Determining the minimal mass of a proto-neutron star with chirally constrained nuclear equations of state

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Research Background & Motivation

Observational Challenge: Discovery of several **low-mass compact objects**

PSR J0453+1559 (1.17$M_{\odot}$)
HESS J1731-347 (0.77 $M_{\odot}$)

Theoretical Tension: Below the predicted lower mass limit for neutron stars from modern supernova simulations (~1.2).

Could these objects be Proto-Neutron Stars (PNS) formed by core-collapse supernova?

Research Goal: Determine the **minimal mass** of a PNS using nuclear Equations of State (EOS) constrained by **chiral effective field theory**

Evolutionary Stages & Physical Parameters

Study of two characteristic evolutionary stages:

1. Neutrino-Trapped Phase (t ~ 0.1 s)

Physical State: $Y_L = 0.4,\ s = 1$

Characteristics: Neutrinos are trapped, lepton-rich environment

2. Neutrino-Free Phase (t ~ 10 s)

Physical State: $Y_\nu = 0,\ s = 2$

Characteristics: Neutrinos have diffused out, star is cooling.

Thermodynamic Contribution of Neutrinos

When neutrinos are trapped, they are treated as an ultra-relativistic Fermi gas

$$p = \frac{g}{3}T^4\left[\frac{7\pi^2}{120} + \frac{1}{4}\left(\frac{\mu}{T}\right)^2 + \frac{1}{8\pi^2}\left(\frac{\mu}{T}\right)^4\right],$$

$$n = \frac{g}{6}T^3\left[\frac{\mu}{T} + \frac{1}{\pi^2}\left(\frac{\mu}{T}\right)^3\right],$$

$$\frac{S}{V} = gT^3\left[\frac{7\pi^2}{90} + \frac{1}{6}\left(\frac{\mu}{T}\right)^2\right],$$

Lepton fraction conservation is enforced: $Y_L = Y_e + Y_{\upsilon_e} = const$

Nuclear EOS: Core and Crust Treatment

All employed EOS are constrained by χEFT for pure neutron matter and use specialized descriptions for different density regimes:

Homogeneous Core Description:

Described by **Relativistic Mean-Field** (**RMF**) models.

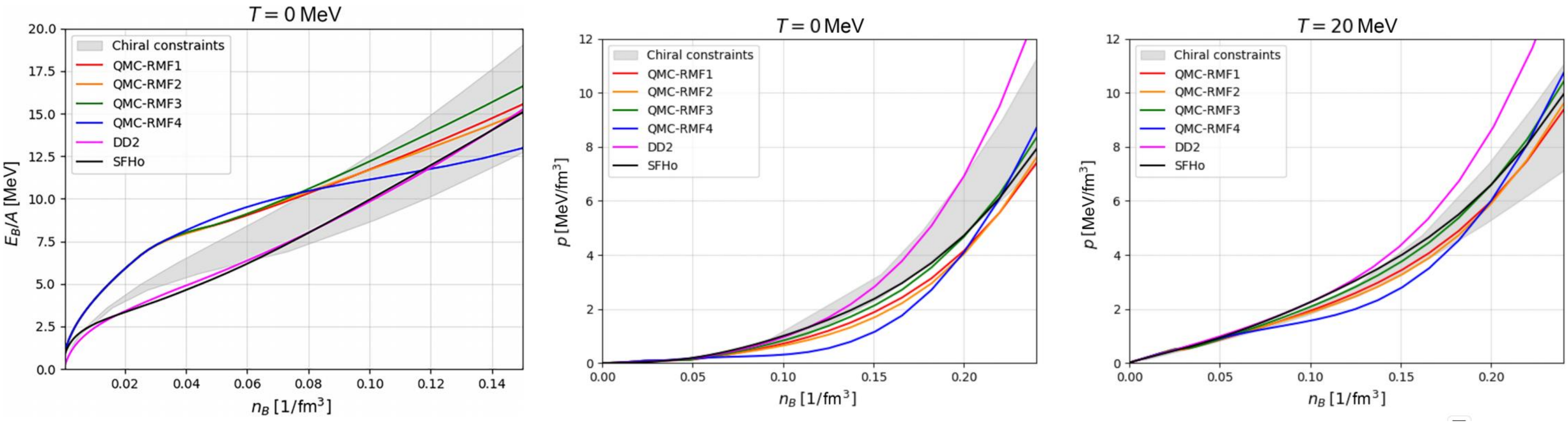
Models used: DD2, SFHo, QMC-RMF1-4.

Inhomogeneous Crust Description:

QMC-RMF series: Use the **HS(IUF)** EOS for consistency in this region.

DD2, SFHo: Employ **specific parameterizations and calculations** for the low-density regime (nuclear lattice, non-uniform matter).

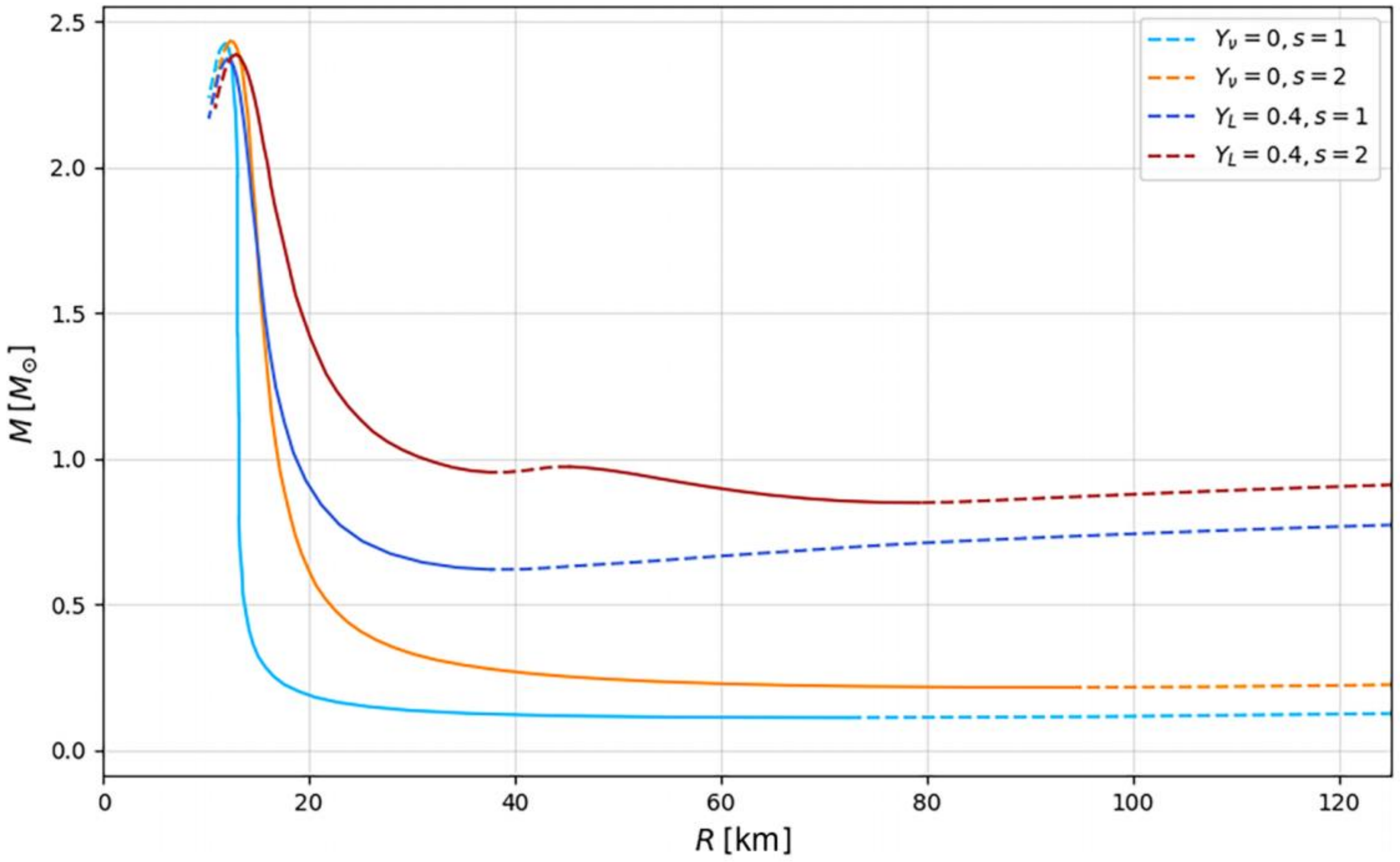
EOS Validation against χEFT Constraints



All selected EOS reasonably satisfy **χEFT** constraints in the relevant density range

($\sim 0.5 n_0$- $1.0 n_0$), providing a reliable foundation for the calculation.

Core Result - Minimal Mass & Radius



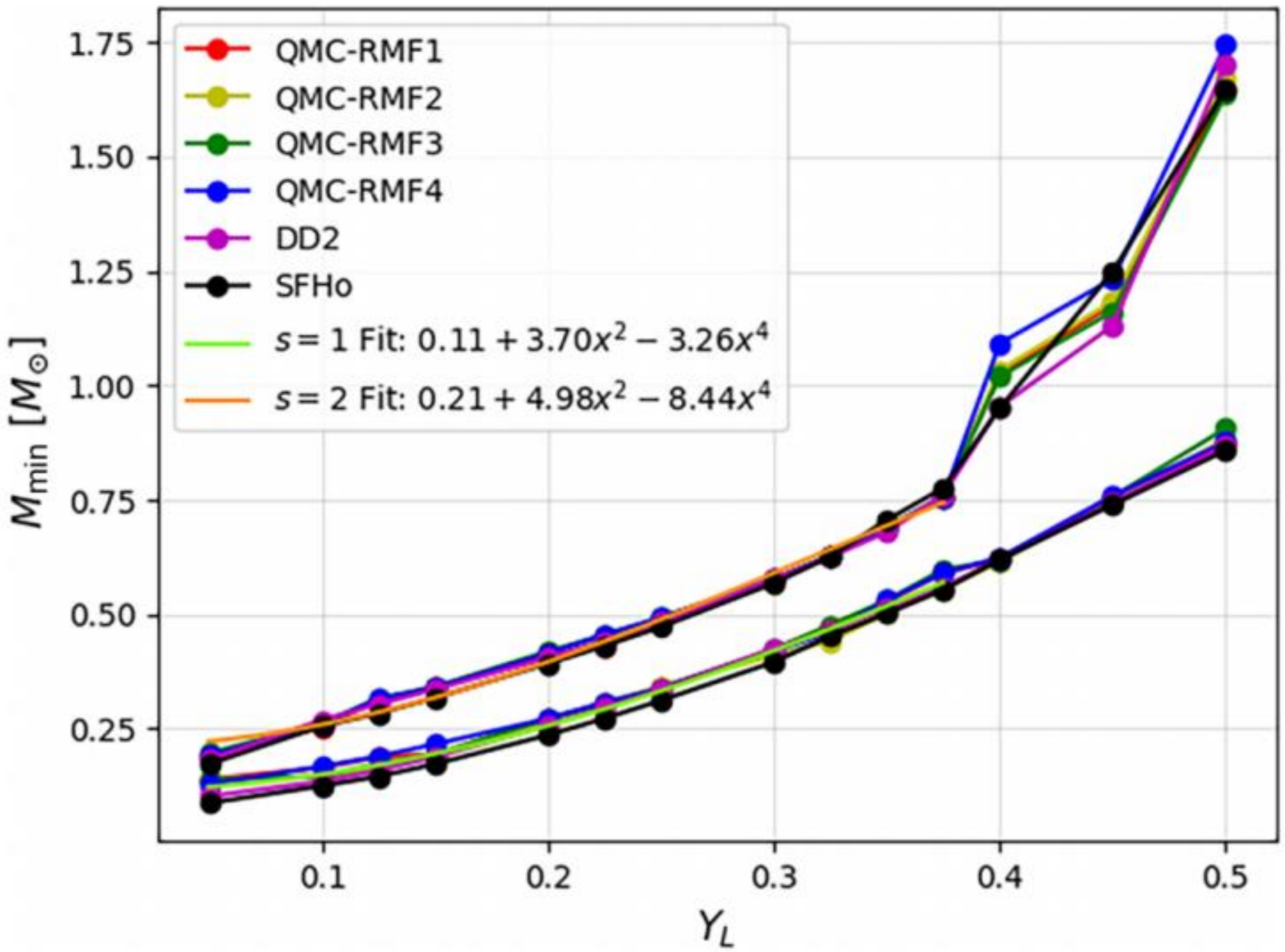
Mass-radius curves for a neutrino-free and neutrino-trapped proto-neutron star, each for a constant entropy per baryon of *s* = 1 and *s* = 2 for DD2

Dominant Neutrino Effect: Additional pressure significantly increases the **minimal mass.**

- $Y_L = 0.4$, $s = 1$: $M_{\min} \sim 0.62\,\mathrm{M}_\odot$, $R \sim 38\,\mathrm{km}$
- $Y_\nu = 0$, $s = 2$: $M_{\min} \sim 0.22\,\mathrm{M}_\odot$, $R \sim 90\,\mathrm{km}$

Thermal effects (increasing *s*) also increase mass, but to a lesser extent.

Universal Relation - M_{min} vs. Y_L



Minimal mass dependence on the lepton fraction for different ratios of the entropy per baryon *s*. *s=1 vs. s=2*

Universal Scaling: The minimal mass increases monotonically with Y_L, following **a model-independent relation.**

$$M_{\rm min}(Y_L) = a + bY_L^2 + cY_L^4$$

Fitting Parameters (Units: M⊙):

s (k_B/baryon)	a	b	c
1	0.11	3.70	-3.26
2	0.21	4.98	-8.44

Summary & Outlook

Robust minimal mass estimates for PNS:

- Neutrino-trapped phase: $M_{\rm min} \sim 0.62\,\mathrm{M}_\odot$
- Neutrino-free phase: $M_{\rm min} \sim 0.22\,\mathrm{M}_\odot$

Key Discovery: A universal relation between M_{min} *and* Y_L

Results are based on EOS constrained by first-principles χEFT calculations.