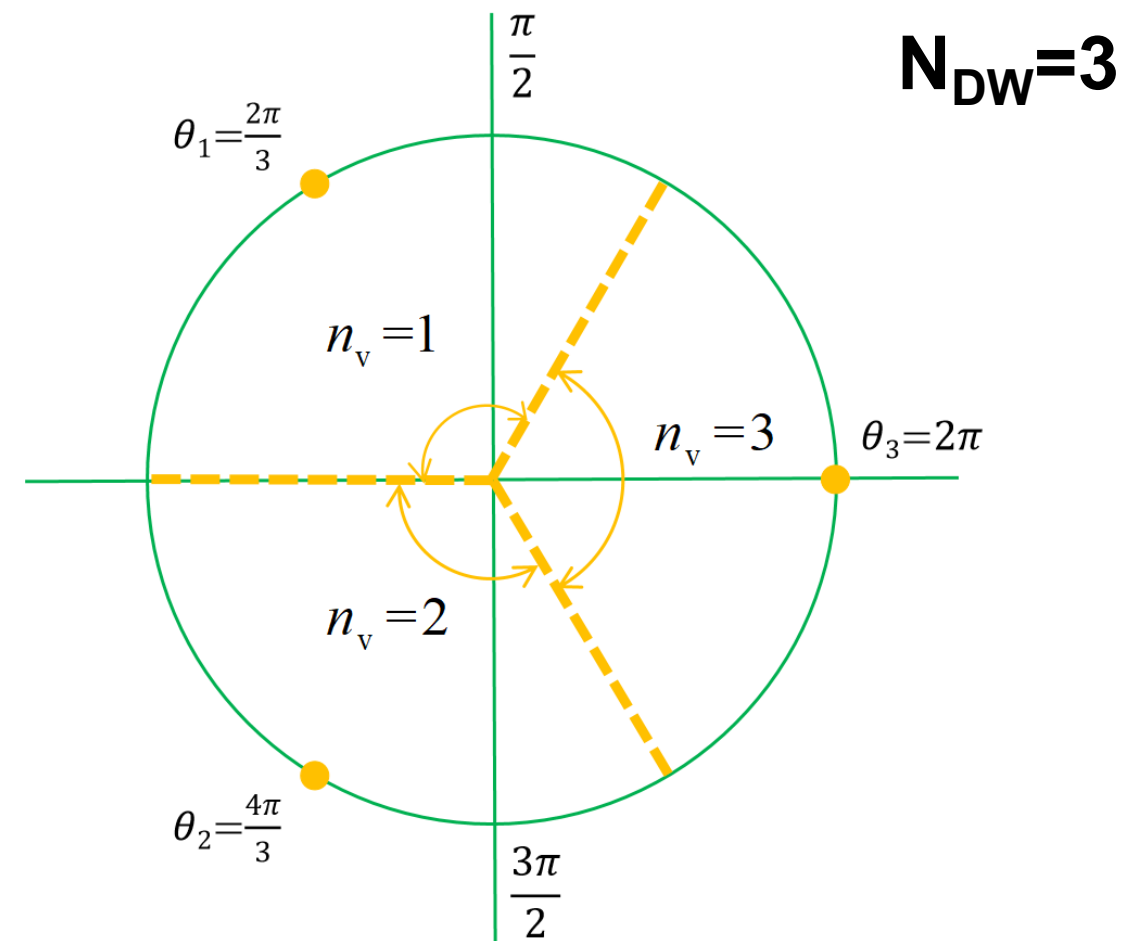
$\theta_{,i}$ ($i = x, y, z$) : spatial derivatives of the dimensionless axion field $\theta(x)$

Area parameter A of DW (scaling parameter of DW) $\xi_{\text{dw}} \equiv \mathcal{A} = \frac{\rho_{\text{wall}}}{\sigma_{\text{wall}}} t$, with $\rho_{\text{wall}} = \frac{\sigma_{\text{wall}} A}{R(t)V}$

$$A = \Delta A \sum_{\text{links}} \delta \frac{|\nabla \theta|}{|\theta_{,x}| + |\theta_{,y}| + |\theta_{,z}|}$$

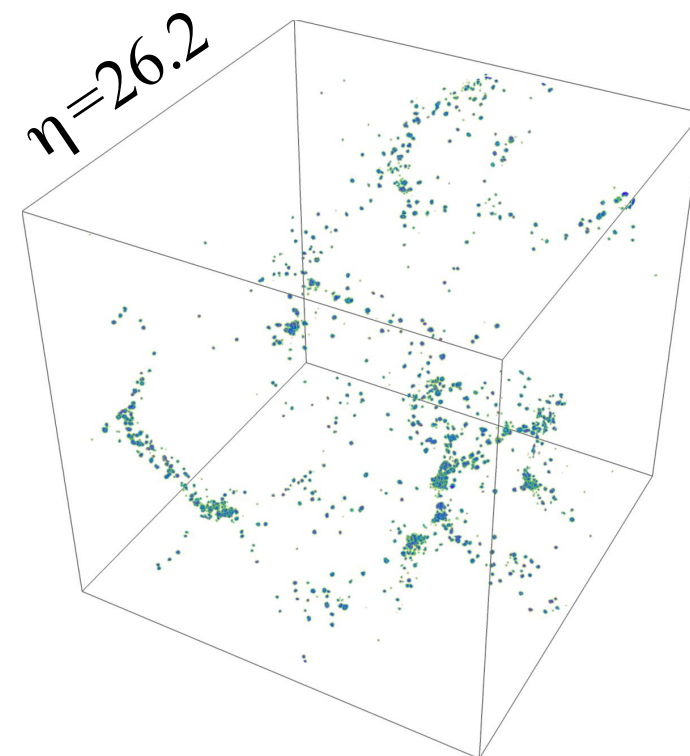
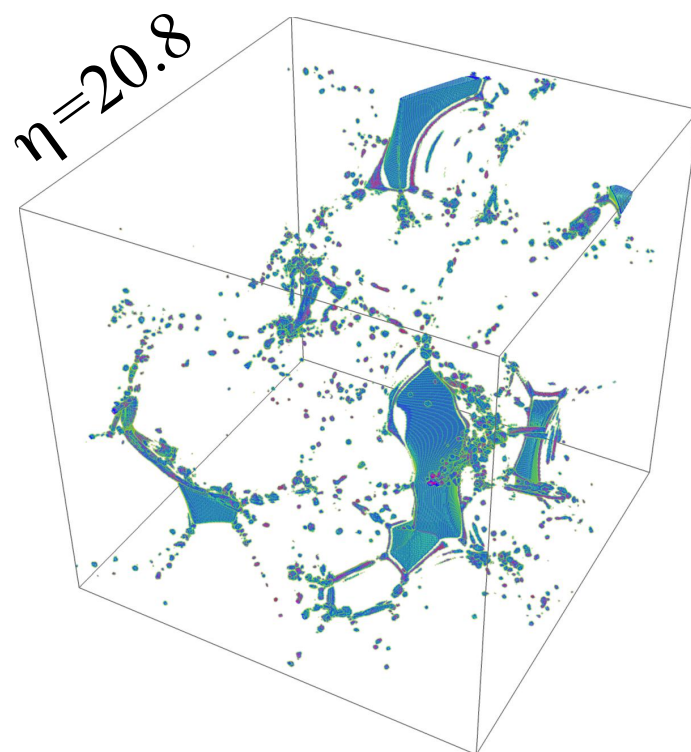
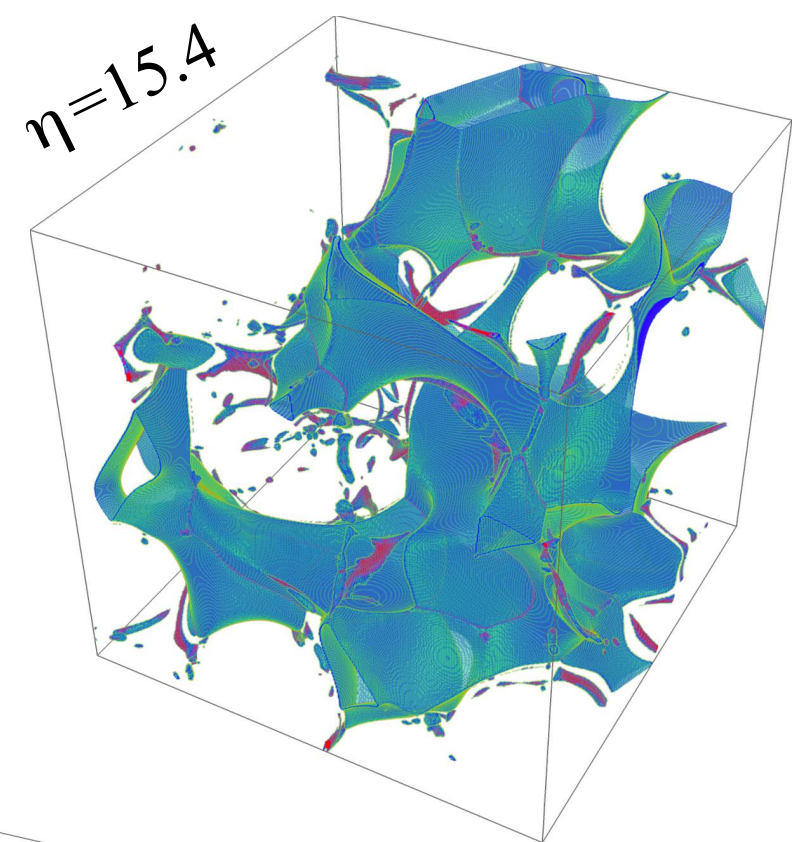
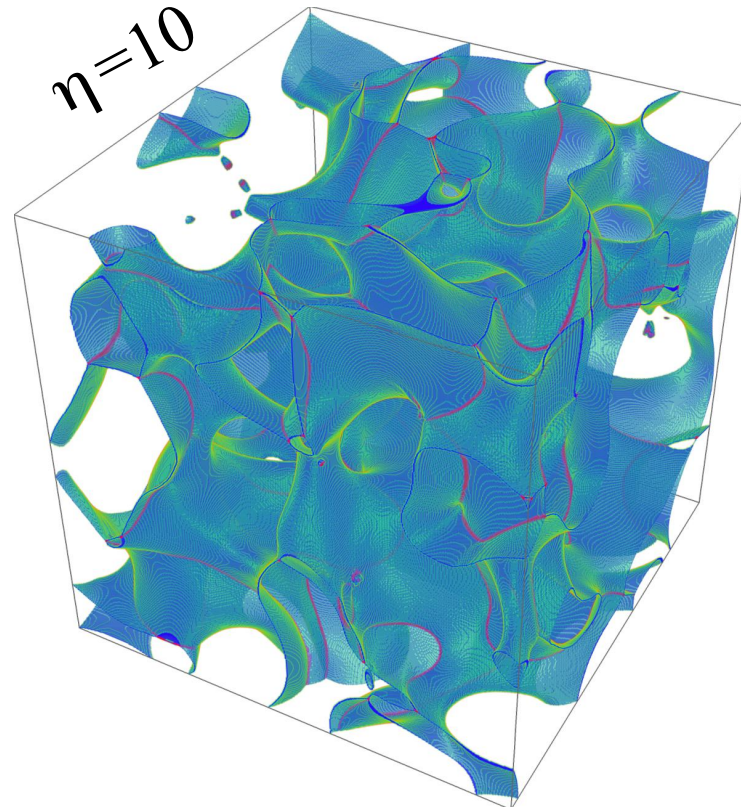
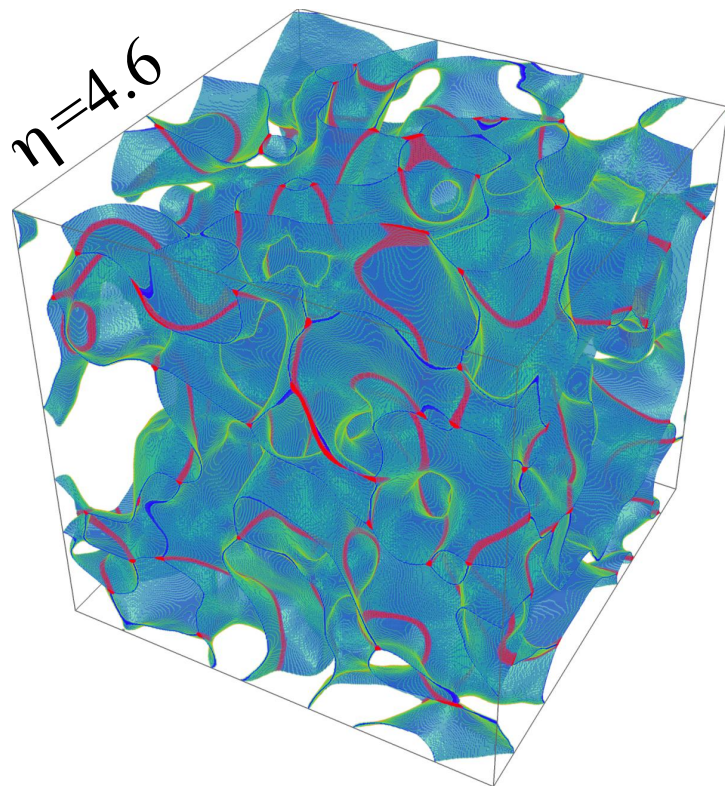
$\Delta A = (\delta x)^2$ is the comoving area of one grid surface

The distribution of fields in phase space



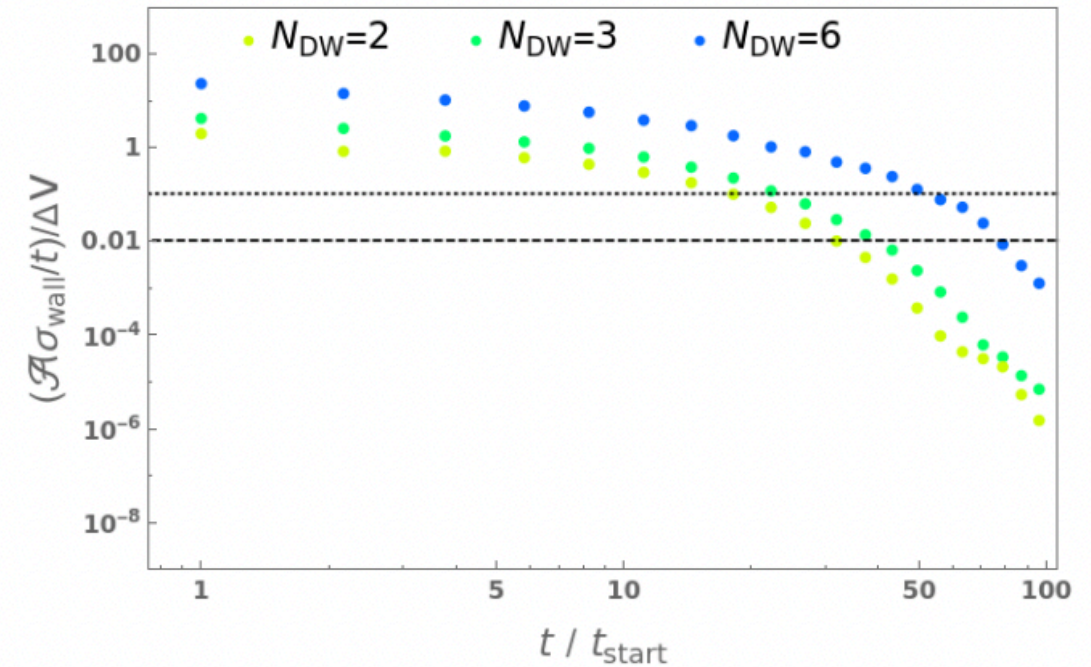
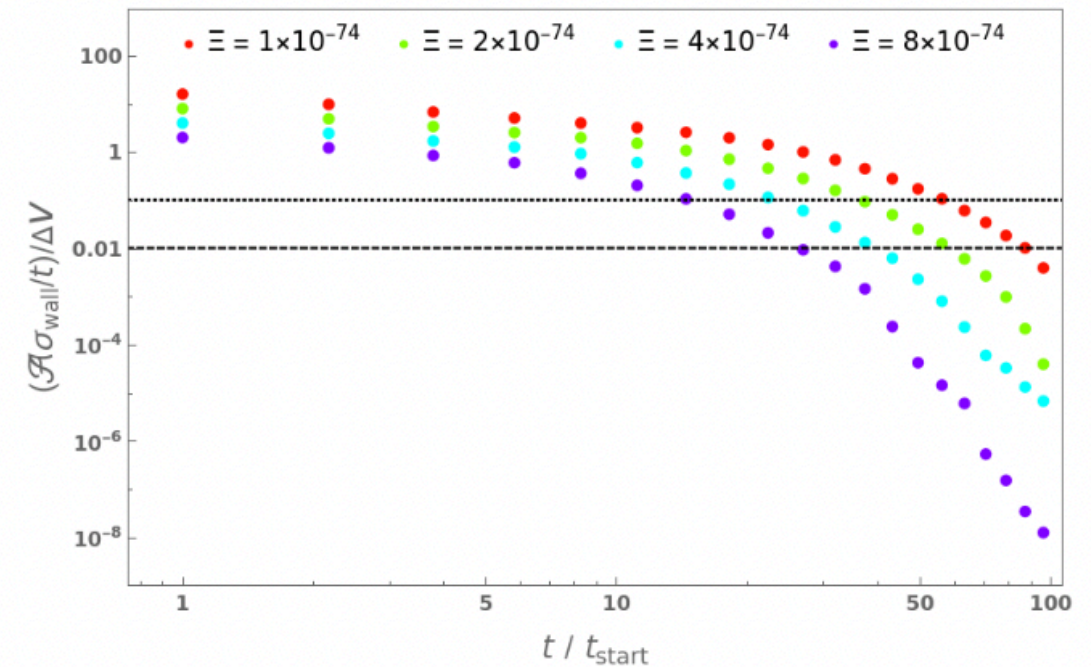
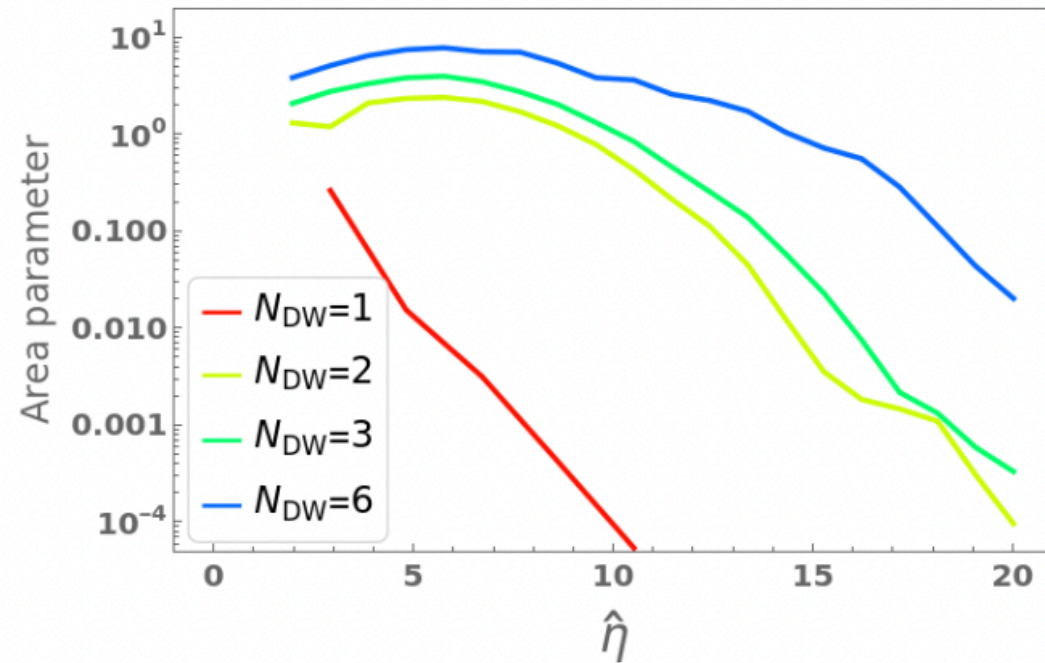
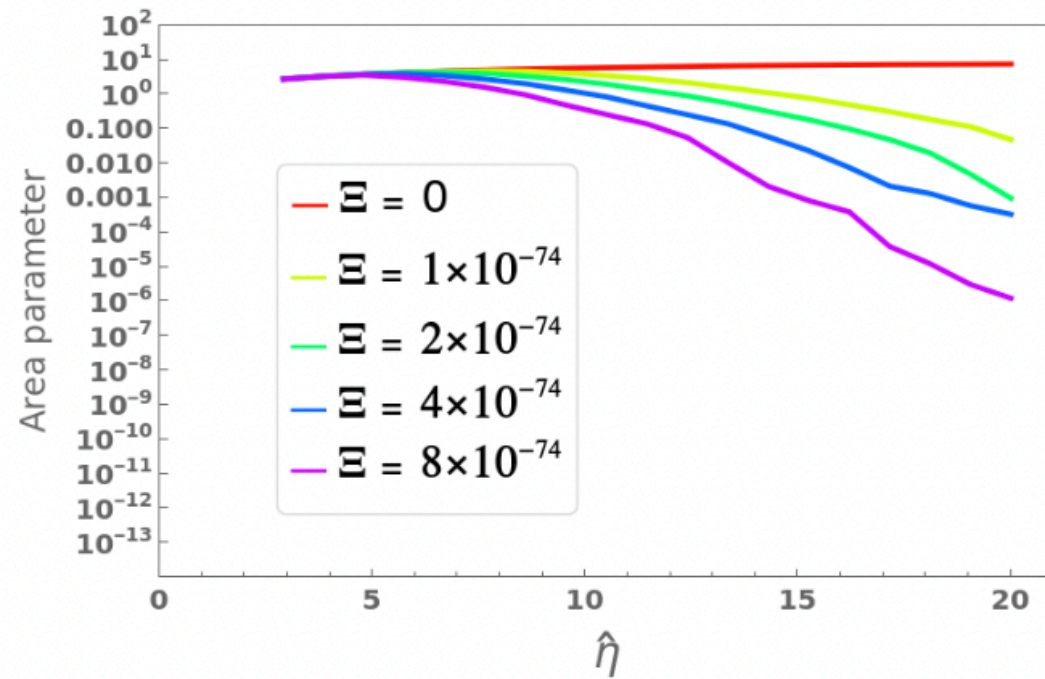
► String-wall evolution

Axion string-domain wall hybrid network destruction with gravitational waves and axions emission



$$N_{\text{DW}} = 3$$

Domain wall Area parameter



DW decay

$$\Delta V = 2\Xi v^4(1 - \cos(2\pi/N_{\text{DW}})) \sim \rho_{\text{wall}} = \mathcal{A}\sigma_{\text{wall}}/t$$

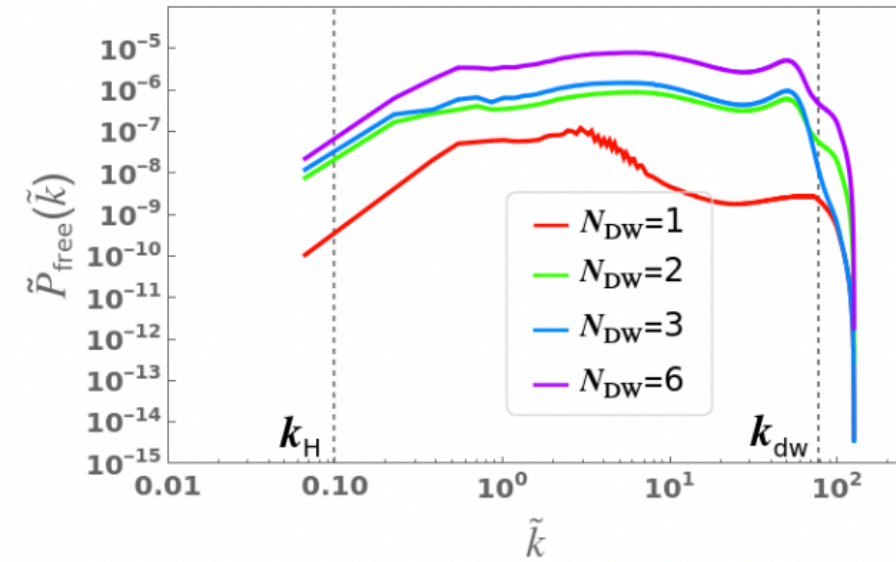
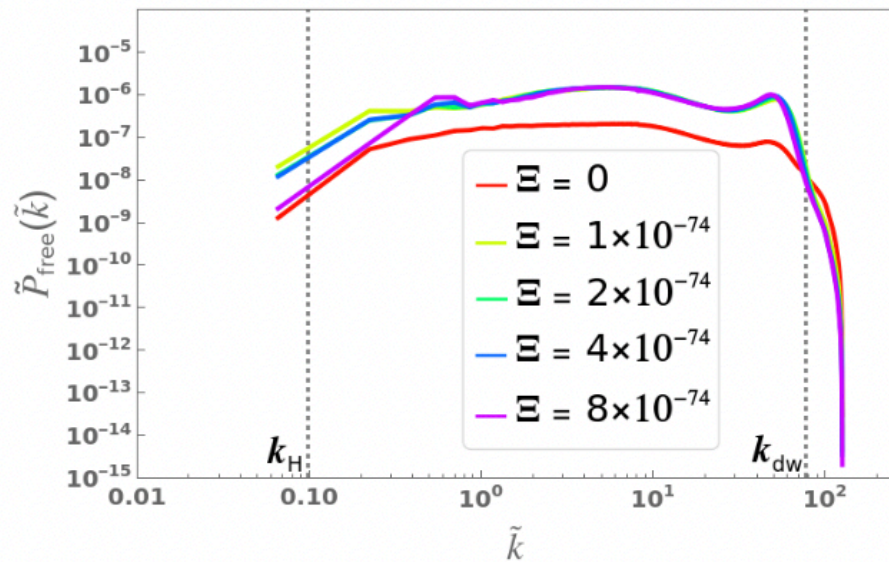


$$t_{\text{dec}} = t_{\text{form}} \left(\frac{C_d \mathcal{A}_{\text{form}} \sigma_{\text{wall}}}{t_{\text{form}} \Xi N_{\text{DW}}^4 f_a^4 (1 - \cos(2\pi/N_{\text{DW}}))} \right)^{1/p}$$

with $p = 1 + \ln C_d / \ln(t_{\text{dec}} / t_{\text{form}})$

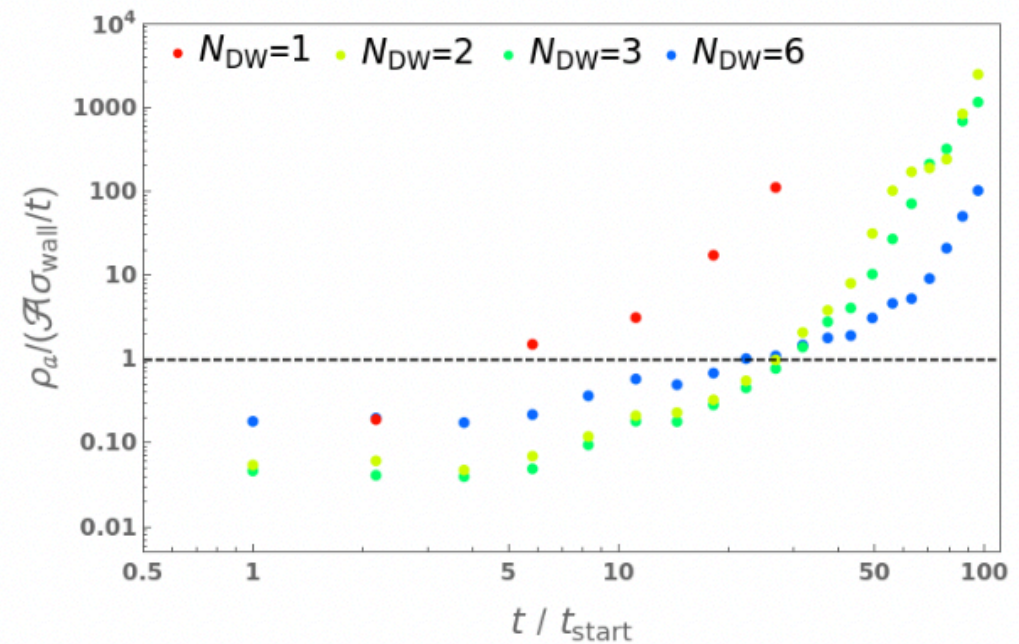
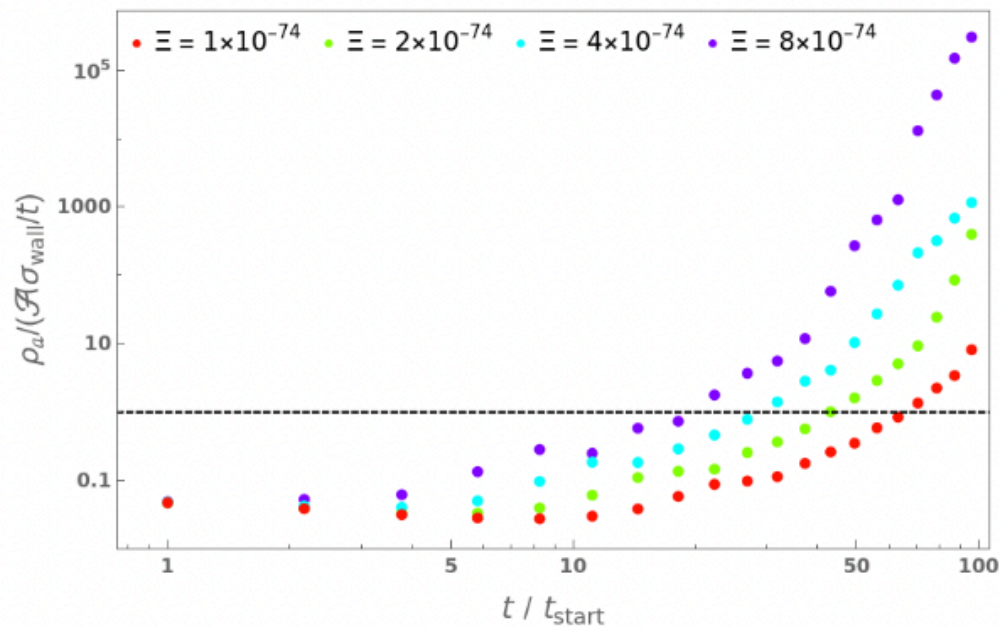
$C_d = \Delta V / (\mathcal{A}\sigma_{\text{wall}}/t) \sim \mathcal{O}(10^2)$

► Domain wall decay to free axions



$$\frac{1}{2} \langle \dot{a}_{\text{free}}(\mathbf{k})^* \dot{a}_{\text{free}}(\mathbf{k}') \rangle = (2\pi)^3 \frac{2\pi^2}{k^3} P_{\text{free}}(k) \delta(\mathbf{k} - \mathbf{k}') \quad W(\mathbf{x}) = W_{\text{st}}(\mathbf{x}) \times W_{\text{dw}}(\mathbf{x}), \quad \dot{a}_{\text{free}}(\mathbf{x}) = W(\mathbf{x}) \dot{a}(\mathbf{x}), \quad \dot{a}(\mathbf{x}) = f_a \frac{\phi_1 \dot{\phi}_2 - \dot{\phi}_1 \phi_2}{\phi_1^2 + \phi_2^2}$$

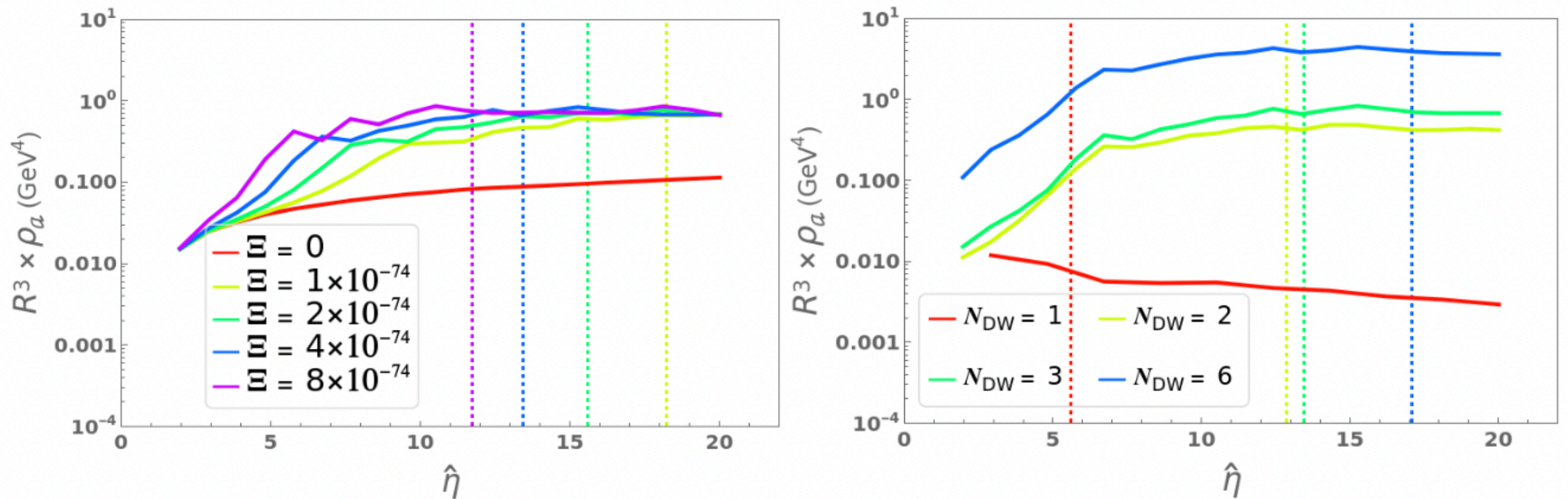
Window function $W_{\text{dw}}(\mathbf{x})$ will take 0 near the DW core and 1 near the true vacuum



$$\rho_a \simeq 2 \times \frac{1}{2} \langle \dot{a}_{\text{free}}^2(\mathbf{x}) \rangle$$

$$\rho_a \sim \mathcal{A} \sigma_{\text{wall}} / t \sim 8 \mathcal{A} m f_a^2 / t_{\text{dec}}$$

► Domain wall decay to free axions



$$\rho_a(t_{\text{dec}}) \simeq \frac{2(2p-1)f_a^4 N_{\text{DW}}^4}{(3-2p)C_d} \Xi \sin^2(\pi/N_{\text{DW}})$$

Axion energy density tends to be a constant at the final moment, and the energy density of the radiated free axion is (almost) proportional to the bias term and N_{DW}

► Domain wall decay to axion Dark matter

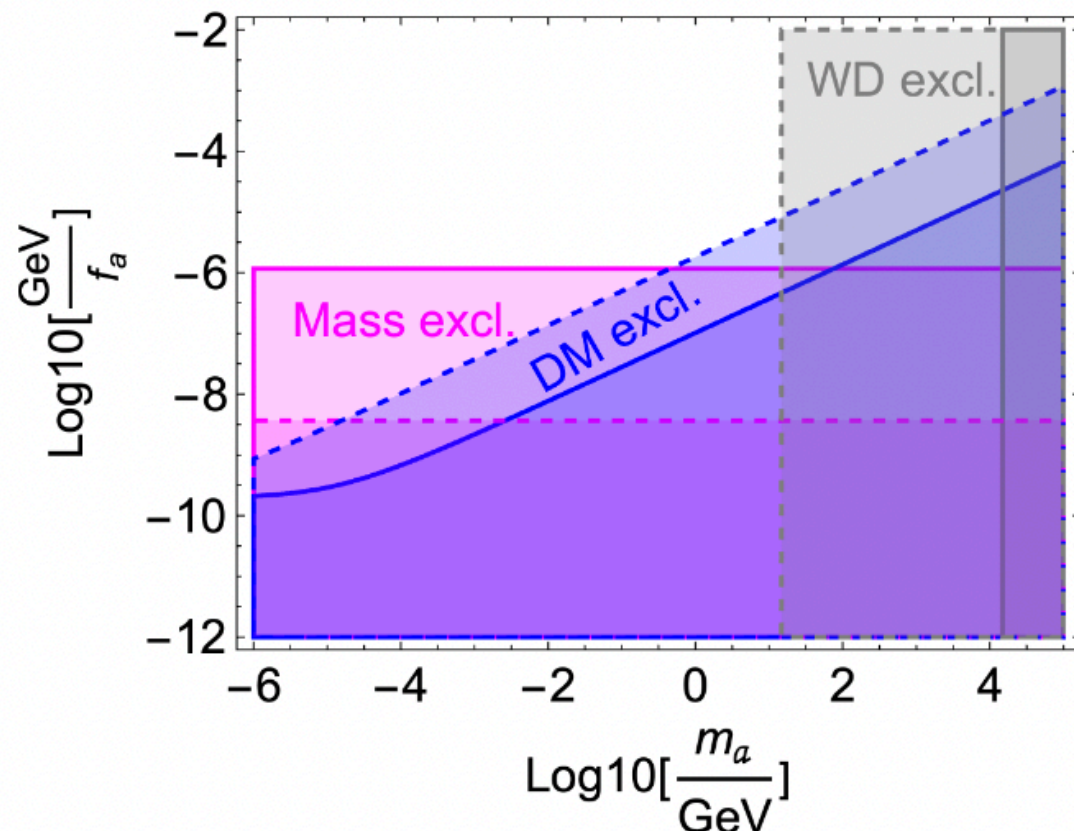
$$\Omega_{a,\text{new}}(t_0)h^2 = \Omega_a^{\text{DW}}h^2 + \Omega_{a,0}(t_0)h^2 + \Omega_{a,\text{st}}(t_0)h^2$$

Axion DM from Wall decay:

$$\Omega_a h^2 = \frac{\rho_a(t_0)}{\rho_{0,c}/h^2} = 1.024 \times 10^{-19} \left(\frac{8}{3}\right)^{3/(2p)} \left(\frac{2p-1}{3-2p}\right) C_d^{3/(2p)-1} \left(\frac{m}{\text{GeV}}\right)^{3/p-3/2} \left(\frac{f_a}{\text{GeV}}\right)^{4-3/p} \Xi^{1-3/(2p)} \left(\frac{\text{csc}(\pi/N_{\text{DW}})}{N_{\text{DW}}^2}\right)^{3/p-2}$$

Misalignment mechanism: $(\Omega_{a,0}(t_0)h^2 \simeq 4.63 \times 10^{-3} (f_{a,\text{new}}/10^{10}\text{GeV})^{1.19})$

Axion DM from pure axion string: $\Omega_{a,\text{st}}(t_0)h^2 = 2.0 \times (f_a/10^{12}\text{GeV})^{1.19}$



WD excl.:

$$t_{\text{dec}} < t_{\text{WD}} \quad t_{\text{WD}} = \frac{3}{16\pi G \sigma_{\text{wall}}}$$

Mass excl.:

bias term dominating over the QCD instanton effect in the contributions to axion mass term

$$\Xi < 2 \times 10^{-45} N_{\text{DW}}^{-2} \left(\frac{10^{10}\text{GeV}}{f_a}\right)^4$$

DM excl.

$$\Omega_a(t_0)h^2 \leq 0.12$$

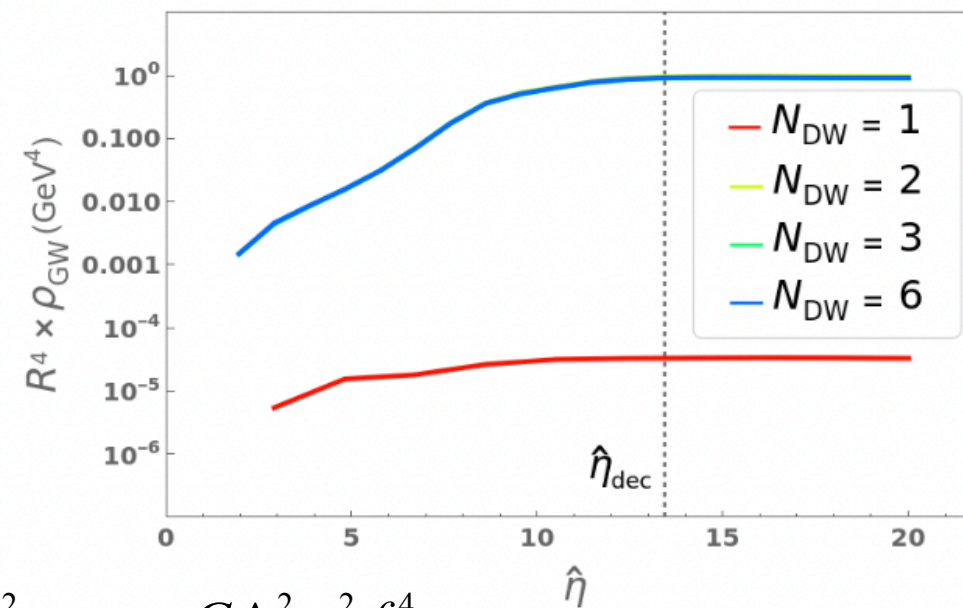
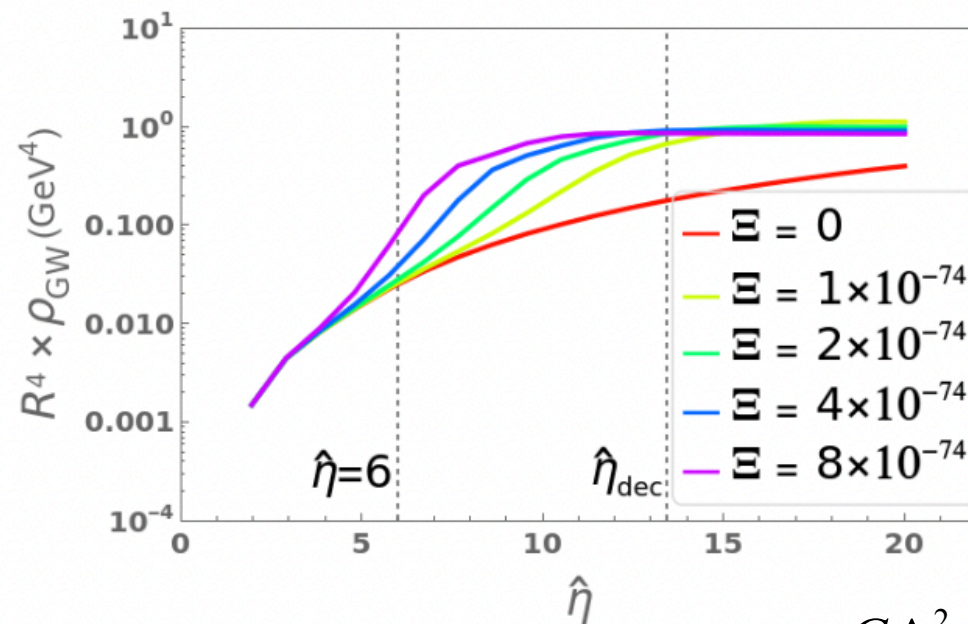
► String-wall GW

$$\ddot{h}_{ij} + 3\frac{\dot{R}}{R}\dot{h}_{ij} - \frac{\nabla^2}{R^2}h_{ij} = \frac{16\pi G}{R^2}\Pi_{ij}^{\text{TT}}$$

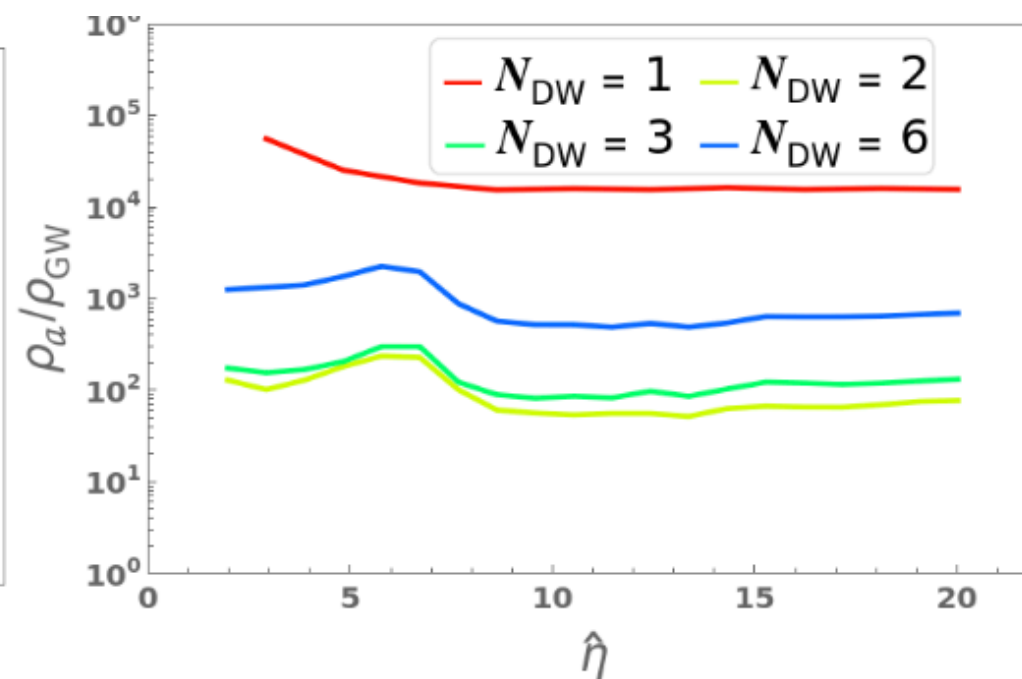
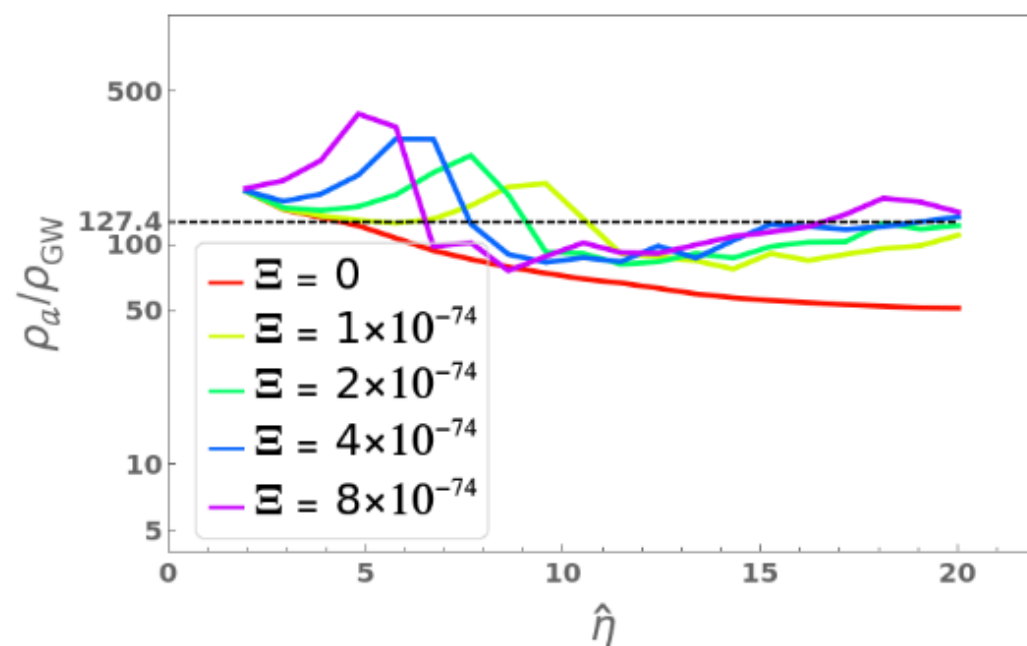
$$\rho_{\text{GW}}(t) = t_{00} = \frac{1}{32\pi G}\langle\partial_\mu h_{ij}^{\text{TT}}(\mathbf{x}, t)\partial_\nu h^{\text{TT},ij}(\mathbf{x}, t)\rangle|_{\mu=\nu=0} = \frac{1}{32\pi G}\langle\dot{h}_{ij}(\mathbf{x}, t)\dot{h}_{ij}(\mathbf{x}, t)\rangle$$

$$\Pi_{ij} \equiv T_{ij} - pg_{ij} = T_{ij} - \frac{\delta_{ij}}{3}\sum_l T_{ll}$$

$$= (\partial_i\phi_1\partial_j\phi_1 + \partial_i\phi_2\partial_j\phi_2) - \frac{\delta_{ij}}{3}\sum_l(\partial_l\phi_1\partial_l\phi_1 + \partial_l\phi_2\partial_l\phi_2)$$

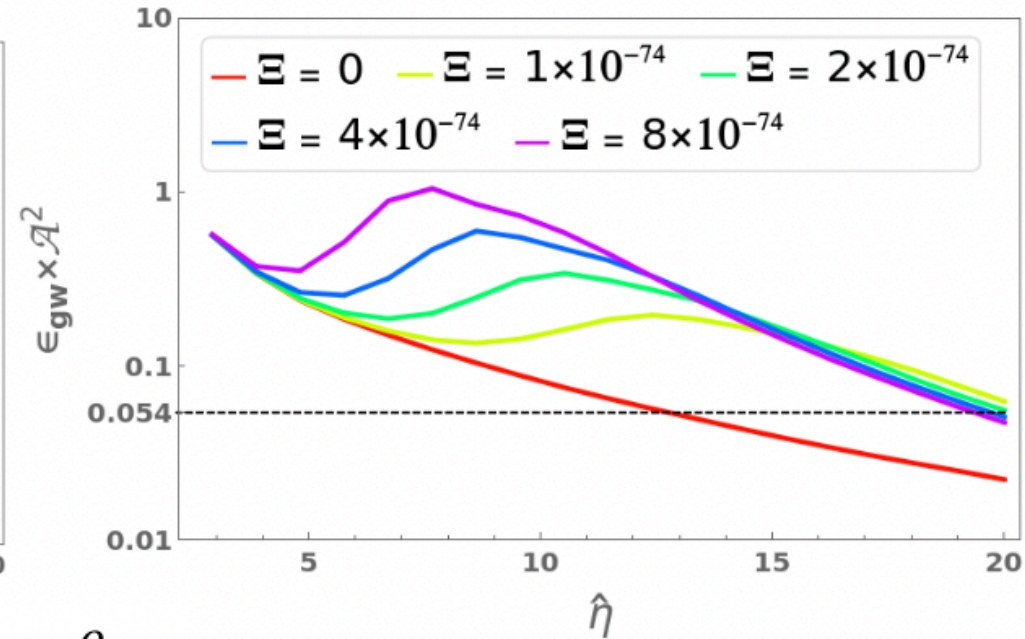
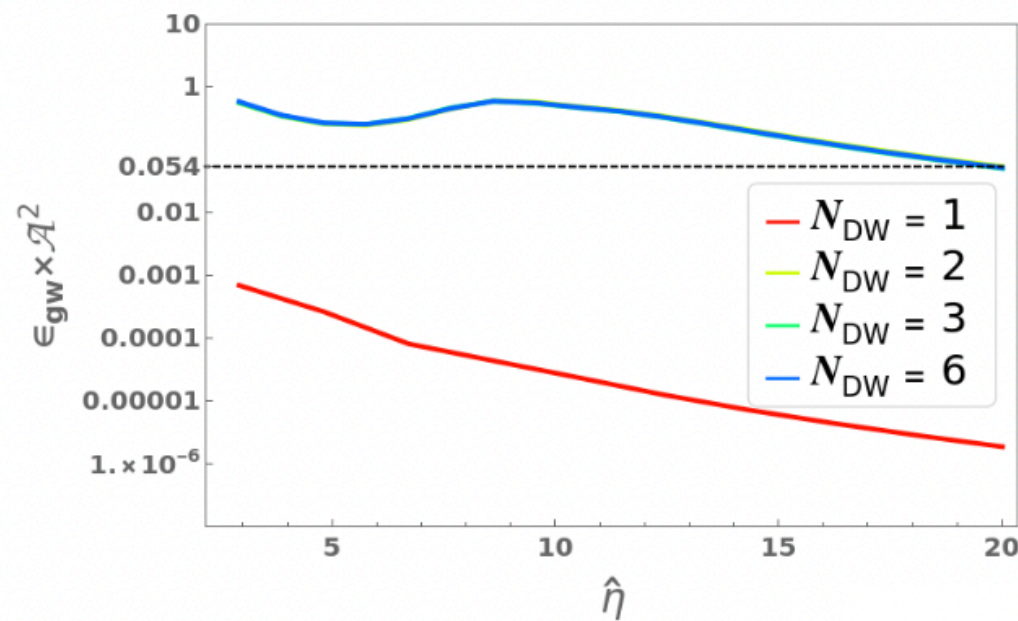


$$\rho_{\text{gw}} = \varepsilon_{\text{gw}} GA^2 \sigma_{\text{wall}}^2 \propto \varepsilon_{\text{gw}} GA^2 m^2 f_a^4$$

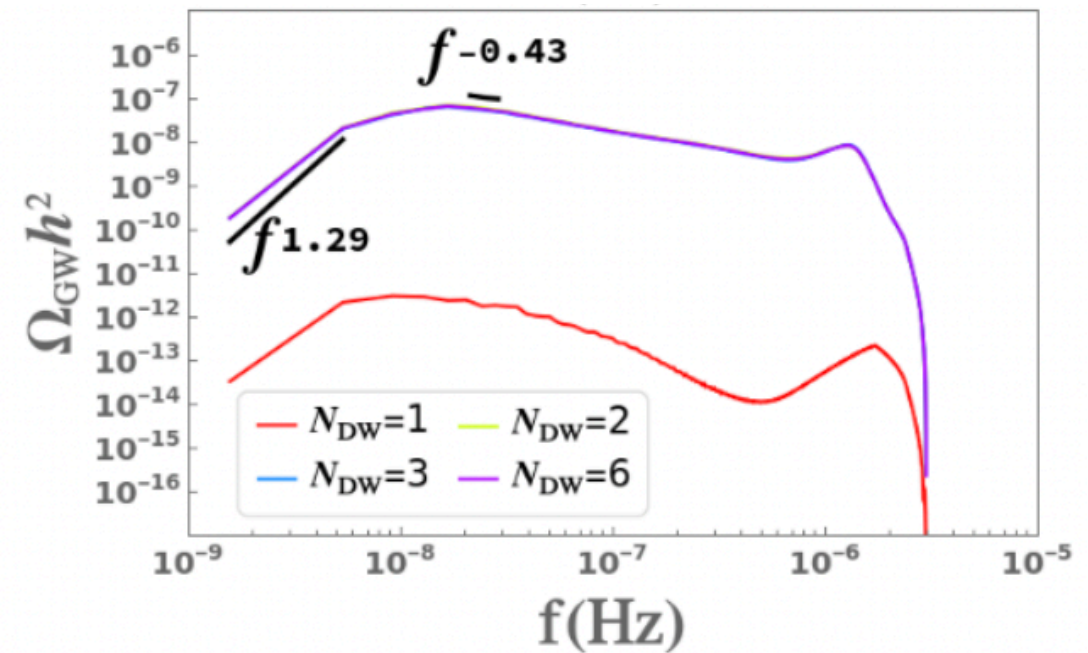
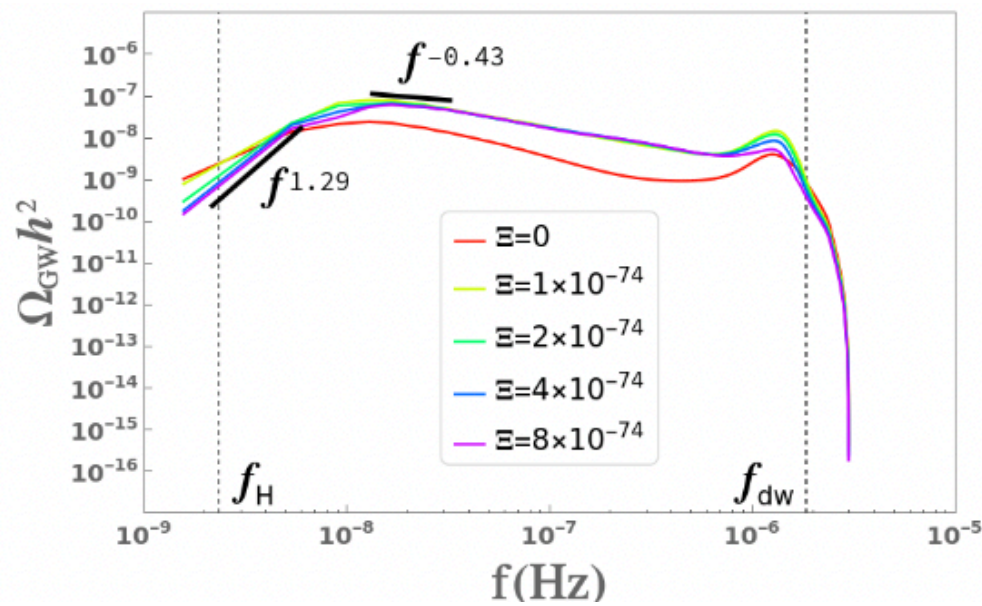


axion string-wall networks mainly decay to axion

► String-wall GW



Efficiency parameter $\epsilon_{\text{gw}} \mathcal{A}^2 = \frac{\rho_{\text{gw}}}{G\sigma_{\text{wall}}^2} \sim \text{Const.}$ $\rho_{\text{gw}} \propto Gm^2 f_a^4$



$$\Omega_{\text{GW}} \equiv \frac{1}{\rho_c} \frac{d\rho_{\text{GW}}}{d\ln k}$$

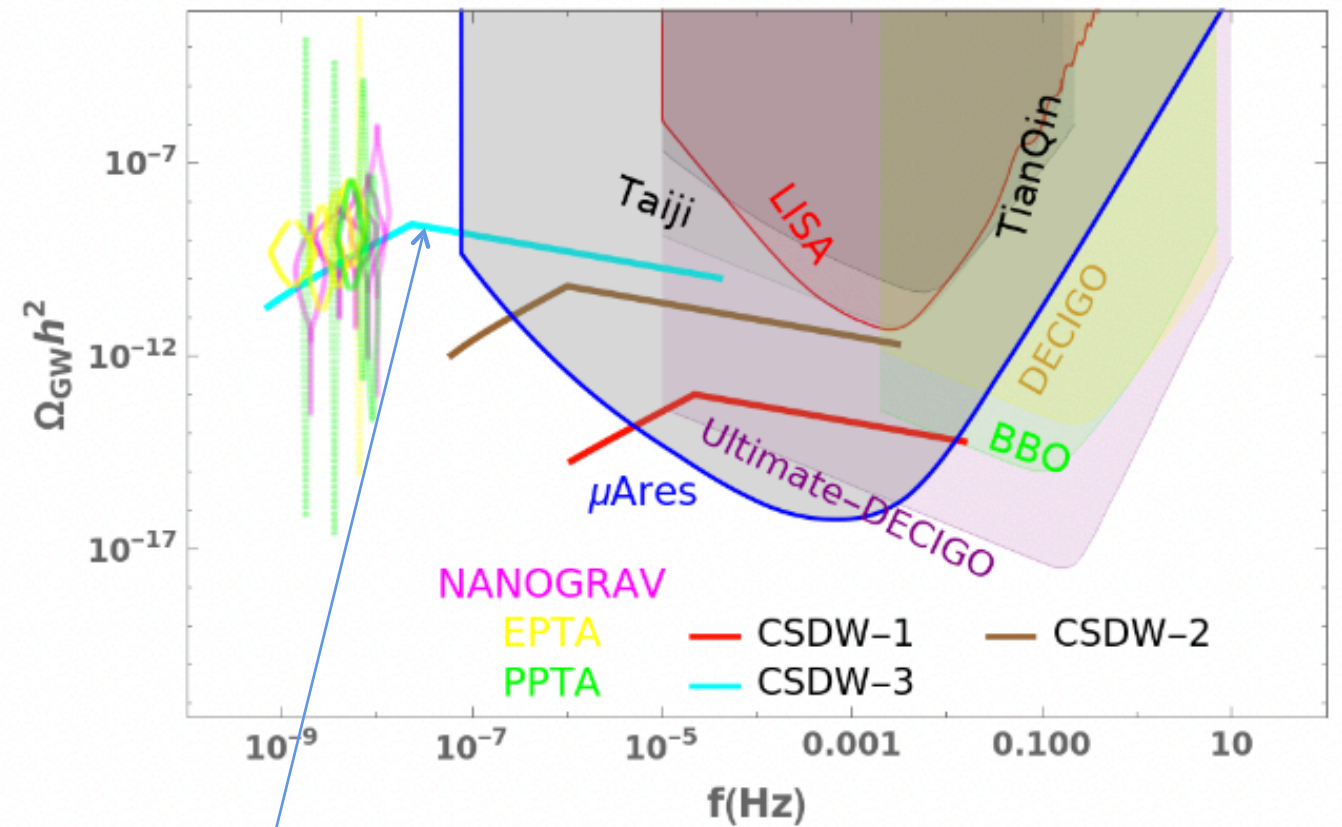
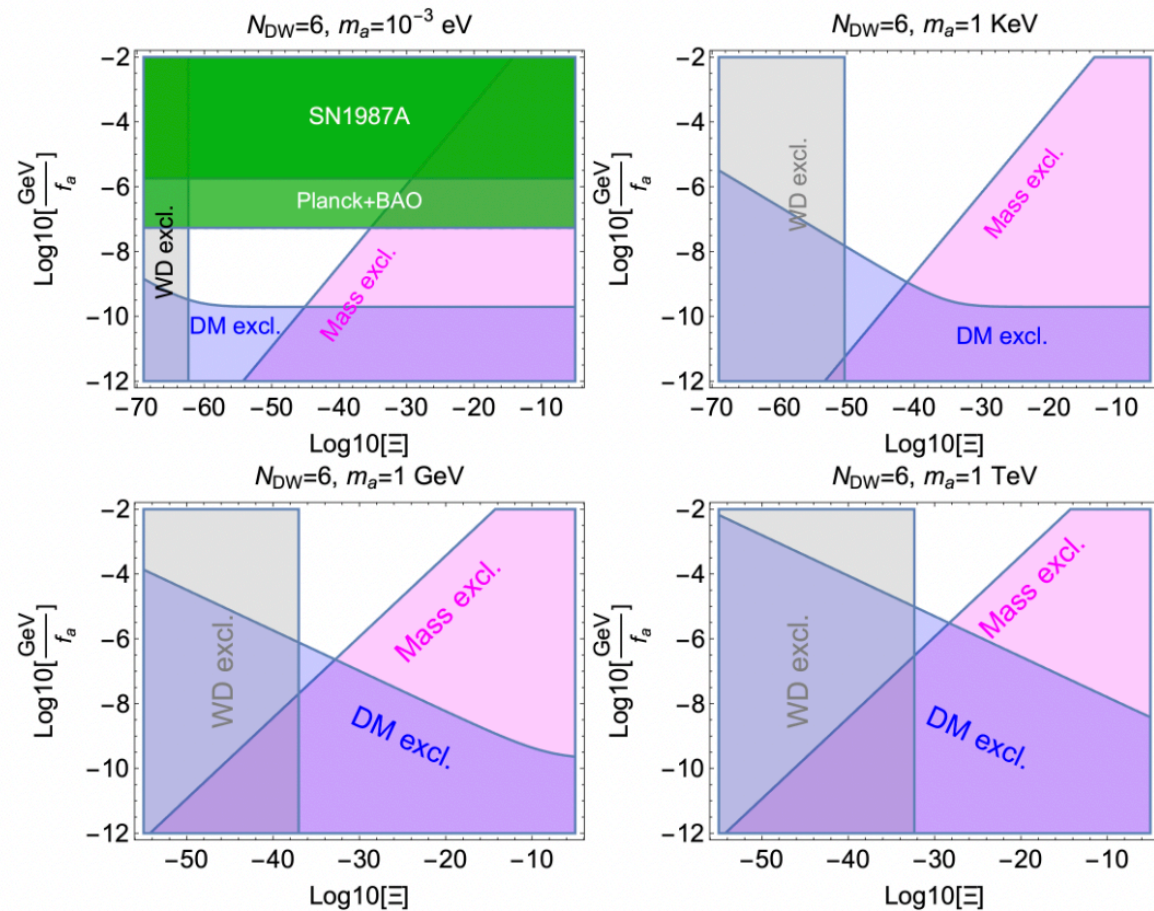
$$= \frac{1}{32\pi G} \frac{1}{(2\pi)^3 V} \int d\Omega |\mathbf{k}|^3 |h_{ij}(\mathbf{k}, t)|^2$$

$$\Omega_{\text{GW},0} h^2 = \frac{h^2}{\rho_{c,0}} \frac{d\rho_{\text{GW},0}}{d\ln k_{p,0}}$$

$$= \Omega_{\text{rad},0} h^2 \left(\frac{g_0}{g_e} \right)^{1/3} \left\{ \frac{1}{\rho_{c,e}} \frac{d\rho_{\text{GW},e}}{d\ln k_{\text{co},e}} \right\}$$

► Axion DM and GW from String-wall

Our research provides the possibility of searching for axion models or constraining model parameters through GW detection experiments



Simultaneously explanation of DM and GWs with axion string-wall networks, **only for $N_{DW} > 1$ of ALP scenarios.**

For $N_{DW} = 1$, the GW energy density appears undetectable for QCD axions and axion-like particles.

► Cosmic string simulation: the KSVZ axion model

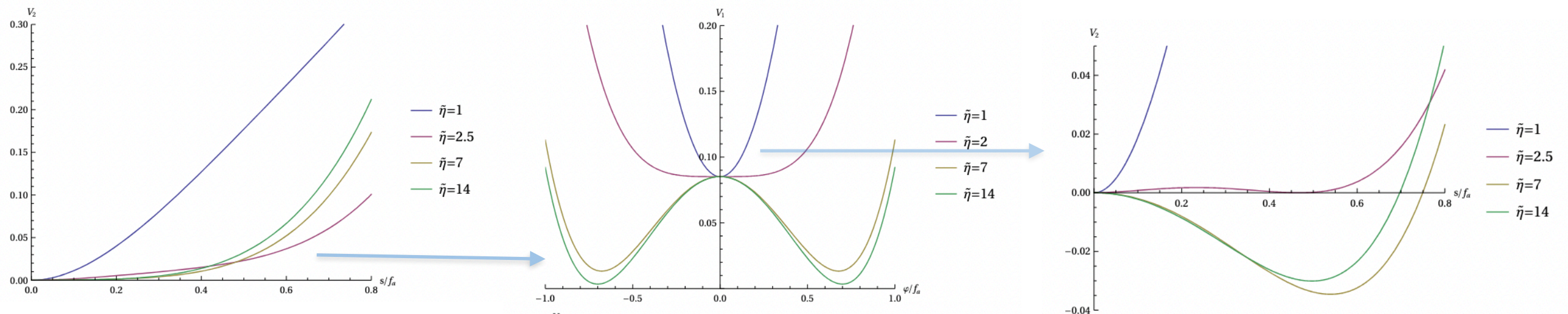
The action in expanding universe with spatially flat FLRW metric:

$$S = - \int d^4x \sqrt{-g} \left(\partial_\mu \varphi^* \partial^\mu \varphi + \frac{1}{2} \partial_\mu h \partial^\mu h + V(\varphi, h, T) \right)$$

Thermal effective potential

$$V(\varphi, h, T) = V_1(\varphi, T) + V_2(\varphi, h, T)$$

$$V_1(\varphi, T) = \lambda_\varphi \left(|\varphi|^2 - \frac{f_a^2}{2} \right)^2 + \left(\frac{\lambda_\varphi}{3} + \frac{\lambda_{\varphi h}}{6} \right) T^2 |\varphi|^2 \quad V_2(\varphi, s, T) = \frac{1}{2} \gamma (T^2 - T_0^2) h^2 + \frac{1}{3} A T h^3 + \frac{1}{4} \lambda_h h^4 + \frac{1}{2} \lambda_{\varphi h} |\varphi|^2 h^2$$



$$T_c = \sqrt{\lambda_\phi / (\lambda_\phi / 3 + \lambda_{\phi h})} f_a$$

Potential Shape

► Simulation setup

Equations of motion

Only PQ era

$$\left\{ \begin{array}{l} \tilde{\varphi}'' - \tilde{\nabla}^2 \tilde{\varphi} + 2 \frac{a'}{a} \tilde{\varphi}' = -a^2 \frac{\partial \tilde{V}}{\partial |\tilde{\varphi}|} \frac{\tilde{\varphi}}{2|\tilde{\varphi}|}, \\ \tilde{h}'' - \tilde{\nabla}^2 \tilde{h} + 2 \frac{a'}{a} \tilde{h}' = -a^2 \frac{\partial \tilde{V}}{\partial \tilde{h}}, \end{array} \right. \quad \begin{array}{l} \tilde{\varphi} = \frac{\varphi}{f_*} \quad \tilde{h} = \frac{h}{f_*} \\ \tilde{V} = \frac{V(f_* \tilde{\varphi}, f_* \tilde{h}, T)}{f_*^2 \omega_*^2} \end{array} \quad \begin{array}{l} f_* = f_a \\ \omega_* = a_i H_i \end{array}$$

Initial condition

thermal spectrum

$$\mathcal{P}_{\phi_i}(k) = \frac{1}{\omega_k (e^{\omega_k/T} - 1)} \quad \mathcal{P}_{\pi_{\phi_i}}(k) = \frac{\omega_k}{e^{\omega_k/T} - 1}$$

$$w_k = \sqrt{k^2/R^2 + m_{\text{eff}}^2} \quad m_\phi^2 = (\lambda_\phi/3 + \lambda_{\phi h}/6)T^2 - \lambda_\phi v_\phi^2$$

two-point correlation functions

$$\begin{aligned} \langle \phi_i(\mathbf{k}) \phi_j(\mathbf{k}') \rangle &= (2\pi)^3 \mathcal{P}_\phi(k) \delta(\mathbf{k} - \mathbf{k}') \delta_{ij}, \\ \langle \dot{\phi}_i(\mathbf{k}) \dot{\phi}_j(\mathbf{k}') \rangle &= (2\pi)^3 \mathcal{P}_{\dot{\phi}}(k) \delta(\mathbf{k} - \mathbf{k}') \delta_{ij}, \\ \langle \phi_i(\mathbf{k}) \dot{\phi}_j(\mathbf{k}') \rangle &= 0. \end{aligned}$$

$$\begin{aligned} \langle |\phi_i(\mathbf{k})|^2 \rangle &= \left(\frac{N}{\delta x_{\text{phy}}} \right)^3 \mathcal{P}_{\phi_i}(k), & \langle \phi_i(\mathbf{k}) \rangle &= 0, \\ \langle |\dot{\phi}_i(\mathbf{k})|^2 \rangle &= \left(\frac{N}{\delta x_{\text{phy}}} \right)^3 \mathcal{P}_{\dot{\phi}_i}(k), & \langle \dot{\phi}_i(\mathbf{k}) \rangle &= 0, \end{aligned}$$

► Field configurations and the surface tension

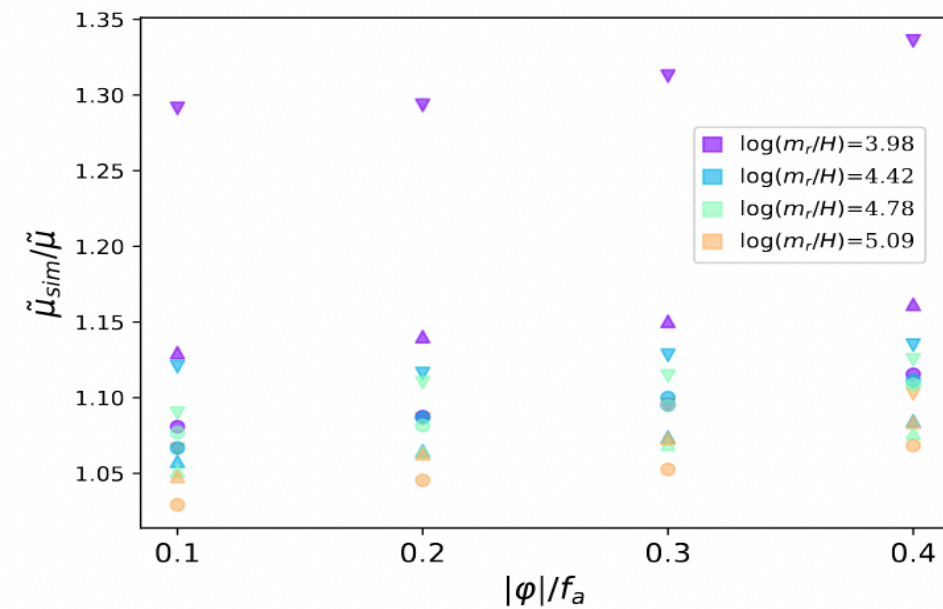
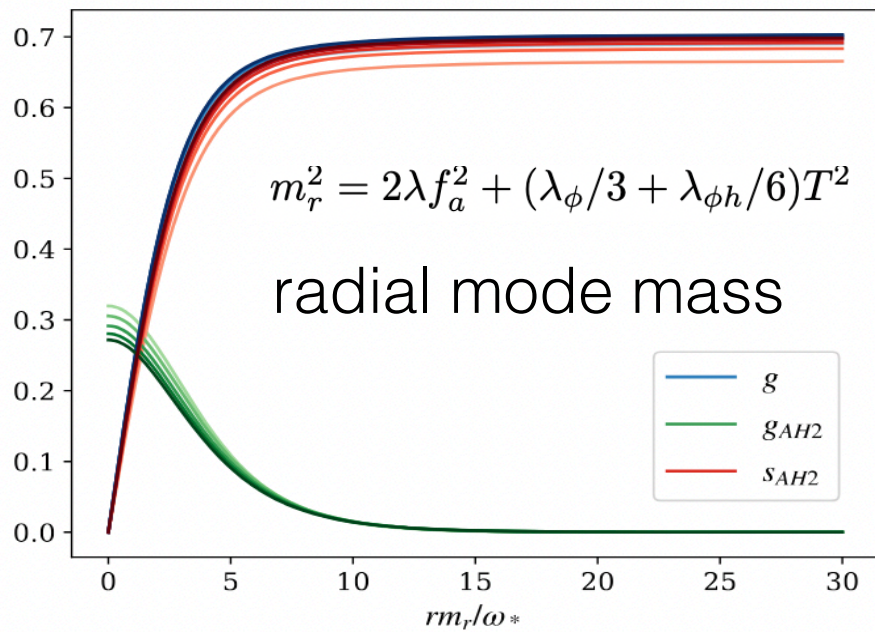
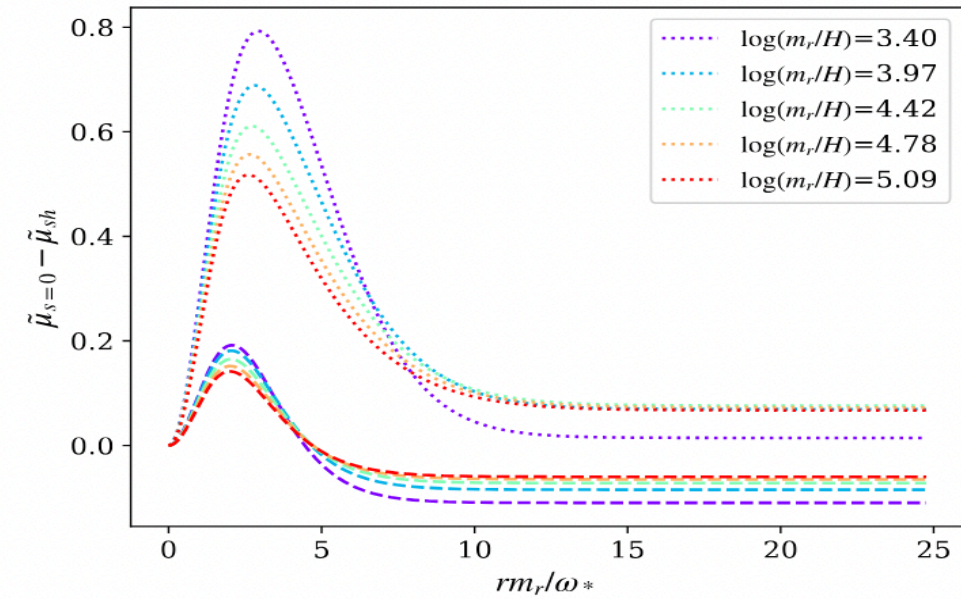
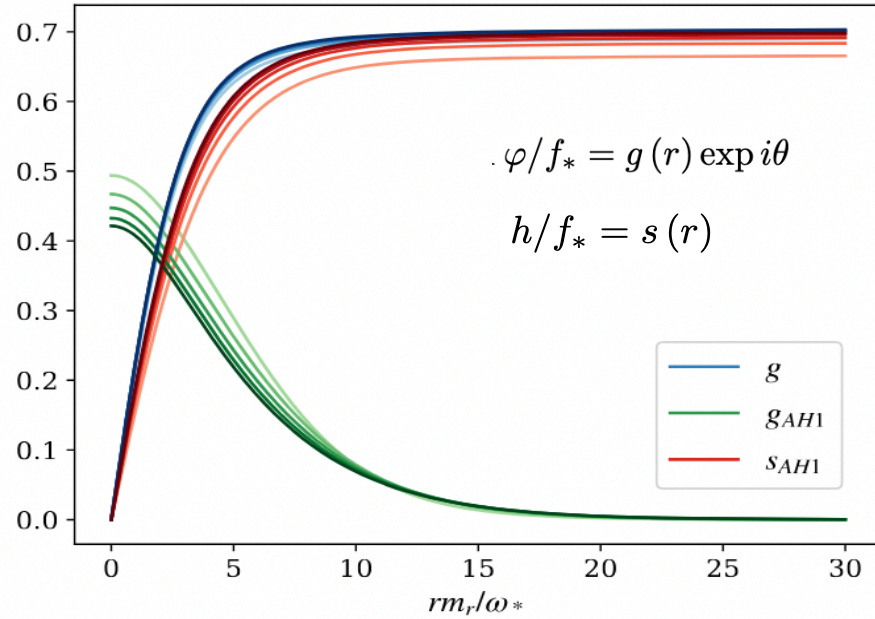
$$g''(r) + \frac{1}{r}g'(r) - \frac{1}{r^2}g(r) = \lambda_\varphi \frac{f_*^2}{\omega_*^2} g(g^2 - \frac{1}{2}) + (\frac{\lambda_\varphi}{3} + \frac{\lambda_{\phi h}}{6}) \frac{T^2}{\omega_*^2} g + \frac{\lambda_{\phi h}}{2} \frac{f_*^2}{\omega_*^2} g s^2 ,$$

$$s''(r) + \frac{1}{r}s'(r) = \lambda_{\phi h} \frac{f_*^2}{\omega_*^2} g^2 s + \gamma \frac{T^2 - T_0^2}{\omega_*^2} s + AT \frac{f_*}{\omega_*^2} s^2 + \lambda_h \frac{f_*^2}{\omega_*^2} s^3 .$$

Surface tension

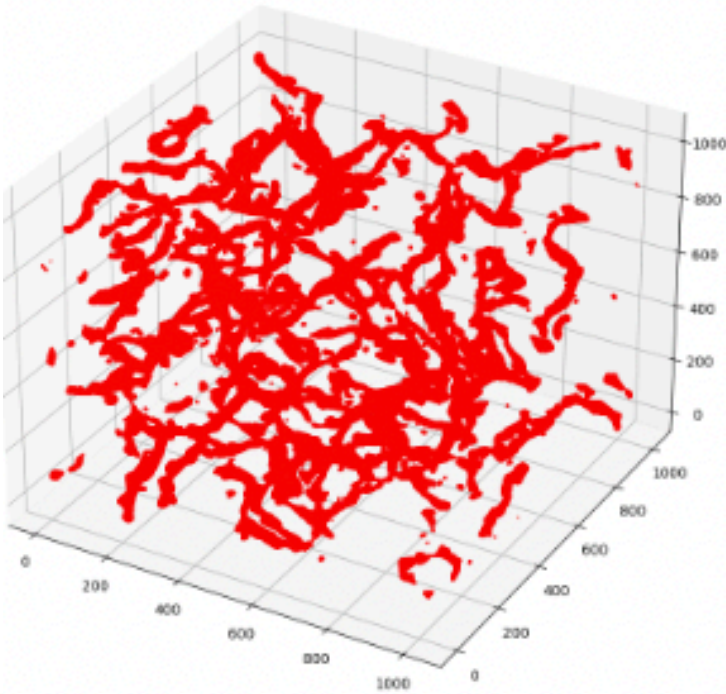
$$\tilde{\mu} = \mu/f_*^2 = \int \tilde{\rho} r dr d\theta$$

$$\tilde{\mu} = 2\pi \int \left((g'^2 + \frac{g^2}{r^2} + (s'^2 + \tilde{V}(g, s)) \right) r dr$$



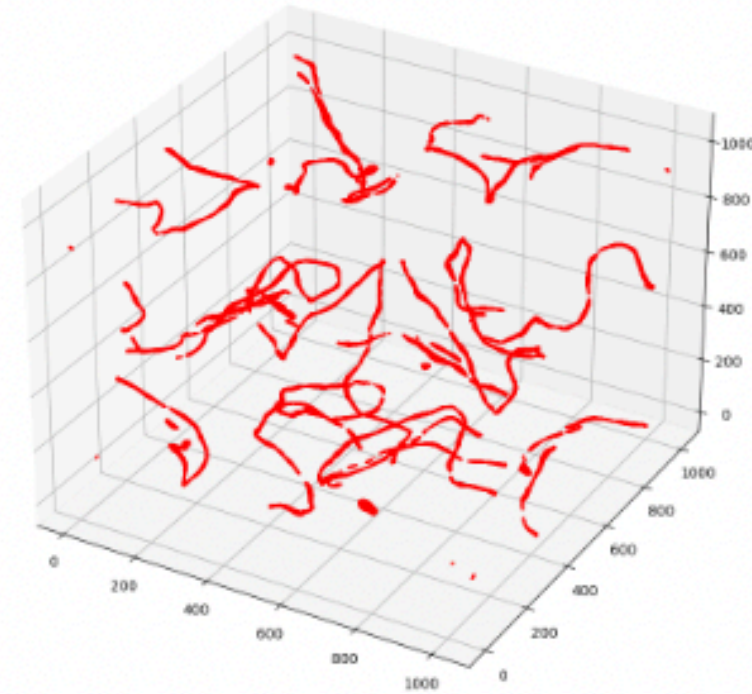
► Axion-Higgs String evolution

$$\tilde{\eta} = 9.0$$

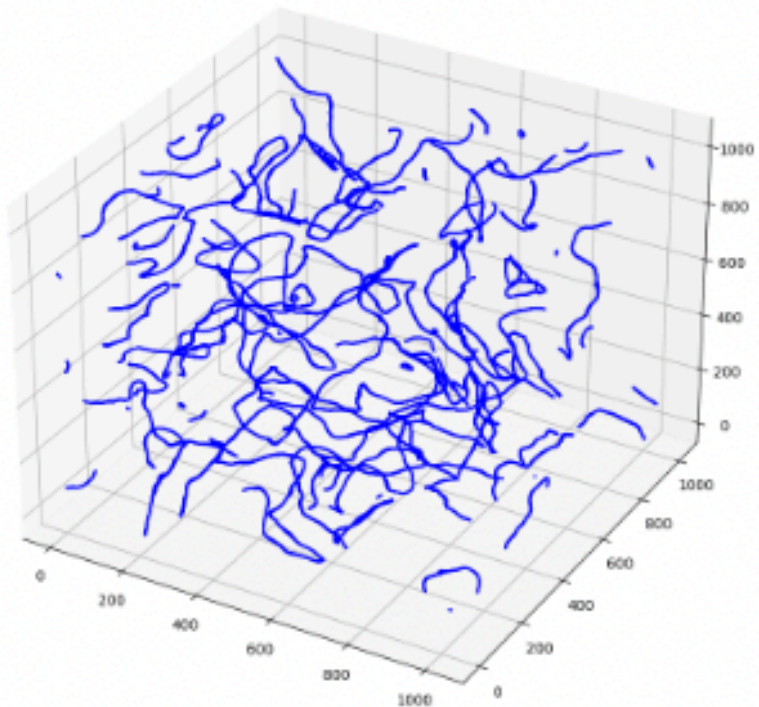


$$s > 0.35 f_a$$

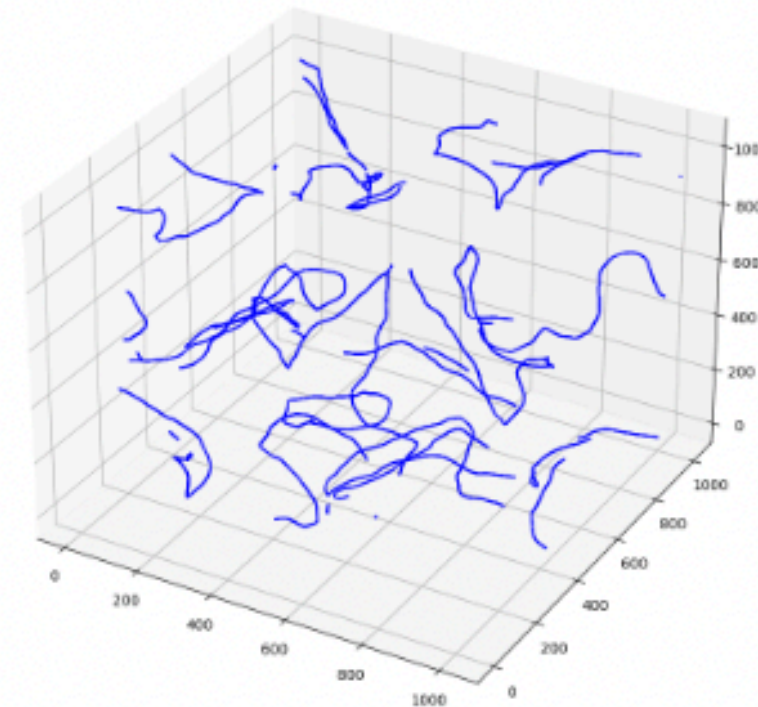
$$\tilde{\eta} = 14.0$$



1024^3

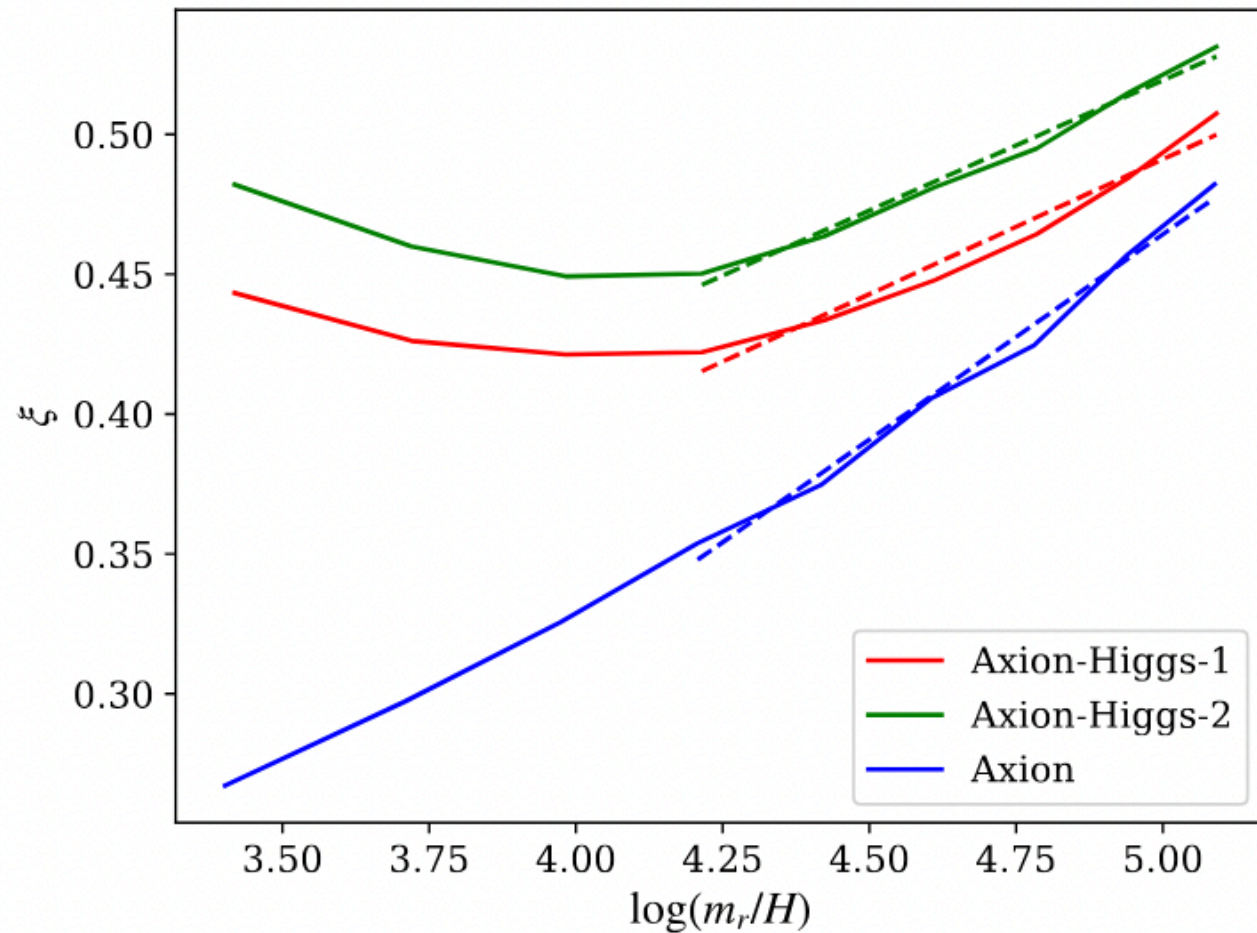


$$|\varphi| < 0.1 f_a$$



► Scaling parameter and energy density

Scaling parameters



$$\xi = \alpha \log(m_r/H) + \beta$$

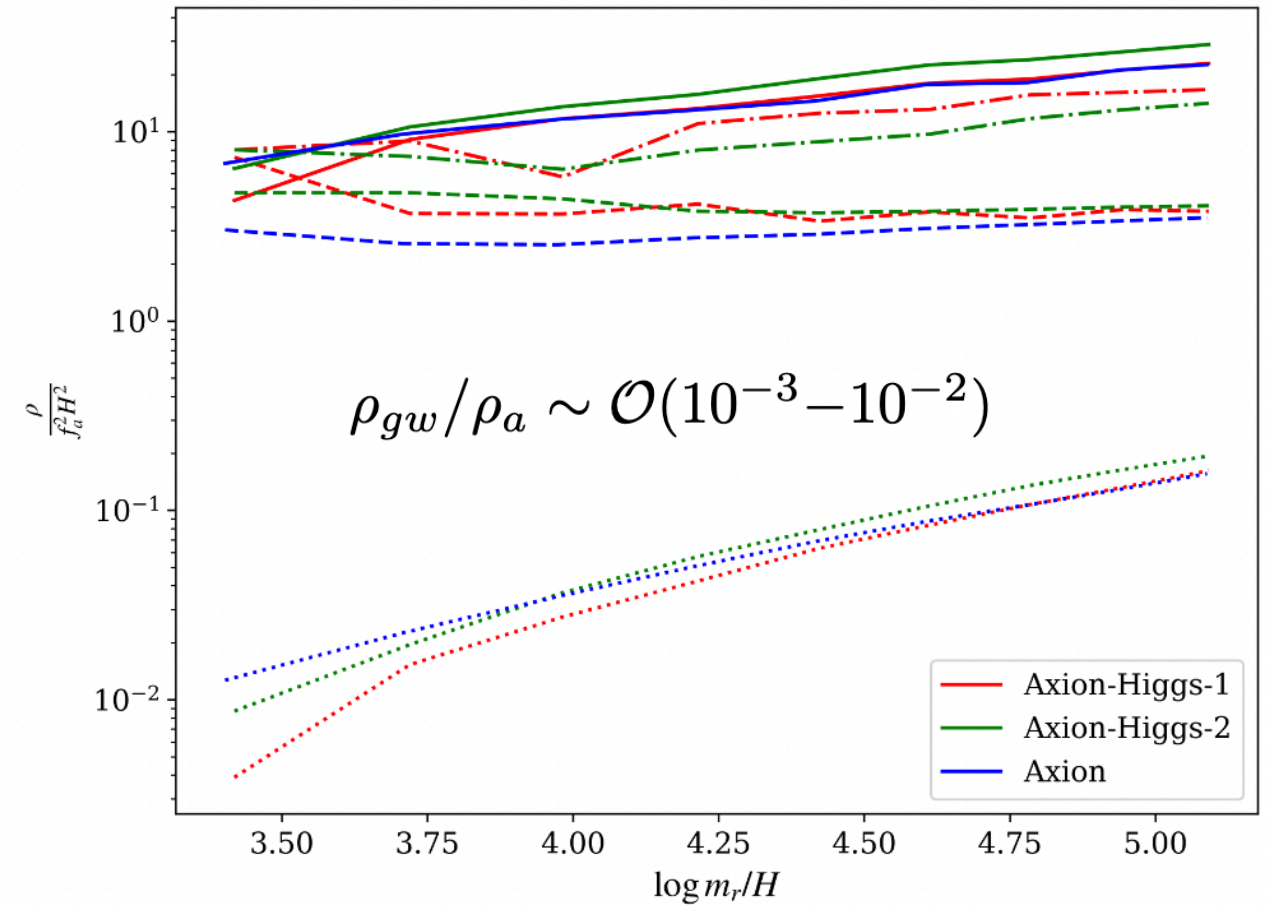
$$\alpha = 0.096, 0.093, 0.15$$

$$\beta = -0.009, 0.054, -0.27$$

linear growth of $\log(m_r/H)$

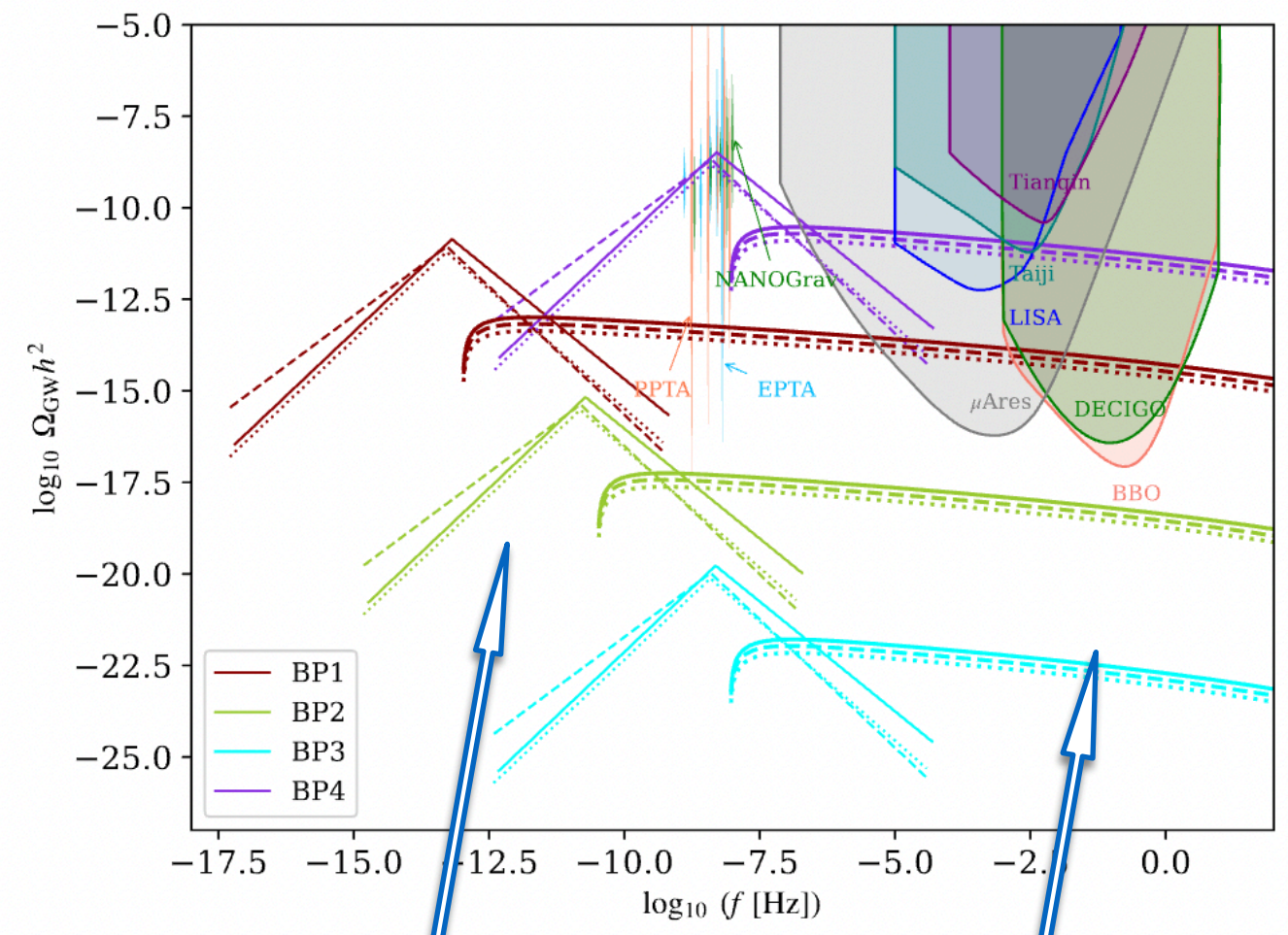
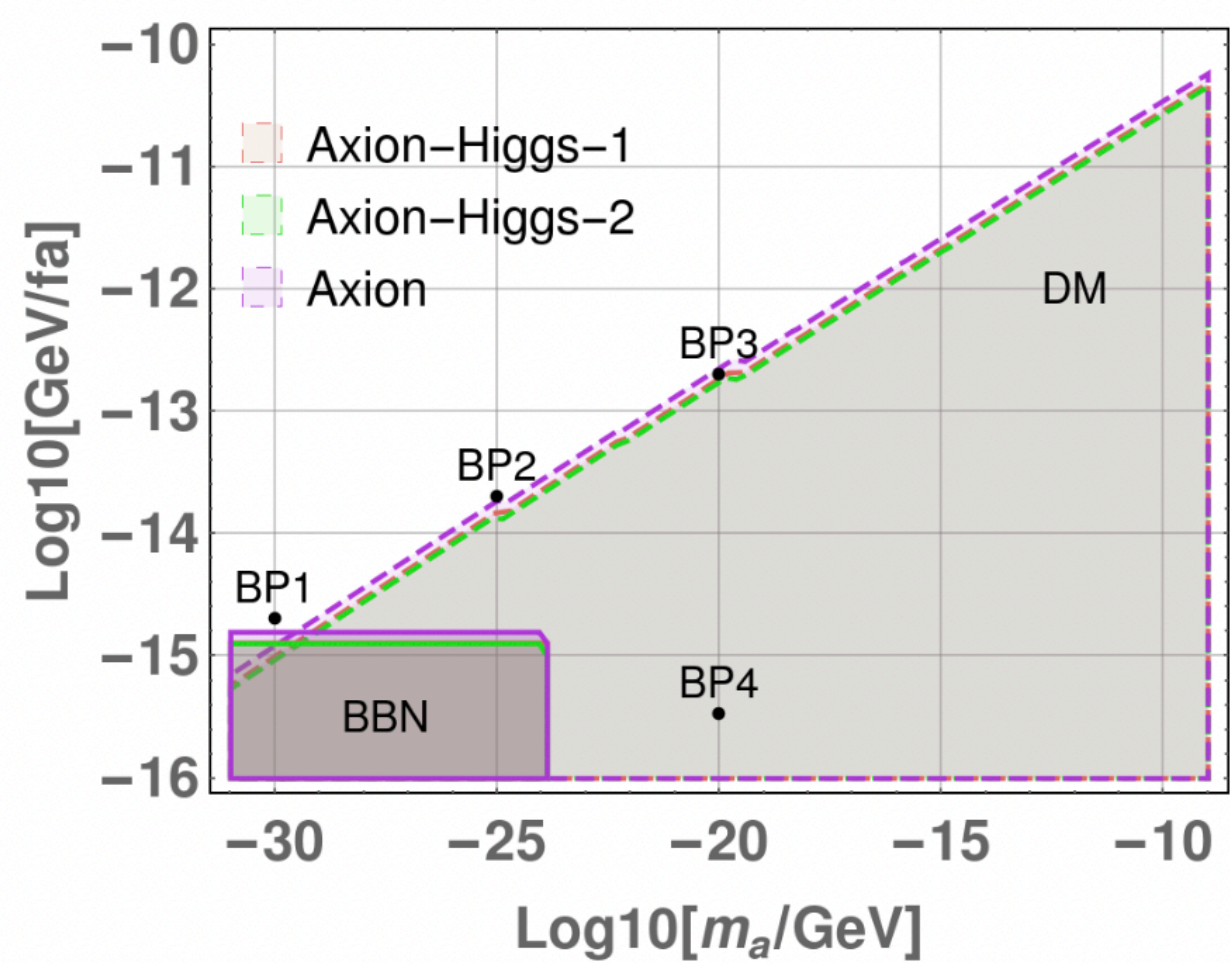
the same as: arXiv:2007.04990, 1806.04677

Energy density



$$\begin{aligned} \frac{d\rho_{str}(t)}{dt} &= -2H(t)\rho_{str}(t) - \left[\frac{d\rho_{str}(t)}{dt} \right]_{emi}, \\ \frac{d\rho_a(t)}{dt} &= -4H(t)\rho_a(t) + \left[\frac{d\rho_{str}(t)}{dt} \right]_{emi1}, \\ \frac{d\rho_g(t)}{dt} &= -4H(t)\rho_g(t) + \left[\frac{d\rho_{str}(t)}{dt} \right]_{emi2}. \end{aligned}$$

► Observations



DM relic abundance

$$\Omega_{DM} h^2 \approx 1.6 \times 10^9 f_a^2 m_a^2 \xi (-2.25 + \log \frac{f_a}{m_a \xi^{1/2}}) \times \epsilon^{-1} (m_a^2 m_{pl}^2 g_*^{1/3})^{-\frac{3}{4}} .$$

Dark Radiation

$$\Delta N_{eff} = \xi \left(\frac{f_a}{10^{15} \text{GeV}} \right)^2 \left(-1.21 \times 10^{-3} + 1.67 \times 10^{-5} \log^2 \left(\frac{1.12 \times 10^{39} \frac{f_a}{10^{15} \text{GeV}}}{\xi^{1/2}} \right) \right)$$

$$(d(a^4 \rho_g)/dt)/(d(a^4 \rho_a)/dt) = \gamma G \mu^2 / f_a^2$$

Method 1

$$F_g(x, y) = \frac{H}{\Gamma_g} \frac{1}{a^3} \frac{\partial}{\partial t} \left(a^3 \frac{\rho_g}{\partial k} \right)$$

$$\Gamma_g \simeq 8\pi \xi r G f_a^4 H^3 \log^3 m_r / H$$

Method 2

► Cosmic string simulation: the local string

- The action and potential are

$$\mathcal{L} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(\Phi, T)$$

EoMs

$$\begin{aligned} \Phi'' + 2\mathcal{H}\Phi' - D_i D_i \Phi + a^2 \frac{\partial V}{\partial \Phi^\dagger} &= 0, \\ E'_i + \partial_j F_{ij} - ea^2 \text{Im}(\Phi^\dagger D_i \Phi) &= 0, \\ \partial_i E_i - ea^2 \text{Im}(\Phi^\dagger \Phi') &= 0, \end{aligned}$$

$$V(\Phi, T) = \frac{\lambda}{4} (|\Phi|^2 - v^2)^2 + \frac{\lambda}{6} T^2 |\Phi|^2,$$

Initial conditions

$$\begin{aligned} \langle \phi(\mathbf{k}) \phi(\mathbf{k}') \rangle &= (2\pi)^3 P_\phi(|\mathbf{k}|) \delta^3(\mathbf{k} - \mathbf{k}'), \\ \langle \pi(\mathbf{k}) \pi(\mathbf{k}') \rangle &= (2\pi)^3 P_\pi(|\mathbf{k}|) \delta^3(\mathbf{k} - \mathbf{k}') \end{aligned}$$

$$P_\phi(k) = \frac{1}{w_k} \frac{1}{e^{w_k/T_0} - 1}, \quad P_\pi(k) = \frac{w_k}{e^{w_k/T_0} - 1}$$

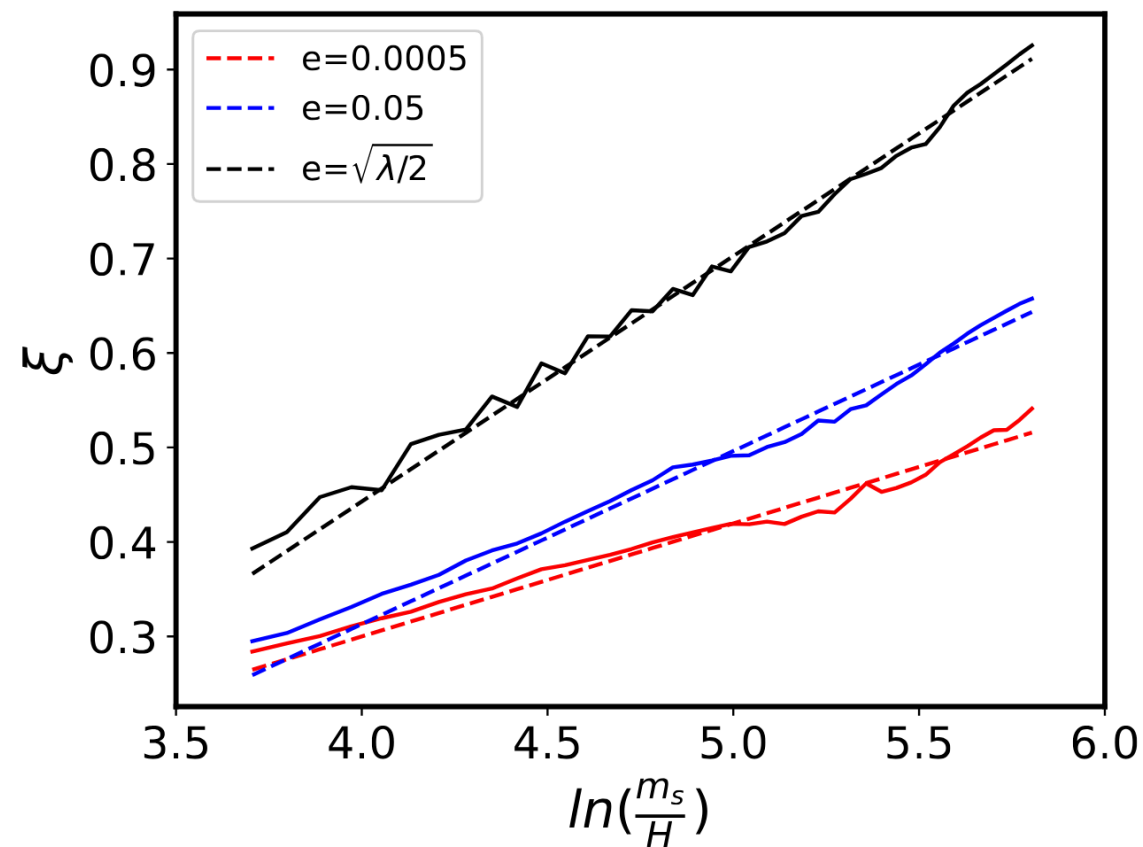
- With NG approximation, GW spectrum is calculated using number density distribution of loops $n(t, l)$

$$\Omega_{\text{gw}}^j(t, f_0) = \frac{j^{-q}}{\epsilon} \frac{8\pi}{3} \left(\frac{G\mu}{H(t)} \right) \frac{2j}{f_0 a_0} \Gamma \int_{a_{\min}}^{a_{\max}} \left(\frac{a(t)}{a} \right) \frac{da}{H(a)} n(t(a), l_j(a)).$$

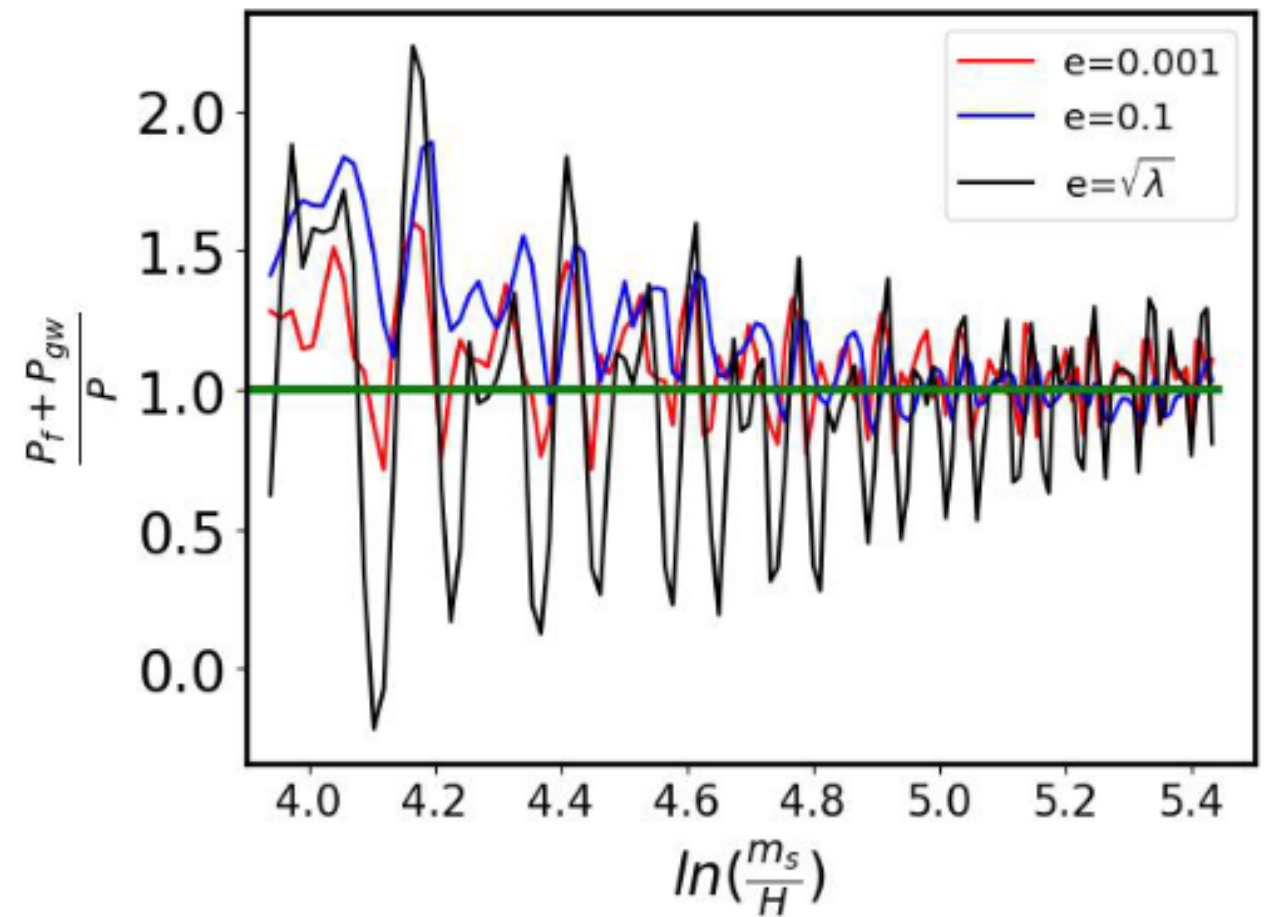
String tension

$$\mu_g = \pi v^2 \ln \left(\frac{m_s}{H} \frac{\gamma}{\sqrt{\xi}} \right), \quad \mu_l = \pi v^2 \ln \left(\frac{m_s}{m_v} \gamma \right)$$

► Cosmic string simulation: the local string



$$\xi = \frac{l_s t^2}{V a^2} \propto \ln(m_s/H)$$



Energy emission power of string network

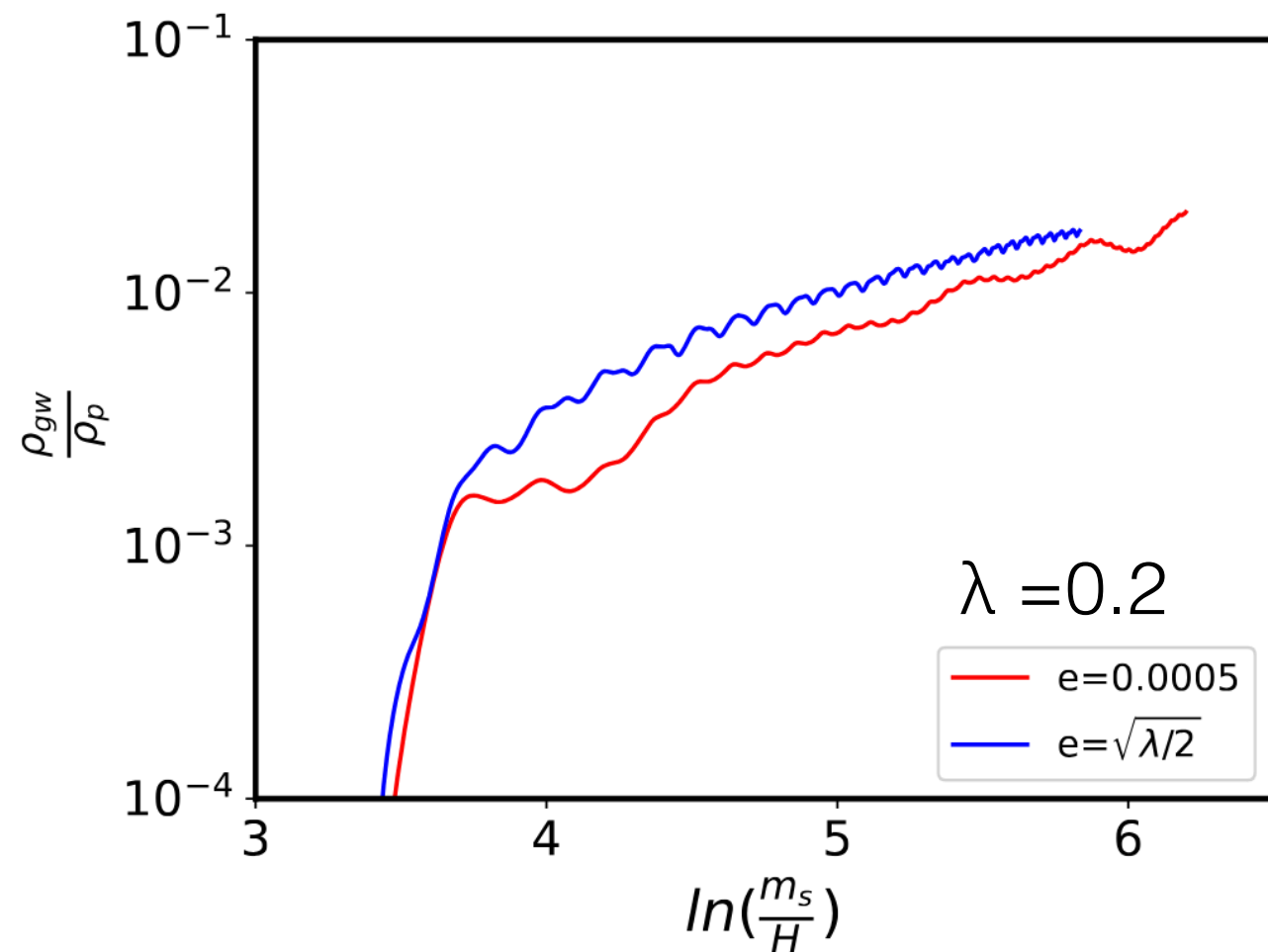
$$P(t_0) = [\dot{\rho}^{\text{free}}(t) - \dot{\rho}(t)]_{t=t_0} = \rho(t_0) \left[2H - \frac{\dot{\xi}}{\xi} - \frac{\mu_0}{\mu} \left(H + \frac{\dot{\gamma}}{\gamma} - \frac{\dot{\xi}}{2\xi} \right) \right]$$

$$\xrightarrow{\ln(m_r/H) \gg 1} 8H^3(t)\xi(t)\mu(t), \quad \frac{P_f + P_{gw}}{P} \sim 1$$

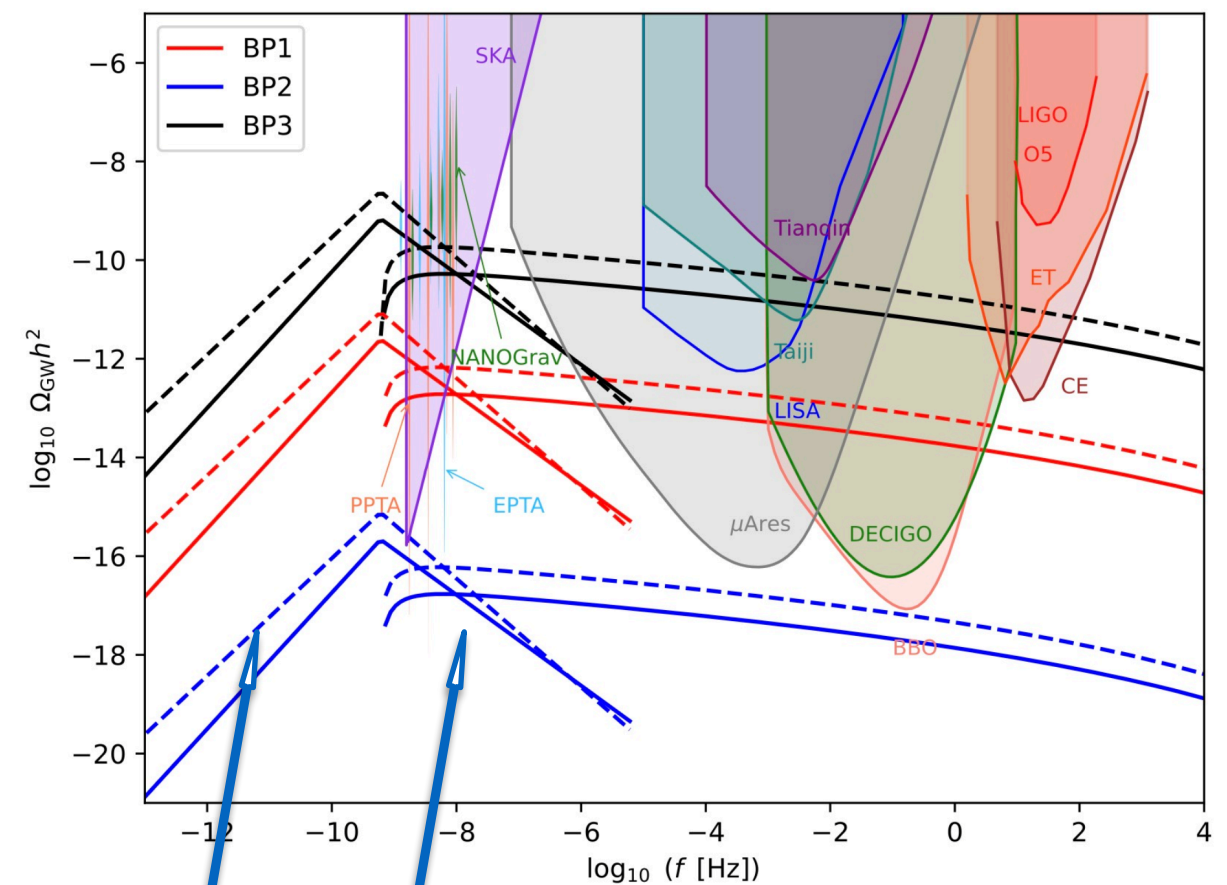
$$\sqrt{4\pi\gamma} = 2.0, \quad 1.3, \quad 4.4 \quad \text{for } e=0.001, \quad 0.1, \quad \sqrt{\lambda}$$

► Cosmic string simulation: the local string

GW to particle
energy density ratio



GW for different scenarios
At $H_{\text{dec}} \sim \text{MeV}$



$e = 0.05$ $e = 0.0005$

❖ Summary

- **Updated a semi-realistic DM emission from two-stage simulation of axion string-wall network**
- **GW of ALP particles in the DFSZ case can be probed by GW detectors**
- **GW of Axion/ALP dark matter scenario in KSVZ cannot be probed**

❖ Future

- **Dark matter from PT**
- **Topological defects: Magnetic monopoles, cosmic strings, domain walls**
- **Magnetic fields on Axion**

Thanks

谢谢！

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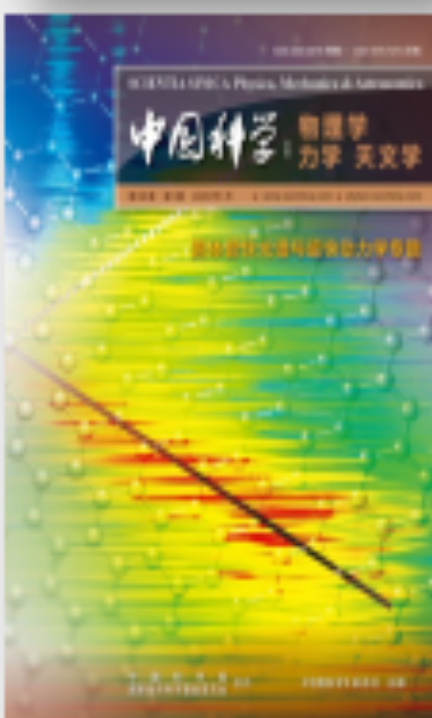
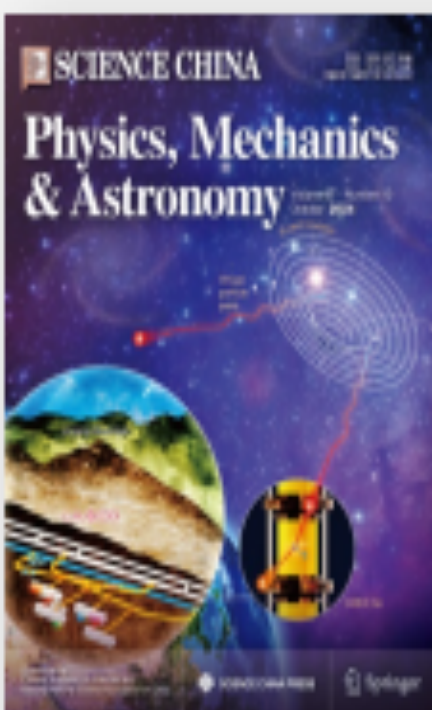
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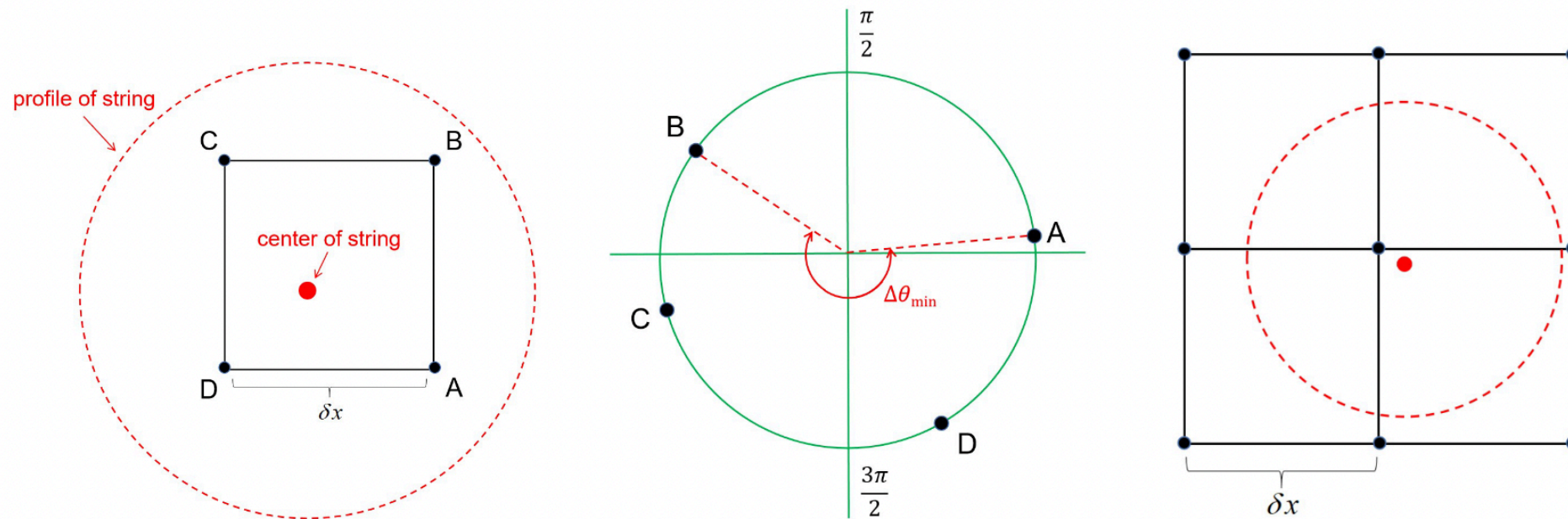


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► Identification of Cosmic string

String penetrates the square loop if the minimum phase range which contains the four points is greater than π and the phase changes continuously



For a specific square loop, assuming that the minimum phase at four points is $\theta_{\min}$

- (1) $\theta_{\min} < \pi$.
- (2) There exists at least one phase at another point minus $\theta_{\min}$ is greater than π .
- (3) There exists at least one phase at another point minus $\theta_{\min}$ is smaller than π .
- (4) Denote the phase closest to π in all phases greater than π as θ_a , and denote the phase closest to π in all phases smaller than π as θ_b , it is required to meet $\theta_a - \theta_b < \pi$.
- (5) Calculate the difference between the phases at each of two adjacent points in a counterclockwise direction, the multiplication of the four differences is required to be negative.