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Fully charm tetraquark production and decay at LHC

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2025.9.17 @ 中科院高能所

Heavy Flavor Physics

- ## New physics

  indirect detection

$$\mathcal{H}_{NP}^{\Delta C=2} = \frac{1}{\Lambda_{NP}^2}\sum_{i=1}^{8} z_i(\mu) Q'_i$$

$$\Lambda_b^0 \to pK^-\pi^+\pi^-$$

[LHCb, Nature 2025]

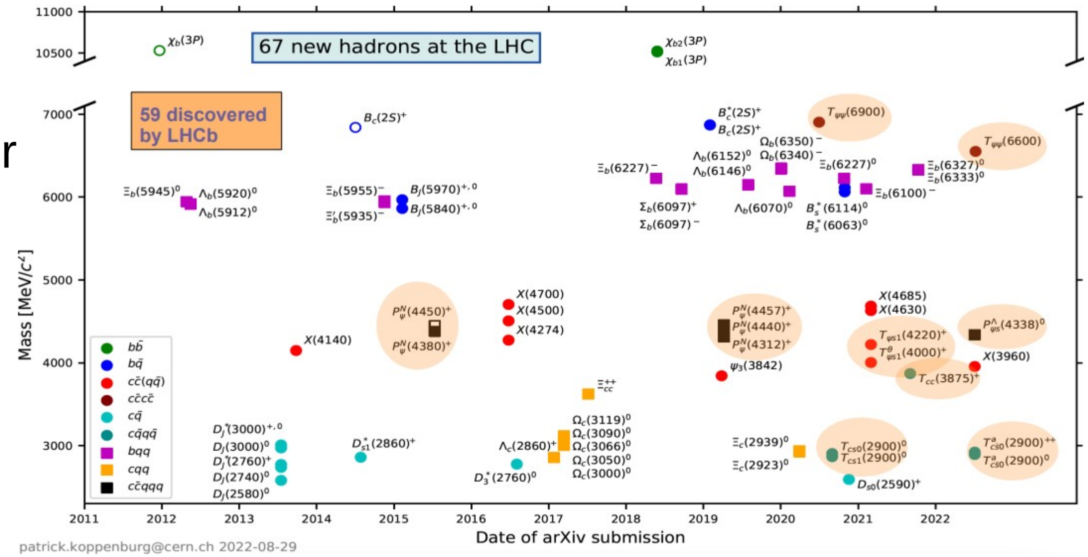
$$\mathcal{A}_{CP} = (2.45 \pm 0.46 \pm 0.10)\% \quad 5.2\sigma$$

- ## CP violation

  Understand its origin:
  Strong phase, new sources

- ## Exotic hadrons

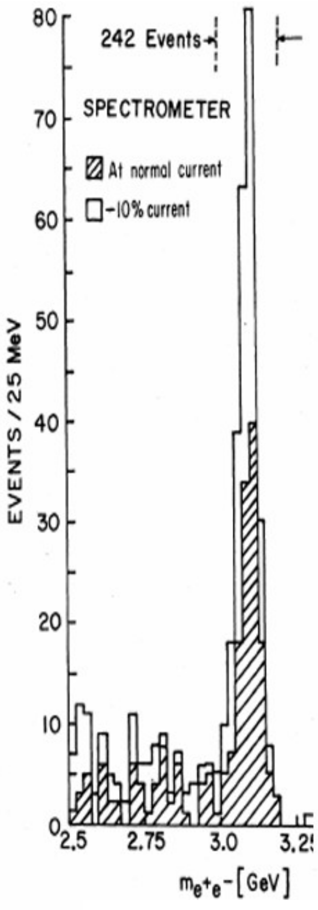
  Inner structure; Exploring Color confinement mechanism

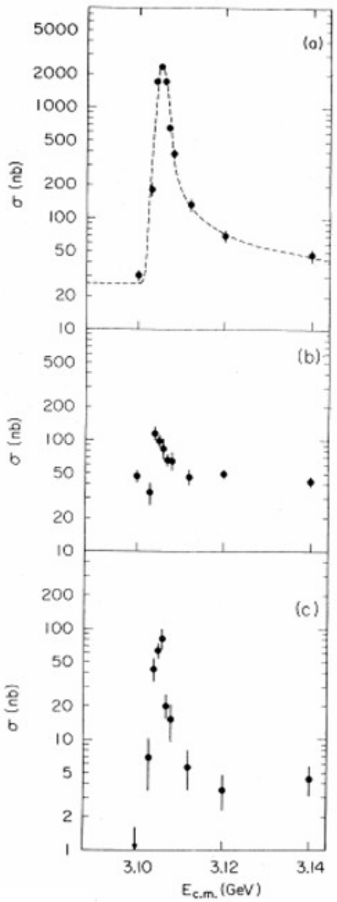


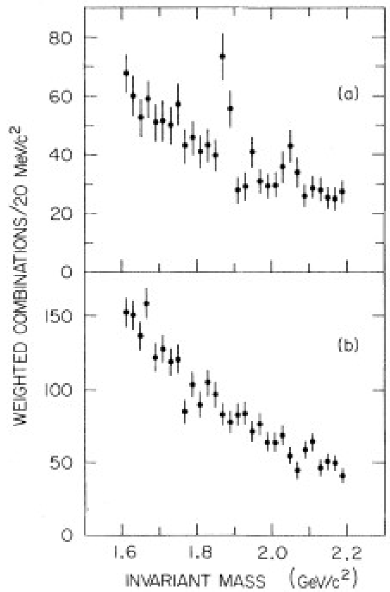
- ## Heavy flavor theories

  Heavy quark symmetry;
  nonrelativistic at Fermi-scale;
  test effective theory of QCD;

Charm family

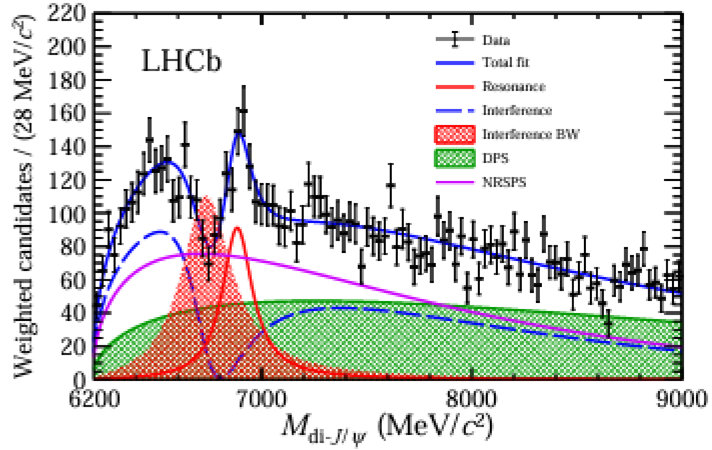






Ω_{ccc} (CCC)???

T_{3c} $(CC\bar{C}\bar{q})$???

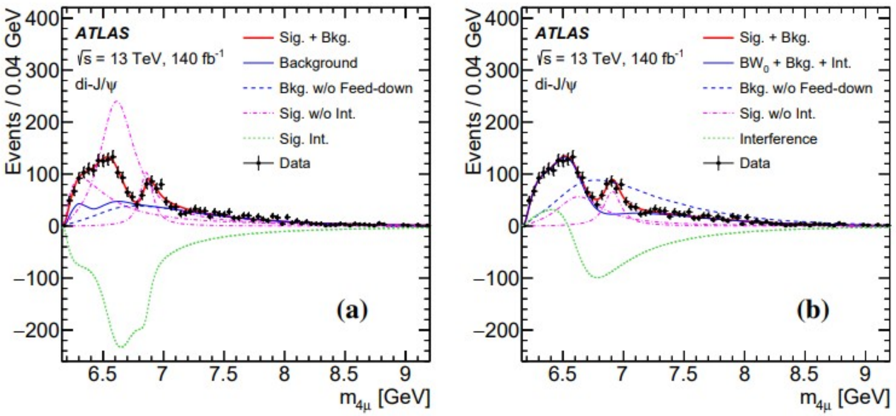


J/ψ $(C\bar{C})$  **(1976)** D $(C\bar{q})$ T_{4c} $(CC\bar{C}\bar{C})$

1974 by Ting and Richter 1976 at SLAC LHCb, 2006.16957

Latest data on fully charm tetraquarks

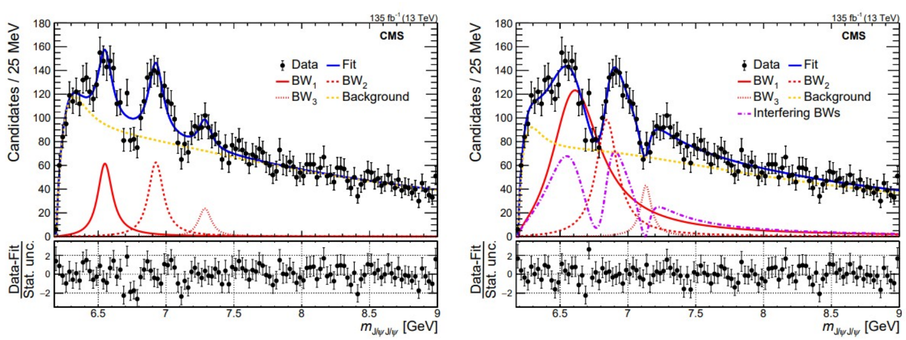
Exp.	Fit	M_{BW_1}	Γ_{BW_1}	$M_{X(6900)}$	$\Gamma_{X(6900)}$	M_{BW_3}	Γ_{BW_3}
LHCb	No interf.	–	–	$6905 \pm 11 \pm 7$	$80 \pm 19 \pm 33$	–	–
CMS	No interf.	$6552 \pm 10 \pm 12$	$124^{+32}_{-26} \pm 33$	$6927 \pm 9 \pm 4$	$122^{+24}_{-21} \pm 184$	$7287^{+20}_{-18} \pm 5$	$95^{+59}_{-40} \pm 19$
LHCb	Interf.	6741 ± 6	288 ± 16	$6886 \pm 11 \pm 11$	$168 \pm 33 \pm 69$	–	–
CMS	Interf.	6638^{+43+16}_{-38-31}	$440^{+230+110}_{-200-240}$	6847^{+44+48}_{-28-20}	191^{+66+25}_{-49-17}	7134^{+48+41}_{-25-15}	97^{+40+29}_{-29-26}
ATLAS	Fit-A	$6630 \pm 50^{+80}_{-10}$	$350 \pm 110^{+110}_{-40}$	$6860 \pm 30^{+10}_{-20}$	$110 \pm 50^{+20}_{-10}$	$7220 \pm 30^{+10}_{-40}$	$90 \pm 60^{+60}_{-50}$
ATLAS	Fit-B	$6650 \pm 20^{+30}_{-20}$	$440 \pm 50^{+60}_{-50}$	$6910 \pm 10 \pm 10$	$150 \pm 30 \pm 10$	–	–



ATLAS, 2304.08962; 140fb-1 data;

P_t(mu1,2,3,4)>4,4,3,3GeV; |eta(mu1,2,3,4)|<2.5;

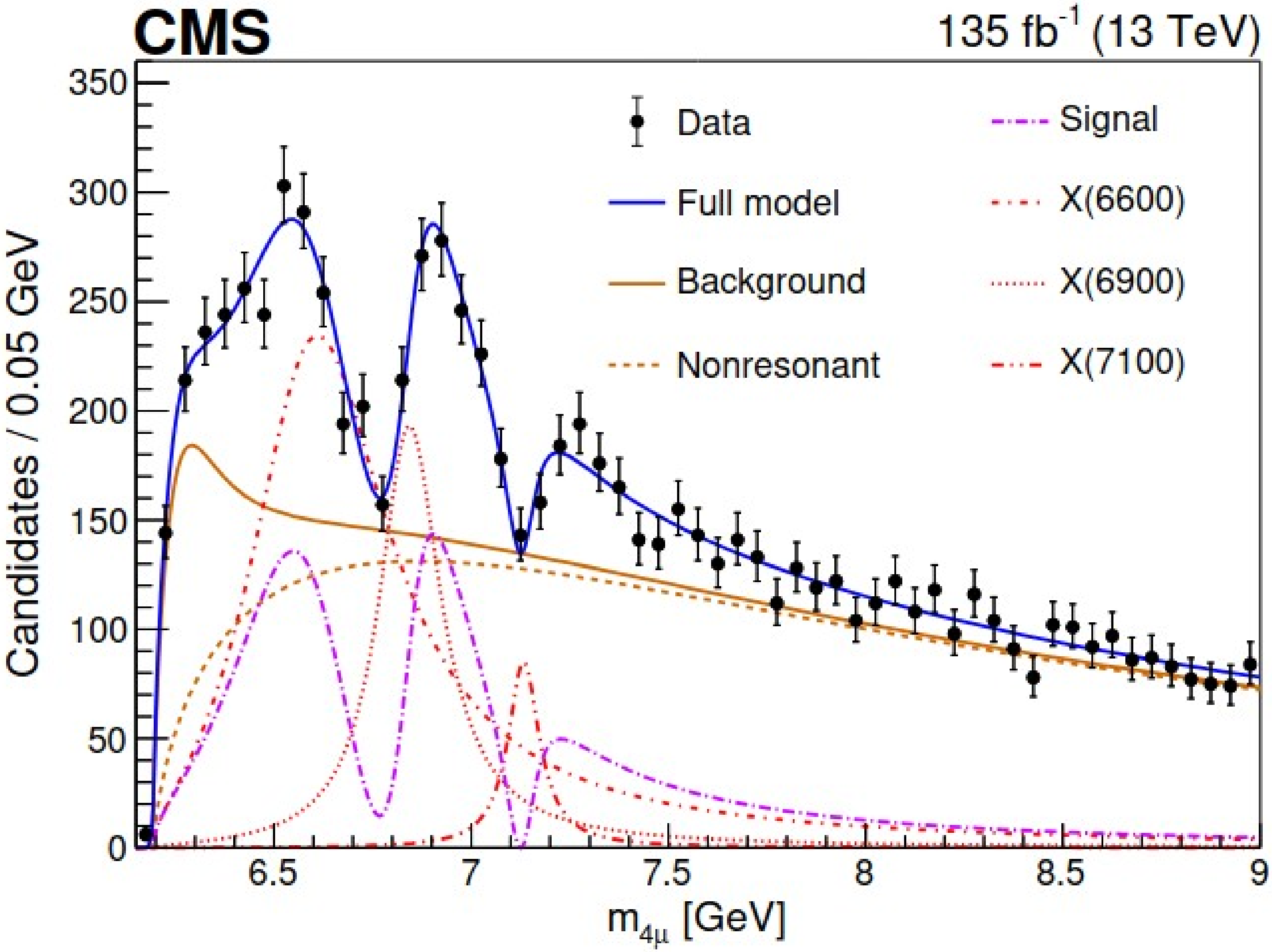
2.94(3.56)GeV<M(dimuon)<3.25(3.80)GeV



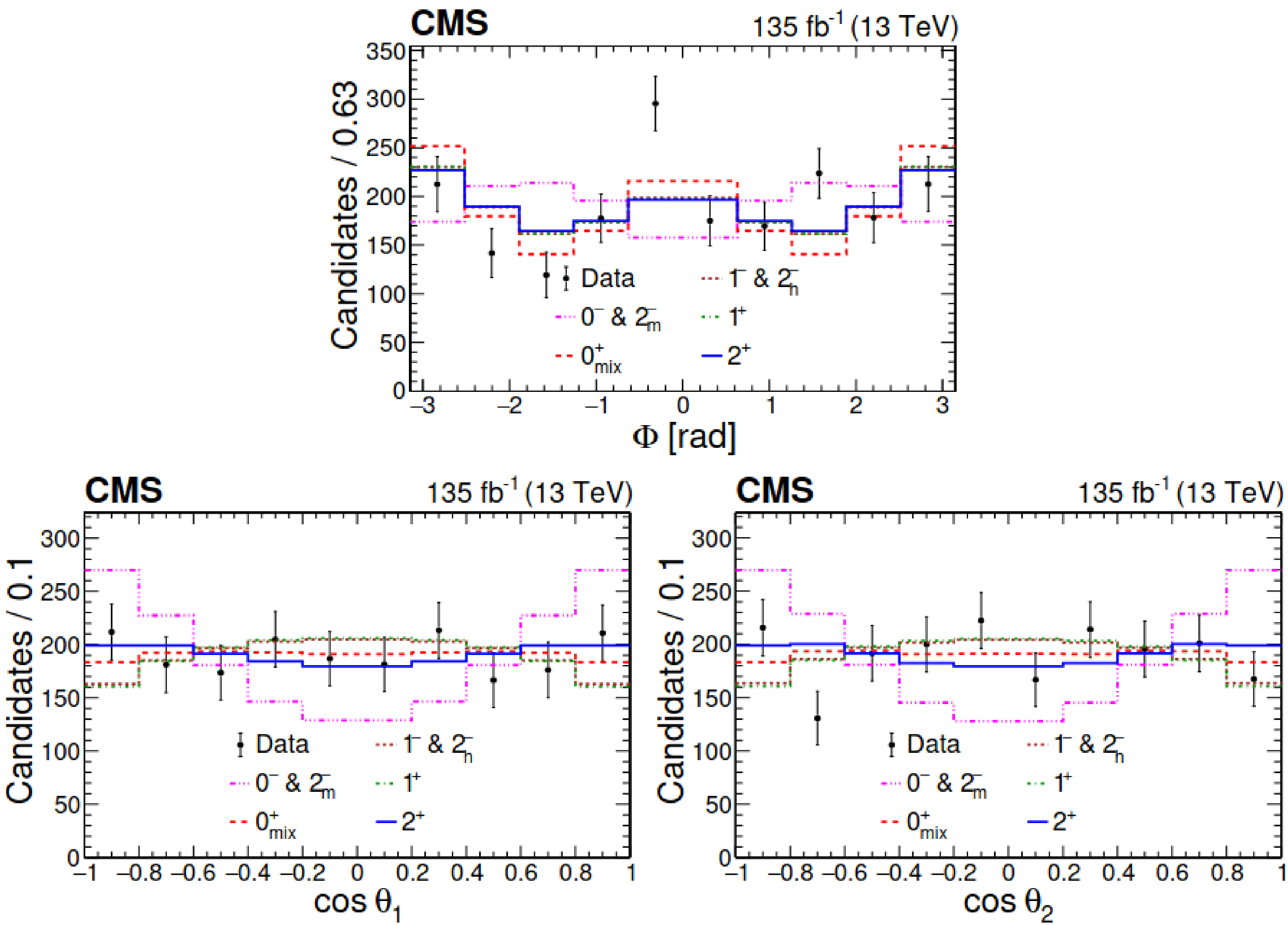
CMS, 2306.07164; 135fb-1 data;

P_t(muon)>2GeV;|eta(muon)|<2.4;

P_t(di muon)>3.5GeV;

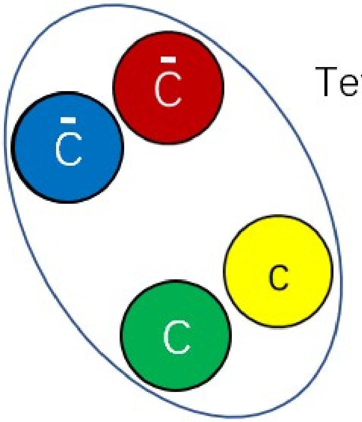


CMS,2506.07944 (submitted to Nature)

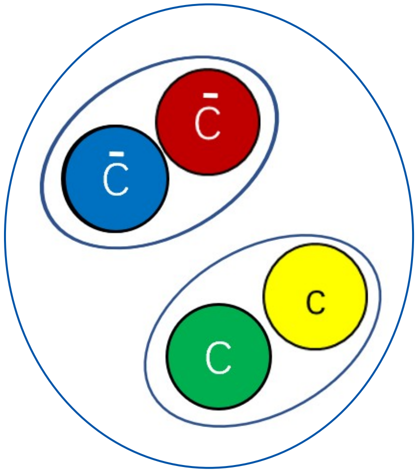


CMS data supports spin-parity 2^++

How to explain these exotic states?

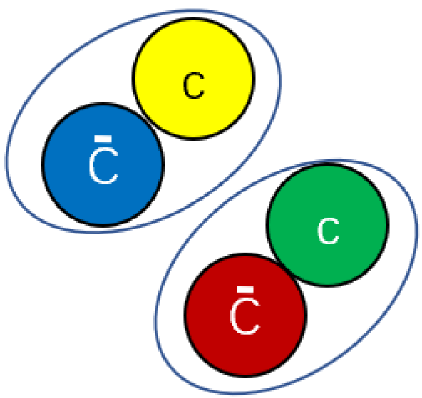


Tetraquark

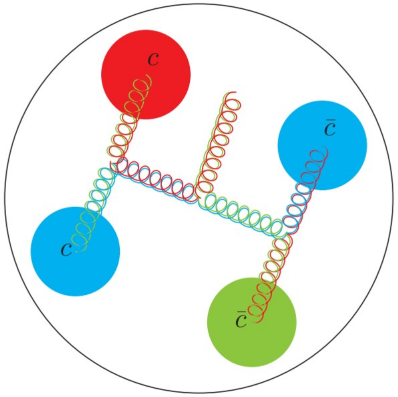


Diquark-antidiquark

Zhu, Bauer, Yi, 2410.11210



Charmonia Molecule



Gluonic Tetracharm Hybrid

Previous theoretical studies

Y. Iwasaki, Phys. Rev. Lett. 36, 1266 (1976)
K. T. Chao, Z. Phys. C 7, 317 (1981)
J. P. Ader, J. M. Richard and P. Taxil, Phys. Rev. D 25, 2370 (1982)
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Z. G. Wang, Eur. Phys. J. C 77, no.7, 432 (2017)
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C. Deng, H. Chen and J. Ping, [arXiv:2003.05154]
F. Feng, Y. Huang, Y. Jia, W. L. Sang, X. Xiong and J. Y.Zhang, arXiv:2009.08450
Y.Q. Ma, H.F. Zhang, arXiv:2009.08376
……

Diquark-antidiquark or charmonium molecule?

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Fully-heavy tetraquark spectra and production at hadron colliders

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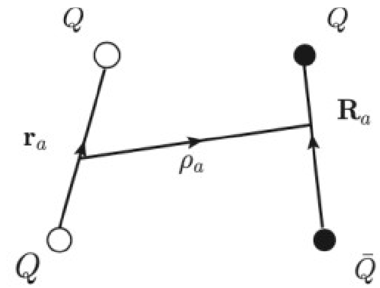
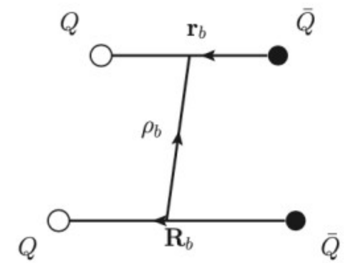
Abstract

Motivated by the observation of exotic structure around 6900 MeV in the J/ψ-pair mass spectrum using proton-proton collision data by the LHCb collaboration, we study the spectra of fully-heavy tetraquarks within Bethe-Salpeter equation and Regge trajectory relation. The $X(6900)$ may be explained as a radially excited state with quark content $cc\bar{c}\bar{c}$ and spin-parity $0^{++}(3S)$ or $2^{++}(3S)$ or an orbitally excited $2P$ state. New $cc\bar{c}\bar{c}$ structures around 6.0 GeV, 6.5 GeV, and 7.1 GeV are predicted together. Other $bb\bar{b}\bar{b}$ and $bc\bar{b}\bar{c}$ structures which may be experimentally prominent are discussed. On the other hand, the fully-heavy S-wave tetraquark production at hadron colliders is investigated and their cross sections are obtained.

$$\bar{3}_c \otimes 3_c = 1_c \text{ and } 6_c \otimes \bar{6}_c = 1_c.$$

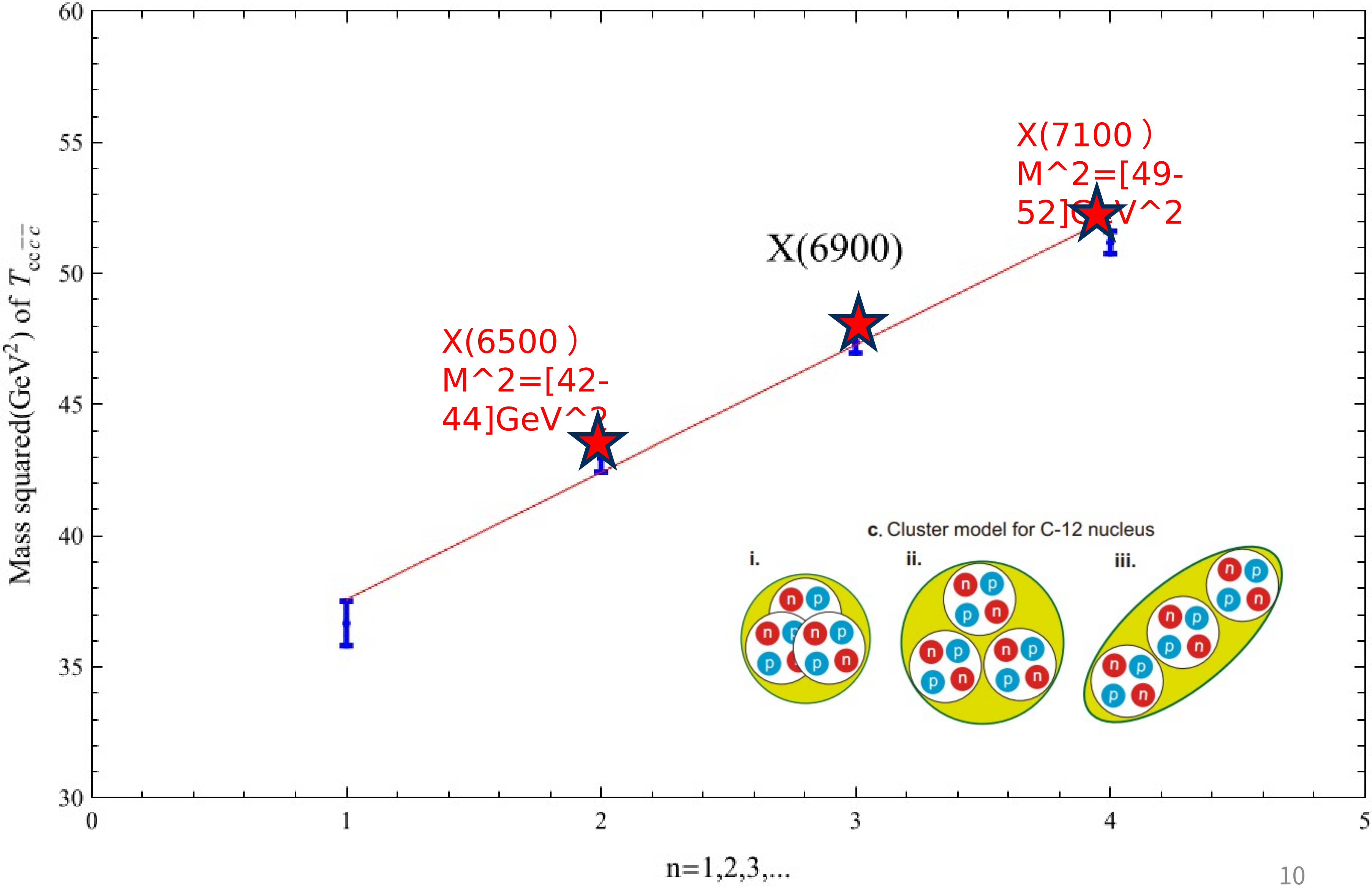


$$|\mathbf{1}\rangle \equiv |(Q_1\bar{Q}_3)_{\mathbf{1}}(Q_2\bar{Q}_4)_{\mathbf{1}}\rangle,$$
$$|\mathbf{8}\rangle \equiv |(Q_1\bar{Q}_3)_{\mathbf{8}}(Q_2\bar{Q}_4)_{\mathbf{8}}\rangle,$$



The masses (6.5GeV, 7.1GeV) are predicted previously and confirmed by CMS.

Fully charm tetraquark family: Linear Regge trajectories?



Outline

- Fully charm tetraquarks production properties

- Fully charm tetraquarks decay properties

- Summary and Outlook

- **Fully charm tetraquarks production properties**

Fully charm tetraquark production

QCD factorization:

$$\sigma\left(pp \to T_{4c} + X\right)$$
$$= \sum_{i,j=q,g} \int_0^1 dx_1 dx_2 f_{i/p}\left(x_1, \mu_F\right) f_{j/p}\left(x_2, \mu_F\right) \hat{\sigma}(i + j \to T_{4c} + X)\left(\hat{s}, \mu_F\right)$$

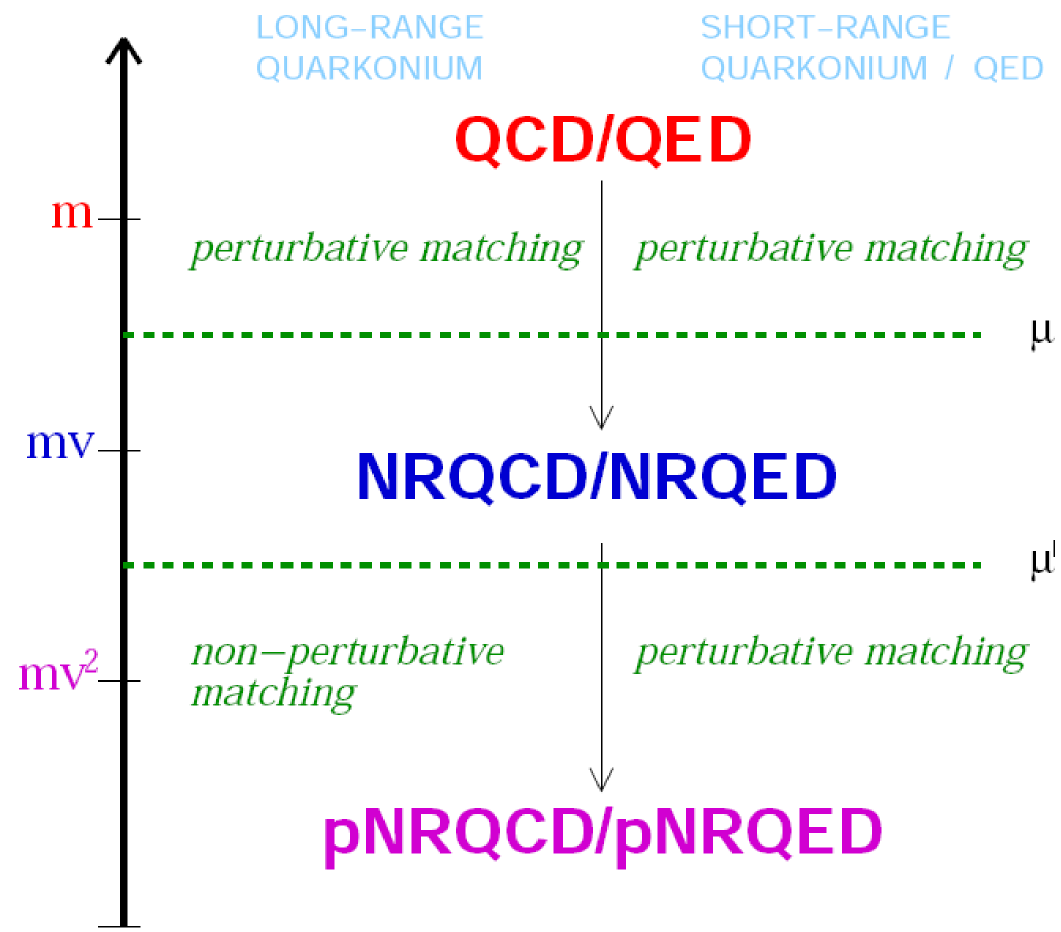
Partonic processes:

LO+NLO virtual: $g + g \to T_{4c}$ and $q + \bar{q} \to T_{4c}$.

NLO real: $g + g \to T_{4c} + g,\ q + \bar{q} \to T_{4c} + g$

$q + g \to T_{4c} + q,\ \bar{q} + g \to T_{4c} + \bar{q}$

NRQCD/pNRQCD factorization



$$\alpha_s(mv) \sim v$$

$v^2 \approx 0.1$ for the Υ

Bodwin-Braaten-Lapage
1995

Pineda-Soto-Brambilla-Vairo
2000

NRQCD Lagrangian

- **QCD Lagrangian**

$$\mathcal{L}_{\rm QCD} = \sum_{q=u,d,s,\ldots} \overline{\psi}_{qi}(x)\big[(i\gamma_\mu D^\mu)_{ij} - m_q\delta_{ij}\big]\Psi_{qj}(x) - \frac{1}{4}F^a_{\mu\nu}(x)F^{\mu\nu\,a}(x)\ ,$$

- **Rewrite heavy quark field and do the NR expansion**

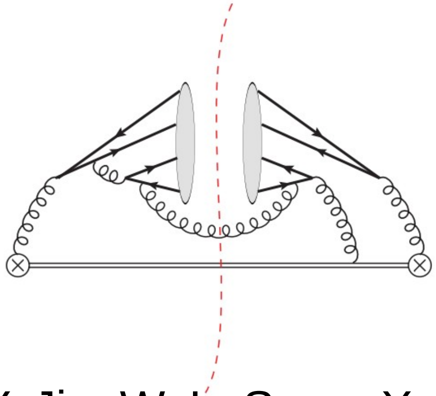
$$\Psi = e^{-iMt}\tilde{\Psi} = e^{-iMt}\begin{pmatrix}\psi\\ \chi\end{pmatrix}, \qquad \Psi' = e^{iM't}\tilde{\Psi} = e^{iM't}\begin{pmatrix}\psi'\\ \chi'\end{pmatrix},$$

- **Obtain NRQCD Lagrangian**

$$\begin{aligned}\mathcal{L}_{NRQCD} =&\psi^\dagger\left(iD_t - \frac{1}{2M}(i\mathbf{D})^2\right)\psi + \frac{c_F}{2M}\psi^\dagger\boldsymbol{\sigma}\cdot g\mathbf{B}\psi\\ &+\frac{c_D}{8M^2}\psi^\dagger(\mathbf{D}\cdot g\mathbf{E} - g\mathbf{E}\cdot\mathbf{D})\psi^\dagger + \frac{c_S}{8M^2}\psi^\dagger(i\boldsymbol{\sigma}\cdot\mathbf{D}\times g\mathbf{E} - i\boldsymbol{\sigma}\cdot g\mathbf{E}\times\mathbf{D})\psi^\dagger\\ &+\frac{c_4}{8M^3}\psi'^\dagger\left(\mathbf{D}^2\right)^2\psi' + \mathcal{O}\left(1/M^3\right)\\ &+[\psi\to i\sigma^2\chi'*, A_\mu\to -A^T_\mu, M\to M'] + \mathcal{L}_{light}].\end{aligned}$$

Previous studies (selected)

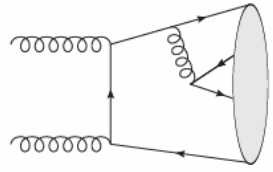
➢ **Fragmentation mechanism (based on NRQCD), equally considering the NLO real diagrams (valid for large Pt)**



F. Feng, Y. Huang, Y. Jia, W. L. Sang, X. Xiong and J. Y.Zhang, arXiv:2009.08450; F. Feng et al, 2304.11142
Y.Q. Ma, H.F. Zhang, arXiv:2009.08376
I. Belov, A. Giachino, and E. Santopinto, arXiv:2409.12070

➢ **NRQCD LO (Pt=0)**

R.L. Zhu, arXiv: 2010.09082



Differential cross section in NRQCD

$$\frac{\mathrm{d}\hat{\sigma}(T_{4c}^{(J)}+X)}{\mathrm{d}\hat{t}} = \frac{2M_{T_{4c}}}{m_c^{14}}\left[F_{3,3}^{(J)}\left\langle O_{3,3}^{(J)}\right\rangle + 2F_{3,6}^{(J)}\left\langle O_{3,6}^{(J)}\right\rangle + F_{6,6}^{(J)}\left\langle O_{6,6}^{(J)}\right\rangle\right],$$

$$O_{3,3}^{(J)} = \mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(J)}\sum_X |T_{4c}^J+X\rangle\langle T_{4c}^J+X|\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(J)\dagger},$$

$$O_{6,6}^{(0)} = \mathcal{O}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{(0)}\sum_X |T_{4c}^0+X\rangle\langle T_{4c}^0+X|\mathcal{O}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{(0)\dagger},$$

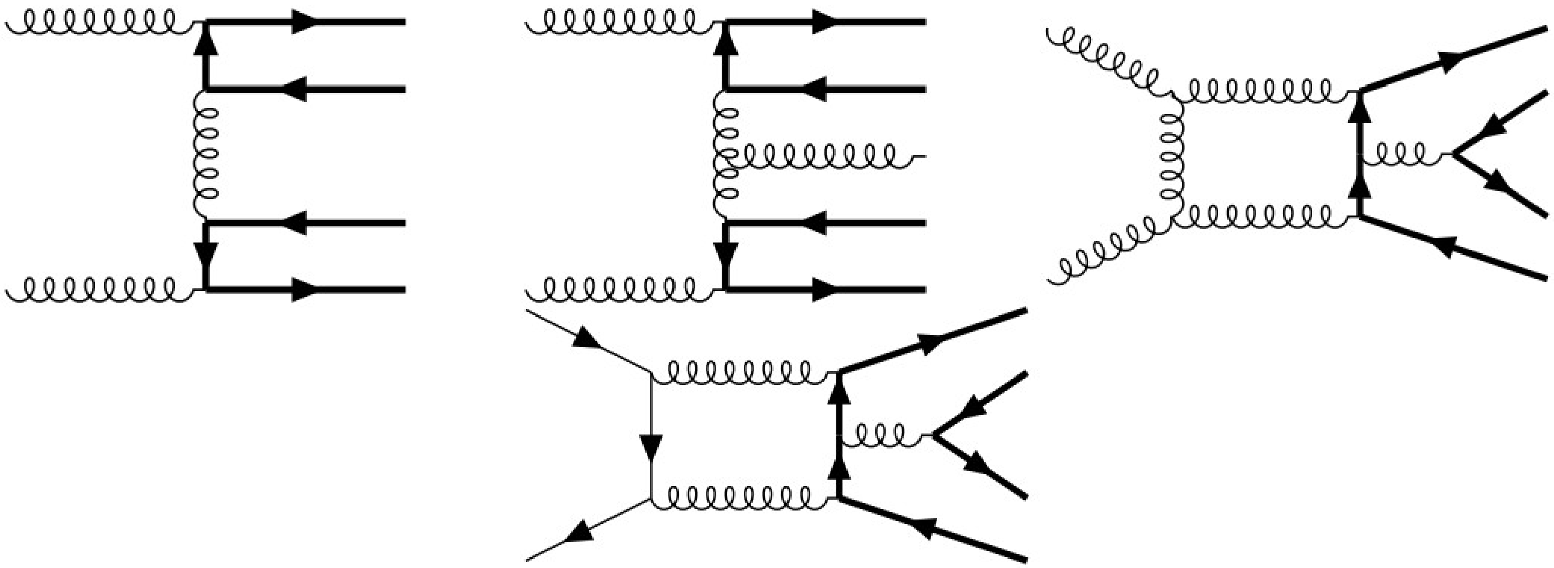
$$O_{3,6}^{(0)} = \mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(0)}\sum_X |T_{4c}^0+X\rangle\langle T_{4c}^0+X|\mathcal{O}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{(0)\dagger},$$

$$\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(0)} = -\frac{1}{\sqrt{3}}[\psi_a^T(i\sigma^2)\sigma^i\psi_b][\chi_c^\dagger\sigma^i(i\sigma^2)\chi_d^*]\ \mathcal{C}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{ab;cd},$$

$$\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{ij;(2)} = [\psi_a^T(i\sigma^2)\sigma^m\psi_b][\chi_c^\dagger\sigma^n(i\sigma^2)\chi_d^*]\ \Gamma^{ij;mn}\ \mathcal{C}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{ab;cd},$$

$$\mathcal{O}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{(0)} = [\psi_a^T(i\sigma^2)\psi_b][\chi_c^\dagger(i\sigma^2)\chi_d^*]\ \mathcal{C}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{ab;cd}.$$

Full NLO calculation in NRQCD



LO:64(gg)， 4(qqbar)

NLO Virtual:2008(gg)， 170(qqbar)

NLO Real: 618(gg)， 98(qqbar, qg)

Exact IR cancellation

$$K^{\mathrm{LH3}}_{\mathrm{gg,virtual}} = \frac{3}{\epsilon^2} - \frac{1}{\epsilon}\left(3\log\left(\frac{\mu}{4m_c}\right)^2 - \frac{n_l}{3} + \frac{11}{2}\right) - \frac{3}{2}\log^2\left(\frac{\mu}{4m_c}\right)^2 + \left(\frac{11}{2} - \frac{2n_h + n_l}{3}\right)\log\left(\frac{\mu}{m_c}\right)^2 + \frac{4719}{256}\left(\mathrm{Li}_2\left(2\sqrt{2}-2\right) + \mathrm{Li}_2\left(-2\sqrt{2}-2\right)\right) + \cdots$$

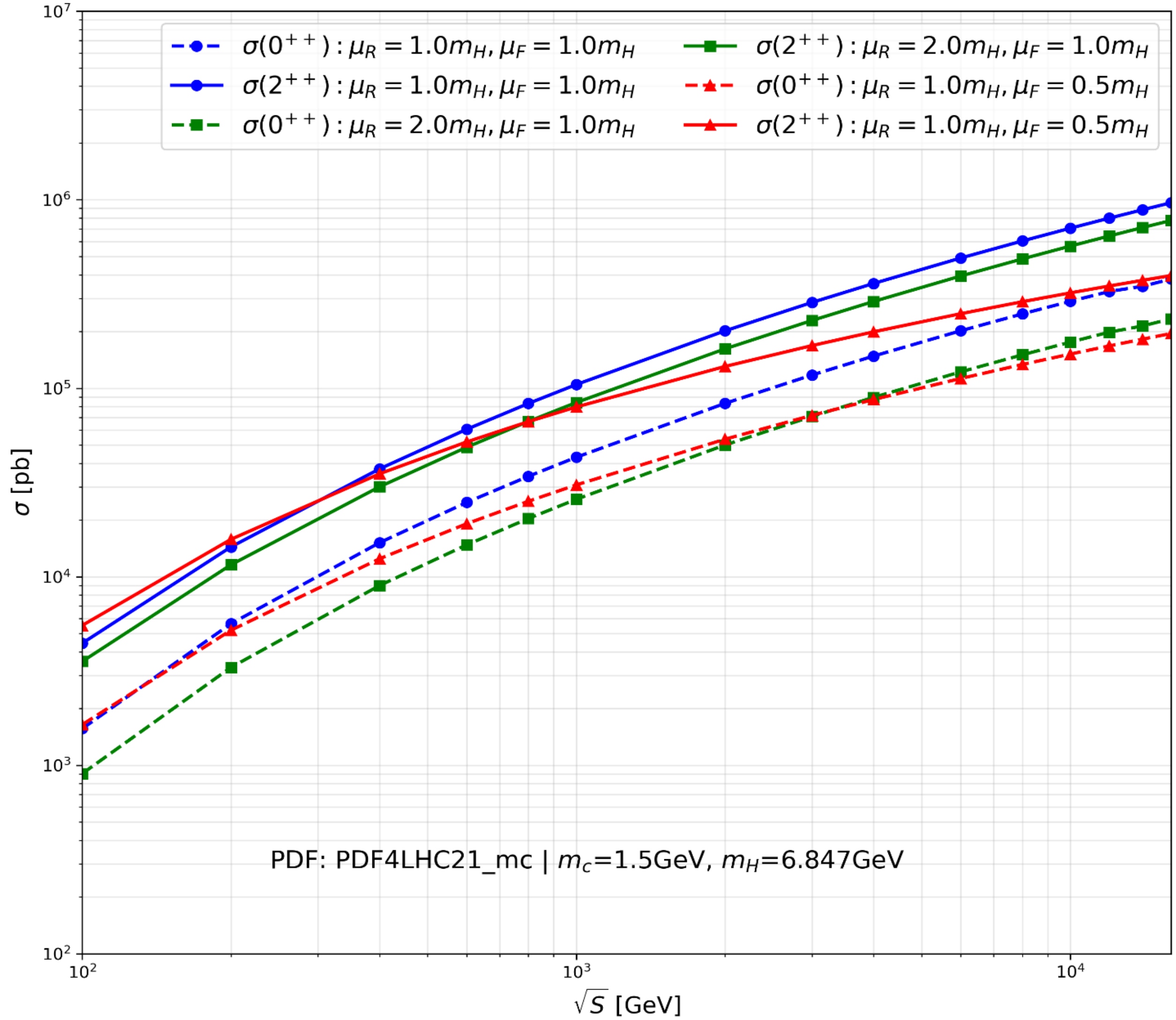
There are soft divergences when $k_g = 0$ or $z = 1$, and collinear divergences when $y_\theta = \cos\theta_{k_n k_g} = \pm 1$ with $k_n = k_1, k_2, k_T$.

$$\begin{aligned}\hat{\sigma}_{\mathrm{soft}}\ (i + j \to T_{4c} + k) &= -C\frac{1}{2\epsilon_{\mathrm{IR}}}\delta(1-z)\frac{4^{-\epsilon}\Gamma(1-\epsilon)\Gamma(-\epsilon)}{\Gamma(1-2\epsilon)}\mathcal{B}_{ij}\left(z=1, y_\theta\right)\\ &= \Gamma(1+\epsilon)\left(\frac{4\pi\mu_R^2}{\hat{s}}\right)^{\epsilon}\frac{\alpha_s}{\pi}\hat{\sigma}^{(0)}_{ij}\delta(1-z)\left(\frac{1}{\epsilon^2_{\mathrm{IR}}} - \frac{\pi^2}{3}\right)C_{ij}\end{aligned}$$

$$\begin{aligned}&\hat{\sigma}_{\text{hard col. } y_\theta=\pm 1}\,(i + j \to T_{4c} + k)\\ &= -C\frac{4^{-\epsilon}}{2\epsilon_{\mathrm{IR}}}\left[\left(\frac{1}{1-z}\right)_+ - 2\epsilon\left(\frac{\log(1-z)}{1-z}\right)_+\right]\mathcal{B}_{ij}\left(z, y_\theta = \pm 1\right)\\ &= -C\frac{4^{-\epsilon}}{2\epsilon_{\mathrm{IR}}}\left[\left(\frac{1}{1-z}\right)_+ - 2\epsilon\left(\frac{\log(1-z)}{1-z}\right)_+\right]\mathcal{B}_{ij}\left(z=1, y_\theta\right)\frac{b_{ij}^{\mathrm{collinear}}}{z^3}\\ &= \Gamma(1+\epsilon)\left(\frac{4\pi\mu_R^2}{\hat{s}}\right)^{\epsilon}\frac{\alpha_s}{\pi}\hat{\sigma}^{(0)}_{ij}\delta(1-z)\left[-2C_{ij}b_{ij}^{\mathrm{collinear}}\left(\left(\frac{1}{1-z}\right)_+\frac{1}{\epsilon_{\mathrm{IR}}} - 2\left(\frac{\log(1-z)}{1-z}\right)_+\right)\right]\end{aligned}$$

$$\hat{\sigma}^{\mathrm{AP-CT}} = \frac{1}{\epsilon}\frac{\alpha_s}{2\pi}\left(\frac{4\pi\mu_R^2}{\mu_F^2}\right)^{\epsilon}\Gamma(1+\epsilon)\hat{\sigma}^{(0)}zP_{ij}(z),$$

Collider energy dependence for different spin and scales

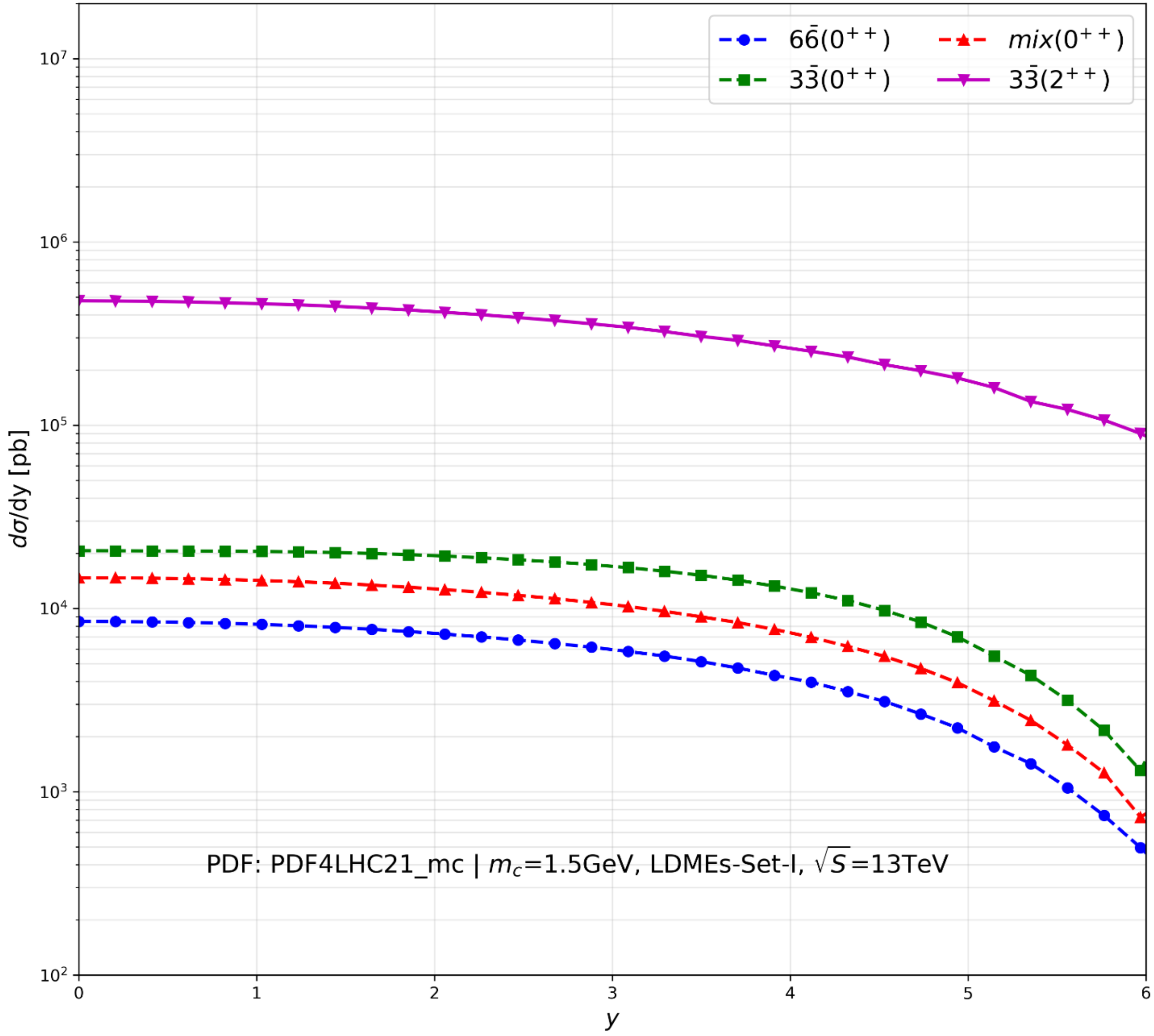


Reference at 7TeV:
σ(eta_c)~10^6nb

σ(2J/psi)~10nb

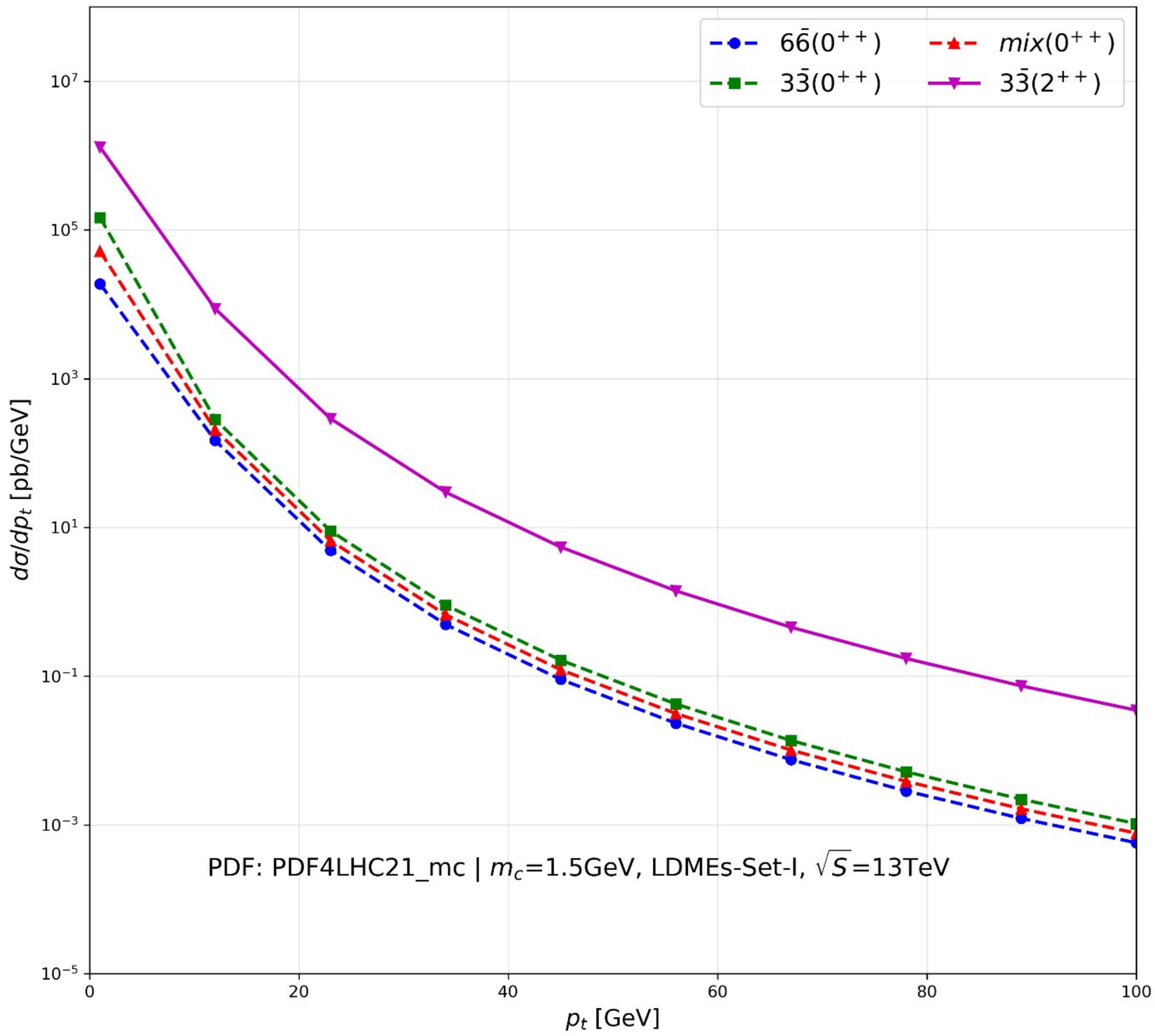
σ(T_4c)
~[10-100]nb

Rapidity dependence for different diquark configurations



[3 3_bar]
plays a major role

Transverse momentum distribution



Divergence
when pt -> 0

Soft gluon radiation produces large logarithms

$$\frac{d\sigma}{dyd^2p_t}\Big|^{gg}_{p_t\ll M} = \sigma_0(T^{J,i}_{4c})\frac{\alpha_s C_A}{2\pi^2}\int f(x)dx f(x')dx' \times \frac{1}{p_t^2}\left[\frac{2(1-\xi_1+\xi_1^2)^2}{(1-\xi_1)_+}\delta(1-\xi_2)+\frac{2(1-\xi_2+\xi_2^2)^2}{(1-\xi_2)_+}\delta(1-\xi_1) + \left(2\ln\frac{M^2}{p_t^2}\right)\delta(1-\xi_2)\delta(1-\xi_1)\right],$$

At low P_perp, the soft gluon radiations generate the divergence, which should be resumed to obtain reliable predictions.

Collins-Soper-Sterman resummation

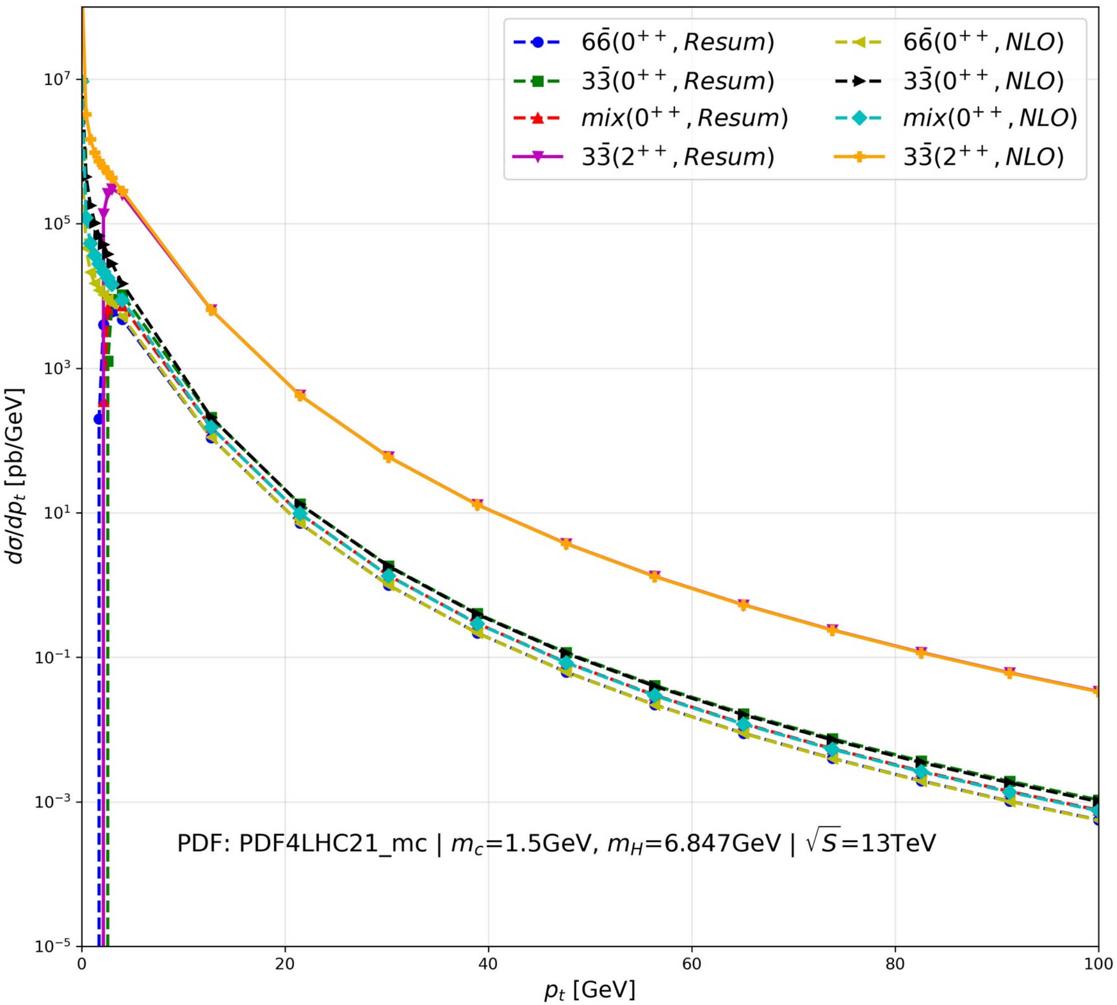
$$\frac{d\sigma}{dp_t^2} \sim \frac{(L+1)}{p_t^2}\Big\{\alpha_s + \alpha_s^2 L^2 + \alpha_s^3 L^4 + \alpha_s^4 L^6 + \alpha_s^2 + \alpha_s^3 L^2 + \alpha_s^4 L^4 + \alpha_s^3 + \alpha_s^4 L^2 \cdots \Big\},$$

$$\frac{\partial W(b,M^2)}{\partial \ln M^2} = (K+G')W(b,M^2) \ ,$$

Collins, Soper 81
Collins, Soper, Sterman 85

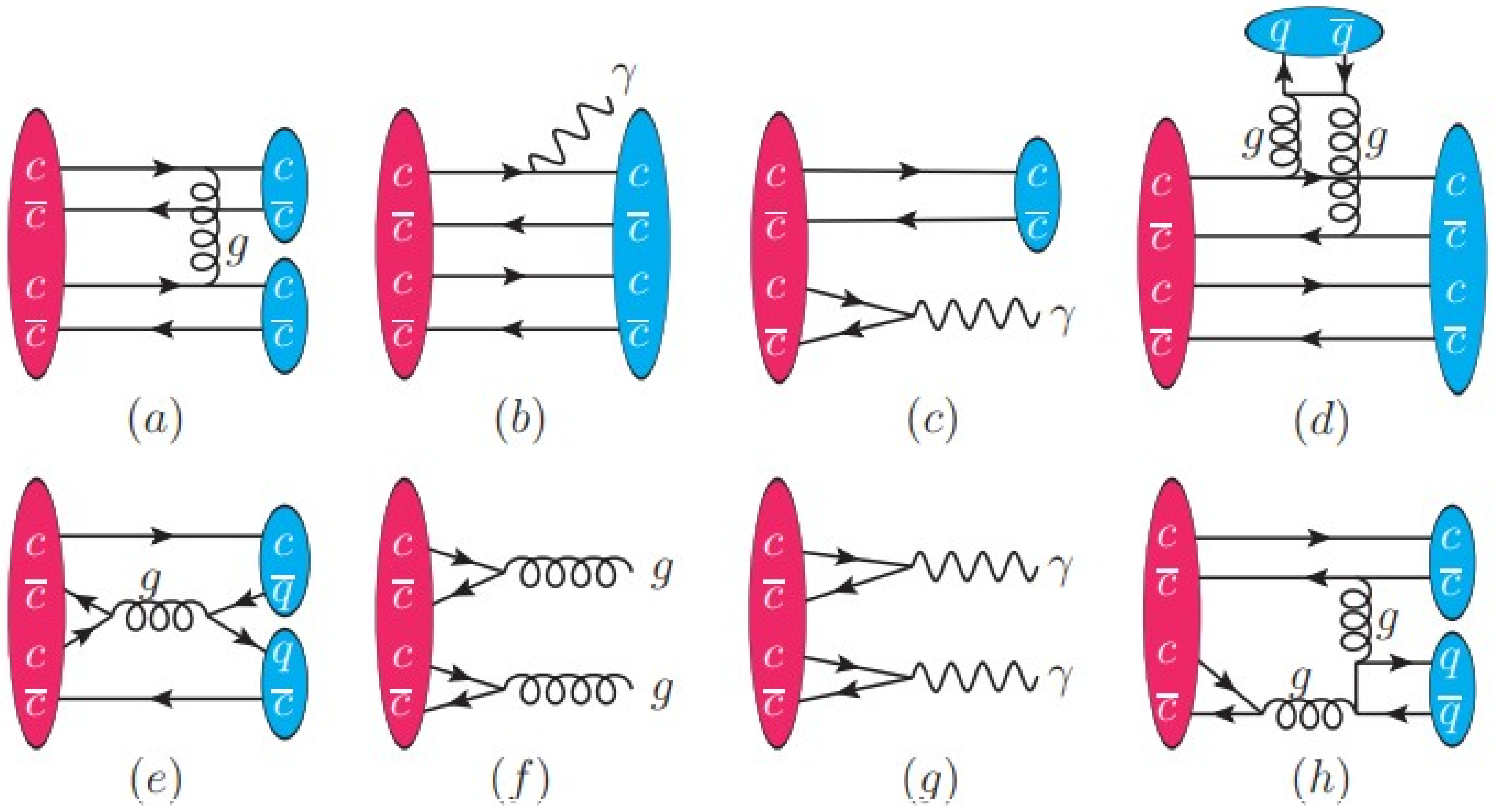
$$\frac{d\sigma}{dy dp_t^2} \sim \int d^2 b e^{ip_t\cdot b} \times \sum_a \int_{x_A}^1 \frac{d\xi_A}{\xi_A} f_{a/A}(\xi_A;1/b) \sum_b \int_{x_B}^1 \frac{d\xi_B}{\xi_B} f_{b/B}(\xi_B;1/b) \times \exp\left\{-\int_{1/b^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\left(\frac{Q^2}{\bar{\mu}^2}\right) A\left(g(\bar{\mu})\right) + B\left(g(\bar{\mu})\right)\right]\right\} \times C_{ja}\left(\frac{x_A}{\xi_A}; g(1/b)\right) C_{jb}\left(\frac{x_B}{\xi_B}; g(1/b)\right) + Y(p_t; Q, x_A, x_B).$$

NLL Resummation results

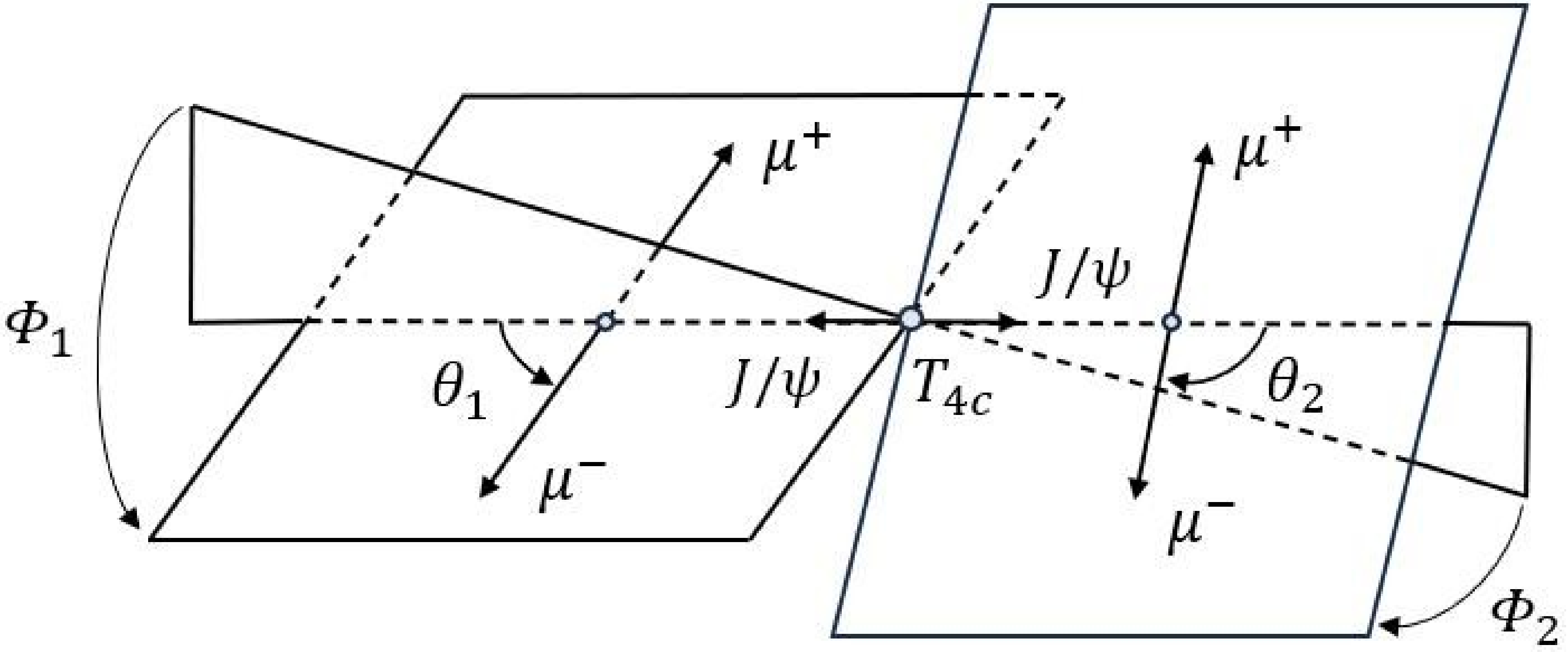


- **Fully charm tetraquarks decay properties**

Major decay modes



Cascade decay angles



Helicity amplitudes

$$\langle \vec{p}_1 \lambda_{a_1}, -\vec{p}_1 \lambda_{a_2} | H_1 | s_1 \lambda_1 \rangle = 4\pi \sqrt{\frac{\omega_1}{p_1}} \langle \Omega_1 \lambda_{a_1} \lambda_{a_2} | H_1 | s_1 \lambda_1 \rangle$$

$$= 4\pi \sqrt{\frac{\omega_1}{p_1}} \sum_{\lambda'_{a_1} \lambda'_{a_2}} \langle \Omega_1 \lambda_{a_1} \lambda_{a_2} | | s_1 \lambda_1 \lambda'_{a_1} \lambda'_{a_2} \rangle \langle s_1 \lambda_1 \lambda'_{a_1} \lambda'_{a_2} | H_1 | s_1 \lambda_1 \rangle$$

$$= 4\pi \sqrt{\frac{\omega_1}{p_1}} N_{s_1} D^{s_1 *}_{\lambda_1 \lambda_a}(-\Phi_1, \theta_1, \Phi_1) \langle s_1 \lambda_1 \lambda_{a_1} \lambda_{a_2} | H_1 | s_1 \lambda_1 \rangle$$

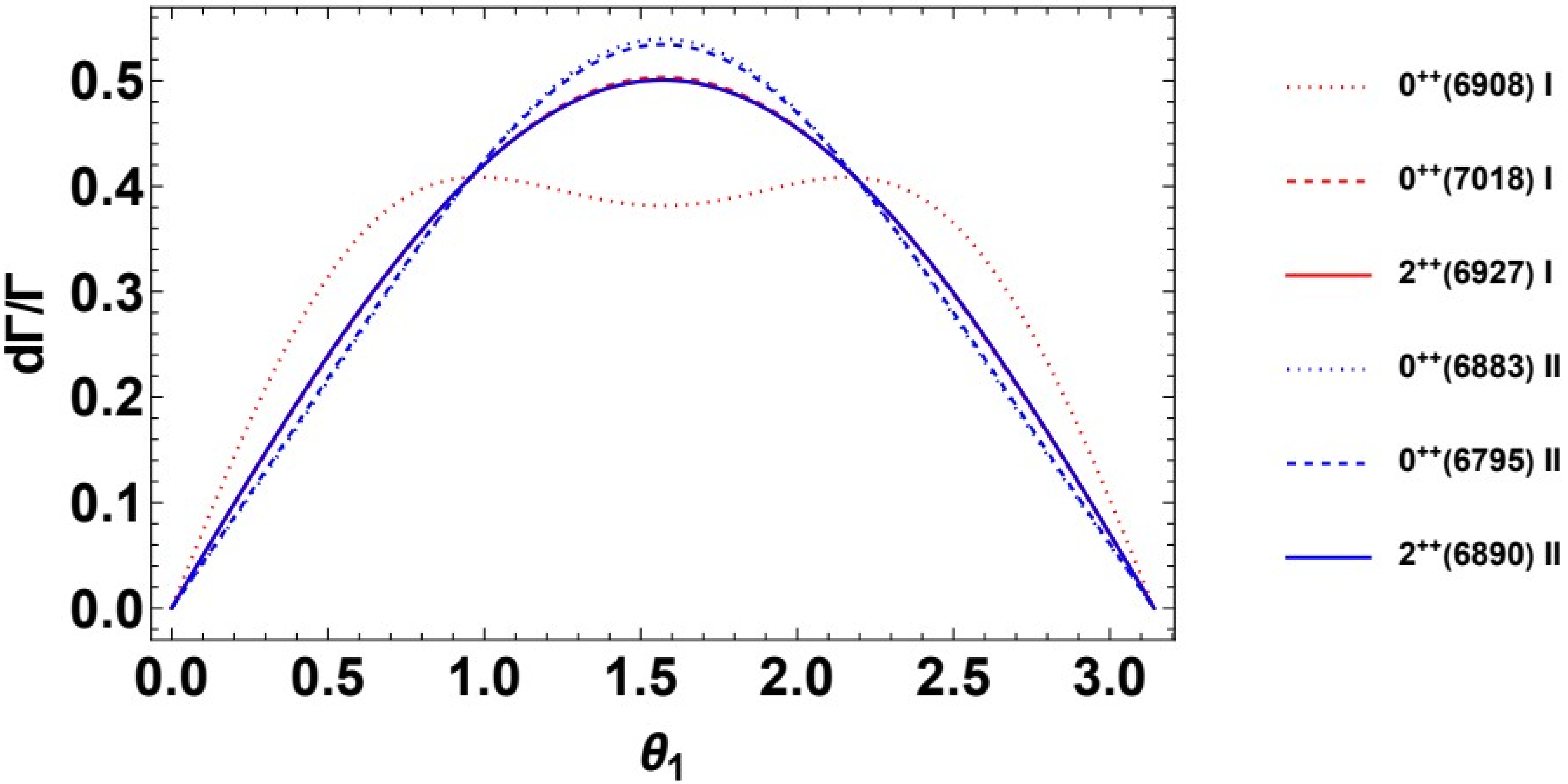
$$= N_{s_1} F^{s_1}_{\lambda_{a_1} \lambda_{a_2}} D^{s_1 *}_{\lambda_1 \lambda_a}$$

Symmetry constraint:
Parity conservation;
Identical particles

$$F^s_{\lambda_1 \lambda_2} = \eta \eta_1 \eta_2 (-1)^{s - s_1 - s_2} F^s_{-\lambda_1 - \lambda_2}$$

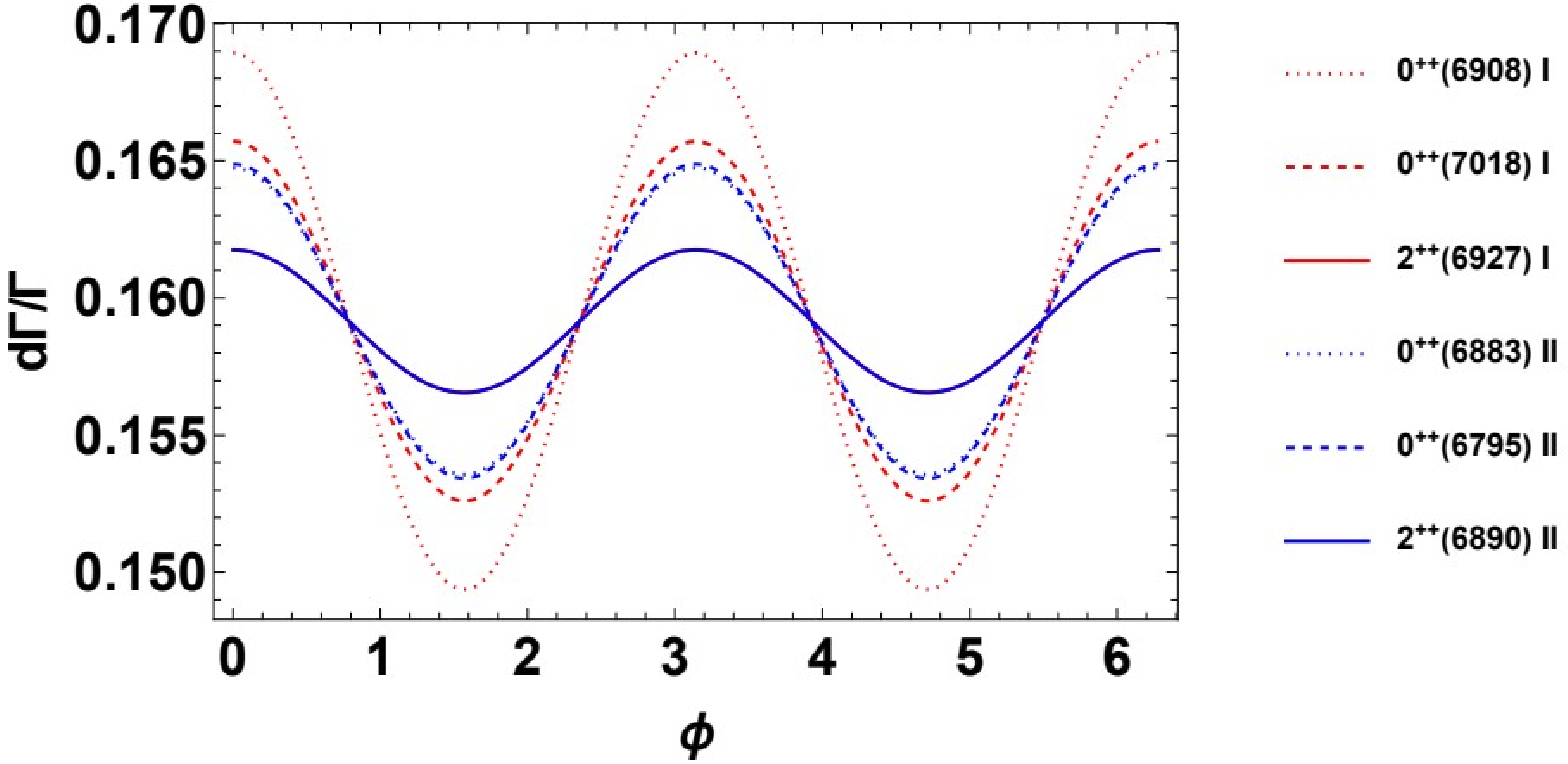
$$F^s_{\lambda_1 \lambda_2} = (-1)^s F^s_{\lambda_2 \lambda_1}$$

Polar angular distribution

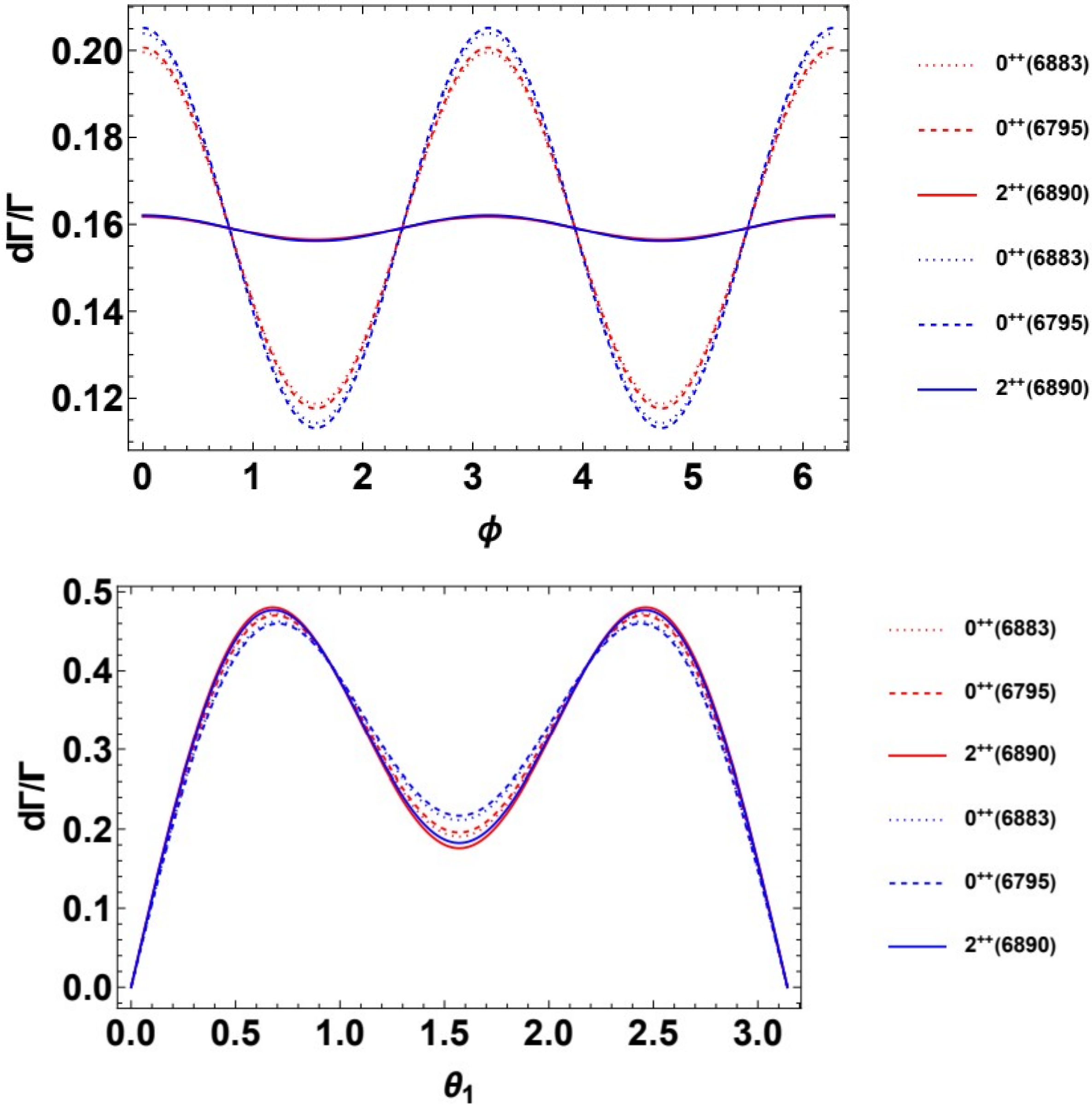


Model I: quark model (QM);
Model II: diquark model+heavy quark effective theory(HQET)

Plane angular distribution

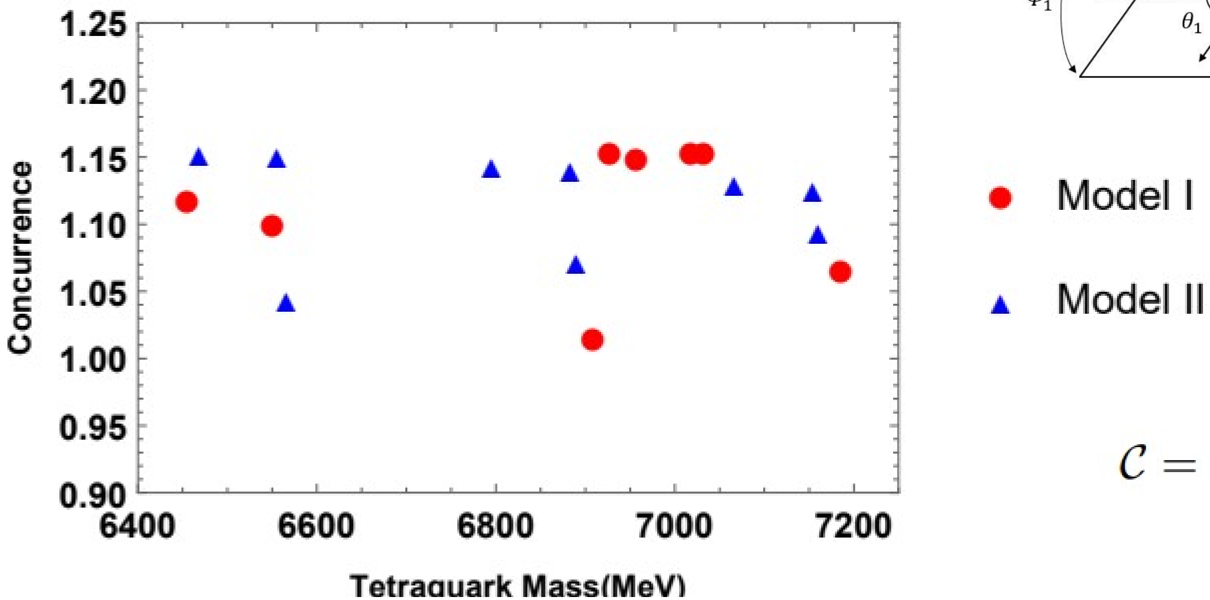


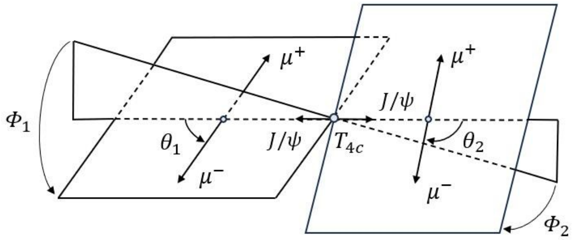
Charmed meson pair channel



The θ_1 and Φ distributions for various tetraquarks near 6.9 GeV into $D^*(\to D\pi)$ and $\bar{D}^*(\to \bar{D}\pi)$ using Model II (HQET).

Quantum entanglement





$$\mathcal{C} = \sqrt{2(1 - \mathrm{Tr}\rho_A^2)},$$

$$max\left(0, \sqrt{\frac{1}{3}\left[\frac{h_{00}^{00} + 2h_{01}^{01} + 4h_{11}^{00} + 2h_{11}^{11}}{N} - 1\right]}\right) \leq \sqrt{2(1 - \frac{\left(h_{00}^{00}\right)^2 + 2\left(h_{11}^{11}\right)^2}{N^2})} \leq \frac{2}{\sqrt{3}}.$$

$$max\,(0, LB_2) \leq \sqrt{2(1 - \left(\frac{h_{00}^{00} + 2h_{10}^{10}}{N}\right)^2 - \left(\frac{h_{10}^{10} + h_{11}^{11} + h_{1-1}^{1-1}}{N}\right)^2 - \left(\frac{h_{10}^{10} + h_{1-1}^{1-1} + h_{11}^{11}}{N}\right)^2)} \leq \frac{2}{\sqrt{3}}.$$

If assume quantum entanglement as a basic principle, then a constraint formula for helicity amplitudes.

Summary and outlook

- ✓ The full NLO calculation for T4c production spectrum is given.
- ✓ The angular distribution and entanglement are studied in T4c decay processes

Outlook: measuring the (differential) cross section (or) and the decay angular distribution shall tell us the inner structure of fully charm tetraquarks; a lot of tasks in there

Thank you a lot!

Backup

	LDME	Model I [15]	Model II [16]
0^{++}	$\left\langle O_{3,3}^{(0)} \right\rangle$ [GeV^9]	0.0347	0.0187
	$\left\langle O_{3,6}^{(0)} \right\rangle$ [GeV^9]	0.0211	-0.0161
	$\left\langle O_{6,6}^{(0)} \right\rangle$ [GeV^9]	0.0128	0.0139
1^{+-}	$\left\langle O_{3,3}^{(1)} \right\rangle$ [GeV^9]	0.0780	0.0480
2^{++}	$\left\langle O_{3,3}^{(2)} \right\rangle$ [GeV^9]	0.072	0.0628

fully charmed tetraquark decay widths

Exp.	Fit method	M_{BW_1}	Γ_{BW_1}	M_{BW_2}	Γ_{BW_2}	M_{BW_3}	Γ_{BW_3}
LHCb [3]	No interf.	-	-	$6905 \pm 11 \pm 7$	$80 \pm 19 \pm 33$	-	-
LHCb [3]	Interf.	6741 ± 6	288 ± 16	$6886 \pm 11 \pm 11$	$168 \pm 33 \pm 69$	-	-
ATLAS [4]	Fit-A	$6630 \pm 50^{+80}_{-10}$	$350 \pm 110^{+110}_{-40}$	$6860 \pm 30^{+10}_{-20}$	$110 \pm 50^{+20}_{-10}$	$7220 \pm 30^{+10}_{-40}$	$90 \pm 60^{+60}_{-50}$
ATLAS [4]	Fit-B	$6650 \pm 20^{+30}_{-20}$	$440 \pm 50^{+60}_{-50}$	$6910 \pm 10 \pm 10$	$150 \pm 30 \pm 10$	-	-
CMS [5]	No interf.	$6552 \pm 10 \pm 12$	$124^{+32}_{-26} \pm 33$	$6927 \pm 9 \pm 4$	$122^{+24}_{-21} \pm 184$	$7287^{+20}_{-18} \pm 5$	$95^{+59}_{-40} \pm 19$
CMS [5]	Interf.	6638^{+43+16}_{-38-31}	$440^{+230}_{-200-240}$	6847^{+44}_{-28-20}	191^{+66+25}_{-49-17}	7134^{+48+41}_{-25-15}	97^{+40+29}_{-29-26}
Theo.	$n, {}^{2S+1}L_J, J^{PC}$	M_T	Γ_T	M_T	Γ_T	M_T	Γ_T
QM	$1, {}^1S_0, 0^{++}$	6455	20.5	-	-	-	-
	$1, {}^1S_0, 0^{++}$	6550	7.5	-	-	-	-
	$1, {}^5S_2, 2^{++}$	6524	31.0	-	-	-	-
	$2, {}^1S_0, 0^{++}$	-	-	6908	38.7	-	-
	$2, {}^1S_0, 0^{++}$	-	-	6957	83.3	-	-
	$2, {}^1S_0, 0^{++}$	-	-	-	-	7018	34.6
	$2, {}^1S_0, 0^{++}$	-	-	-	-	7185	36.9
	$2, {}^5S_2, 2^{++}$	-	-	6927	27.1	-	-
	$2, {}^5S_2, 2^{++}$	-	-	-	-	7032	670.9
HQET	$2, {}^1S_0, 0^{++}$	6555^{+36}_{-37}	15.5	-	-	-	-
	$3, {}^1S_0, 0^{++}$	-	-	6883^{+27}_{-27}	16.2	-	-
	$4, {}^1S_0, 0^{++}$	-	-	-	-	7154^{+22}_{-22}	15.9
	$2, {}^1S_0, 0^{++}$	6468^{+35}_{-35}	23.6	-	-	-	-
	$3, {}^1S_0, 0^{++}$	-	-	6795^{+26}_{-26}	21.3	-	-
	$4, {}^1S_0, 0^{++}$	-	-	-	-	7066^{+21}_{-22}	27.1
	$2, {}^5S_2, 2^{++}$	6566^{+34}_{-35}	39.4	-	-	-	-
	$3, {}^5S_2, 2^{++}$	-	-	6890^{+27}_{-26}	36.0	-	-
	$4, {}^5S_2, 2^{++}$	-	-	-	-	7160^{+21}_{-22}	29.0

fully charmed tetraquark major decay modes

Theo.	$n,^{2S+1}L_J,J^{PC},M_T$	$J/\psi J/\psi$	$H_c H_c'$	$D^{(*)}\bar{D}^{(*)}$	$D_s^{(*)}\bar{D}_s^{(*)}$	gg	$\gamma\gamma(\times 10^{-3})$
QPM[a]	$1,^1S_0,0^{++},6455$	0.7	1.45	$9.6(\frac{\xi_D}{0.1})^2$	$6.9(\frac{\xi_{D_s}}{0.1})^2$	$1.9(\frac{f_0'^1}{100})^2$	$1.3(\frac{f_0'^1}{100})^2$
	$1,^1S_0,0^{++},6550$	1.78	0.12	$3.0(\frac{\xi_D}{0.1})^2$	$2.1(\frac{\xi_{D_s}}{0.1})^2$	$0.5(\frac{f_0^1}{100})^2$	$0.3(\frac{f_0^1}{100})^2$
	$1,^5S_2,2^{++},6524$	-	-	$15(\frac{\xi_D}{0.1})^2$	$14.2(\frac{\xi_{D_s}}{0.1})^2$	$1.8(\frac{f_2^2}{100})^2$	$1.3(\frac{f_2^2}{100})^2$
	$2,^1S_0,0^{++},6908$	0.12	23.75	$7.6(\frac{\xi_D}{0.1})^2$	$5.5(\frac{\xi_{D_s}}{0.1})^2$	$1.7(\frac{f_0'^2}{100})^2$	$1.3(\frac{f_0'^2}{100})^2$
	$2,^1S_0,0^{++},6957$	4.66	74.03	$2.4(\frac{\xi_D}{0.1})^2$	$1.8(\frac{\xi_{D_s}}{0.1})^2$	$0.4(\frac{f_0^2}{100})^2$	$0.3(\frac{f_0^2}{100})^2$
	$2,^1S_0,0^{++},7018$	1.87	18.01	$7.8(\frac{\xi_D}{0.1})^2$	$5.2(\frac{\xi_{D_s}}{0.1})^2$	$1.7(\frac{f_0'^2}{100})^2$	$1.2(\frac{f_0'^2}{100})^2$
	$2,^1S_0,0^{++},7185$	0.48	32.25	$2.2(\frac{\xi_D}{0.1})^2$	$1.6(\frac{\xi_{D_s}}{0.1})^2$	$0.4(\frac{f_0^2}{100})^2$	$0.3(\frac{f_0^2}{100})^2$
	$2,^5S_2,2^{++},6927$	0.36	1.45	$12(\frac{\xi_D}{0.1})^2$	$11.6(\frac{\xi_{D_s}}{0.1})^2$	$1.7(\frac{f_2^2}{100})^2$	$1.3(\frac{f_2^2}{100})^2$
	$2,^5S_2,2^{++},7032$	7.12	640.06	$11(\frac{\xi_D}{0.1})^2$	$11(\frac{\xi_{D_s}}{0.1})^2$	$1.7(\frac{f_2^2}{100})^2$	$1.2(\frac{f_2^2}{100})^2$
HQET	$1,^1S_0,0^{++},6055$	-	-	$4.0(\frac{\xi_D}{0.1})^2$	$2.8(\frac{\xi_{D_s}}{0.1})^2$	$2.0(\frac{f_0'^1}{100})^2$	$1.4(\frac{f_0'^1}{100})^2$
	$1,^1S_0,0^{++},5984$	-	-	$16.4(\frac{\xi_D}{0.1})^2$	$8.7(\frac{\xi_{D_s}}{0.1})^2$	$0.5(\frac{f_0^1}{100})^2$	$0.4(\frac{f_0^1}{100})^2$
	$1,^5S_2,2^{++},6090$	-	-	$19.3(\frac{\xi_D}{0.1})^2$	$17.6(\frac{\xi_{D_s}}{0.1})^2$	$2.0(\frac{f_2^1}{100})^2$	$1.4(\frac{f_2^1}{100})^2$
	$2,^1S_0,0^{++},6555$	$2.6(\frac{\xi_\psi}{0.1})^2$	$6.0(\frac{\xi_{\eta_c}}{0.1})^2$	$3.0(\frac{\xi_D}{0.1})^2$	$2.1(\frac{\xi_{D_s}}{0.1})^2$	$1.8(\frac{f_0'^2}{100})^2$	$1.3(\frac{f_0'^2}{100})^2$
	$2,^1S_0,0^{++},6468$	$5.5(\frac{\xi_\psi}{0.1})^2$	$1.2(\frac{\xi_{\eta_c}}{0.1})^2$	$9.6(\frac{\xi_D}{0.1})^2$	$6.8(\frac{\xi_{D_s}}{0.1})^2$	$0.5(\frac{f_0^2}{100})^2$	$0.3(\frac{f_0^2}{100})^2$
	$2,^5S_2,2^{++},6566$	$8.5(\frac{\xi_\psi}{0.1})^2$	$0.01(\frac{\xi_{\eta_c}}{0.1})^2$	$14.9(\frac{\xi_D}{0.1})^2$	$14(\frac{\xi_{D_s}}{0.1})^2$	$2.0(\frac{f_2^2}{100})^2$	$1.4(\frac{f_2^2}{100})^2$
	$3,^1S_0,0^{++},6883$	$3.6(\frac{\xi_\psi}{0.1})^2$	$5.7(\frac{\xi_{\eta_c}}{0.1})^2+0.8(\frac{\xi_{\chi_{c0}}}{0.1})^2$	$2.6(\frac{\xi_D}{0.1})^2$	$1.8(\frac{\xi_{D_s}}{0.1})^2$	$1.7(\frac{f_0'^3}{100})^2$	$1.3(\frac{f_0'^3}{100})^2$
	$3,^1S_0,0^{++},6795$	$6.3(\frac{\xi_\psi}{0.1})^2$	$0.7(\frac{\xi_{\eta_c}}{0.1})^2$	$8(\frac{\xi_D}{0.1})^2$	$5.9(\frac{\xi_{D_s}}{0.1})^2$	$0.4(\frac{f_0^3}{100})^2$	$0.3(\frac{f_0^3}{100})^2$
	$3,^5S_2,2^{++},6890$	$9.6(\frac{\xi_\psi}{0.1})^2$	$0.03(\frac{\xi_{\eta_c}}{0.1})^2$	$12.5(\frac{\xi_D}{0.1})^2$	$11.9(\frac{\xi_{D_s}}{0.1})^2$	$2.0(\frac{f_2^3}{100})^2$	$1.4(\frac{f_2^3}{100})^2$
	$4,^1S_0,0^{++},7154$	$4.1(\frac{\xi_\psi}{0.1})^2$	$5.3(\frac{\xi_{\eta_c}}{0.1})^2+1.8(\frac{\xi_{\chi_{c0}}}{0.1})^2+0.4(\frac{\xi_{h_c}}{0.1})^2$	$2.2(\frac{\xi_D}{0.1})^2$	$0.8(\frac{\xi_{D_s}}{0.1})^2$	$1.7(\frac{f_0'^4}{100})^2$	$1.2(\frac{f_0'^4}{100})^2$
	$4,^1S_0,0^{++},7066$	$6.2(\frac{\xi_\psi}{0.1})^2$	$0.4(\frac{\xi_{\eta_c}}{0.1})^2+4(\frac{\xi_{\chi_{c0}}}{0.1})^2+4(\frac{\xi_{h_c}}{0.1})^2$	$7.0(\frac{\xi_D}{0.1})^2$	$5.1(\frac{\xi_{D_s}}{0.1})^2$	$0.4(\frac{f_0^4}{100})^2$	$0.3(\frac{f_0^4}{100})^2$
	$4,^5S_2,2^{++},7160$	$9.8(\frac{\xi_\psi}{0.1})^2$	$0.04(\frac{\xi_{\eta_c}}{0.1})^2+1.4(\frac{\xi_{h_c}}{0.1})^2$	$10.9(\frac{\xi_D}{0.1})^2$	$10.3(\frac{\xi_{D_s}}{0.1})^2$	$2.0(\frac{f_2^4}{100})^2$	$1.4(\frac{f_2^4}{100})^2$

$$\mathcal{M}(0^{++}) = \epsilon^*_{1\mu}\epsilon^*_{2\nu}\left(ag^{\mu\nu} + \frac{bp_1^\mu p_2^\nu}{m_1 m_2} + \frac{ic\epsilon^{\mu\nu\alpha\beta}p_{1\alpha}p_\beta}{m_1 m_2}\right), \tag{A.29}$$

where m_i, p_i and ϵ_i are the mass, momentum and polarization vector for the two daughter particles, respectively. The relationship between the parameters a,b,c and the helicity amplitude can be expressed as

$$F^0_{11} = a + \sqrt{x^2-1}c, \quad F^0_{-1-1} = a - \sqrt{x^2-1}c,$$
$$F^0_{00} = -ax - b(x^2-1), \tag{A.30}$$

$$\mathcal{C} = \sqrt{2(1 - \mathrm{Tr}\rho_A^2)},$$

where

$$x^2 = \frac{p_m^2 M_T^2}{m_1^2 m_2^2} + 1. \tag{A.31}$$

$$\varrho = \frac{1}{|H|^2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{+1}h^*_{+1} & 0 & h_{+1}h^*_0 & 0 & h_{+1}h^*_{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_0h^*_{+1} & 0 & h_0h^*_0 & 0 & h_0h^*_{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{-1}h^*_{+1} & 0 & h_-h^*_0 & 0 & h_{-1}h^*_{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$|\Psi\rangle = \frac{1}{\sqrt{|H|^2}}\left[h_{+1}|J/\psi(+1)\rho(+1)\rangle + h_0|J/\psi(0)\rho(0)\rangle + h_{-1}|J/\psi(-1)\rho(-1)\rangle\right],$$

$$W(b) = W(b_*)W^{NP}(b) \ ,$$

$$b_* = b/\sqrt{1 + (b/b_{max})^2}.$$

$$W^{NP}(b) = \exp\left[-g_1 - g_2 \ln\left(\frac{Q}{2Q_0}\right) - g_1 g_3 \ln(100 x_1 x_2)\right] b^2 \ ,$$