國科大杭州高等研究院
Hangzhou Institute for Advanced Study, UCAS

Workshop on Gravitational Waves, Cosmology and Fundamental Physics

PBHs from bubble nucleation in double-field inflation

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2025. 11. 1

Based on: H. Wang, YZ, T. Suyama, 2510.19233

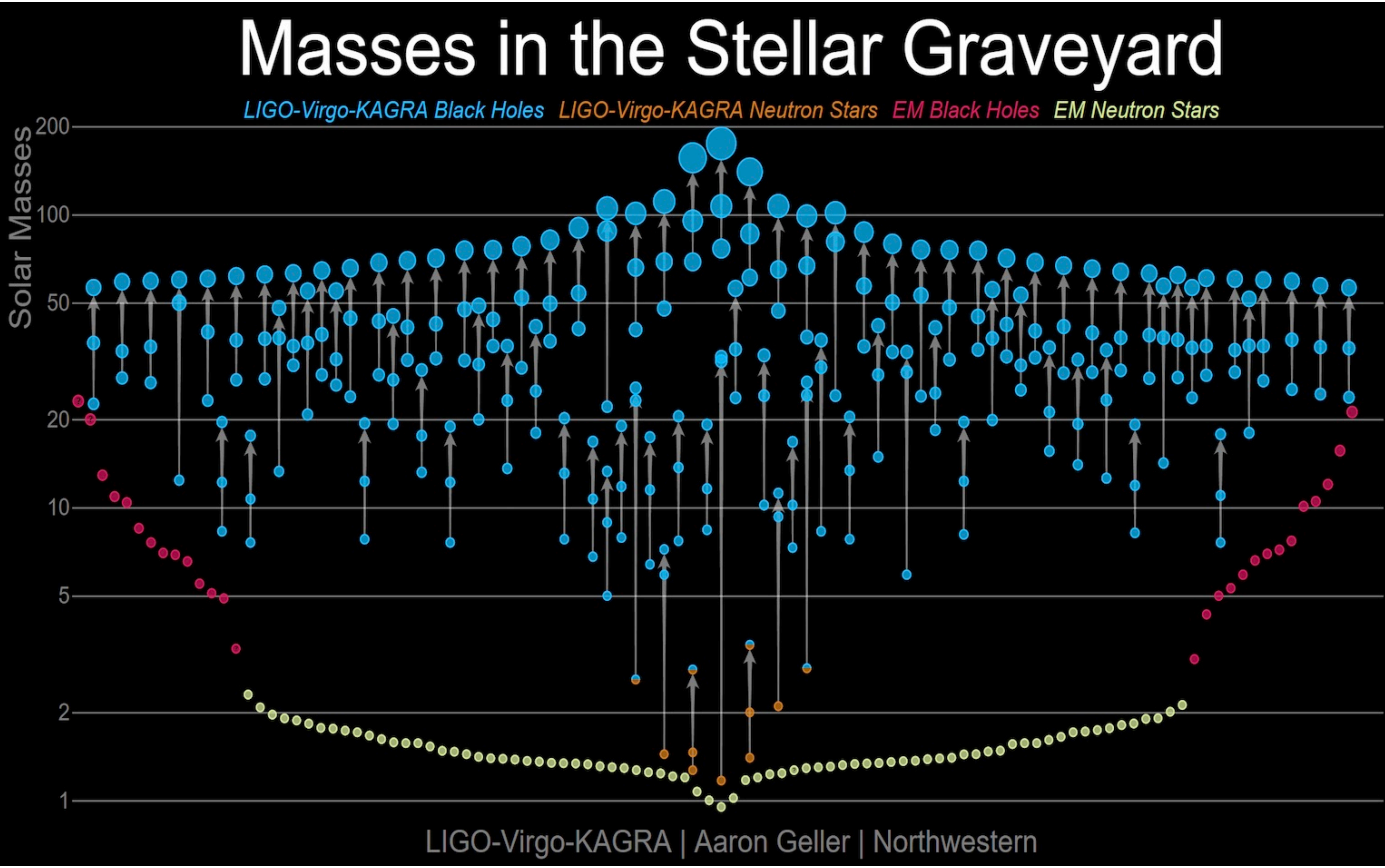
H. Wang, YZ, T. Suyama, 2511.xxxxx

Why Primordial Black Hole (PBH)?

- BHs exist in the universe
- No need for new physics
- PBHs may dominate Dark Matter
- Detected GW events from LIGO may originate from the merger of PBH binaries

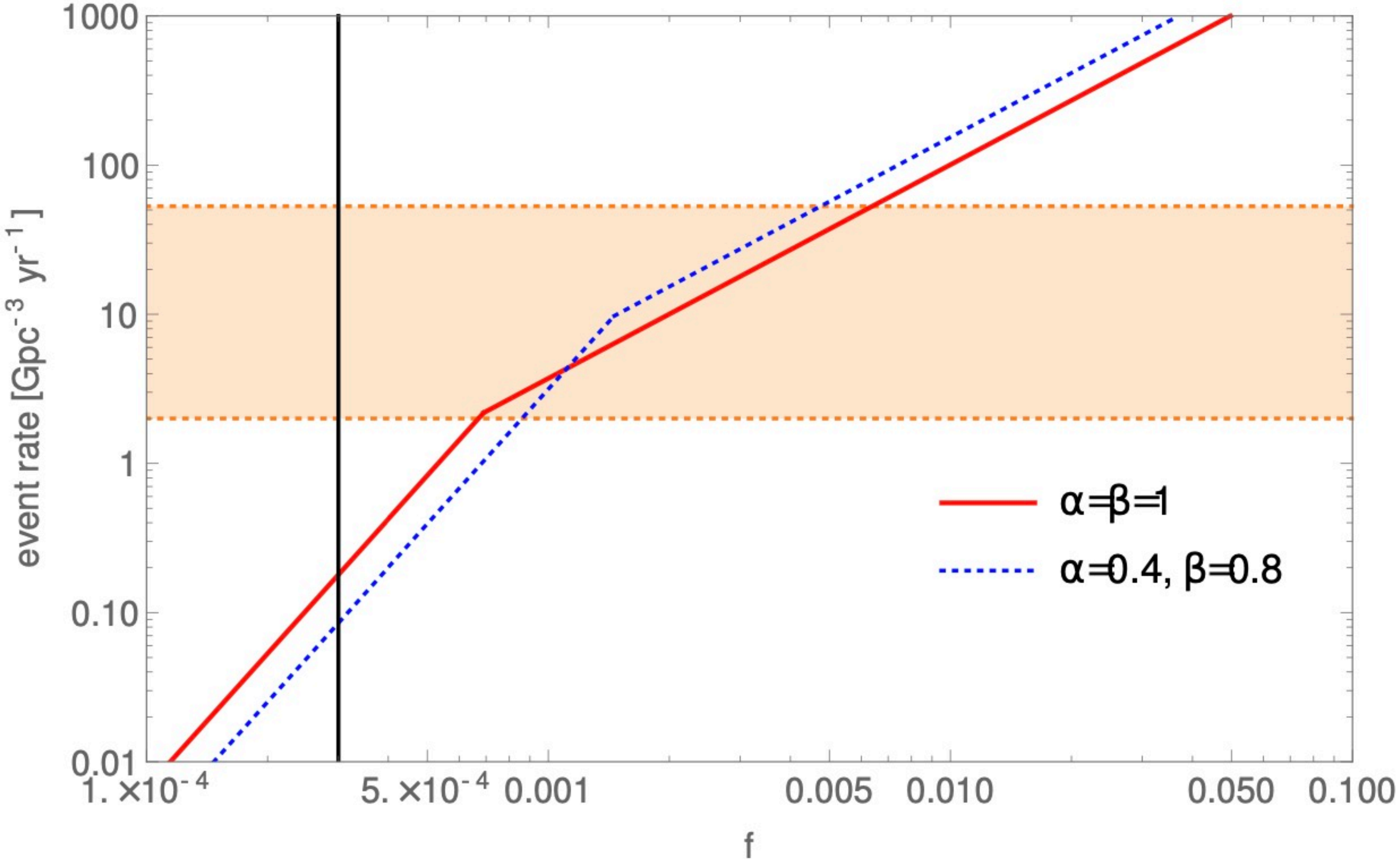
M. Sasaki, T. Suyama, T. Tanaka, S. Yokoyama, PRL 117, no. 6, 061101 (2016)

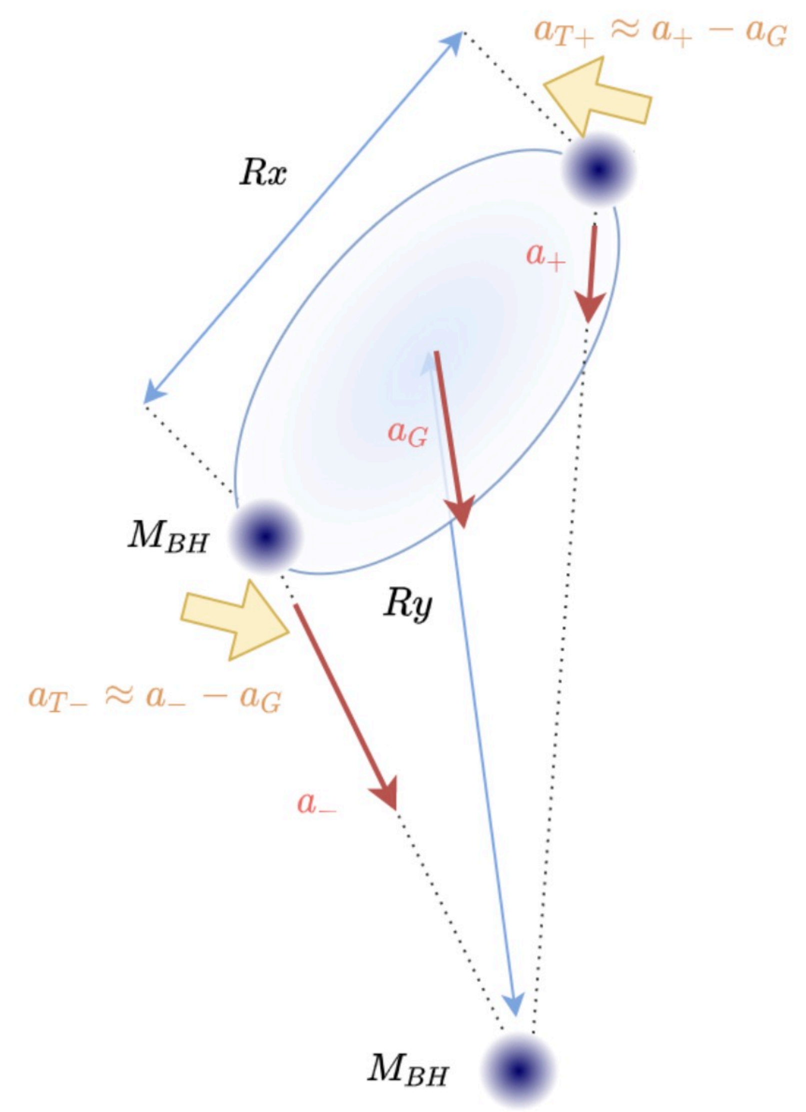
- A possible way to probe the primordial power spectrum of curvature perturbation on small scales
- …



https://media.ligo.northwestern.edu

Massive PBHs for explanation of LIGO events

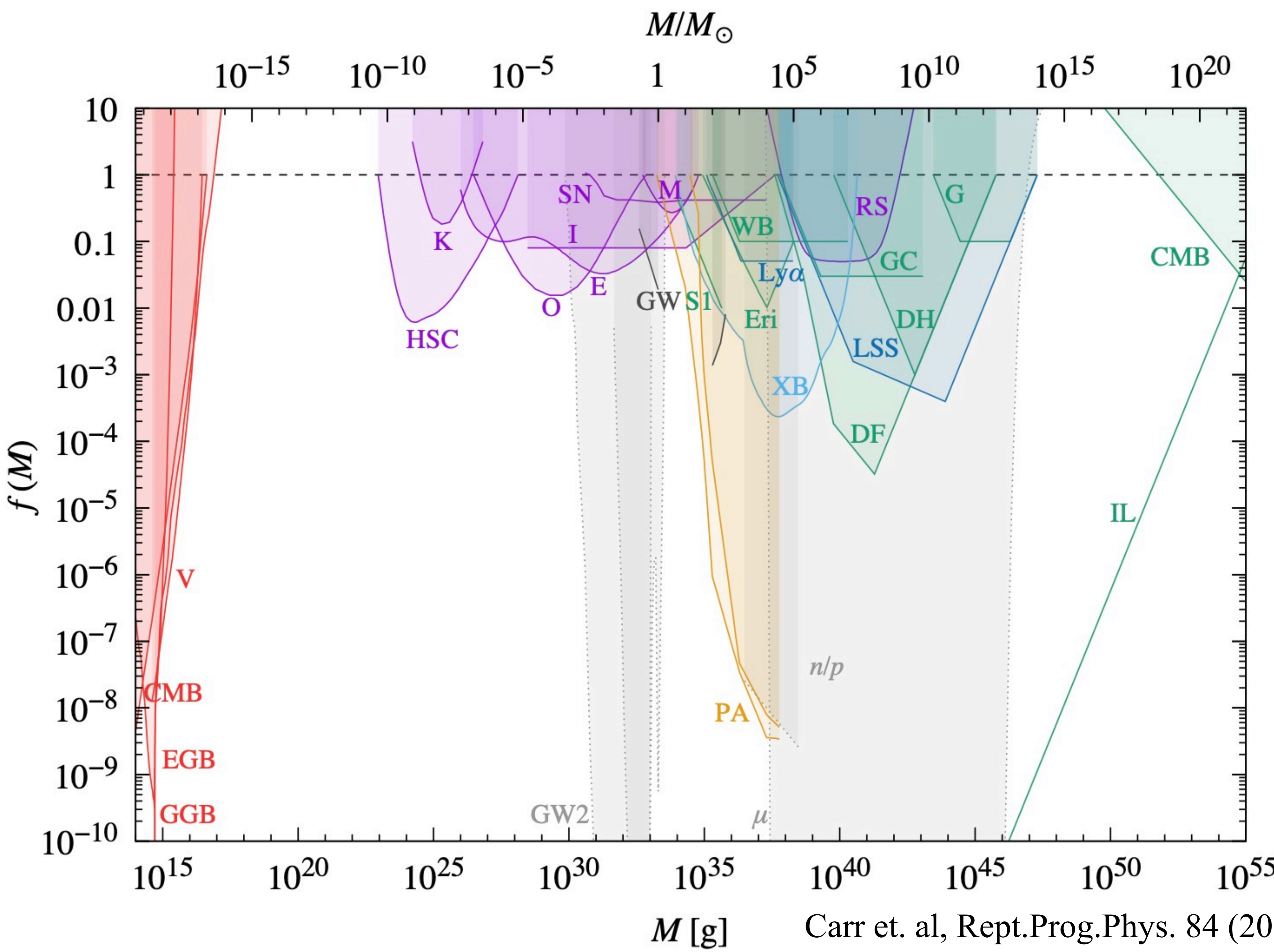




Sasaki, Suyama, Tanaka, Yokoyama, PRL 117 (2016) 6, 061101

Illustrated by Xinpeng

Moreover , small PBHs are candidate for DM



Carr et. al, Rept.Prog.Phys. 84 (2021) 11, 116902

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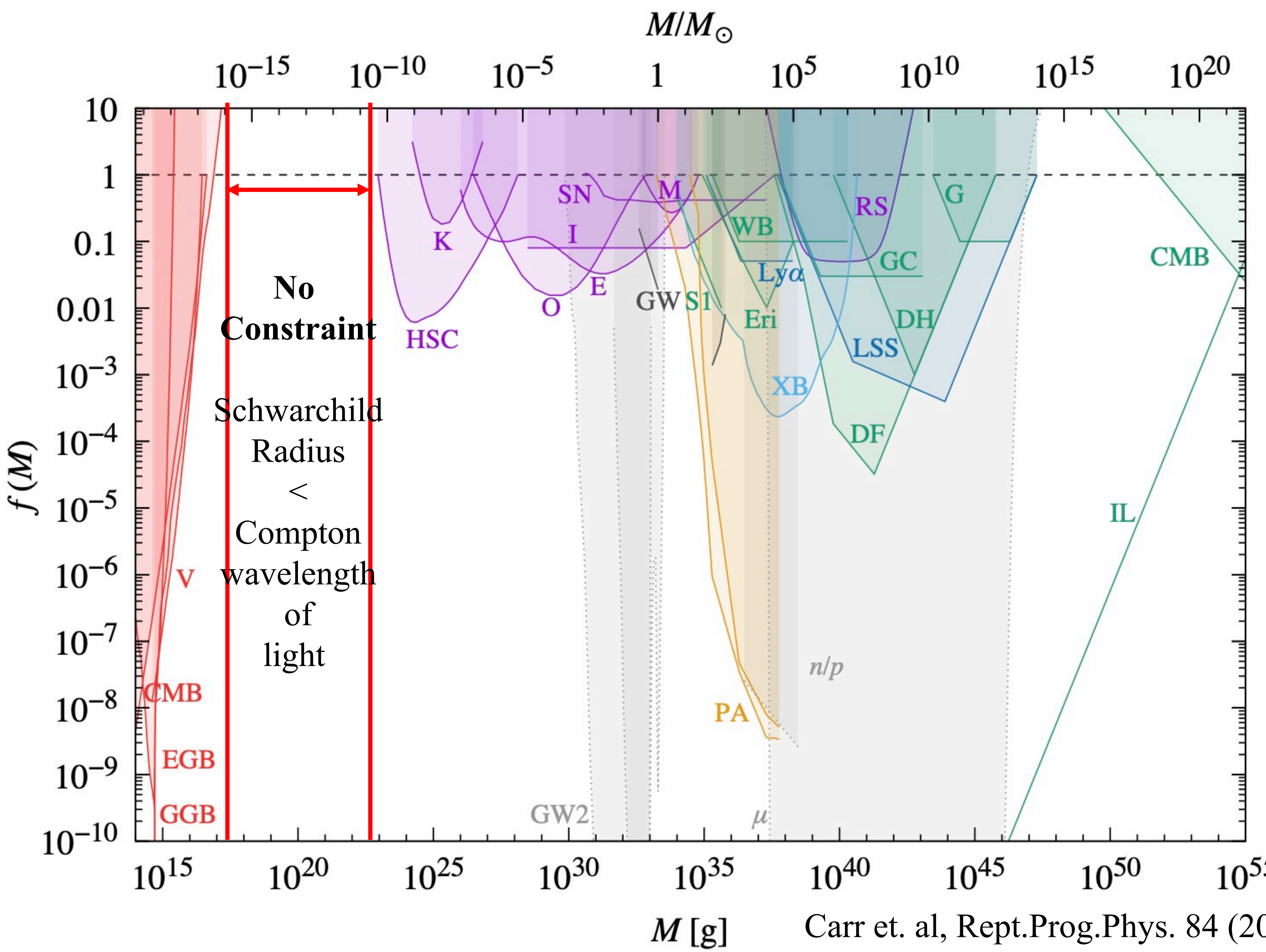
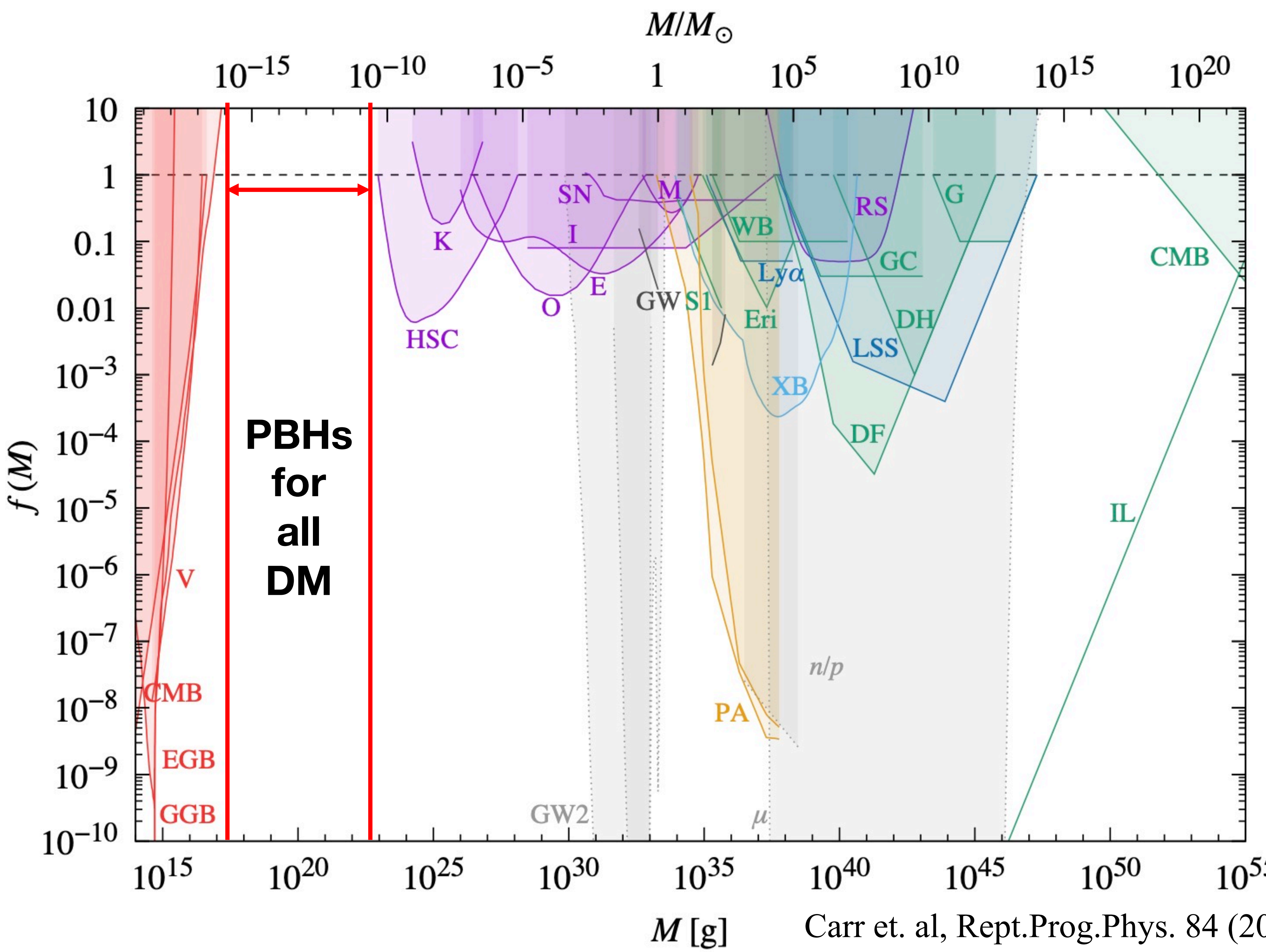


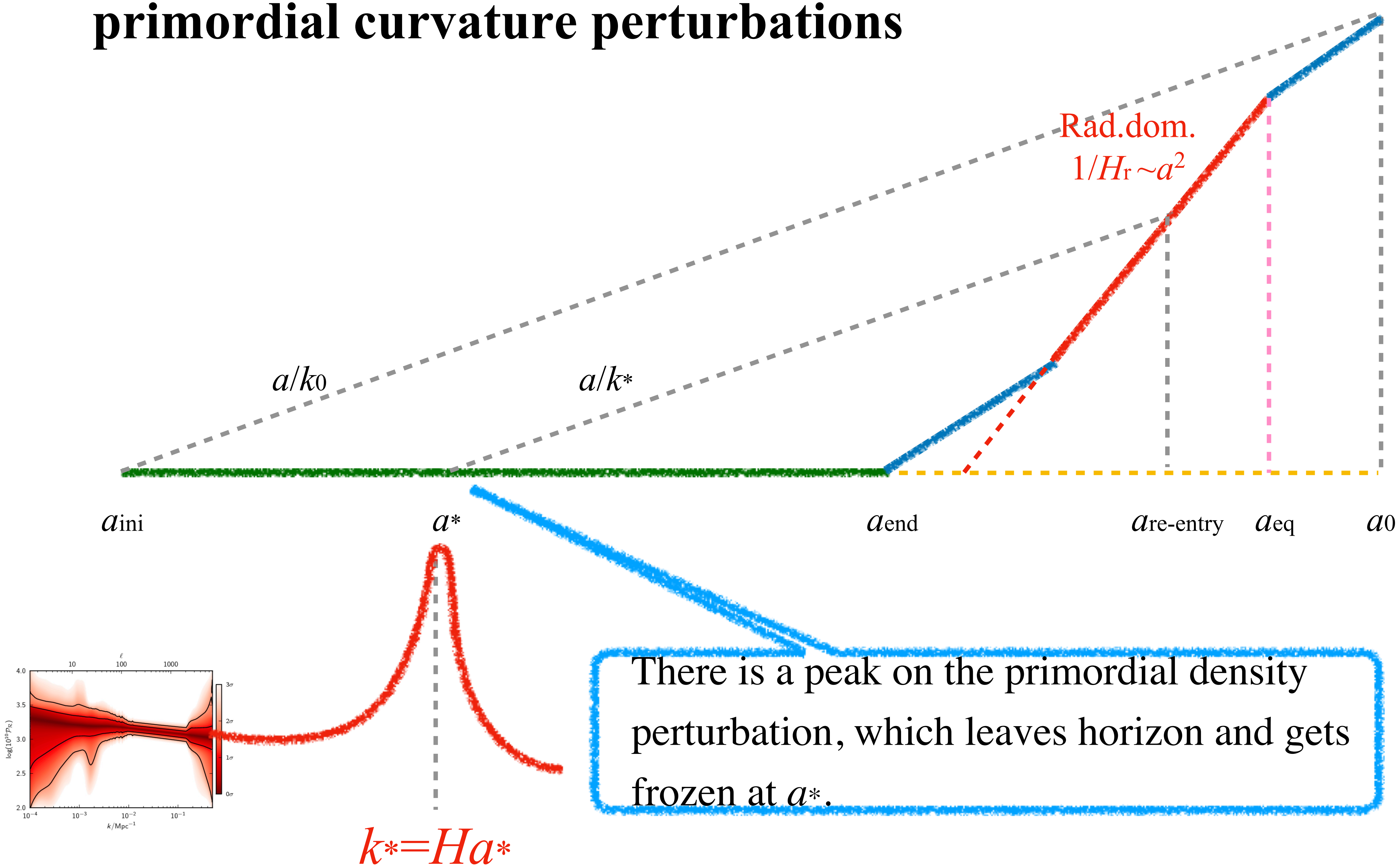
Carr et. al, Rept.Prog.Phys. 84 (2021) 11, 116902

Moreover , small PBHs are candidate for DM

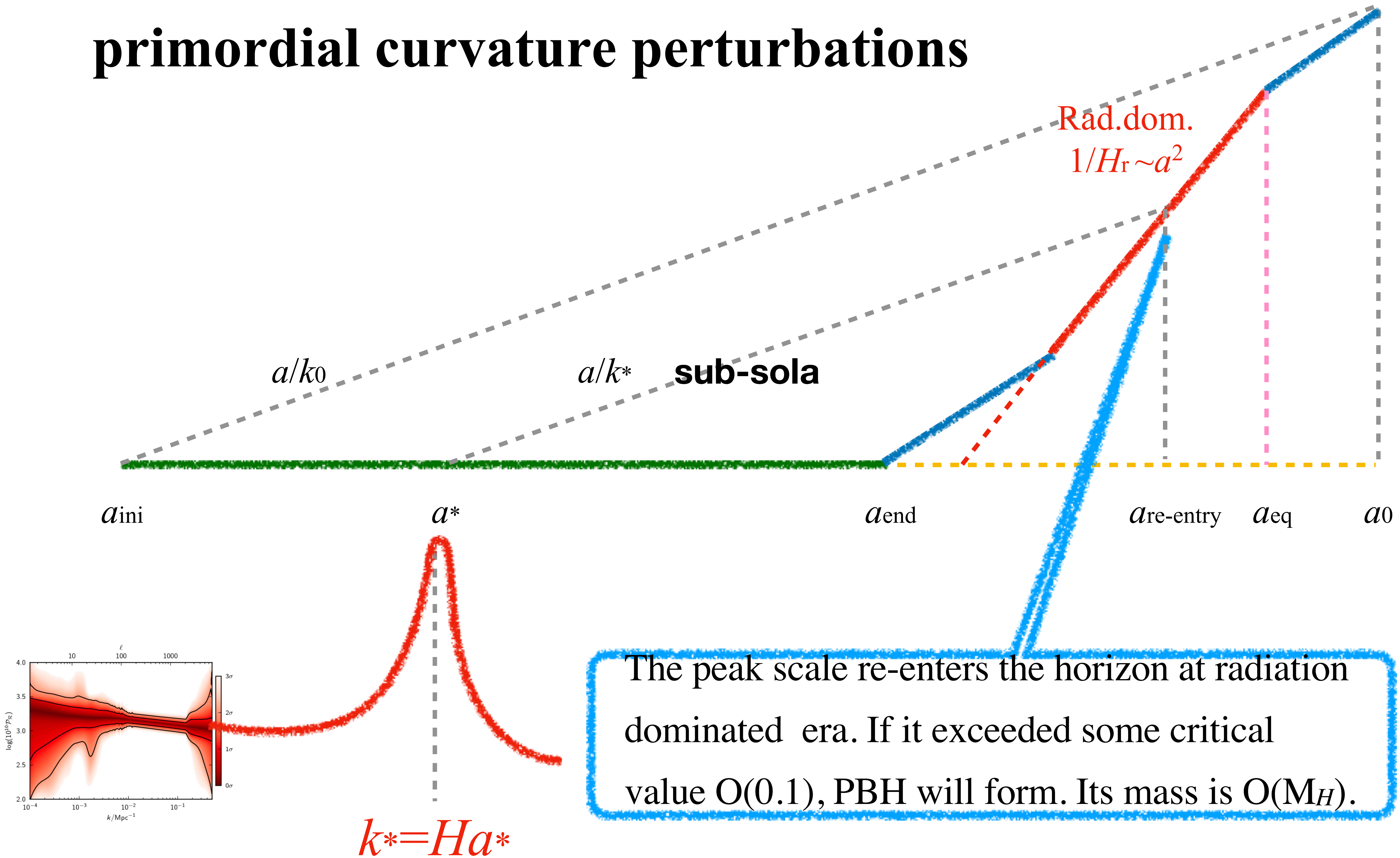


Carr et. al, Rept.Prog.Phys. 84 (2021) 11, 116902

Formation of PBHs from enhancement of primordial curvature perturbations



Formation of PBHs from enhancement of primordial curvature perturbations



The peak scale re-enters the horizon at radiation dominated era. If it exceeded some critical value O(0.1), PBH will form. Its mass is O(M_H).

Single Field Inflation:

Yokoyama 1998
Leach, Sasaki, Wands, Liddle 2001
Byrnes, Cole, Patil 2019
Liu, Guo, Cai 2020
Ozsoy, Tasinato 2020
Pi, Wang 2022
Cai, Ma, Sasaki, Wang, Zhou 2022……

Multi-field Inflation:

Garcia-Bellido, Linde, Wands 1996
Gordon, Wands, Bassett, Maartens 2000
Inomata, Kawasaki, Mukaida, Tada, Yanagida 2017
Pi, Zhang, Huang, Sasaki 2018
Anguelova 2020
Cai, Chen, Fu 2021
Meng, Yuan, Huang 2022……

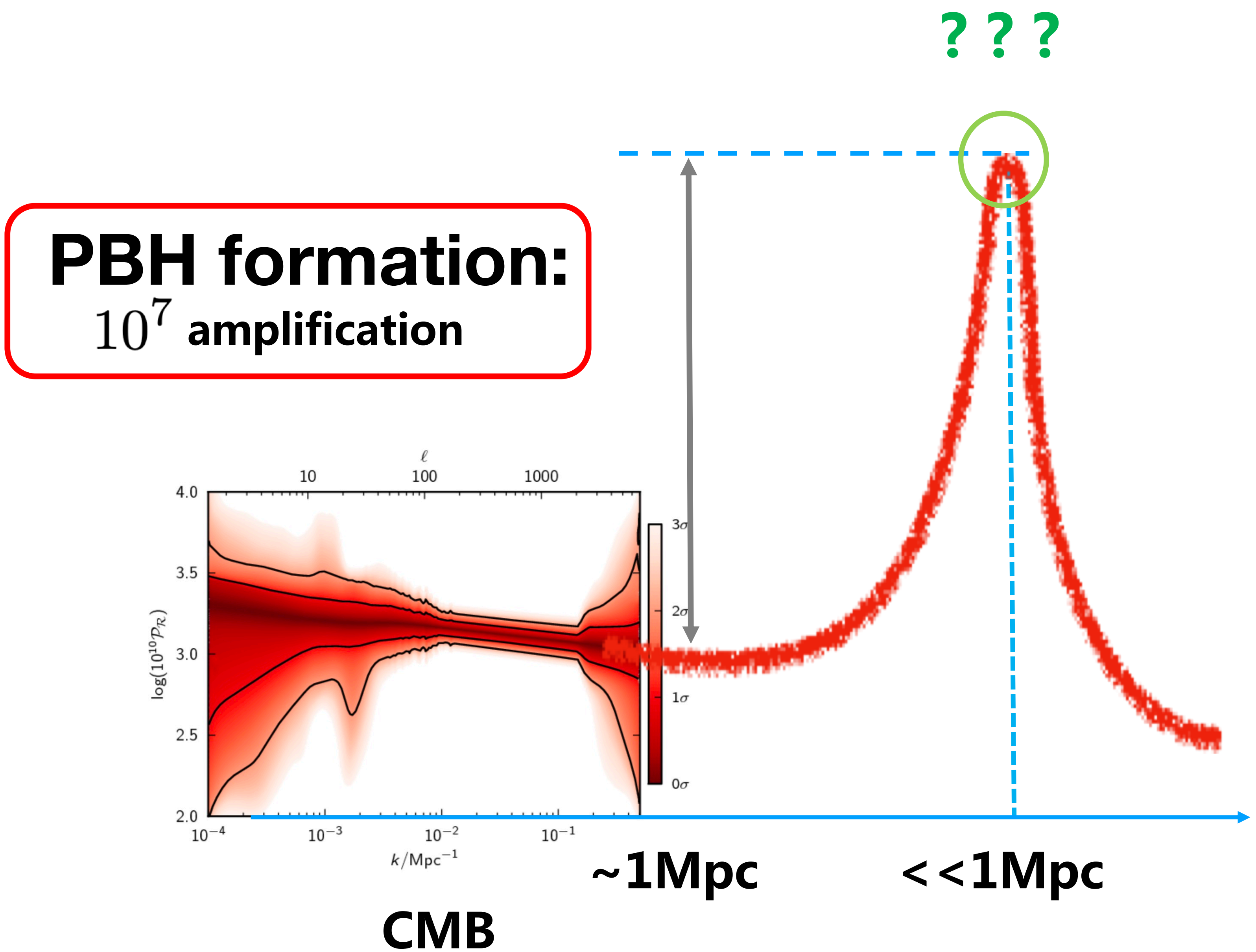
Supergravity Model:

Kawasaki, Sugiyama, Yanagida 1997
Yamaguchi 2001
Aldabergenov, Addazi, Ketov 2020……

Curvaton Model:

Kawasaki, Kitajima, Yanagida 2012
Kohri, Lin, Matsuda 2012
Ando, Inomata, Kawasaki, Mukaida 2017
Pi, Sasaki 2021……

Question： Any hints on such huge amplification?



Wang, YZ, Kimura, Yamaguchi, SCPMA 66 (2023) 6, 260462

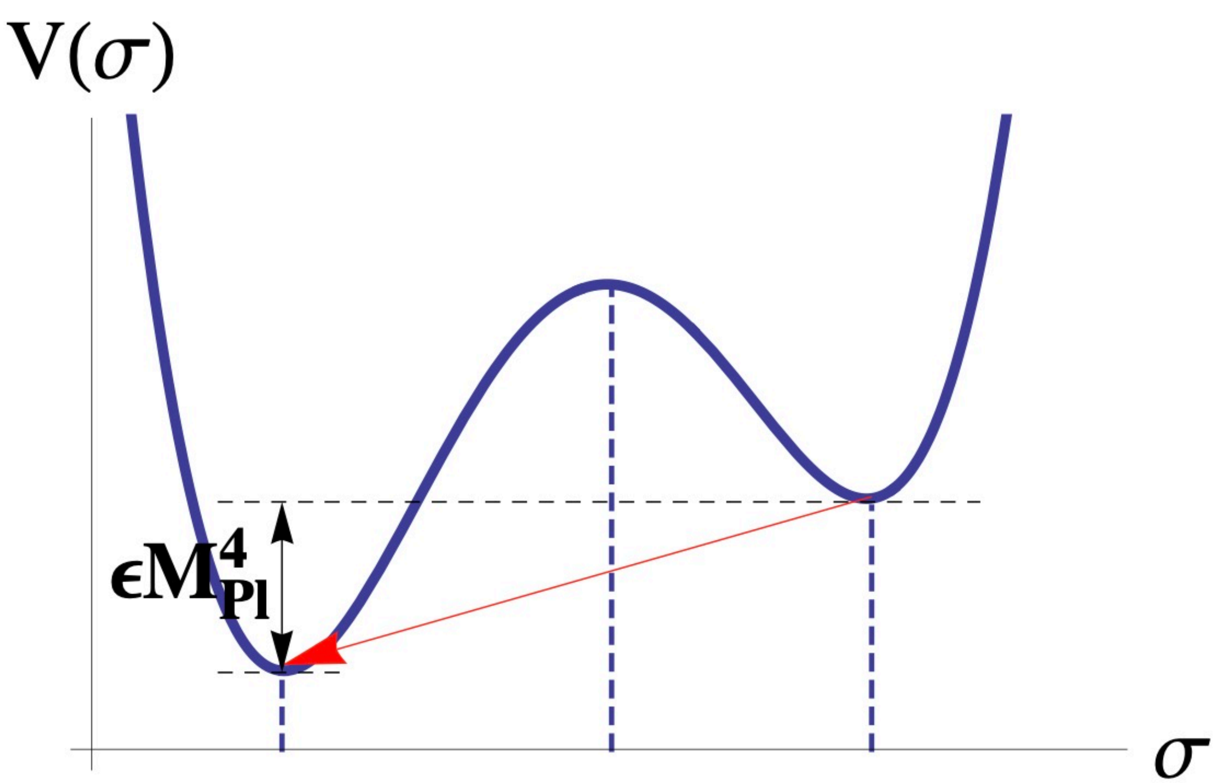
Question: If no IGW observed, can we rule out PBH scenario?

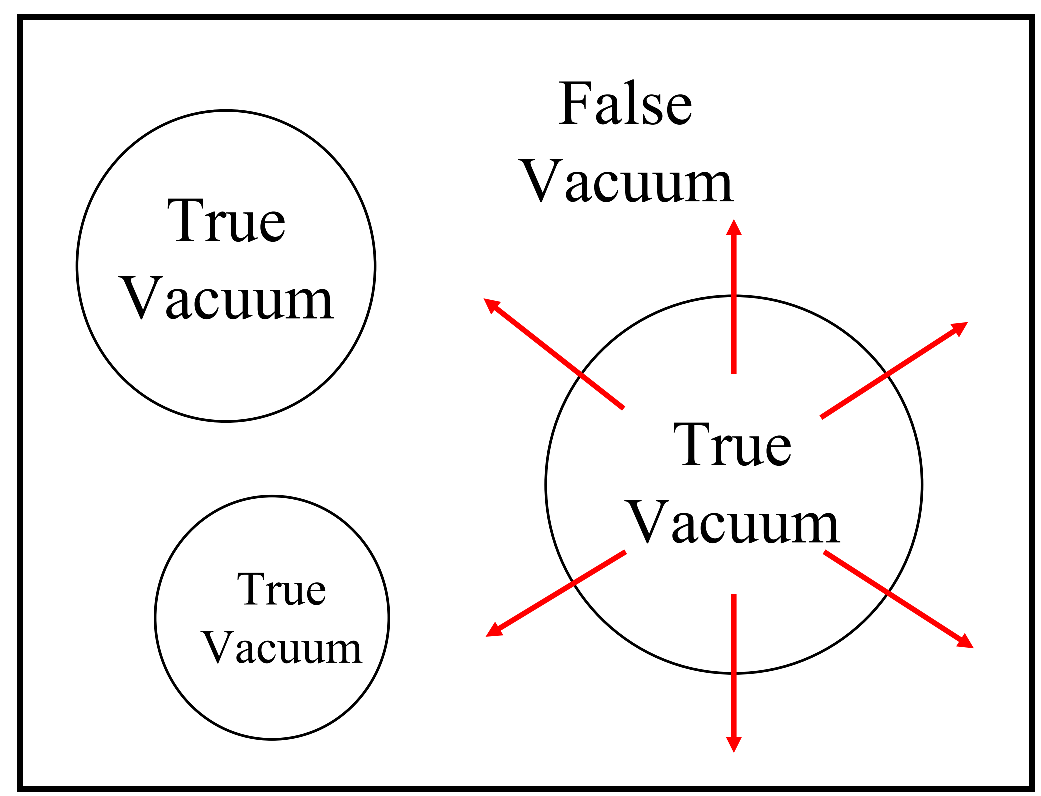
↕

Question: Is there any PBH scenario without amplification of primordial power spectrum?

Another mechanism: The Bubble Nucleation

Bubble will be nucleated via tunneling process in early universe

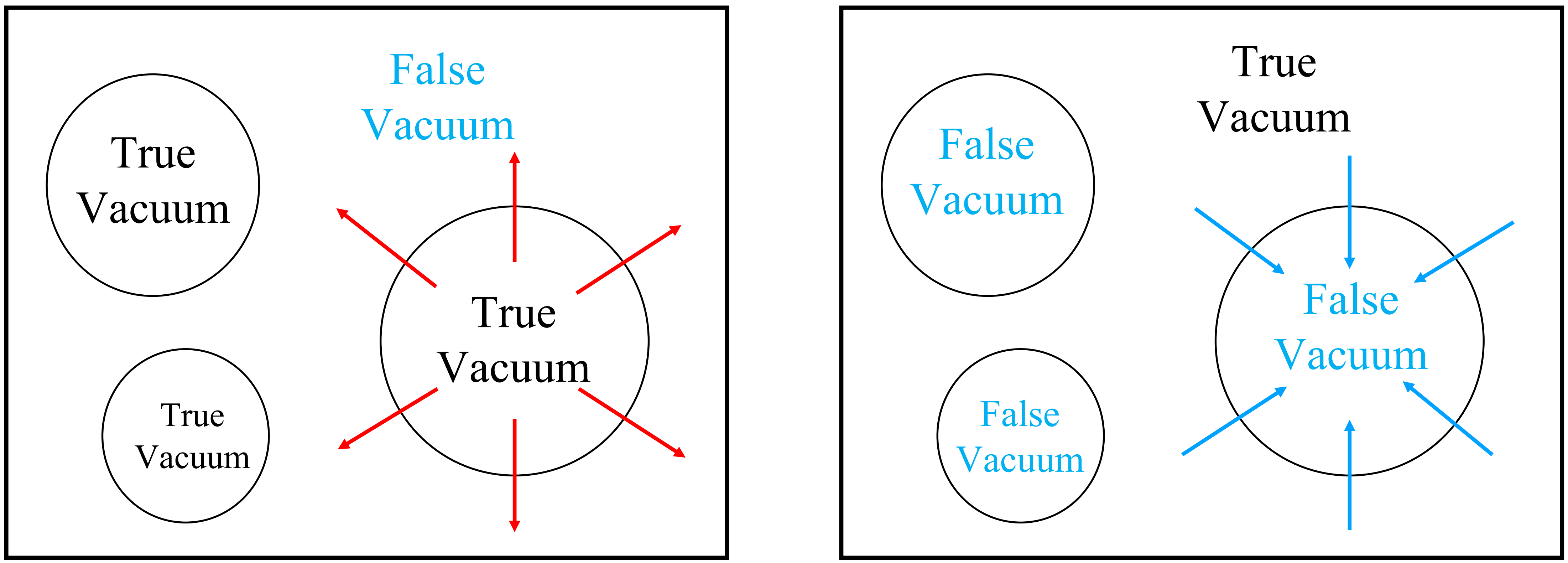




During Inflation

The Bubble Nucleation

Bubble will be nucleated via tunneling process in early universe



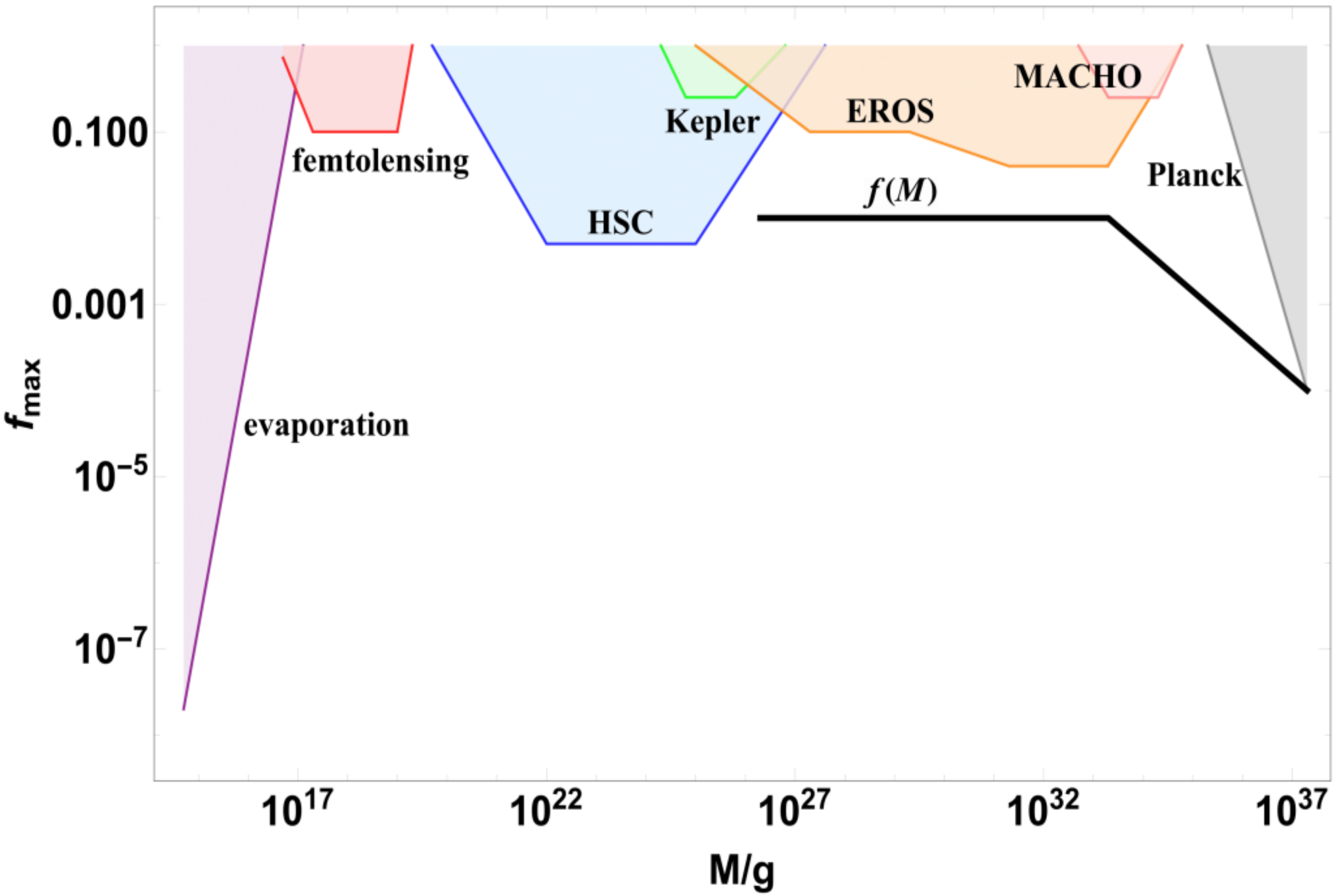
During Inflation

After Inflation

Question: How to realize this scenario?

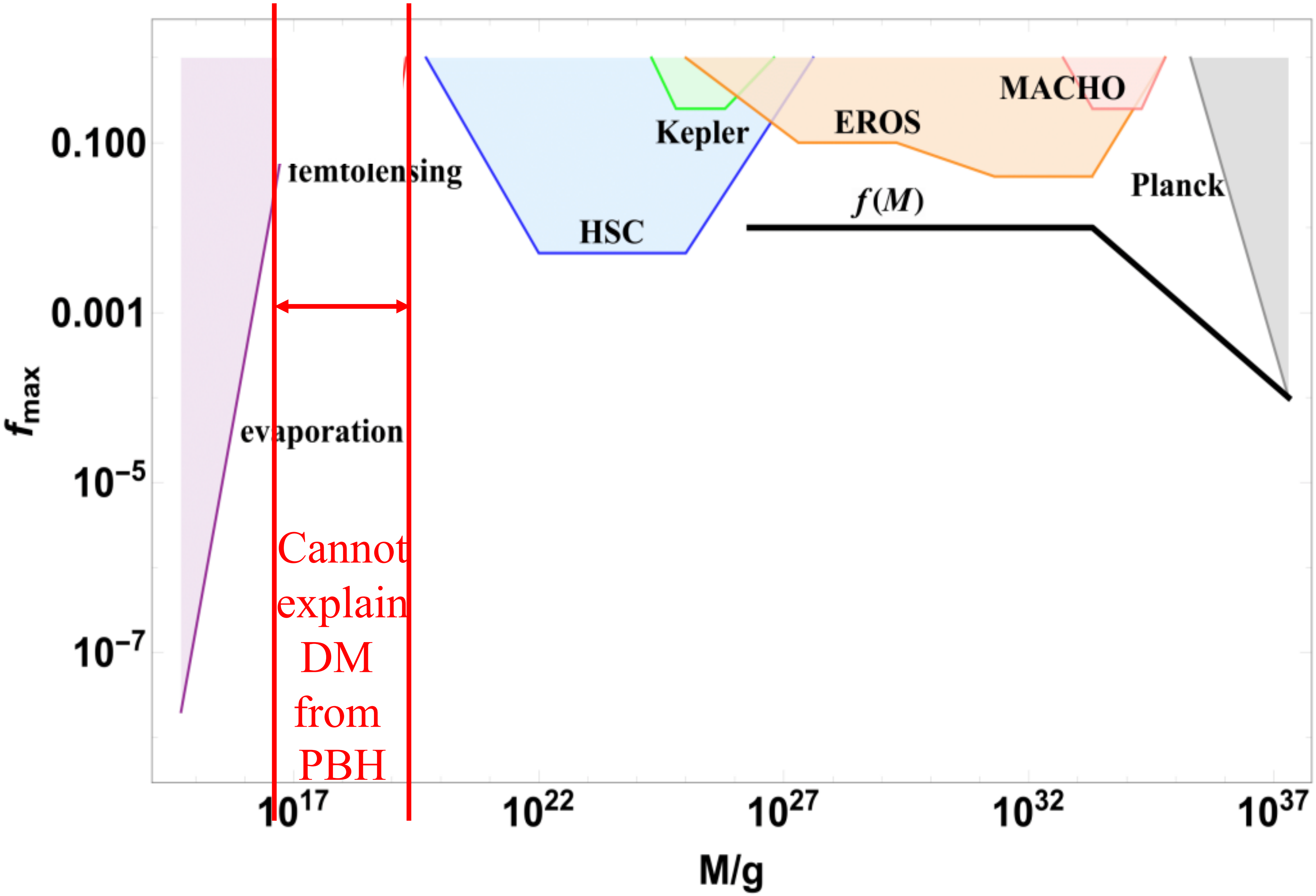
Case of a constant tunneling rate

H. Deng, A. Vilenkin, JCAP 2017(12): 044.



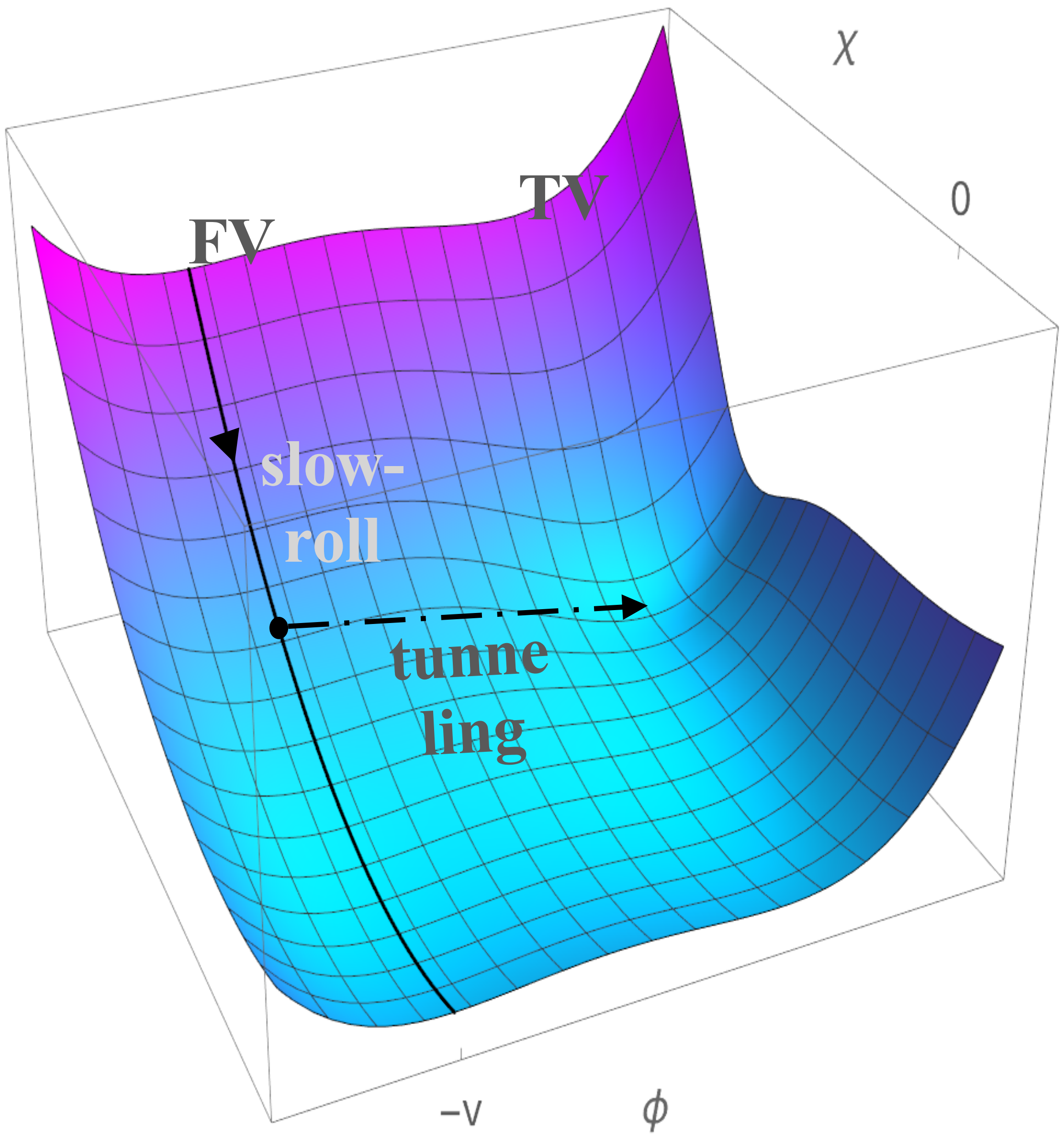
Case of a constant tunneling rate

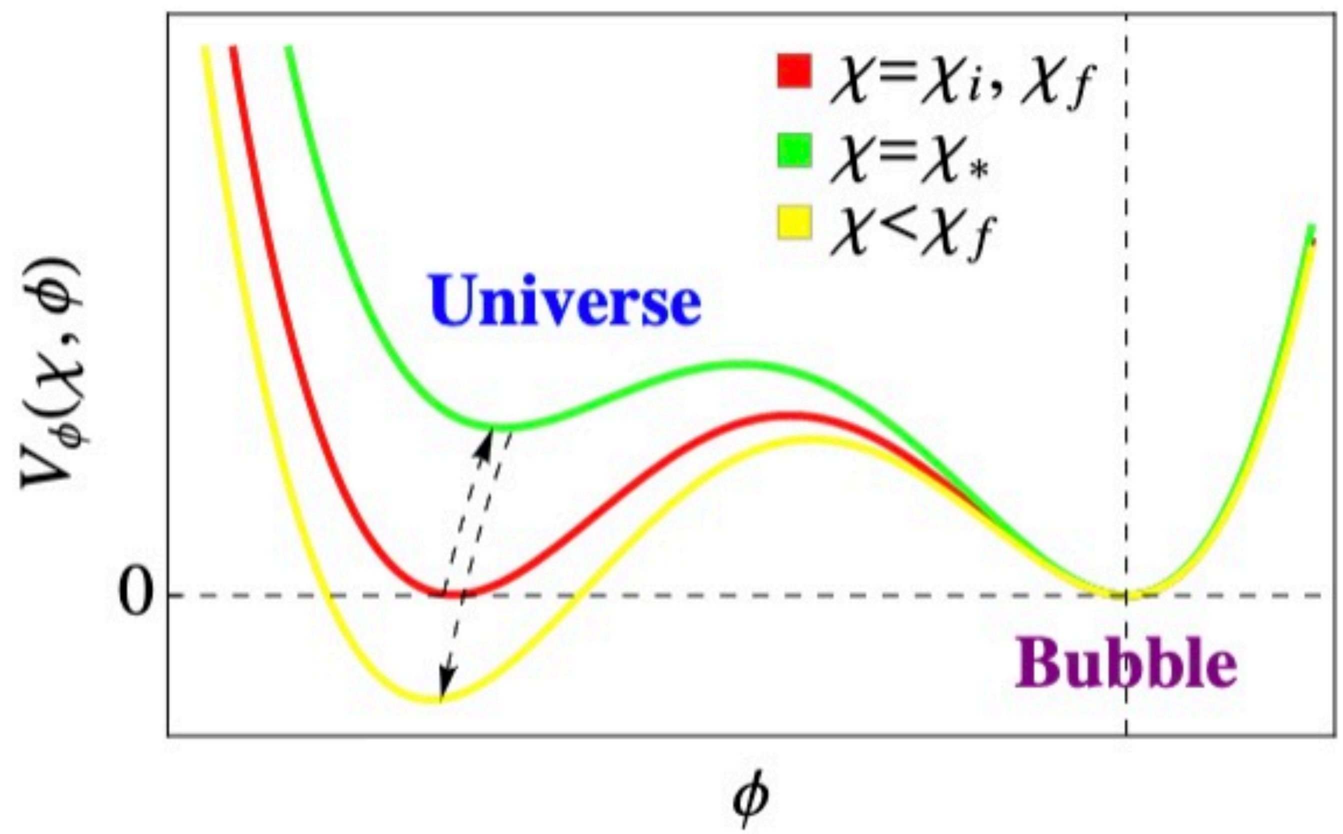
H. Deng, A. Vilenkin, JCAP 2017(12): 044.

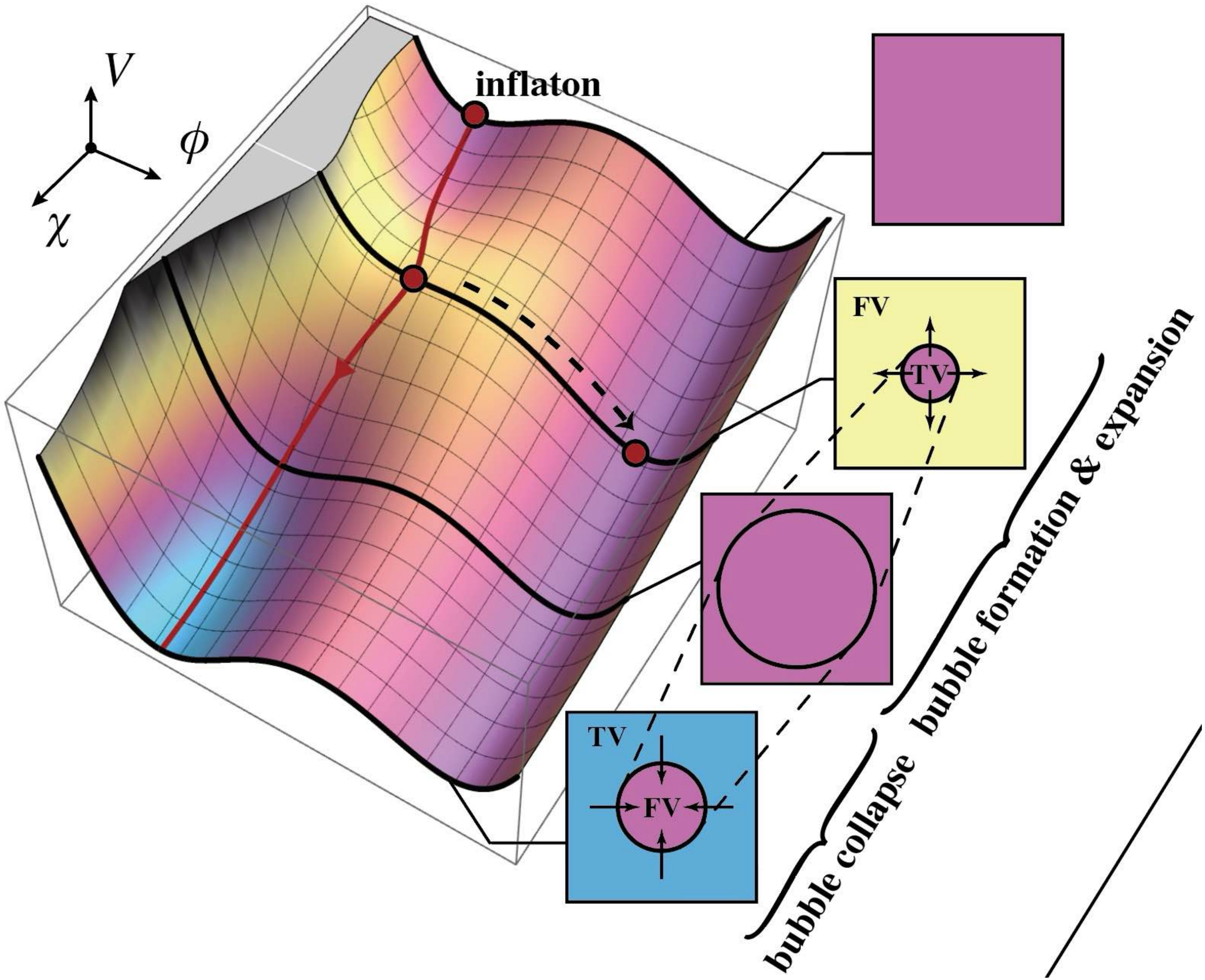


We need a varying tunneling rate to explain DM

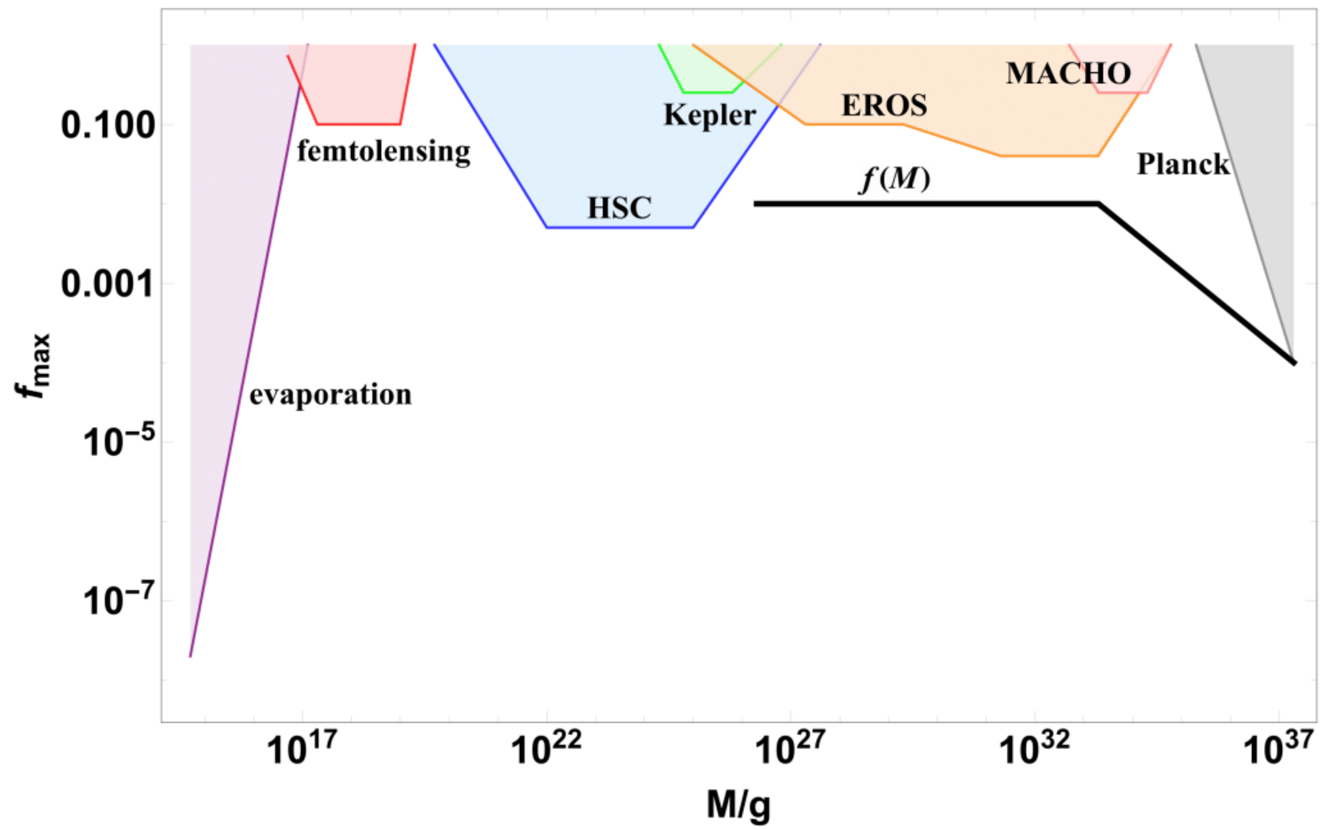
Running of local minima





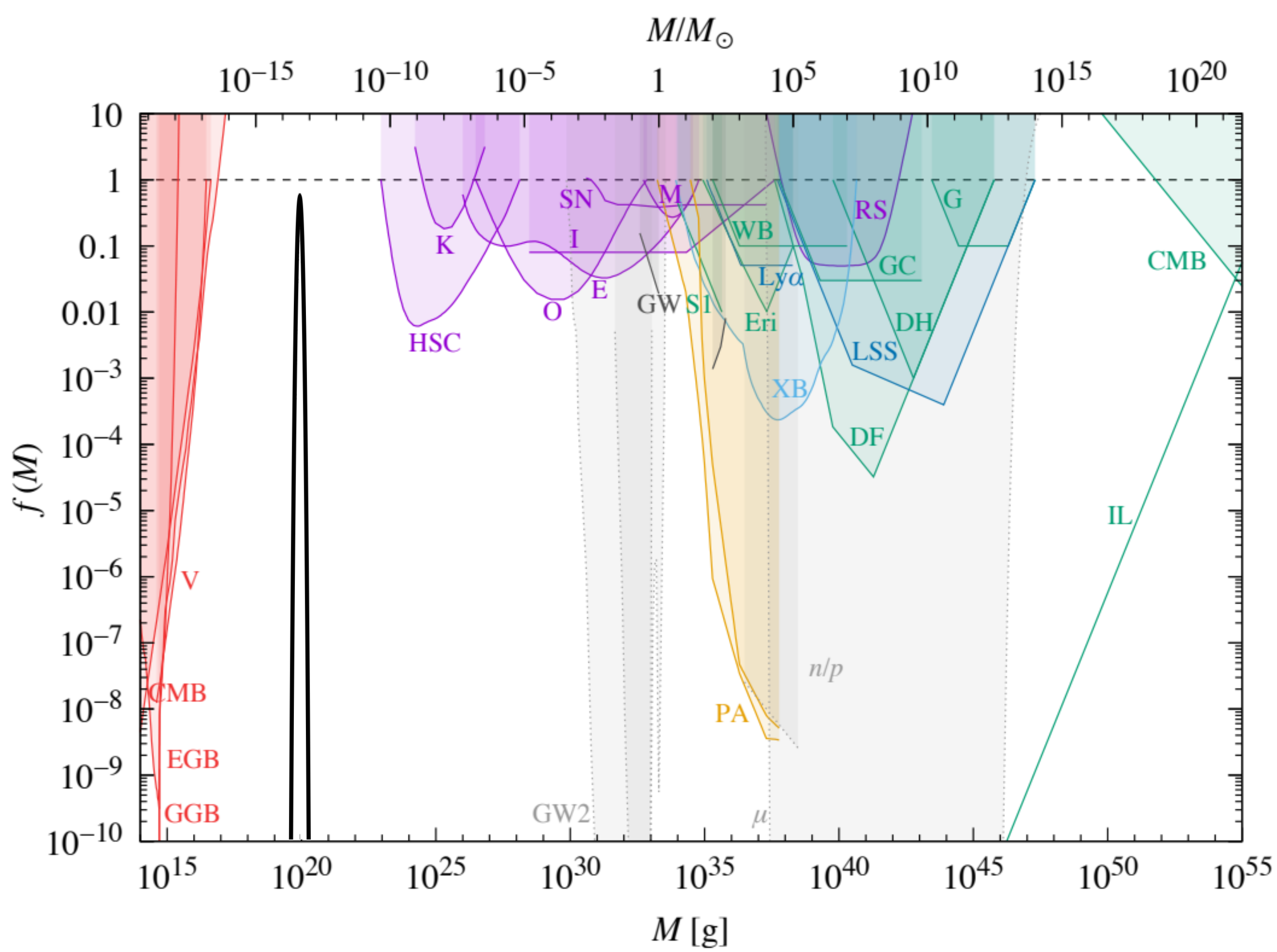


V
ϕ
χ
inflaton
FV
TV
TV
FV
bubble formation & expansion
bubble collapse



Single Field case

H. Deng, A. Vilenkin, JCAP 2017(12): 044.



Expectation: Two field with nonminimally coupling?

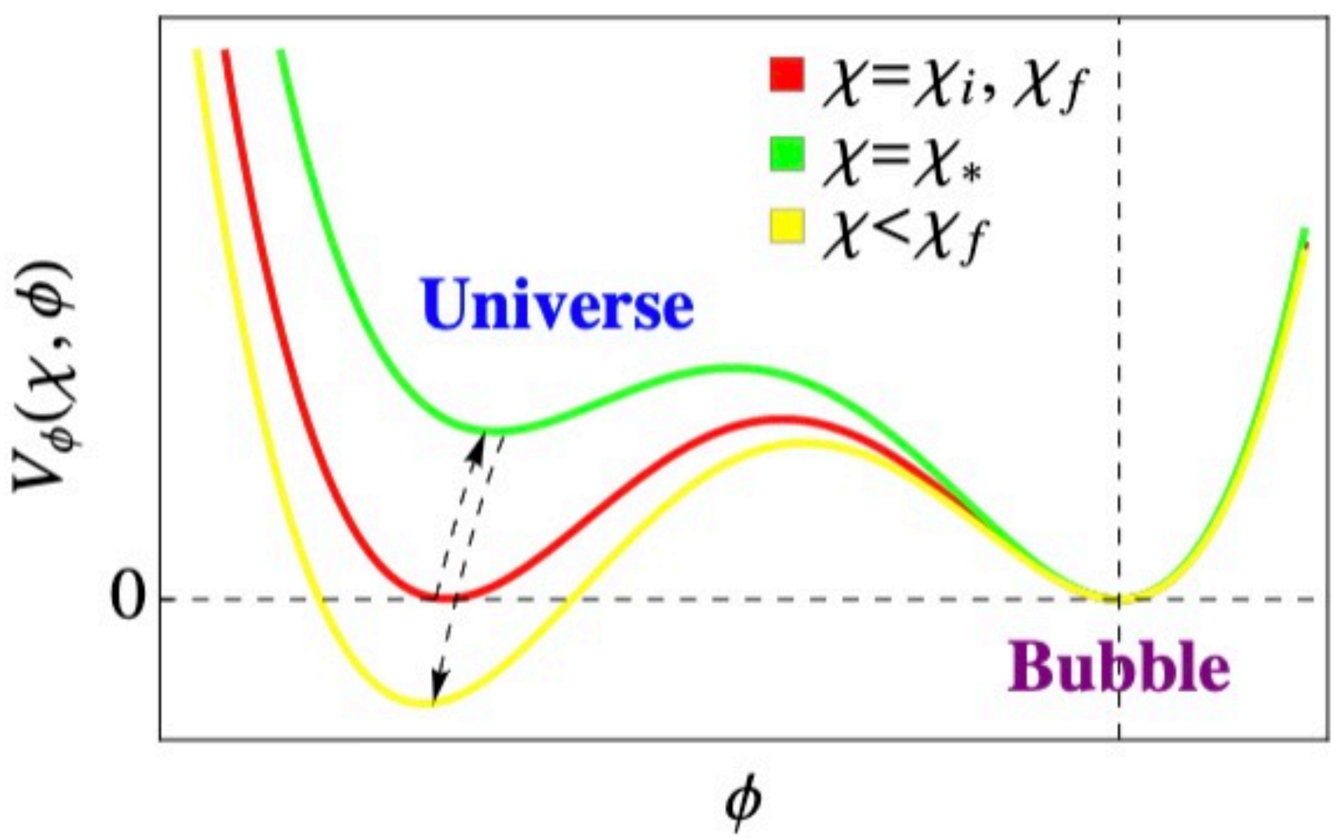
Setup of Model

$$\mathcal{L} = -\frac{1}{2}\partial_\mu \chi \partial^\mu \chi - \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\chi, \phi) \quad \left\{ \begin{array}{l} V_\chi(\chi) = \frac{1}{2} m^2 \chi^2 \\ \\ V_\phi(\chi, \phi) = \lambda(\phi^2 - v^2)^2 + \boxed{g(\chi)} v^2 (\phi - v)^2 \end{array} \right.$$

inflaton

instanton

nonminimal coupling



Setup of Model

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\chi,\phi)$$

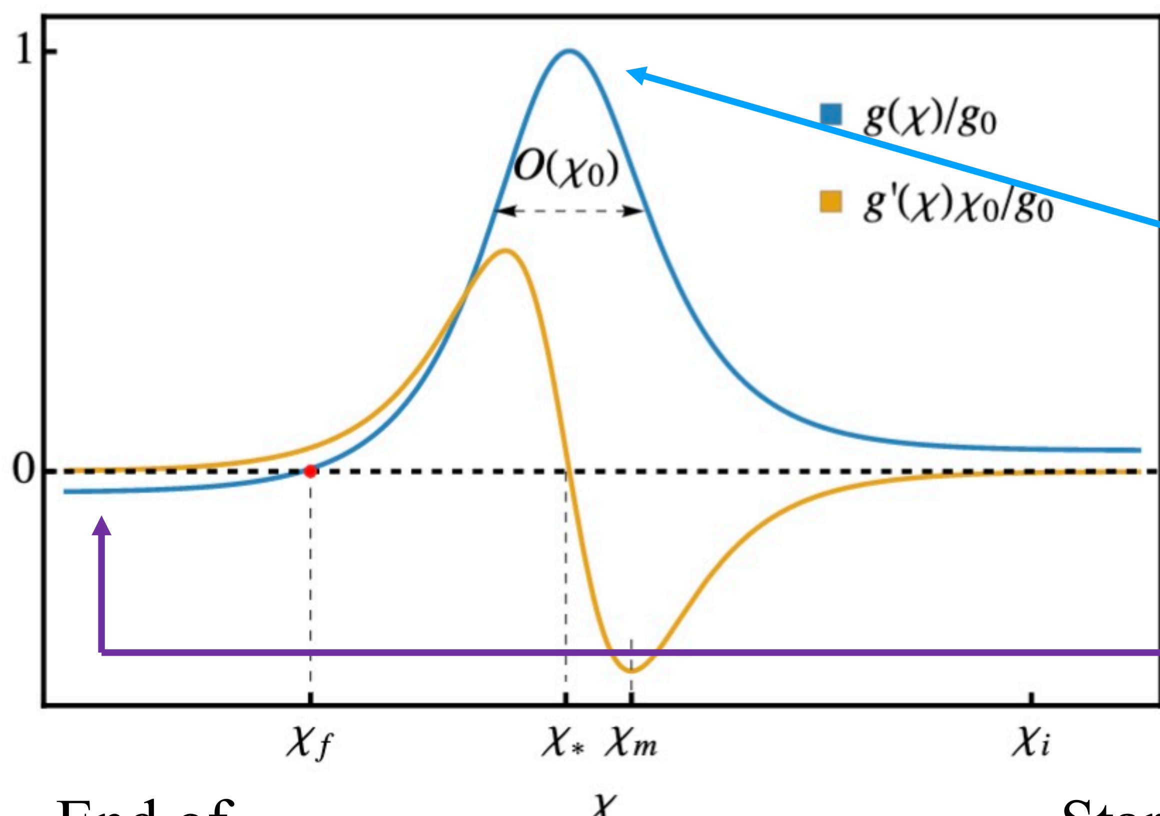
$$V_\chi(\chi) = \frac{1}{2}m^2\chi^2$$

$$V_\phi(\chi,\phi) = \lambda(\phi^2 - v^2)^2 + \boxed{g(\chi)}v^2(\phi - v)^2$$

inflaton

instanton

nonminimal coupling



Peak

$$g(\chi) = g_0\left[\boxed{\text{sech}\left(\frac{\chi-\chi_*}{\chi_0}\right)} + \boxed{\alpha\ \tanh\left(\frac{\chi-\chi_*}{\chi_0}\right)}\right]$$

Collapse of Bubbles

End of inflation

Start of inflation

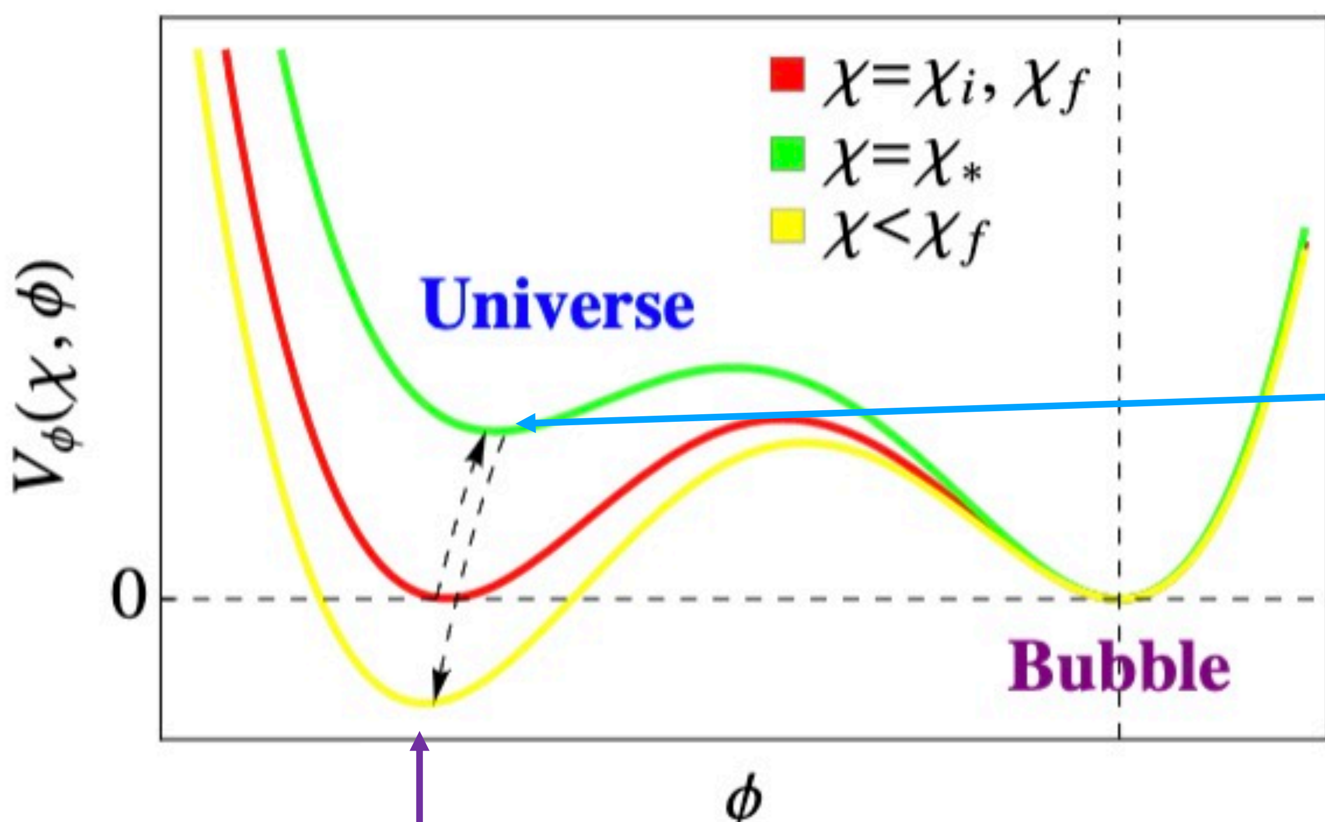
Setup of Model

$$\mathcal{L} = -\frac{1}{2}\partial_\mu \chi \partial^\mu \chi - \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\chi,\phi) \begin{cases} V_\chi(\chi) = \frac{1}{2}m^2\chi^2 \\ V_\phi(\chi,\phi) = \lambda(\phi^2 - v^2)^2 + \boxed{g(\chi)}\, v^2(\phi - v)^2 \end{cases}$$

inflaton

instanton

nonminimal coupling



$$g(\chi) = g_0 \left[\boxed{\mathrm{sech}\left(\frac{\chi - \chi_*}{\chi_0}\right)} + \boxed{\alpha \tanh\left(\frac{\chi - \chi_*}{\chi_0}\right)} \right]$$

Peak

Collapse of Bubbles

Setup of Model

Question: Process of Inflation?

Effective potential for Inflation: $V_{\text{eff}}(\chi) \equiv V_{\chi}(\chi) + V_{\phi}(\chi, \phi = \phi_{-})$

Necessary condition for Inflation: $\dfrac{\mathrm{d}V_{\text{eff}}}{\mathrm{d}\chi} > 0$

$$m^2 > \frac{2g_0 v^4}{\chi_0 \chi_f \left(\alpha + \sqrt{1+\alpha^2}\right)}$$

Setup of Model

Question: Process of Inflation?

$$H^2 \approx \frac{V_\chi}{3M_{\rm Pl}^2}\,, \qquad 3H\dot{\chi} \approx -\frac{\mathrm{d}V_\chi}{\mathrm{d}\chi}$$

$$\Downarrow$$

$$\chi(t) \approx \chi_i - \sqrt{\frac{2}{3}} M_{\rm Pl} m (t - t_i)$$

$$\boxed{\begin{gathered} m = 10^{-5} M_{\rm Pl} \\ \chi_i = 15 M_{\rm Pl} \\ \chi_f = \sqrt{2} M_{\rm Pl} \end{gathered}}$$

Question 2: From tunneling rate to PBH?

Express Euclidean action S in terms of PBH mass M

$$S = S(\chi) \longrightarrow \chi = \chi(t)$$

$$\downarrow$$

$$t - t_f = -\frac{1}{H}\ln(H R) \longrightarrow HR^2 = M$$

R: bubble radius at the end of inflation

Question 2: From tunneling rate to PBH?

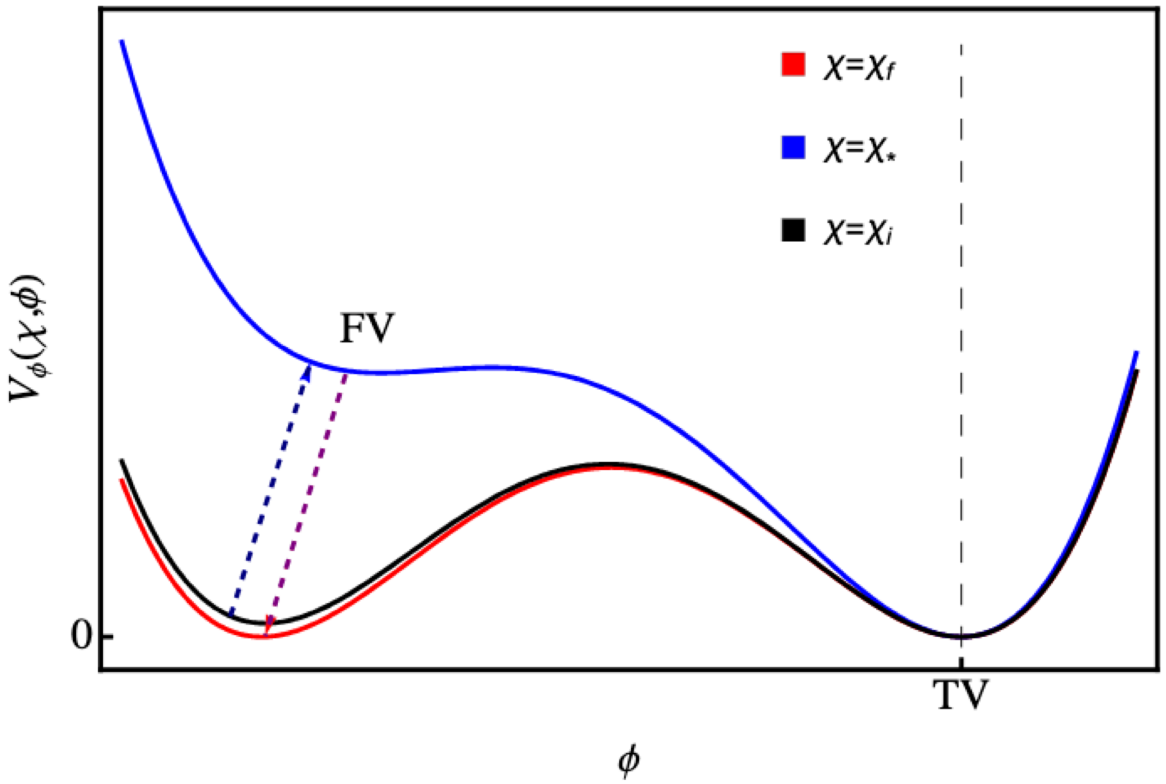
$$\left.\begin{aligned} f_{\rm PBH} &\equiv \frac{\rho_{\rm PBH}(t)}{\rho_{\rm CDM}(t)} = \int \frac{dM}{M} f(M) \\ \rho_{\rm PBH}(t) &= \int M dn(t) \end{aligned}\right\} \quad f(M) = \frac{M^2}{\rho_{\rm CDM}(t)} \frac{dn(t)}{dM}$$

$$dN = \lambda H_i^4 \underbrace{e^{3H_i t_{\rm n}} d^3\mathbf{x}}_{\text{dV}} dt_{\rm n} \longrightarrow \text{n(t)} \longrightarrow \text{n(R)} \longrightarrow \text{n(M)}$$

$$\Rightarrow \boxed{f(M) \sim \exp(-S) B \left(\frac{\mathcal{M}_{\rm eq}}{M}\right)^{\frac{1}{2}}}$$

- Open window for PBH DM

$$M/(1\text{g}) \in [10^{17}, 10^{22}]$$



- Set $M_* = 10^{20}\text{g}$

$$\chi(M) \approx \chi_f - \sqrt{\frac{2}{3}} m M_{\text{Pl}}(t - t_f)$$
$$= \chi_f + \frac{m M_{\text{Pl}}}{\sqrt{6}H} \ln(HM)$$

- $\chi_* \approx 4.66 M_{\text{Pl}}$ $\longrightarrow$ $g(\chi_*) = g_0$

Question 3: Analysis of tunneling rate?

(1) Coleman-de Luccia case

For a general potential $V(\phi) = \lambda\phi^4 - \mu\phi^3 + \frac{1}{2}\nu^2\phi^2 + V_0$

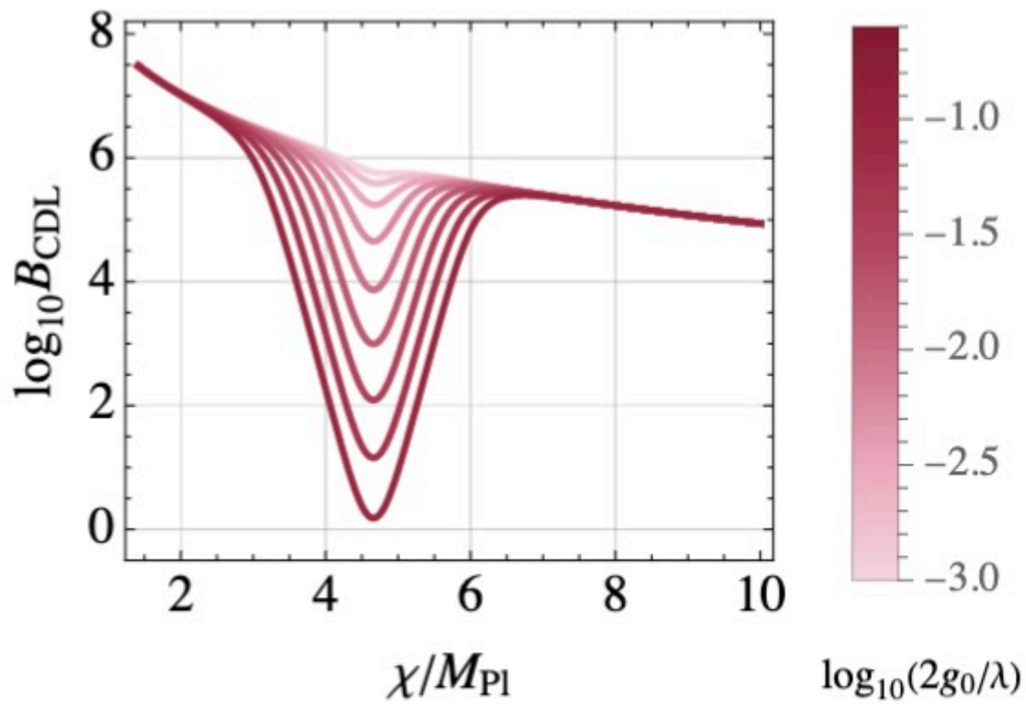
$$\boxed{B_{\rm CDL} = \frac{\pi^2}{12}\frac{1-\delta}{2\lambda}\frac{4(1+10.07\delta+16.55\delta^2)}{\omega(\delta+\omega)^2(1+10\delta)} \times \left[1+\frac{11}{2}\delta+3\frac{\delta^2}{\omega}+\frac{3}{2}\frac{\delta^3}{\omega^2}\right] \times \left(1+\sum_{i,j=0}^{4} a_{ij}\delta^i\omega^j\right)}$$

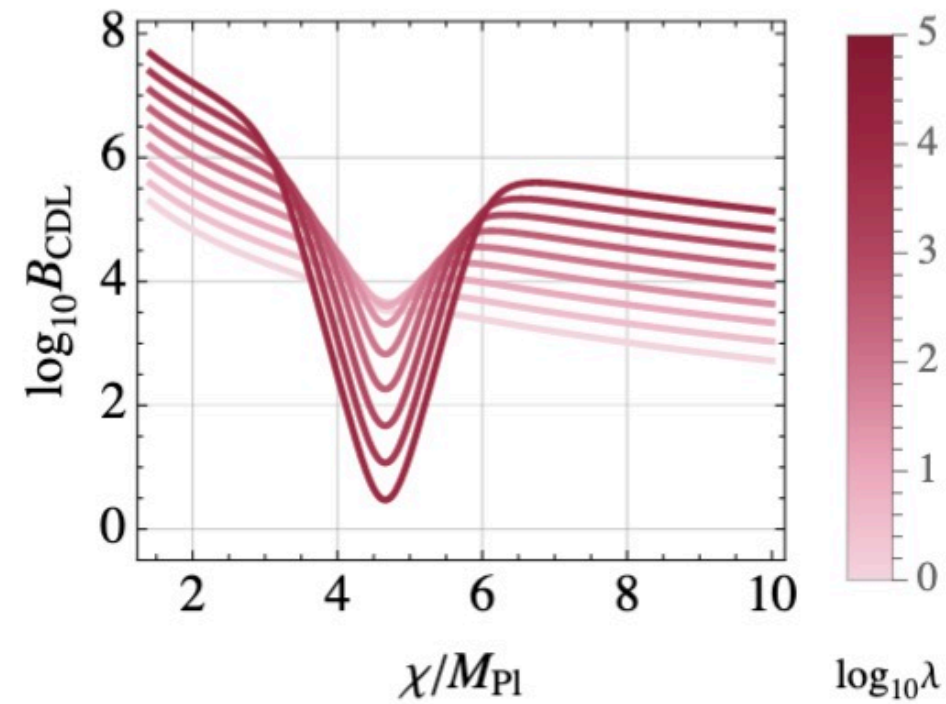
$$\omega \equiv \sqrt{\delta^2 + H_0^2} \qquad\qquad \delta \equiv 1 - \frac{2\lambda\nu^2}{\mu^2} \in (0,1)$$

$$H_0 \equiv \frac{1}{\nu}\left(\frac{V_0}{3M_{\rm Pl}^2}\right)^{1/2}$$

M. Sasaki, E. Stewart, T. Tanaka, PRD 50 (1994) 941-946

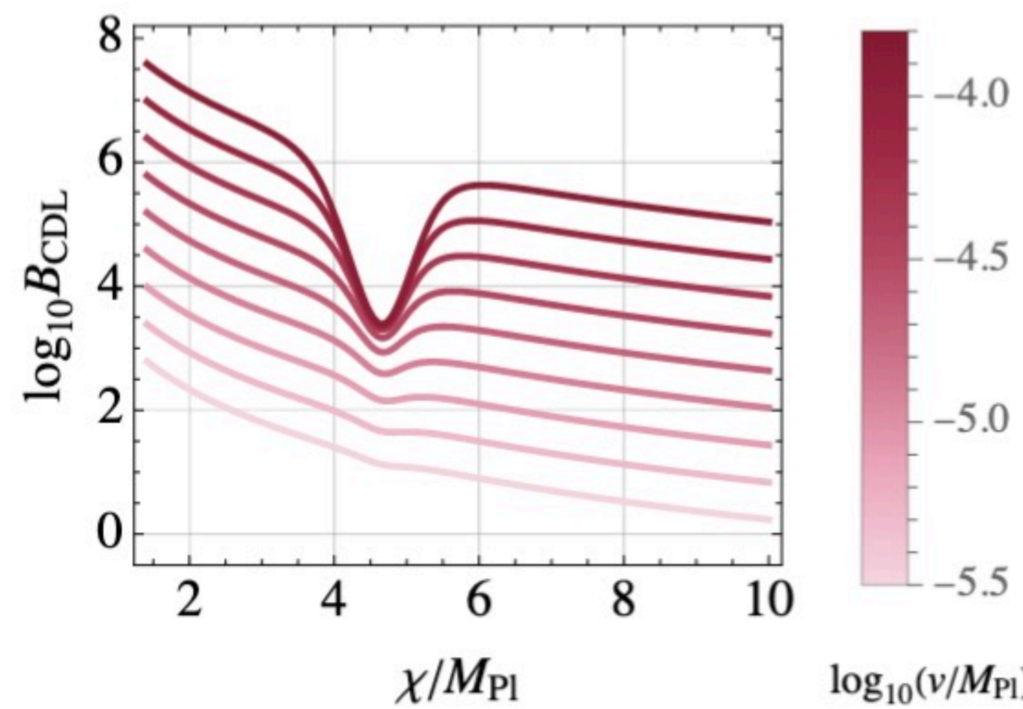
We can shift our potential to fit the fitting function

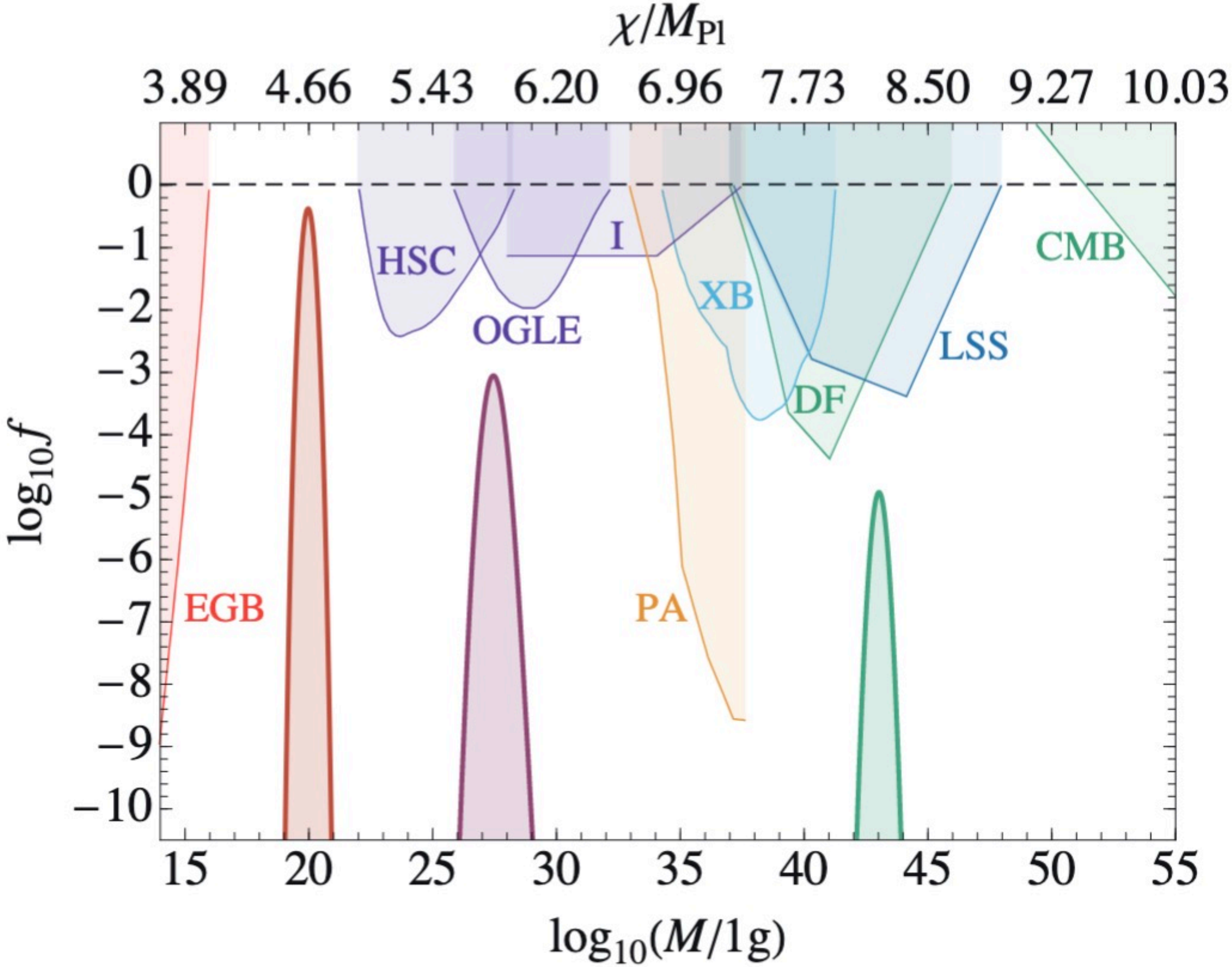




$\chi_* \approx 4.66 M_{\rm Pl}$

$B_{\rm CDL}(\chi)$ is minimized at χ_*



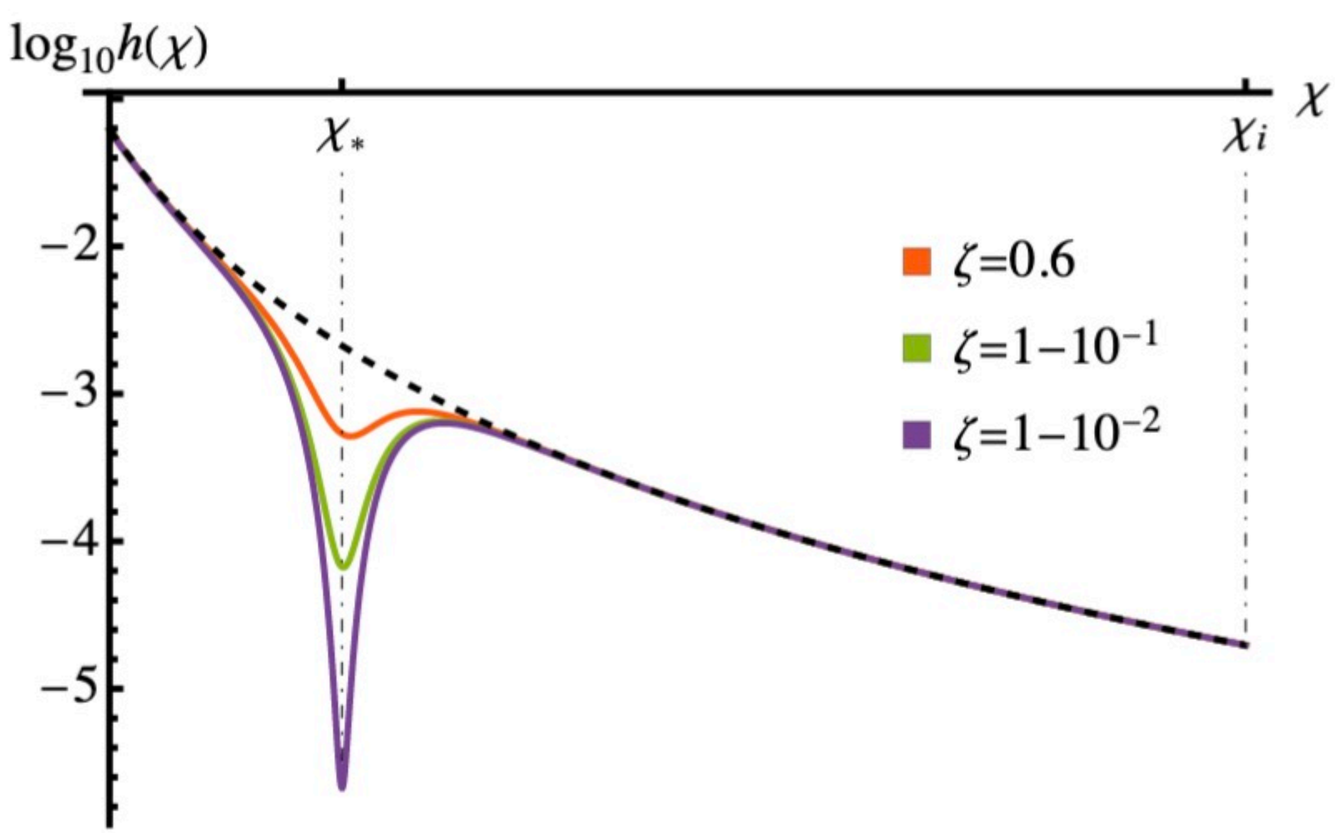


$\chi/M_{\rm Pl}$
3.89
4.66
5.43
6.20
6.96
7.73
8.50
9.27
10.03
$\log_{10} f$
0
−1
−2
−3
−4
−5
−6
−7
−8
−9
−10
HSC
OGLE
I
XB
LSS
DF
CMB
EGB
PA
15
20
25
30
35
40
45
50
55
$\log_{10}(M/1\mathrm{g})$

(2) Hawking-Moss case

$$B_{\rm HM} = 24\pi^2 M_{\rm Pl}^4 \left(\frac{1}{V(\phi_-,\chi)} - \frac{1}{V(\phi_{\rm top},\chi)} \right)$$
$$\approx 96\lambda\pi^2 v^4 \left(\frac{M_{\rm Pl}}{m}\right)^4 h(\chi)$$

$$\boxed{h(\chi) \equiv \frac{1}{\chi^4}\left[1 - \frac{2g_0}{\lambda}\,\mathrm{sech}\left(\frac{\chi-\chi_*}{\chi_0}\right)\right]^{3/2}}$$



Similar with CDL case!

Summary

We constructed a double-field model to realize a varying tunneling rate during inflation.

From analysis of HM and CdL cases, this model results monochromatic PBHs from the collapse of bubbles. This may greatly relieve the constraints from observations.

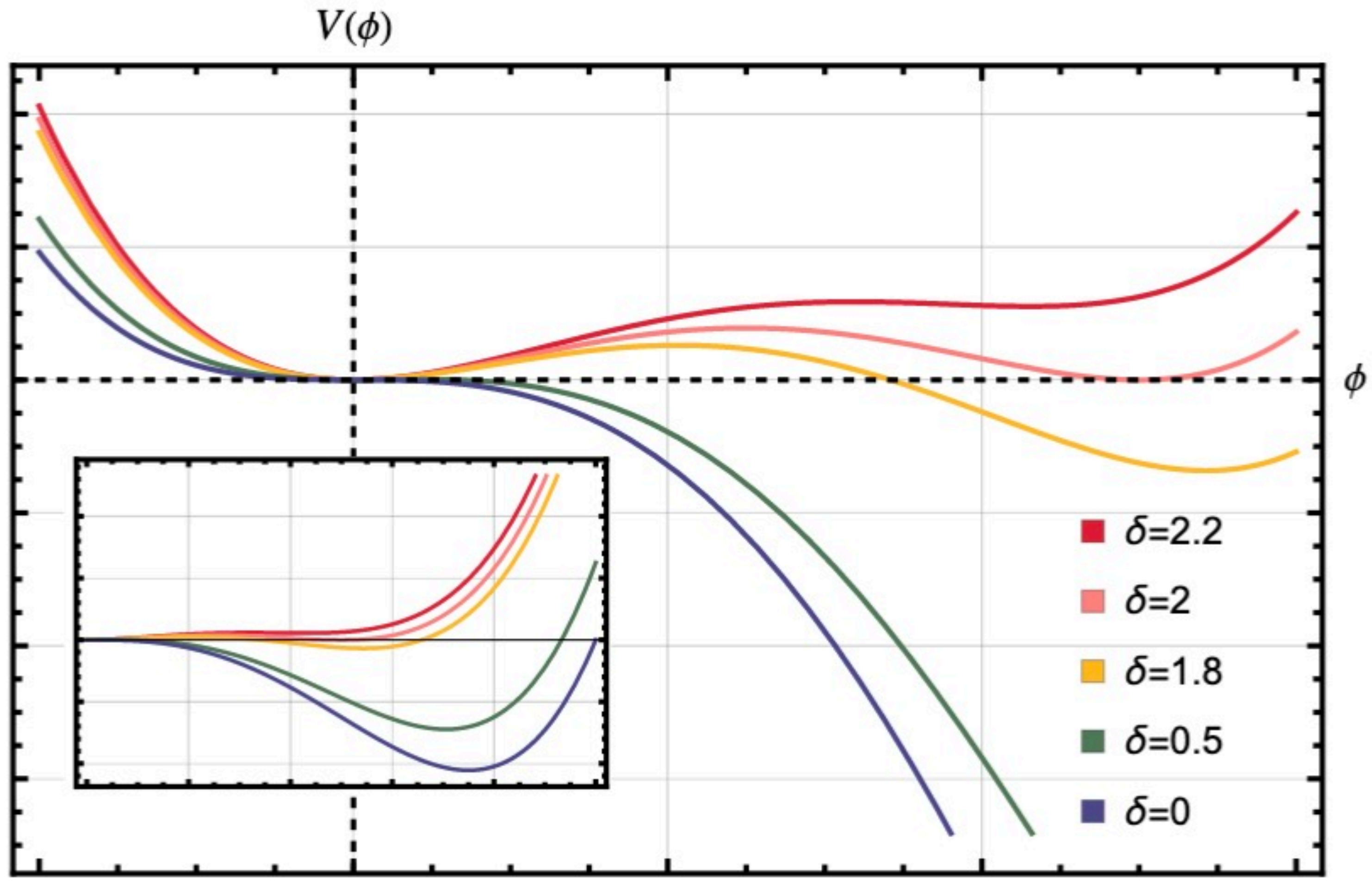
We can realize either PBHs DM, sub-solar PBHs or supermassive PBHs.

A first taste: without gravity

There exists a fitting formula for estimation of Euclidean action

$$V(\psi) = \lambda\psi^4 - a\psi^3 + b\psi^2$$

Defining $$\delta \equiv \frac{8\lambda b}{a^2} \in (0,2)$$



We firstly hope to fix g_0

When $\chi = \chi_*$

$\downarrow$

$g(\chi_*) = g_0$

$\downarrow$

$$\delta = 2\times \frac{1 - \frac{2g_0}{\lambda} + 3\sqrt{1 - \frac{2g_0}{\lambda}}}{2 - \frac{2g_0}{\lambda} + 2\sqrt{1 - \frac{2g_0}{\lambda}}}$$

CL case

- $g_{0,\mathrm{CL}} = \frac{\lambda}{2}\left(1 - \frac{\epsilon_2^2}{36}\right)$

$\downarrow$

- $\delta_{\mathrm{CL}} \approx \epsilon_2$

$\downarrow$

- $S_{*,\mathrm{CL}} \approx \frac{\pi^2 \alpha_1 \epsilon_2}{24\lambda}$

$\downarrow$

- $\epsilon_2 \approx 6.54\lambda$

DL case

- $g_{0,\mathrm{DL}} = 2\epsilon_1\lambda$

$\downarrow$

- $\delta_{\mathrm{DL}} \approx 2 - \epsilon_1$

$\downarrow$

- $S_{*,\mathrm{DL}} \approx \frac{\pi^2}{3\epsilon_1^3\lambda}$

$\downarrow$

- $\epsilon_1^3\lambda \approx 8.85\times10^{-2}$

An example: DL limit case (Coleman-de Luccia)

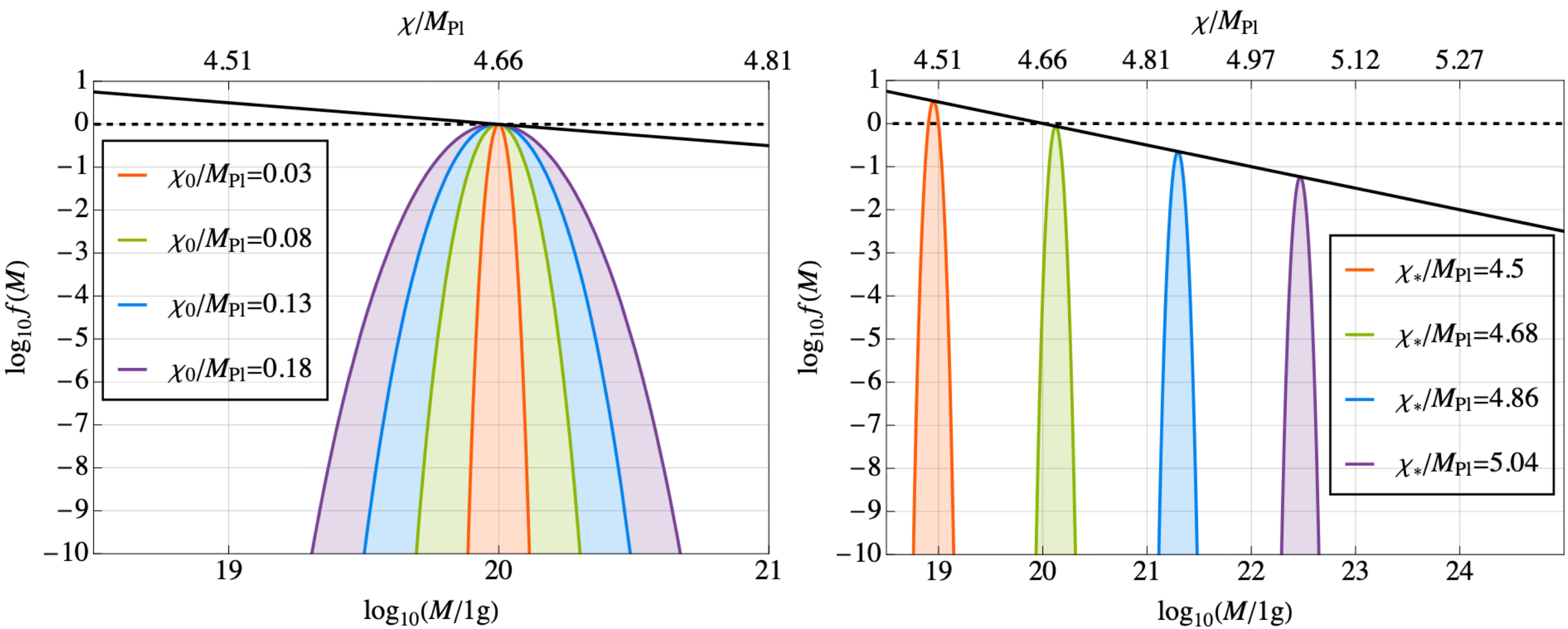
$$g(\chi) = \frac{g_0}{\cosh\left(\frac{\chi - \chi_*}{\chi_0}\right)} \longrightarrow \delta = 2\times\frac{1 - \frac{2g}{\lambda} + 3\sqrt{1 - \frac{2g}{\lambda}}}{2 - \frac{2g}{\lambda} + 2\sqrt{1 - \frac{2g}{\lambda}}}$$

$$S_{\mathrm{DL}} \approx \underset{\uparrow}{S_{*,\mathrm{DL}}} \cosh^3\left(\frac{mM_{\mathrm{Pl}}}{\sqrt{6}\chi_0 H}\ln\frac{M}{M_*}\right)$$

$$S_{*,\mathrm{DL}} = 37.18$$

$$\boxed{\ln f(M) = -S_{*,\mathrm{DL}}\cosh^3\left(\frac{mM_{\mathrm{Pl}}}{\sqrt{6}\chi_0 H}\ln\frac{M}{M_*}\right) + \ln B + \frac{1}{2}\ln\left(\frac{\mathcal{M}_{\mathrm{eq}}}{M}\right)}$$

An example: DL limit case (Coleman-de Luccia)



$$\ln f\,(M) = -S_{*,\mathrm{DL}} \cosh^3\left(\frac{m M_{\mathrm{Pl}}}{\sqrt{6}\chi_0 H}\ln\frac{M}{M_*}\right) + \ln B + \frac{1}{2}\ln\left(\frac{\mathcal{M}_{\mathrm{eq}}}{M}\right)$$

In case with gravity, no fitting formula, but we can work in the HM and CdL cases analytically

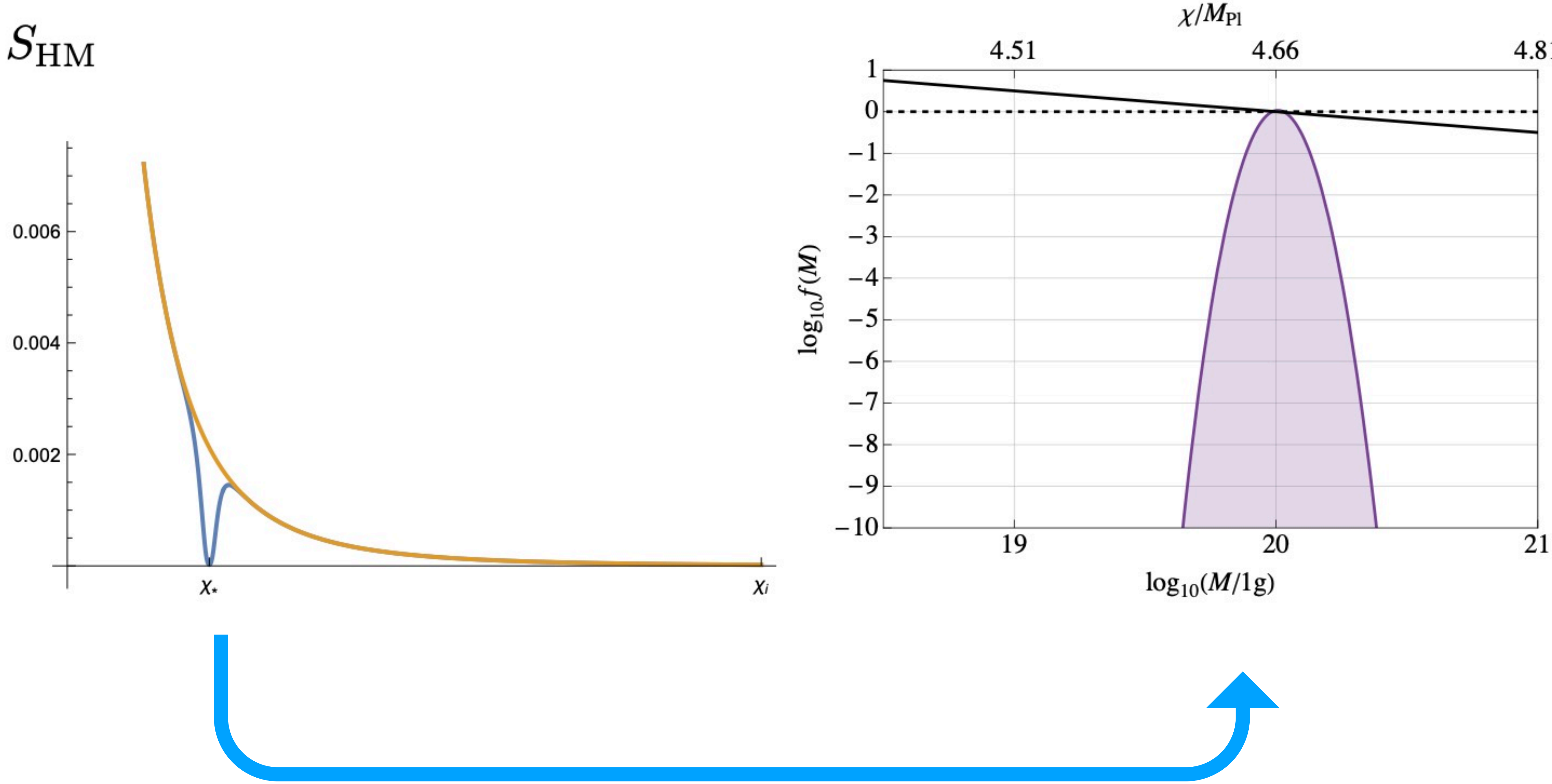
Numerical calculations are in process

$$S_{\rm HM} = 24\pi^2 M_{\rm Pl}^4 \left(\frac{1}{V(\phi_{\rm fv})} - \frac{1}{V(\phi_{\rm top})} \right)$$

$$= 96\pi^2 M_{\rm Pl}^4 \frac{\lambda v^4}{m^4 \chi^4} \left(1 - \frac{2g}{\lambda}\right)^{\frac{3}{2}}$$

We first fix the point at χ_* so that $S_{\rm HM}(\chi_*) = 37.18$

this fits $\lambda v^4 = 1.9845 \times 10^{-17}$



$S_{\rm HM}$
0.006
0.004
0.002
$\chi_\star$
χ_i
$\chi/M_{\rm Pl}$
4.51
4.66
4.81
$\log_{10}f(M)$
1
0
−1
−2
−3
−4
−5
−6
−7
−8
−9
−10
19
20
21
$\log_{10}(M/1{\rm g})$