Low Transverse Momentum Distribution of Heavy Quarkonium Production in Hadronic Collisions

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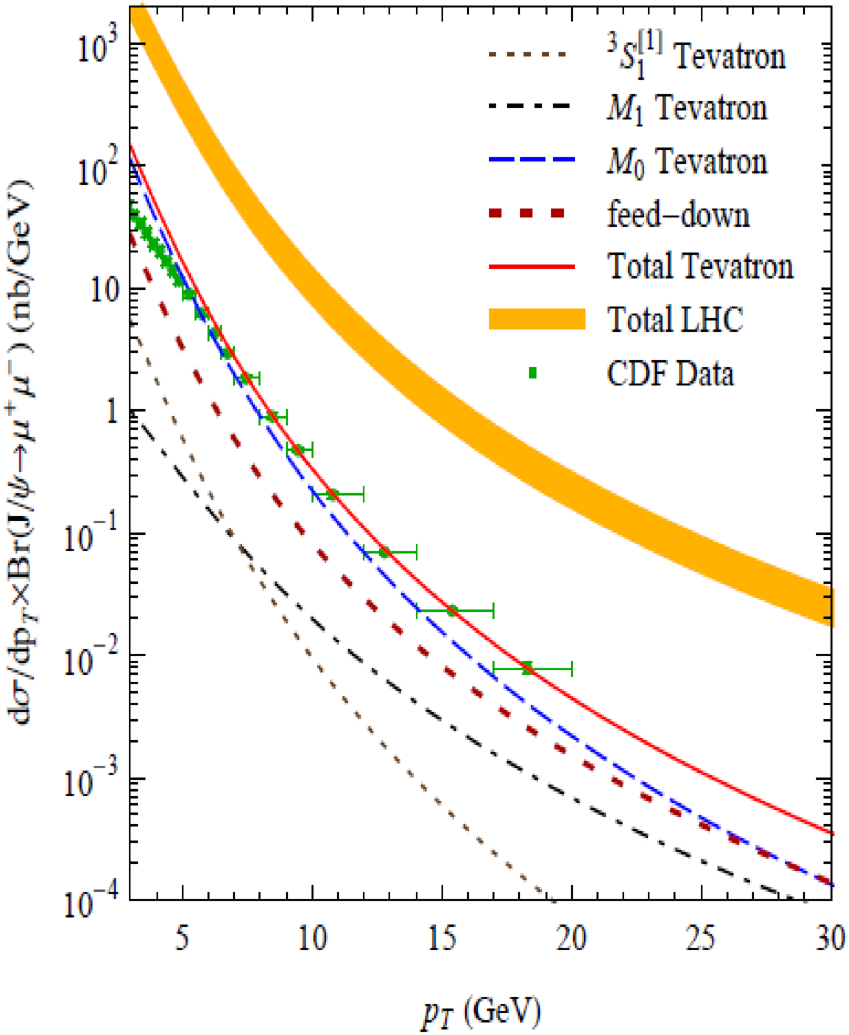
Outline

- Heavy quarkonium production in NRQCD and its distribution in high Pt region
- QCD resummation
- Non-perturbative Sudakov factor
- QCD resummation in heavy quarkonium production
- Our result for low Pt distribution of heavy quarkonium production by using QCD resummation



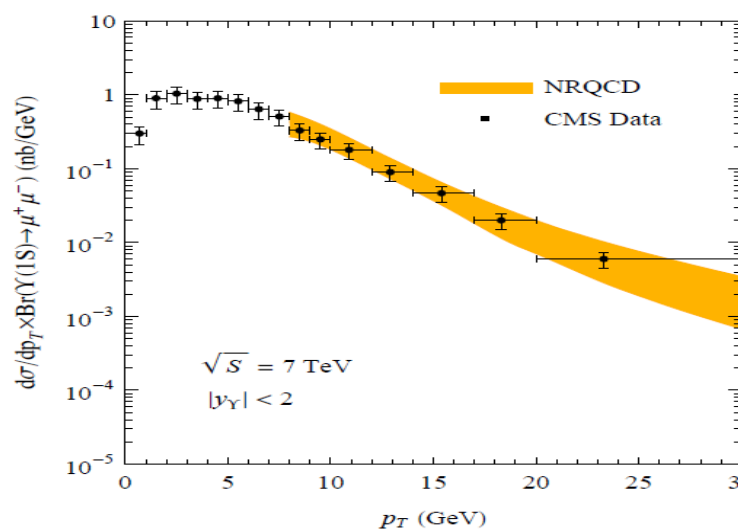
- In high pT, the cross section only depends on the linear combinations of color-octet matrix elements.

$$M_{0,r_0}^{J/\psi} = \langle \mathcal{O}^{J/\psi}(^1S_0^{[8]})\rangle + \frac{r_0}{m_c^2}\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]})\rangle$$

$$M_{1,r_1}^{J/\psi} = \langle \mathcal{O}^{J/\psi}(^3S_1^{[8]})\rangle + \frac{r_1}{m_c^2}\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]})\rangle$$

- Here, the values of M_1 and M_0 are:

H	$\langle \mathcal{O}^H \rangle$ (GeV^3)	M_{1,r_1}^H (10^{-2} GeV^3)	M_{0,r_0}^H (10^{-2} GeV^3)
J/ψ	1.16	$0.05 \pm 0.02 \pm 0.02$	$7.4 \pm 1.9 \pm 0.4$
ψ'	0.76	$0.12 \pm 0.03 \pm 0.01$	$2.0 \pm 0.6 \pm 0.2$



K. T Chao et al PRD 84, 114001

QCD resummation

- Consider the production process $h_1h_2 \to V+X$

$$\frac{d\sigma}{dQ_T^2} \sim \frac{1}{Q_T^2}\{\, \alpha_S(L+1) \; +\alpha_S^2(L^3+L^2) \; +\alpha_S^3(L^5+L^4)+\alpha_S^4(L^7+L^6)+\ldots$$

$$+\alpha_S^2(\,L+\;\;1) \; +\alpha_S^3(L^3+L^2)+\alpha_S^4(L^5+L^4)+\ldots$$

$$+\alpha_S^3(\,L+\;\;1)+\alpha_S^4(L^3+L^2)+\ldots\,\}$$

Where Q_T is the transverse momentum, and Q the mass, of V, and $L = \mathrm{Log}[Q^2 / Q_T^2]$.

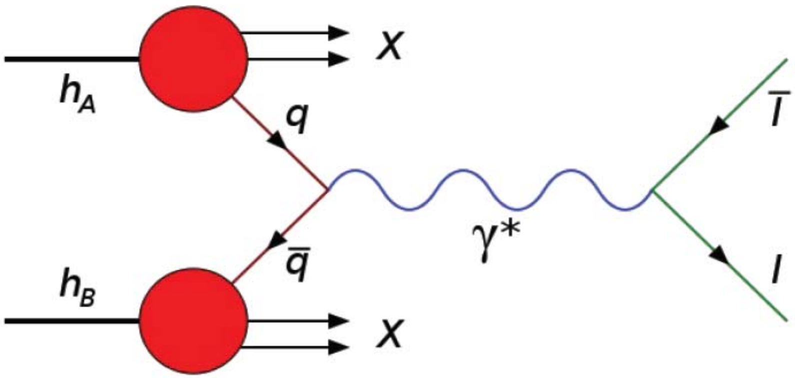
- We have to resum these large logs to make reliable predictions

$$W(Q,b) = e^{-\int_{1/b}^{Q}\frac{d\mu}{\mu}\left(\ln\frac{Q}{\mu}A+B\right)} C\otimes f_1 C\otimes f_2$$

- For Drell-Yan process

$$A^{(1)} = C_F$$

$$B^{(1)} = -3\, C_F / 2$$

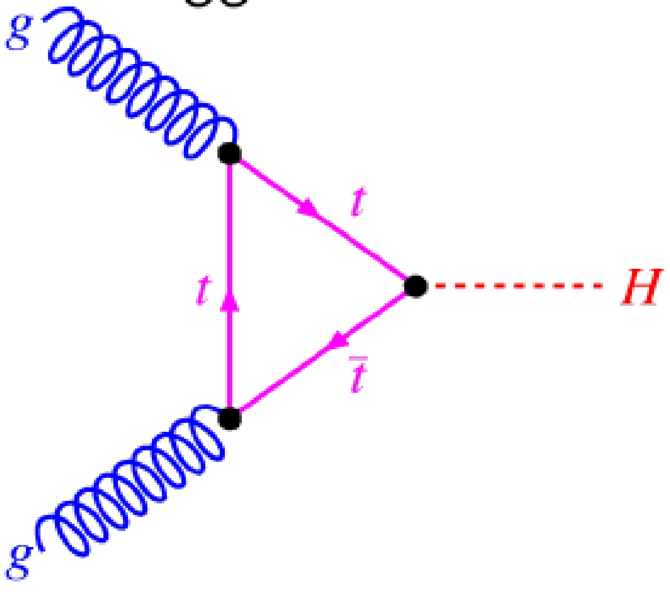


- For Higgs production by di-gluon fusion

$$A^{(1)} = C_A$$

$$B^{(1)} = -\, \beta_0\, C_A$$

gg Fusion



- b is integrated from 0 to ∞. For b >> 1/Λ_{QCD} , the perturbative calculation is no longer reliable. Collins and Soper postulated:

$$W_{j\bar{k}}(b) = W_{j\bar{k}}(b_*)\widetilde{W}_{j\bar{k}}^{NP}(b)$$

and made a cutoff on large b by :

$$b_* = \frac{b}{\sqrt{1+(b/b_{max})^2}}$$

and

$$\widetilde{W}_{j\bar{k}}^{NP}(b,Q,Q_0,x_1,x_2) = \exp\left[-F_1(b)\ln\left(\frac{Q^2}{Q_0^2}\right) - F_{j/h_1}(x_1,b) - F_{\bar{k}/h_2}(x_2,b)\right]$$

with the constraint that $\widetilde{W}_{j\bar{k}}^{NP}(b=0) = 1$

Non-perturbative Sudakov factor

- Davies-Webber-Stirling (DWS) model

$$\exp\left[-g_1-g_2\ln\left(\frac{Q}{2Q_0}\right)\right]b^2$$

C. Davies et al Nucl. Phys. B256, 413

- Ladinsky-Yuan (LY) model

C. P. Yuan et al PRD 50, 4239

$$\exp\left\{\left[-g_1-g_2\ln\left(\frac{Q}{2Q_0}\right)\right]b^2-[g_1g_3\ln(100x_1x_2)]b\right\}$$

- Brock-Landry-Nadolsky-Yuan (BLNY) model

$$\exp\left[-g_1-g_2\ln\left(\frac{Q}{2Q_0}\right)-g_1g_3\ln(100x_1x_2)\right]b^2$$

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Here $x_1x_2 = Q^2 / S$

Parameter	DWS-G fit	LY-G fit	BLNY fit
g_1	0.016	0.02	0.21
g_2	0.54	0.55	0.68
g_3	0.00	-1.50	-0.60
CDF Z Run-0 N_{fit}	1.00 (fixed)	1.00 (fixed)	1.00 (fixed)
R209 N_{fit}	1.02	1.01	0.86
E605 N_{fit}	1.15	1.07	1.00
E288 N_{fit}	1.23	1.28	1.19
DØ Z Run-1 N_{fit}	1.01	1.01	1.00
CDF Z Run-1 N_{fit}	0.89	0.90	0.89
χ^2	416	407	176
χ^2/DOF	3.47	3.42	1.48

Experiment	Reaction	$\sqrt{S}$(GeV)
R209	$p+p\to\mu^+\mu^-+X$	62
E605	$p+Cu\to\mu^+\mu^-+X$	38.8
E288	$p+Cu\to\mu^+\mu^-+X$	27.4
CDF-Z (Run-0)	$p+\bar{p}\to Z+X$	1800
DØ-Z (Run-1)	$p+\bar{p}\to Z+X$	1800
CDF-Z (Run-1)	$p+\bar{p}\to Z+X$	1800

Drell-Yan (R209, E605, E288)

Z production (CDF-Z Run-0, DØ-Z Run-1, CDF-Z Run-1)

Annihilation of quark ant-quark

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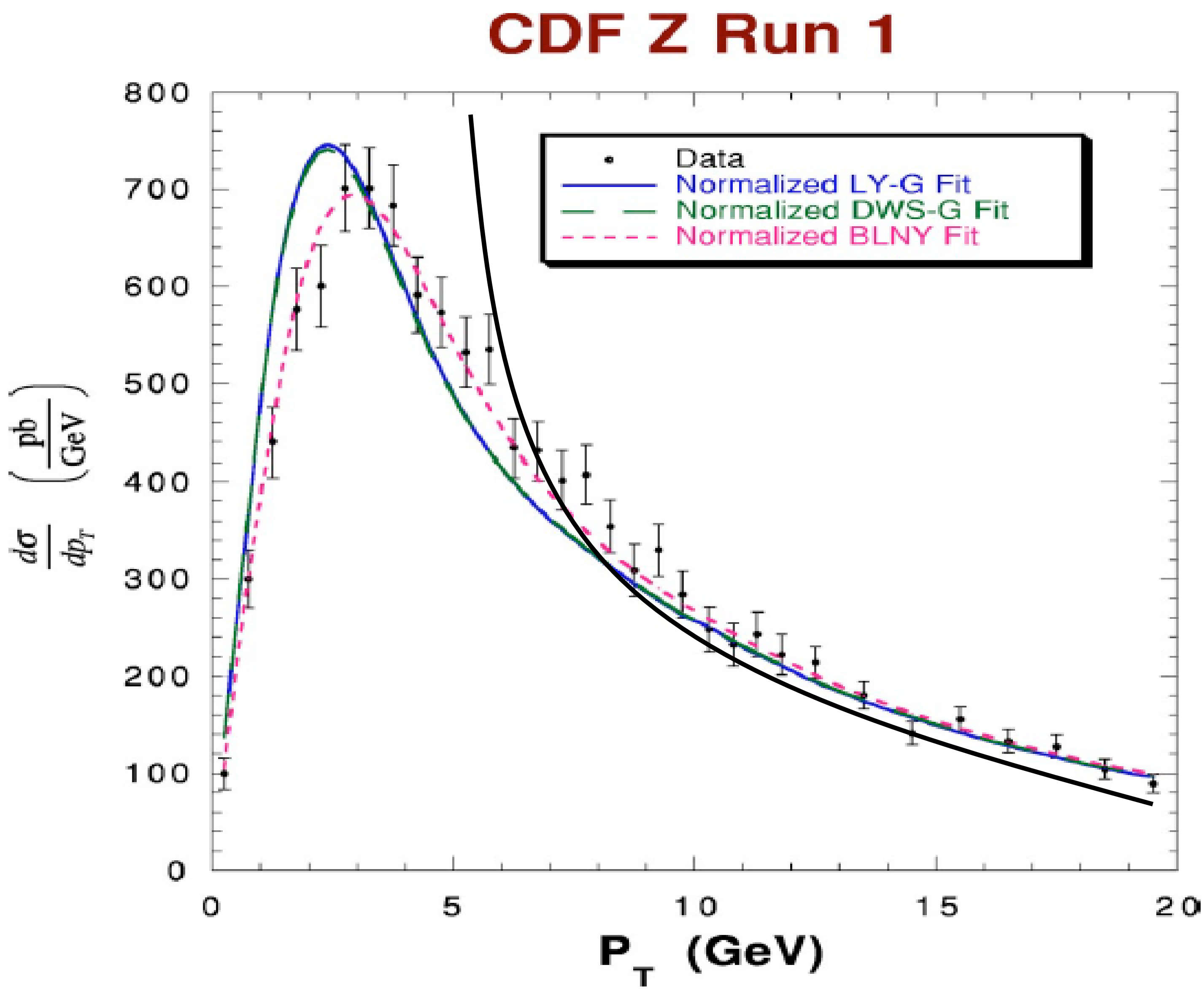

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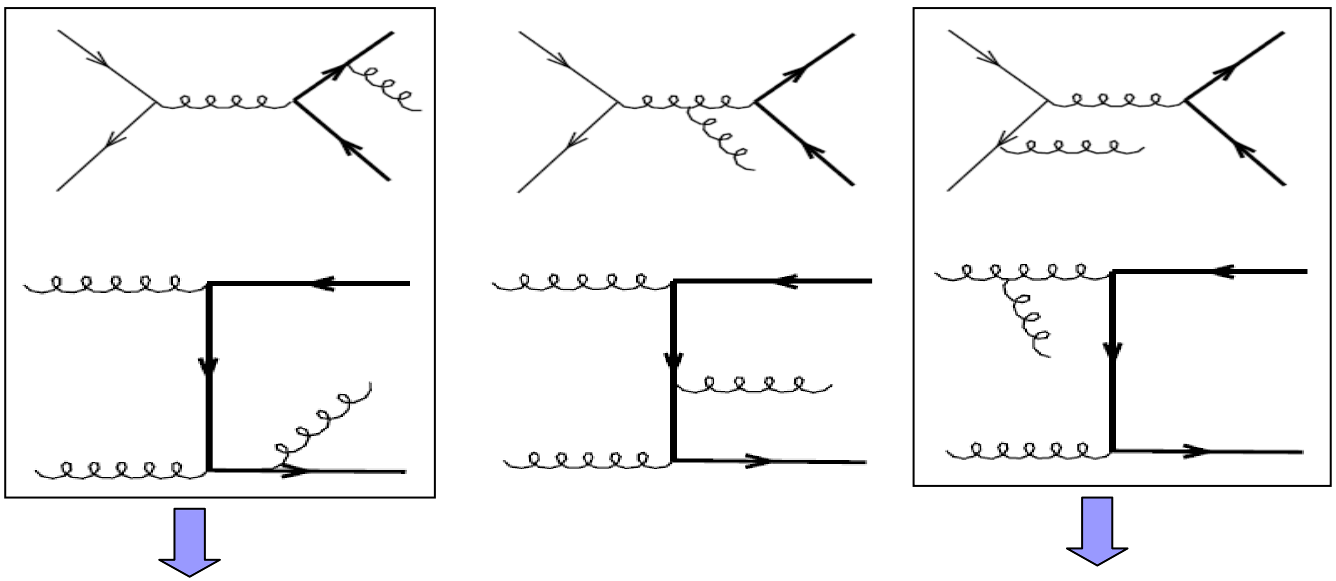
- The differential cross sections of ${}^3S_1^8$, ${}^1S_0^8$, ${}^3P_J^8{}_{(J=0 \text{ or } 2)}$ and $\chi_{(b,c)}$ production contain large logarithm at the NLO.

${}^3S_1^8$:

${}^1S_0^8$, ${}^3P_J^8{}_{(J=0 \text{ or } 2)}$ and $\chi_{(b,c)}$:



a soft divergence deduced by soft gluon radiated from final state.

a soft collinear divergence deduced by soft or collinear gluon radiated from initial state.

In general, for gluon-gluon fusion process:

$$W(b, M, x_1, x_2) = x_1 g(x_1) x_2 g(x_2) \Sigma_{IJ} H_{IJ} S_{IJ}$$

The $\boldsymbol{H_{IJ}}$ and $\boldsymbol{S_{IJ}}$ should be 3×3 matrix

$$c_1 = \delta^{ab}\delta_{ij} \ , \quad c_2 = i f^{abc} T^c_{ij} \ , \quad c_3 = d^{abc} T^c_{ij} \ ,$$

But in our case, for ${}^1S_0^8$, ${}^3P_J^8$ $_{(J=0 \text{ or } 2)}$,in the framework of NRQCD, we have

$$H_{IJ} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & H_{33} \end{pmatrix} \qquad \Gamma = \frac{\alpha_s}{\pi} \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\frac{C_A}{2} & 0 \\ 0 & 0 & -\frac{C_A}{2} \end{pmatrix}$$

$$\left(\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g}\right) S_{IJ} = -\Gamma^\dagger_{IB} S_{BJ} - S_{IA} \Gamma_{AJ} \ \longrightarrow \ \left(\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g}\right) S_{33} = -(\Gamma^\dagger_{33} + \Gamma_{33}) S_{33}$$

For $gg \to {}^1S_0^8, {}^3P_J^8$

$$W^{(1)}(x_i, b, M^2) = \frac{\alpha_s C_A}{\pi}\left\{\left[x_1 \mathcal{P}_{gg}(x_1)\delta(x_2-1)\left(-\frac{2}{\epsilon}-\gamma_E+\ln\frac{4}{4\pi\mu^2 b^2}\right)+(x_1\to x_2)\right]\right.$$
$$\left.+\,\delta(x_1-1)\delta(x_2-1)\left[\boxed{\underbrace{(b_0+\frac{1}{2})}_{only\ b_0\ in\ gg\to H}}\ln\frac{b^2M^2}{4}e^{2\gamma_E}\boxed{-\frac{1}{2}\ln^2\left(\frac{M^2b^2}{4}e^{2\gamma_E}\right)}\boxed{-\frac{\pi^2}{6}+\frac{B_Q^{[8]}}{C_A}}\right]\right\}. \tag{27}$$

$B^{(1)}$ $A^{(1)}$ $C^{(1)}$

Where

$$\mathcal{P}_{gg}(x) = \frac{x}{(1-x)_+}+\frac{1-x}{x}+x(1-x)+\delta(x-1)\frac{b_0}{2}, \tag{28}$$

For $q\bar{q} \to {}^3S_1^8$

$$W^{(1)}(x_i, b, M^2) = \frac{\alpha_s C_F}{\pi}\left\{\left[x_1 \mathcal{P}_{qq}(x_1)\delta(x_2-1)\left(-\frac{2}{\epsilon}-\gamma_E+\ln\frac{4}{4\pi\mu^2 b^2}\right)+(x_1\to x_2)\right]\right.$$
$$\left.+\,\delta(x_1-1)\delta(x_2-1)\left[\boxed{\underbrace{(\frac{3}{2}+\frac{9}{8})}_{only\ \frac{3}{2}\ in\ Drell-Yan}}\ln\frac{b^2M^2}{4}e^{2\gamma_E}\boxed{-\frac{1}{2}\ln^2\left(\frac{M^2b^2}{4}e^{2\gamma_E}\right)}\boxed{-\frac{\pi^2}{6}+\frac{A_Q^{[8]}}{C_F}}\right]\right\}. \tag{29}$$

$B^{(1)}$ $A^{(1)}$ $C^{(1)}$

Where

$$\mathcal{P}_{qq}(x) = \frac{1}{2}\frac{1+x^2}{(1-x)_+}+\delta(x-1)\frac{3}{4}, \tag{30}$$

- For color-octet state production by di-gluon fusion

$$A^{(1)} = C_A$$

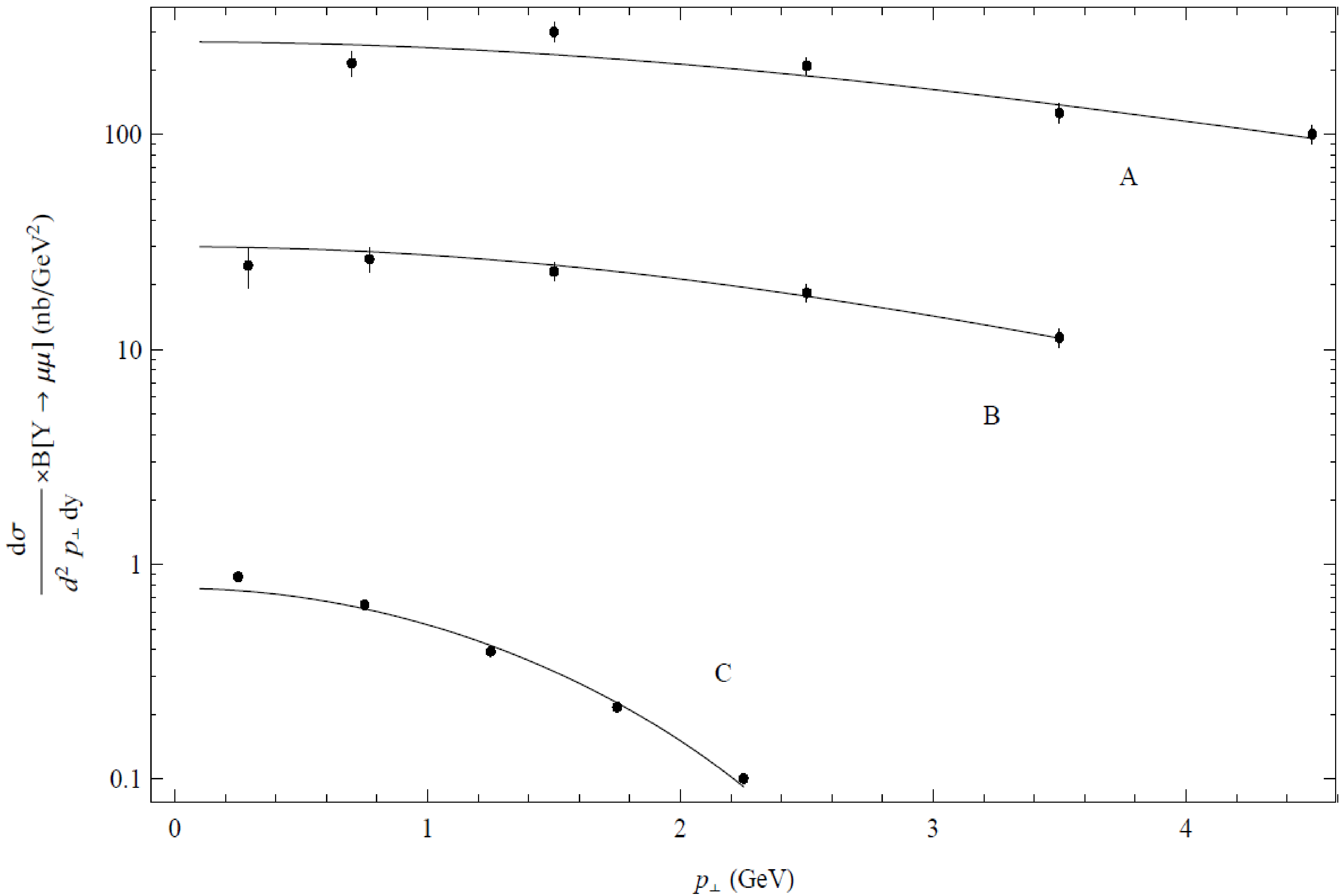
$$B^{(1)} = -(\beta_0+1/2)\, C_A$$

- For color-octet state production by quark-antiquark annihilation

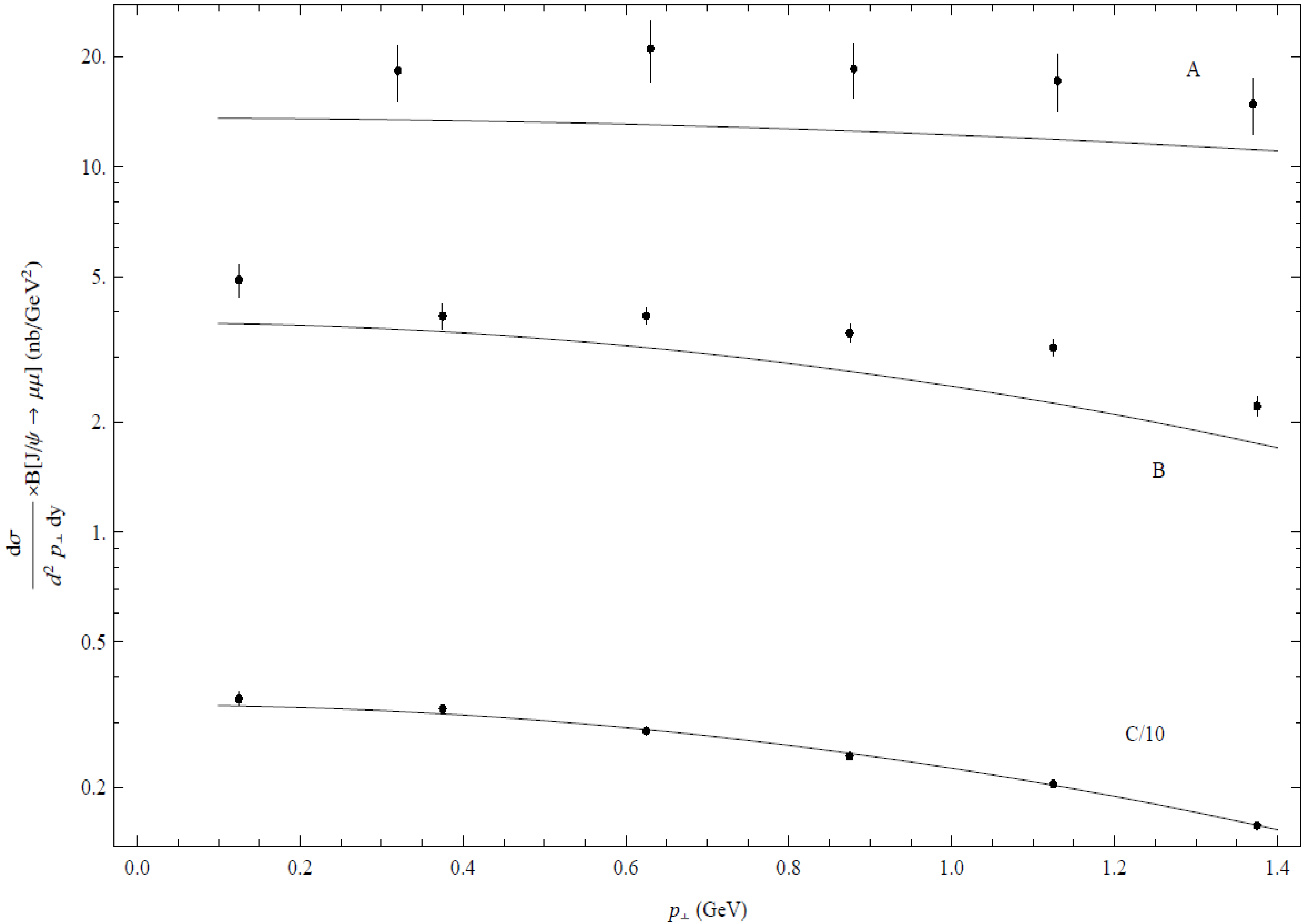
$$A^{(1)} = C_F$$

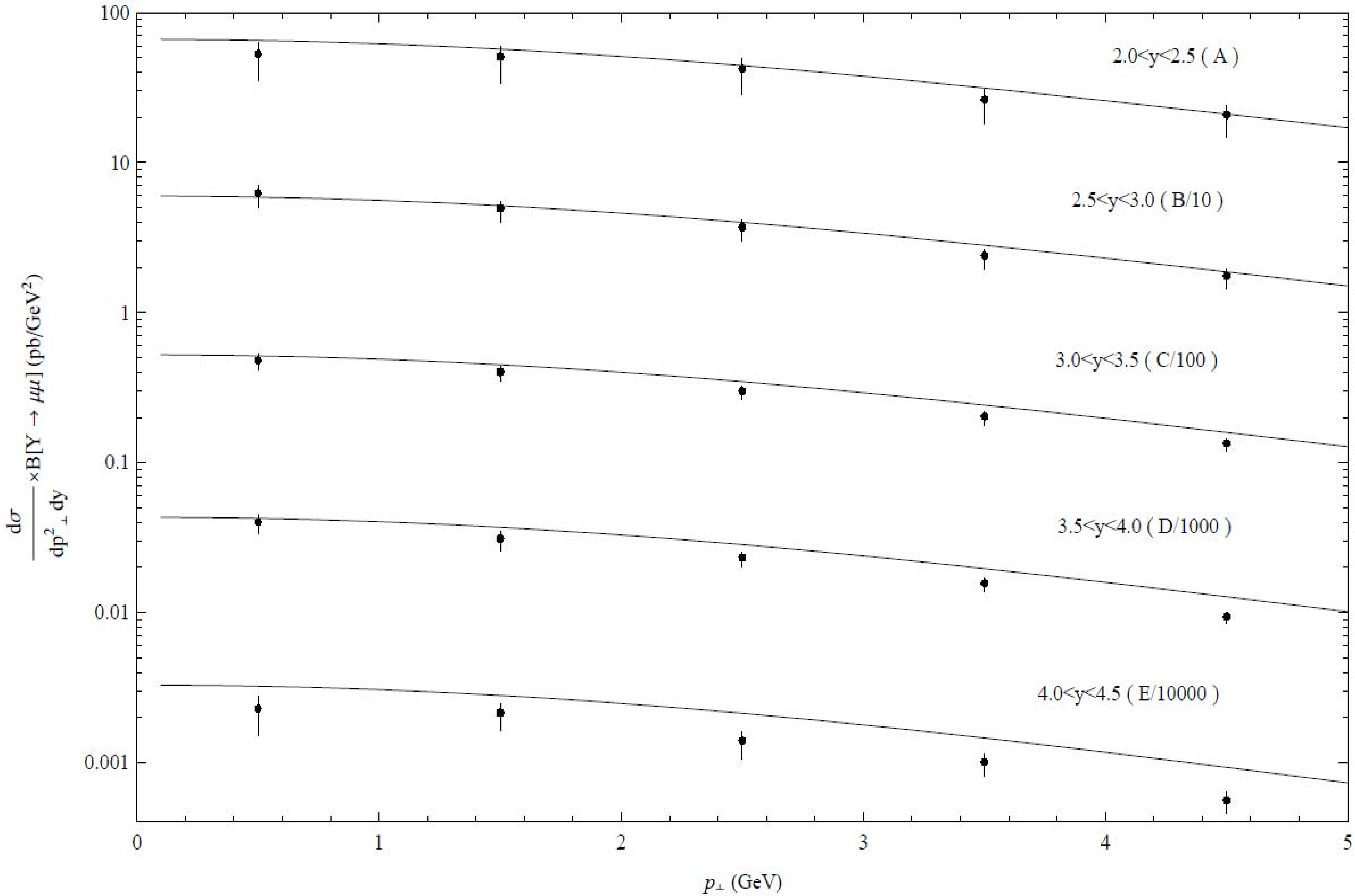
$$B^{(1)} = -(3/2 + 9/8)C_F$$

- For color-singlet state production, $A^{(1)}$ and $B^{(1)}$ are the same as those in Higgs production.



100
10
1
0.1
0
1
2
3
4
A
B
C
$\frac{d\sigma}{d^2 p_\perp dy} \times B[\Upsilon \to \mu\mu]$ (nb/GeV^2)
$p_\perp$ (GeV)


A
B
C/10
20.
10.
5.
2.
1.
0.5
0.2
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
$\frac{d\sigma}{d^2 p_\perp dy} \times B[J/\psi \to \mu\mu]$ (nb/GeV^2)
$p_\perp$ (GeV)



100
10
1
0.1
0.01
0.001
0
1
2
3
4
5
2.0<y<2.5 (A)
2.5<y<3.0 (B/10)
3.0<y<3.5 (C/100)
3.5<y<4.0 (D/1000)
4.0<y<4.5 (E/10000)
$\frac{d\sigma}{dp_\perp^2 dy} \times B[\Upsilon \to \mu\mu]$ (pb/GeV^2)
$p_\perp$ (GeV)

- The QCD resummation calculation can well simulate the behavior of heavy quarkonium hadronic production in the small transverse momentum region.
- In the small transverse momentum region, the non-perturbative Sudakov factor is important, which decides the shape of the curve.
- The nonperturbative Sudakov factor obtained by fitting to heavy quarkonium production in hadronic collisions.

- It may be used to predict other particle production, dominantly through di-gluon fusion, such as Higgs boson production at the LHC.

	BNLY	ours
g_1	$0.21(C_A/C_F)$ =0.4725	0.03
g_2	$0.68(C_A/C_F)$ =1.53	0.9
$g_1 * g_3$	-0.28	-0.17

- From our fit, we found:

$$\langle \mathcal{O}^{J/\psi}[^1S_0^8]\rangle + \frac{7}{m_c^2}\langle \mathcal{O}^{J/\psi}[^3P_0^8]\rangle = 0.0197 \pm 0.0009\,\mathrm{GeV}^3$$

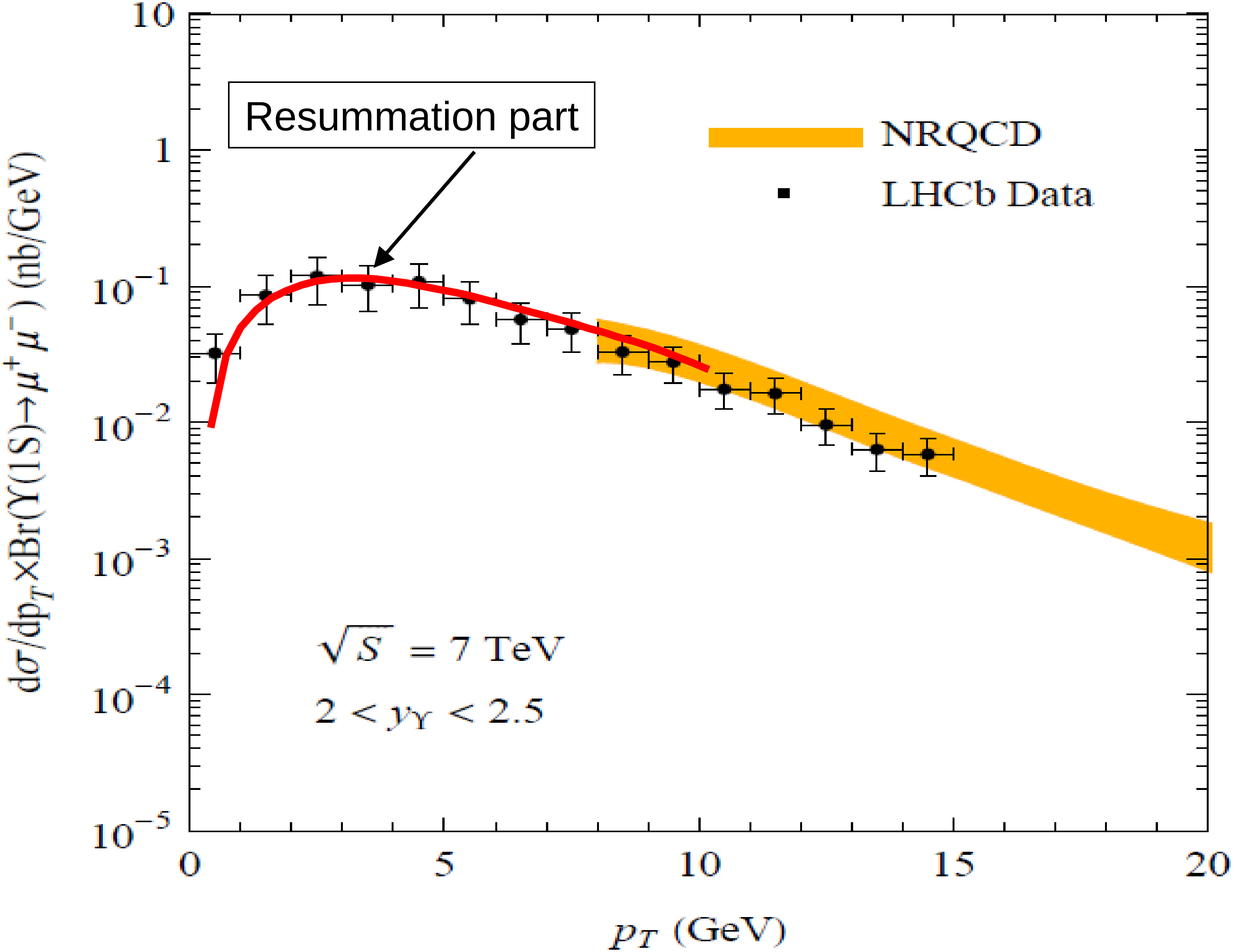
$$\langle \mathcal{O}^{\Upsilon}[^1S_0^8]\rangle + \frac{7}{m_b^2}\langle \mathcal{O}^{\Upsilon}[^3P_0^8]\rangle = 0.0321 \pm 0.0014\,\mathrm{GeV}^3$$

They agree well with those determined by comparing to the total cross section in photoproduction and hadroproduction.

- Combining results from the fixed order fits, we find:

	other 's	ours
$\langle O^{J/\psi}[^1S_0^8]\rangle$	0.050 GeV^3	0.14 GeV^3
$\langle O^{J/\psi}[^3S_1^8]\rangle$	0.022 GeV^3	-0.01 GeV^3
$\langle O^{J/\psi}[^3P_J^8]\rangle$	-0.016 GeV^5	-0.04 GeV^5

Other :M.Butenschoen and B.A.Kniehl, Phys.Rev. D84 (2011) 051501



Resummation part
NRQCD
LHCb Data
$\sqrt{S}$ = 7 TeV
$2 < y_\Upsilon < 2.5$
$d\sigma/dp_T \times Br(\Upsilon(1S) \to \mu^+\mu^-)$ (nb/GeV)
p_T (GeV)

Thank you very much!