Shattering Neutron Stars

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Natural systems of units

Particle, nuclear, and atomic physicists are used to *natural units* constructed by setting $c = \hbar = 1$. The first choice gives $[T] = [L]$ and Einstein's famous formula $E = m$. The second choice tells us that $[M] = 1/[L]$. The choice of units is usually eV, with

$$m_p = 0.938 \text{ GeV} \qquad \text{and} \qquad G = 6.708 \times 10^{-39} \text{ GeV}^{-2}$$

Using $k_B = 1$, one also has the conversion $1 \text{ GeV} = 11.6 \times 10^{12} K$.
For gravitation dominated physics *natural units* are constructed by setting $c = G = 1$. The first choice makes $[T] = [L]$ and the second tells us $[L] = [M]$. For NS, an appropriate mass unit is

$$2M_\odot = 2.95 \text{ Km} = 10^{57} m_p.$$

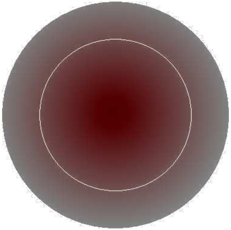
The compactness of a star $x = 2M/R = (M/M_\odot) \times (2.95 \text{ Km}/R) = v_{esc}^2$.
For a black hole $v_{esc} = 1$.

A typical Neutron Star

Typical neutron star mass is about $1.5 M_{\odot}$, and radius about 12 Kms. $v_{esc} \simeq 0.85$: must be studied in GR. M-R relation can be obtained numerically using the Tolman-Oppenheimer-Volkov (TOV) equation for radially symmetric objects.

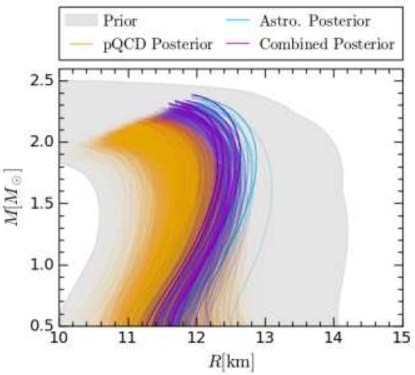
Neutron stars are seen as radio pulsars: they are rotating. Several neutron stars are seen to rotate with periods of order ms. TOV equations need to be corrected for them.

All observed neutron stars have strong magnetic fields: 10^{12}–10^{15} Gauss. They can affect many observables, but are not strong enough to change M-R relations.



Structure: core+crust. Crust: iron nuclei at the top (remnant of synthesis in supernovae), more compressed structures at the bottom. Core is nuclear matter and perhaps quark matter. 99% of mass in 30% of volume.

Nuclear Equations of State and M-R relations



Cuceu and Robles 2410.23407

Current state of the art: EoS at low densities follows χPT, at high density dense thermal weak-coupling QCD, constrained by NICER and LIGO/VIRGO data.

Binary neutron star merger: GW 170817

◐ Merger first observed by LIGO and VIRGO in gravity waves from time $T_0 - 100$s to T_0.
◐ Independently seen in Fermi and INTEGRAL γ-ray telescopes as a short GRB (sGRB) at $T_0 + 2$s.
◐ At time $T_0 + 11.5$ hrs a 1 m^2 telescope of the Swope Supernova Survey saw the transient and localized it to a galaxy NGC 4993.
◐ Up to $T_0 + 2$ d, UV, optical, near-IR telescopes observed a dimming of an initially bright blue source and brightening in the IR.
◐ At $T_0 + 9$ d the first X-ray signals were seen.
◐ At $T_0 + 16$ d the first radio signals were received.

Neutrino telescopes were off-line during the merger.

The birth of multimessenger astronomy: clarifying the relationship between sGRBs, kilonovae, and BNS mergers. Kilonovae are expected to be the sites of nucleosynthesis of heavy elements.

Two-component structure of GW 170817

◑ Two jets from the merger with axis 20 degrees from line of sight: synchrotron radiation first seen in γ-rays, then in X-rays, finally in radio. Pulsar signal continues till now.

◑ 2–5% of $M_{\odot}$ ejected into an isotropic cloud where nucleosynthesis occurs: seen in IR and optical. Cooling curves indicate that there may be more than one episode of energy release into the cloud. Enhanced ratio $[Eu]/[Fe]$.

Neutron stars are special

The stability of neutron stars involve all four known forces in the universe—

- Gravity: overall binding of a nucleus with $A \simeq 10^{57}$. Without gravity the heaviest (metastable) nuclei have $A < 400$.
- Strong interactions: nuclear liquid properties due to specifics of inter-nucleon forces.
- EM forces: local charge neutrality is required to prevent instability
- Weak interactions: stability against β decays requires departure from isoscalarity

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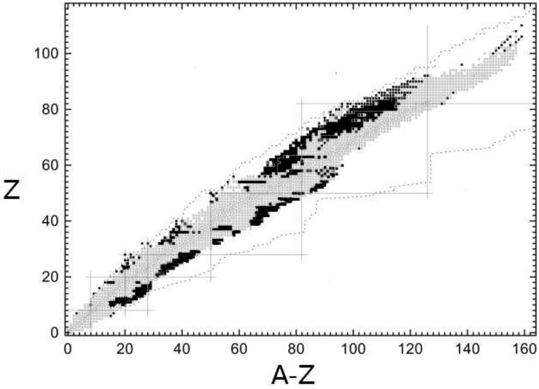
Neutron star collisions involve very large energy in total but very small energy per nucleon. In the bulk they involve the nuclear liquid-gas transition: till now only studied in microscopic nuclear systems, not macroscopic matter.

(Meta) Stable nuclides: drip line physics



doi:10.1103/RevModPhys.75.1021

(Meta) stable means lifetimes greater than size divided by bound nucleon speed. Limits of stability due to nucleon drip: finite range potentials support only a finite number of bound states.

Large Z instability of the Bohr atom

When $\hbar = 1$ then dimensionally $[e^2] = L/T$. So in a Bohr atom the electron's velocity $v \sim e^2$. When the nuclear charge is Z one has

$$v \sim Ze^2.$$

For increasing $Z > 100$ power corrections in v need to be taken into account (NRQED). With the Bohr Hamiltonian (LO-NRQED), $H = mv^2/2 - Ze^2/r$, a natural length scale in the problem is the Bohr radius

$$a_0 \sim \frac{1}{Zme^2} = \frac{1}{mv}.$$

When $v = O(1)$ then this scale approaches the Compton wavelength of the electron. The typical energy scale is then the Rydberg

$$\mathrm{Ry} \sim mv^2 \sim mZ^2e^4.$$

For $v = O(1)$, Ry approaches threshold for e^+e^- pair production and full Dirac equation is needed: shows vacuum breakdown for $Z \simeq 170$.

Stability of atoms with $Z \gg 100$

Instability against pair production is due to the fact that the nuclear radius $R_A = r_0 \sqrt[3]{A}$, with $r_0 \simeq 1.1$ fm is much smaller than $a_0(Z) \simeq 52.9 \times 10^6/Z$ fm and for $Z > 100$ there are extremely strong fields between them. Consider $Z \simeq 10^4$ and $A \simeq 10^{10}$. Then $R_A \simeq a_0(Z) \simeq 10^3$ fm and this instability may be avoided.

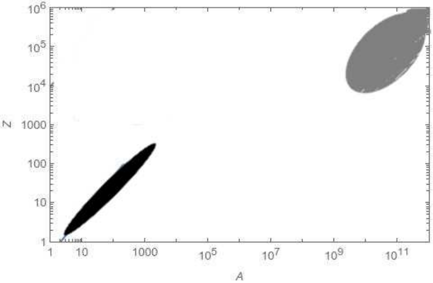
So, for nuclei with $Z > 10^4$ one may take as a better starting approximation local charge neutrality, and stability against β decay as well as electron capture by protons. Two conditions for the Fermi energies arise

$$m_p E_p^F = m_e E_e^F, \quad \text{and} \quad E_n^F = E_p^F + E_e^F.$$

These give $E_e^F = \mu E_p^F$ and $E_n^F = (1+\mu)E_p^F$ where $\mu = m_p/m_e$. Then

$$A = \left(1 + \mu^{3/2}\right) Z \simeq 8.9 \times 10^5 Z.$$

Stable monsters?



- Does this give a new island of stable nuclei with $A \gg 100$? (These monsters would have size greater than pm, perhaps much greater.)
- Are they stable? (Examine possible decay modes)
- How would you construct them? (Gap in the mass spectrum means that the bottom up synthesis route is blocked)

Decay modes

◐ Stable against β decay and inverse-β decays by construction

◐ Fission of heavy nuclei proceeds by instabilities in which surface deformations are magnified by Coulomb repulsion: gain in surface energy offset by decrease of Coulomb energy. Local charge neutrality removes this surface instability.

◑ Low energy neutrons may evaporate from the surface, since potential vanishes outside the surface. This is likely to be the main decay mode.

◐ Evaporation of protons possible, but suppressed due to two factors: $Z \ll A$ and attractive interactions between proton and remainder. Negligible rates for evaporation of light nuclei, such as α-decay.

Neutron evaporation is surface limited

Surface area of monster $S \propto A^{2/3}$. The rate of neutron evaporation is the rate of change of the mass number, A, of the monster. So one has the decay equation

$$\frac{dA}{dt} = -\frac{1}{\tau} A^{2/3}.$$

where the constant τ is the mean time of an attempt to escape. This has the solution

$$A(t) = \left(\frac{(t/\tau) - \sqrt[3]{A_0}}{3} \right)^3,$$

where A_0 is the initial mass number of the monster. The lifetime is bounded by

$$t = \tau \sqrt[3]{A_0}.$$

This is an upper bound, since other channels open up when enough neutrons have evaporated.

Bound on lifetime due to neutron evaporation

Neutrons successfully escape from a monster if their thermal energy exceeds the binding energy per nucleon BE. The probability of finding such neutrons goes as $\exp(-BE/T)$, and hence the lifetime as its inverse. This is an enormous increase in lifetime when $T \ll BE$.

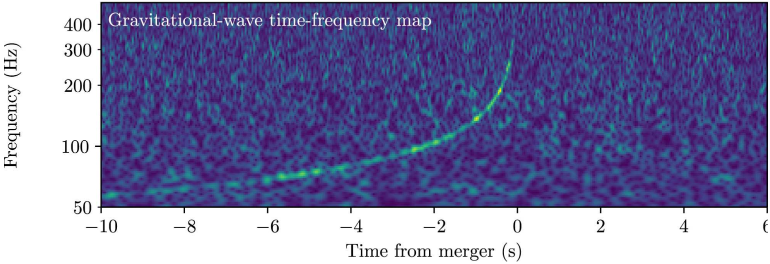
Moreover the nucleon has to be within distance r_0 of the surface. Nucleons further in have to diffuse out to the surface, since the nuclear fluid has a hard core repulsion and its density is close to maximum packing. So the mean time for a neutron to attempt to escape is $2r_0/v$ with velocity $v^2 \geq 2BE/m$.

Averaging over the thermal distribution with $BE = 15{-}17$ MeV, one finds

$$t = R(A_0) \times \begin{cases} 10^{13-15} & (T = 0.5\ \mathrm{MeV}) \\ (1{-}1.4) \times 10^2 & (T = 5\ \mathrm{MeV}) \\ 4.82{-}4.83 & (T = 50\ \mathrm{MeV}) \end{cases}$$

Mind the gap

The only route to monsters is top down: shatter a neutron star into pieces



Before two compact stars collide, the less massive secondary begins to lose mass as material streams from its Roche lobe to the primary, at a loss rate of about 5% of $M_\odot$ per second.

This strips off the crust and gives a first burst of neutrons, but most are captured by the primary.

Shattering of Neutron Stars

A second phase comes when the two NSs make contact. Two liquid drops colliding with each other can either merge, bounce, or shatter into multiple droplets: kinetic energy partly transformed into surface energy.

In lab experiments, two important dimensionless numbers control the dynamics of colliding droplets: Weber number and Reynolds number:

$$\begin{aligned} We &= \frac{\text{Drag force}}{\text{Cohesion force}} = \frac{\rho}{\sigma} v^2 R \longrightarrow \frac{m}{\mathrm{BE}} v^2 \frac{R}{r_0} \simeq 10^{18} \\ Re &= \frac{\text{Viscous force}}{\text{Inertial force}} = \frac{\rho v R}{\eta} \longrightarrow \frac{mv}{r_0^2 \eta} \frac{R}{r_0} \simeq 10^{18} \end{aligned}$$

For colliding water droplets, shattering sets in when the dimensionless Weber number is larger than about 3–10.

Large droplets and causality: some details may change in relativistic hydro.

Gravity causes re-assembly of shattered droplets

The major extra ingredient in NS collisions is the need to include gravity. Due to mutual gravitational attraction, the majority of droplets can reassemble into a NS or BH. (More shattering events may intervene before merger)

At least two widely separated time scales relevant for nucleosynthesis: that of nuclear reactions, and the other of re-assembly time scales.

Also an emergent time scale of droplet decay. A monster of size 1 Km can evaporate in a typical time of μs to much larger, depending on T. A fog of droplets may lead to evaporated neutrons from one droplet being captured by other droplets: dynamic balance between gas of neutrons and monster droplets (cf: equilibrium gas pressure over liquid surface).

As a result, sustained flux of neutrons is possible. Kilonovae can be sites for s-process as well as r-process nucleosynthesis.

Multimessenger signals

Monsters change the early stages of NS mergers. What observables?

1. Shattering creates droplets which are of size smaller than the 10 Kms scale of NSs. Merger of droplets could be prolonged. Are there possible gravity wave signals? Challenge: low strength, high frequency.
2. Extended flux of neutrons could generate extended flux of neutrinos: unfortunately low energies. But studies in progress.
3. A fog of monster droplets could change the early energy distribution in the kilonova, and likely to make it more homogenous and isotropic. Could early optical and IR observations see something interesting? Detailed modelling is needed.
4. Sustained flux of neutrons is likely to change the nucleosynthesis chain and give a different balance between s-process and r-process. Can spectroscopy reveal differences? Challenge: model the chain of reactions.

The phase diagram of physical QCD

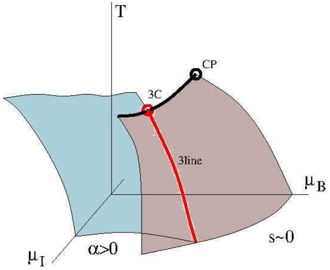
$N_f = 1 + 1$, so flavour symmetry is U(1), because $m \neq 0$ and $\Delta m \neq 0$. Two order parameters:

$$s = \langle \overline{\psi} \psi \rangle, \qquad p = \langle \overline{\psi} \mathrm{e}^{i\gamma_5 \tau_1 \alpha} \psi \rangle$$

Three phases:

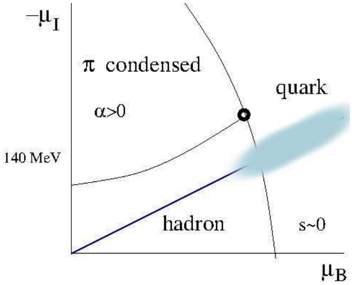
- hadronic phase has $s < 0$ and $\alpha = 0$,
- pion condensed phase has $s < 0$ and $\alpha \simeq m_\pi^2 / \mu_I^2$,
- quark phase has $s \simeq m$.

Sharma and SG, in progress



$m(\pi^+) = m(\pi^-) \neq m(\pi^0)$ because symmetry allows isoscalar mixing with π^0. Define $m = (m_u + m_d)/2$ and $\Delta m = m_d - m_u$.

The real-world phase diagram at small T



$N_f = 1 + 1$: $m > 0$ and $\Delta m > 0$ (fixed T)

NSs charge neutral and stable against β-decay: $\mu_B \simeq \mu_n$ and $\mu_I \simeq -\mu_n$.
Somewhat above saturation density, core may be in the chiral symmetric phase!
If NS cores contain chiral symmetric matter, then there must be a first order phase transition between quark and hadronic phases.

Clausius-Clapeyron in Neutron Stars

Slopes of first order surfaces related to latent heat and jumps in order parameters

$$\left.\frac{\partial T}{\partial \mu_B}\right|_{\chi SB} = \frac{B_\chi - B_{\not\chi}}{S_\chi - S_{\not\chi}} < 0$$

where S is entropy density and B is baryon density. Since latent heat is positive, $B_\chi < B_{\not\chi}$. A mass of baryons increases in volume when heated across the first order transition.

When two NSs collide (eg, GW 170817) the region of collision is immediately heated to more than 10 MeV: gravitational and KE converted to heat. Big jump in temperature so the upper core makes a transition to the chiral symmetric phase. Instant jump in volume: explosion.

Result: a shock wave travels from the region of impact through the NS core, triggering instantaneous heating and expansion!

The central engine of jets

Criteria for viable central engines of GRBs examined before.
Kumar and Zhang (2015)

Central engines of GRBS

- must be able to launch jets with enormous luminosities. In GW 170817 the jets carry 10^{52}–10^{53} GeV of energy, *i.e.*, less than 0.01$ of $M_{\odot}$. Possible to obtain this at a first order transition.
- must not contain massive baryons in order to accelerate to ultrarelativistic speeds. The phase transition eats up massive baryons. Low mass quarks accelerate along magnetic fields to produce jets.
- may need to be intermittent in order to recreate multiplicity of time scales. Propagation of a shock front (forest fire) and the diffusion of energy released by the latent heat behind the front, give multiple scales.

The outlook is bright

- NS radius and mass, and the compressibility strongly constrain the equation of state through the TOV equations. Intense ongoing work. New ideas: theory constraint from χPT at low energy, dense thermal QCD at high energy. Narrow band of remaining possibilities.
- New aspects of the phase diagram of QCD can be explored in BNS collisions. This talk pointed out that
  - the nuclear liquid-gas transition gives rise to multimessenger signals which can be further explored at both early and late times.
  - quark-hadron phase transition can give rise to explosive energy generation in the very early stages of the BNS collisions.
- More ideas will be needed in the coming years as more direct observations are made.