Gravitational waves in thermal environments

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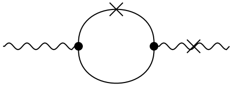
Question:
How do gravitons behave in a thermal plasma?

- Practical importance: relevant to the radiation-dominated universe/thermal or warm inflation.
- Standard approach: hydrodynamics on an FLRW background + linearized GR.

$$h_{ij}'' + 2\mathcal{H}h_{ij} + k^2 h_{ij} = 0.$$

- Missing interactions (in-in formalism):



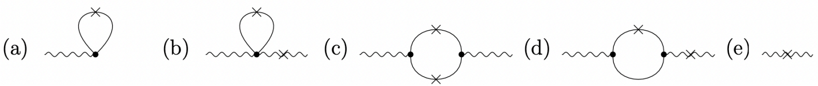
Setup:
GR + thermal scalar χ + interaction pict ($\kappa = \sqrt{32\pi G}$).

$$S_0 = \frac{1}{2}\int d^4x a^2 \left((h^{i\prime}_{\ j})^2 - (\partial_k h^i_{\ j})^2\right),$$
$$S_{\rm int} = -\frac{\kappa}{2}\int d^4x a^2 h^{ij}\partial_i\chi\partial_j\chi + \frac{\kappa^2}{4}\int d^4x a^2 h^{ik}h_k^{\ j}\partial_i\chi\partial_j\chi,$$
$$\langle\mathcal{O}\rangle = {\rm Tr}[\hat{D}_{\rm tot}\mathcal{O}],\ \ \hat{D}_{\rm tot} = \hat{D}_h \otimes \hat{D}^{\rm th}_\chi.$$

Goal:
GW power spectrum:

$$(2\pi)^3\delta(\vec{k}+\vec{k}')P_h(k) = \langle h^i_{\ j}(\vec{k})h^j_{\ i}(\vec{k}')\rangle$$

Previous results (AO,Sasaki,Wang 2023)



(a) Tadpole

(b) Effective mass

(c) Induced GWs

(d) Radiation exchange

(e) Tree level spectrum

We found

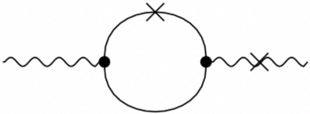
$$(a) + (b) = 0$$

in the local thermal ensemble

$$\mathrm{Tr}\left[\hat{D}^{\mathrm{th}}_{\chi,L}\,\hat{T}^{I}_{\mu\nu}\right] = (\rho + P)u_\mu u_\nu + P g_{\mu\nu}.$$

Radiation exchange

P_h is enhanced by exchanging radiation.

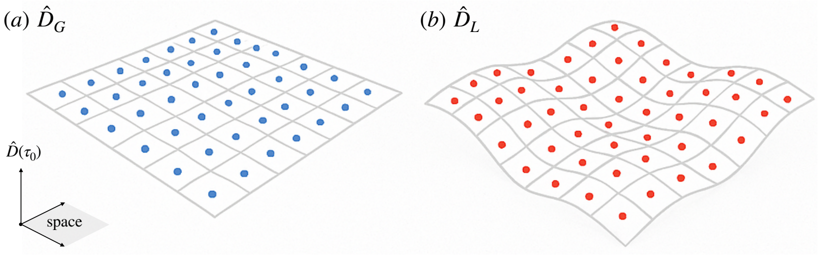
(d)  $$= \left(\ln \frac{\tau}{\tau_{\mathrm{R}}} - 1 + \frac{\tau_{\mathrm{R}}}{\tau} \right) P_h^{\mathrm{tree}}$$

- Reheating time (initial time of RD): τ_R
- Thermal correction dominates over the tree graph: $\ln \frac{\tau}{\tau_{\mathrm{R}}} = \mathcal{O}(10)$
- Negative effective mass. (This talk)
- Super horizon secular growth. (Large diff is explicitly broken in $H_{\mathrm{int}} \sim hT$).

Outline

The issue: initial environmental ensemble

*What do we mean by “initial thermal ensemble”?



(a) : thermal in background ($\kappa \to 0$).

(b) : thermal in local inertial frame:

$$\mathrm{Tr}\left[\hat{D}^{\mathrm{th}}_{\chi,L}\,\hat{T}^{I}_{\mu\nu}\right] = \underbrace{(\rho + P)u_\mu u_\nu + P g_{\mu\nu}}_{\text{perturbed}}\,.$$

Thermal effective action for GWs

*In-in path integral

$$e^{iW[h]} = \oint_{\hat{D}^{th}_{L/G}} \mathcal{D}\chi \, e^{iS[\chi;h]} = e^{i\Gamma[h]}$$

Thermal correction to $S_{\mathrm{EH}}[h]$ is (FLRW)

$$\Gamma[h] = -\frac{1}{2}\int d^4x \left[\kappa a^2 \tau_{\mu\nu} h^{\mu\nu} + \kappa^2 a^4 h^{\mu\nu} \pi_{\mu\nu\rho\sigma} h^{\rho\sigma}\right].$$

The diffeomorphism symmetry under $x^\mu \to x^\mu - \kappa\xi^\mu$ is

$$\delta_\xi \Gamma[h] = 0.$$

This generates Ward identities for $\tau_{\mu\nu}$ and $\pi_{\mu\nu\rho\sigma}$.

Ward identities of diff. symmetry

The Ward identities of diffeormorphism symmetry in a flat FLRW background ($\mathcal{H}$: conformal Hubble):

Ward identities:

$$\mathcal{O}(\kappa^0): \ \partial^\mu(a^2\tau_{\mu\nu}) - \mathcal{H}\, a^2 \underbrace{\tau_{\rho\sigma}\,\eta^{\rho\sigma}}_{=0 \text{ for CFTs}} \delta^0_\nu = 0,$$

$$\mathcal{O}(\kappa^1): \ \partial^\mu(a^4\pi_{\mu\nu\rho\sigma}) + \frac{1}{4}\left[\partial_\rho(a^2\tau_{\sigma\nu}) + \partial_\sigma(a^2\tau_{\rho\nu})\right] - \frac{1}{2}\delta^0_\nu\,\mathcal{H}\, a^2 \underbrace{\left(\tau_{\rho\sigma} + 2\eta^{\mu\nu}\, a^2\pi_{\mu\nu\rho\sigma}\right)}_{=0 \text{ for CFTs}} = 0.$$

*Correct environmental ensemble $\hat{D}$ must satisfy these.

Collision less thermal plasma with the global ensemble $\hat{D}_G$:

Flat spacetime case by [R. Francisco, J. Frenkel, J. Taylor (2016)]

$$\tau_{\mu\nu} = a^2\rho \int \frac{d\Omega}{4\pi}\, Q_\mu Q_\nu, \;\; Q_\mu = (-1, \vec{n}), \;\; \rho \propto a^{-4}.$$

$$\pi^G_{\mu\nu\rho\sigma} = \frac{\rho}{4} \int \frac{d\Omega}{4\pi} \left(\frac{Q_\mu Q_\nu Q_\rho Q_\sigma}{(Q\cdot\partial)^2}\, \partial\cdot\partial - \frac{\partial_{\langle\mu} Q_\nu Q_\rho Q_{\sigma\rangle}}{Q\cdot\partial} \right).$$

$\langle\mu\nu\rho\sigma\rangle = \mu\nu\rho\sigma + \nu\rho\sigma\mu + \rho\sigma\mu\nu + \sigma\mu\nu\rho$

This satisfies the WI:

$$\partial^\mu(a^4\pi^G_{\mu\nu\rho\sigma}) + \frac{1}{4}\left[\partial_\rho(a^2\tau_{\sigma\nu}) + \partial_\sigma(a^2\tau_{\rho\nu})\right]$$
$$- \frac{1}{2}\delta^0_\nu\, \mathcal{H}\, a^2 \left(\tau_{\rho\sigma} + 2\eta^{\mu\nu} a^2 \pi^G_{\mu\nu\rho\sigma}\right) = 0$$

Hard thermal limit effective action

Collision less thermal plasma with the local ensemble $\hat{D}_L$:

$$\pi^L_{\mu\nu\rho\sigma} = \pi^G_{\mu\nu\rho\sigma} - \underbrace{\frac{1}{8}a^{-2}\tau_{\langle\mu\rho}\eta_{\nu\sigma\rangle}}_{Ph_{ij}} .$$

The WI reduces to

$$\delta^0_\nu \mathcal{H}\tau_{\rho\sigma} = 0,$$

which contradicts with $\mathcal{H} \neq 0$ and $\tau_{\rho\sigma} \neq 0$.

EoM becomes integral equation:

$$h_{ij}'' + 2\mathcal{H}\, h_{ij}' + k^2 h_{ij} = -24\mathcal{H}^2 \int_{\tau_0}^{\tau} d\bar{\tau}\, K\big(k(\tau - \bar{\tau})\big)\, \partial_{\bar{\tau}} h_{ij}(\bar{\tau})$$
$$+ \text{(time boundary)},$$

- Tachyonic mass: eliminated by diffeomorphism symmetry.

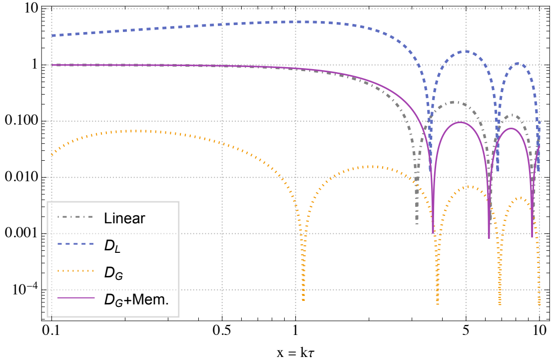
$$m_{\rm eff}^2 : \; -\frac{2}{5}H^2 \to \frac{8}{5}H^2 > 0.$$

- Secular growth: Integral equation requires a proper initial data and history: Time boundary (integral constants) can be chosen such that large diff is respected.

$$\text{Large diff. : } h_{ij} \to h_{ij} + \epsilon_{ij}$$

Consistency with the known results

Weinberg's damping Tensor mode as a special case [Weinberg 2003]:



Weinberg's case respected both small and large diff symmetry (based on kinetic theory).

Summary

- The issues of tachyonic mass and IR secular growth for gravitons in environments (QFT based)
- Tachyonic mass is eliminated when small diff is imposed for the effective action.
- IR secular growth is eliminated when large diff is imposed for the IR effective EoM (possible by tuning integral constant)
- Ward identities in FLRW background are found and used.
- In interaction theory, environmental initial distribution should be global. Local feature appears dynamically.
- First step to QFT based medium effects on GWs.

Things not discussed today

- The field redefinition dependence and off-shellness of WIs.
- Inherent incompleteness of thermal Minkowski limit.
- Nearly instantaneous $(\beta \ll \tau)$ response in

$$\hat{T}_{\mu\nu} = \hat{T}^{\mathrm{I}}_{\mu\nu} + i \int_{\tau_0 \sim \beta}^{\tau} d\tau_1 \underbrace{\left[\hat{H}_{\mathrm{int}}(\tau_1), \hat{T}^{\mathrm{I}}_{\mu\nu}\right]}_{\supset \delta(\tau)} + \mathcal{O}(\kappa^2),$$