# Non-thermodynamics and Non-hydrodynamics in Heavy-ion Collisions
# 重离子碰撞中的非热力学与非流体力学

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- **Non-hydrodynamics 非流体力学 (非平衡态)**
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  - hydro with jets 流体和吸引子
  - hydro with hidden symmetries 流体和隐藏的对称性
- **Recap and outlook 总结和展望**

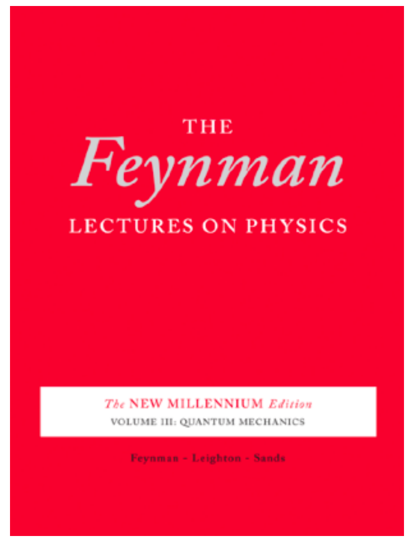
# What is thermodynamics & hydrodynamics?
# 什么是热力学和流体力学?

# What is thermodynamics? 什么是热力学?

- **Thermodynamics** deals with homogeneous and equilibrium systems with large volume and number of particles (Boyle's law, 1662).

$S(E, V, N)$ $\longleftrightarrow$ $p(T, \mu)$

$$T\mathrm{d}S = \mathrm{d}E + p\mathrm{d}V - \mu\mathrm{d}N \qquad V\mathrm{d}p = S\mathrm{d}T + N\mathrm{d}\mu$$



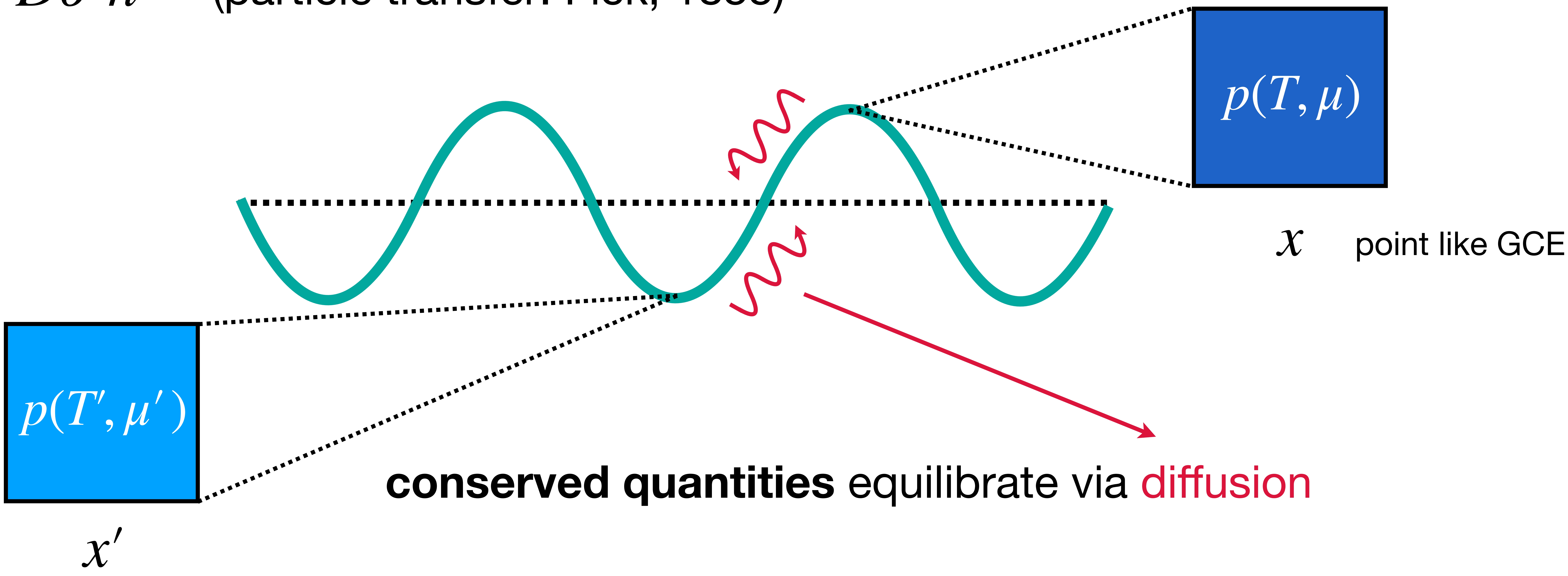

*The subject of thermodynamics is complicated because there are so many different ways of describing the same thing.*
— Richard Feynman

# What is hydrodynamics? 什么是流体力学?

- **Hydrodynamics** deals with a collection of inhomogeneous and nonequilibrium thermodynamics systems.

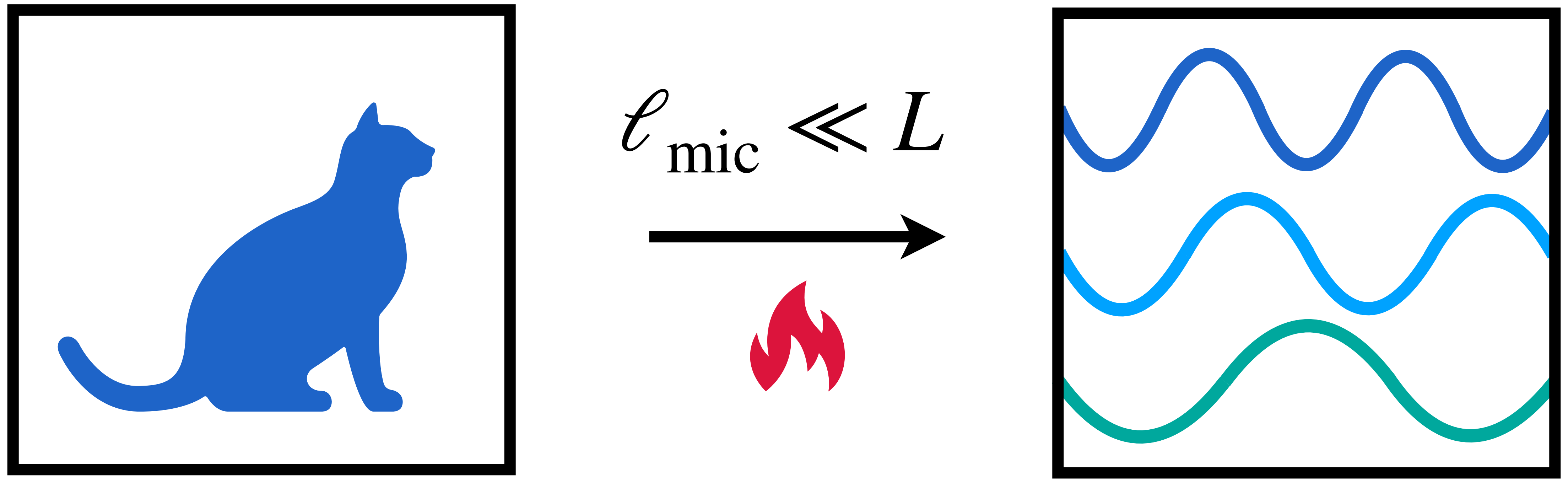
$$\partial_t \vec{v} = -\vec{v}\cdot\vec{\partial}\vec{v} - \vec{\partial}p + \nu\vec{\partial}^2\vec{v}$$ (momentum transfer: Navier-Stokes, 1822)

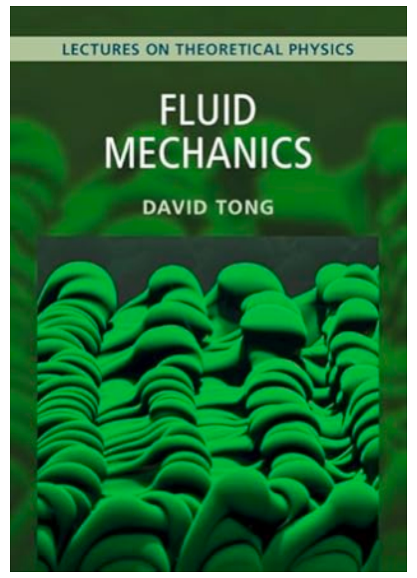
$$\partial_t n = D\partial^2 n$$ (particle transfer: Fick, 1855)



# Why hydrodynamics? 为什么要研究流体力学?

- **Hydrodynamics** is a universal effective theory that
  - associates with conserved quantities (symmetries) dominating at large scales;
  - describes near equilibrium states approximated by small gradient expansion.



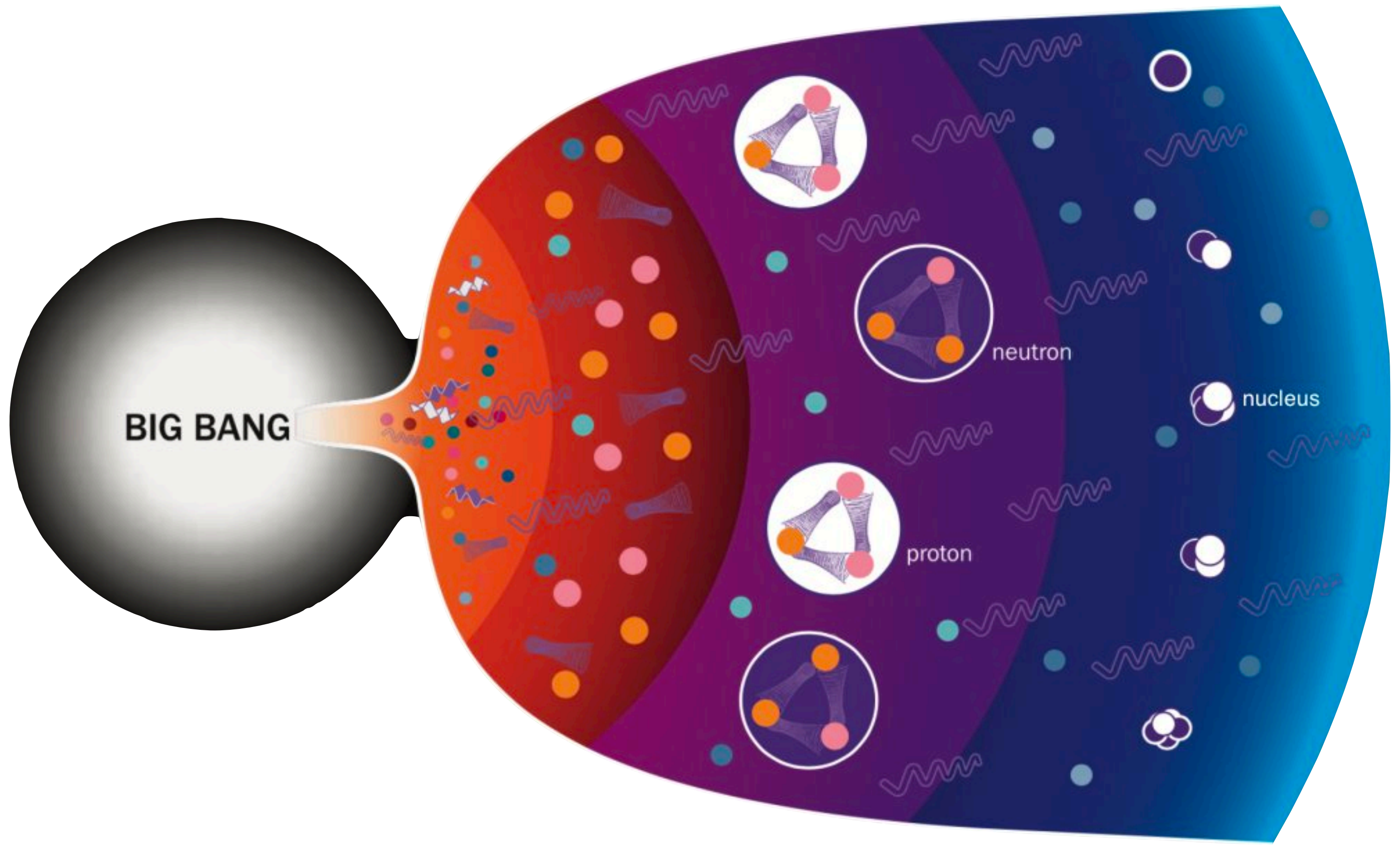




*Take anything in the universe, throw a bunch of it in a box, and turn up the heat. Then it doesn't matter what you started with, the motion of this substance will be governed by the equations of fluid dynamics.*

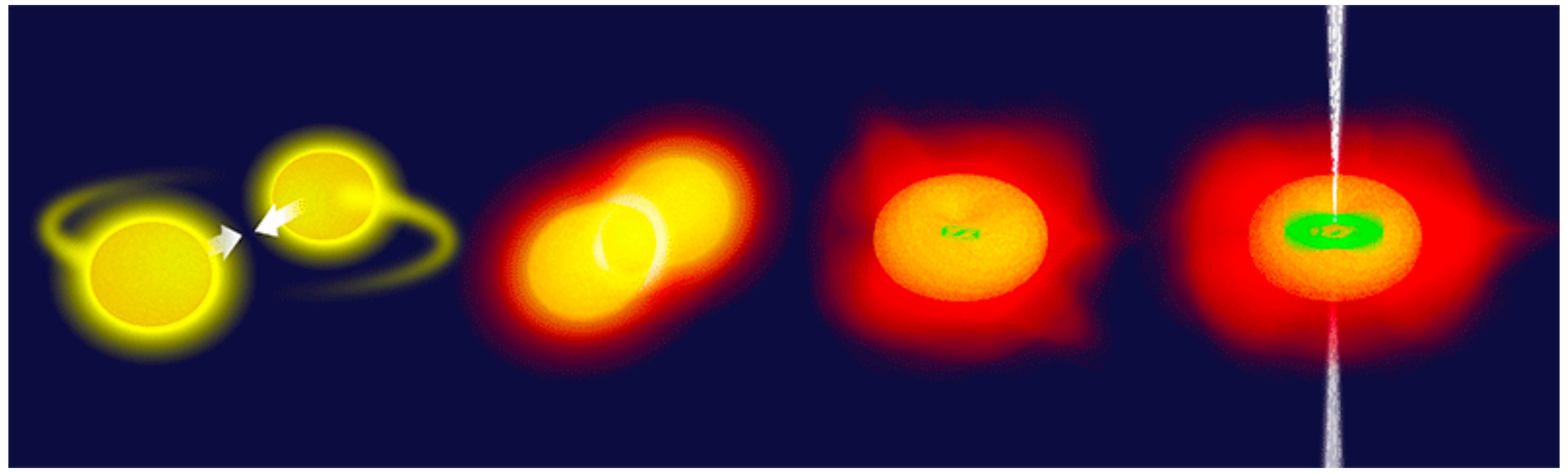
— David Tong

# Universality of thermodynamics/hydrodynamics? 热力学/流体力学的普适性

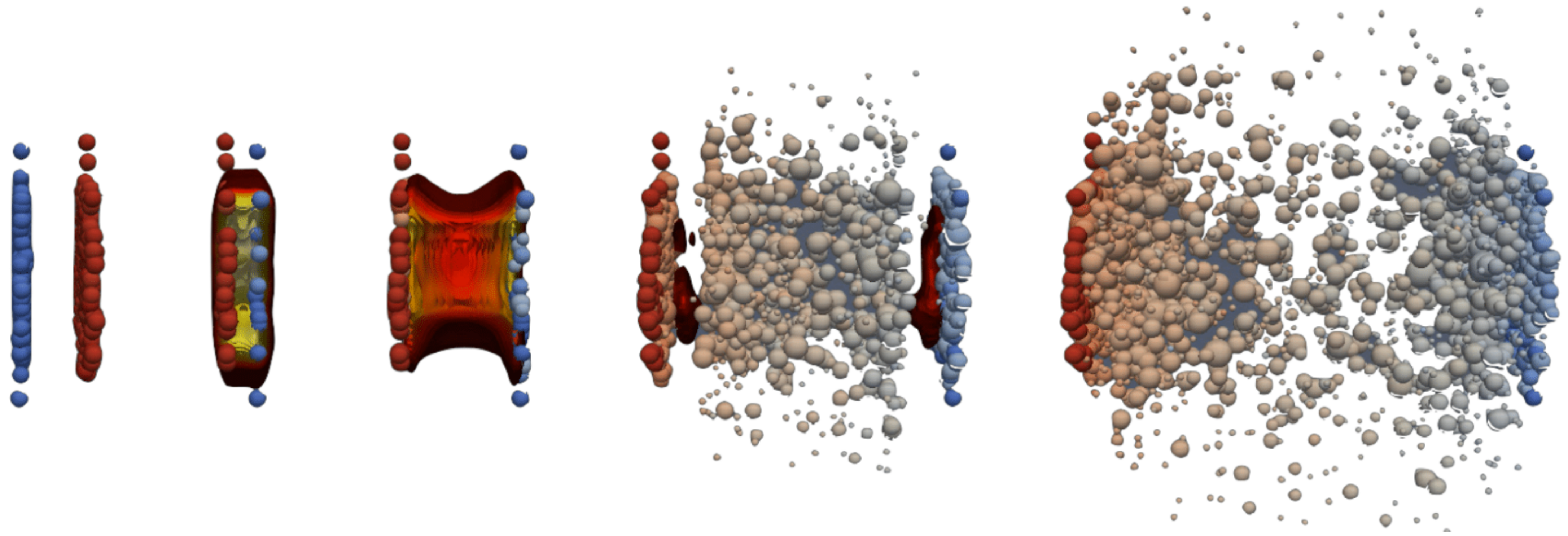
- Thermodynamics/hydrodynamics describes systems across different scales: from **big bang** to **small bang**.



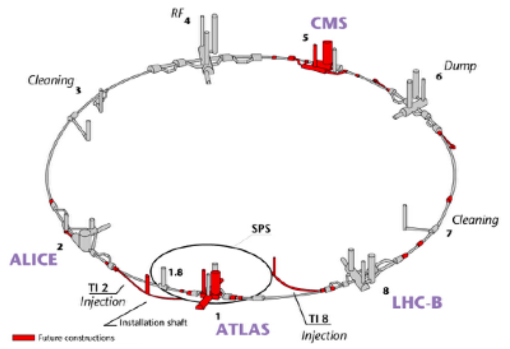

History of Universe



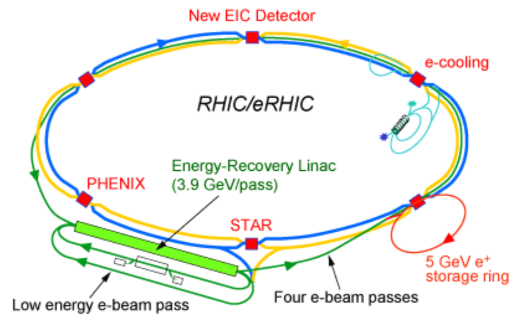
Neutron star merger (L. Rezzolla et al)



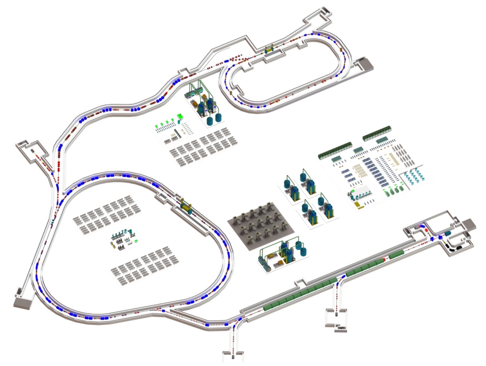

History of a heavy-ion collision (HIC)



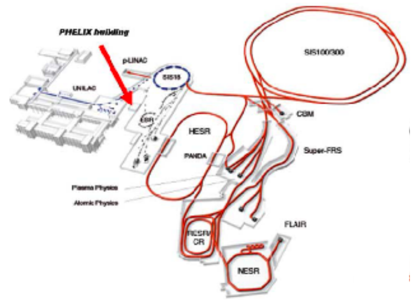

LHC (EU)



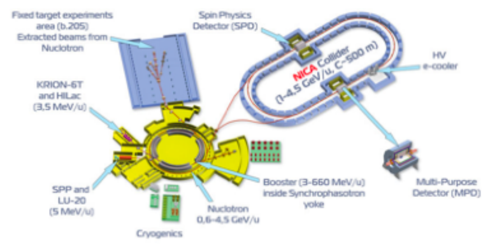

RHIS (US)



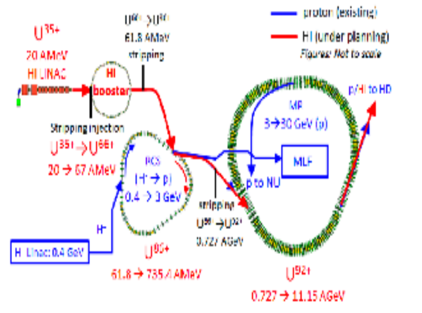
HIAF (CN)



GSI (DE)



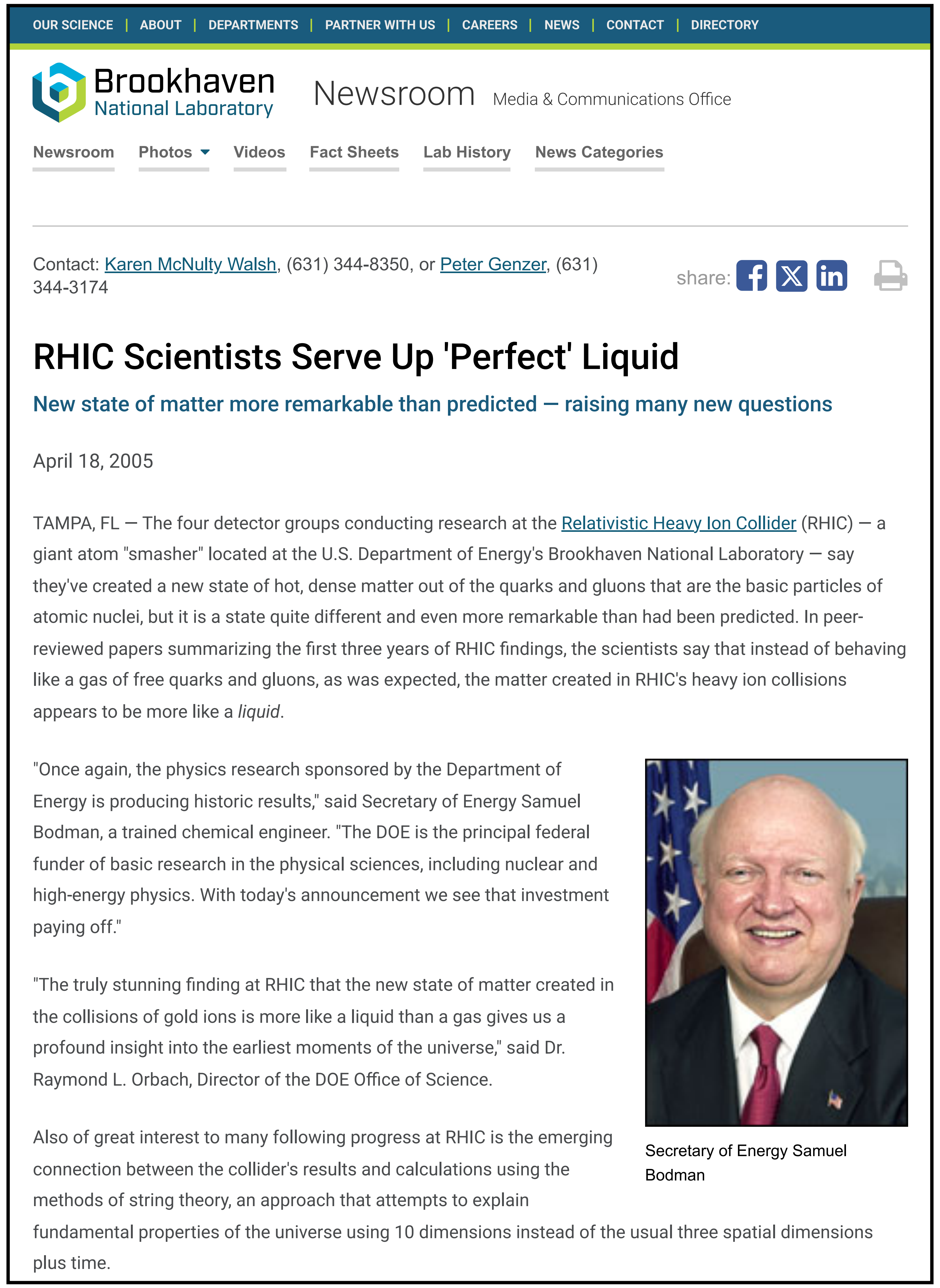
NICA (RU)



J-PARC (JP)

# Thermodynamics and hydrodynamics in HIC 重离子碰撞中的热力学和流体力学

- One of the triumphs of relativistic heavy-ion collisions experiments:



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## RHIC Scientists Serve Up 'Perfect' Liquid

New state of matter more remarkable than predicted — raising many new questions

April 18, 2005

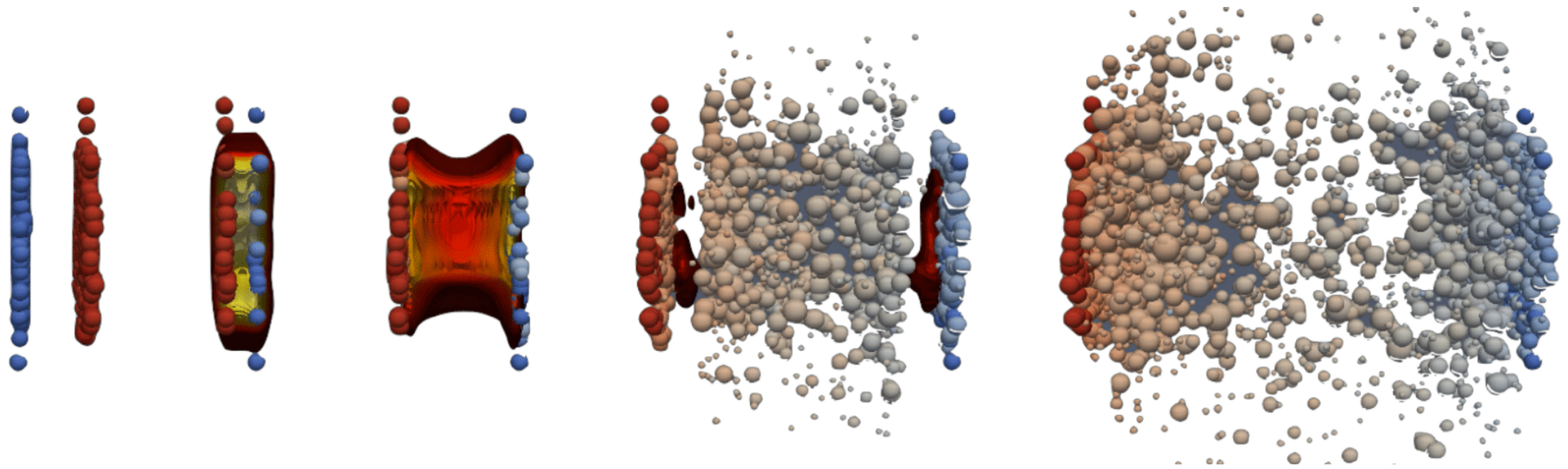
TAMPA, FL — The four detector groups conducting research at the Relativistic Heavy Ion Collider (RHIC) — a giant atom "smasher" located at the U.S. Department of Energy's Brookhaven National Laboratory — say they've created a new state of hot, dense matter out of the quarks and gluons that are the basic particles of atomic nuclei, but it is a state quite different and even more remarkable than had been predicted. In peer-reviewed papers summarizing the first three years of RHIC findings, the scientists say that instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC's heavy ion collisions appears to be more like a *liquid*.

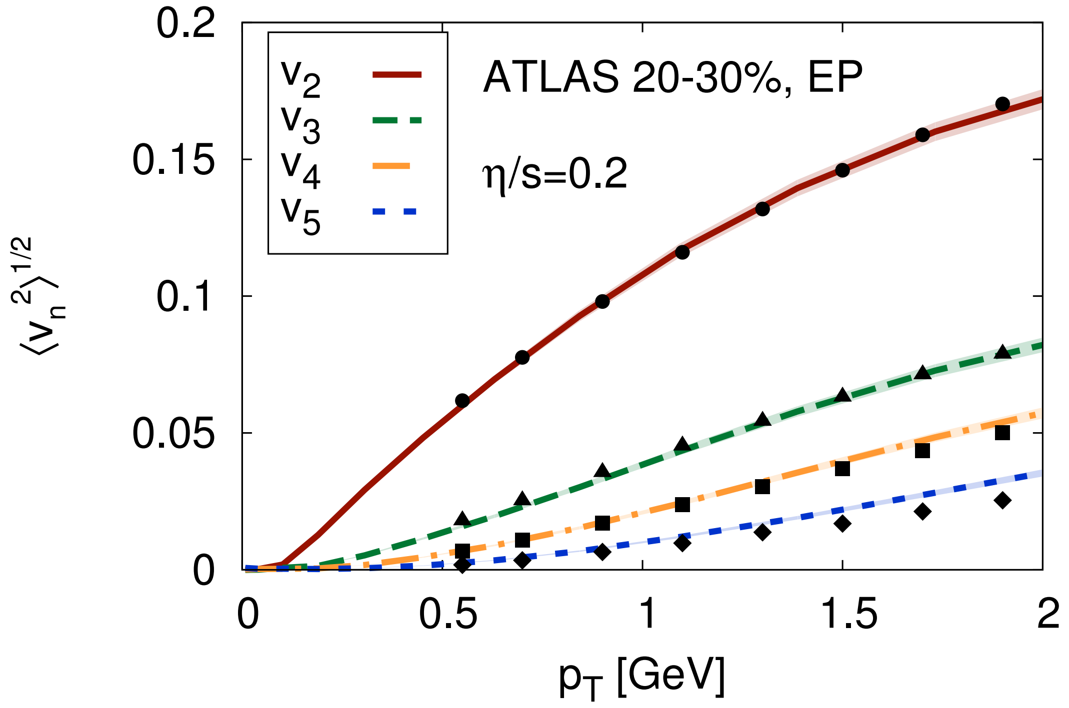
"Once again, the physics research sponsored by the Department of Energy is producing historic results," said Secretary of Energy Samuel Bodman, a trained chemical engineer. "The DOE is the principal federal funder of basic research in the physical sciences, including nuclear and high-energy physics. With today's announcement we see that investment paying off."

Secretary of Energy Samuel Bodman

"The truly stunning finding at RHIC that the new state of matter created in the collisions of gold ions is more like a liquid than a gas gives us a profound insight into the earliest moments of the universe," said Dr. Raymond L. Orbach, Director of the DOE Office of Science.

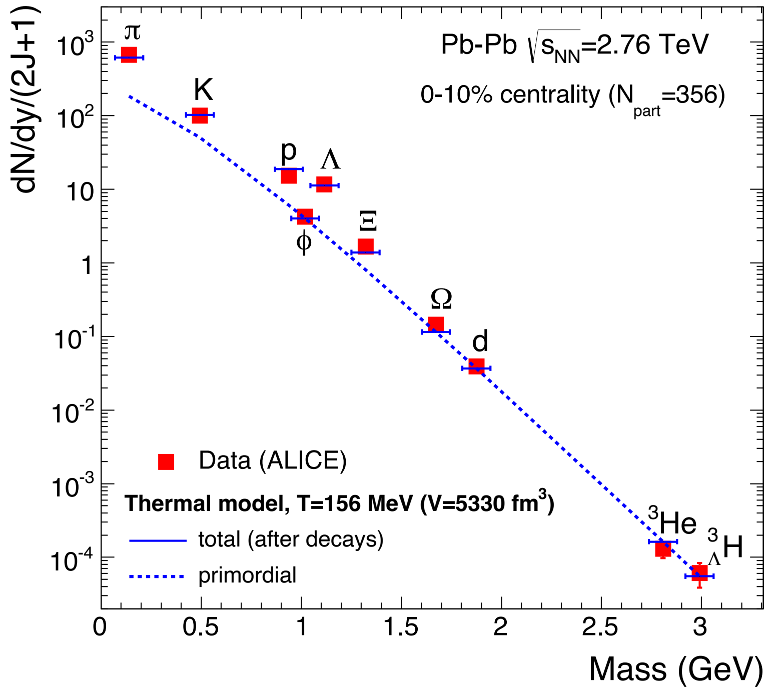
Also of great interest to many following progress at RHIC is the emerging connection between the collider's results and calculations using the methods of string theory, an approach that attempts to explain fundamental properties of the universe using 10 dimensions instead of the usual three spatial dimensions plus time.






hydrodynamization

Gale et al, 1301.5893




thermalization

Andronic, 1407.5003

# Unsolved problems in physics 未解决的物理学问题

- There are **102** unsolved problems in physics, including **4** in nuclear physics and **3** of them are directly relevant to this talk:

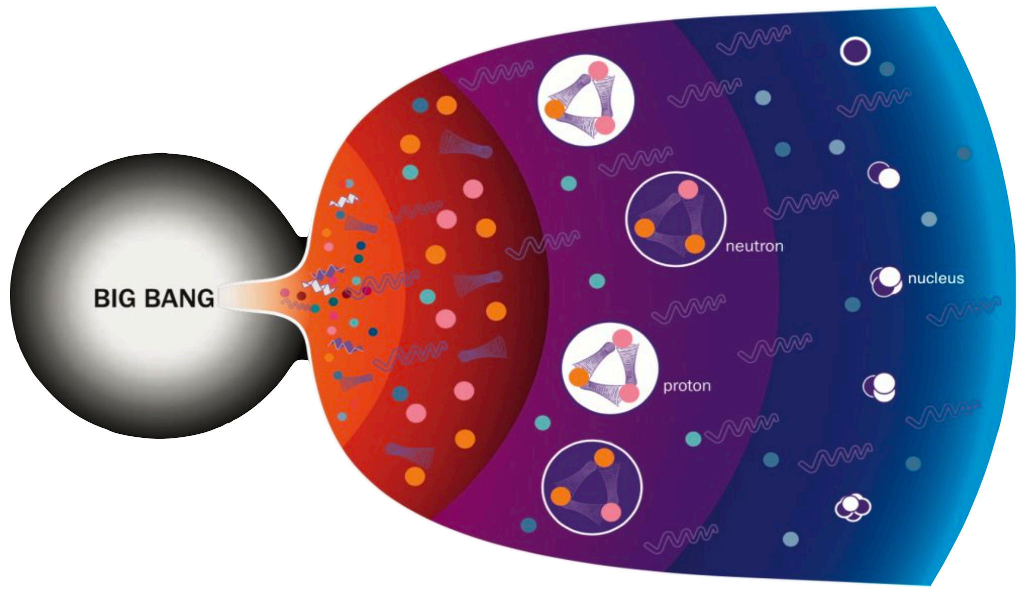
List of unsolved problems in physics

Article Talk

From Wikipedia, the free encyclopedia

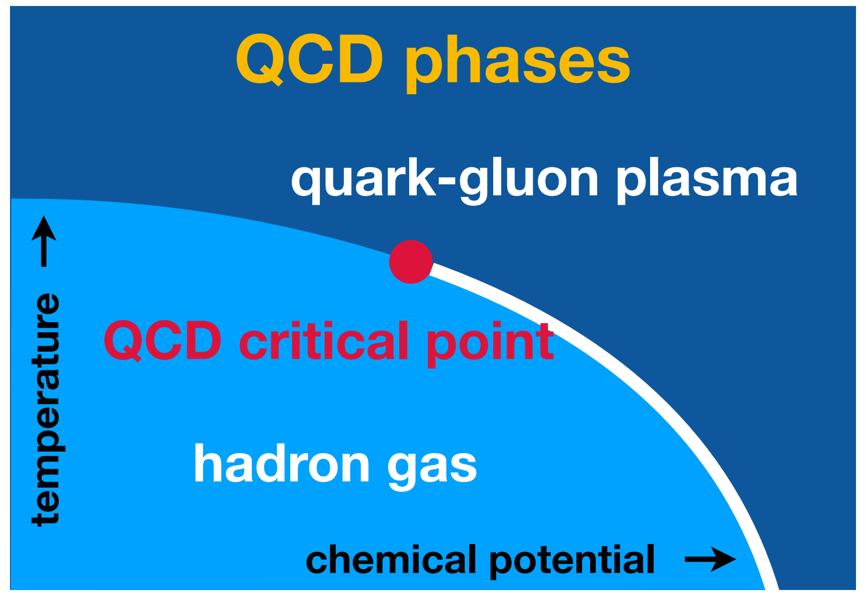
**Q1**

What are the phases of strongly interacting matter, and what roles do they play in the evolution of the cosmos? What is their relation to the nature of gravity/spacetime?



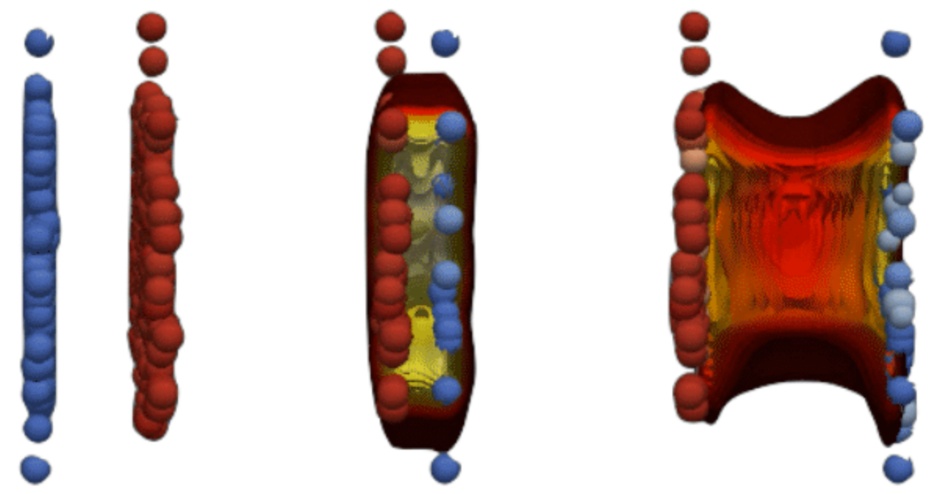

**Q2**

Where is the onset of deconfinement 1) as a function of temperature and chemical potentials? 2) as a function of relativistic heavy-ion collision energy and system size?




**Q3**

How does quark–gluon plasma form? Why does deconfined matter show ideal flow?

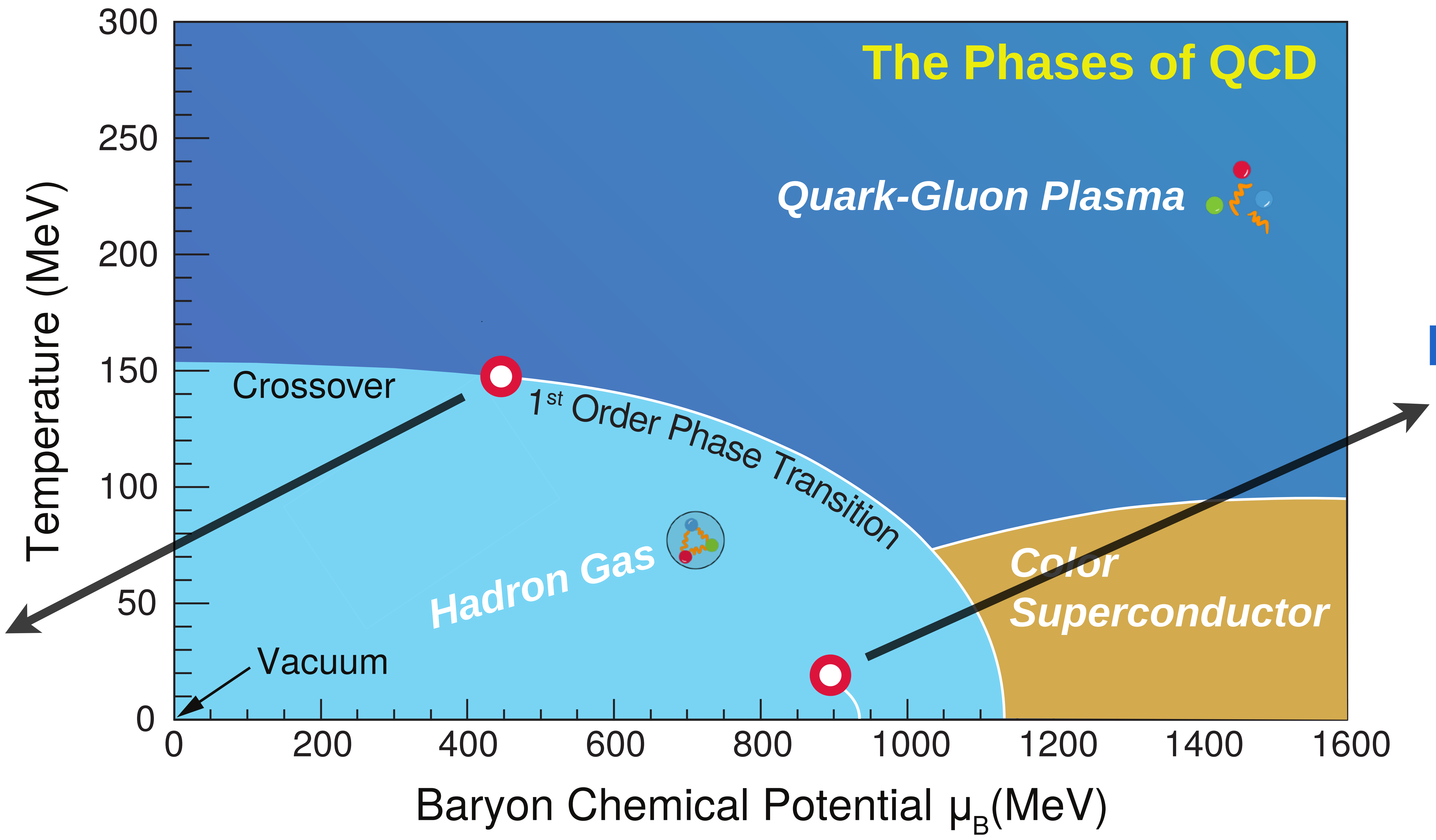


xinan@ugent.be

# Relevant goals & status of HIC experiments
# 重离子碰撞实验的相关目标和现状

# Understand QCD phase diagram 理解量子色动力学相图

- Heavy-ion collisions → QCD phase diagram — very little is known yet.




**QCD critical point**
(hadron-QGP)
TBA

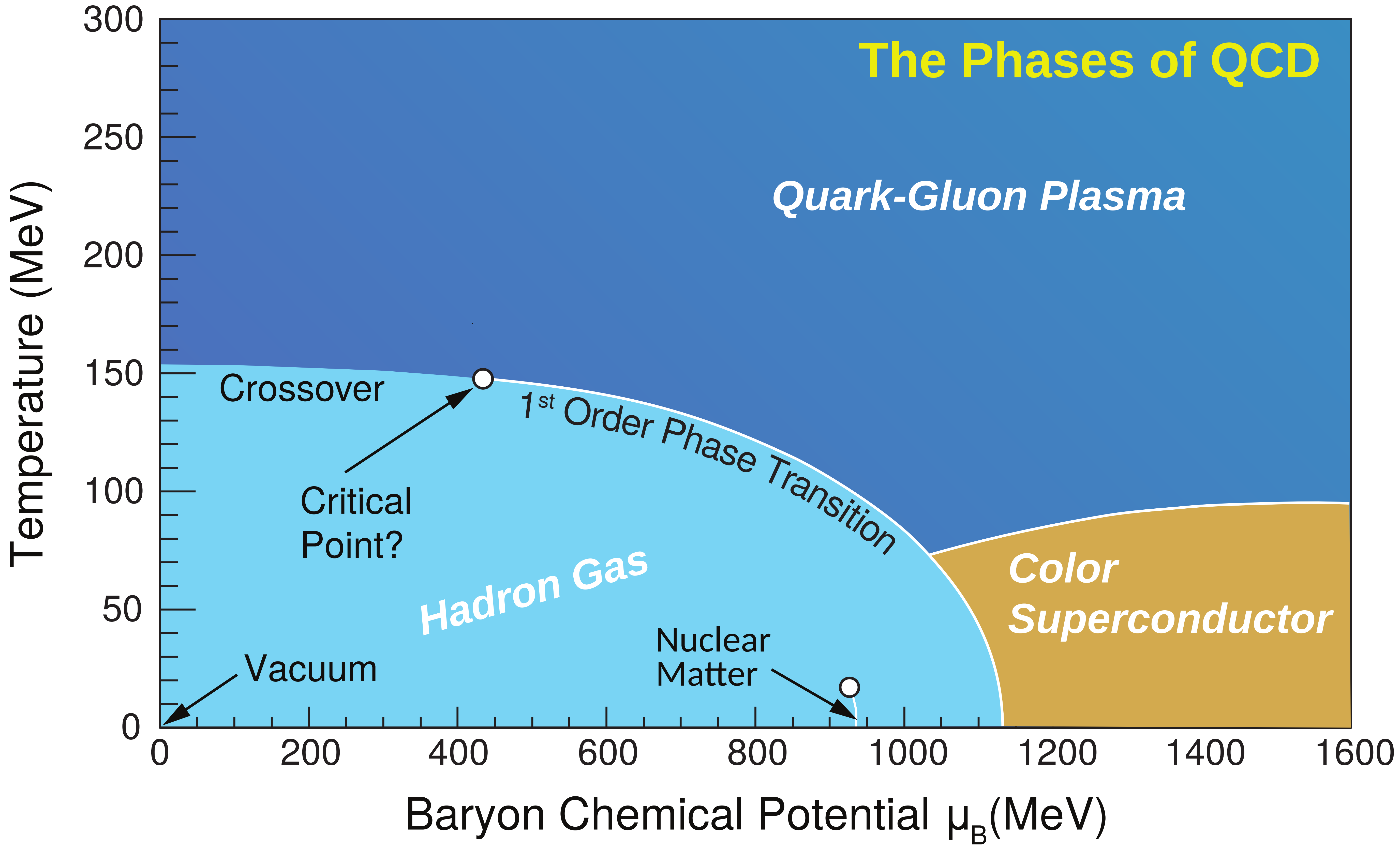
**Nuclear critical point**
(nuclear liquid-gas)
Pochodzalla et al, 1995

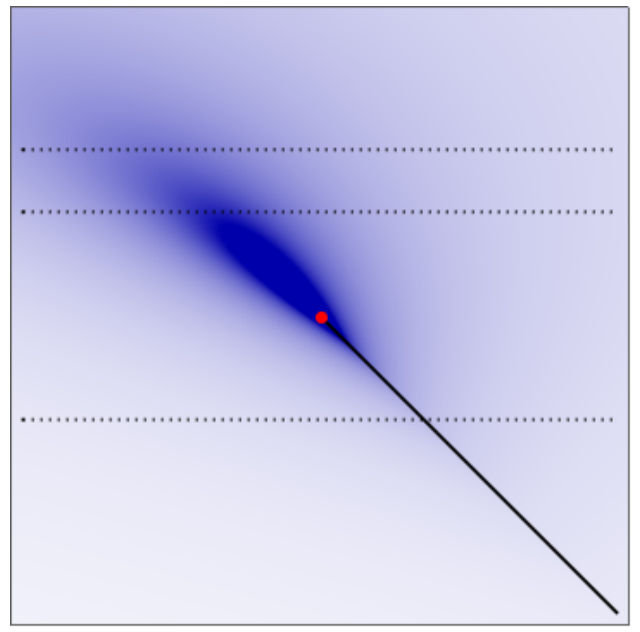
- Understanding the finite temperature and density regime is challenging (sign problem).
- A simpler task: search the landmark (critical point) where singularity occurs.

# Understand QCD phase diagram 理解量子色动力学相图

- QCD thermodynamics

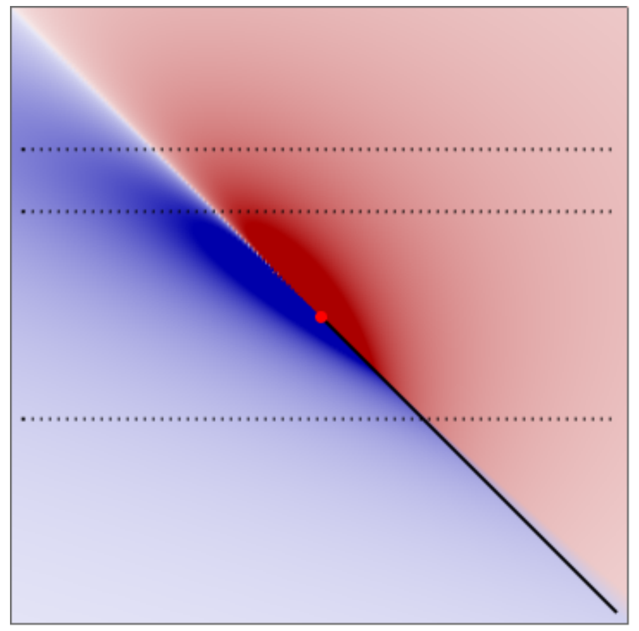
$P(T, \mu)$



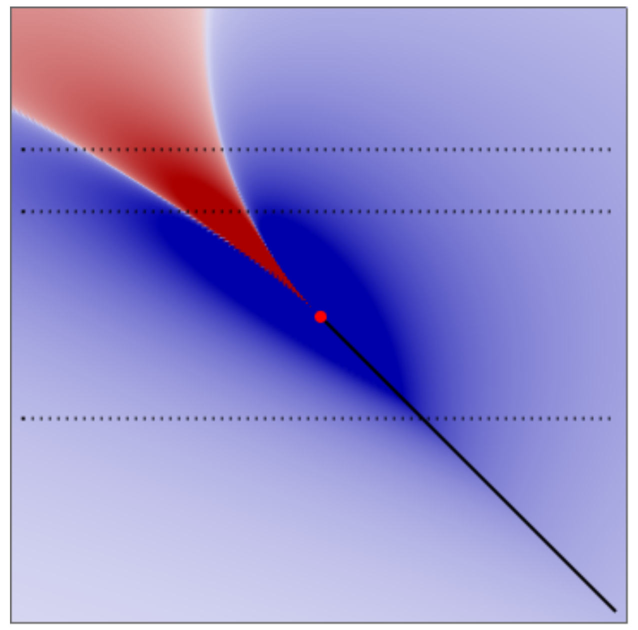
variance

$\chi_2 = \partial^2 P / \partial \mu^2$



skewness

$\chi_3 = \partial^3 P / \partial \mu^3$

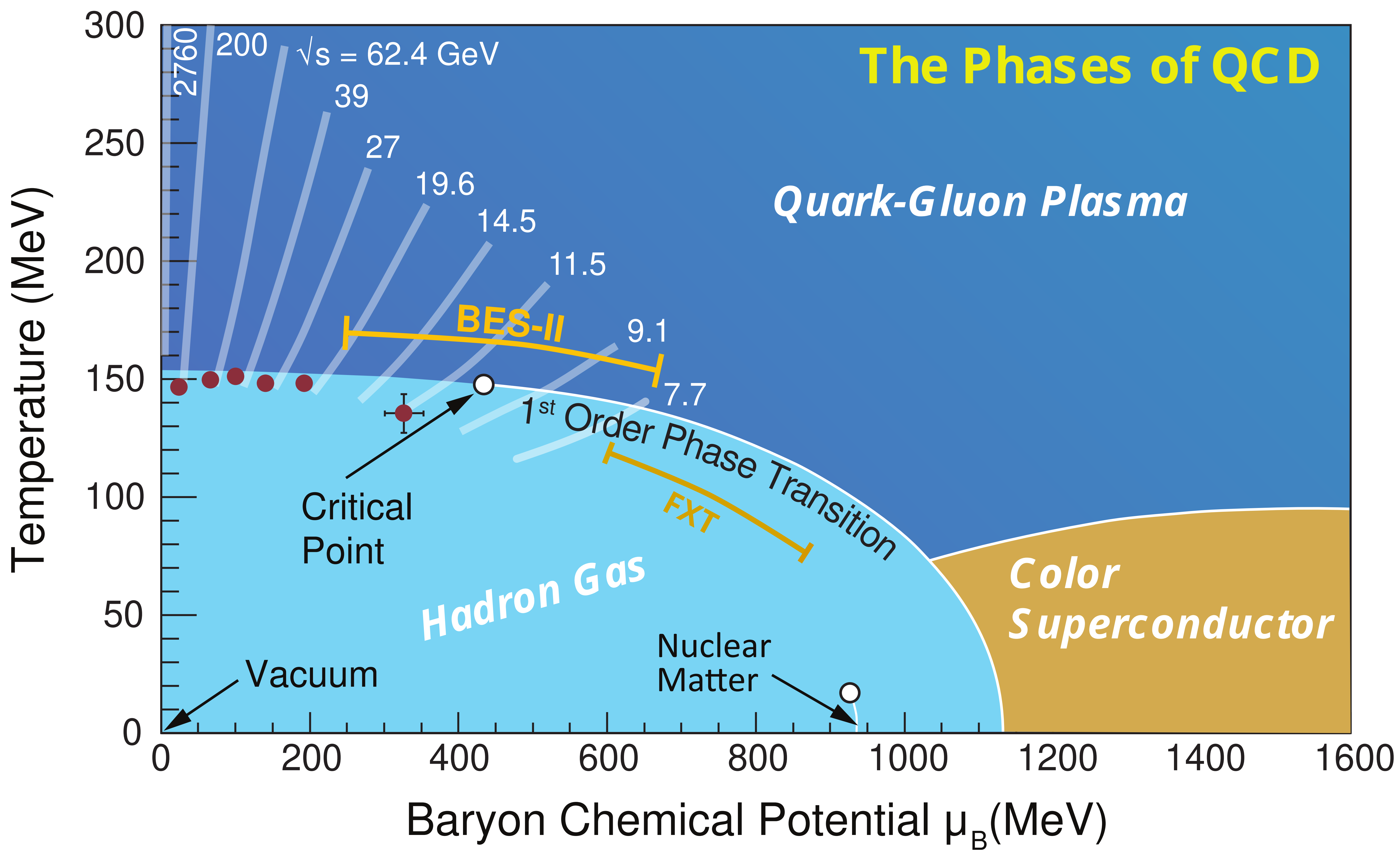


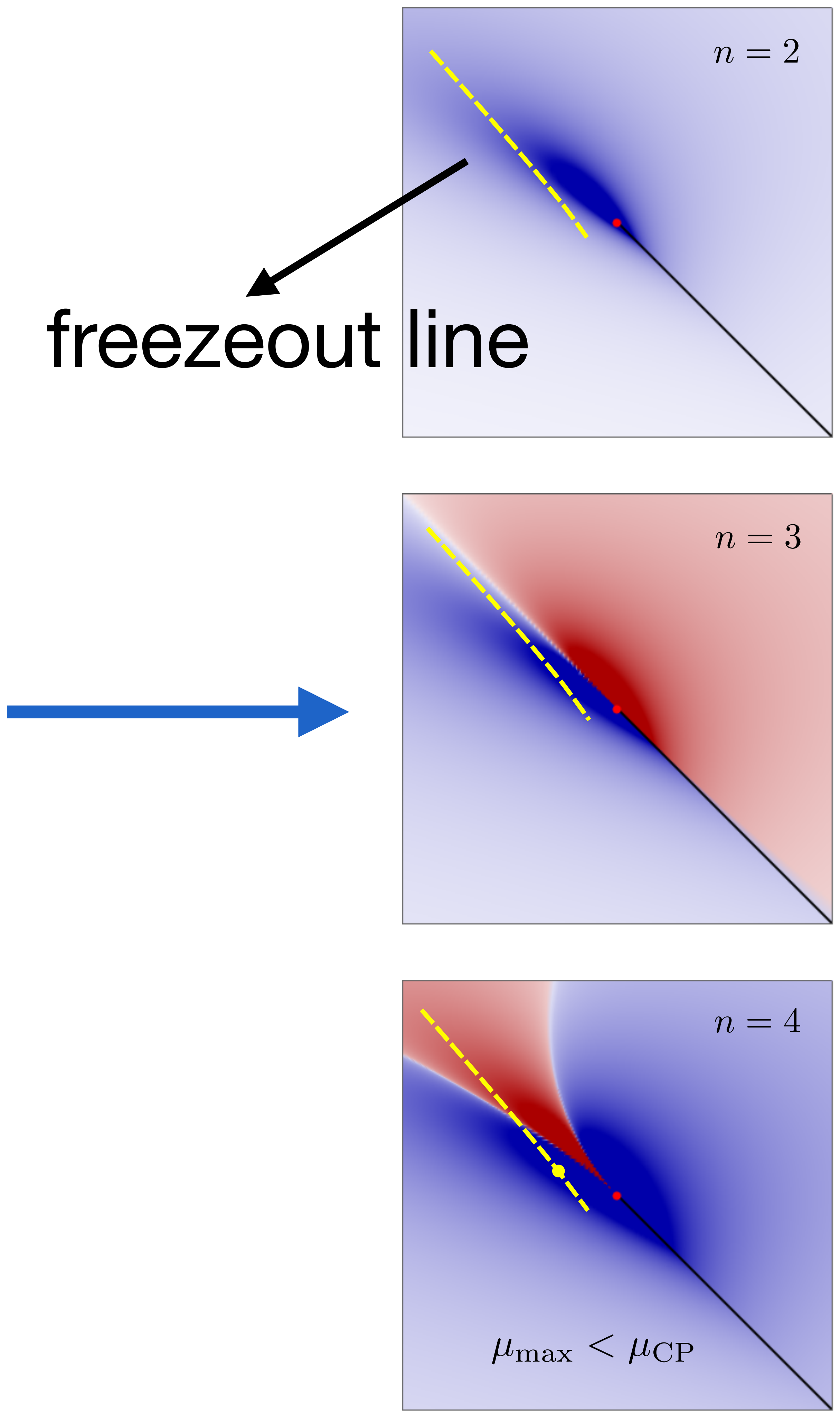
kurtosis

$\chi_4 = \partial^4 P / \partial \mu^4$

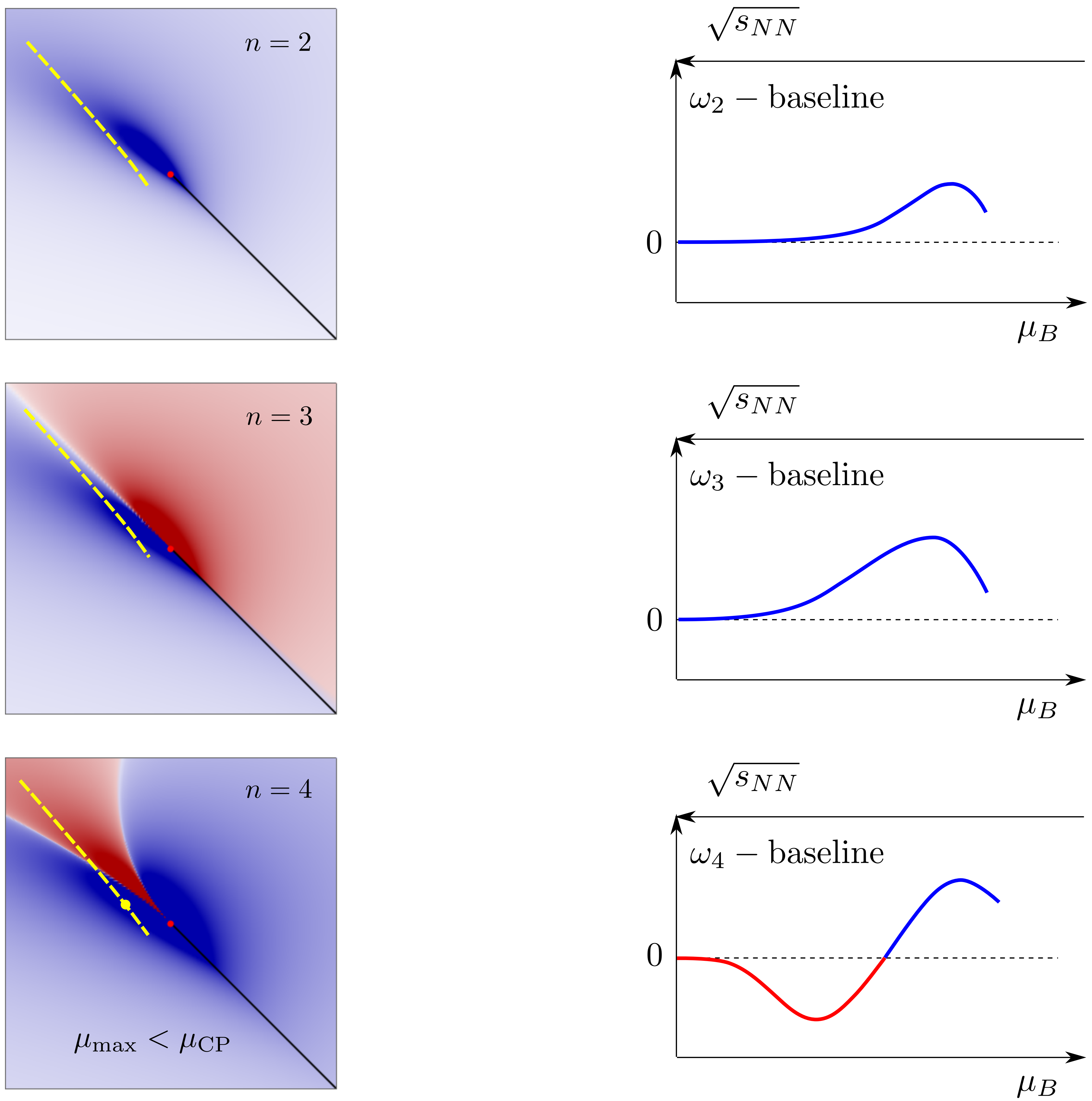
# Understand QCD phase diagram 理解量子色动力学相图

- Beam-energy scan: system with different beam energies freezes out at different thermodynamic state.

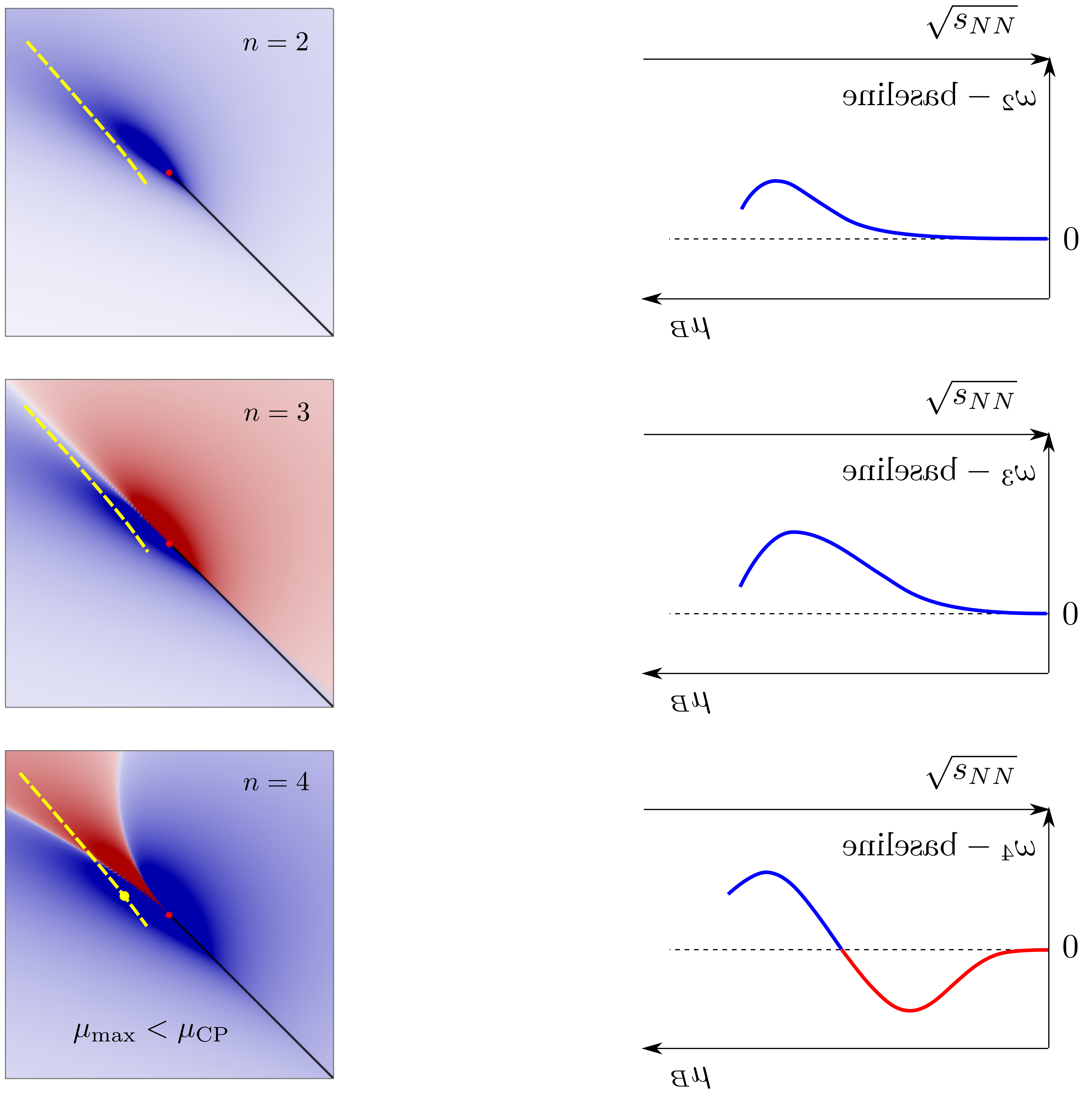


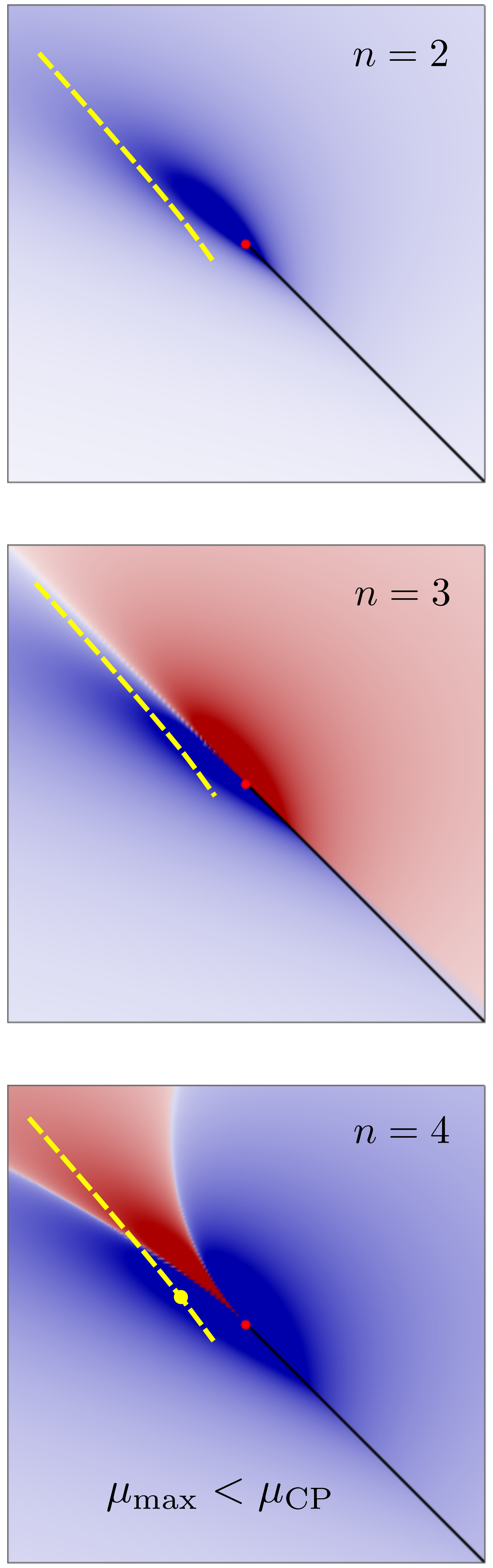
# Understand QCD phase diagram 理解量子色动力学相图

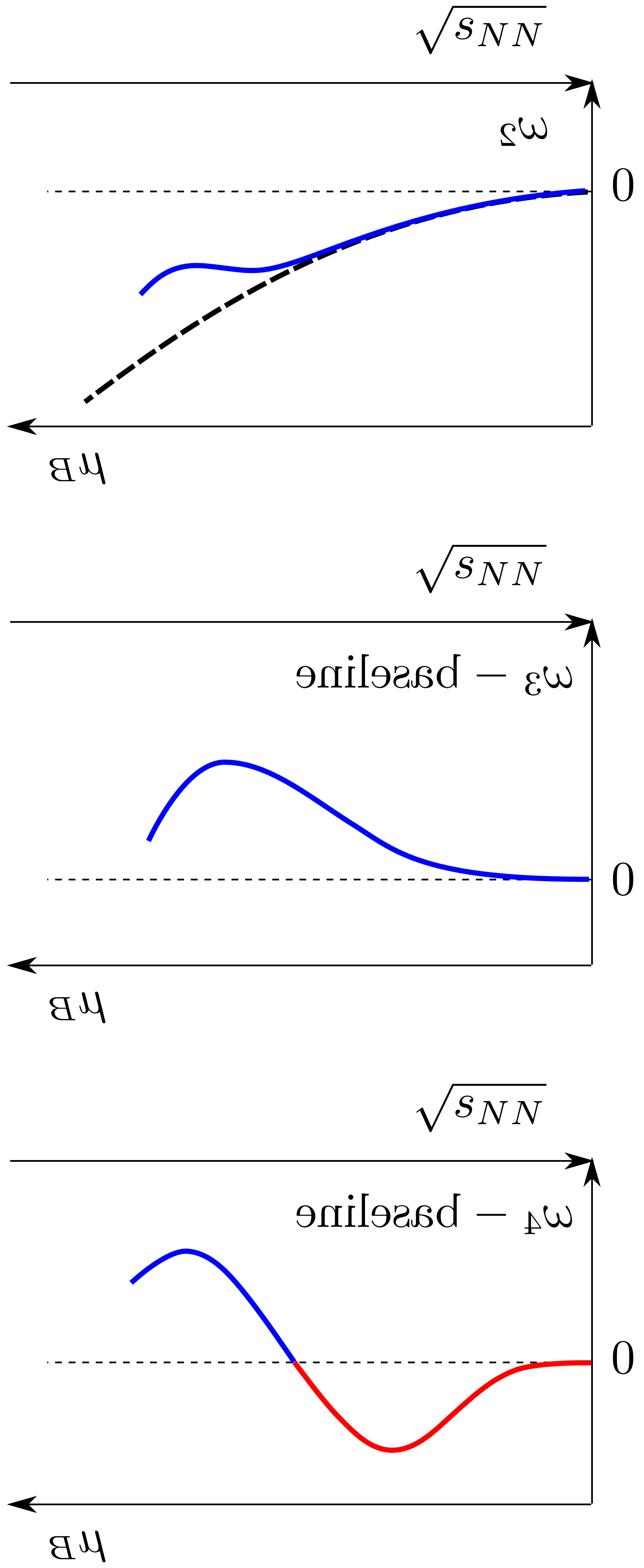


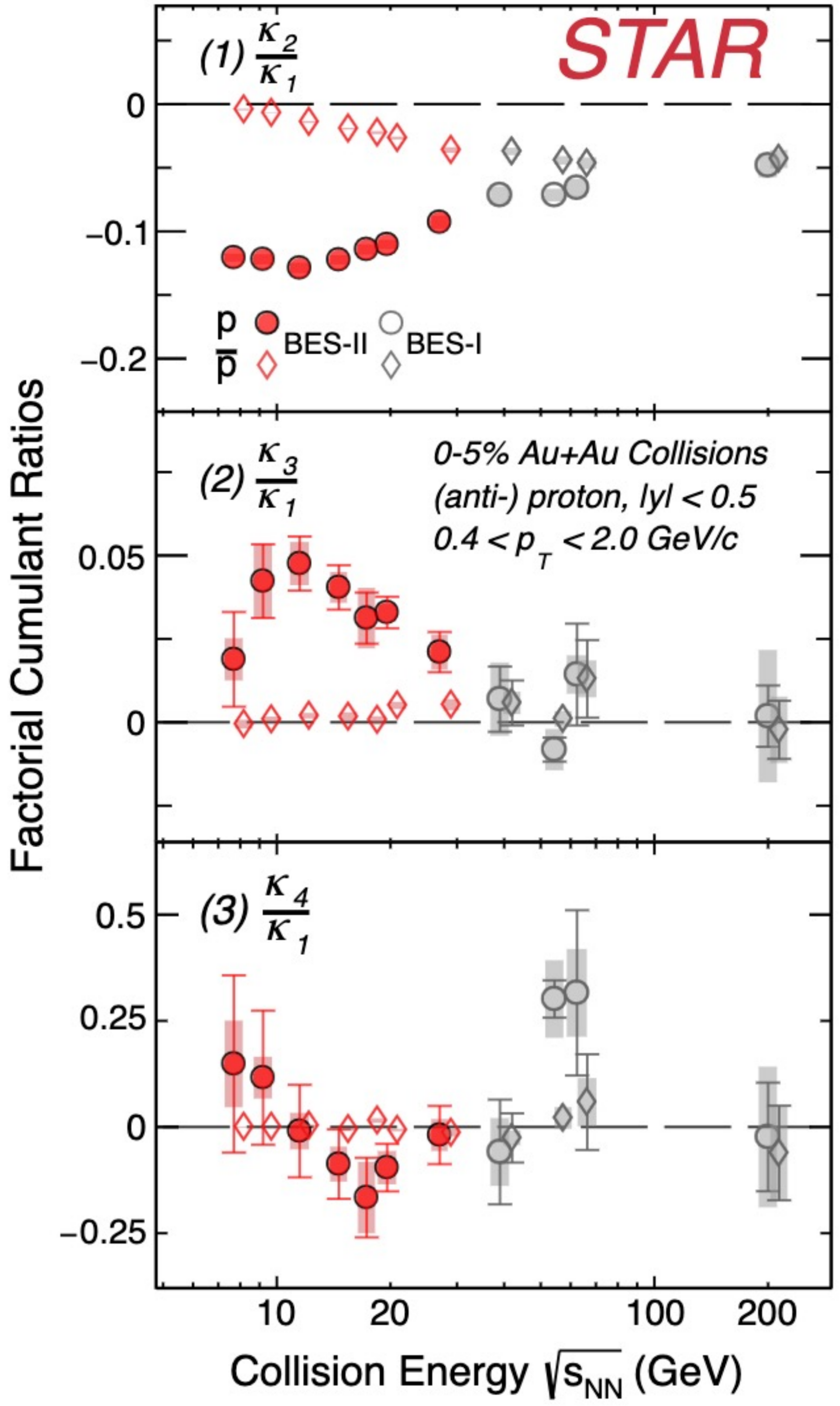
# Understand QCD phase diagram 理解量子色动力学相图



# Understand QCD phase diagram 理解量子色动力学相图



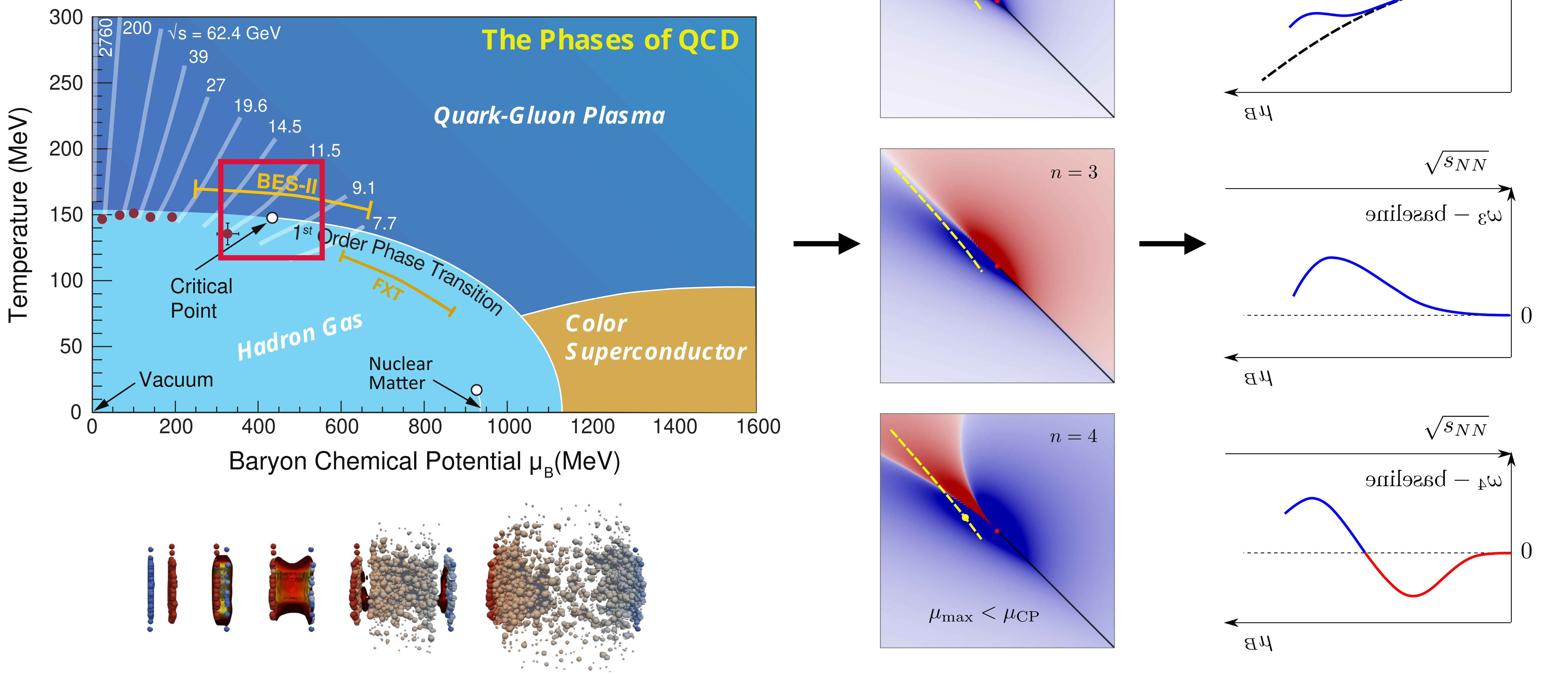



# RHIC STAR BES-II results 相对论重离子对撞机STAR BES-II 结果

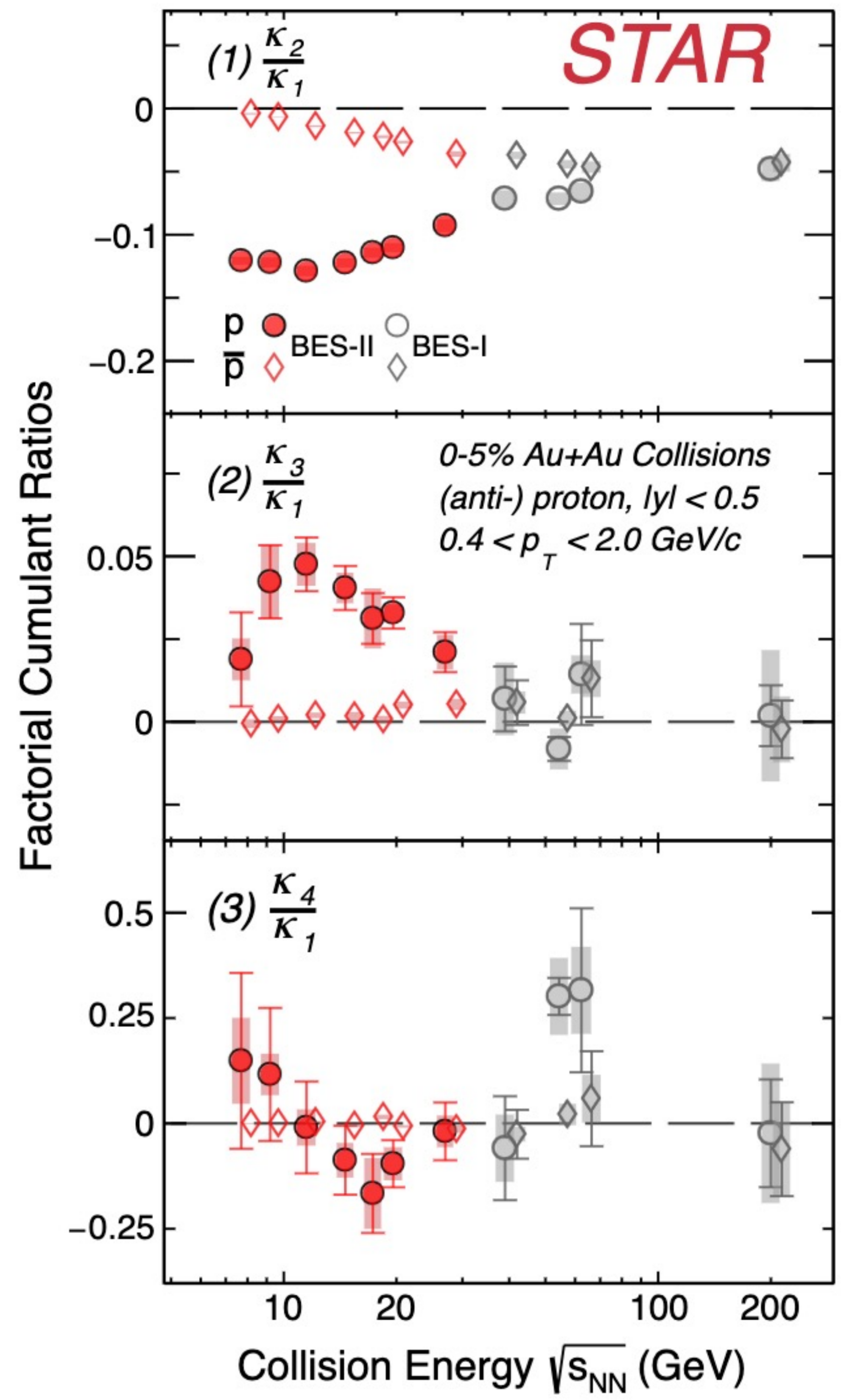
- BES-II Data from **RHIC STAR at BNL** seem to advocate the intriguing hint of QCD critical point from BES-I based on equilibrium assumption, in a qualitative level.

STAR, 2504.00817; Stephanov, 2410.02861

**theory** (thermodynamics)




**experiment**

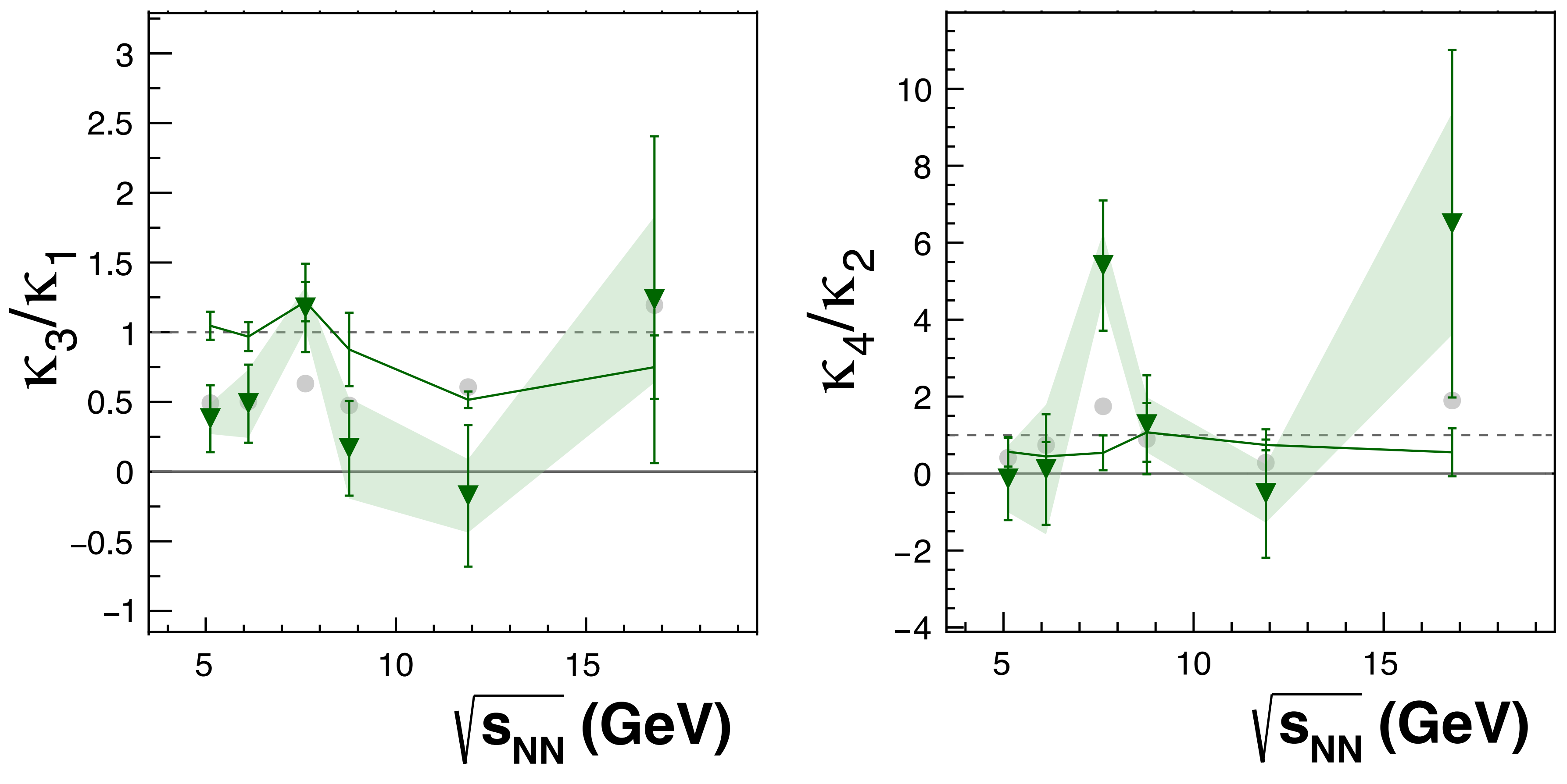



non-monotonicity of cumulants

# Other experiment results 其它实验结果

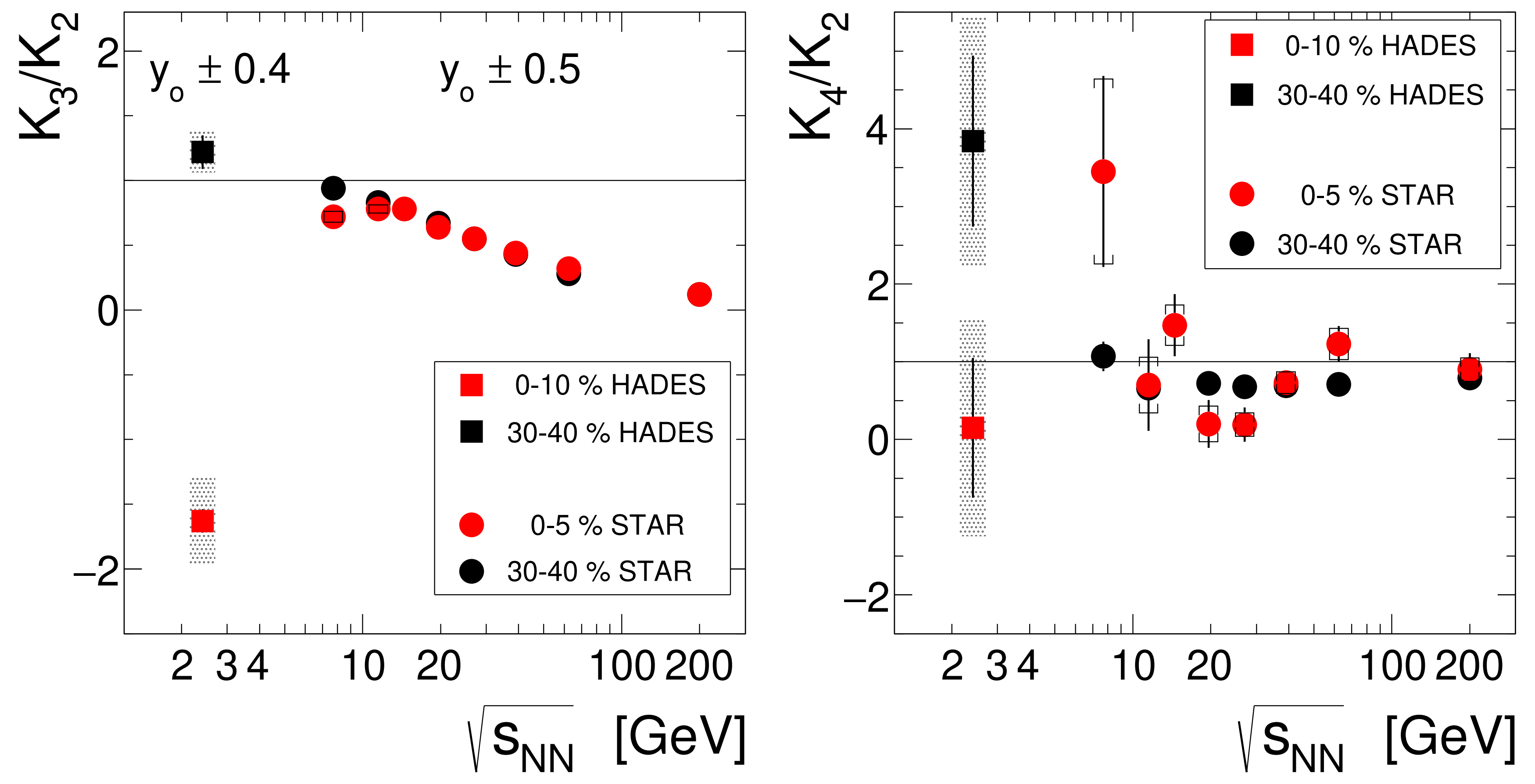
- **NA61/SHINE at CERN SPS**

NA61/SHINE, 2503.22484; CERN 2024 report




- **HADES at GSI**

HADES 2002.08701




"No clear critical point signatures either at SPS/GSI energies" (QM25 summary)

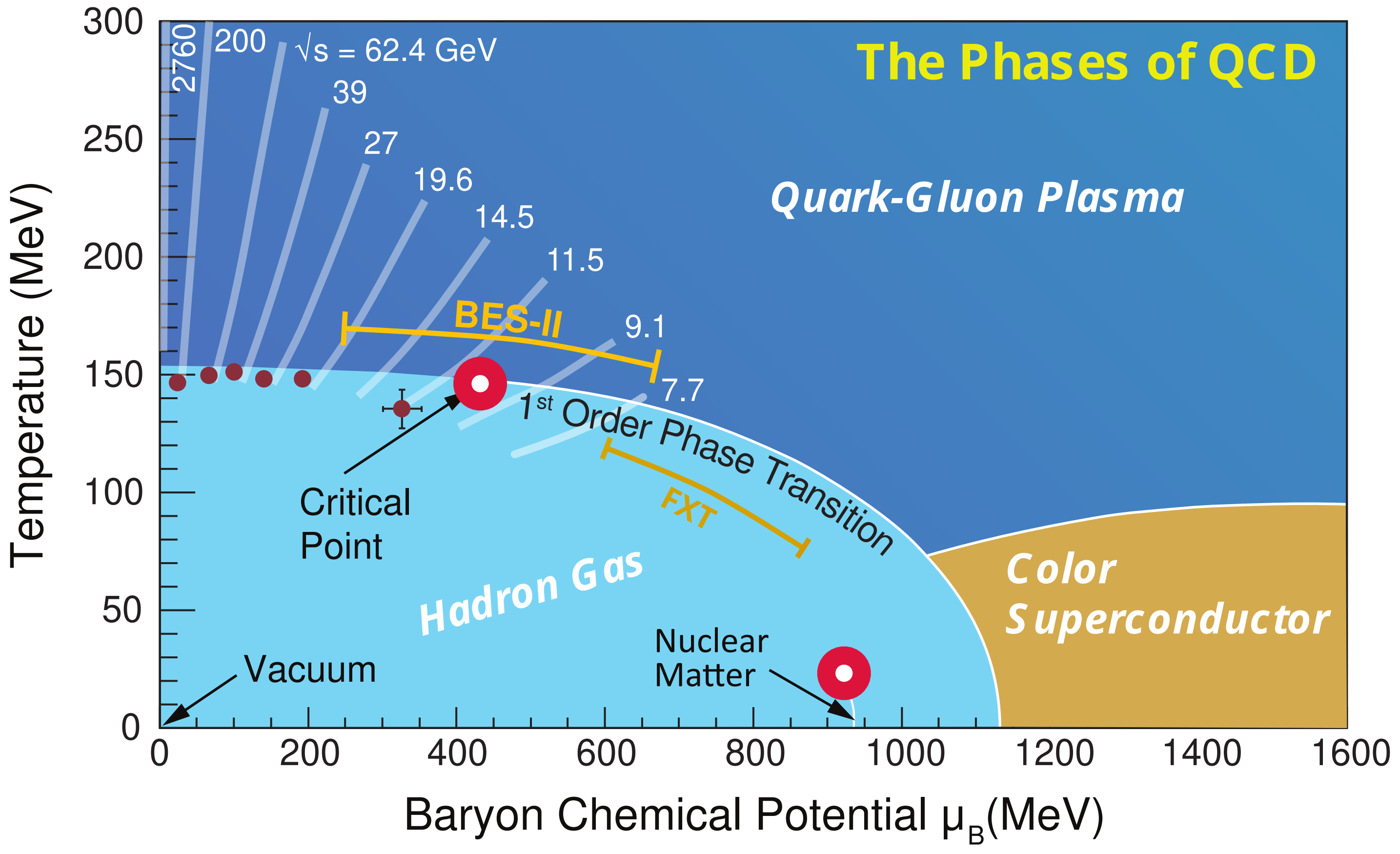
- **Future experiments**: CBM at FAIR (EU), MPD at NICA (RU), HIAF at IMP (CN), …

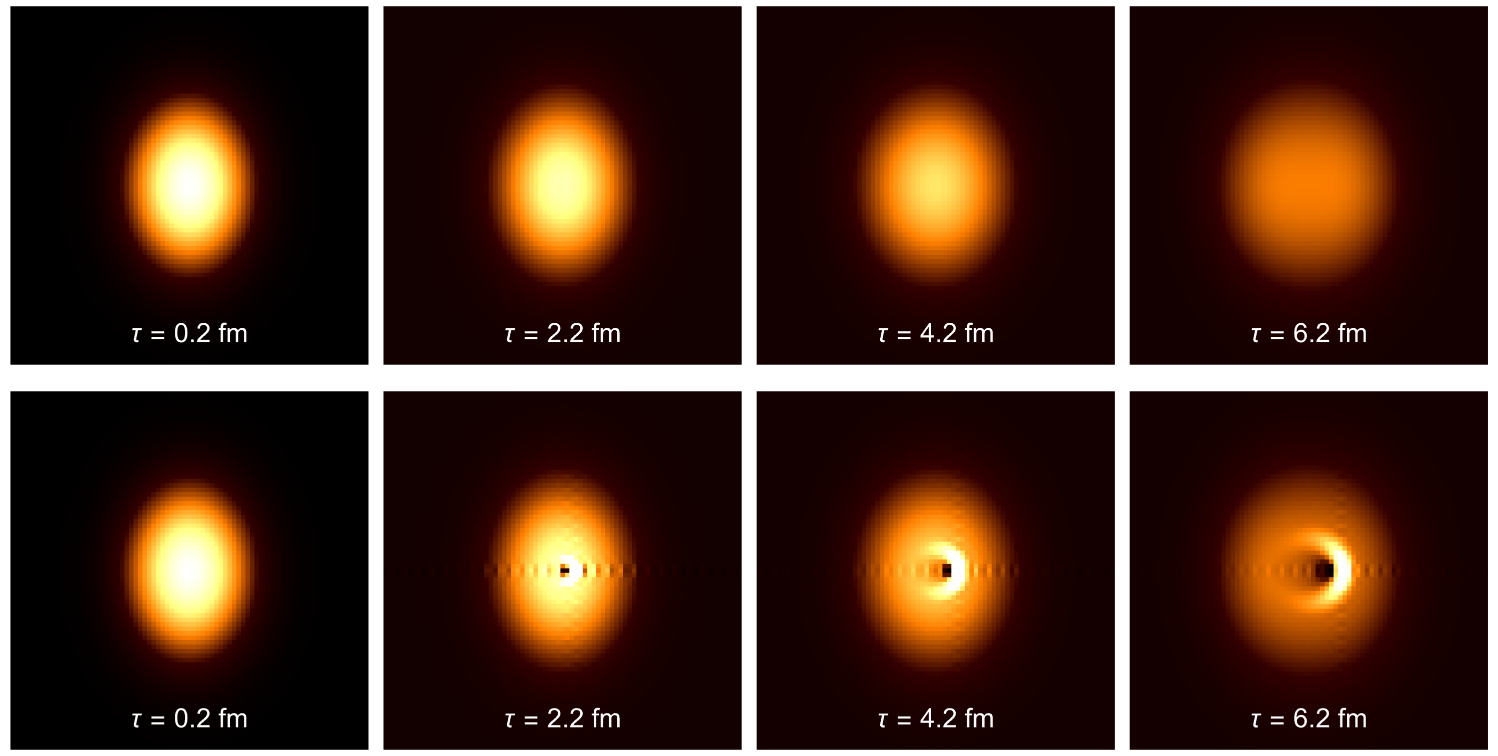
**Next**: "dynamical model calculations including the criticality needed to fully understand the data" (QM25 summary)

# Theory vs experiment 理论 vs 实验

Theoretical idealization

Experimental realization



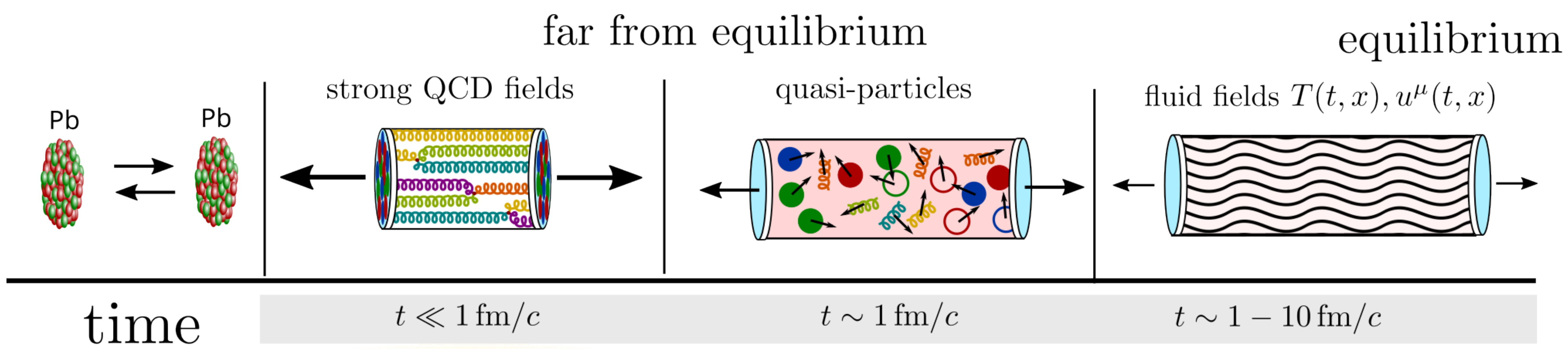

XA and Spalinski, 2312.17237 and in progress

- Infinite time (equilibrium)
- Infinite size (thermodynamic limit)

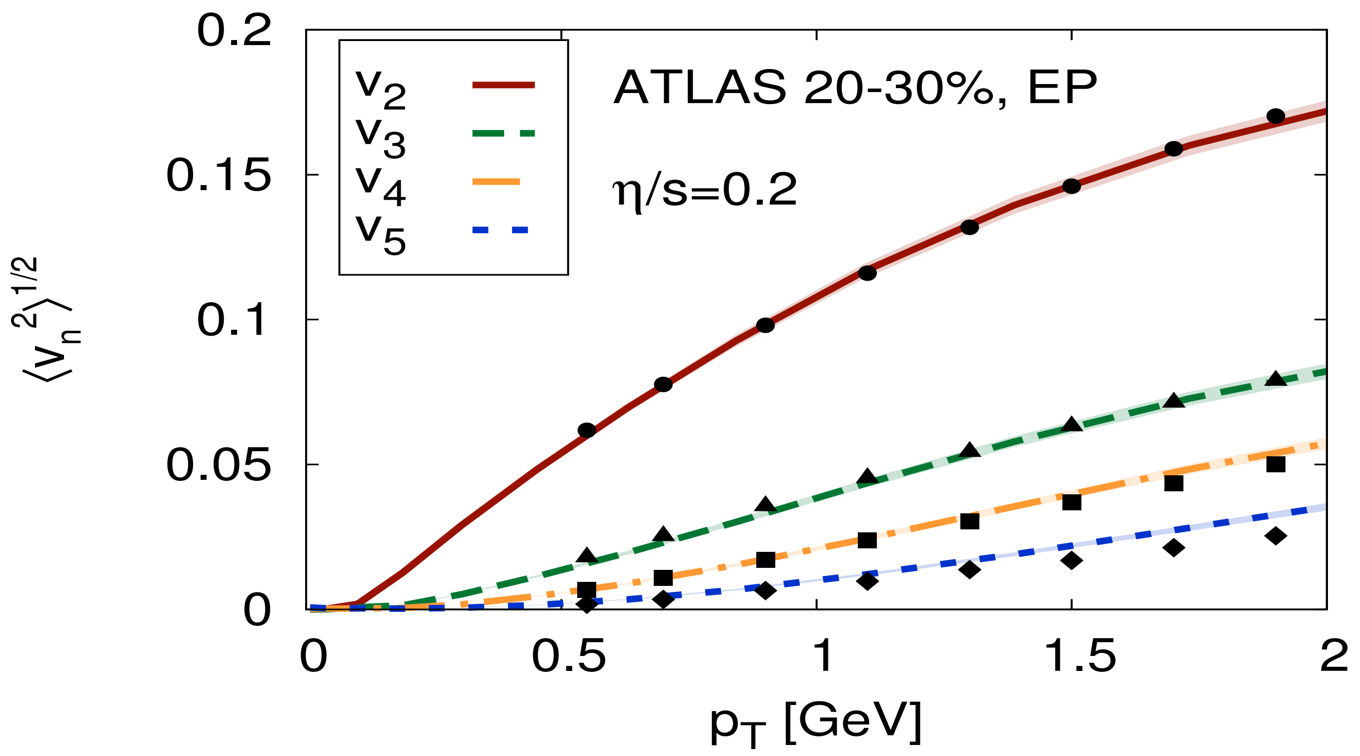
- Finite time (non-equilibrium)
- Finite size (non-thermodynamics)

# Understand the thermalization of QGP 理解夸克胶子等离子体的热化

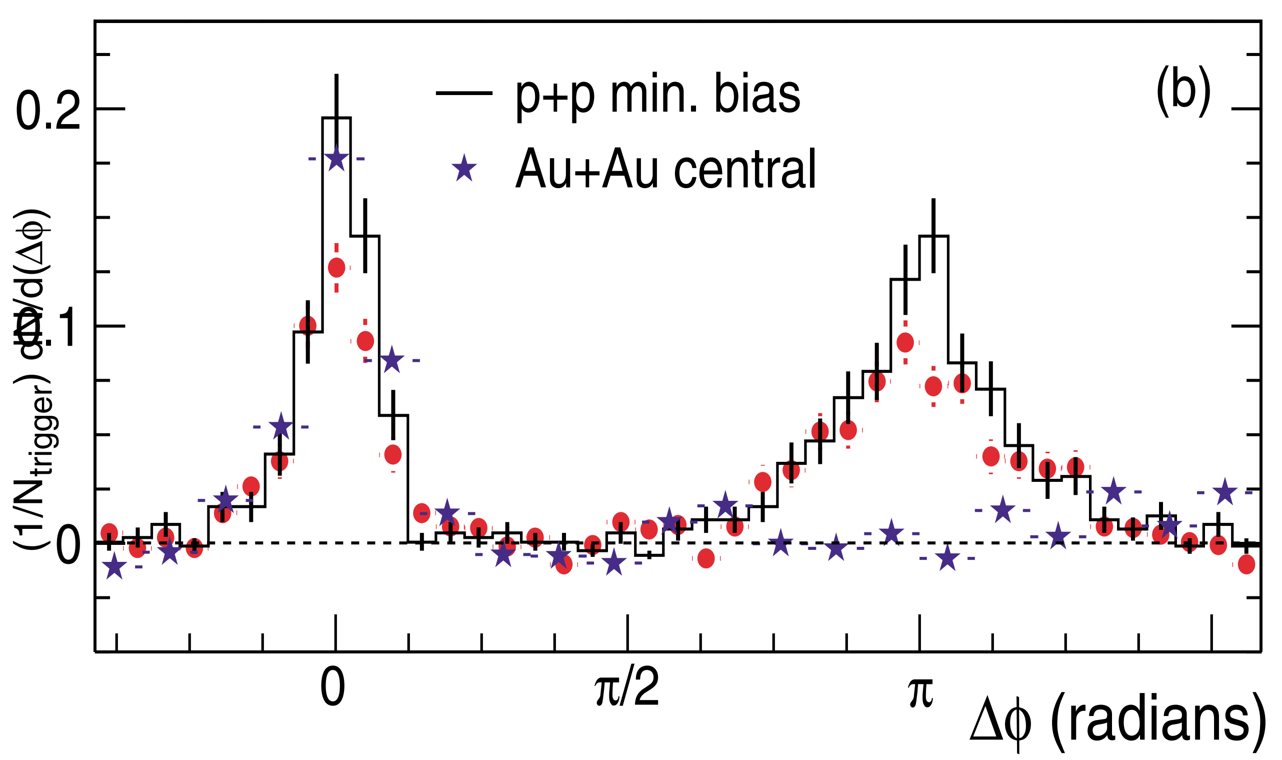
- Heavy-ion collisions → fast hydrodynamization/thermalization — very little is known yet.




Courtesy of A. Mazeliauskas




Soft probe: collectivity




Hard probe: Jet quenching

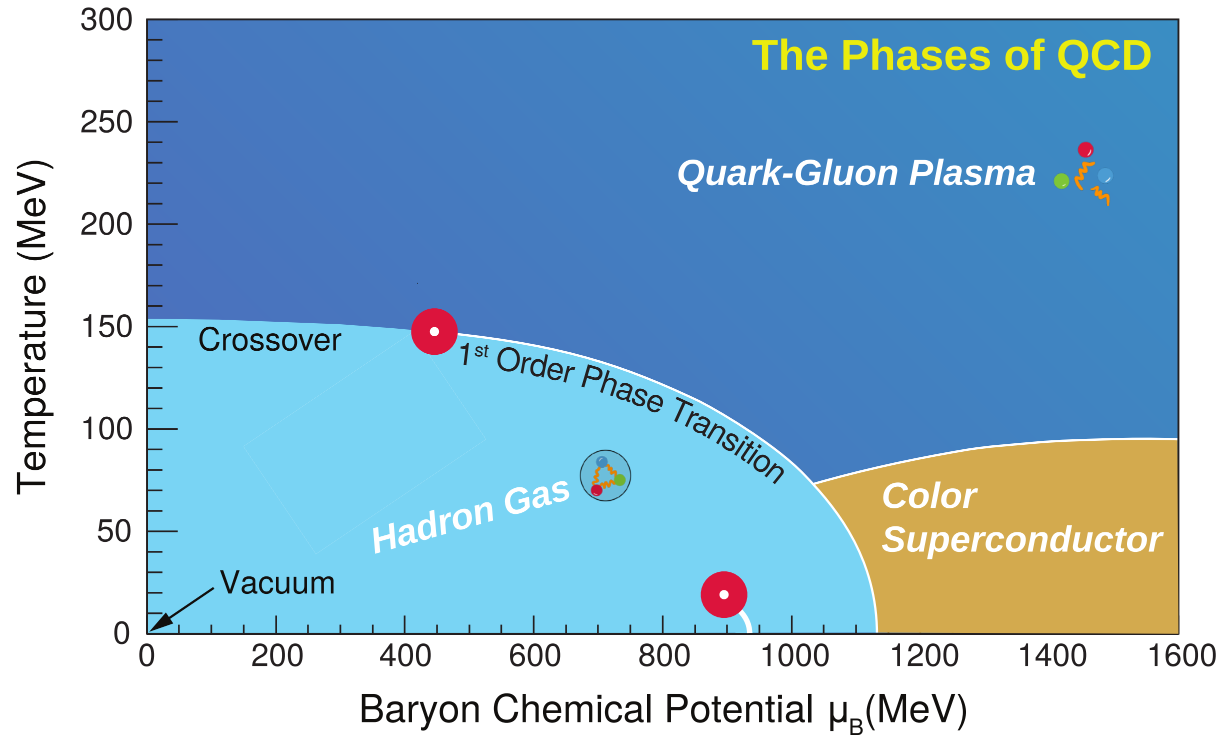
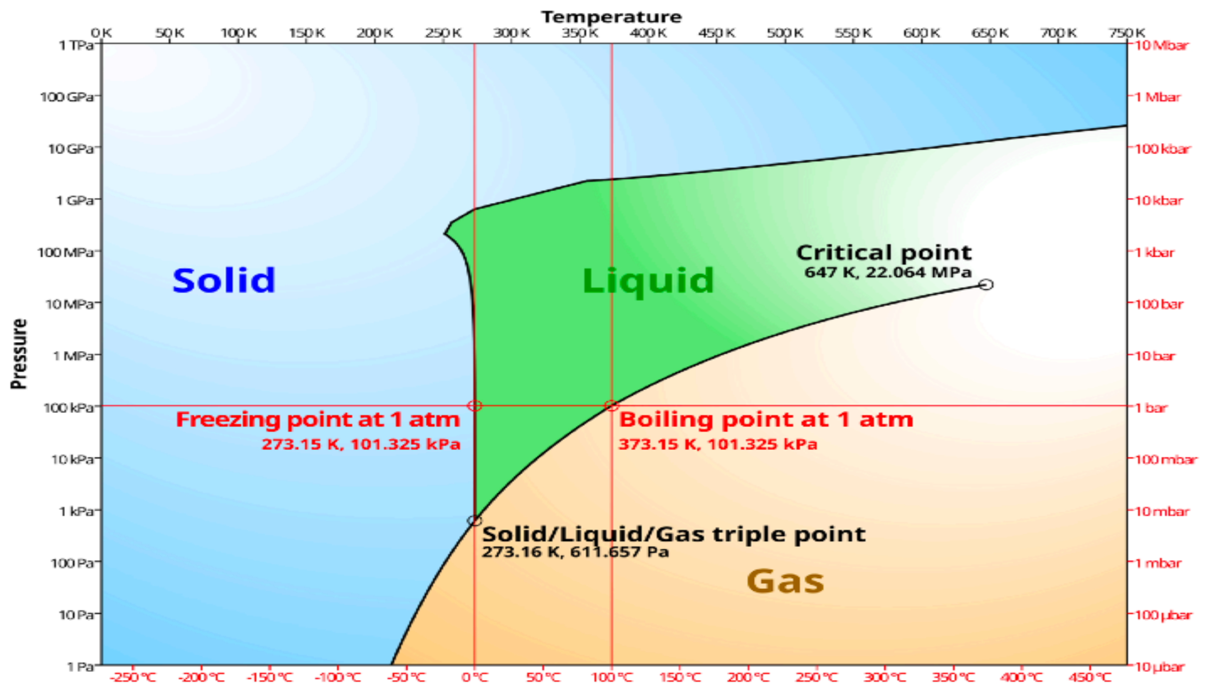
- Understanding the early-time physics is challenging (measurement only at freezeout).
- A simpler task: find the legacy of non-hydrodynamics.

# Non-thermodynamics at finite size
# 有限尺度下的非热力学

# Thermodynamic potentials 热力学势

- Partition function from ensembles summation to continuous limit, with various choices of thermodynamic potential

$$Z(J) = \sum_i e^{-J\psi_i} \quad \longrightarrow \quad Z(J) = \int d\psi \, e^{\Omega(J,\psi)} \qquad \Omega = S(\psi) - J\psi$$

| Landau potential | Planck potential |
|---|---|
| $\Omega(t, V, y) = S(E, V, N) - tE + yN$ | $\Omega(t, x, N) = S(E, V, N) - tE - xV$ |
|  |  |

$$t \equiv 1/T, \quad x \equiv P/T, \quad y \equiv \mu/T$$

# Extended vdW with multiple CPs 多临界点的推广型范德瓦尔斯方程

- The partition function for extended vdW:

XA, F. Giglio and G. Landolfi, 2503.15719, submitted to PRL

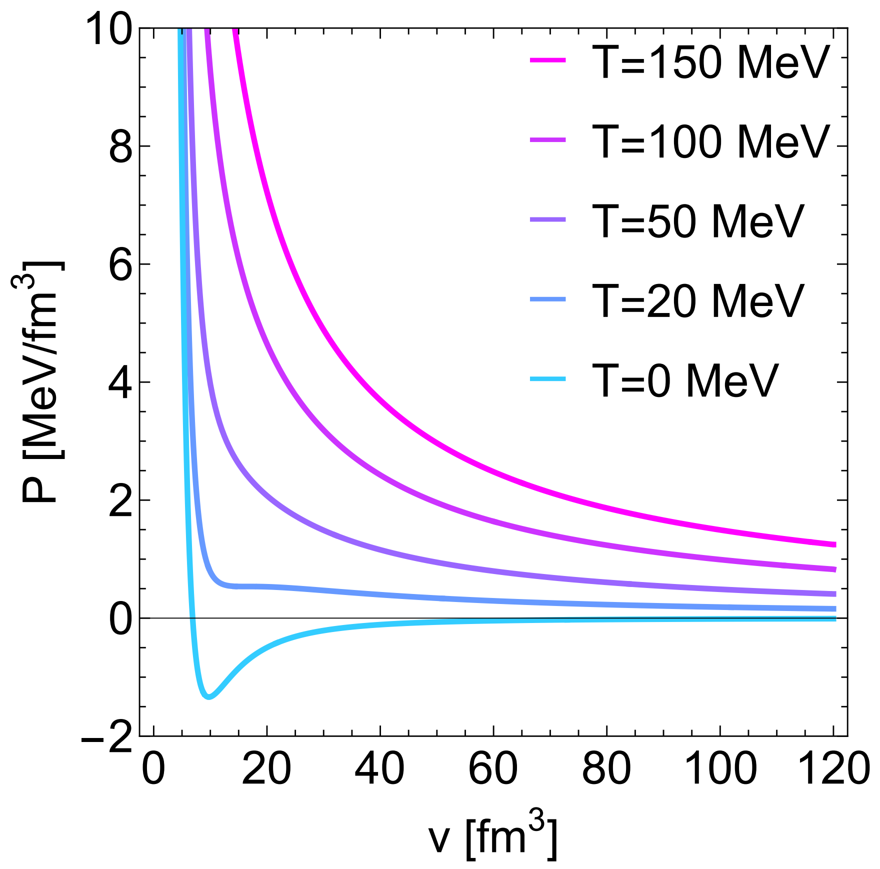
$$Z_N(t,x) \sim \int_b^\infty dv\, e^{N\psi(v)} = \int_b^\infty dv\, e^{N(s(v)-t\epsilon(v)-xv)} \xrightarrow{\text{EOS}} P(v,T) = \frac{T}{v-b} - \sum_{k=2}^{6} \frac{a_k}{v^k}$$

where $s(v) = \log(v-b)$ configuration entropy, kinetic part irrelevant

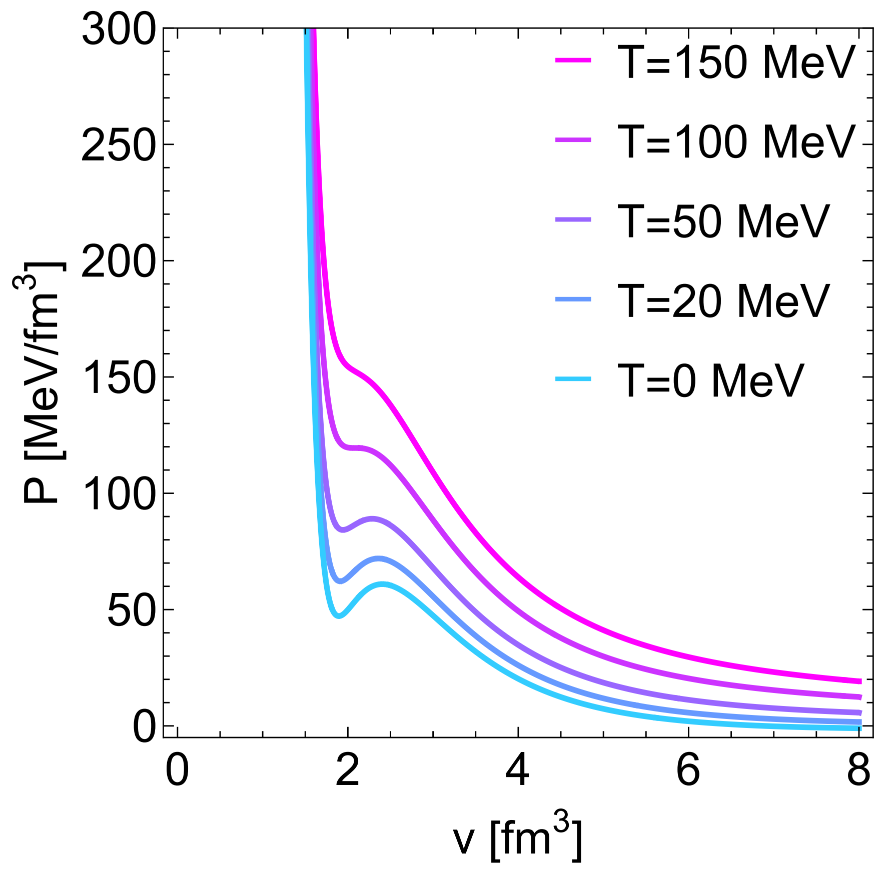
$\epsilon(v) = -\sum_{k=1}^{5} \frac{a_{k+1}}{k v^k}$ virial expansion, multi-particle interactions

We choose

nuclear CP:
$T_c = 20\,\text{MeV}, n_c = 0.06\,\text{fm}^{-3}$
QCD CP:
$T_c = 100\,\text{MeV}, n_c = 0.48\,\text{fm}^{-3}$
spinodal boundary:
$T = 0\,\text{MeV}, n_L = 0.42\,\text{fm}^{-3}, n_R = 0.53\,\text{fm}^{-3}$



nuclear critical point



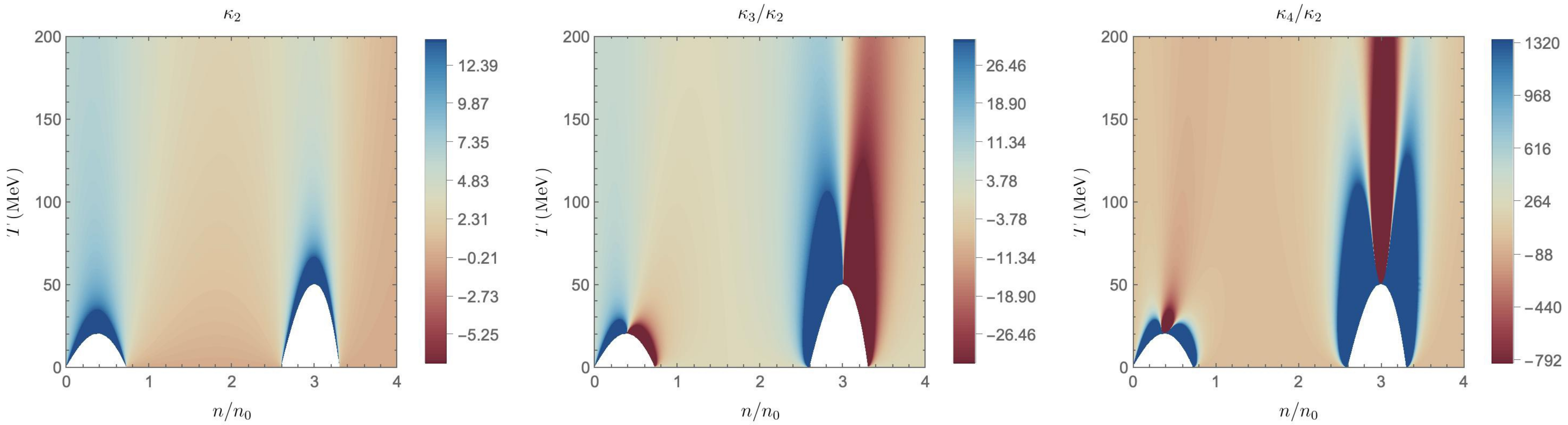
QCD critical point

# Cumulant density plot 累积矩密度图

- Observable can be calculated via

$$\langle O \rangle = \frac{1}{Z_N} \int_b^\infty dv\, e^{N\psi(t,x;v)} O(v)$$

- Cumulant density plot in the global QCD phase diagram:



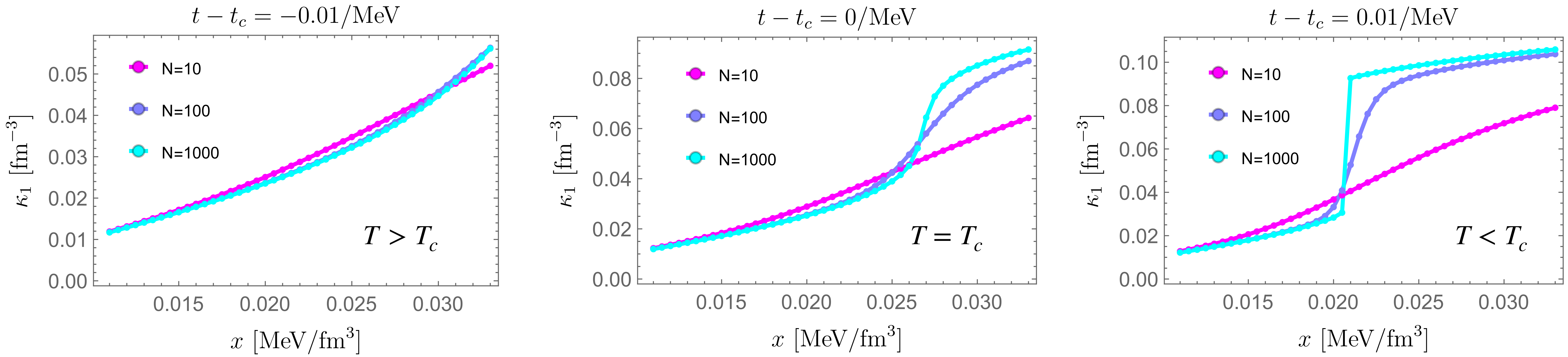

density plot of cumulants $\kappa_m = \langle (n - \langle n \rangle)^m \rangle_c$

cf. Sorensen-Koch, 2011.06635

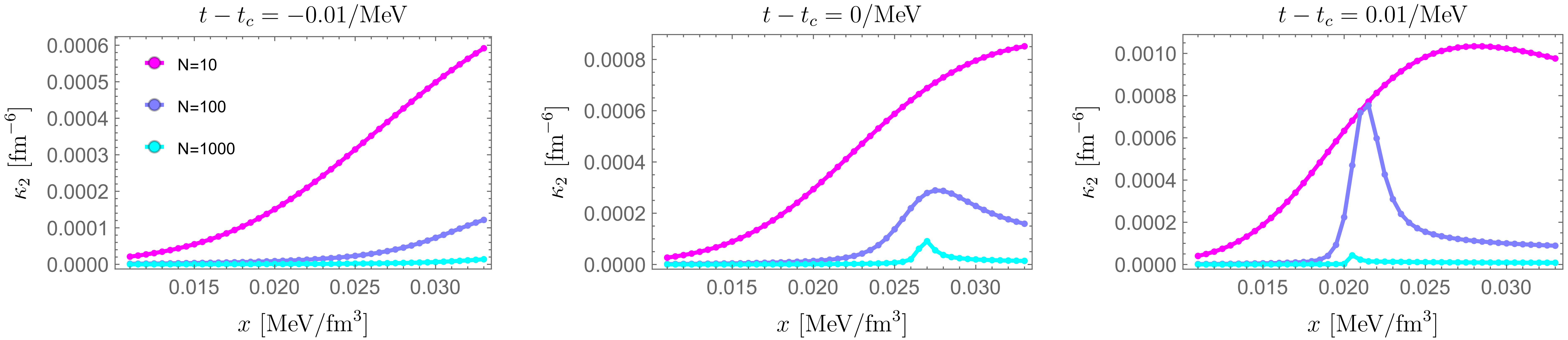
# Numerics at finite N 有限N下的数值结果

- The **nth** cumulant $\kappa_n \equiv \langle \delta n \dots \rangle = \langle \delta v^{-1} \dots \rangle$ as function of pressure $x \equiv P/T$ and temperature $t \equiv 1/T$ at different particle number $N$:

- **1st**




- **2nd**




**s**mearing singularity, **s**hifted location, **s**tronger magnitude at **s**maller N

# PDE for partition function 配分函数的偏微分方程

- Partition function satisfies a linear PDE that are integrable (i.e., exactly solvable):

$$\left( \partial_x^m \partial_t + \sum_{j=1}^{m} \frac{(-N)^{j+1} a_{j+1}}{j} \partial_x^{m-j} \right) Z_N(t, x) = 0$$

$t \equiv 1/T \sim$ time
$x \equiv P/T \sim$ space

For the conventional vdW model it reduces to

$$\left( \partial_x \partial_t + N^2 a_2 \right) Z_N(t, x) = 0$$

(Klein–Gordon equation)

NB: given Cauchy initial conditions at $(t_0, x_0)$, thermodynamics at any $(t, x)$ is inferred.

# PDE for free energy and order parameter 自由能和序参量的偏微分方程

- PDE for Gibbs free energy $G_N$

$$B_K \partial_t G_N + \sum_{j=1}^{K} B_{K-j} \left[ \binom{K}{j} \partial_x^j \partial_t G_N + \frac{(-N)^j a_{j+1}}{j} \right] = 0, \qquad B_n \equiv B_n(\partial_x G_N, \ldots, \partial_x^n G_N)$$

Bell polynomial

- PDE for order parameter $v_N(t,x) \equiv \partial_x G_N(t,x)$ with $\epsilon_N(t,x) \equiv \partial_t G_N(t,x)$:

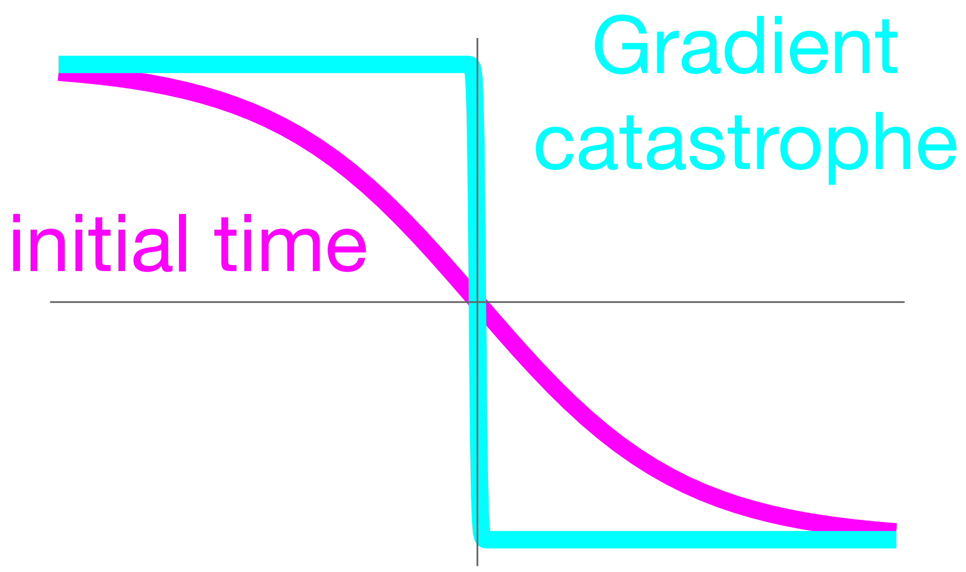
$$\partial_t v_N = \partial_x \left[ \epsilon_N(v_N) + \frac{1}{2N} \partial_x \epsilon_N(v_N) + O(1/N^2) \right]$$

(viscous Burgers equation)

For the conventional vdW model $(N \to \infty)$ it reduces to

$$\partial_t v = \partial_x \epsilon(v) = -\partial_x \left( \frac{a_2}{v} \right)$$

(inviscid Burgers equation)



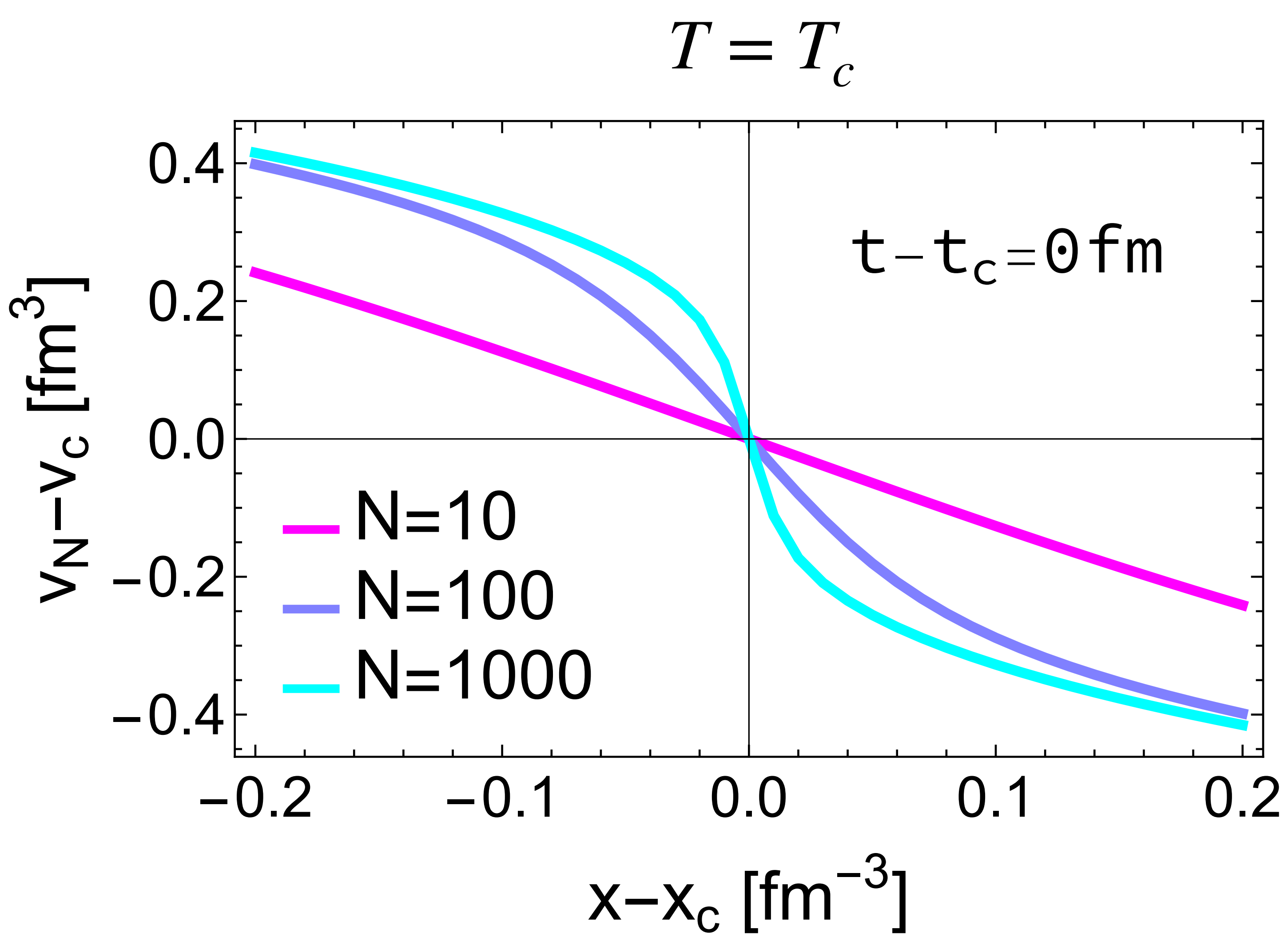
# Critical EOS at finite N 有限N下的临界方程

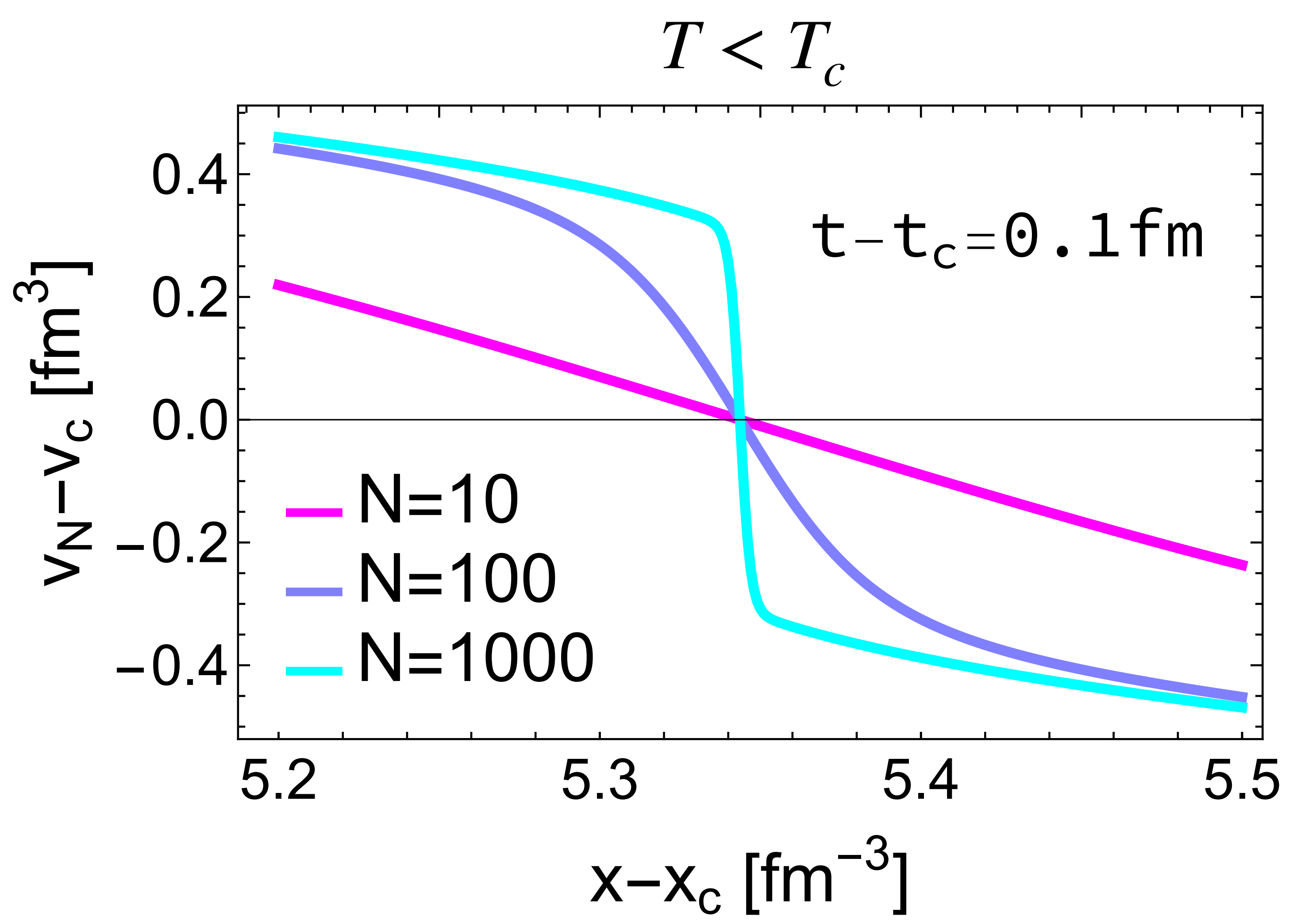
- The universal scaling EOS near critical point at finite N: Dubrovin et al, 2018

$$v_N(t,x) = v_c + \frac{\gamma}{N^{1/4}} U\left(\frac{\Delta x - \epsilon'(v)\Delta t}{\alpha N^{-3/4}}, \frac{\Delta t}{\beta N^{-1/4}}\right) + O(1/N^{1/2})$$

where $\Delta t = t - t_c, \quad \Delta x = x - x_c, \quad U(r_1, r_2) = -2\partial_{r_1} \log \int_{-\infty}^{\infty} dr\, e^{-\frac{1}{8}(r^4 - 2r^2 r_2 + 4 r r_1)}$ (Pearcey integral)

$\alpha, \beta, \gamma$ : determined by thermodynamics

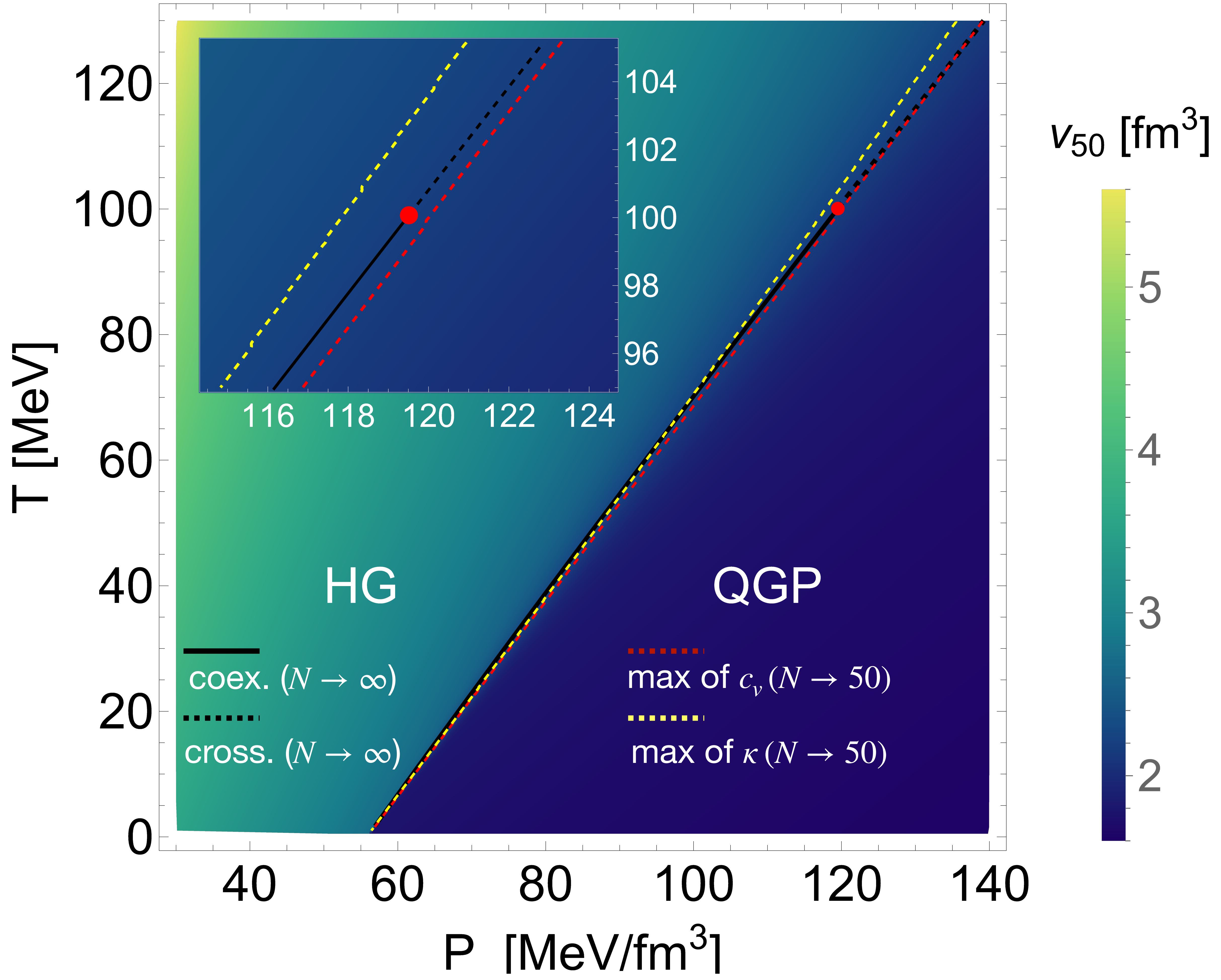


# QCD phase diagram at finite N 有限N下的QCD相图

- The volume density plot (“phase diagram”) at finite N
  - No sharp singularity/boundary: 1st order line and critical point turn to crossover
  - Phase boundary shift, and matters even in the presence of strong finite-time effect
  - Ambiguity to identify the remnant of phase boundary at finite N

Can we really see the critical point?



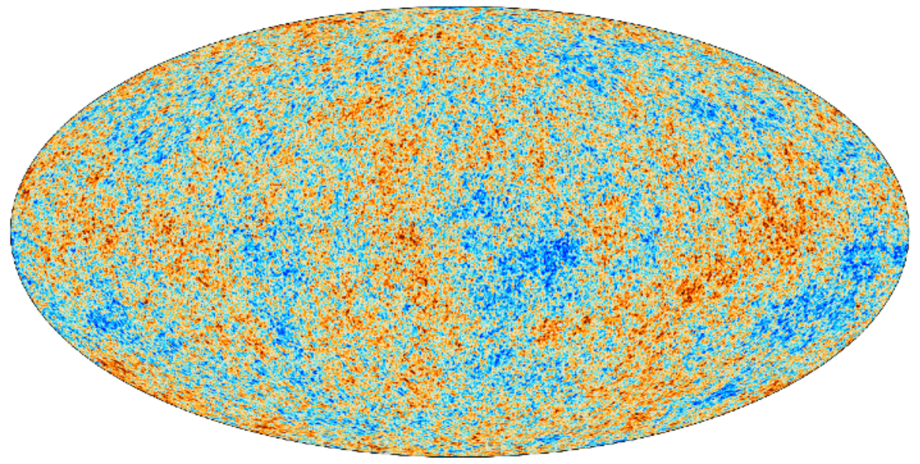

volume density at N=50 near the HG-QGP transition region

# Non-hydro I: hydro with fluctuations
# 非流体力学 I：流体和涨落

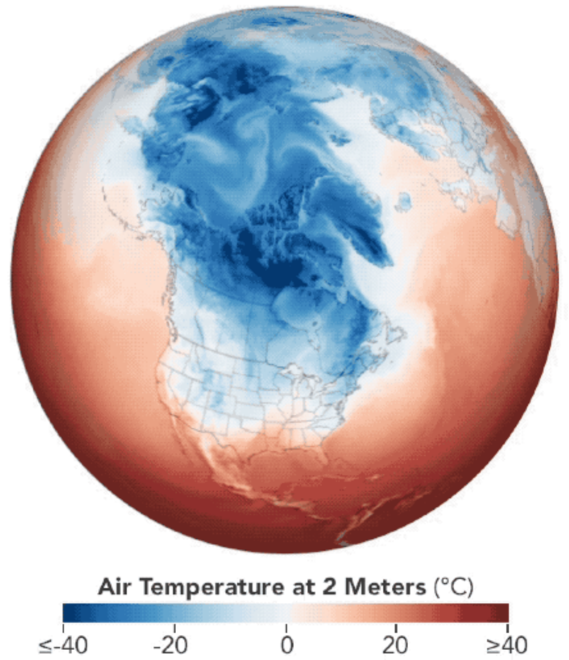
# Why fluctuations? 为什么研究涨落?

- Fluctuations are ubiquitous phenomena emerging on all length scales.

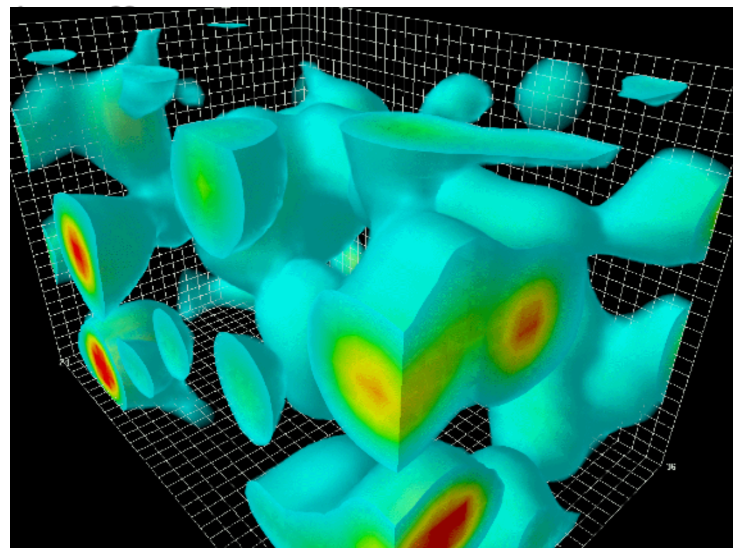
Nobel Prize in Physics 2021
Manabe, Hasselmann, Parisi



CMB

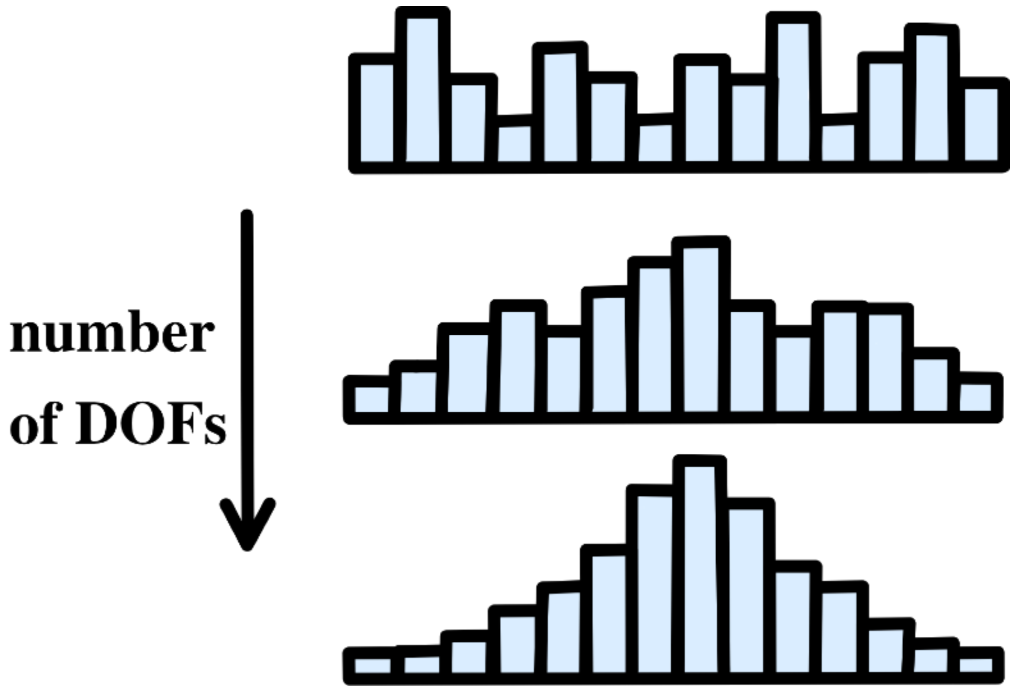


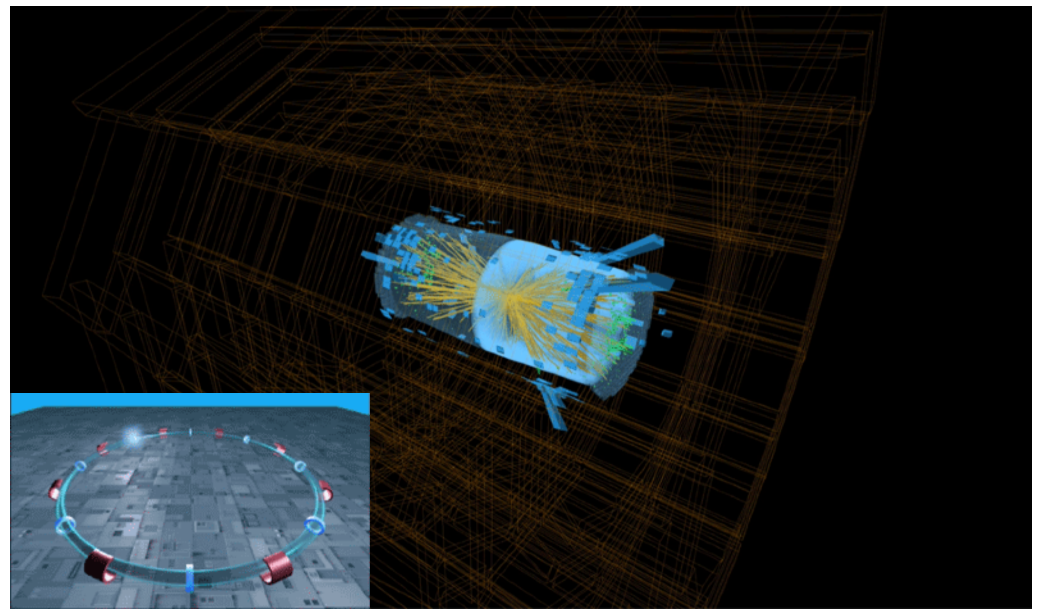

Atmosphere



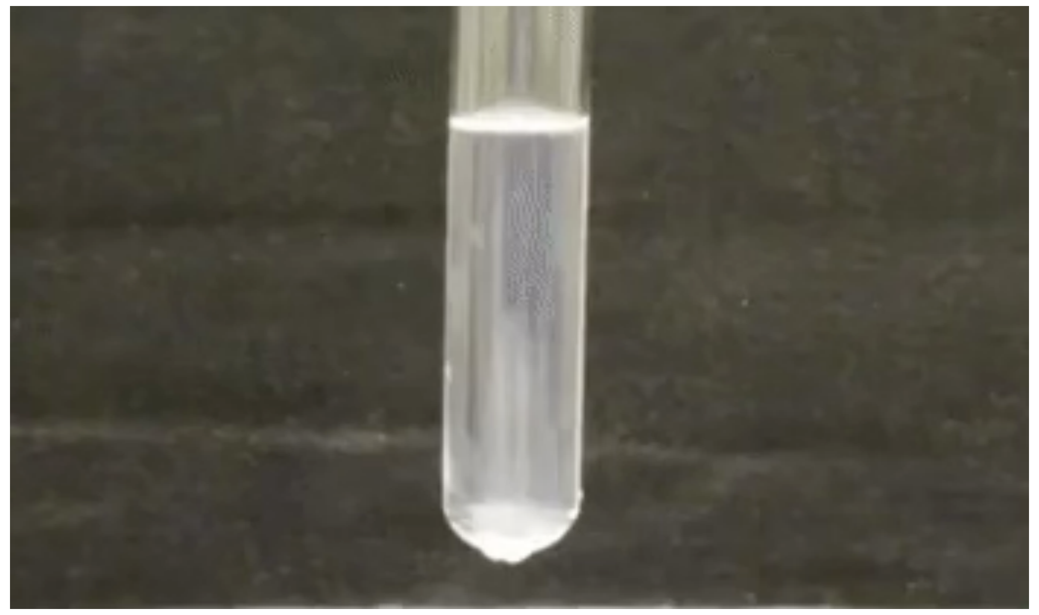
Quantum fluctuations

- Fluctuations are important at small $N$ (central limit theorem $\delta \sim 1/\sqrt{N}$) and develop large deviation from Gaussian.



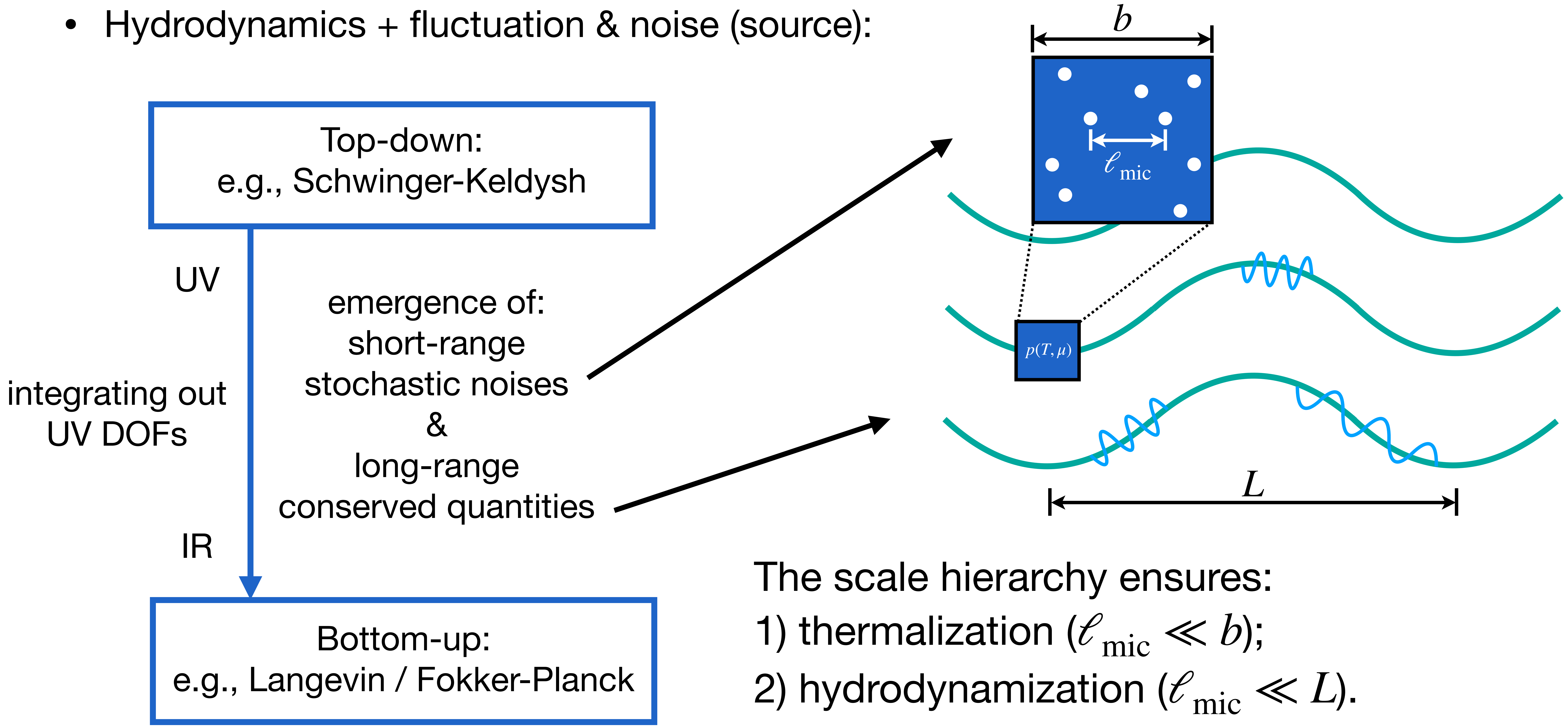
$N \sim 10^2 - 10^4$



$N \sim L^3/\xi^3$

# Scales hierarchy 尺度序列

- Hydrodynamics + fluctuation & noise (source):




The scale hierarchy ensures:

1) thermalization ($\ell_{\rm mic} \ll b$);

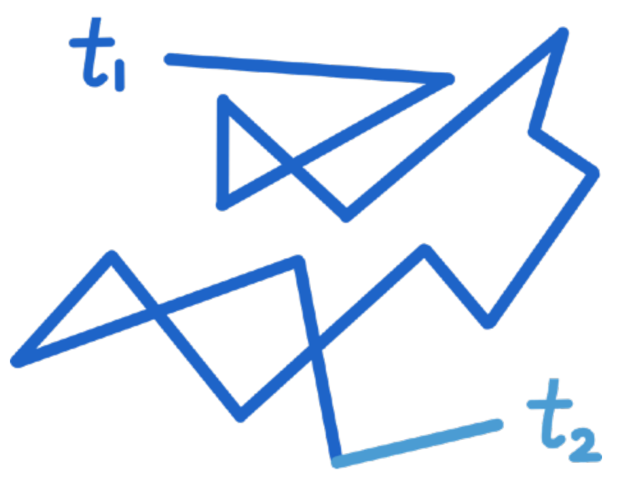
2) hydrodynamization ($\ell_{\rm mic} \ll L$).

# Variant theoretical approaches 不同的理论方法

- Fluctuating hydro can be formulated in variant approaches, to list a few:

### Stochastic Navier-Stokes

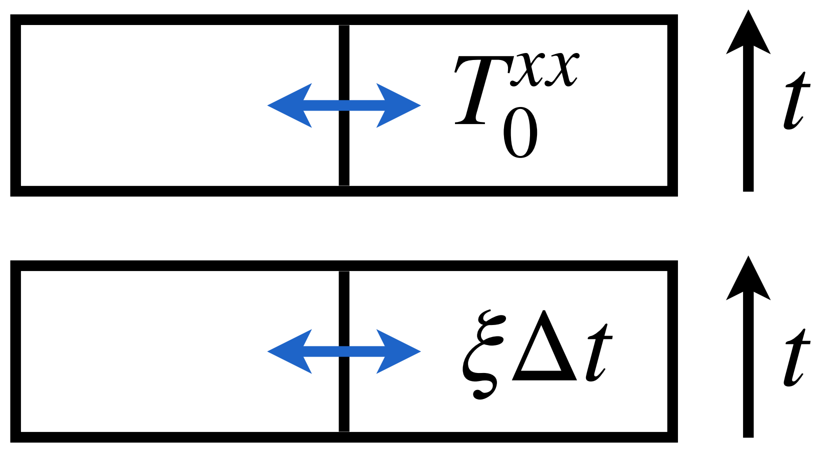
$$\dot{\psi}_i = F_i + \xi_i,$$
$$\langle \xi_i(t)\xi_j(t') \rangle = M_{ij}\delta_{tt'}$$




infinite/multiplicative noise; unstable & acausal

### Metropolis algorithm

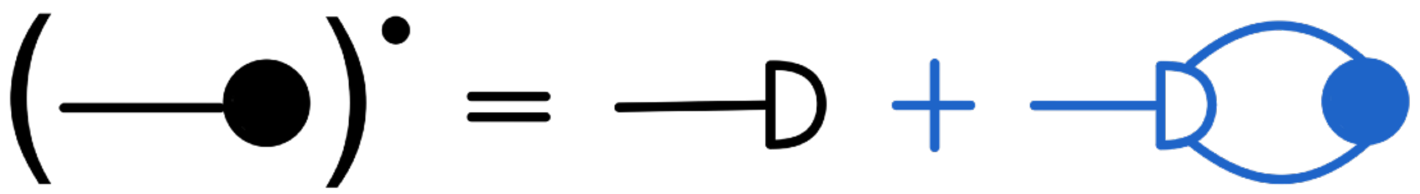
$$\dot{\psi}_i = -M_{ij}H_{,j} + \xi_i,$$
$$P[\psi] = e^{-H[\psi]}$$




stable; need renormalized quantities

### Deterministic equations

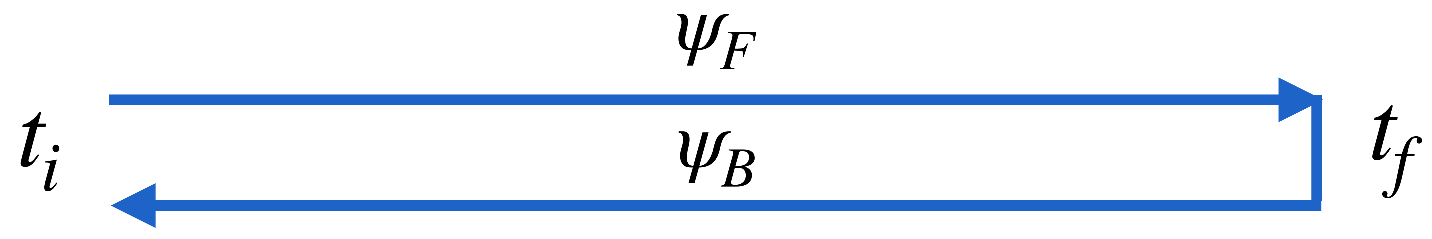
$$\dot{\psi}_i = F_i + F_{i,jk}G_{jk} + \ldots$$



analytic; so far hydro regime

### Schwinger-Keldysh formalism

$$\mathbb{L}(\psi_R, \psi_A) = E(\psi_R)\psi_A + i\psi_A M(\psi_R)^{-1}\psi_A$$




field-theoretical, top-down

# Deterministic approach 确定性方法

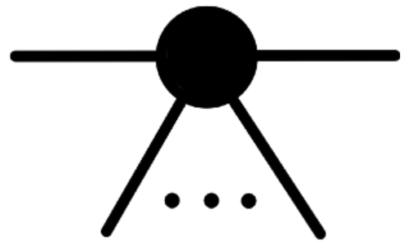
- Partition function from classical fields to thermodynamic variables (infinite time):

$$Z(J) = \int d\psi\, P(\psi)\, e^{-J\psi} \quad \xrightarrow{\psi \to \psi(t,x)} \quad Z[J;t] = \int \mathscr{D}\psi\, P(\psi, t)\, e^{-J\psi}$$

- Deterministic approach describes systems only via averaged quantities

$$\langle \ldots \rangle = \int \mathscr{D}\psi \;\ldots\; P[\psi; t] \qquad \text{where} \qquad \partial_t P = (-F_i P + (M_{ij} P)_{,j})_{,i} \qquad \text{(Fokker-Planck)}$$

- Correlation functions

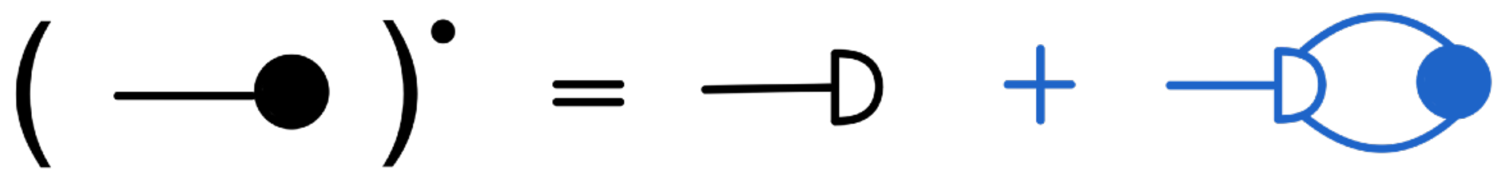


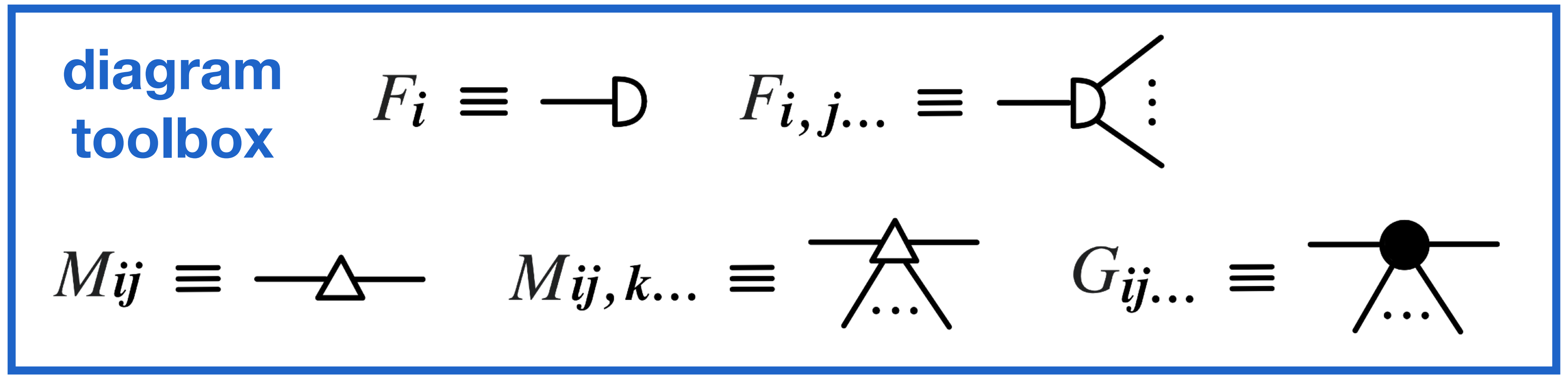
$$G_{i_1\ldots i_n} \equiv \langle \psi_{i_1} \ldots \psi_{i_n} \rangle = (-1)^n \frac{\delta^{(n)} \ln Z}{\delta J_{i_1} \ldots \delta J_{i_n}} \bigg|_{J=0}$$

# Deterministic equations 确定性方程

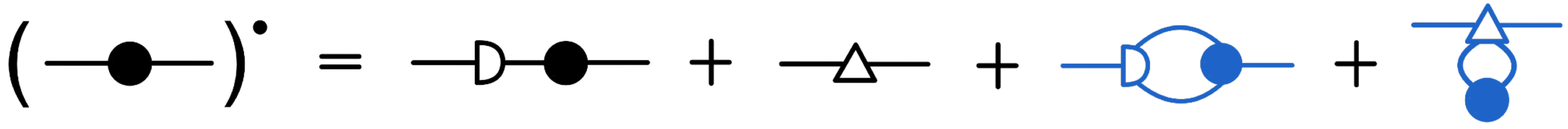
- Diagrammatic form and truncations: XA et al, 2009.10742 (PRL)

**1-pt** $\partial_t \psi_i = F_i + F_{i,jk} G_{jk} + \mathcal{O}(G_3)$



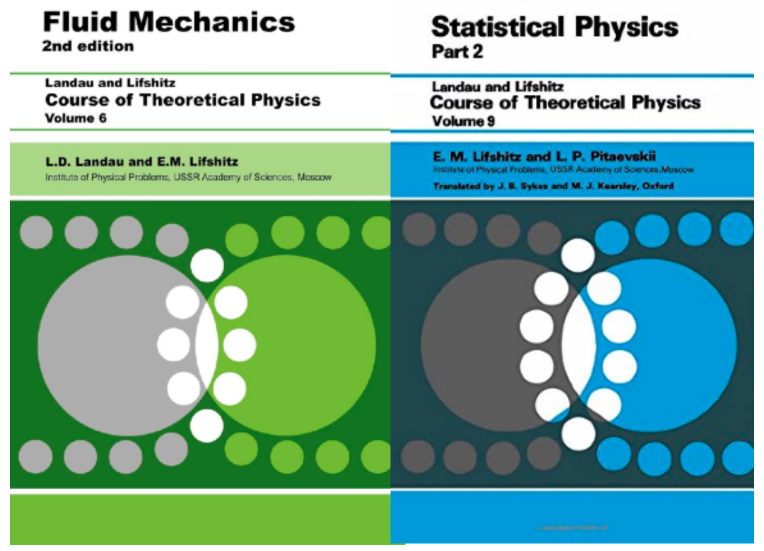



**2-pt** $\partial_t G_{ij} = 2F_{i,k} G_{kj} + 2M_{ij} + F_{i,k\ell} G_{k\ell j} + M_{ij,k\ell} G_{k\ell} + \mathcal{O}(G_4)\Big|_{\text{perm.}}$



Power counting gives rise to leading diagrams trees:

$$G_n \sim \epsilon^{n-1}, \quad \epsilon \sim \frac{1}{N} \qquad \text{(CLT)}$$



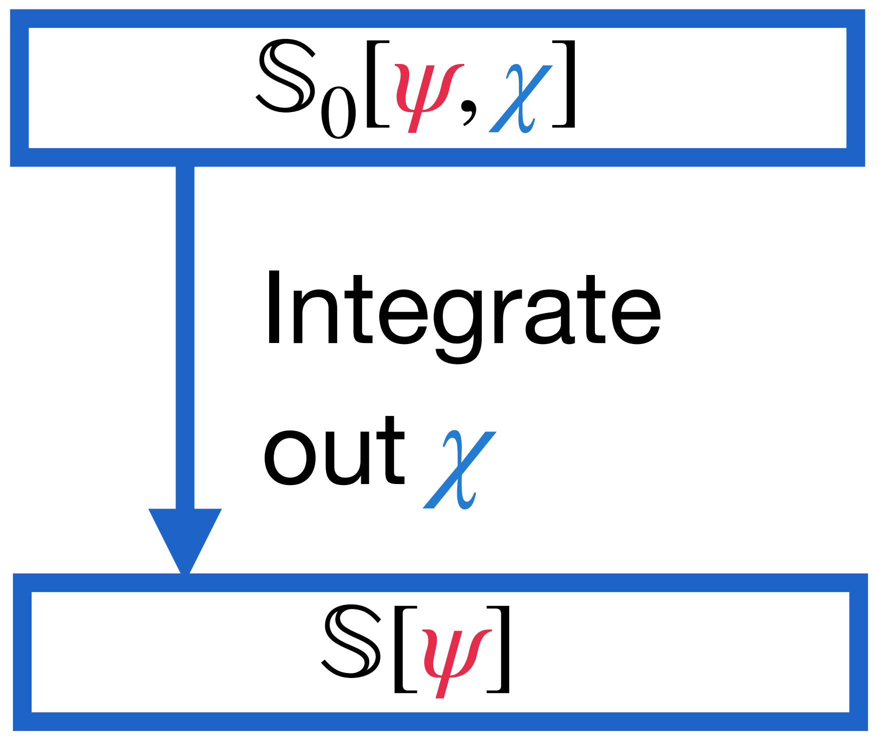

complementary to stochastic approaches in Landau-Lifshitz

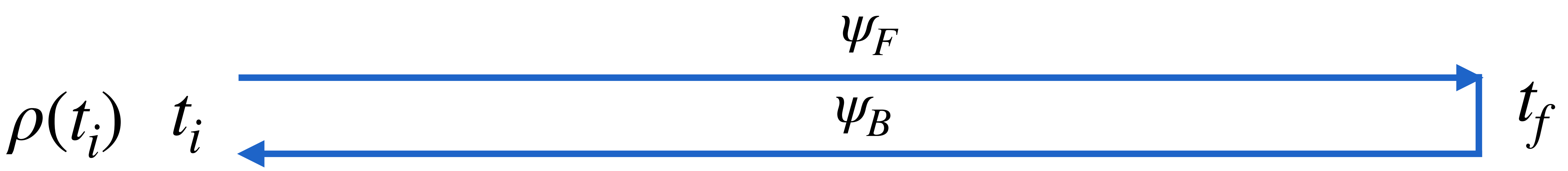
# Schwinger-Keldysh formalism 施温格-凯尔迪什方法

- Starting from a bare action to an effective action (top-down)

$$Z = \int \mathscr{D}\psi_F \mathscr{D}\psi_B \int \mathscr{D}\chi_F \mathscr{D}\chi_B \, e^{i(\mathbb{S}_0[\psi_F,\chi_F]-\mathbb{S}_0[\psi_B,\chi_B])} = \int \mathscr{D}\psi_F \mathscr{D}\psi_B e^{i\mathbb{S}[\psi_F,\psi_B]} = \int \mathscr{D}\psi_R \mathscr{D}\psi_A e^{i\mathbb{S}[\psi_R,\psi_A]}$$

slow (IR) fast (UV)




Schwinger, 1961; Keldysh, 1965

Martin et al, 1973

Glorioso et al, 1805.09331

$$\psi_R = \psi = \frac{1}{2}(\psi_F + \psi_B) \qquad \psi_A = \tilde{\psi} = \psi_F - \psi_B$$

**Unitarity:** $\mathbb{S}^*[\psi_R, \psi_A] = -\mathbb{S}[\psi_R, -\psi_A], \quad \mathbb{S}[\psi_R, \psi_A = 0] = 0, \quad \mathrm{Im}\,\mathbb{S}[\psi_R, \psi_A] \geq 0$

Kubo-Martin-Schwinger **Z(2) symmetry:** $\mathbb{S}[\psi_R, \psi_A] = \mathbb{S}[\tilde{\psi}_R, \tilde{\psi}_A] \quad \Longrightarrow$

$$\tilde{\psi}_R(-t) = \psi_R(t), \quad \tilde{\psi}_A(-t) = \psi_A(t) + i\beta_0 \dot{\psi}_R(t)$$

# Relation of variant approaches 不同方法之间的联系

**Path-integral** $Z = \int \mathscr{D}\psi_R \mathscr{D}\psi_A \, e^{i\int_t \mathbb{L}}, \ \mathbb{L}(\psi_R, \psi_A) = (F - \dot{\psi}_R)Q^{-1}\psi_A + i\psi_A Q^{-1}\psi_A$

HS transform
$\psi_A \to \xi$

Interpret $Z$ as probability

$Z = \int \mathscr{D}\psi_R \mathscr{D}\eta \delta(F - \dot{\psi}_R + \xi) e^{-\int_t \xi Q^{-1} \xi}$

$Z = P[\psi] = \int_{\psi_r = \psi(t)} \mathscr{D}\psi_R \mathscr{D}\psi_A \, J(\psi_R) \, e^{i\int_{-\infty}^{t} d\tau \mathbb{L}}$

**Langevin** $\dot{\psi} = F[\psi] + \xi$

**Fokker-Plank** $\partial_t P = (-F_i P + (M_{ij} P)_{,j})_{,i}$

# Why covariance? 为什么需要协变性?

- Covariance is a fundamental requirement of physics.

> *The requirement of general covariance takes away from space and time the last remnant of physical objectivity.*

Einstein

- Convenient for ultra-relativistic processes.


$$\gamma = \frac{1}{\sqrt{1 - \mathbf{v}^2}} \quad \to \quad \infty$$

- Covariance admits the choice of local rest frame (albeit *ambiguous*), where thermodynamics become simpler.

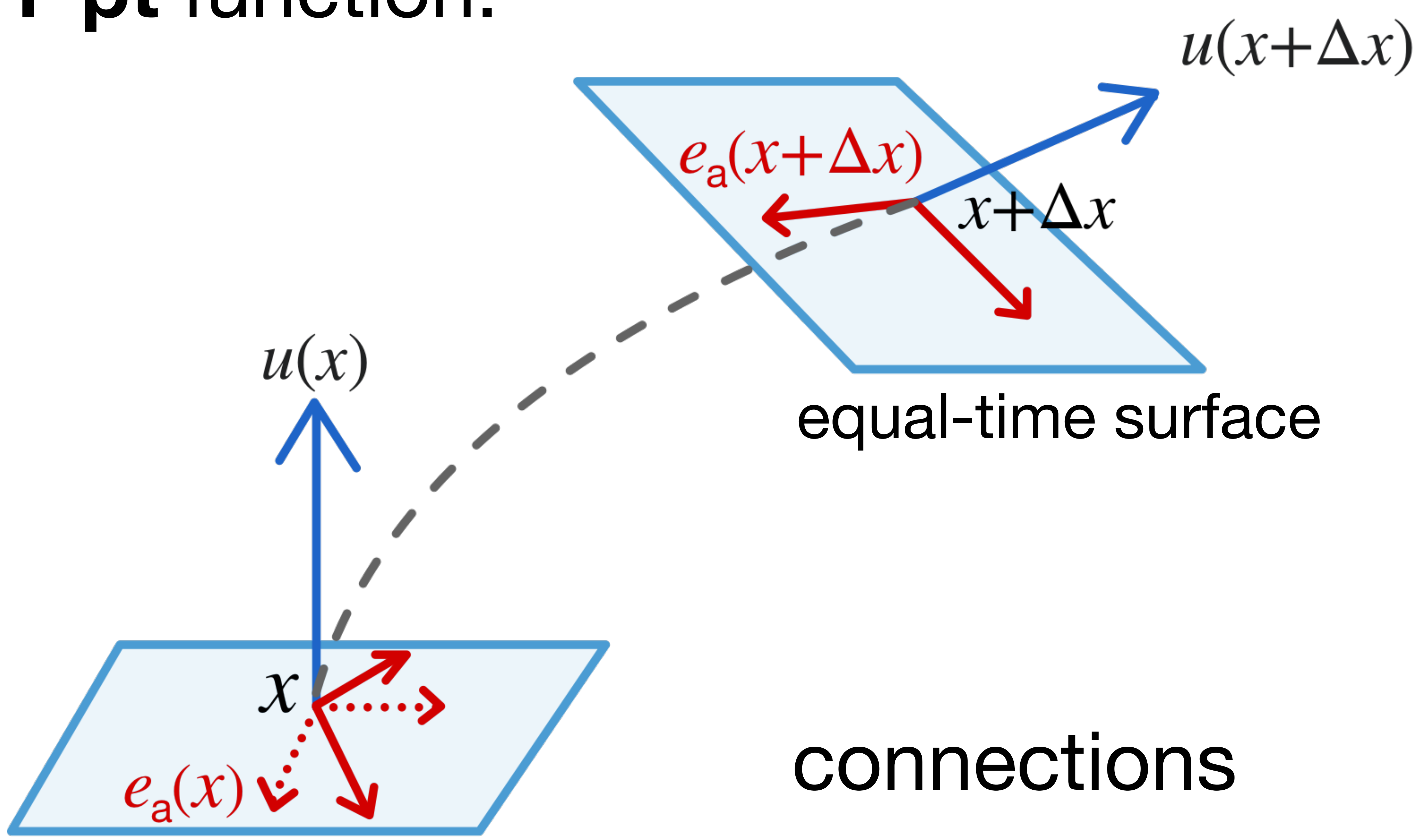
$$\underline{T}\mathrm{d}\underline{s} = \mathrm{d}\underline{\epsilon} - \underline{\mu}\mathrm{d}\underline{n} - \underline{\mathbf{v}} \cdot \mathrm{d}\underline{\pi} \quad \to \quad T\mathrm{d}s = \mathrm{d}\epsilon - \mu \mathrm{d}n$$

# Relativistic dynamics: 1-pt function 相对论动力学：1点函数

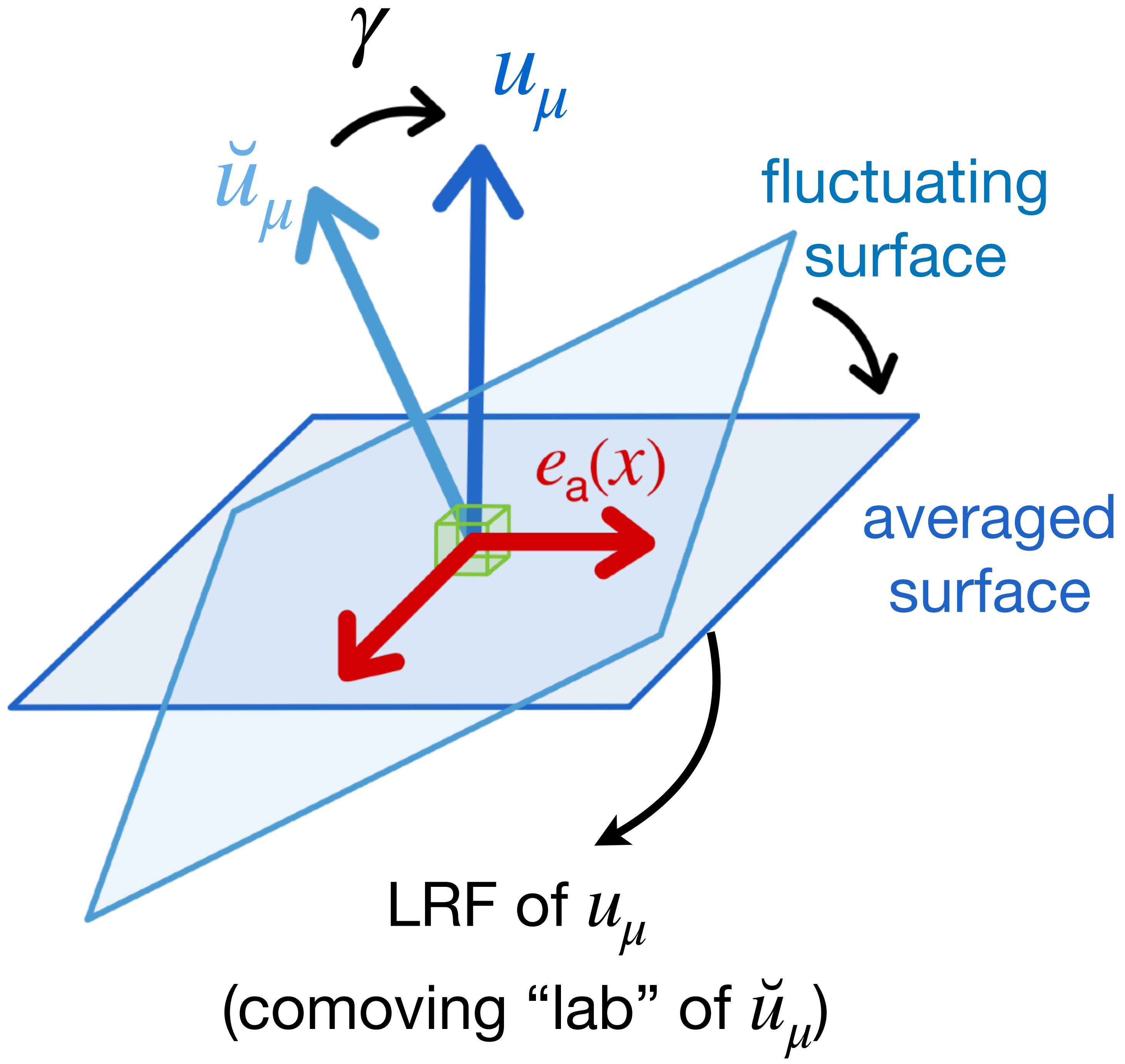
- Goal 1: deal with relativity/covariance as if one knows nothing about relativity (i.e., like how one deals with non-relativistic theories in the lab).

- Goal 2: deal with stochastic process as if one knows nothing about randomness (i.e., like how one deals with non-fluctuating theories).

- **1-pt** function:



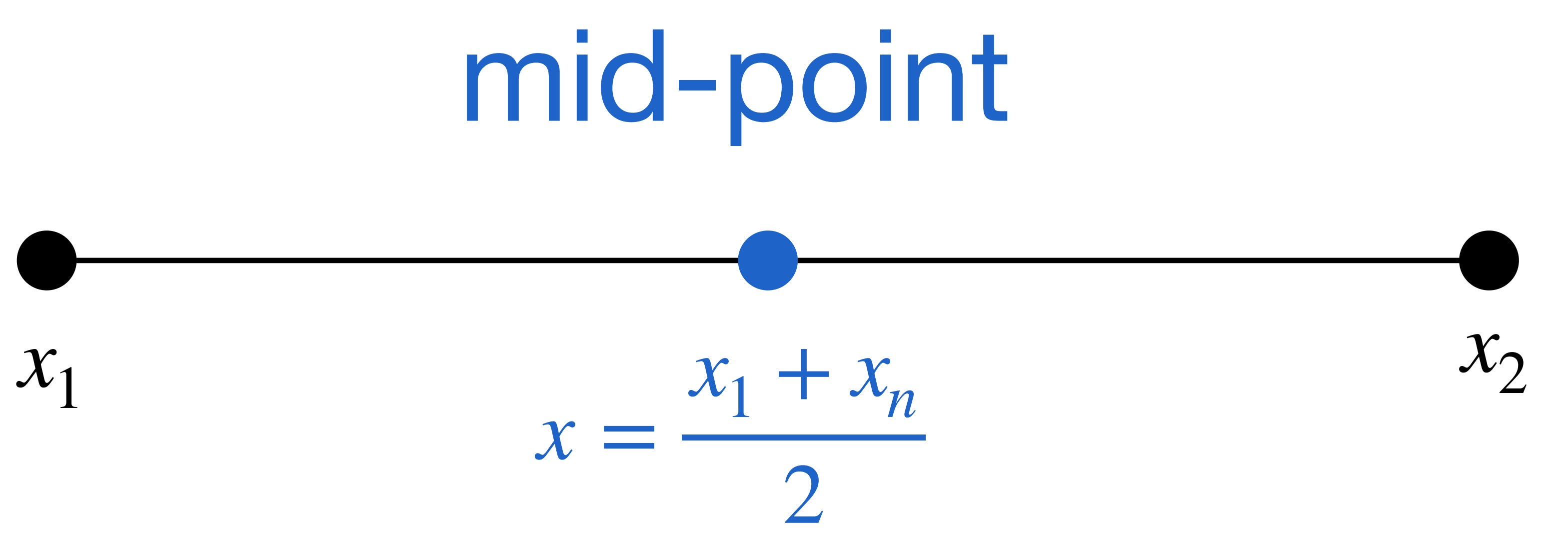

connections

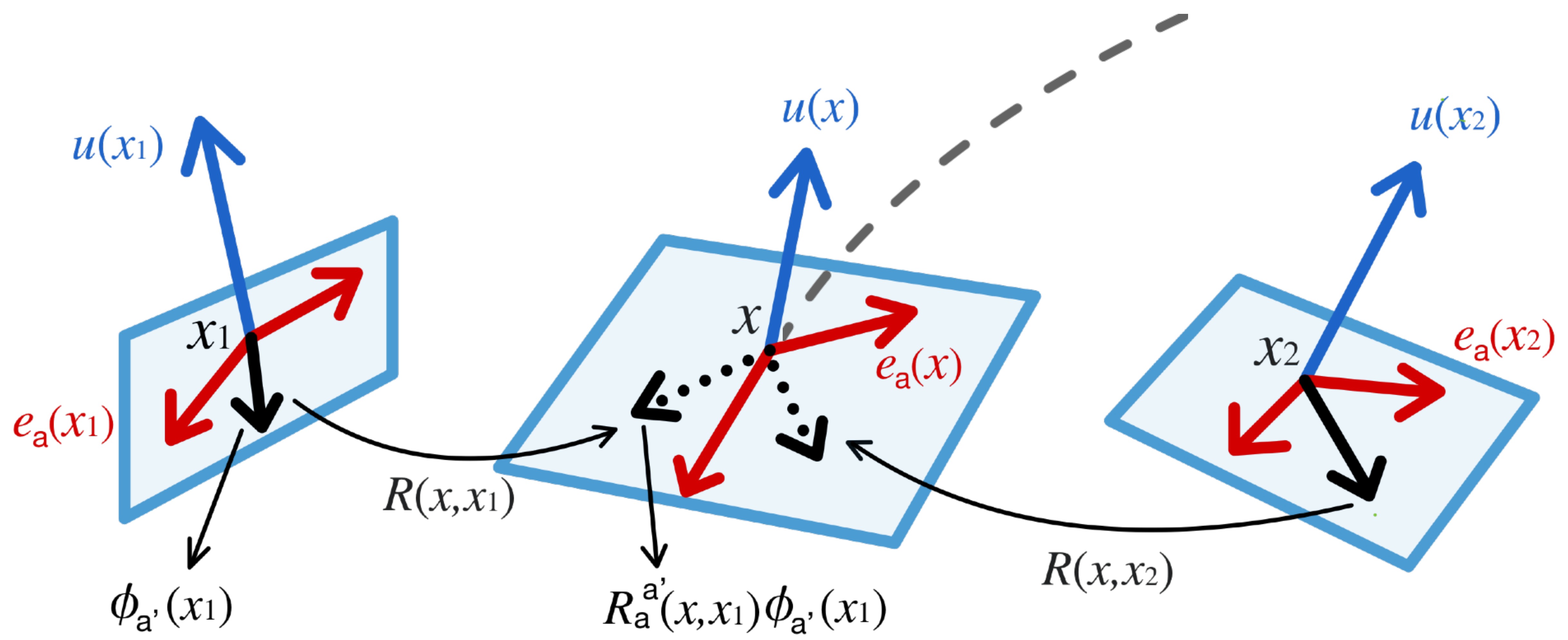



XA et al, 2212.14029 and working in progress

# Relativistic dynamics: n-pt function 相对论动力学：n点函数

- **n-pt**: relative motion to the midpoint needs to be described by 1-pt-like EOM.




more connections

consider 2-pt for illustration purpose

XA et al, 2212.14029 and working in progress

# Relativistic dynamics: Wigner function 相对论动力学：魏格纳函数演化方程

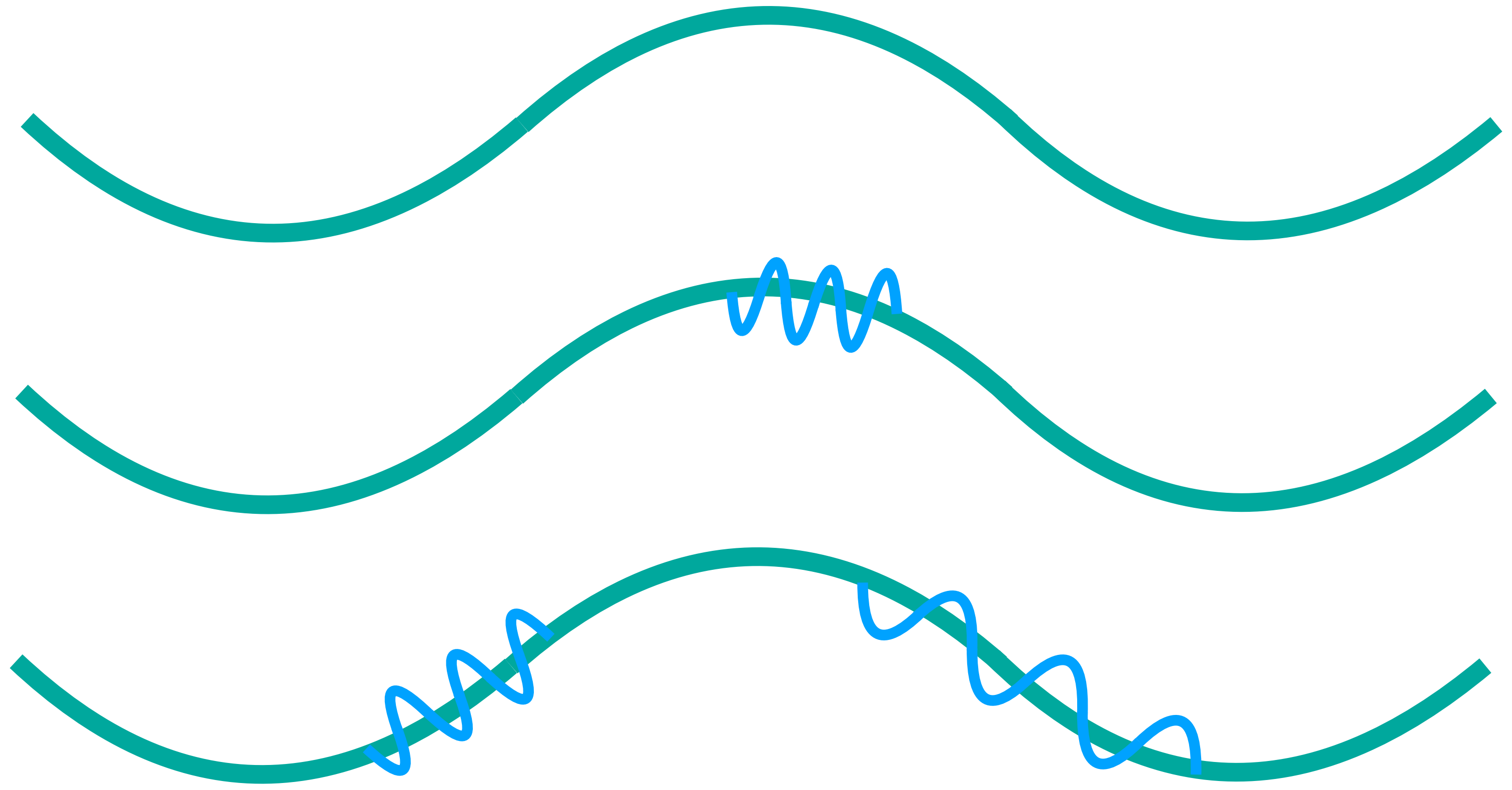
- Confluent $n$-pt Wigner transform between 3-vectors $y^a$ and $q^a$ (*as if you are dealing with non-relativistic theories*).

$$W_n(x; q_1^a, \ldots, q_n^a) = \underbrace{\int \prod_{i=1}^{n} \left(d^3 y_i^a \, e^{-i q_{ia} y_i^a}\right) \delta^{(3)}\left(\frac{1}{n}\sum_{i=1}^{n} y_i^a\right)}_{x \text{ independent integration kernal}} \bar{G}_n(\underbrace{x + e_a y_1^a}_{x_1}, \ldots, \underbrace{x + e_a y_n^a}_{x_n})$$

$$(u \cdot \bar{\nabla} + f \cdot \nabla_q) W_L(q_1, q_2) = -\gamma_L q^2 \left(W_L - \frac{T}{E}\right)$$

kinetic equation for phonons

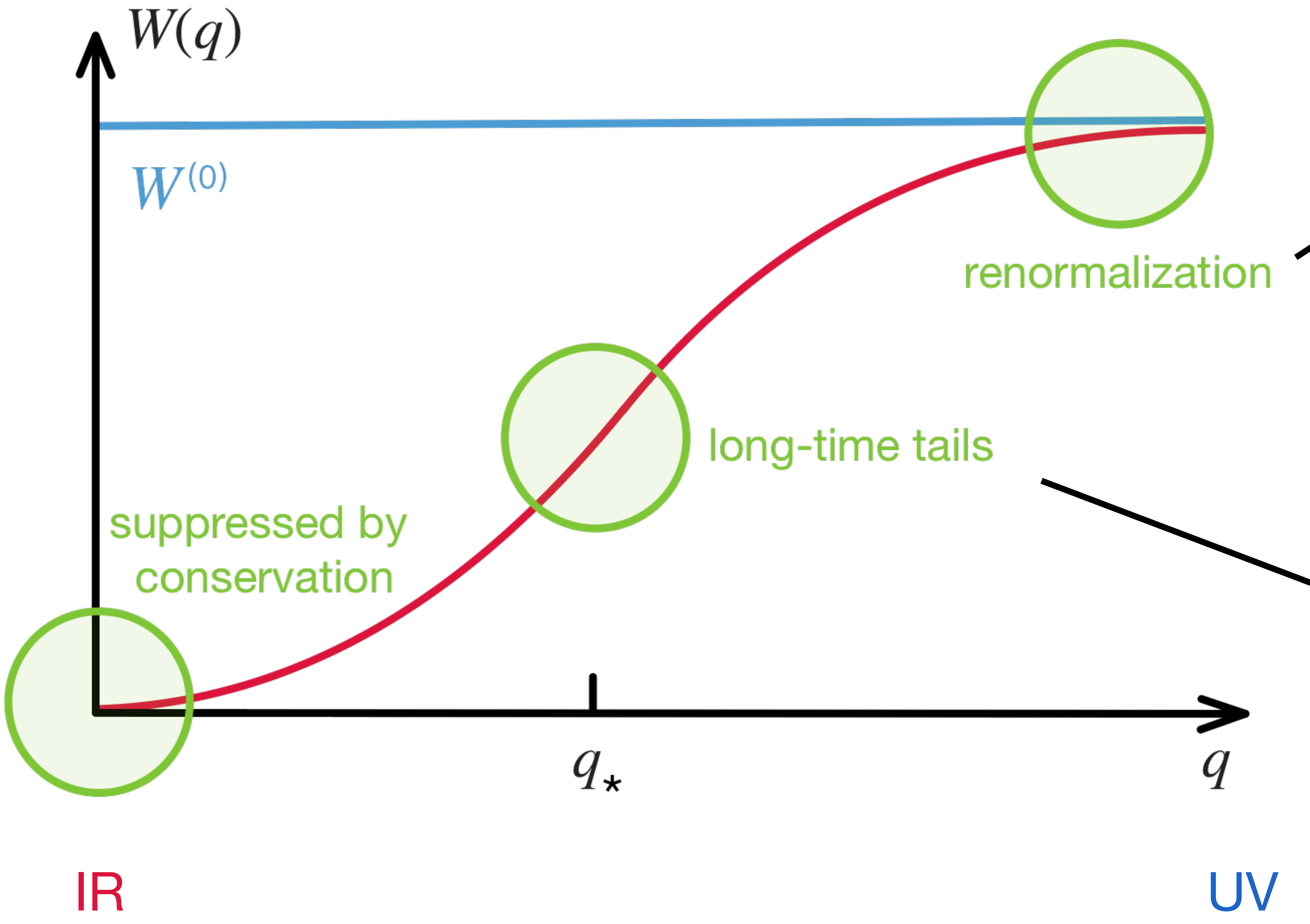
this is one equation among **about 100** equations to be solved

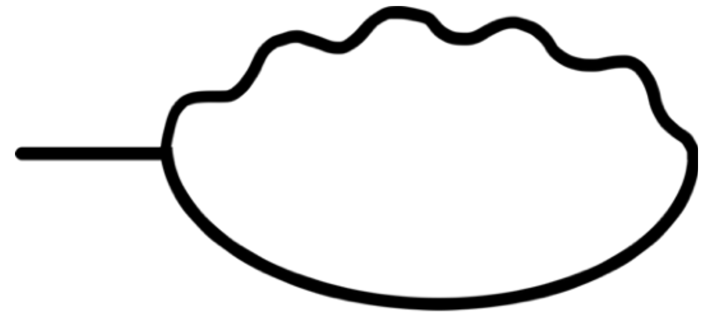



XA et al, 2212.14029 and working in progress

# Renormalization and dynamic feedback 重整化和动力学反馈

- Solutions in wavenumber space



one-loop tadpole

$\langle \delta n \delta n \rangle$

$\langle \delta u \delta u \rangle$

Hohenberg-Halperin, 1977
XA et al, 1912.13456

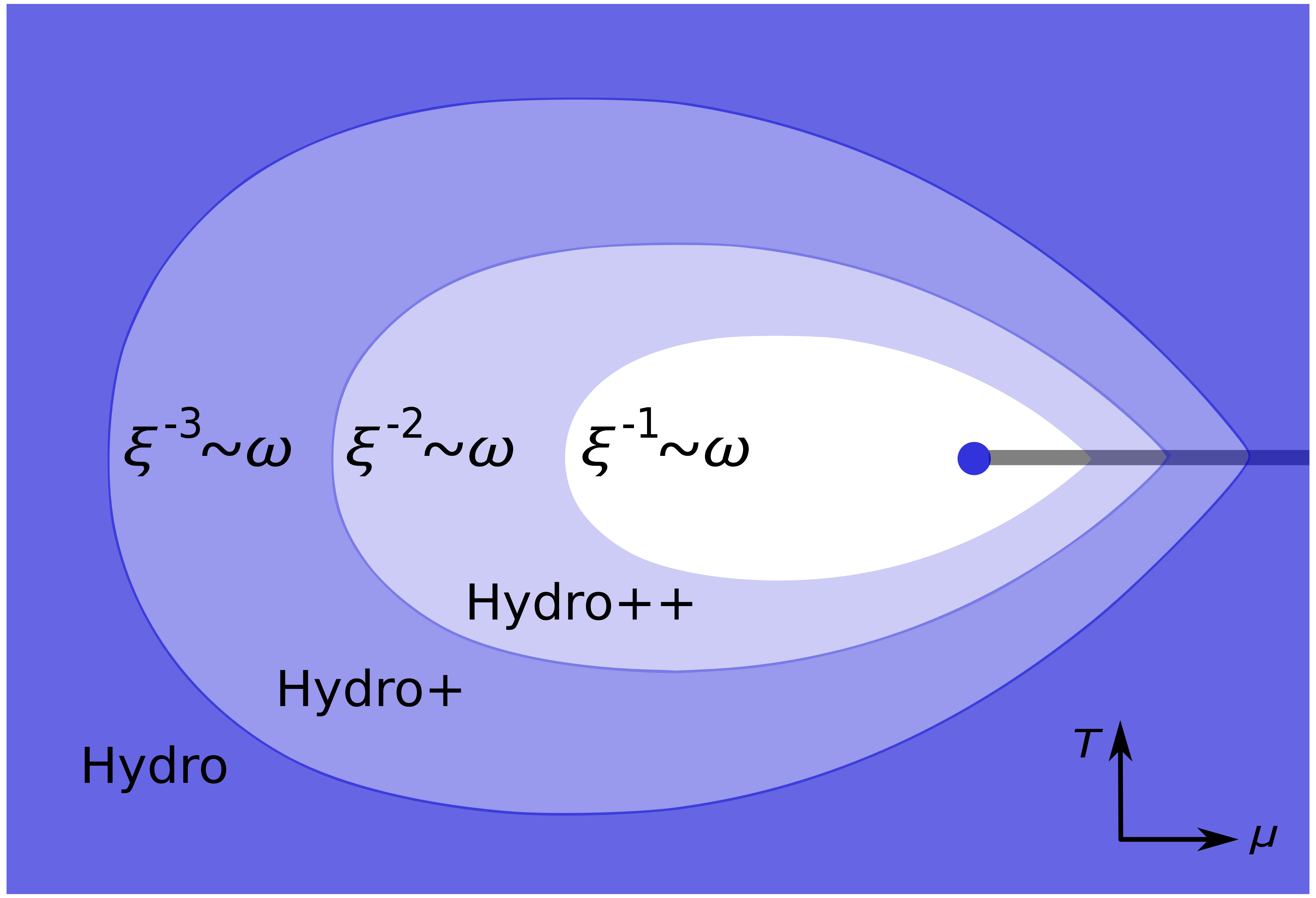
$$t^{-3/2} \gg e^{-t}, \quad t \to \infty$$

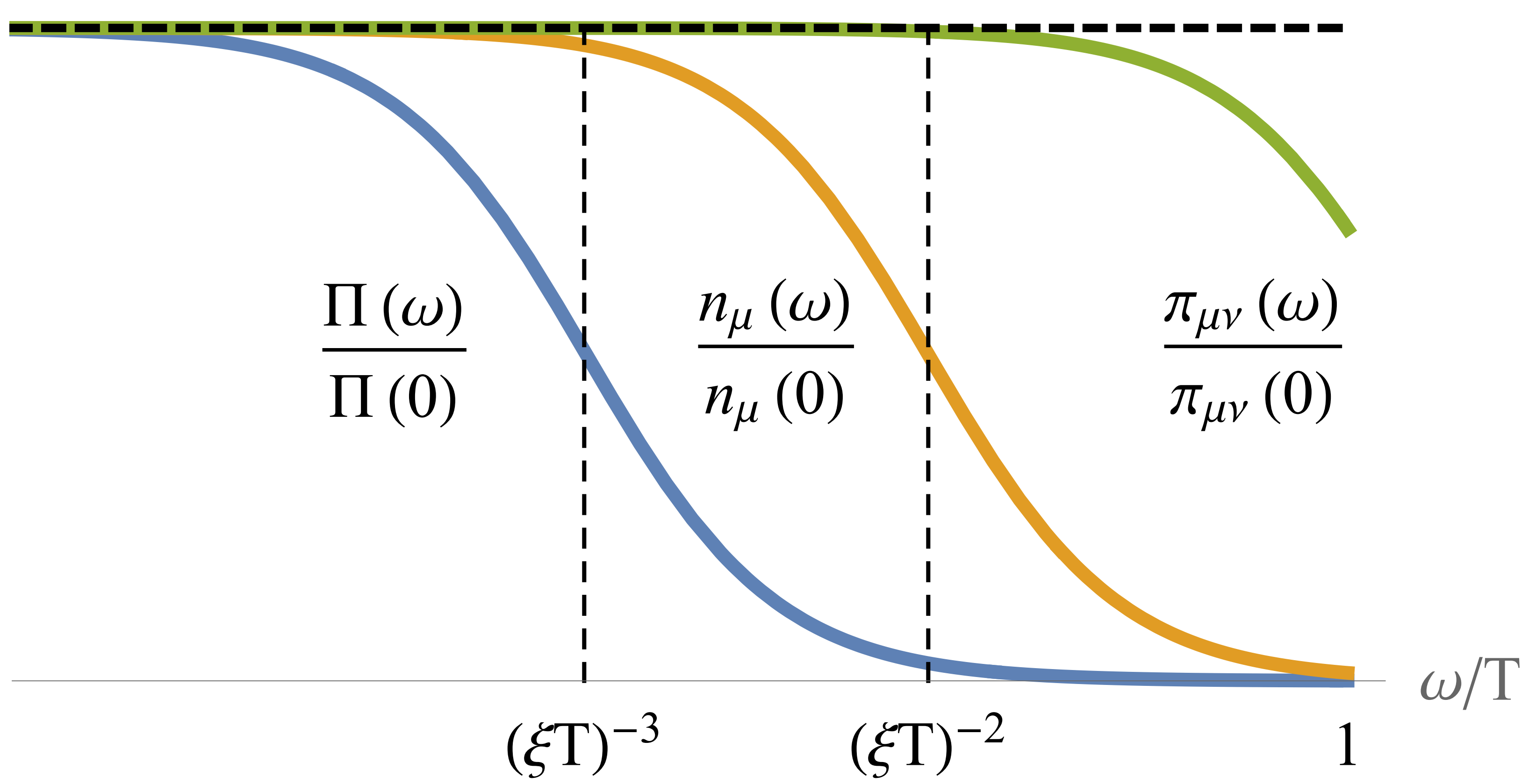
Kovtun-Yaffe, 0303010
XA et al, 1912.13456

# Fluctuations as non-hydrodynamic modes 涨落和非流体模式

- Dynamic feedback demonstrates the relaxation of fluctuations slows down near CP:

$$\text{relaxation rate } \gamma \sim \xi^{z}, \quad \xi \to \infty$$



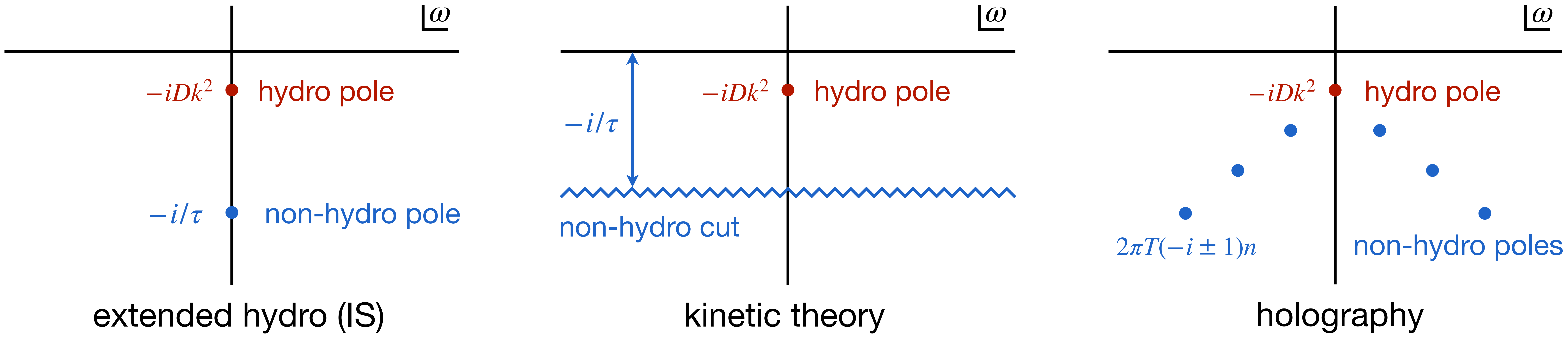

XA, 2003.02828
Du, XA, Heinz, 2107.02302

cf Y. Shen, 2404.02397

closer to CP, more and more modes becomes **non-hydrodynamic**

# Various non-hydro modes 不同的非流体模式

- Analytic structures of retarded Green functions differ beyond hydro regime.




**Hydro modes**: governed by symmetries associated with conservation laws.

**Non-hydro modes**: no such manifest symmetries to associate with. We argue that, there are symmetries hidden behind, in theories involving non-hydro sectors such as extended hydro, kinetic theory, and holography.

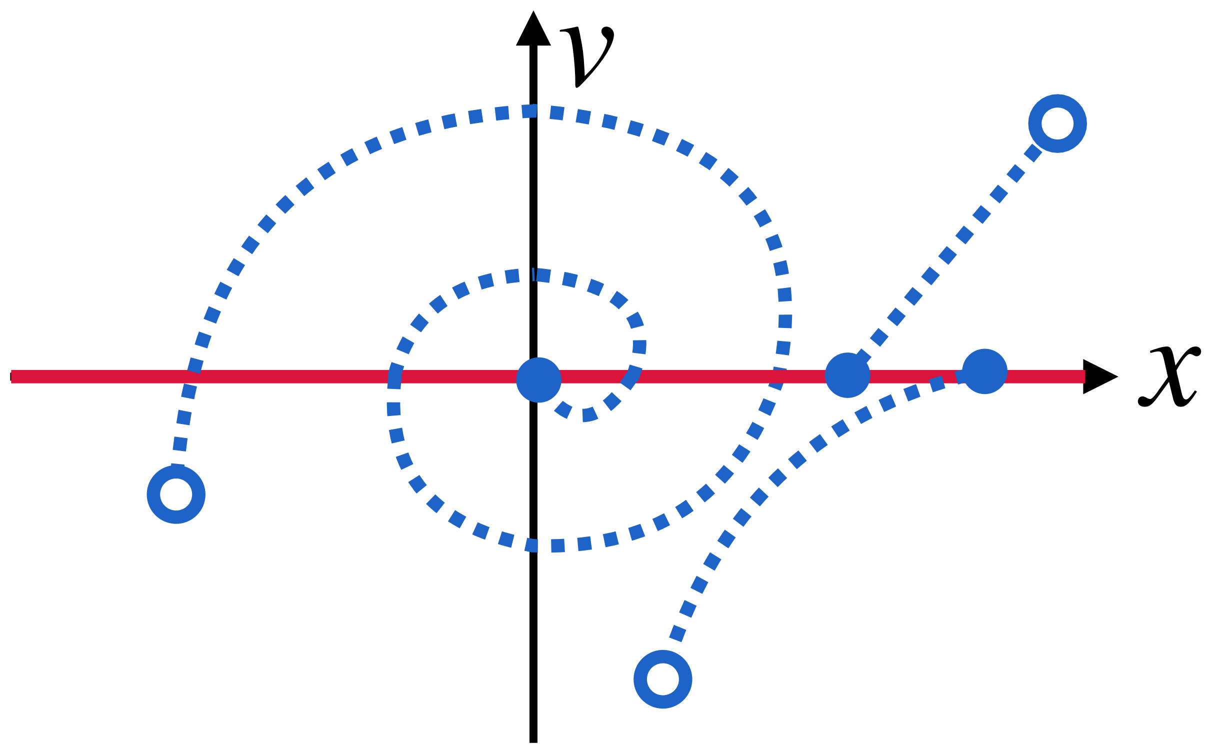
# Non-hydro 2: hydro with attractor
# 非流体力学 2：流体和吸引子

# Attractor 吸引子

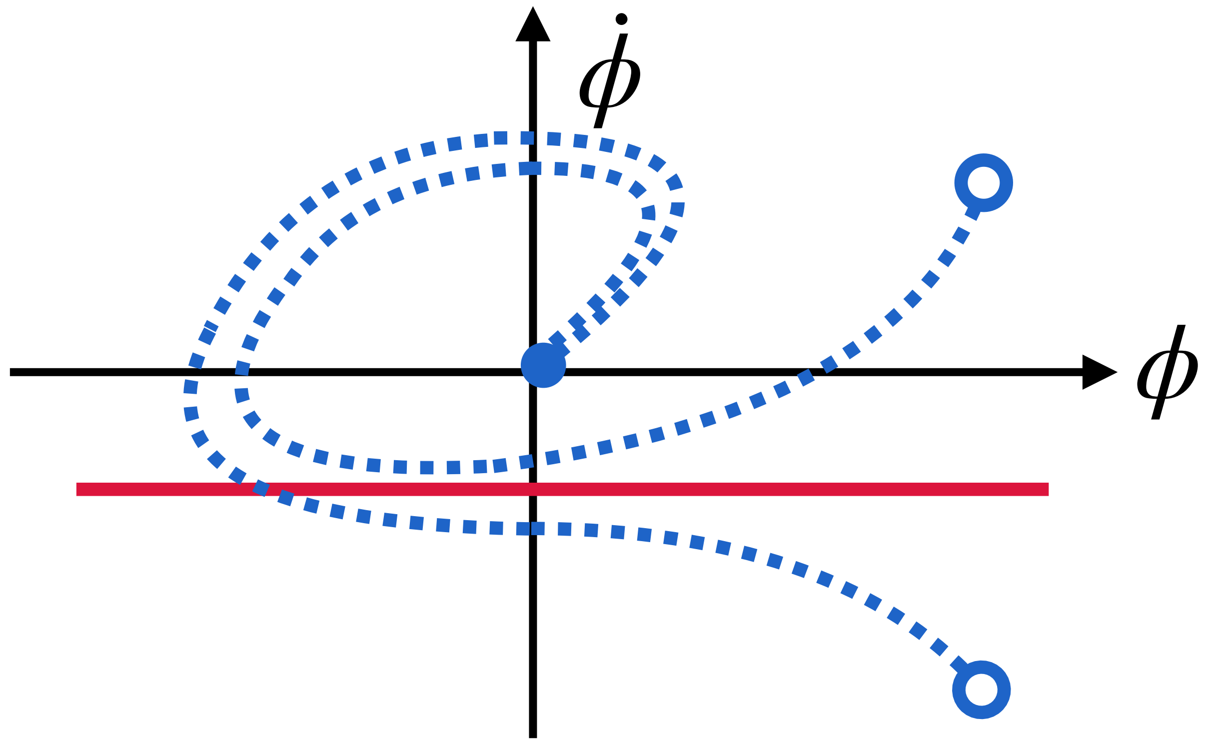
- An **attractor** is a set of states toward which a dynamic system tends to evolve, for a wide variety of initial conditions.

Examples:

1. Aristotle's law of motion, albeit wrong, implies a **dissipative attractor**.

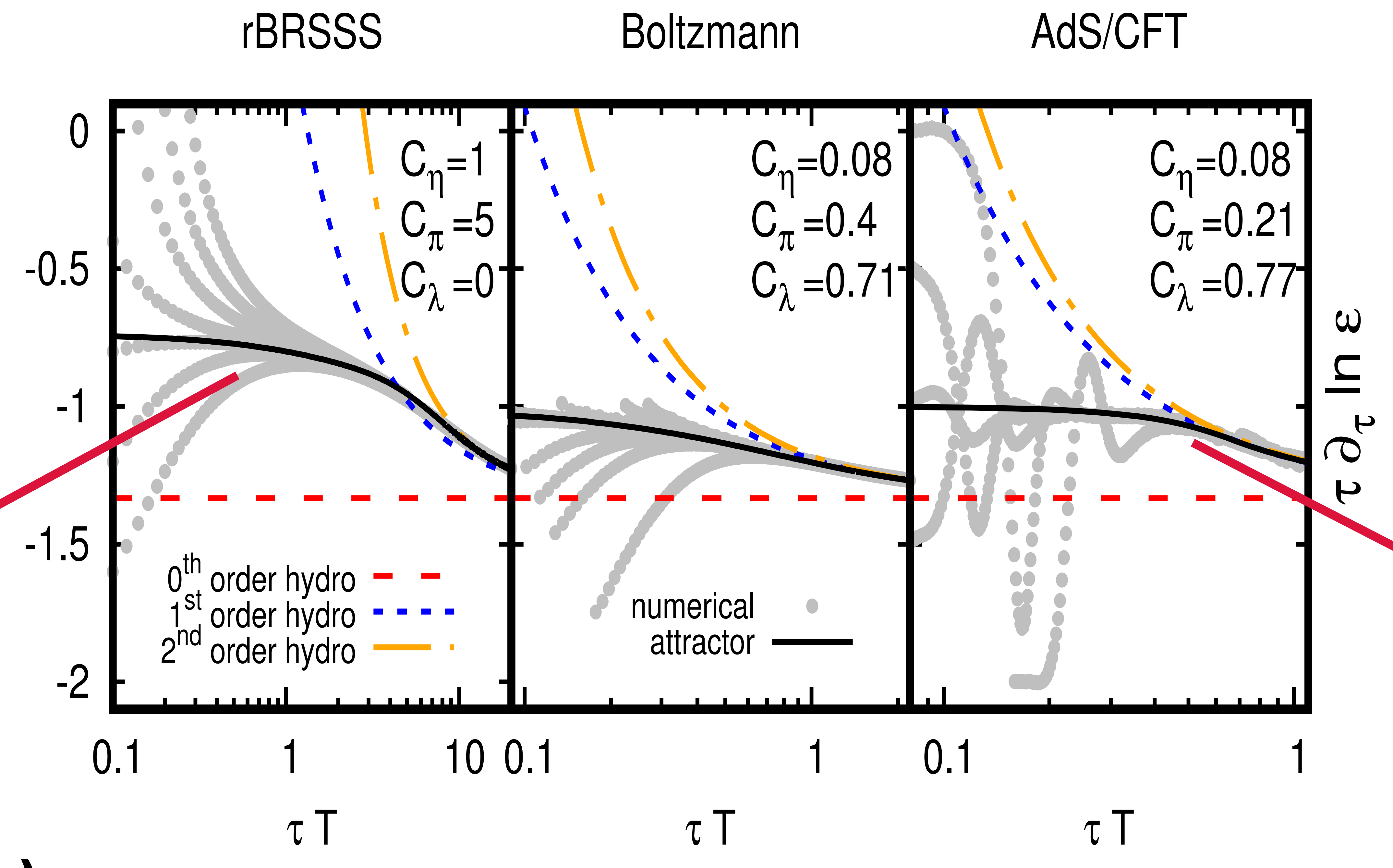



2. Inflation of the Universe at its early time implies a **slow-roll attractor**.



# Hydro attractor and transseries 流体吸引子和跨级数

- Hydrodynamic attractor in heavy-ion collisions




**Slow-roll attractor**: onset of hydrodynamization at early time

**geometry-driven (fast longitudinal expansion)**

**Dissipative attractor**: lost of initial information at later time

**equilibration-driven**

Florkowski et al, 1707.02282, Romatschke, 1712.05815

## Central lemma of non-equilibrium fluid dynamics:

**Attractor** offers a valid and quantitatively reliable description of the non-equilibrium systems (via resummation) as long as the contribution from all **non-hydrodynamic** modes can be neglected.

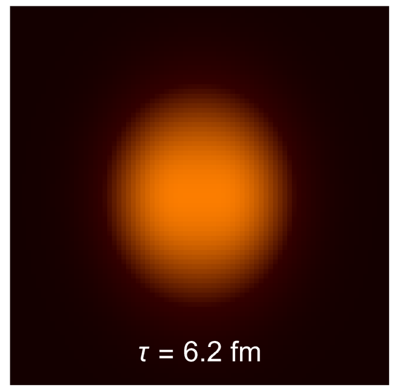
# Non-hydro perturbations 非流体微扰

- Consider non-hydro modes as perturbations

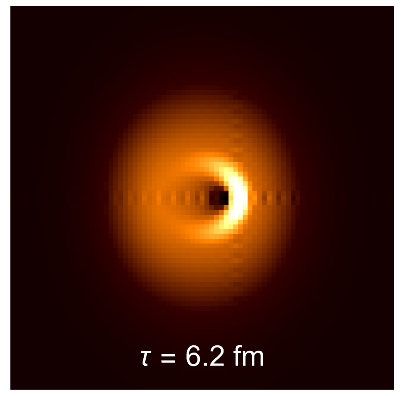
XA-Spalinski, 2312.17237 and in progress

$$\partial_\nu T^{\mu\nu} = \partial_\nu (T^{\mu\nu}_{\text{attractor}} + \delta T^{\mu\nu}) = 0 \quad \longrightarrow \quad \begin{cases} \partial_\nu T^{\mu\nu}_{\text{attractor}} = 0, \\ \partial_\nu \delta T^{\mu\nu} = J^\mu \,. \end{cases}$$

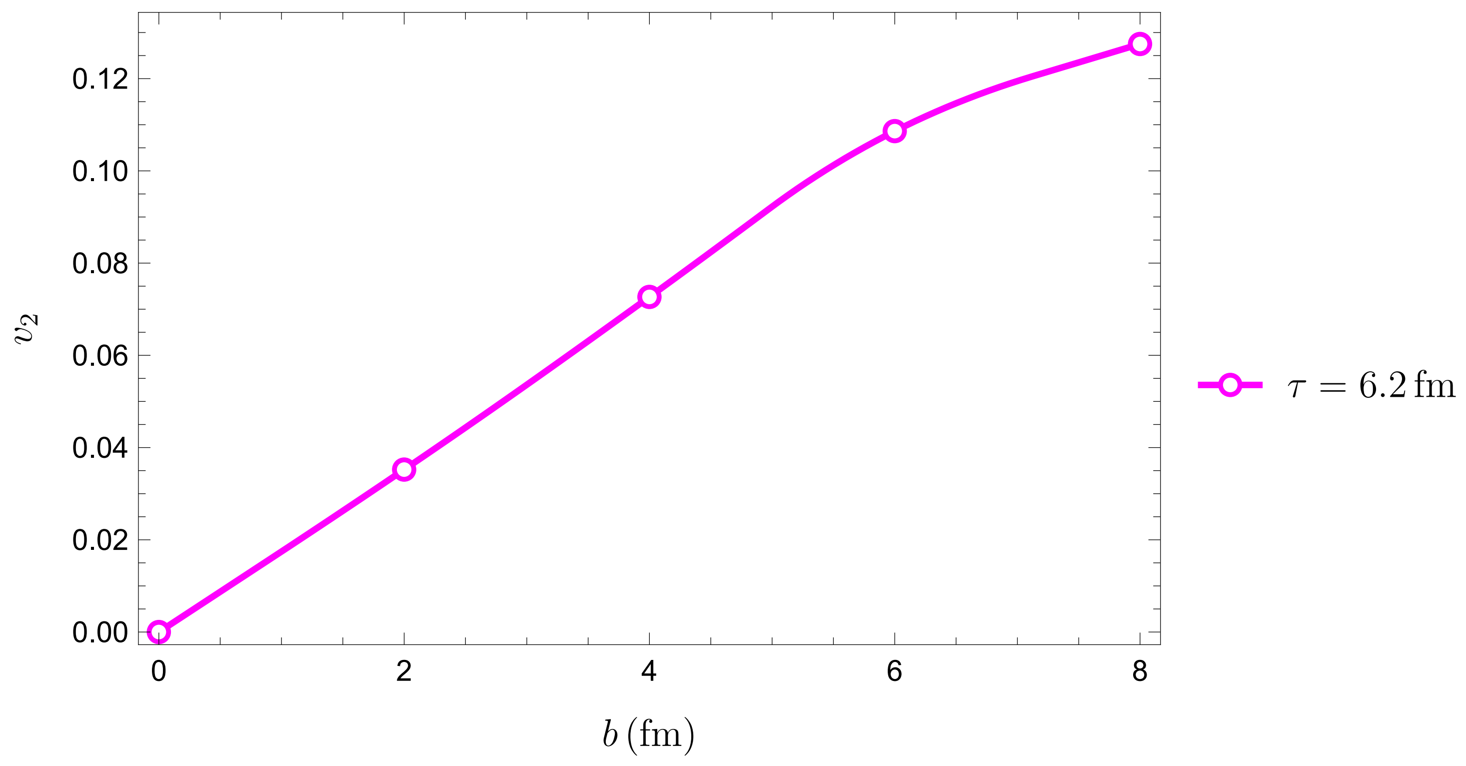
- From semi-analytic calculations we find



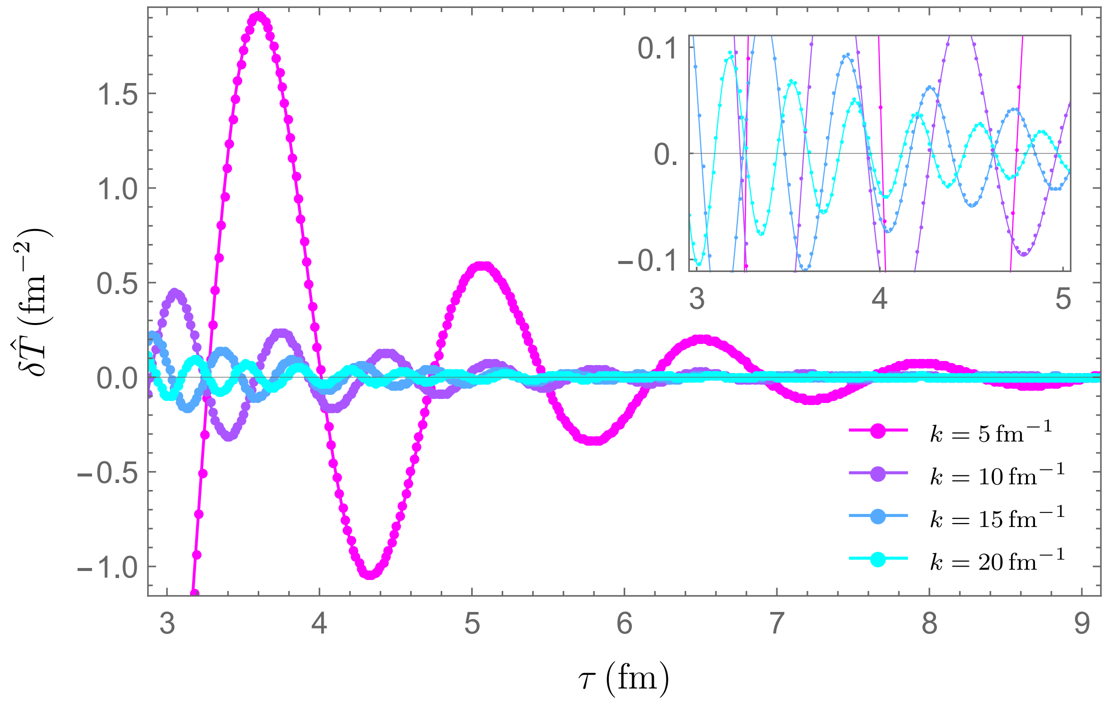

flow



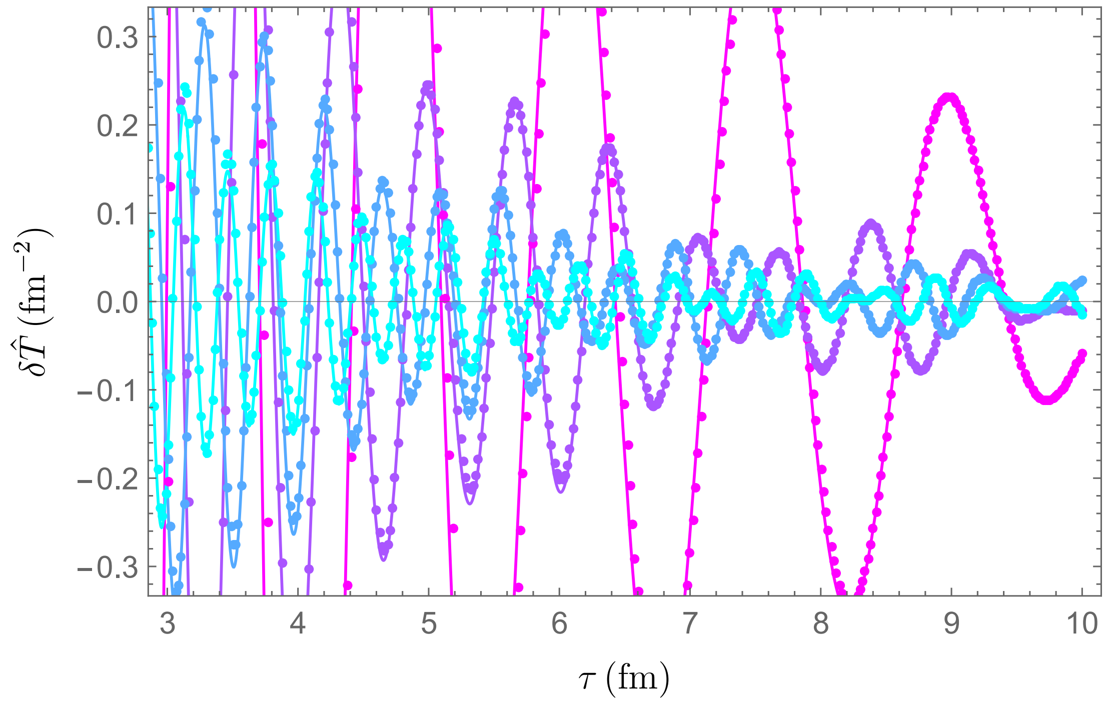

jet wake




v2 from perturbations




temperature w/o jet




temperature w/ jet

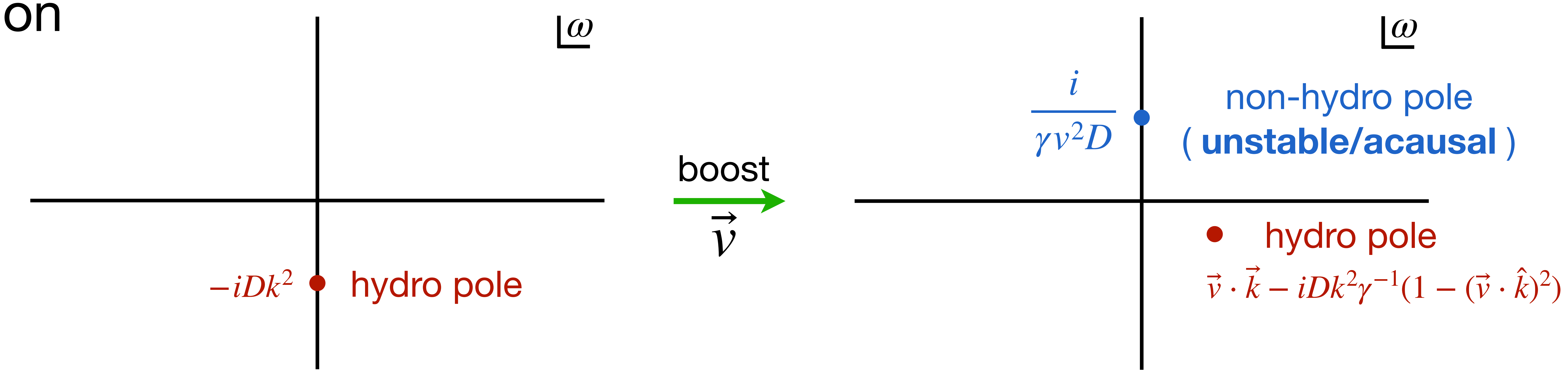
# Non-hydro 3: hydro with hidden symmetries
# 非流体力学 3：流体和隐藏对称性

# Israel-Stewart (IS) theory 伊斯雷尔-斯图尔特理论

- Conventional diffusion

$$\partial_t n + \vec{\partial} \cdot \vec{J} = 0$$

$$\vec{J} = -D\vec{\partial} n$$

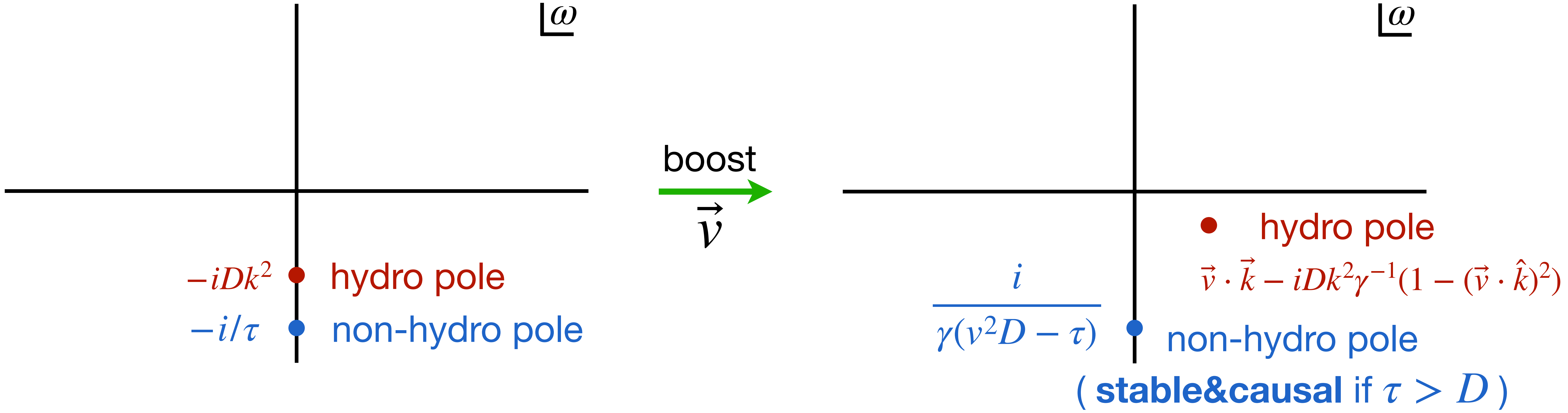



- Treating current $\vec{J}$ as dynamic DOF with relaxation time $\tau$

Maxwell 1867, Cattaneo 1948
Y. Ahn et al, 2506.00926

$$\partial_t n + \vec{\partial} \cdot \vec{J} = 0$$

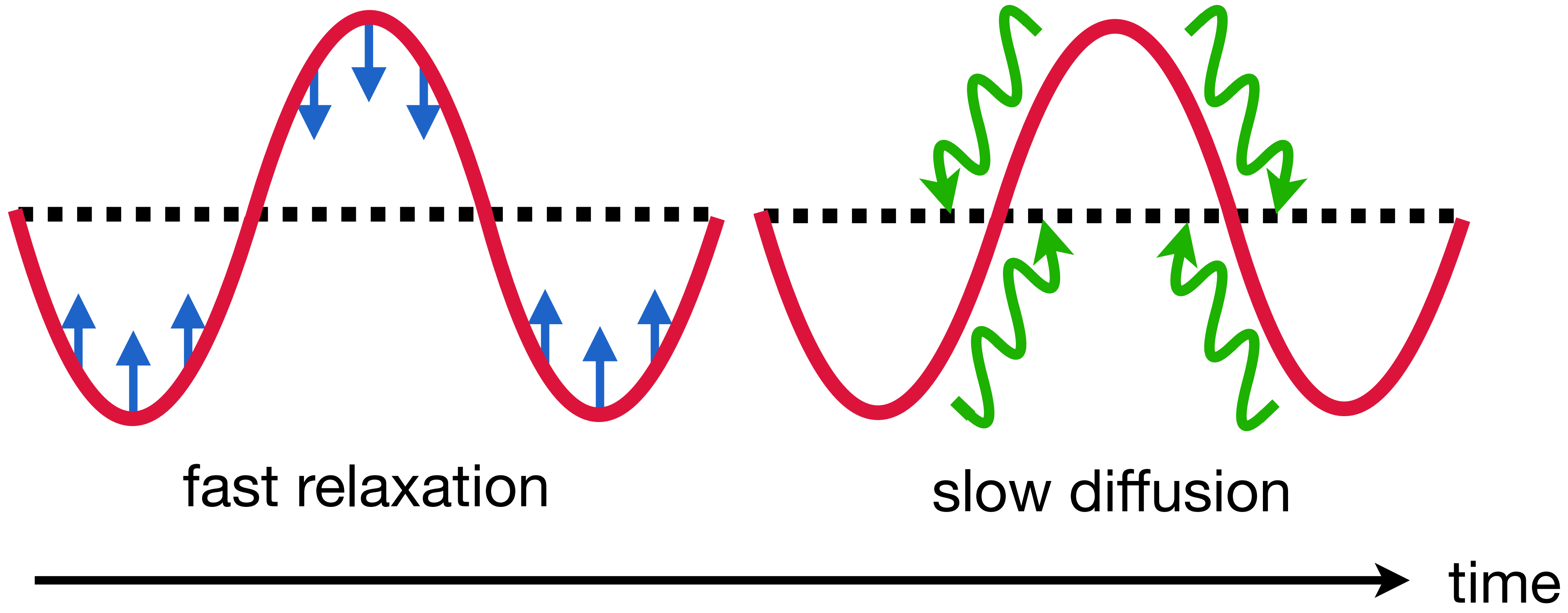
$$\tau \partial_t \vec{J} = -(\vec{J} + D\vec{\partial} n)$$

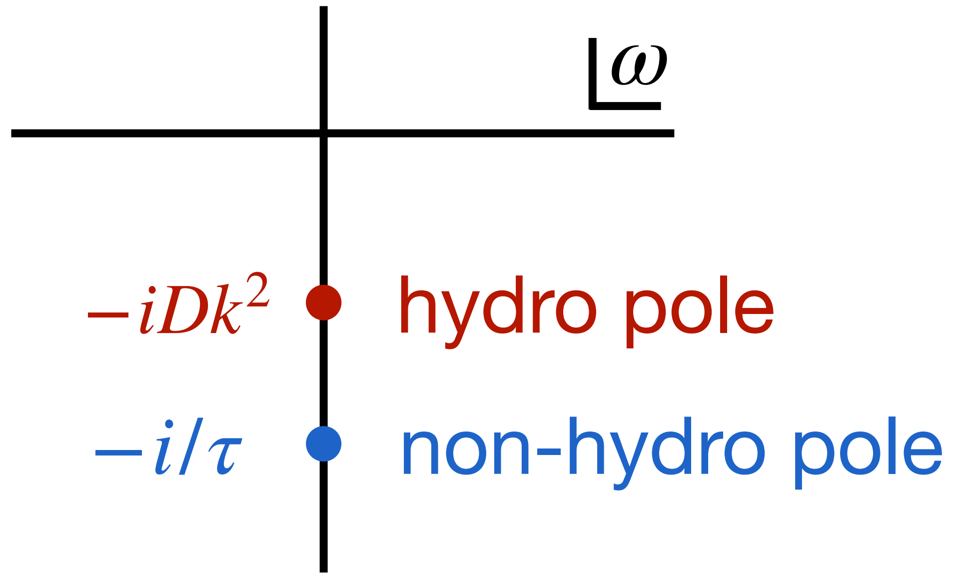



It is unclear how $\tau$'s role in UV regularization relates to any underlying symmetry.

# Diffusion near critical point 临界点附近的扩散

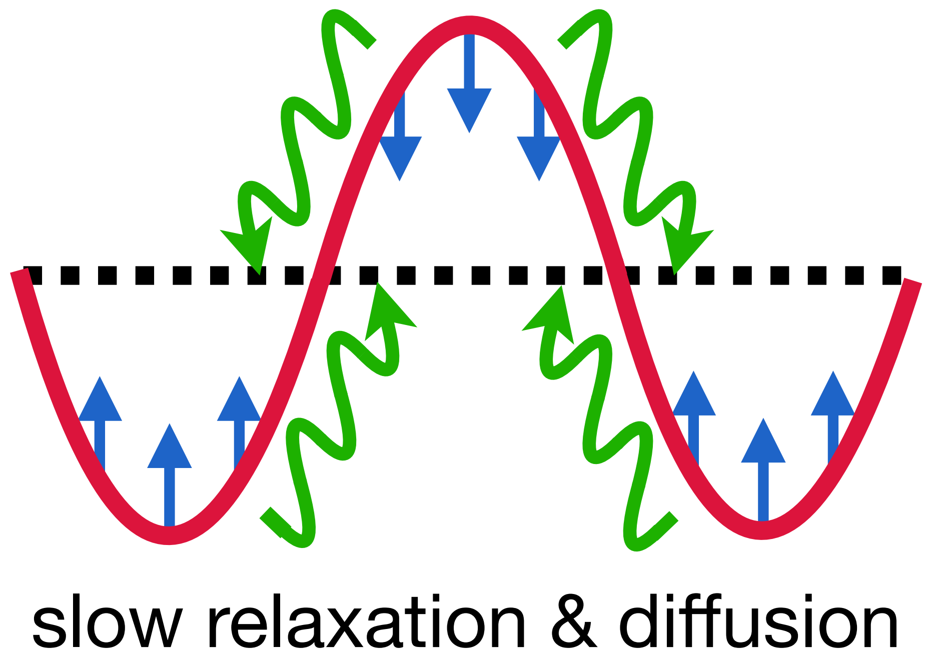
- Away from the critical point (gapped modes are transient)

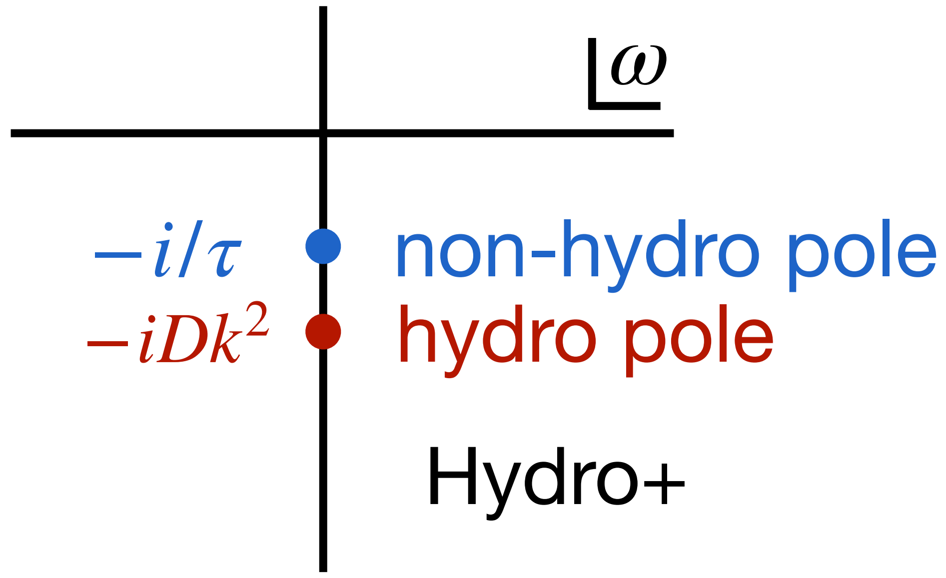


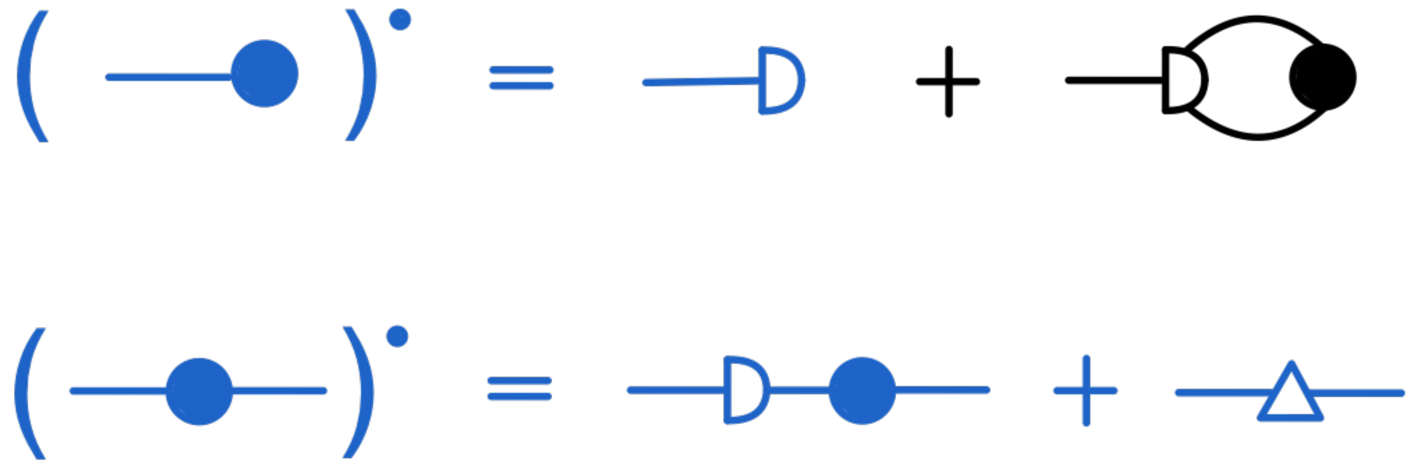

$$\partial_t n + \vec{\partial} \cdot \vec{J} = 0 \qquad \vec{J} = -D\vec{\partial} n$$

- Near the critical point (critical slowing down $\tau\partial_t \gg 1$)



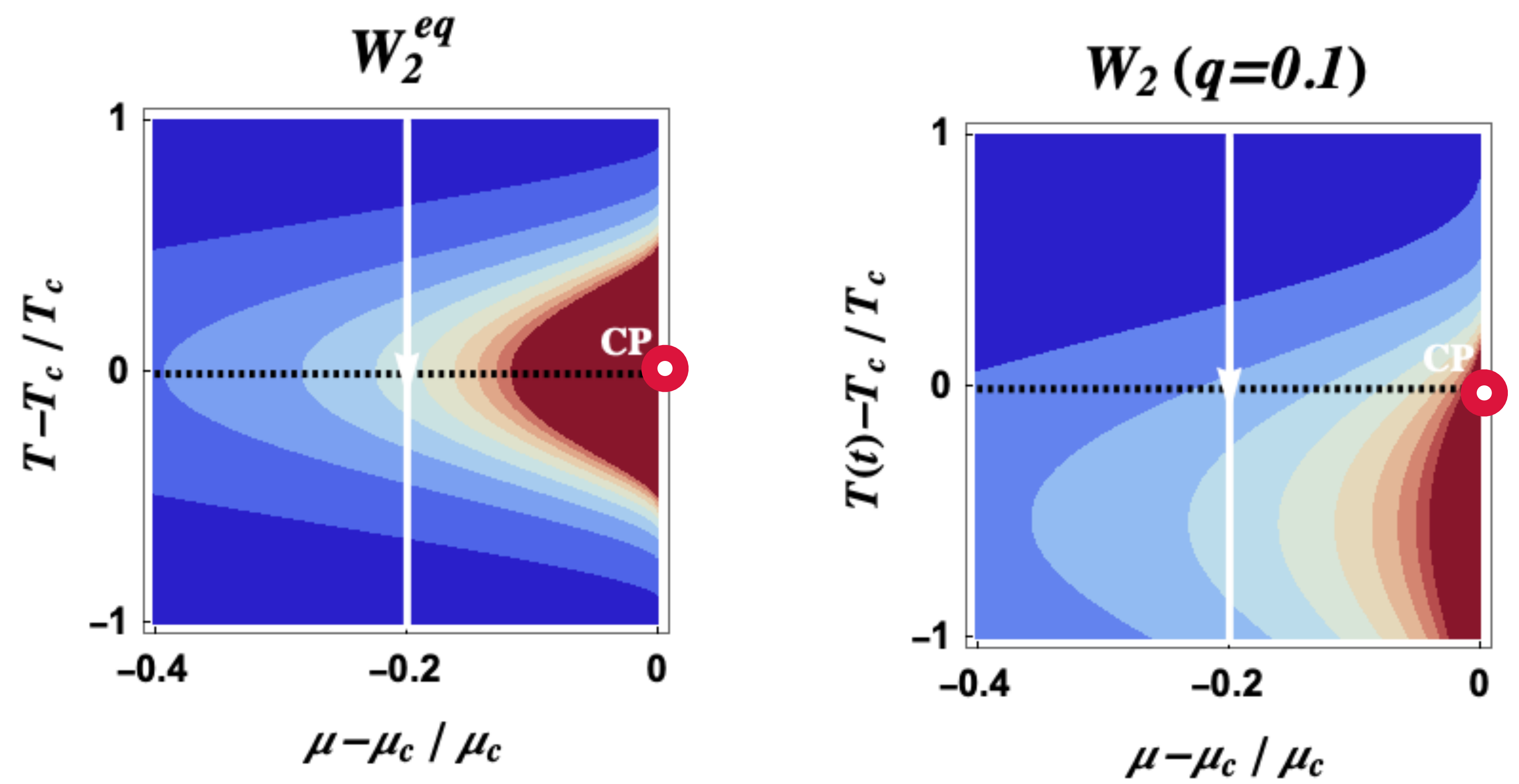

$$\partial_t n + \vec{\partial} \cdot \vec{J} = 0 \qquad \tau\partial_t \vec{J} = -(\vec{J} + D\vec{\partial}\, n)$$



# Finite-time effect near critical point 临界点附近的有限时间效应

- Evolution near QCD critical point

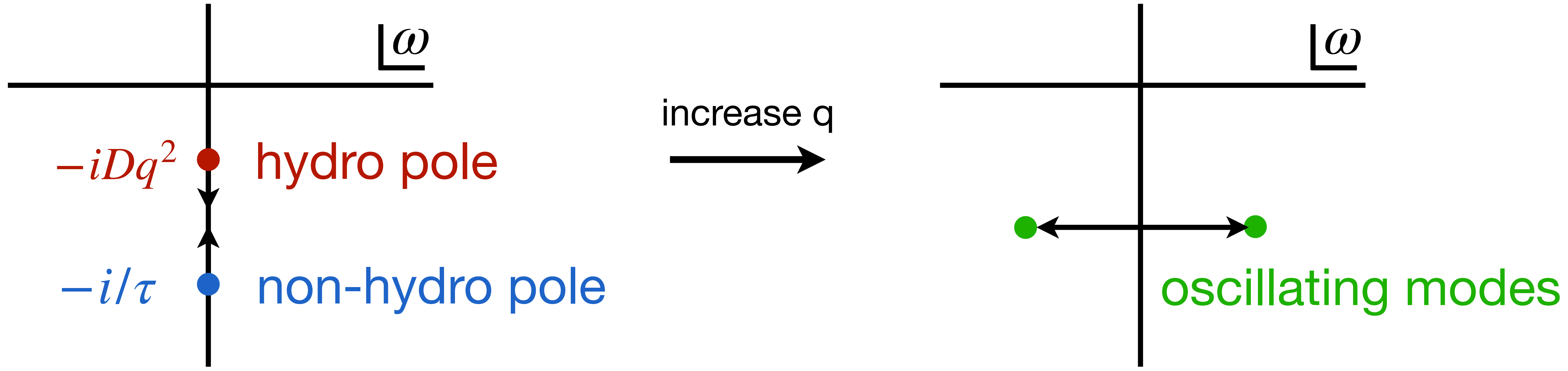
$$(1+\tau\partial_t)\partial_t W(q) = -Dq^2(W(q) - (1+\tau\partial_t)T\chi) \qquad W(q) \equiv \langle n(x+y/2)n(x-y/2)\rangle_{\text{W.T.}}$$

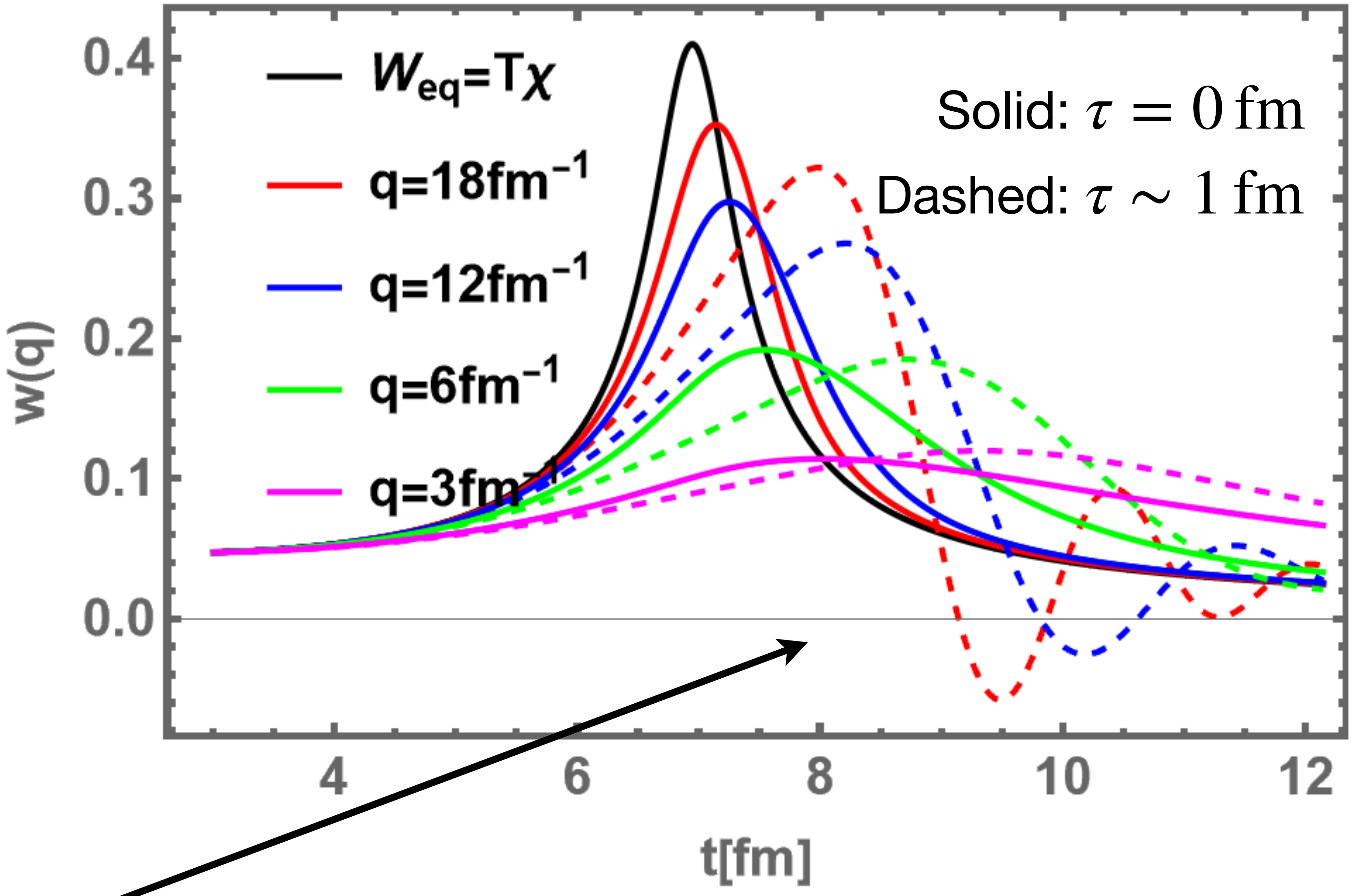



equilibrium

finite q ($\tau = 0$)

XA, 2209.15005



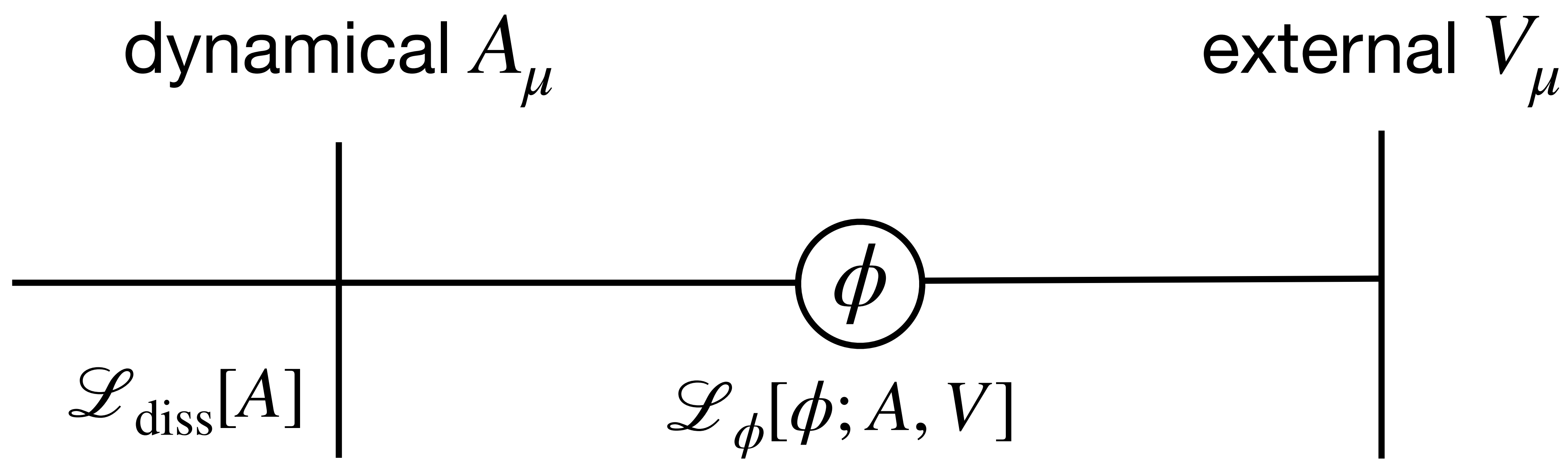

with N. Abbasi and S. Wu, in progress, see also 2312.17237

# Nickel-Son (NS) theory 尼克尔-谭青山理论

- Constructed to describe the emergence of hydro in holographic liquid  Nickel-Son, 1009.3094

charge $U_V(1) \times$ hidden $U_A(1) \quad \longrightarrow \quad U_{\text{diag}}(1)(\delta V = \delta A)$

environment
(black hole)




$$A_\mu \to A_\mu - \partial_\mu \lambda_A$$
$$V_\mu \to V_\mu - \partial_\mu \lambda_V$$
$$\phi \to \phi + \lambda_A - \lambda_V$$

- DOFs: Goldstone $\phi$ + gauge field $A_\mu$ (massive $A_\mu$)

$f \sim$ decaying constant
$g \sim$ Higgs mass
$\lambda \sim$ conductivity

$$\mathscr{L} = \mathscr{L}_\phi[\phi; A, V] + \mathscr{L}_{\text{diss}}[A]$$

$$\mathscr{L}_\phi = \frac{1}{2} f^2 \left(\partial_t \phi - V_t + A_t\right)^2 - \frac{1}{2} g^2 (\vec{\partial}\phi - \vec{V} + \vec{A})^2$$

Stueckelberg-Proca

$$\mathscr{L}_{\text{diss}}[A] \to \vec{J}_{\text{diss}} = \lambda(\partial_t \vec{A} - \vec{\partial} A_t)$$

Ohm's law

# Duality between NS and IS NS理论和IS理论之间的对偶

- The duality suggests non-hydro can be described by hidden but breaking symmetry.

| | |
|---|---|
| Goldstone $\phi$ | $J_0 \equiv n = f^2(\partial_t\phi - V_t + A_t)$ |
| hidden gauge field $A$ | $J_i = -g^2(\vec{\partial}\phi - \vec{V} + \vec{A})$ |

$$U_{\mathrm{diag}}(1) \ \rightarrow \ \partial_t J_0 + \partial_i J^i = 0 \qquad\qquad U_A(1) \ \rightarrow \ \tau\partial_t\vec{J} = -(\vec{J} + D\vec{\partial} J_0)$$

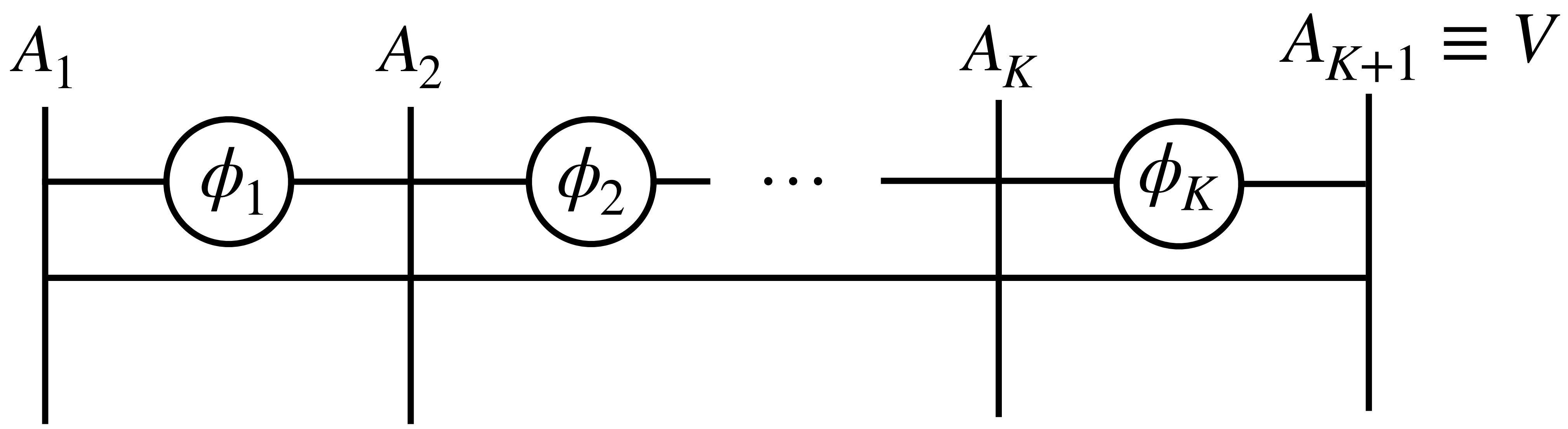
- The saddle point of NS action is equivalent to IS with the matching

$$f^2 = \chi = \frac{\lambda}{D}, \qquad g^2 = \frac{\lambda}{\tau}$$

Causality constraint $\quad u^2 = \dfrac{g^2}{f^2} = \dfrac{D}{\tau} \leq 1$

# Fields and action with HLS 满足隐藏对称性的场和作用量

- Introducing hidden local symmetries (HLS) $U^n_{\rm diag}(1)$ with extra layers labelled by $n = 1,2,\ldots,K$, assuming gauge field $A^n$ only couple with the *nearest neighbor* $A^{n\pm1}$ through Goldstone $\phi^n$.




$K$ hidden massive photons propagating in medium

$$\mathscr{L} = \sum_n \mathscr{L}^n_\phi[\phi^n; A^n, A^{n+1}] + \mathscr{L}^n_{\rm kin}[A^n] + \mathscr{L}^n_{\rm diss}[A^n], \qquad n = 1,\ldots,K$$

where $$\mathscr{L}^n_\phi = \frac{1}{2} f_n^2(\partial_t\phi^n - A_t^{n+1} + A_t^n)^2 - \frac{1}{2} g_n^2(\vec{\partial}\phi^n - \vec{A}^{n+1} + \vec{A}^n)^2$$

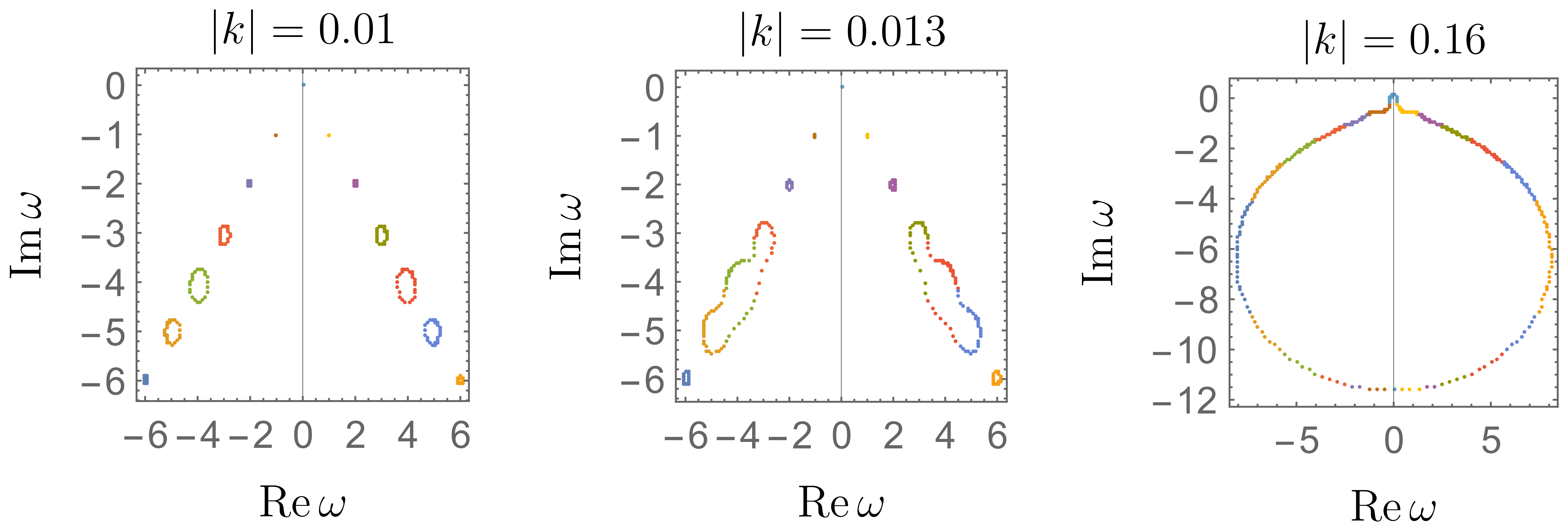
$\epsilon \sim$ permittivity
$\kappa^{-1} \sim$ permeability

$$\mathscr{L}_{n,\rm kin}[A^n] = \frac{1}{2}\left[\,\epsilon_n (F^n_{0i})^2 - \kappa_n\,(F^n_{ij})^2\right], \qquad \mathscr{L}^n_{\rm diss}[A^n] \to \vec{J}^n_{\rm diss} = -\lambda_n F^n_{0i}$$

Analogy: vector meson abundance due to breaking hidden local $SU(2)$ Son-Stephanov, 0304182

# Quasi-normal modes as massive photons 有质量光子的准简正模

- Bottom-up holography: QNM by tuning $f_n, g_n$ (at $k = 0$) :




Christmas tree QNM and level crossing

One Goldstone corresponds to hydro mode while the other hidden Goldstones coupled to hidden plasmon (massive hidden photon in medium).

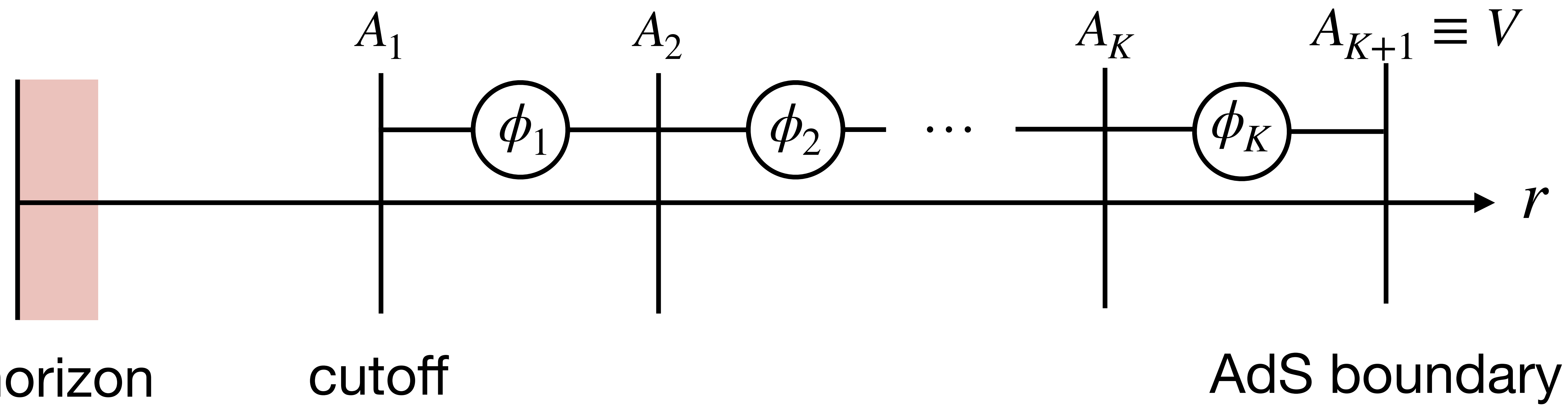
# Continuum limit 连续极限

- The 5th dimension can be **deconstructed** with a large number of gauge fields.

Arkani-Hamed et al, 0104005

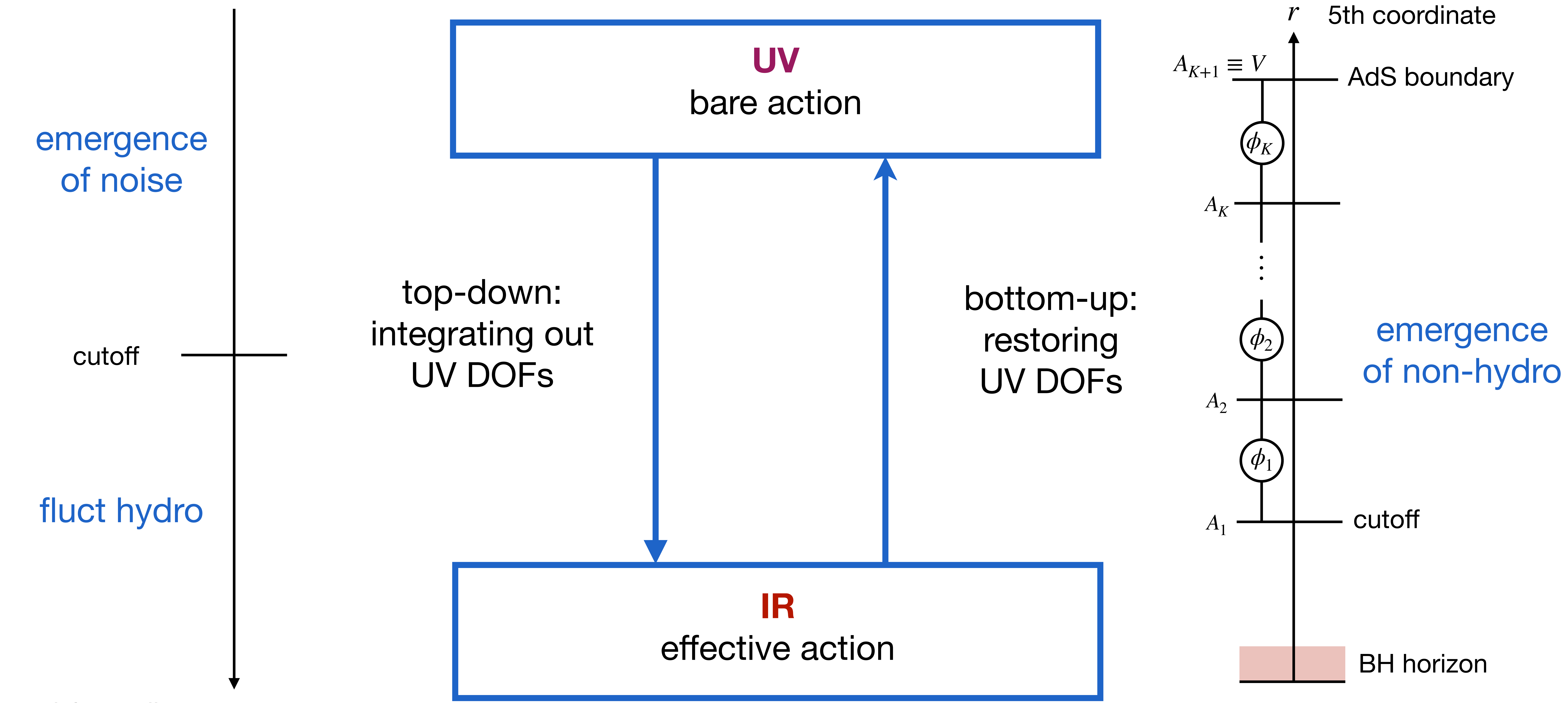
  - $n$ labels lattice site of 5th coordinate $na \to r$
  - Gauge field $A_\mu^n \to A_\mu(r)$, Goldstone field $\phi^n/a \to A_r(r)$
  - $f_n, g_n \to f(r), g(r)$ specify the metric

$$g_n^2(\vec{\partial}\phi_n - \vec{A}_{n+1} + \vec{A}_n)^2 \quad \longrightarrow \quad g^2(r)F_{ri}F_{ri} \sim h^{rr}h^{ii}F_{ri}F_{ri} \quad \longrightarrow \quad g^{MM'}g^{NN'}F_{MN}F_{M'N'}$$




$\partial_t A_\mu(r \to 0) \sim$ dissipative current (membrane paradigm) $\quad A_\mu(r \to \infty) = V_\mu \sim$ external field

# General framework of non-equilibrium EFT 非平衡有效场论的一般架构

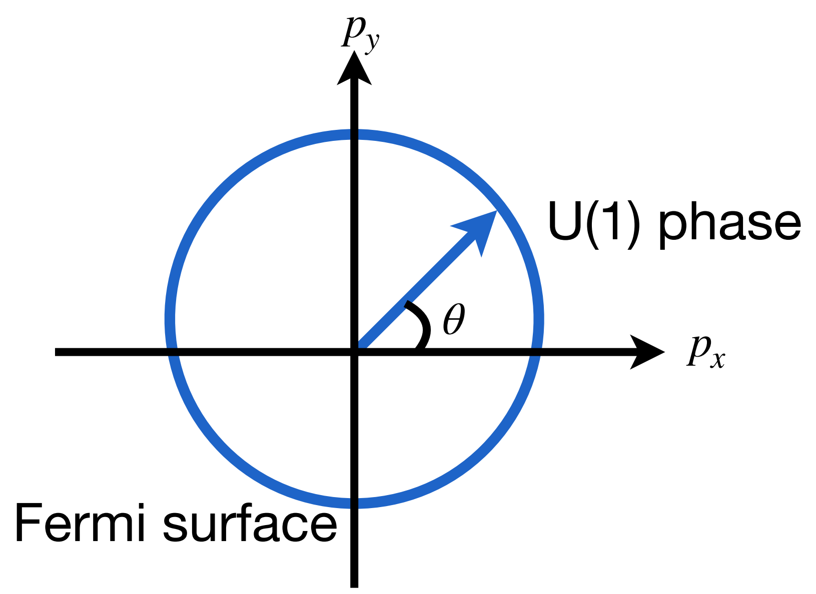



with R. Brants, M. Heller and Y. Yin, to appear

# Discussion and application 讨论与应用

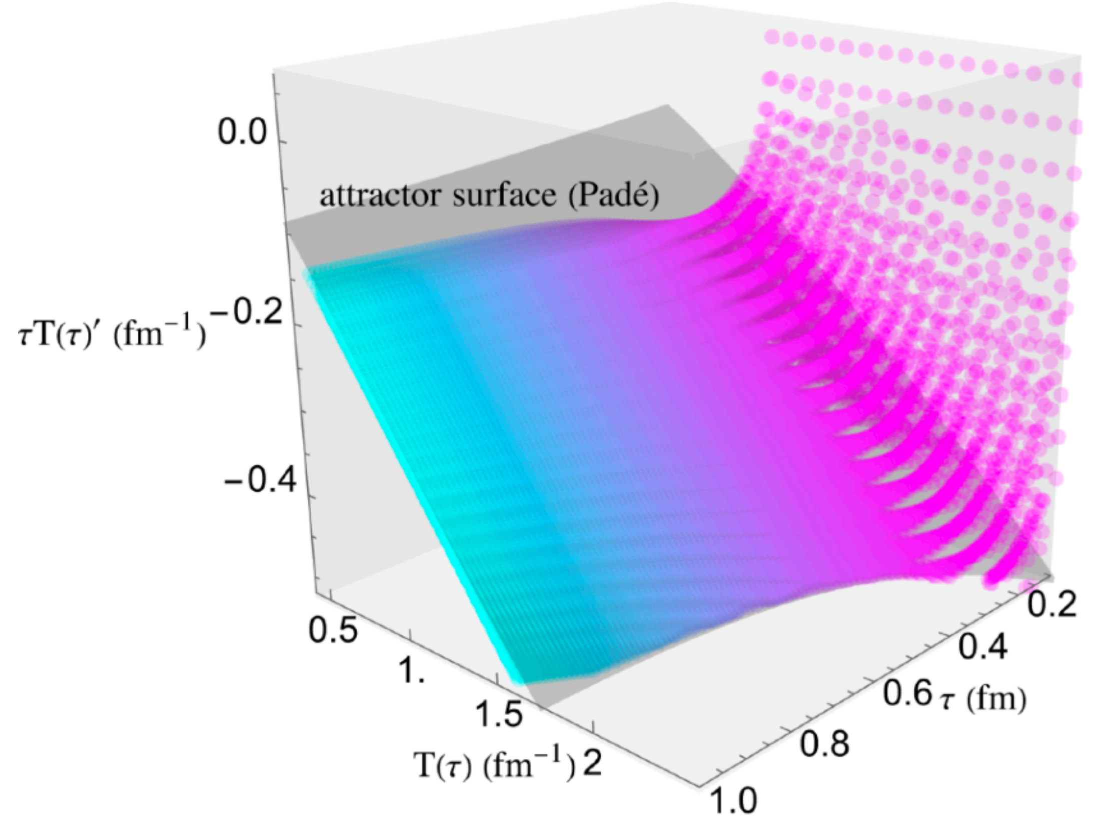
- In high-frequency regime, relevant fields are infinite Goldstones, HLS theory matches to collision-less kinetic theory.

Delacretaz et al, 2203.05004
York et al, 0811.0729



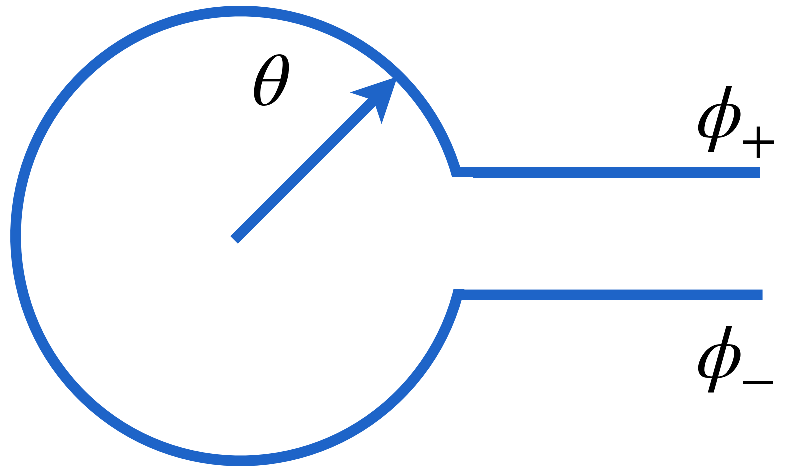

$$-\frac{1}{4}\int d^5x\sqrt{-g}\,g^{MM'}g^{NN'}F_{MN}F_{M'N'} \longrightarrow (\partial_t + \hat{n}\cdot\vec{\partial})\delta\hat{f} + \vec{F}\cdot\hat{n} = 0$$

- HLS modes dominate early-time (high frequency) attractor behavior. Attractor is related to symmetry of theory, not that of geometry.




attractor from IS equations
XA et al, 2312.17237

- Bottom-up SK: the idea of MSR formalism




$$Z = \int \mathcal{D}\psi_r \mathcal{D}\psi_a \exp[i\int_x -\varphi_a(\partial\cdot J) - V_a^\mu E_\mu + i\sigma V_{a\mu}V_a^\mu]$$

hydro EOM: $\partial\cdot J(\psi_r) = 0$ non-hydro EOM: $E_\mu(\psi_r) = 0$

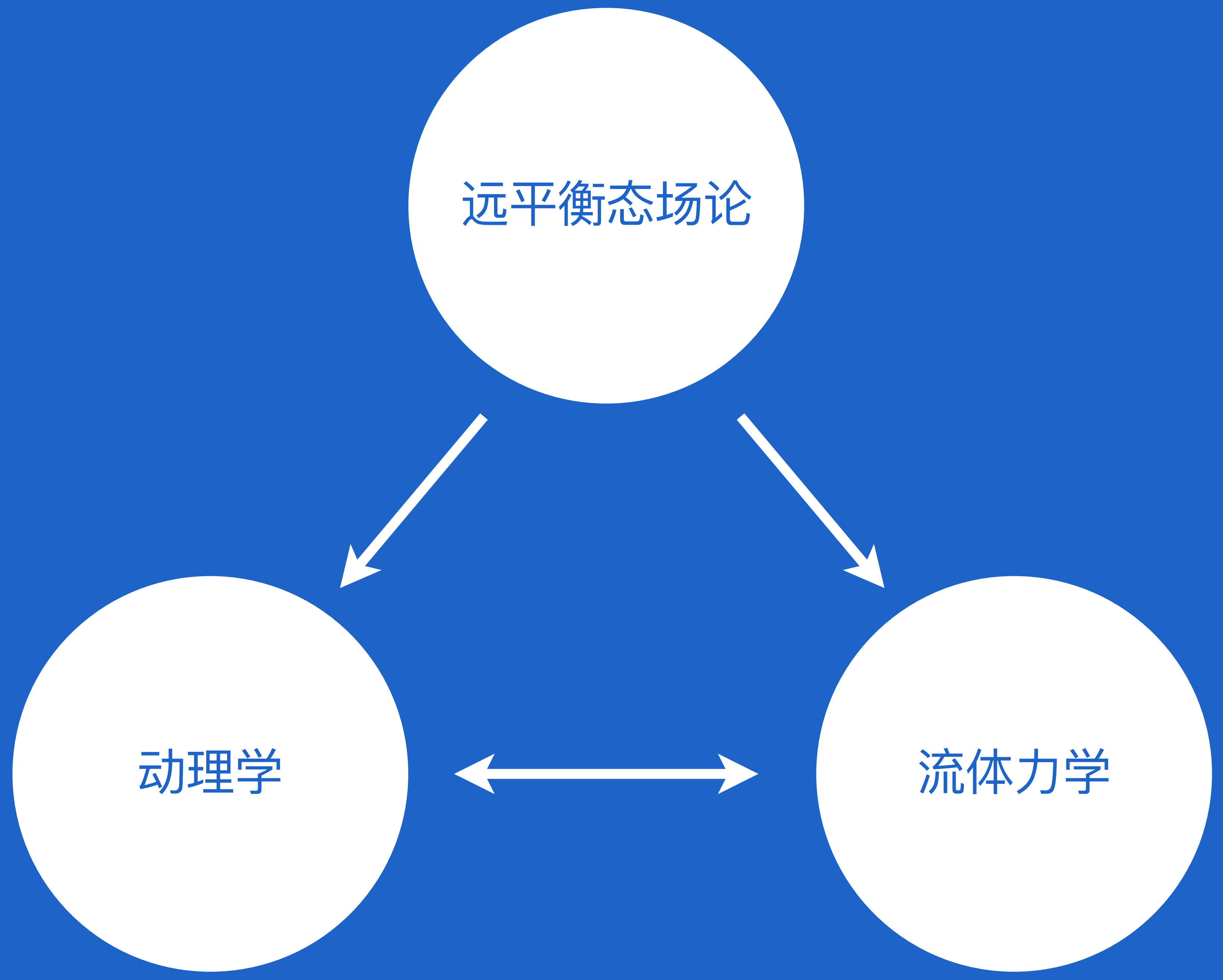
# Recap 总结

- Non-thermodynamics (finite-size effect) smears and change critical point signature.
- Non-hydro behavior emerges at different scenarios. Hidden symmetry can describe diverse non-hydro behavior in one and the same conceptual framework.
- Towards understanding of non-hydrodynamic behaviors at fundamental level.

| **system** | **non-hydro modes** | **emergence mechanism** |
|---|---|---|
| **with fluctuation** | correlation function | critical lowing down |
| **with attractor** | dissipative tensor | resummation |
| **with hidden symmetries** | Goldstone boson | Dim. deconstruction |

# Outlook 展望

- Non-thermodynamics and observables in experiments.
- A unified framework for everything and for far-from equilibrium regime.
- AI?




Thank You