α_s, m_c, and m_b from Quarkonium Correlators

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For the HPQCD Collaboration

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Introduction

- Since 2003 the HPQCD collaboration has been computing the properties of mesons containing charm and “bottom” quarks using improved staggered fermions.
- I will report progress on calculations with HISQ quarks for both the heavy and light quarks.

The talk will largely be based on results from

- Precision tests of the J/ψ from full lattice QCD: mass, leptonic width and radiative decay rate to η_c. Donald et al. (1208.2855).
- High-Precision c and b Masses, and QCD Coupling from Current-Current Correlators in Lattice and Continuum QCD (1004.4285)

Basic idea of lattice QCD

What needs to be done

The equations of lattice QCD are numerically solved in Euclidean space via a Monte Carlo process.

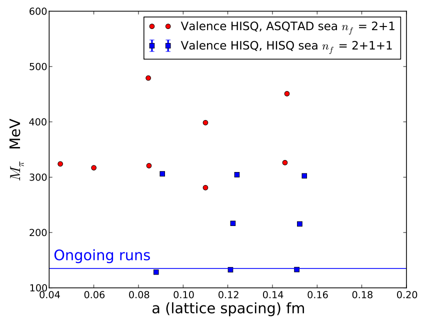
Supercomputer	Generate gauge configurations (“snapshots of the QCD vacuum”)
Supercomputer	Compute quark propagators.
Supercomputer	Combine quark propagators into meson correlators and average over spatial volume $c(t)$.
PC	Fit the meson correlators $c(t) \sim Ae^{-mt}$ to get masses m and amplitude A (from which decay constants).
PC	Repeat calculations for different quark masses and lattice spacings and take the continuum limit.

The HISQ action

- There are many different choices for the lattice version of the Dirac operatror.
- The HISQ (Highly Improved Staggered Action) action was designed to reduced lattice spacing dependence over previous improved staggered actions.
- The leading lattice spacing corrections to continuum for the HISQ action are $O(\alpha_s a^2)$.
- The next "best" fermion action has leading lattice spacing corrections to continuum of $O(a^2)$.
- For example at a = 0.09 fm, am_c = 0.4, the discretization errors in the decay constant of the η_c are $O(2\%)$ (1203.3862).

Parameters of lattice QCD calculation

- In 2001 MILC collaboration started generating gauge configs with ASQTAD staggered sea quarks (0903.3598).
- In 2008 MILC collaboration started generating gauge configurations with HISQ staggered sea quarks.



Moments method

$j_5 = \overline{\psi}_h \gamma_5 \psi_h$:

$$G(t) = a^6 \sum_{\mathbf{x}} (am_{0h})^2 \langle 0 | j_5(\mathbf{x}, t) j_5(0,0) | 0 \rangle$$

where m_{0h} is the heavy quark's bare mass.
The moments of $G(t)$ are simple to analyze:

$$G_n \equiv \sum_t (t/a)^n G(t)$$

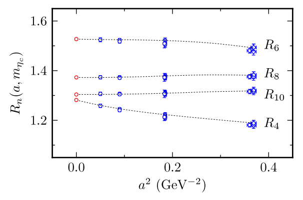
$$G_n = \frac{g_n(\alpha_{\overline{\mathrm{MS}}}(\mu), \mu/m_h)}{(am_h(\mu))^{n-4}} + \mathcal{O}((am_h)^m)$$

for small $n \geq 4$,

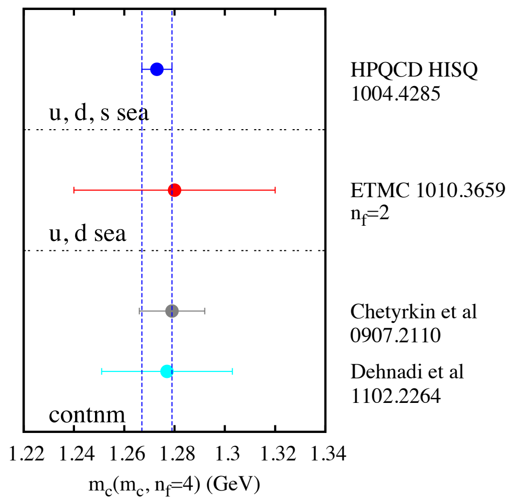
- $m_h(\mu)$ is the heavy quark's $\overline{\mathrm{MS}}$ mass at scale μ.
- The dimensionless factor g_n is computed using continuum perturbation theory known through α_s^3 for low moments (lattice perturbation theory is hard).
- Lattice spacing from r_1 tuned from f_π, Υ.

More moments

- First paper with unquenched QCD (0805.2999) HPQCD + Chetyrkin, Kuhn, Steinhauser,Sturm.
- In the second paper (1004.4285) HPQCD used heavier quarks $am_Q < 0.86$ to approach the mass of the bottom quark.
- Ratio further normalised by tree level lattice results.



Summary of m_c masses (1301.7202)



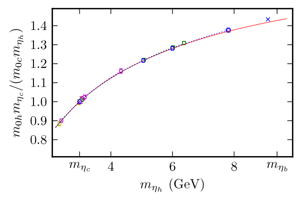
Cross-check on m_b/m_c

To reduce lattice spacing errors slightly extrapolate

$$\frac{(m_b m_{\eta_c})}{(m_c m_{\eta_b})}$$

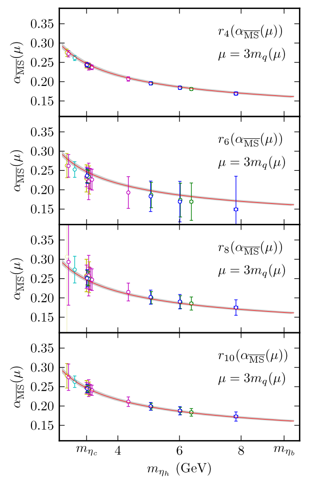
to the η_b mass. M_{η_b} = 9.395(5) Gev and M_{η_c} = 2.985(3) GeV (M_{η_b}/M_{η_c} = 3.15).

$$m_b/m_c \to 4.49(4)$$

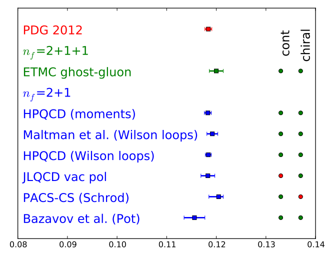


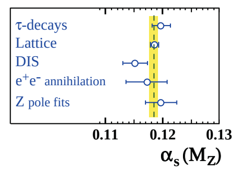
α_s from moments (1004.4285)

4th moment is insensitive to charm mass, hence good for α_s.



Summary of α_s from lattice QCD





(a) From 2012 PDG

α_s and lattice QCD

> *A value "from PDG" that summarizes these different results is*
>
> $$\alpha_s = 0.1185 \pm 0.0007.$$
>
> *With all due respect to lattice people I think this small error is totally umplausible* [2]*.*

The QCD Running Coupling and its Measurement Guido Altarelli (1303.6065).

- The PDG summary for α_s is stable on removing the lattice numbers.
- Altarelli's comments illustrates that it is important to continue to test lattice QCD against experiment (CLEO-C → BESIII...).

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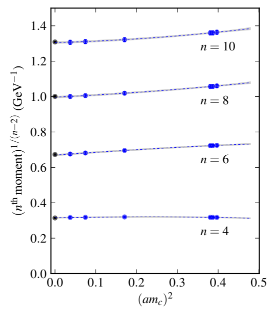
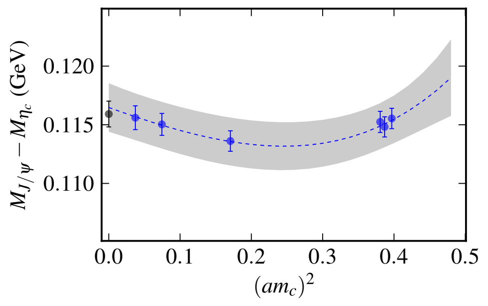
[2] There is no explanation for this statement

Lattice vs expt for moments (1208.2855)

- Compare the lattice results against expt. moments from Kühn et al. hep-ph/0702103
- $R(s) = \frac{\sigma(e+e- \rightarrow \text{hadrons})}{\sigma_{pth}}$ and $\mathcal{M}_n = \int \frac{ds}{s^{n+1}} R(s)$



Charmonium hyperfine splitting.



Donald et al., 1208.2855 connected diagrams only

$$M_{J/\psi} - M_{\eta_c} = 116.5(3.2) MeV$$

compared to the 2012 PDG summary value of 115.9(1.2) MeV.

Decay constant of J/ψ

$$\Gamma(v_h \to e^+e^-) = \frac{4\pi}{3}\alpha_{QED}^2 e_h^2 \frac{f_v^2}{m_v}$$

where e_h is the electric charge of the heavy quark in units of e (2/3 for c).

$$\langle 0|\overline{\psi}\gamma^i\psi|v\rangle = f_v m_v \epsilon^i$$

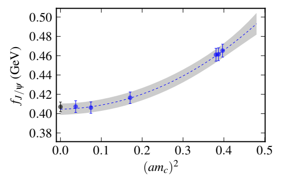
$$\frac{f_v}{Z_v} = a_0\sqrt{\frac{2}{M_0}},$$

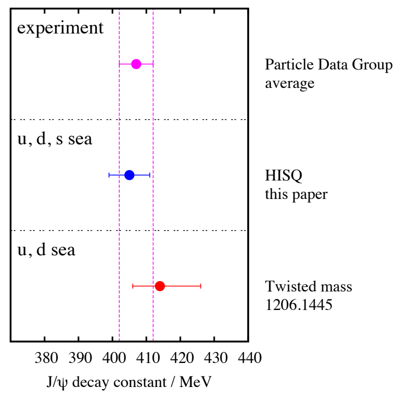
$$c(t) \sim a_0 e^{-M_0 t}$$

- Renormalization Z_v computed using a moments method.

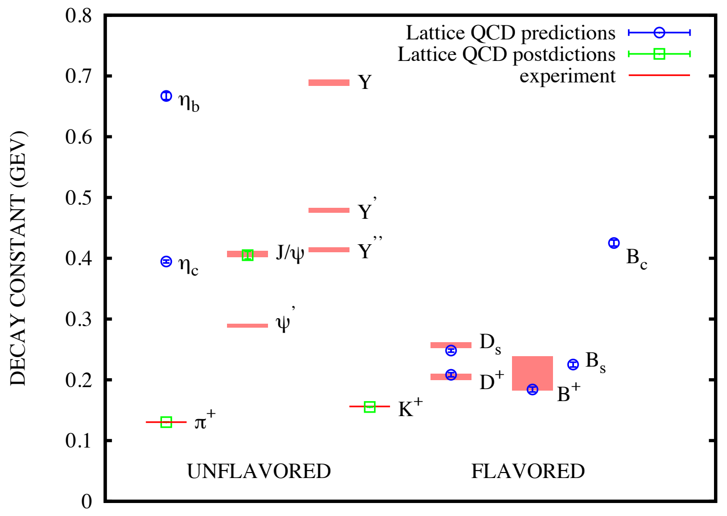
J/ψ decay constant

Donald et al., 1208.2855





Decay constants (1207.0994)

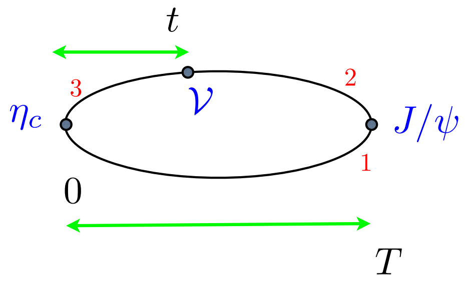


Radiative tranistion $J/\psi \to \eta_c\gamma$

Donald et al., 1208.2855

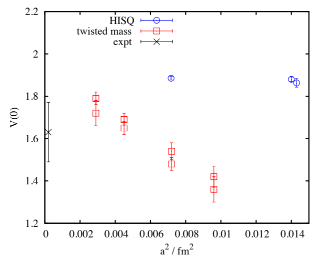
$$\langle \eta_c(p')|\bar{c}\gamma^\mu c|J/\psi(p)\rangle = \frac{2V(q^2)}{(M_{J/\psi}+M_{\eta_c})}\varepsilon^{\mu\alpha\beta\gamma}p'_\alpha p_\beta \epsilon_{J/\psi,\gamma}$$

$$\Gamma(J/\psi \to \eta_c\gamma) = \alpha_{QED}\frac{64|\vec{q}|^3}{27(M_{\eta_c}+M_{J/\psi})^2}|V(0)|^2,$$

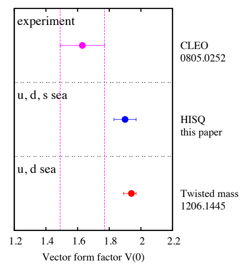


Radiative tranistion $J/\psi \to \eta_c\gamma$

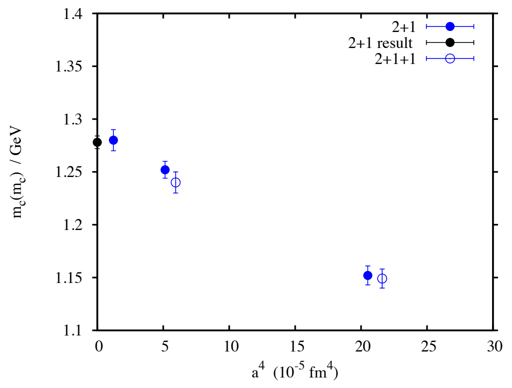
Donald et al., 1208.2855 Signal seen for radiative decays: $h_c \to \eta_c$ and $\chi_0 \to J/\psi$.



(b) Continuum extrap.

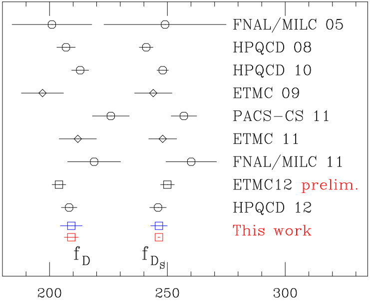


Preliminary results from HPQCD $m_c(m_c)$ from 2+1+1



Decay constants from 2+1+1

Preliminary results (1210.8431) from the MILC and Fermilab lattice collaboration.



Conclusions

- The HPQCD collaboration has computed m_c, α_s and m_b.
- To further validate the calculation HPQCD has computed $M_{J/\psi}$ - M_{η_c}, $f_{J/\psi}$ and $\Gamma(J/\psi \to \eta_c \gamma)$.

The future

- The first preliminary results with 2+1+1 sea quarks are consistent with those with 2+1 sea quarks.
- The new 2+1+1 lattice QCD calculations include “physical pion masses”.
- Updated results for m_c and α_s require finer lattice spacings. These runs are ongoing.

Backup

Backup

Summary of lattice methods to compute α_s

Method	scale	range GeV	pert.	non-perturb
Wilson loops	Υ f_π	2.1 - 14.7	α_s^3	$a^4\langle\alpha_s G^2/\pi\rangle$
Charm moments	Υ f_π	3	α_s^3	$a^4\langle\alpha_s G^2/\pi\rangle$
Light vaccum pol	r_0 f_π Ω	1.8	α_s^3	$\overline{\psi}\psi$, $\frac{\langle\bar{s}s\rangle}{\langle\bar{u}u\rangle}$,$a^4\langle\alpha_s G$
Static energy	r_1 f_π	0.8 - 2.9	N^3LL	renormalons
Schrödinger funct.	Ω	16	α_s^3	-
Glue/ghost	f_π	1.7 - 6.8	4 loop	$\frac{d}{p^6}$

Table: Summary of lattice methods to compute α_s