

Determination of the spin and parity of all-charm tetraquarks

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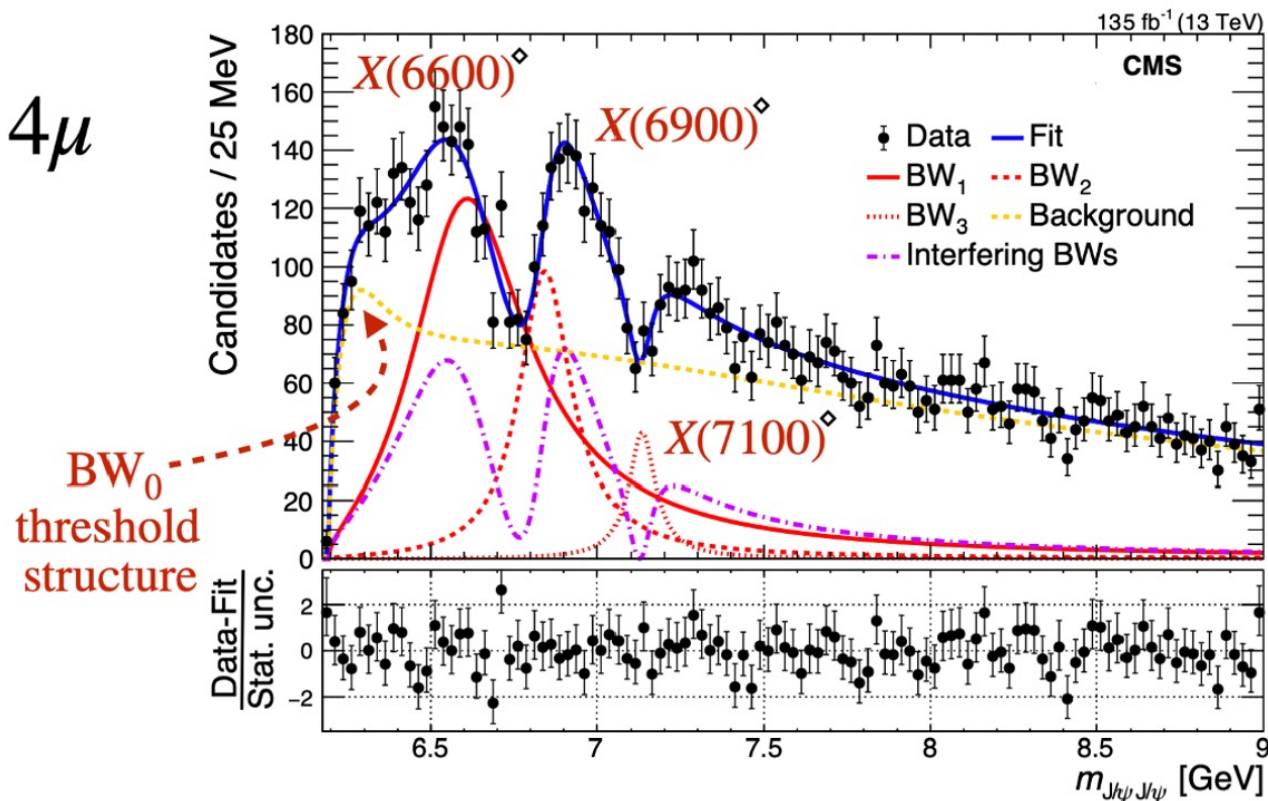
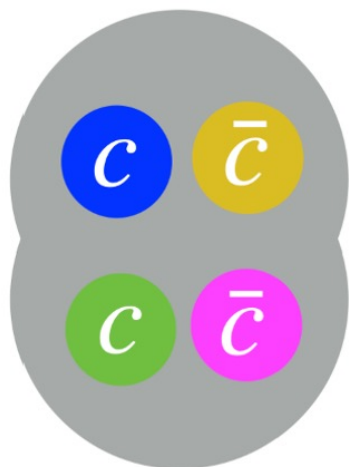
理论与实验联合研讨会--新粒子寻找

Sep 13, 2025

All-charm Tetraquarks on CMS

- CMS: family of 3 tetraquark candidates in 6 - 8 GeV range:

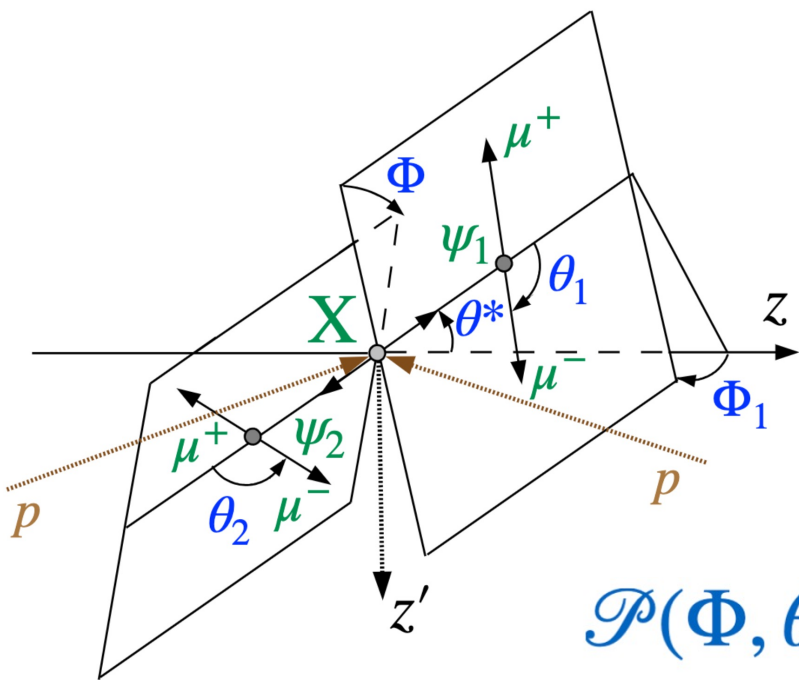
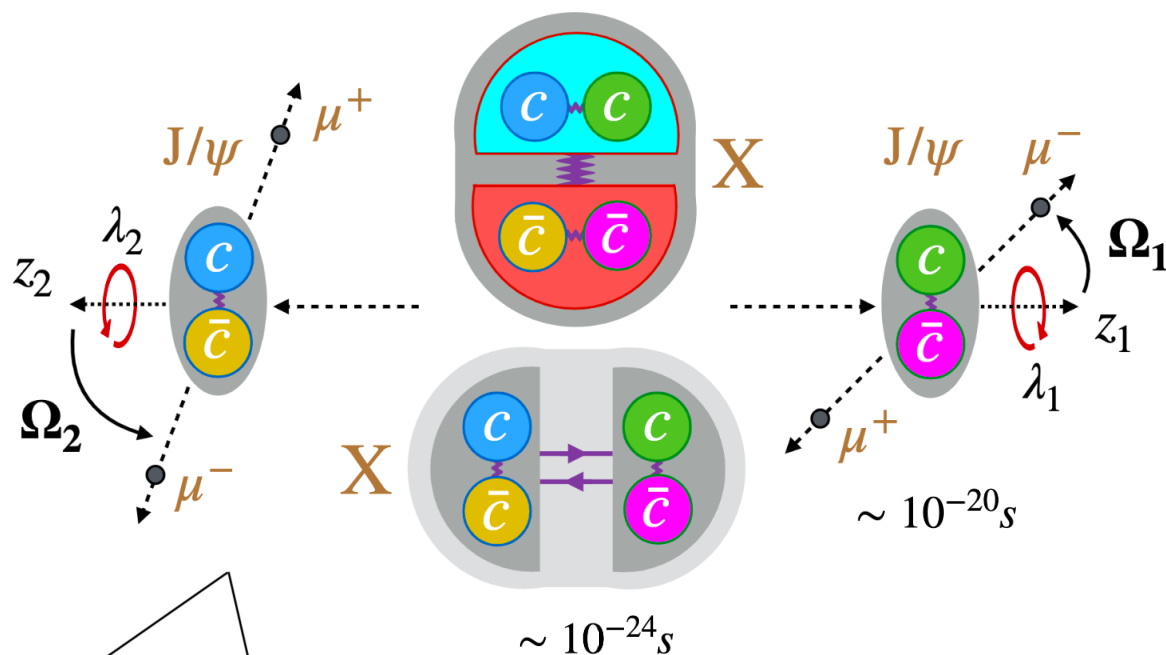
$$X \rightarrow J/\psi J/\psi \rightarrow 4\mu$$



- Analysis with Run 2 data
- Interference among three resonances with same J^P — simplify
 - Strongly motivated by dips observed in data
 - Indicate coherent production process and share same J^P

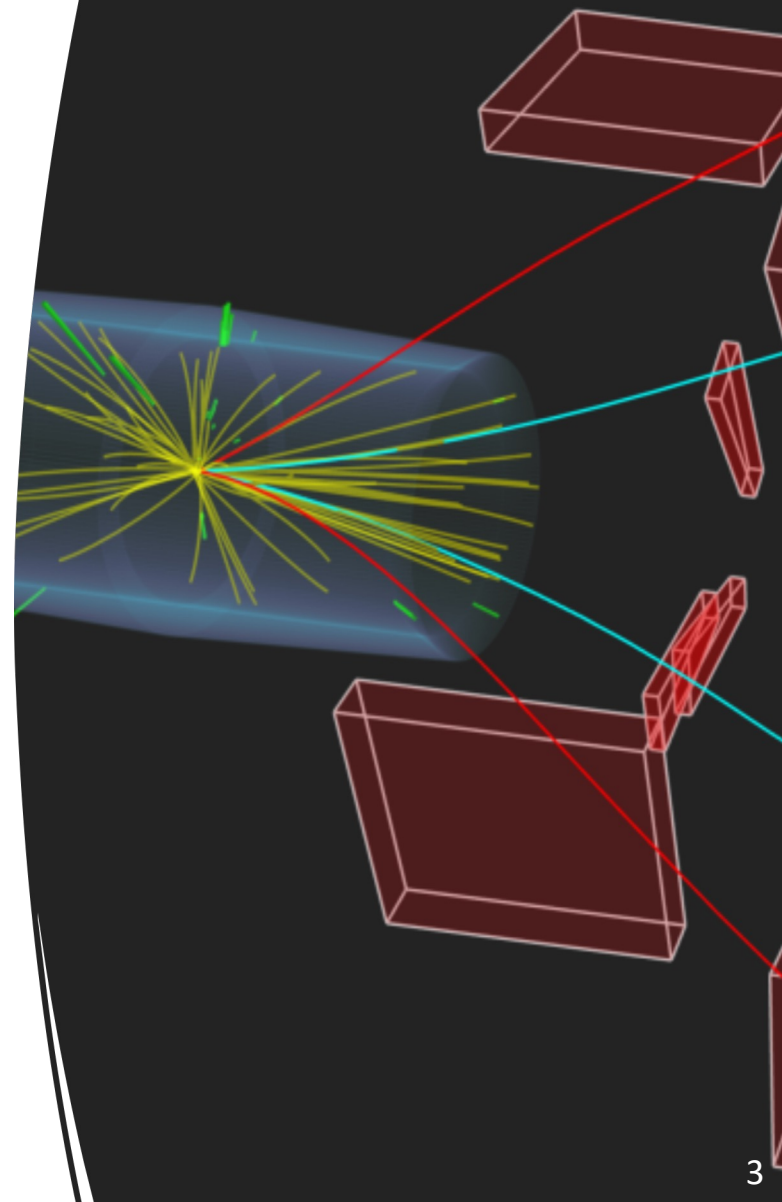
Spin parity analysis

Experiment at the LHC, CERN
 Recorded: 2016-Aug-13 06:39:34.675328 G
 Event / LS: 278769 / 13873923 / 76

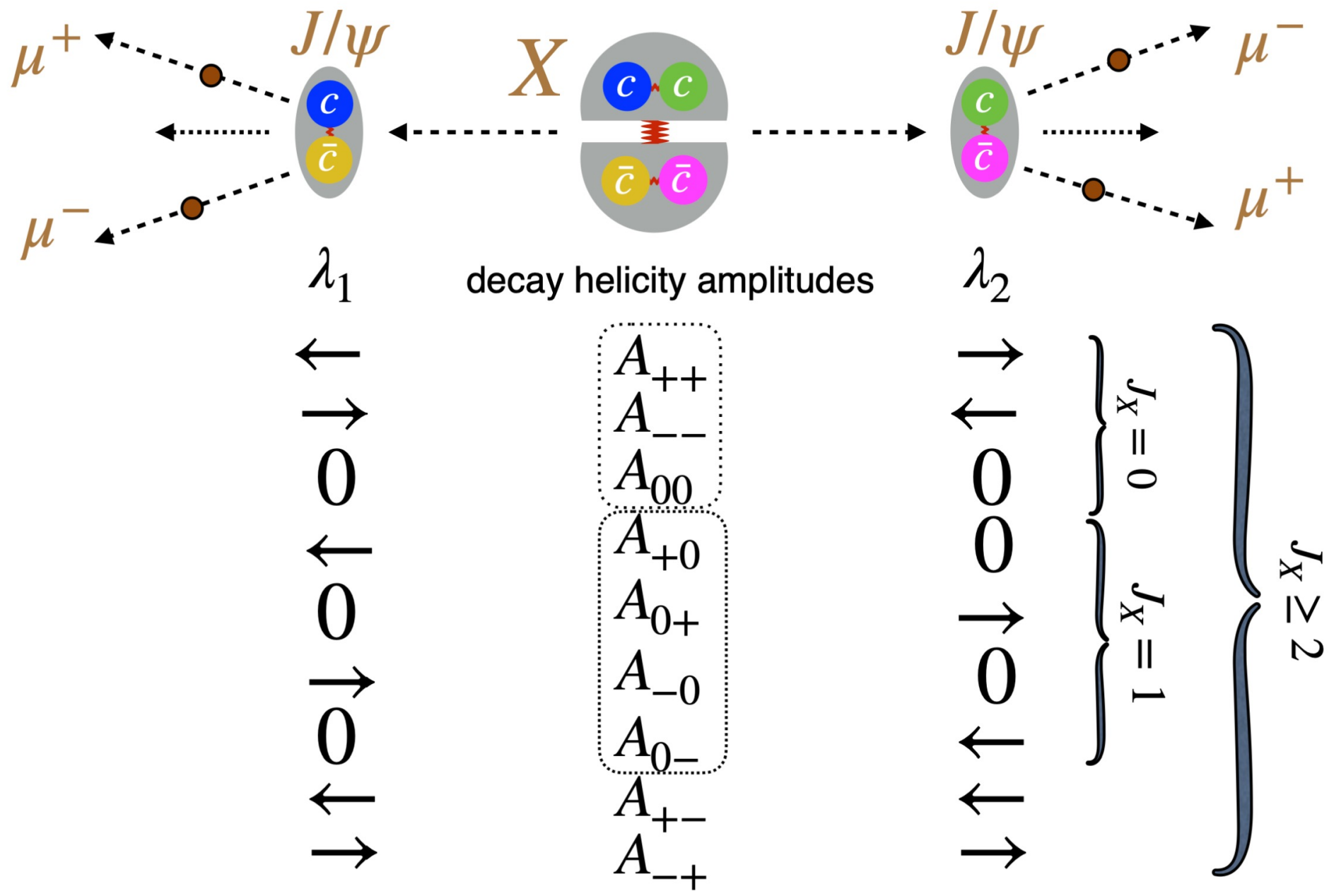


$$J^{PC} = ?$$

$$\mathcal{P}(\Phi, \theta_1, \theta_2; m_{4\mu})$$



J/ψ polarizations



J/ψ polarizations

- Symmetries:

- angular momentum: $|\lambda_1 - \lambda_2| \leq J$

- identical J/ψ bosons $A_{\lambda_1\lambda_2} = (-1)^J A_{\lambda_2\lambda_1}$

- P & C conserved in QCD:

X with definite J^{PC}

$C = +1$

$A_{\lambda_1\lambda_2} = P(-1)^J A_{-\lambda_1-\lambda_2}$

$J_X = 0$
 $J_X = 1$
 $J_X \geq 2$

A_{++}
 A_{--}
 A_{00}
 A_{+0}
 A_{0+}
 A_{-0}
 A_{0-}
 A_{+-}
 A_{-+}

Test 8+ J_X^P models:

0^{-+}	0^-	$A_{++} = -A_{--}$
0^{++}	0_m^+ and 0_h^+	$A_{++} = A_{--}$ and A_{00} ← note 2 d.o.f.
1^{-+}	1^-	$A_{+0} = -A_{0+} = A_{-0} = -A_{0-}$
1^{++}	1^+	$A_{+0} = -A_{0+} = -A_{-0} = A_{0-}$
2^{-+}	2_m^- and 2_h^-	$A_{++} = -A_{--}$ and $A_{+0} = A_{0+} = -A_{-0} = -A_{0-}$ ← note 2 d.o.f.
2^{++}	2_m^+	$A_{++} = A_{--}, A_{00}, A_{+0} = A_{0+} = A_{-0} = A_{0-},$ and $A_{+-} = A_{-+}$

note 4 d.o.f. for 2^{++} , test one model

Angular Analysis

$$\begin{array}{c}
 A_{++} \\
 A_{--} \\
 A_{00} \\
 A_{+0} \\
 A_{0+} \\
 A_{-0} \\
 A_{0-} \\
 A_{+-} \\
 A_{-+}
 \end{array}$$

$$\begin{aligned}
 F_{0,0}^J(\theta^*) \times & \left[4|A_{00}|^2 \sin^2 \theta_1 \sin^2 \theta_2 + 2|A_{++}| |A_{--}| \sin^2 \theta_1 \sin^2 \theta_2 \cos(2\Phi - \phi_{--} + \phi_{++}) \right] \\
 & + |A_{++}|^2 (1 + 2A_{f_1} \cos \theta_1 + \cos^2 \theta_1) (1 + 2A_{f_2} \cos \theta_2 + \cos^2 \theta_2) \\
 & + |A_{--}|^2 (1 - 2A_{f_1} \cos \theta_1 + \cos^2 \theta_1) (1 - 2A_{f_2} \cos \theta_2 + \cos^2 \theta_2) \quad \text{spin} = 0 \ \& \ \geq 1 \\
 & + 4|A_{00}| |A_{++}| (A_{f_1} + \cos \theta_1) \sin \theta_1 (A_{f_2} + \cos \theta_2) \sin \theta_2 \cos(\Phi + \phi_{++}) \\
 & + 4|A_{00}| |A_{--}| (A_{f_1} - \cos \theta_1) \sin \theta_1 (A_{f_2} - \cos \theta_2) \sin \theta_2 \cos(\Phi - \phi_{--})
 \end{aligned}$$

$$\begin{aligned}
 +F_{1,1}^J(\theta^*) \times & \left[2|A_{+0}|^2 (1 + 2A_{f_1} \cos \theta_1 + \cos^2 \theta_1) \sin^2 \theta_2 + 2|A_{0-}|^2 \sin^2 \theta_1 (1 - 2A_{f_2} \cos \theta_2 + \cos^2 \theta_2) \right. \\
 & + 2|A_{-0}|^2 (1 - 2A_{f_1} \cos \theta_1 + \cos^2 \theta_1) \sin^2 \theta_2 + 2|A_{0+}|^2 \sin^2 \theta_1 (1 + 2A_{f_2} \cos \theta_2 + \cos^2 \theta_2) \\
 & + 4|A_{+0}| |A_{0-}| (A_{f_1} + \cos \theta_1) \sin \theta_1 (A_{f_2} - \cos \theta_2) \sin \theta_2 \cos(\Phi + \phi_{+0} - \phi_{0-}) \\
 & + 4|A_{0+}| |A_{-0}| (A_{f_1} - \cos \theta_1) \sin \theta_1 (A_{f_2} + \cos \theta_2) \sin \theta_2 \cos(\Phi + \phi_{0+} - \phi_{-0}) \left. \right] \quad \text{spin} \geq 1
 \end{aligned}$$

$$\begin{aligned}
 +F_{1,-1}^J(\theta^*) \times & \left[4|A_{+0}| |A_{0+}| (A_{f_1} + \cos \theta_1) \sin \theta_1 (A_{f_2} + \cos \theta_2) \sin \theta_2 \cos(2\Psi - \phi_{+0} + \phi_{0+}) \right. \\
 & + 4|A_{0-}| |A_{-0}| (A_{f_1} - \cos \theta_1) \sin \theta_1 (A_{f_2} - \cos \theta_2) \sin \theta_2 \cos(2\Psi - \phi_{0-} + \phi_{-0}) \\
 & + 4|A_{+0}| |A_{-0}| \sin^2 \theta_1 \sin^2 \theta_2 \cos(2\Psi - \Phi - \phi_{+0} + \phi_{-0}) + 4|A_{0-}| |A_{0+}| \sin^2 \theta_1 \sin^2 \theta_2 \cos(2\Psi + \Phi - \phi_{0-} + \phi_{0+}) \left. \right]
 \end{aligned}$$

$$\begin{aligned}
 +F_{2,2}^J(\theta^*) \times & \left[|A_{+-}|^2 (1 + 2A_{f_1} \cos \theta_1 + \cos^2 \theta_1) (1 - 2A_{f_2} \cos \theta_2 + \cos^2 \theta_2) \right. \\
 & + |A_{-+}|^2 (1 - 2A_{f_1} \cos \theta_1 + \cos^2 \theta_1) (1 + 2A_{f_2} \cos \theta_2 + \cos^2 \theta_2) \left. \right] \quad \text{spin} \geq 2
 \end{aligned}$$

$$+F_{2,-2}^J(\theta^*) \times \left[2|A_{+-}| |A_{-+}| \sin^2 \theta_1 \sin^2 \theta_2 \cos(4\Psi - \phi_{+-} + \phi_{-+}) \right] + \text{other 26 interference terms for spin}$$

$$\text{where } \Psi = \Phi_1 + \Phi/2 \quad \text{and} \quad F_{ij}^J(\theta^*) = \sum_{m=0,\pm 1,\pm 2} f_m d_{im}^J(\theta^*) d_{jm}^J(\theta^*)$$

Valid
for any J

Lorentz-Invariant Amplitude

- Expect three X resonances to have the same **tensor structure**:

$$A(X_{J=0} \rightarrow V_1 V_2) = \left(a_1(q^2) m_V^2 \epsilon_1^* \epsilon_2^* + a_2(q^2) f_{\mu\nu}^{*(1)} f^{*(2),\mu\nu} + a_3(q^2) f_{\mu\nu}^{*(1)} \tilde{f}^{*(2),\mu\nu} \right)$$

0_m^+

0_h^+

0^-

$A_{00} = A_{++} = A_{--}$ at $2m_{J/\psi}$ threshold

A_{00} at large m_X $A_{++} = A_{--}$

$A_{++} = -A_{--}$

[arXiv:1001.3396](https://arxiv.org/abs/1001.3396)

empirical **form factors** ($m_{4\mu}^2$)

$$A(X_{J=1} \rightarrow V_1 V_2) = \left(b_1(q^2) \left[(\epsilon_1^* q)(\epsilon_2^* \epsilon_X) + (\epsilon_2^* q)(\epsilon_1^* \epsilon_X) \right] + b_2(q^2) \epsilon_{\alpha\mu\nu\beta} \epsilon_X^\alpha \epsilon_1^{*\mu} \epsilon_2^{*\nu} \tilde{q}^\beta \right)$$

1^-

1^+

more for spin-2

$A_{+0} = -A_{0+} = A_{-0} = -A_{0-}$

$A_{+0} = -A_{0+} = -A_{-0} = A_{0-}$

Lorentz-Invariant Amplitude

- Expect three X resonances to have the same **tensor structure**:

$$\begin{aligned}
 A(X_{J=2} \rightarrow V_1 V_2) = & 2c_1(q^2) t_{\mu\nu} f^{*1,\mu\alpha} f^{*2,\nu\alpha} + 2c_2(q^2) t_{\mu\nu} \frac{q_\alpha q_\beta}{\Lambda^2} f^{*1,\mu\alpha} f^{*2,\nu,\beta} \\
 & + c_3(q^2) \frac{\tilde{q}^\beta \tilde{q}^\alpha}{\Lambda^2} t_{\beta\nu} (f^{*1,\mu\nu} f_{\mu\alpha}^{*2} + f^{*2,\mu\nu} f_{\mu\alpha}^{*1}) + c_4(q^2) \frac{\tilde{q}^\nu \tilde{q}^\mu}{\Lambda^2} t_{\mu\nu} f^{*1,\alpha\beta} f_{\alpha\beta}^{*(2)} \\
 & + m_V^2 \left(2c_5(q^2) t_{\mu\nu} \epsilon_1^{*\mu} \epsilon_2^{*\nu} + 2c_6(q^2) \frac{\tilde{q}^\mu q_\alpha}{\Lambda^2} t_{\mu\nu} (\epsilon_1^{*\nu} \epsilon_2^{*\alpha} - \epsilon_1^{*\alpha} \epsilon_2^{*\nu}) + c_7(q^2) \frac{\tilde{q}^\mu \tilde{q}^\nu}{\Lambda^2} t_{\mu\nu} \epsilon_1^* \epsilon_2^* \right) \\
 & + c_8(q^2) \frac{\tilde{q}_\mu \tilde{q}_\nu}{\Lambda^2} t_{\mu\nu} f^{*1,\alpha\beta} \tilde{f}_{\alpha\beta}^{*(2)} + c_{10}(q^2) \frac{t_{\mu\alpha} \tilde{q}^\alpha}{\Lambda^2} \epsilon_{\mu\nu\rho\sigma} q^\rho \tilde{q}^\sigma (\epsilon_1^{*\nu} (q\epsilon_2^*) + \epsilon_2^{*\nu} (q\epsilon_1^*)) ,
 \end{aligned}$$

[arXiv:1001.3396](https://arxiv.org/abs/1001.3396)

2_m^- ($A_{++} = -A_{--}$) 2_h^- ($A_{+0} = A_{0+} = -A_{-0} = -A_{0-}$)

2_m^+ — minimal representative model including all amplitudes:

4 d.o.f. $A_{00}, A_{++} = A_{--}, A_{+0} = A_{0+} = A_{-0} = A_{0-}, A_{+-} = A_{-+}$ for 2^{++} (or $J \geq 2$)

unique

basis of 2^{++} could be equivalent to $2_m^+, 0_m^+, 0_h^+, 1^+$

if data consistent with $2_m^+ \Rightarrow$ unambiguously 2^{++} (or $J \geq 2$)

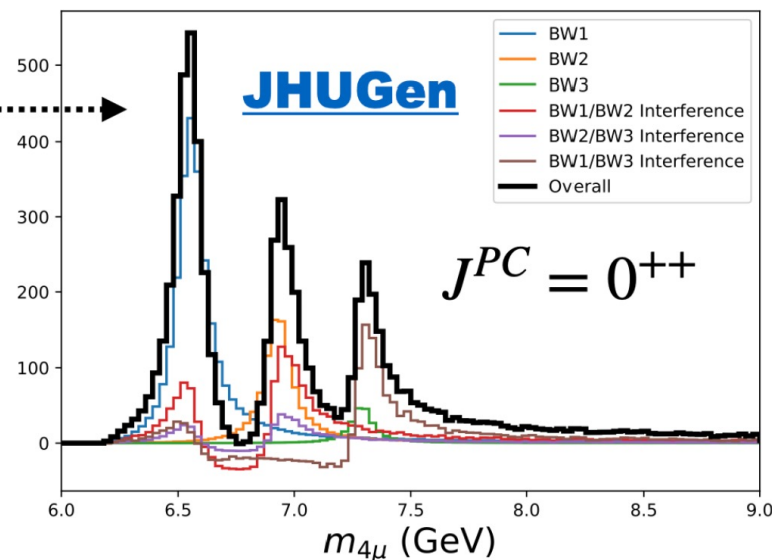
Simplification in Angular Analysis

- Full model possible, but very complex

$$\mathcal{P}(\Phi, \theta_1, \theta_2; m_{4\mu})$$

- (1) Same properties of **3 resonances**:

$$\mathcal{P}(m_{4\mu}, \vec{\Omega}) = \underbrace{\mathcal{P}(m_{4\mu})}_{\text{empirical}} \cdot \underbrace{T(\vec{\Omega} | m_{4\mu})}_{\text{angular}}$$



- (2) Pairwise tests of J_X^P hypotheses i and j :

[arXiv:1208.4018](https://arxiv.org/abs/1208.4018)

$$\text{MELA} \quad \mathcal{D}_{ij}(\vec{\Omega} | m_{4\mu}) = \frac{\mathcal{P}_i(\vec{\Omega} | m_{4\mu})}{\mathcal{P}_i(\vec{\Omega} | m_{4\mu}) + \mathcal{P}_j(\vec{\Omega} | m_{4\mu})}$$

1 optimal observable

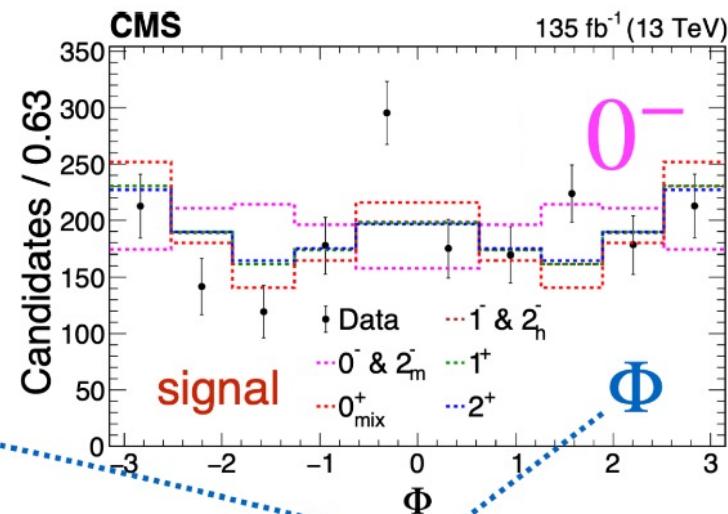
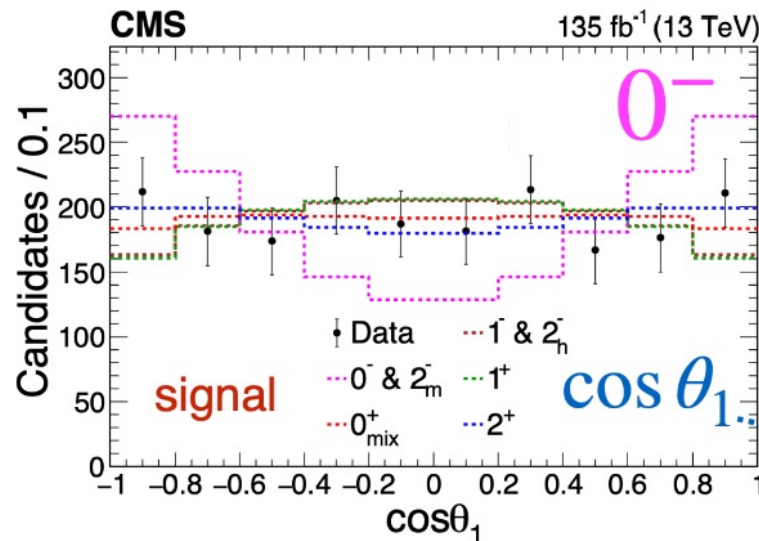
- Final 2D model:

$$\mathcal{P}_{ijk}(m_{4\mu}, \mathcal{D}_{ij}) = \mathcal{P}_k(m_{4\mu}) \cdot T_{ijk}(\mathcal{D}_{ij} | m_{4\mu})$$

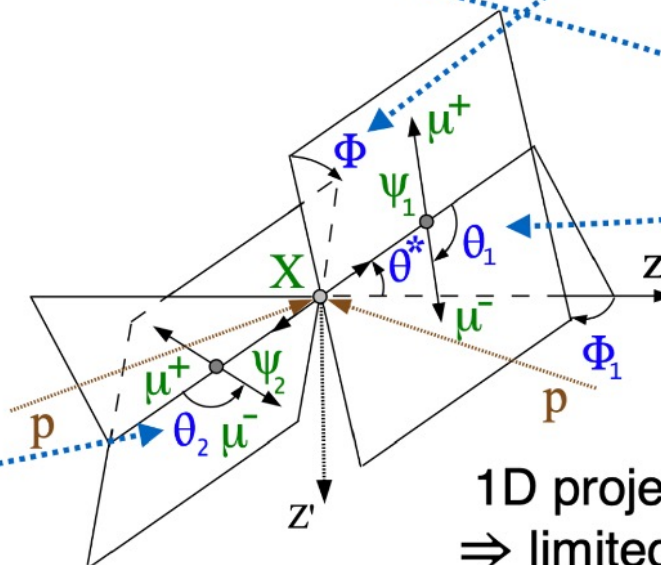
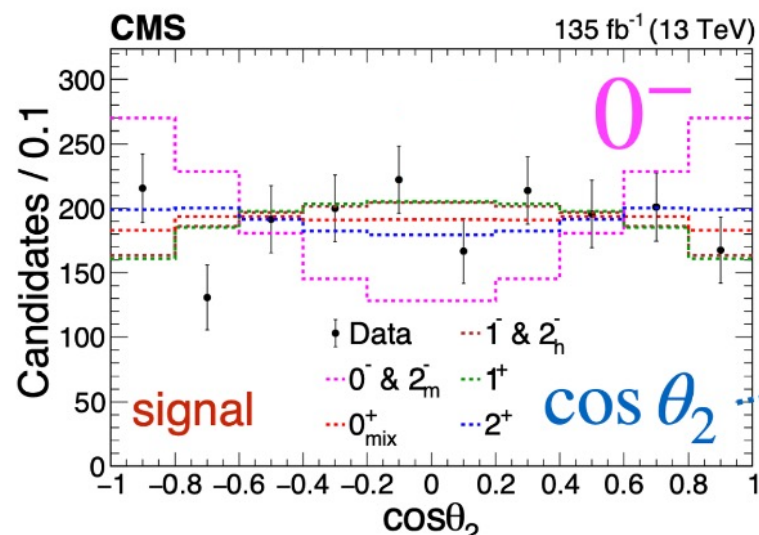
Decay Angles

Production angles not use
Consistent with unpolarized (backup)

decay angles (consistency check): **distinguish** models



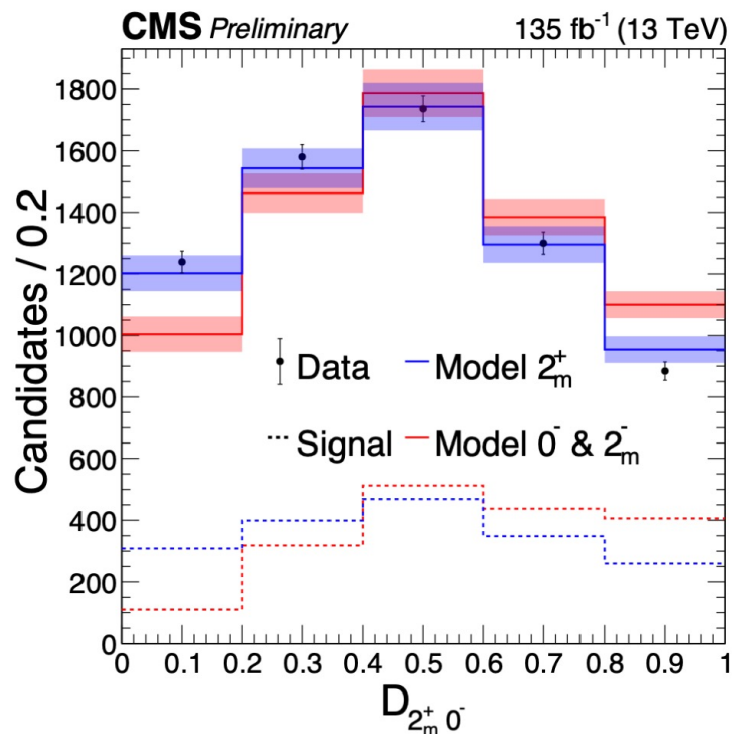
background-subtracted



1D projections from 4D
 \Rightarrow limited information

Optimal Observable

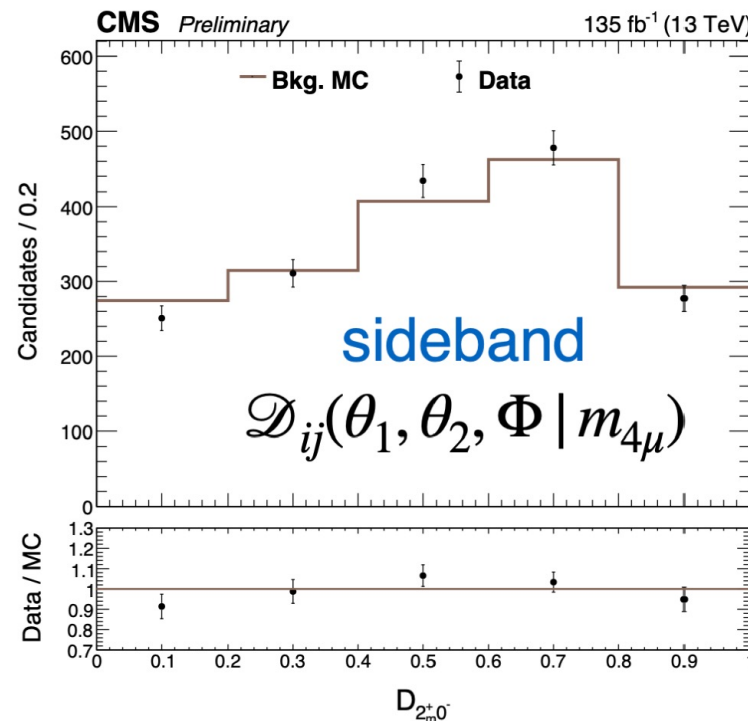
- 1D projection of data, optimal for $j = 0^-(2_m^-)$ vs $i = 2_m^+$



optimal observable

$$\mathcal{D}_{ij}(\vec{\Omega} | m_{4\mu}) = \frac{\mathcal{P}_i(\vec{\Omega} | m_{4\mu})}{\mathcal{P}_i(\vec{\Omega} | m_{4\mu}) + \mathcal{P}_j(\vec{\Omega} | m_{4\mu})}$$

1D projections from 2D
 \Rightarrow limited information



background model from MC
 control in sidebands
 systematic variations

2D parameterization:

$$\mathcal{P}_{ijk}(m_{4\mu}, \mathcal{D}_{ij}) = \mathcal{P}_k(m_{4\mu}) \cdot T_{ijk}(\mathcal{D}_{ij} | m_{4\mu})$$

Statistical Analysis

- Hypothesis test with toy MC for $J_1^P = 2_m^+$ vs $J_2^P = 0^-$

- Test statistic $q = -2\ln(\mathcal{L}_{J_2^P} / \mathcal{L}_{J_1^P})$

- Consistency of data with J_1^P / J_2^P using p-value:

$$p = P(q \leq q_{obs} | J_1^P + bkg)$$

$$p = P(q \geq q_{obs} | J_2^P + bkg)$$

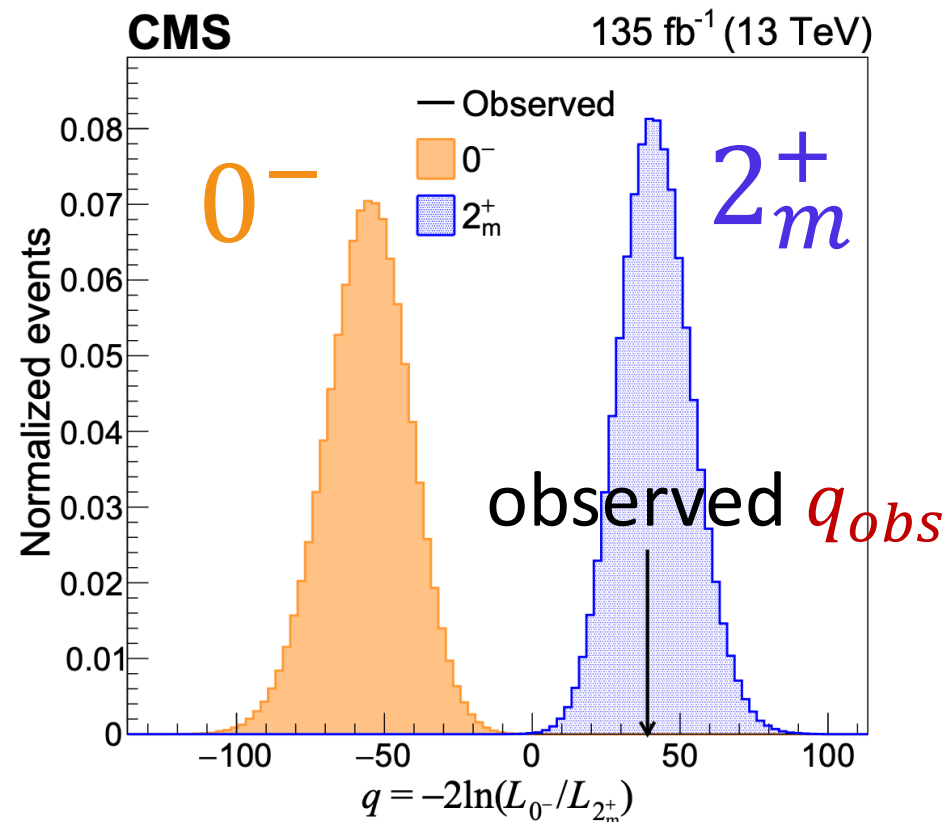
- Significance:

Converted from p-value

via Gaussian one-sided tail integral

- Confidence level

$$CL_s = \frac{P(q \geq q_{obs} | J_2^P + bkg)}{P(q \geq q_{obs} | J_1^P + bkg)}$$

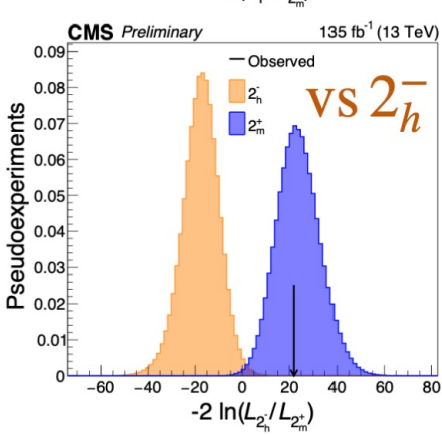
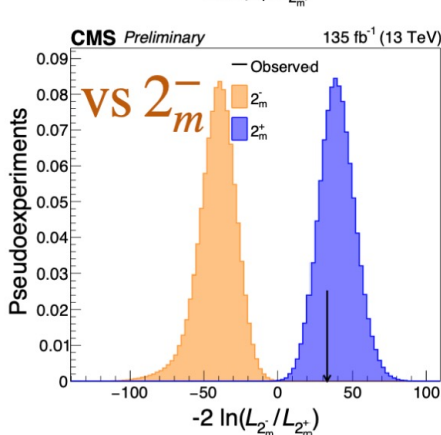
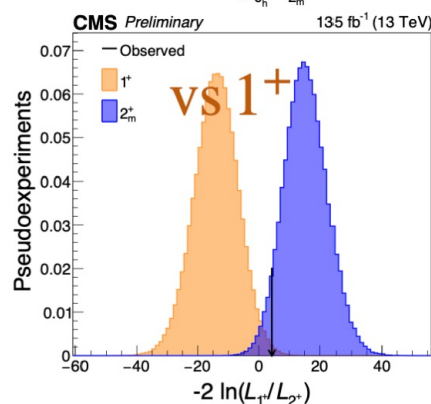
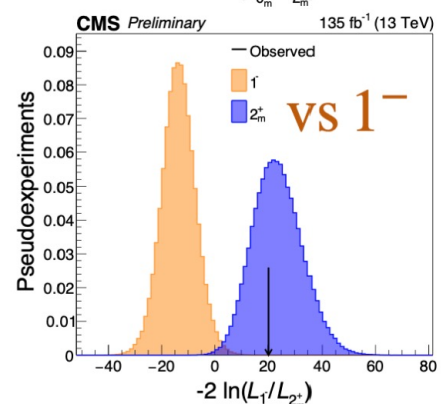
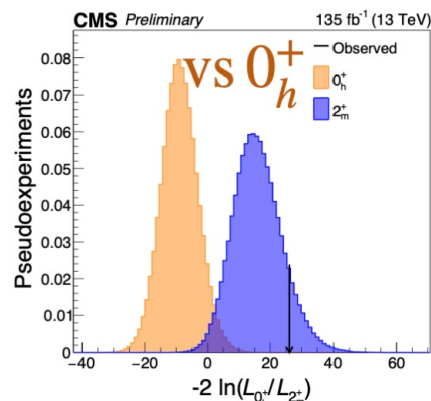
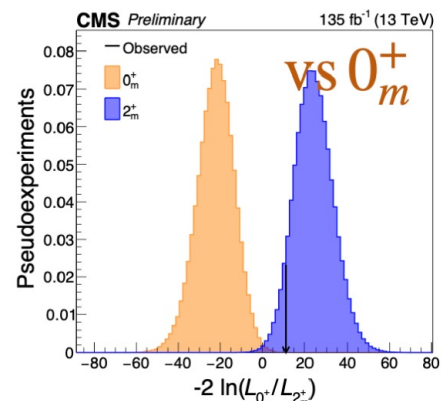


		Observed		Expected	
		p-value	Z-score	p-value	Z-score
0^- vs 2_m^+	0^-	2.7×10^{-13}	7.2	6.5×10^{-14}	7.4
	2_m^+	4.2×10^{-1}	0.2	0.50	0.0

Hypothesis test

- Combine 2D fit: $\mathcal{P}_{ijk}(m_{4\mu}, \mathcal{D}_{ij})$

— $J^P = 2_m^+$ model survives

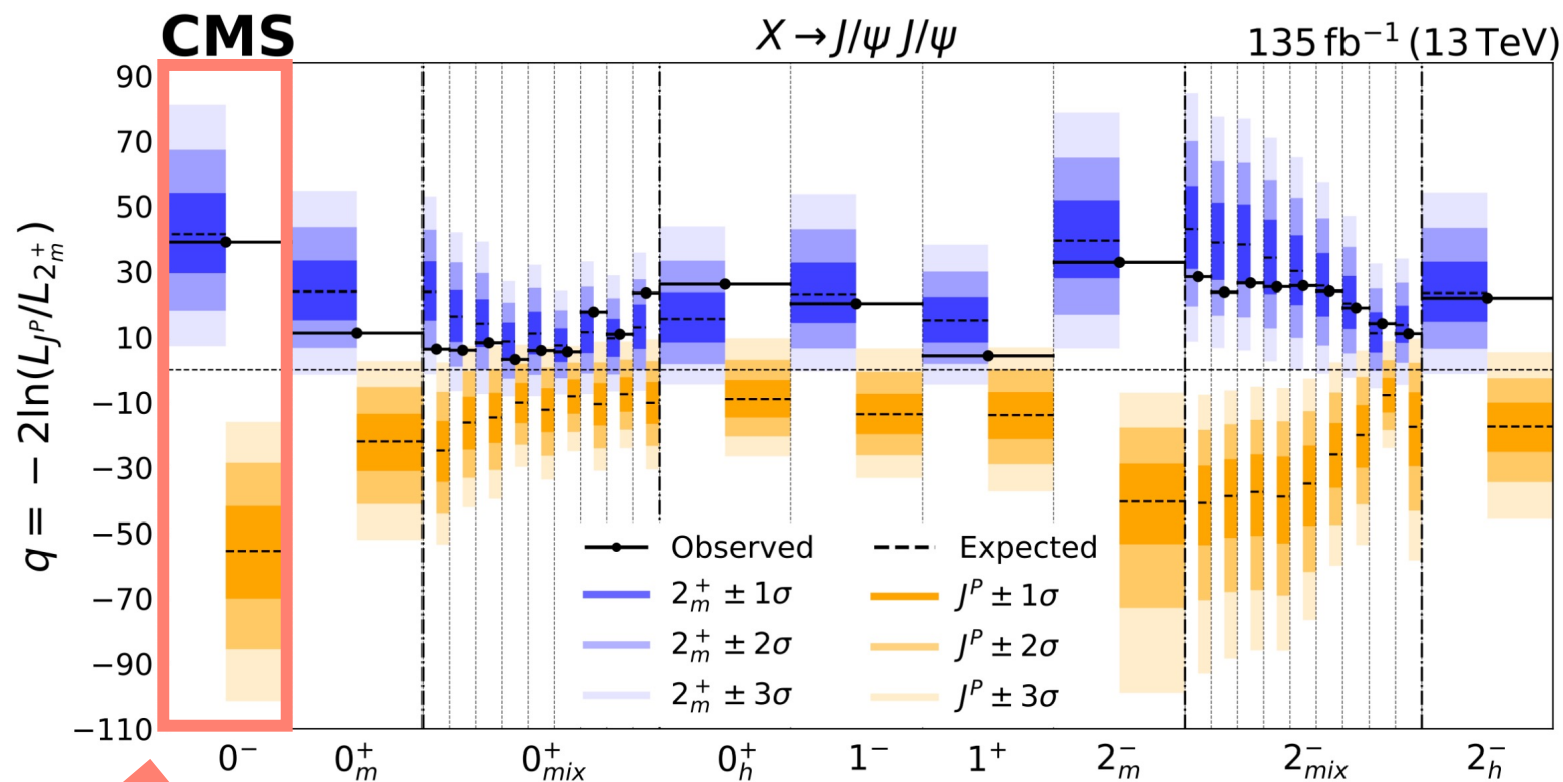
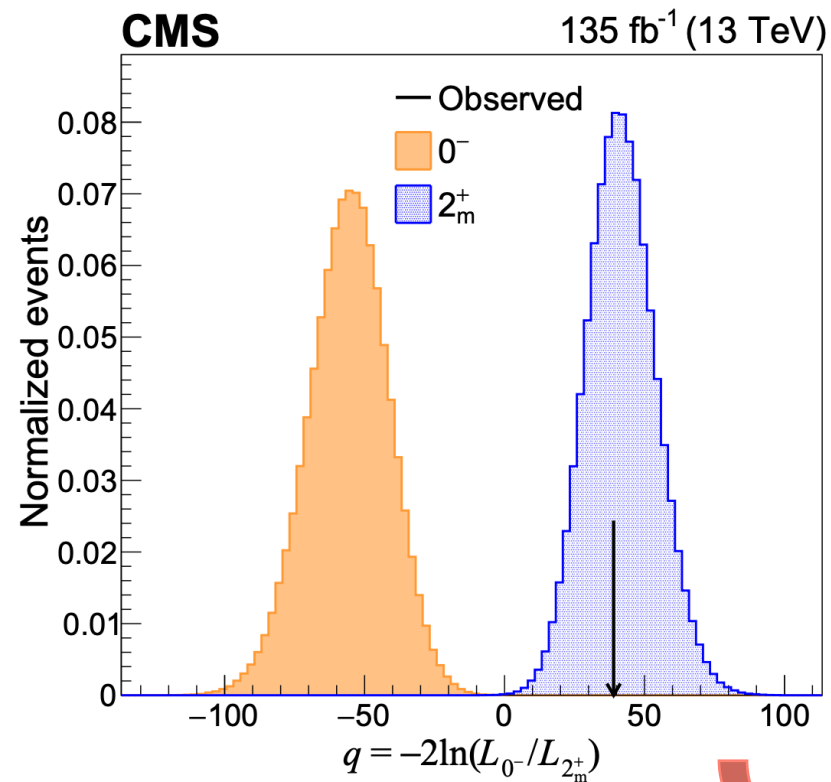


J_X^P	p-value	Z-score	reject J_X^P
0^-	2.7×10^{-13}	7.2	
0_m^+	4.3×10^{-5}	3.9	
0_{mix}^+	1.4×10^{-2}	2.2	
0_h^+	3.1×10^{-9}	5.8	
1^-	8.0×10^{-8}	5.2	
1^+	4.7×10^{-3}	2.6	
2_m^-	4.1×10^{-12}	6.8	
2_{mix}^-	6.5×10^{-4}	3.2	
2_h^-	2.2×10^{-8}	5.5	

mix

mix

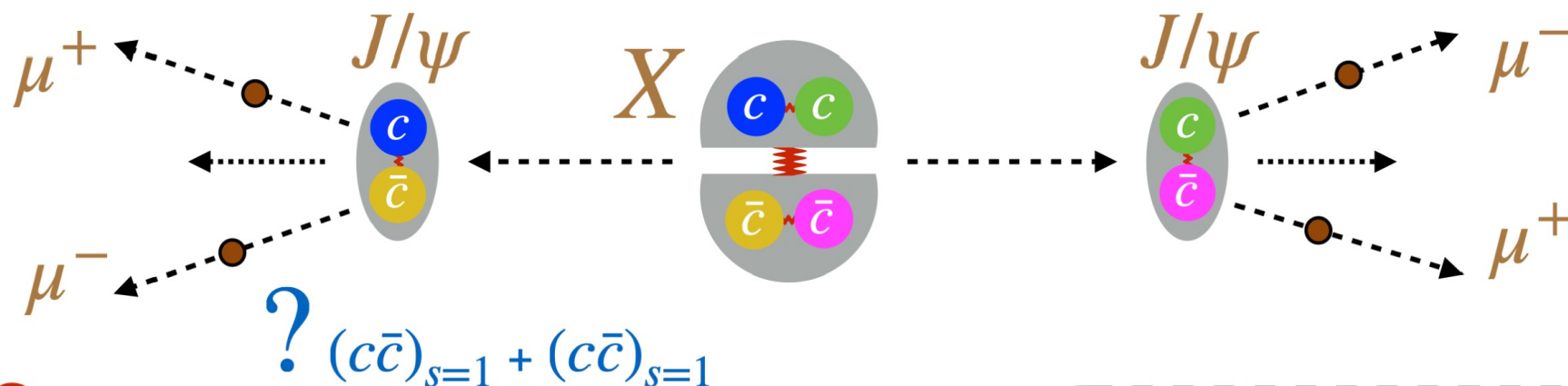
Statistical Analysis



Scan mixture of 0⁺⁺ amplitudes

Scan mixture of 2⁻⁺ amplitudes

Summary



? $(cc)_{s=1} + (\bar{c}\bar{c})_{s=1} \Rightarrow L = 0 \text{ (nS)} : S = 2 \Rightarrow J^{PC} = 2^{++}, n = (1,)2,3,4,\dots$

- J^{PC} analysis of **exotic hadron decays** at LHC (production-independent)
 - consistent picture: set of **3 exotic tetraquark resonances** with the same J^{PC}
 - $PC = ++$ very certain $n = (1,)2,3,4$
 - $J \neq 1$ at $> 99\%$ CL
 - $J \neq 0$ at $> 95\%$ CL
 - $J > 2$ possible, but highly unlikely, require $L \geq 2$
 - $J = 2$ consistent, rare in nature, naively expected $J = 0$

THANKS!

CMS is painting a coherent picture of $J/\psi J/\psi$ structures

BACKUP

J/ψ polarizations

(1) $m_{4\mu}$ spectrum $X \rightarrow 4\mu$ — [arXiv:2306.07164](https://arxiv.org/abs/2306.07164)

(2) p_T and p_Z of $X \rightarrow 4\mu$ — match to data

(3) polarization J_z or $J_{z'}$ of X — unpolarized

for $J = 0$ exact

for $J = 1, 2, \dots$ depends on production mechanism

— vary J_z or $J_{z'}$ systematics or test

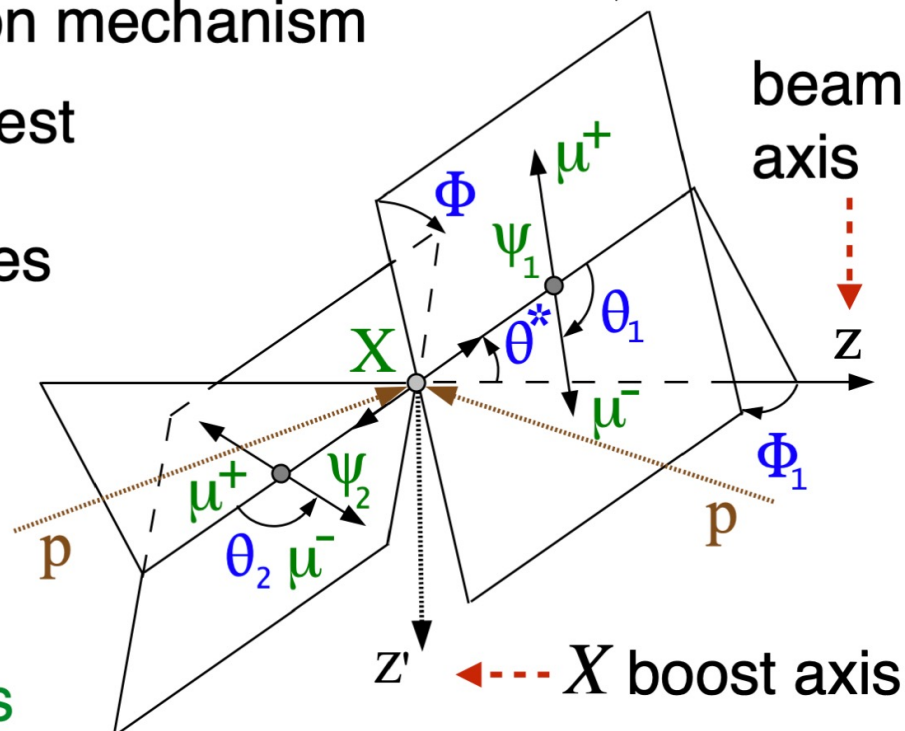
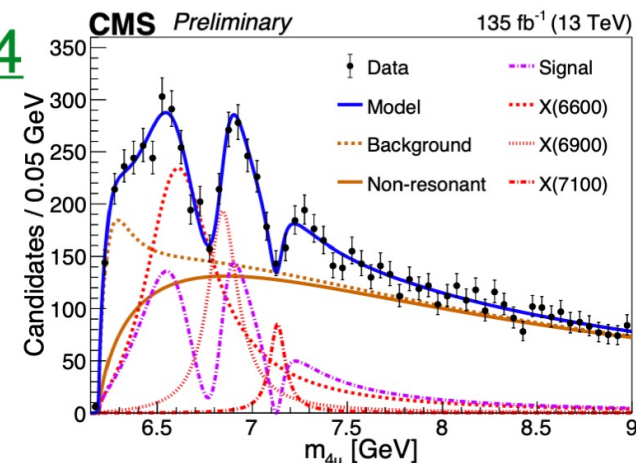
(4) Φ_1, θ^* or Φ'_1, θ'^* production angles

flat for unpolarized — test in data

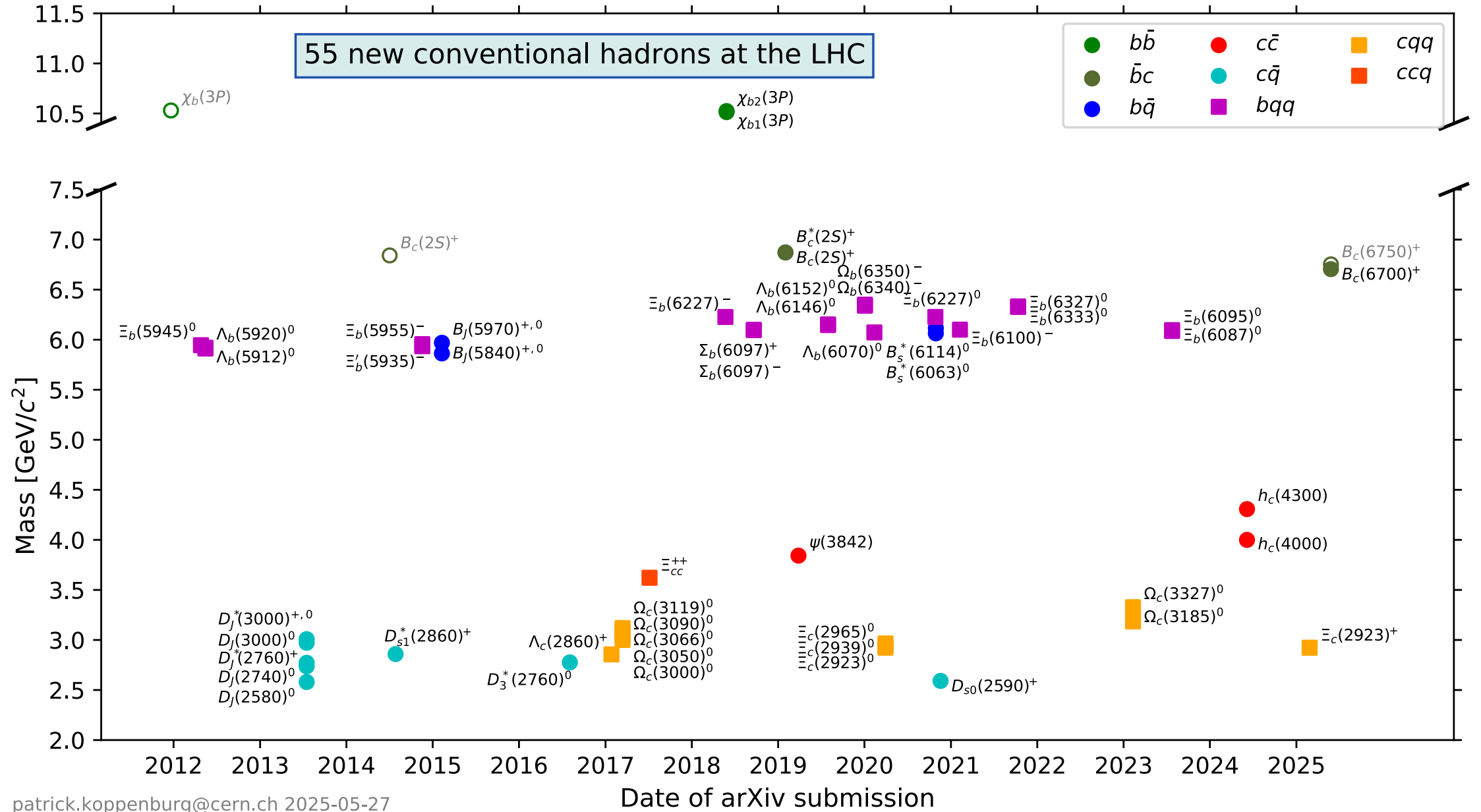
non-flat for polarized

do not use in the primary analysis

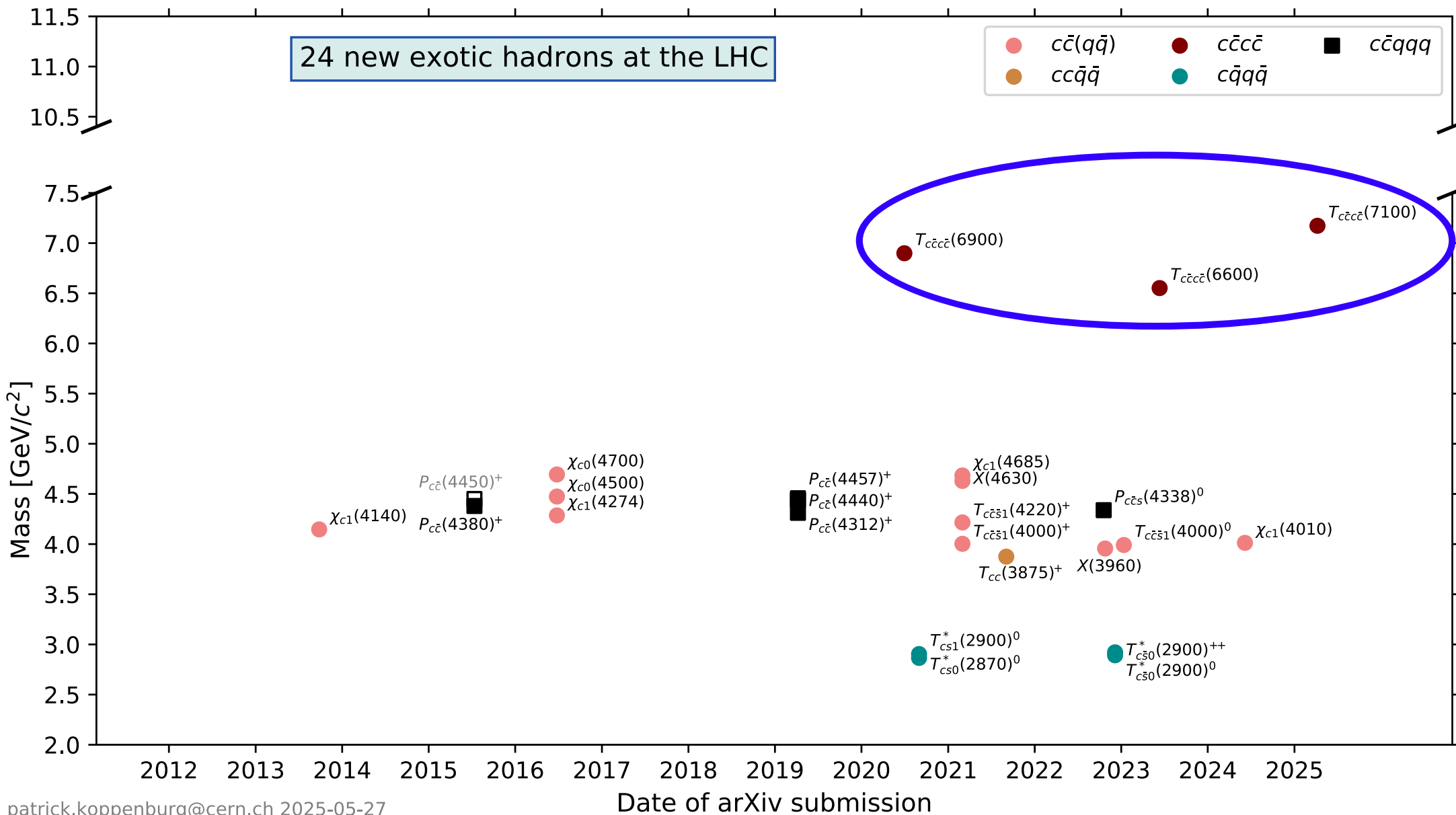
(5) Φ, θ_1, θ_2 decay angles — analysis




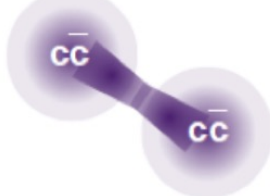
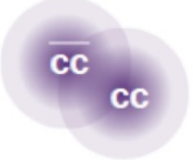
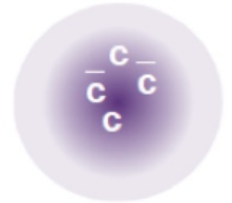

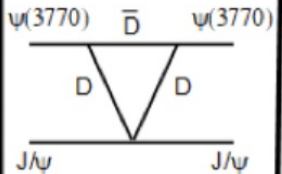
New conventional hadrons at LHC



New exotic hadrons at LHC



Status

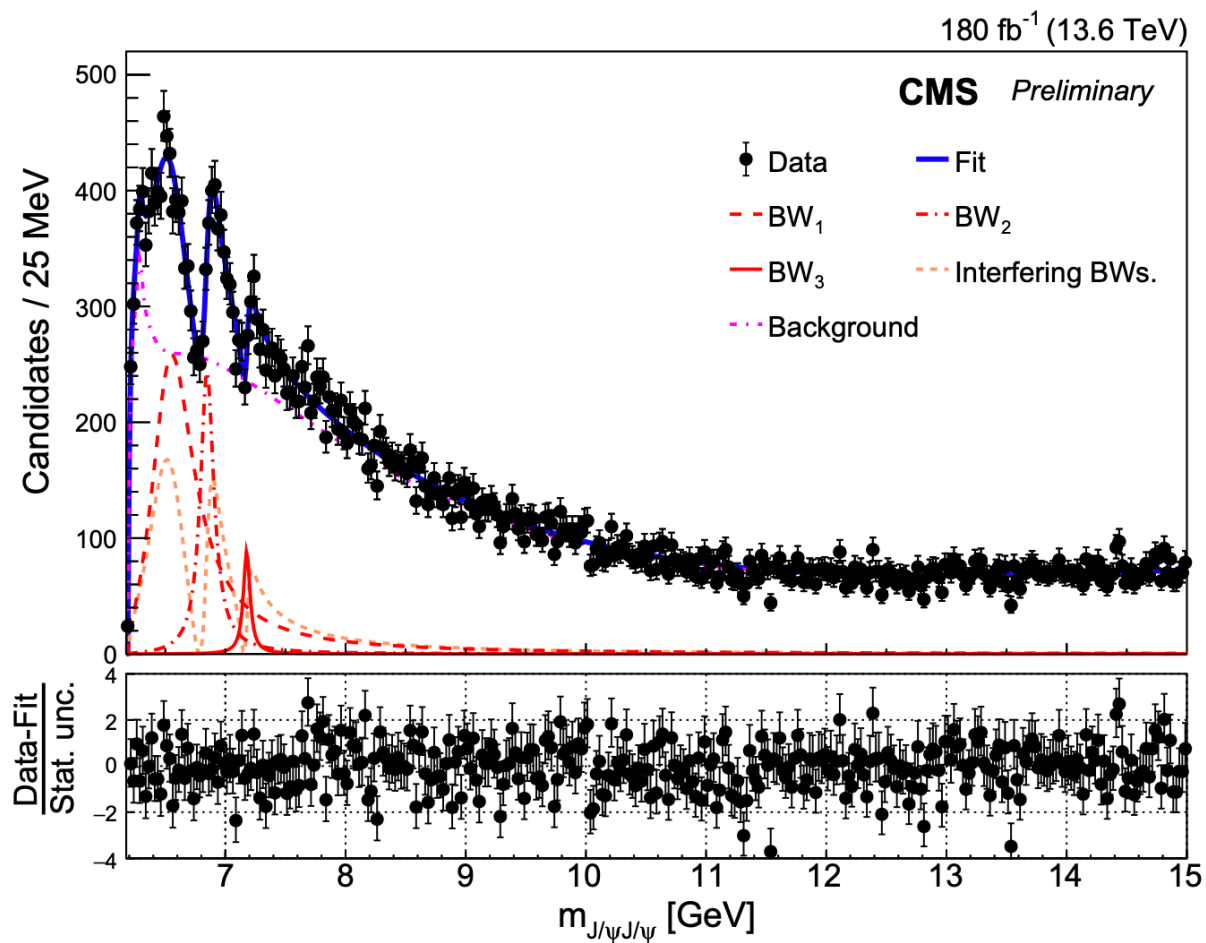
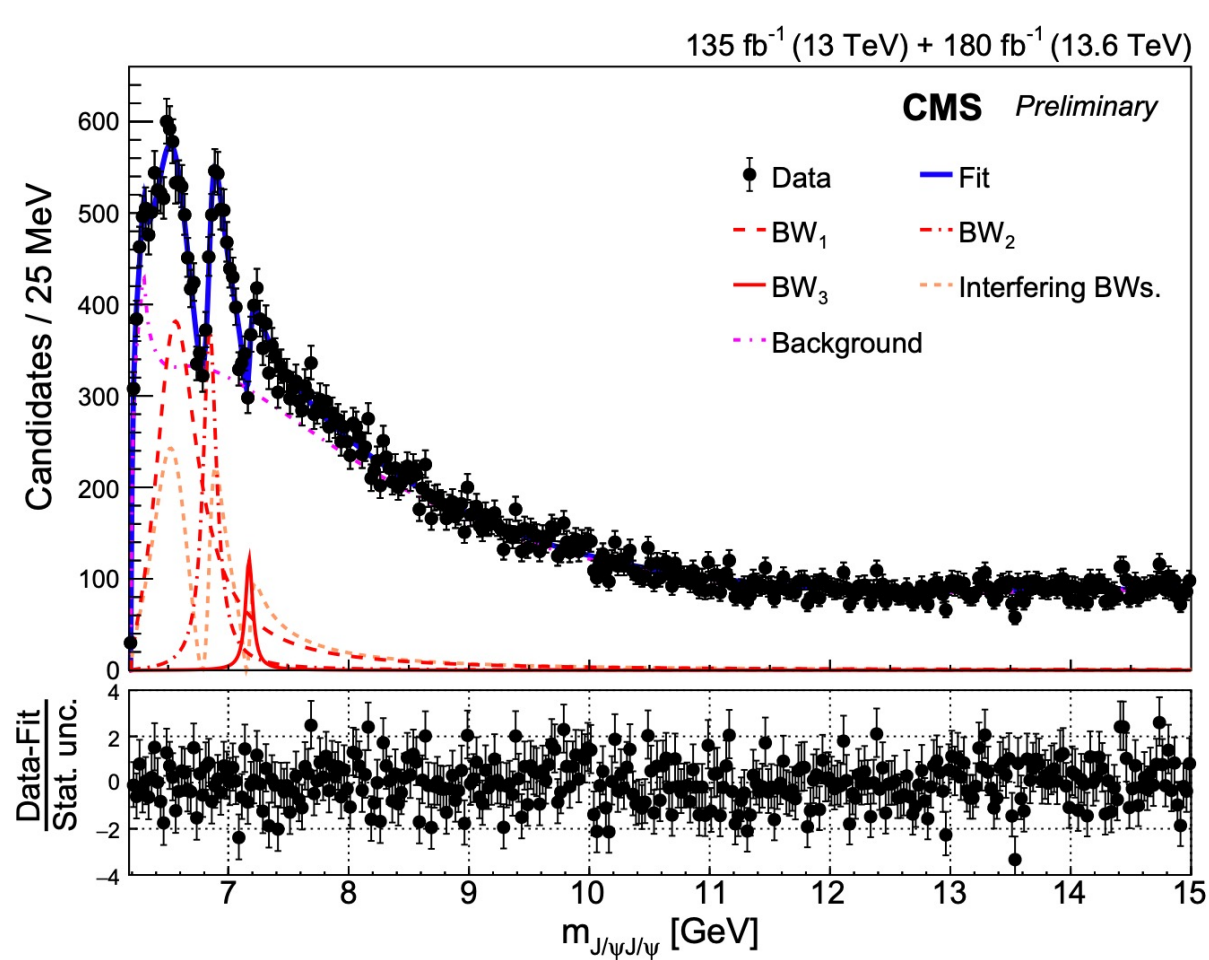
Standard Mesons	Exotic Mesons: Tetracharm				Threshold Effects
	Molecule	Diquark	Compact (Amorphous)	Hybrid	e.g. Triangle Singularity
					

❖ Models of potential quark configurations for $J/\psi J/\psi$ mesons.

- Meson-meson “molecule” ($c\bar{c} - c\bar{c}$)
- Pair of diquarks ($cc - \bar{c}\bar{c}$)
- Hybrid with a valence gluon
- Peaks as artifact of dicharmonia production thresholds
-

*Family of all-charm tetraquarks with same J^{PC}
offers new perspectives on interpretation for **exotics***

$J/\psi J/\psi$: 6-15 GeV fits



Fit model

□ **Final 2D fit model (0^+ vs. 0^-):**

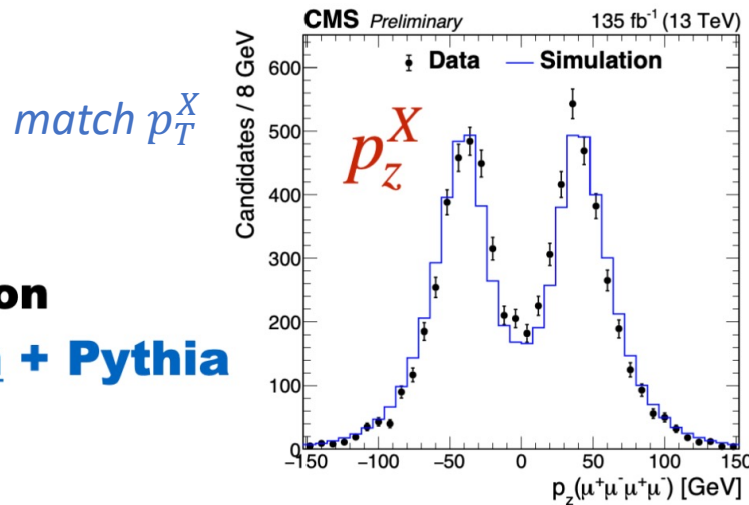
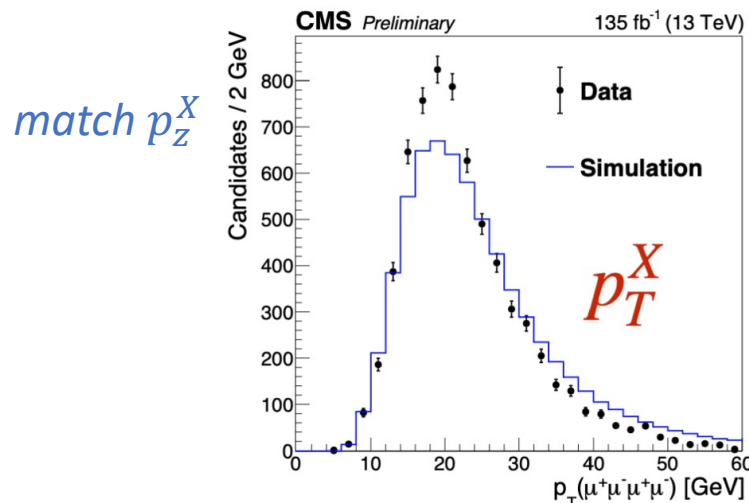
$$P(m_X, D_{0^-}) = \dots\dots$$

$$+ N_{(interf-BW1BW2BW3)} * [f_{0_m^+} * P_{0_m^+}(interf-BW1BW2BW3)(m_X, D_{0^-}) \\ + (1 - f_{0_m^+}) * P_{0^-}(interf-BW1BW2BW3)(m_X, D_{0^-})]$$

$f_{0_m^+}$: fraction of 0_m^+ signal component

Concept of Analysis: Production

- We do not know the production mechanism
 - empirical model to reproduce p_T^X and p_z^X in data
 - tune **Pythia** to match p_T^X in **sideband** and **signal region**
 - fine-tune re-weighting p_T^X
 - residual p_T^X and p_z^X consistency tests coverage in systematics
 - essential to model **detector acceptance**



Simulation
JHUGen + Pythia

Concept of Analysis: Production

- We do not know the production mechanism
 - empirical model to reproduce p_T^X and p_z^X in data

- Monte Carlo tools:

JHUGen

to model spin correlations

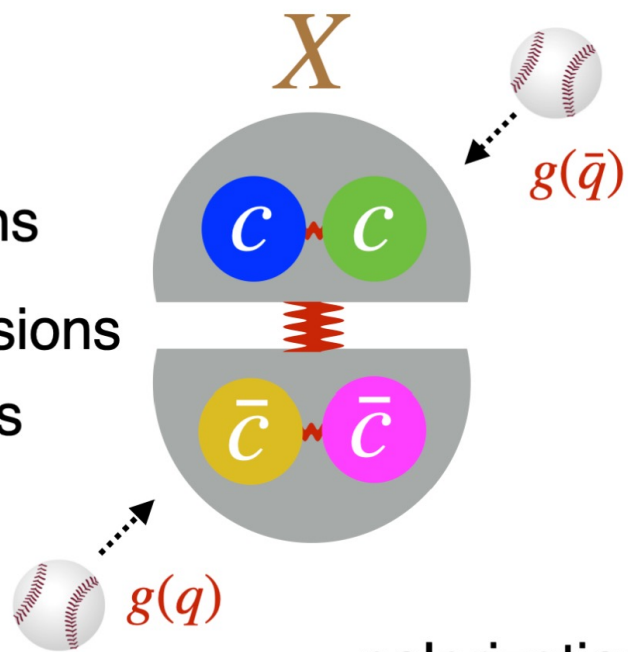
(A) parton ($gg/q\bar{q}$) collisions
polarization J_z beam axis

[arXiv:2109.13363](https://arxiv.org/abs/2109.13363)

MELA

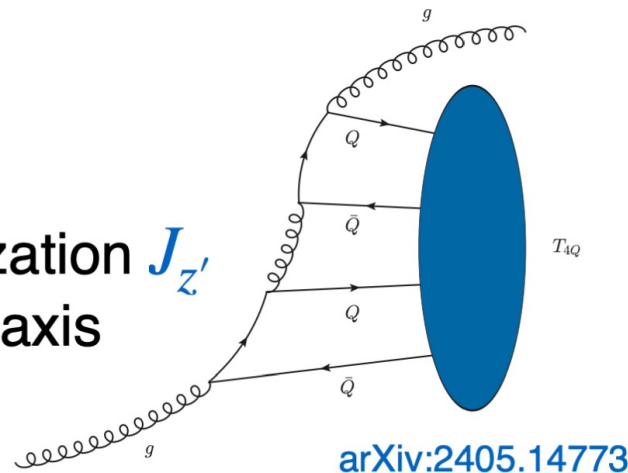
matrix elements

- re-weight $J = 1, 2$ to unpolarized
- re-weight J_z or $J_{z'}$ for systematics



(B) gluon (quark) fragmentation

polarization $J_{z'}$
boost axis



[arXiv:2405.14773](https://arxiv.org/abs/2405.14773)

Angular Analysis

- Observations:
 - 1^+ & 1^- identical in 1D, differ in 3D
 - 0^+ & 1^+ cannot be distinguished from general 2^+
 - unique to 2^+ (or $J \geq 2$): A_{+-} , A_{-+} , “mixture” of 0^+ & 1^+
 - 0^- & 2_m^- identical
 - 1^- & 2_h^- identical
 - unique to 2^- : “mixture”
 - for $J \geq 3$

$$J^P \Leftrightarrow 2^P$$
 - polarized $J \geq 1$
 - unique Φ_1, θ^*
 - not used here...

[arXiv:1001.3396](https://arxiv.org/abs/1001.3396)

$$F_{0,0}^J(\theta^*) \times \left[4|A_{00}|^2 \sin^2 \theta_1 \sin^2 \theta_2 + 2|A_{++}| |A_{--}| \sin^2 \theta_1 \sin^2 \theta_2 \cos(2\Phi - \phi_{--} + \phi_{++}) \right. \\ \left. + |A_{++}|^2 (1 + 2A_{f1} \cos \theta_1 + \cos^2 \theta_1) (1 + 2A_{f2} \cos \theta_2 + \cos^2 \theta_2) \right. \\ \left. + |A_{--}|^2 (1 - 2A_{f1} \cos \theta_1 + \cos^2 \theta_1) (1 - 2A_{f2} \cos \theta_2 + \cos^2 \theta_2) \right. \\ \left. + 4|A_{00}| |A_{++}| (A_{f1} + \cos \theta_1) \sin \theta_1 (A_{f2} + \cos \theta_2) \sin \theta_2 \cos(\Phi + \phi_{++}) \right. \\ \left. + 4|A_{00}| |A_{--}| (A_{f1} - \cos \theta_1) \sin \theta_1 (A_{f2} - \cos \theta_2) \sin \theta_2 \cos(\Phi - \phi_{--}) \right] \quad \text{spin} = 0 \ \& \ \geq 1$$

$$+F_{1,1}^J(\theta^*) \times \left[2|A_{+0}|^2 (1 + 2A_{f1} \cos \theta_1 + \cos^2 \theta_1) \sin^2 \theta_2 + 2|A_{-0}|^2 \sin^2 \theta_1 (1 - 2A_{f2} \cos \theta_2 + \cos^2 \theta_2) \right. \\ \left. + 2|A_{-0}|^2 (1 - 2A_{f1} \cos \theta_1 + \cos^2 \theta_1) \sin^2 \theta_2 + 2|A_{+0}|^2 \sin^2 \theta_1 (1 + 2A_{f2} \cos \theta_2 + \cos^2 \theta_2) \right. \\ \left. + 4|A_{+0}| |A_{-0}| (A_{f1} + \cos \theta_1) \sin \theta_1 (A_{f2} - \cos \theta_2) \sin \theta_2 \cos(\Phi + \phi_{+0} - \phi_{-0}) \right. \\ \left. + 4|A_{+0}| |A_{-0}| (A_{f1} - \cos \theta_1) \sin \theta_1 (A_{f2} + \cos \theta_2) \sin \theta_2 \cos(\Phi + \phi_{+0} - \phi_{-0}) \right] \quad \text{spin} \geq 1$$

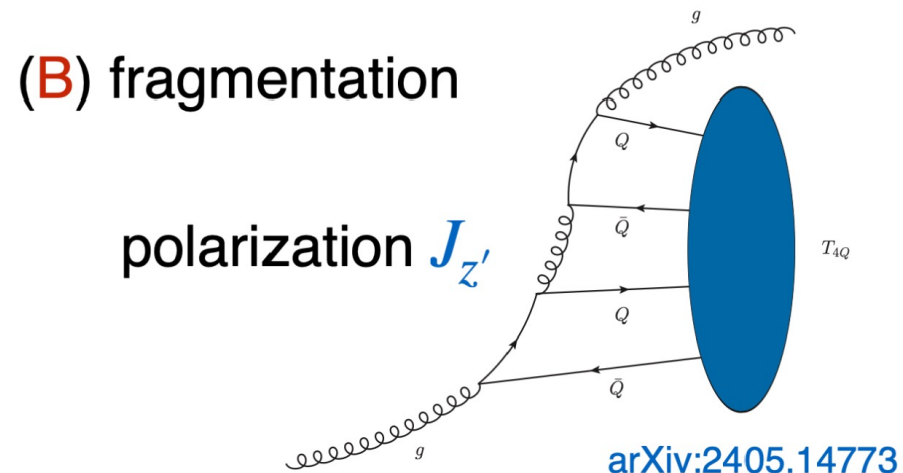
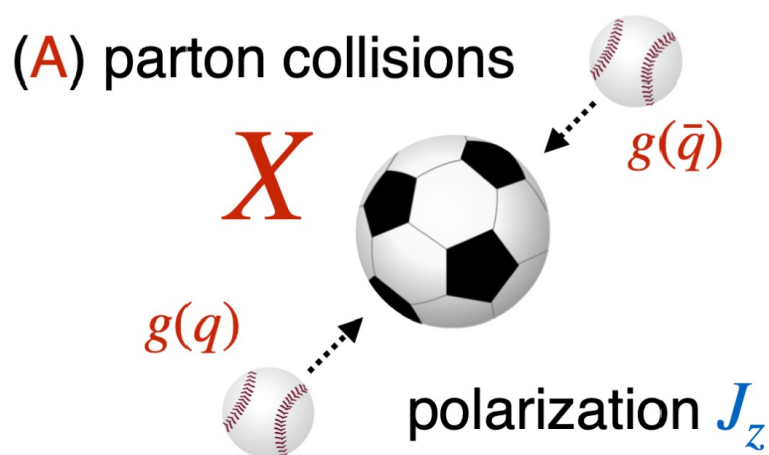
$$+F_{1,-1}^J(\theta^*) \times \left[4|A_{+0}| |A_{0+}| (A_{f1} + \cos \theta_1) \sin \theta_1 (A_{f2} + \cos \theta_2) \sin \theta_2 \cos(2\Psi - \phi_{+0} + \phi_{0+}) \right. \\ \left. + 4|A_{-0}| |A_{0-}| (A_{f1} - \cos \theta_1) \sin \theta_1 (A_{f2} - \cos \theta_2) \sin \theta_2 \cos(2\Psi - \phi_{0-} + \phi_{-0}) \right. \\ \left. + 4|A_{+0}| |A_{-0}| \sin^2 \theta_1 \sin^2 \theta_2 \cos(2\Psi - \Phi - \phi_{+0} + \phi_{-0}) + 4|A_{-0}| |A_{0+}| \sin^2 \theta_1 \sin^2 \theta_2 \cos(2\Psi + \Phi - \phi_{0-} + \phi_{0+}) \right]$$

$$+F_{2,2}^J(\theta^*) \times \left[|A_{+-}|^2 (1 + 2A_{f1} \cos \theta_1 + \cos^2 \theta_1) (1 - 2A_{f2} \cos \theta_2 + \cos^2 \theta_2) \right. \\ \left. + |A_{-+}|^2 (1 - 2A_{f1} \cos \theta_1 + \cos^2 \theta_1) (1 + 2A_{f2} \cos \theta_2 + \cos^2 \theta_2) \right] \quad \text{spin} \geq 2$$

$$+F_{2,-2}^J(\theta^*) \times \left[2|A_{+-}| |A_{-+}| \sin^2 \theta_1 \sin^2 \theta_2 \cos(4\Psi - \phi_{+-} + \phi_{-+}) \right] + \text{other 26 interference terms for spin}$$

$$\text{where } \Psi = \Phi_1 + \Phi/2 \quad \text{and} \quad F_{ij}^J(\theta^*) = \sum_{m=0,\pm 1,\pm 2} f_m d_{im}^J(\theta^*) d_{jm}^J(\theta^*)$$

Polarization in Production



- Helicity amplitudes appear in production. For parton collision:
 - spin-0: unpolarized in any case, e.g. $gg \rightarrow X$
 - spin-1: $q\bar{q} \rightarrow X$ produce $J_z = \pm 1$ (not 0!)
 - spin-2: $gg \rightarrow X$ produce $J_z = 0, \pm 2$, minimal coupling: $J_z = \pm 2$
 $q\bar{q} \rightarrow X$ produce $J_z = \pm 1$
- Similar ideas in fragmentation of g or Q
 - re-weight MELA to any model: unpolarized, polarized z' or z

Lorentz-Invariant Amplitude

- Expect three X resonances to have the same **tensor structure**:

$$A(X_{J=0} \rightarrow V_1 V_2) = \left(a_1(q^2) m_V^2 \epsilon_1^* \epsilon_2^* + a_2(q^2) f_{\mu\nu}^{*(1)} f^{*(2),\mu\nu} + a_3(q^2) f_{\mu\nu}^{*(1)} \tilde{f}^{*(2),\mu\nu} \right)$$

0_m^+

0_h^+

0^-

$A_{00} = A_{++} = A_{--}$ at $2m_{J/\psi}$ threshold

$A_{++} = -A_{--}$

A_{00} at large m_X $A_{++} = A_{--}$

[arXiv:1001.3396](https://arxiv.org/abs/1001.3396)

empirical **form factors** ($m_{4\mu}^2$)

$$A(X_{J=1} \rightarrow V_1 V_2) = \left(b_1(q^2) \left[(\epsilon_1^* q)(\epsilon_2^* \epsilon_X) + (\epsilon_2^* q)(\epsilon_1^* \epsilon_X) \right] + b_2(q^2) \epsilon_{\alpha\mu\nu\beta} \epsilon_X^\alpha \epsilon_1^{*,\mu} \epsilon_2^{*,\nu} \tilde{q}^\beta \right)$$

1^-

1^+

more for spin-2

$A_{+0} = -A_{0+} = A_{-0} = -A_{0-}$

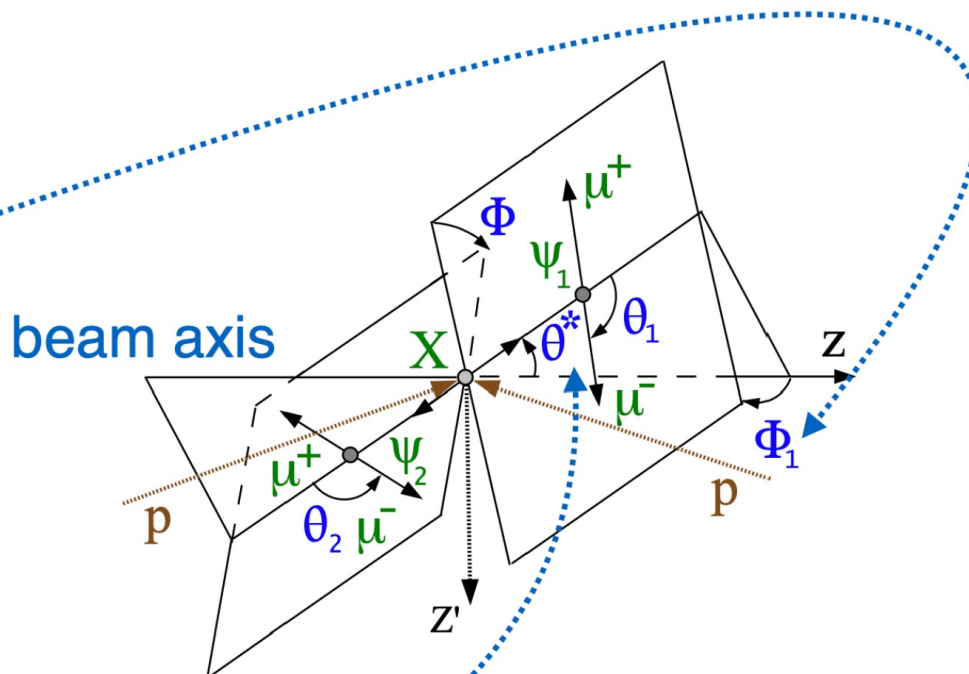
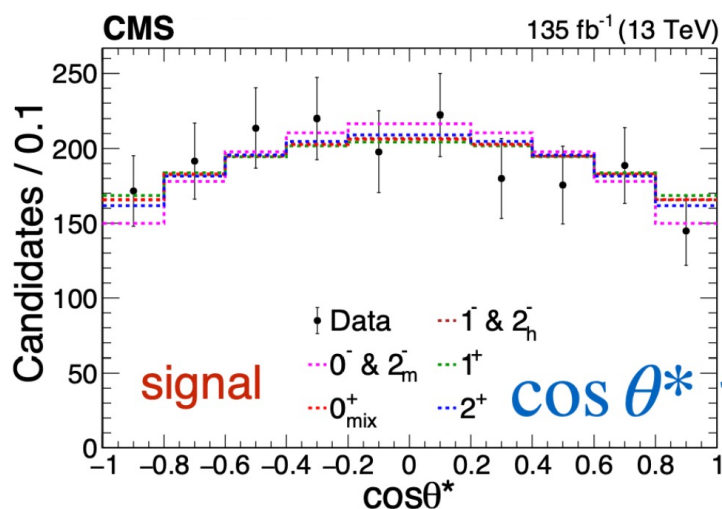
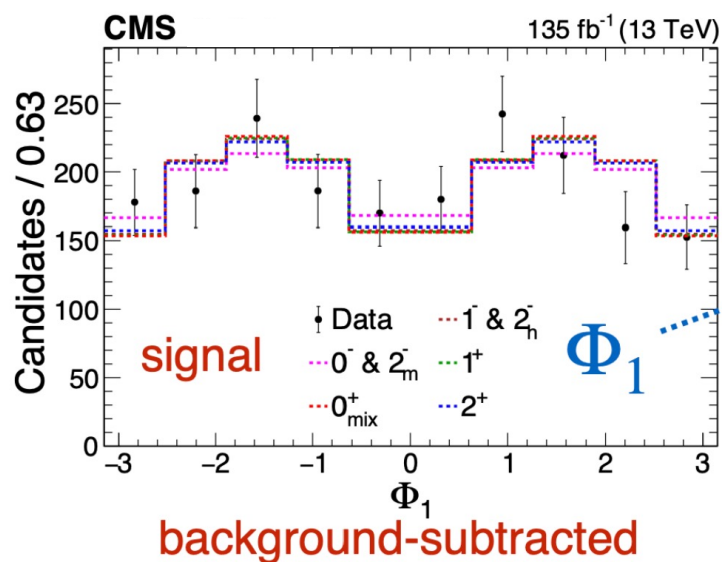
$A_{+0} = -A_{0+} = -A_{-0} = A_{0-}$

Backup

Production Angles

(4) production angles consistent with **unpolarized** resonances

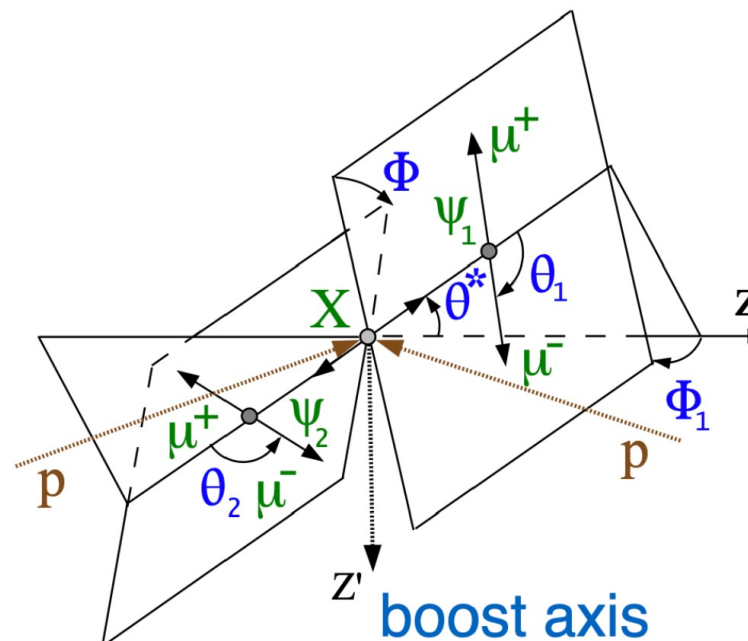
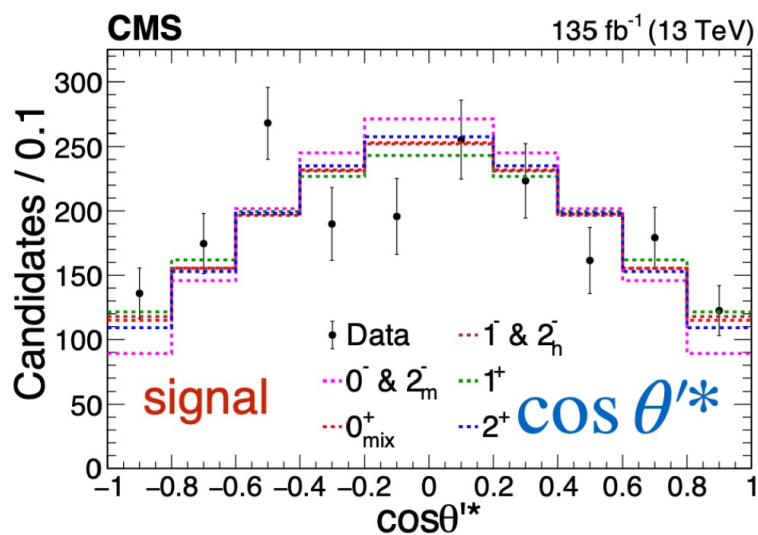
with respect to the **beam axis**



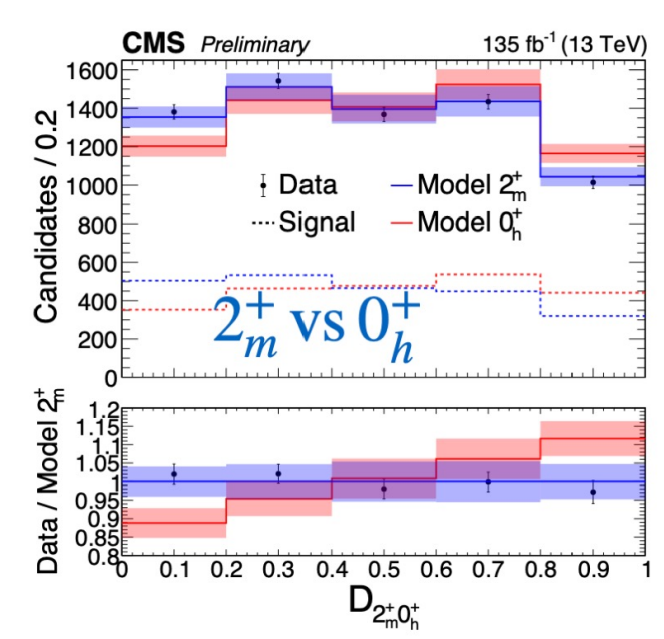
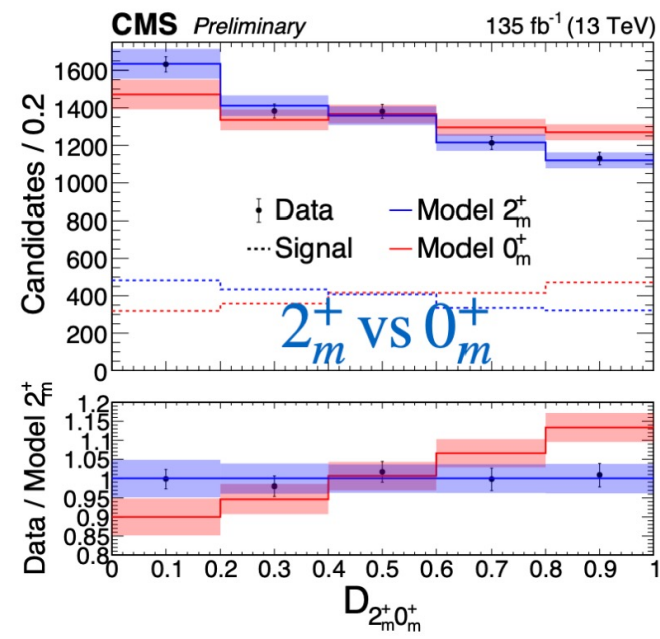
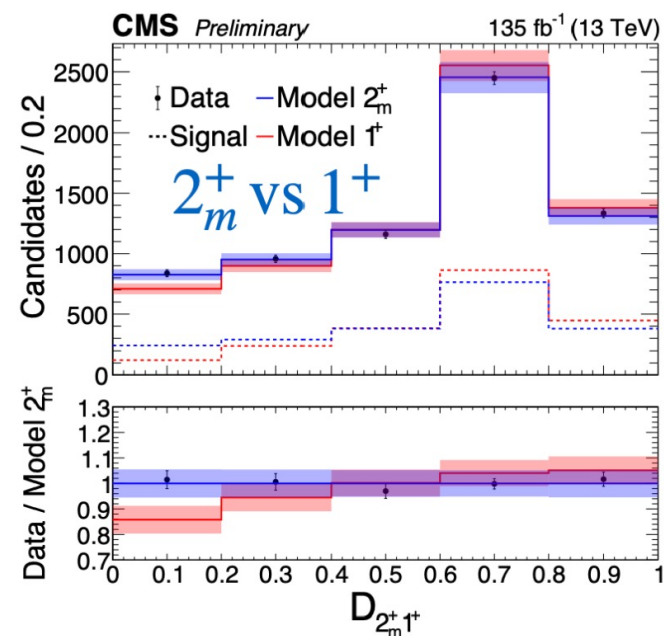
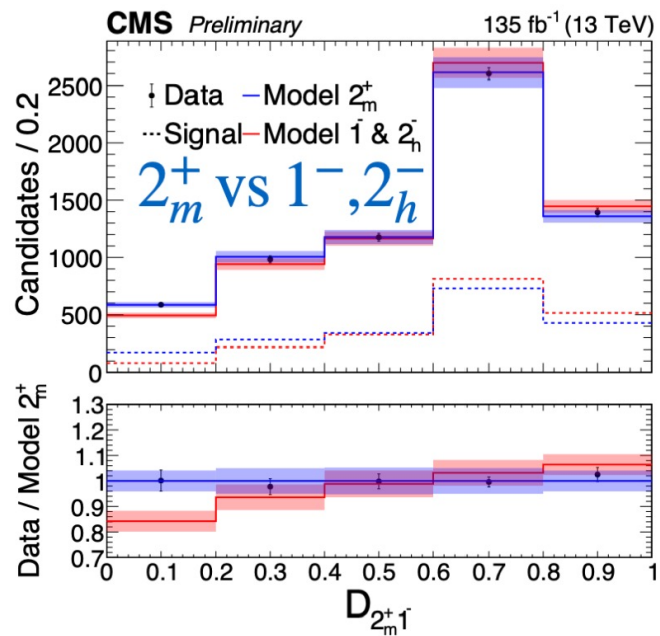
acceptance effects
⇒ distributions not flat

with respect to the **boost axis**

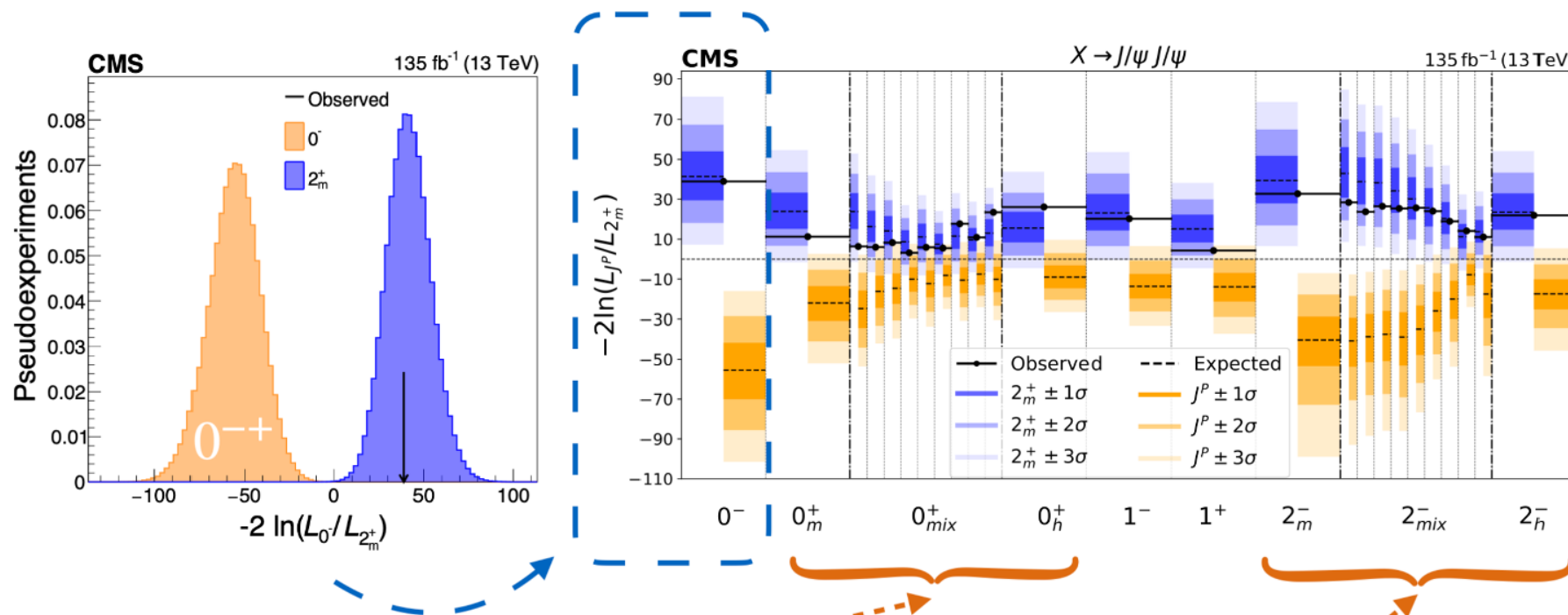
does not prove unpolarized



Discriminant Distributions



Hypothesis test



- Scan mixture of two 0^{++} amplitudes

- Scan mixture of two 2^{--} amplitudes

- Data are consistent with a 2^{++} model, inconsistent with others

Summary of Results

- Full set of results, compared to 2_m^+

$P = -1$

		Observed		Expected	
		p-value	Z-score	p-value	Z-score
0^- vs 2_m^+	0^-	2.7×10^{-13}	7.2	6.5×10^{-14}	7.4
	2_m^+	4.2×10^{-1}	0.2	0.50	0.0
0_m^+ vs 2_m^+	0_m^+	4.3×10^{-5}	3.9	5.6×10^{-9}	5.7
	2_m^+	7.2×10^{-2}	1.5	0.50	0.0
0_{mix}^+ vs 2_m^+	0_{mix}^+	1.4×10^{-2}	2.2	8.4×10^{-4}	3.1
	2_m^+	1.7×10^{-1}	1.0	0.50	0.0
0_h^+ vs 2_m^+	0_h^+	3.1×10^{-9}	5.8	8.5×10^{-5}	3.8
	2_m^+	9.0×10^{-1}	-1.3	0.50	0.0
1^- vs 2_m^+	1^-	8.0×10^{-8}	5.2	6.4×10^{-9}	5.7
	2_m^+	3.8×10^{-1}	0.3	0.50	0.0
1^+ vs 2_m^+	1^+	4.7×10^{-3}	2.6	2.7×10^{-5}	4.0
	2_m^+	5.2×10^{-2}	1.6	0.50	0.0
2_m^- vs 2_m^+	2_m^-	4.1×10^{-12}	6.8	3.9×10^{-14}	7.5
	2_m^+	2.8×10^{-1}	0.6	0.50	0.0
2_{mix}^- vs 2_m^+	2_{mix}^-	6.5×10^{-4}	3.2	1.5×10^{-4}	3.6
	2_m^+	3.1×10^{-1}	0.5	0.50	0.0
2_h^- vs 2_m^+	2_h^-	2.2×10^{-8}	5.5	6.3×10^{-9}	5.7
	2_m^+	4.3×10^{-1}	0.2	0.50	0.0

– $J^{PC} = 2^{++}$
most likely

– $J > 2$ possible
but highly unlikely
require $L \geq 2$

– $J \neq 0$ at $> 95\%$ CL

– confidence level:

$$CL_s = \frac{P(q \geq q_{\text{obs}} | J_j^P + \text{bkg})}{P(q \geq q_{\text{obs}} | J_i^P + \text{bkg})}$$

– $J \neq 1$ at $> 99\%$ CL

– $P \neq -1$ very certain
(exclude J^{-+} including $J \geq 3$)

- Recall: 2^{++} can have a mixture of 2_m^+ and look-alike of $0^+, 1^+$