

α_s from the QCD static energy on the lattice

Xavier Garcia i Tormo

Universität Bern

(work done with Alexei Bazavov, Nora Brambilla, Péter Petreczky,
Joan Soto and Antonio Vairo)

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Introduction

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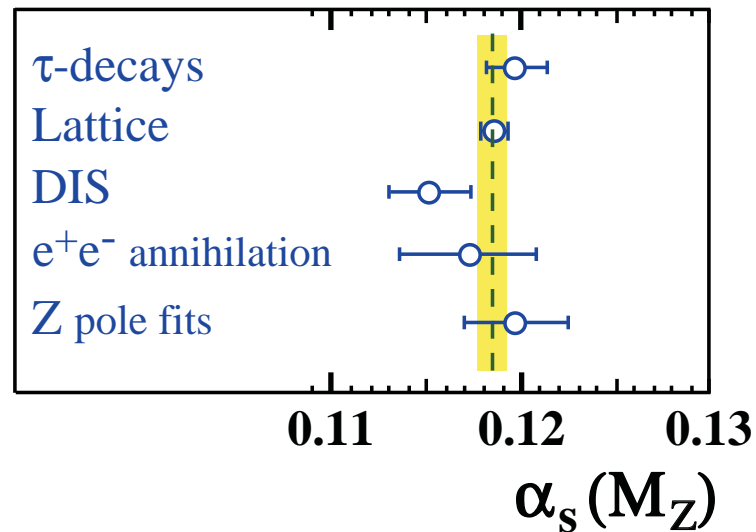
Reminder: current status of α_s world average

Introduction

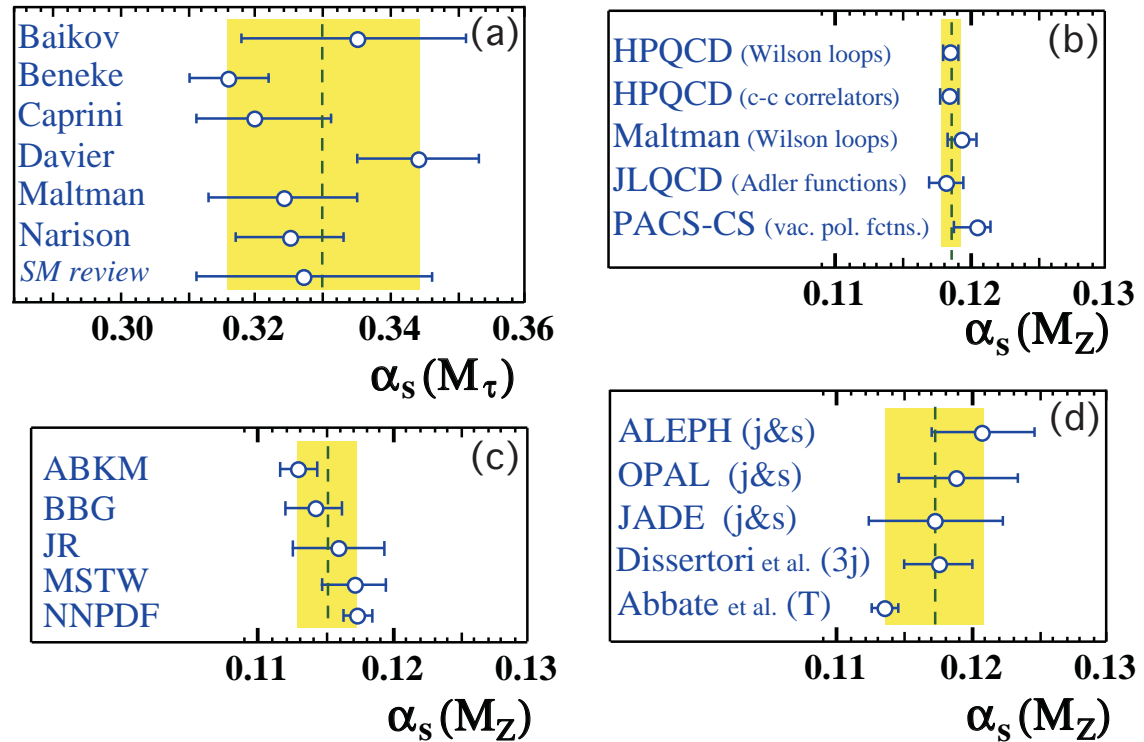
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Reminder: current status of α_s world average

$$\alpha_s(M_Z) = 0.1184 \pm 0.0007$$

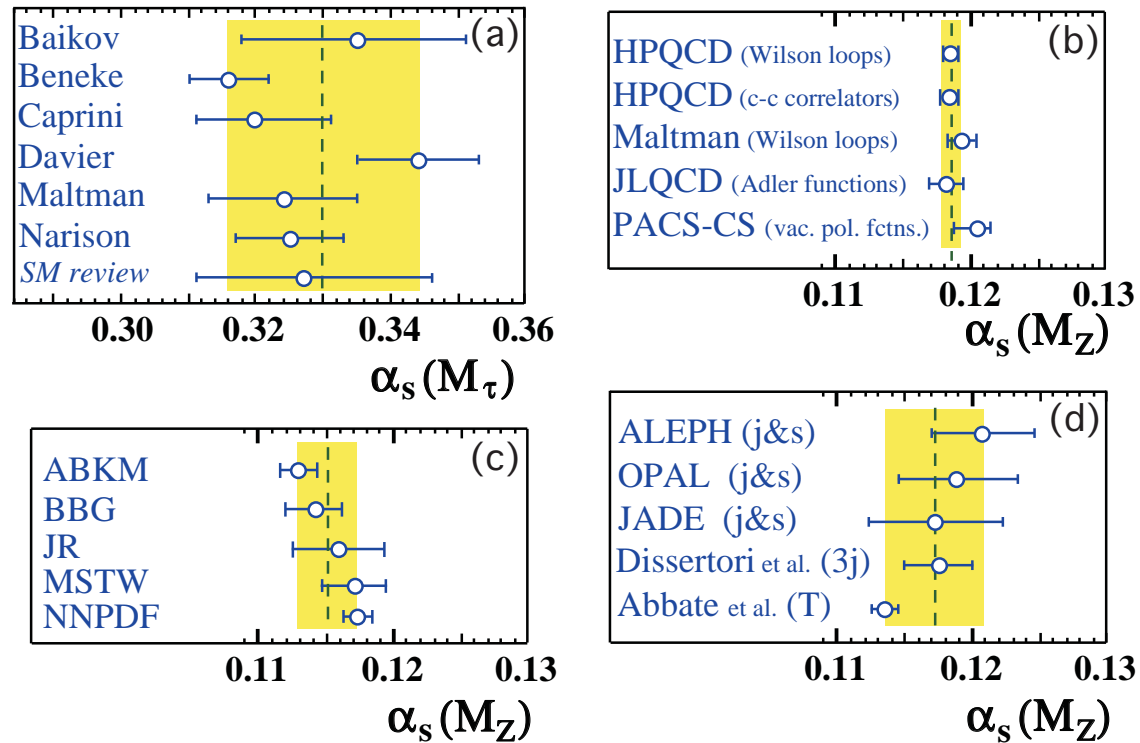


Picture from PDG'12: J. Beringer et al. (Particle Data Group), Phys. Rev. D86, 010001 (2012)



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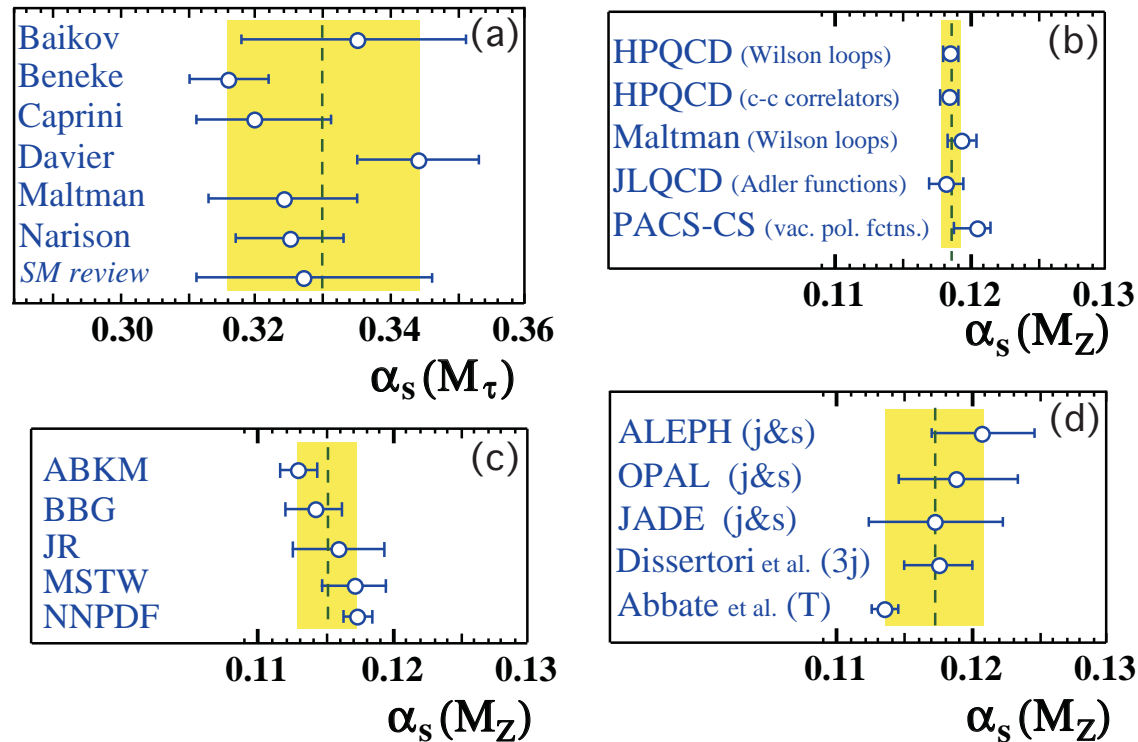
(a) τ decays (b) lattice (c) PDFs (d) e^+e^- jets



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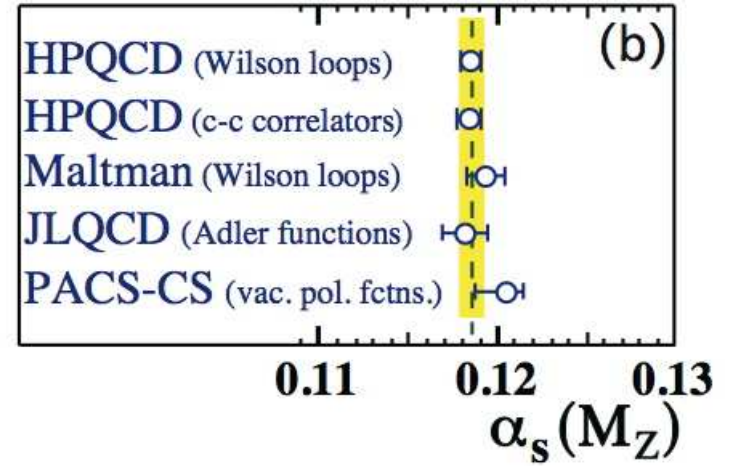
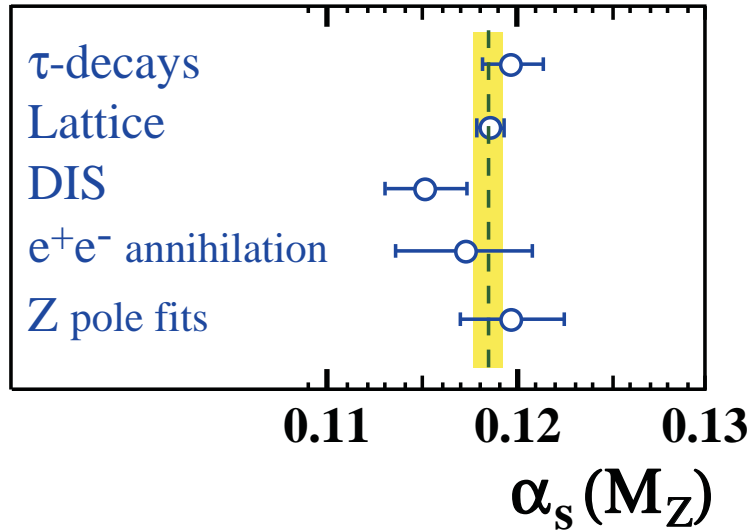
- Many measurements dominated by systematic uncertainties, and many correlated inputs



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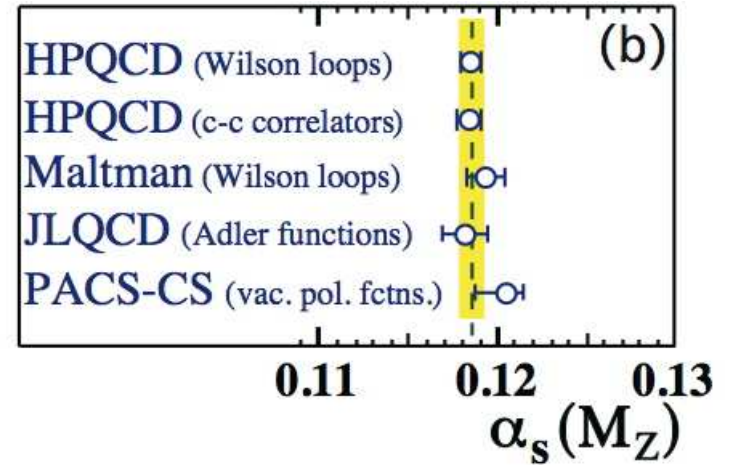
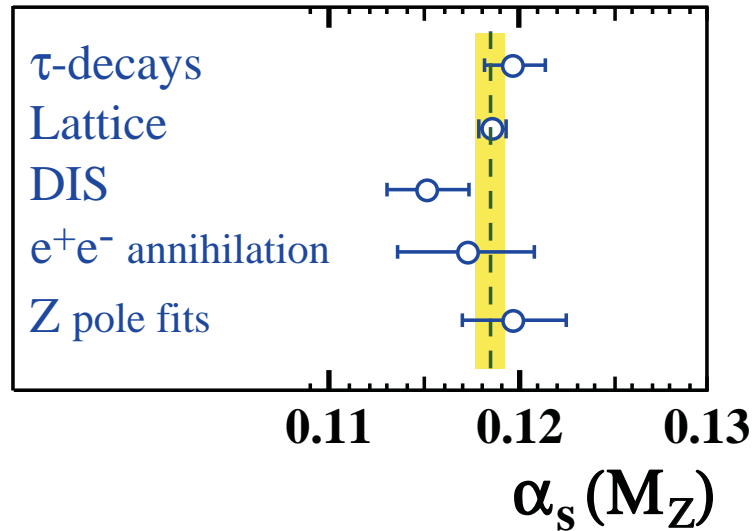
(a) τ decays (b) lattice (c) PDFs (d) e^+e^- jets

- Many measurements dominated by systematic uncertainties, and many correlated inputs
- Some recent precise determinations noticeably different than the average



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Overall uncertainty largely determined by the precise lattice result

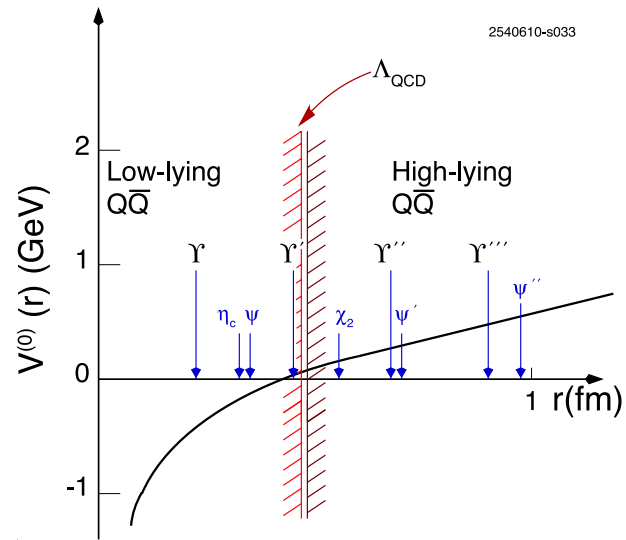


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Overall uncertainty largely determined by the precise lattice result
 We provide a new, independent determination from the static energy on the lattice

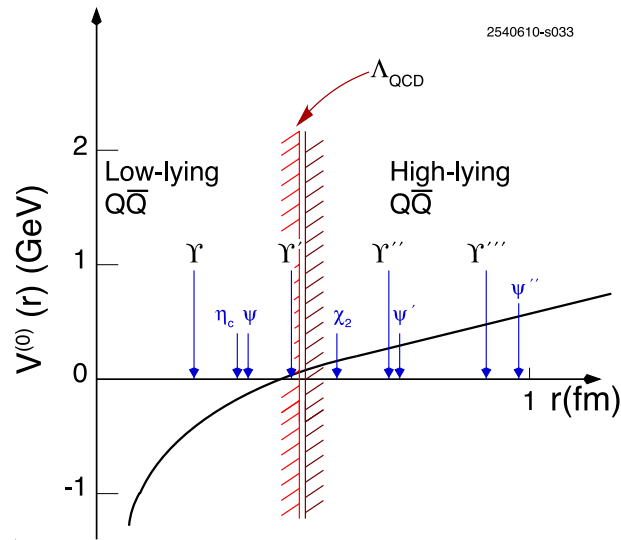
Static energy in QCD

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From N. Brambilla *et al.*, Eur. Phys. J. **C71** (2011) 1534

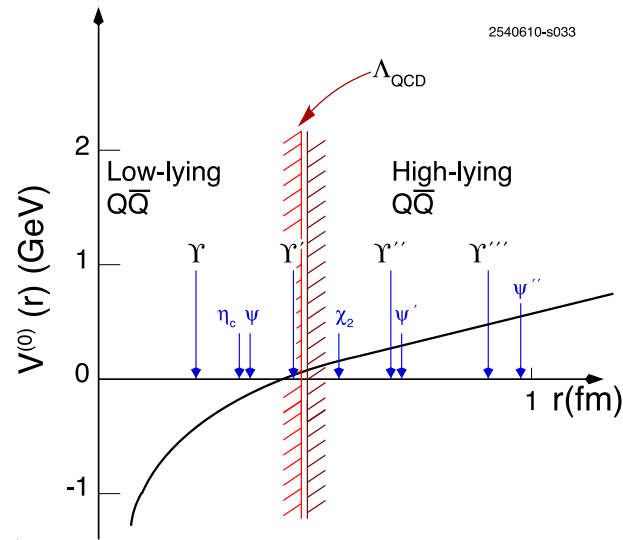
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Short-distance part \longleftrightarrow Long-distance part

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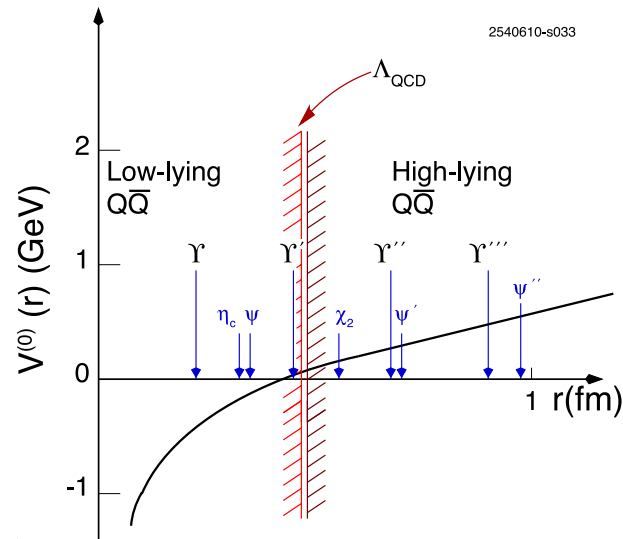


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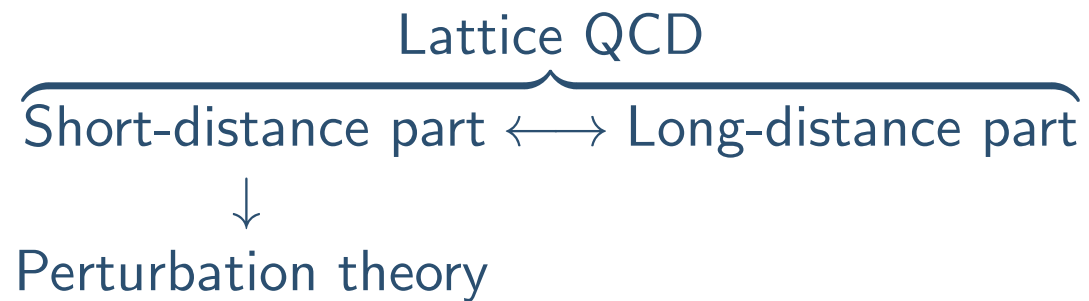
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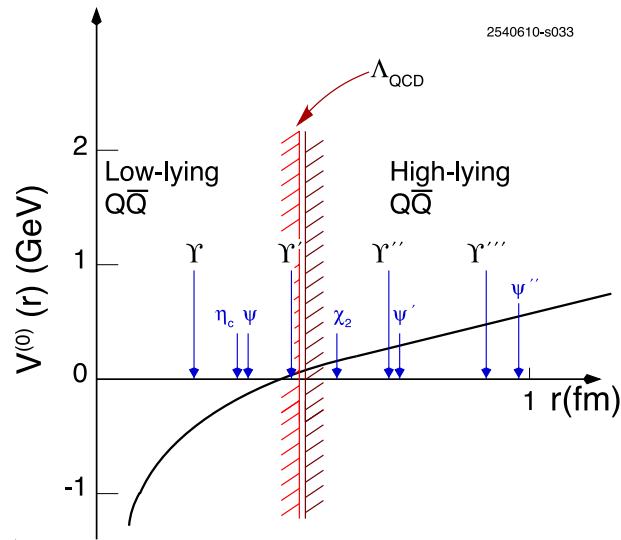
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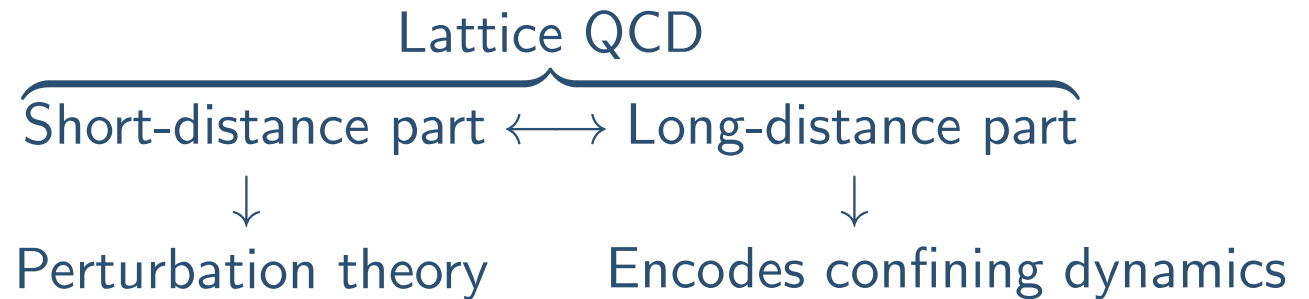
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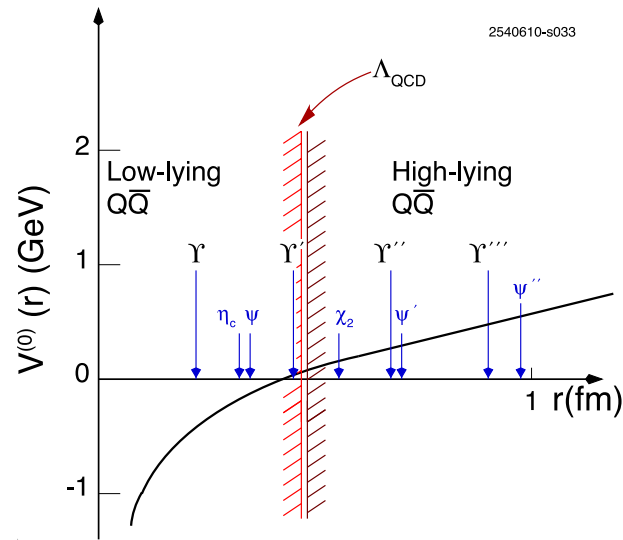
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Short-distance part \longleftrightarrow Long-distance part



Perturbation theory



Encodes confining dynamics

Lattice data

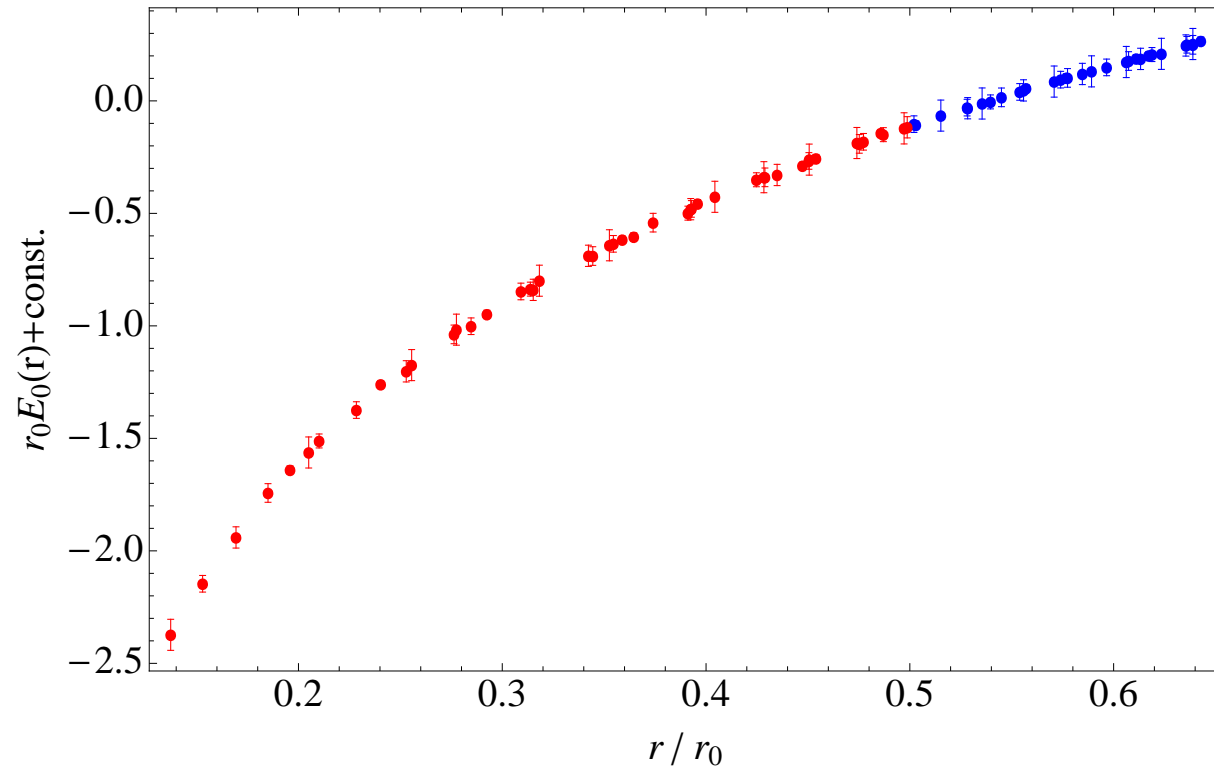
$E_0(r)$ recently calculated on the lattice in 2+1 flavor QCD

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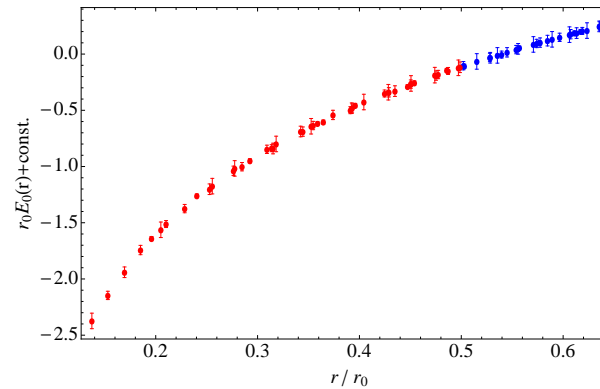
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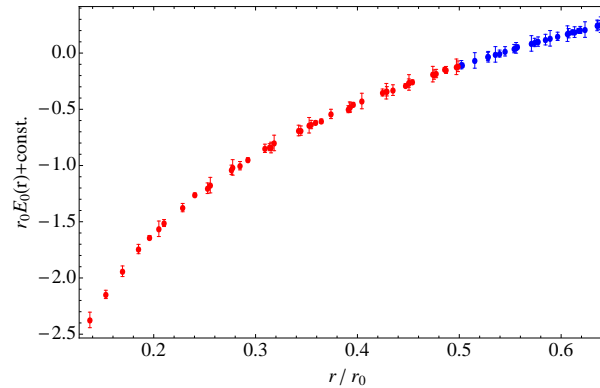


Combination of tree-level improved gauge action and HISQ action

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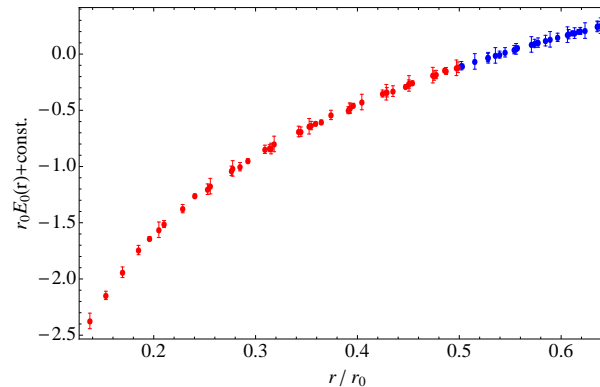
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Energy calculated in units of r_0 Sommer'93

$$r^2 \frac{dE_0(r)}{dr} \Big|_{r=r_0} = 1.65$$

Calculation for a wide range of gauge couplings.

($\beta = 6.664, 6.740, 6.800, 6.880, 6.950, 7.030, 7.150, 7.280$;

corresponds to lattice spacings $3.994/r_0 \leq a^{-1} \leq 6.991/r_0$)

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- Replace r by improved distance $r_I = (4\pi C_L(r))^{-1}$

Necco Sommer'01

$$C_L(r) = \int \frac{d^3k}{(2\pi)^3} D_{00}(k_0 = 0, \vec{k}) e^{i\vec{k}\vec{r}}$$

(D_{00} is the tree-level gluon propagator)

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Aubin *et al.*'04, Booth *et al.*'92

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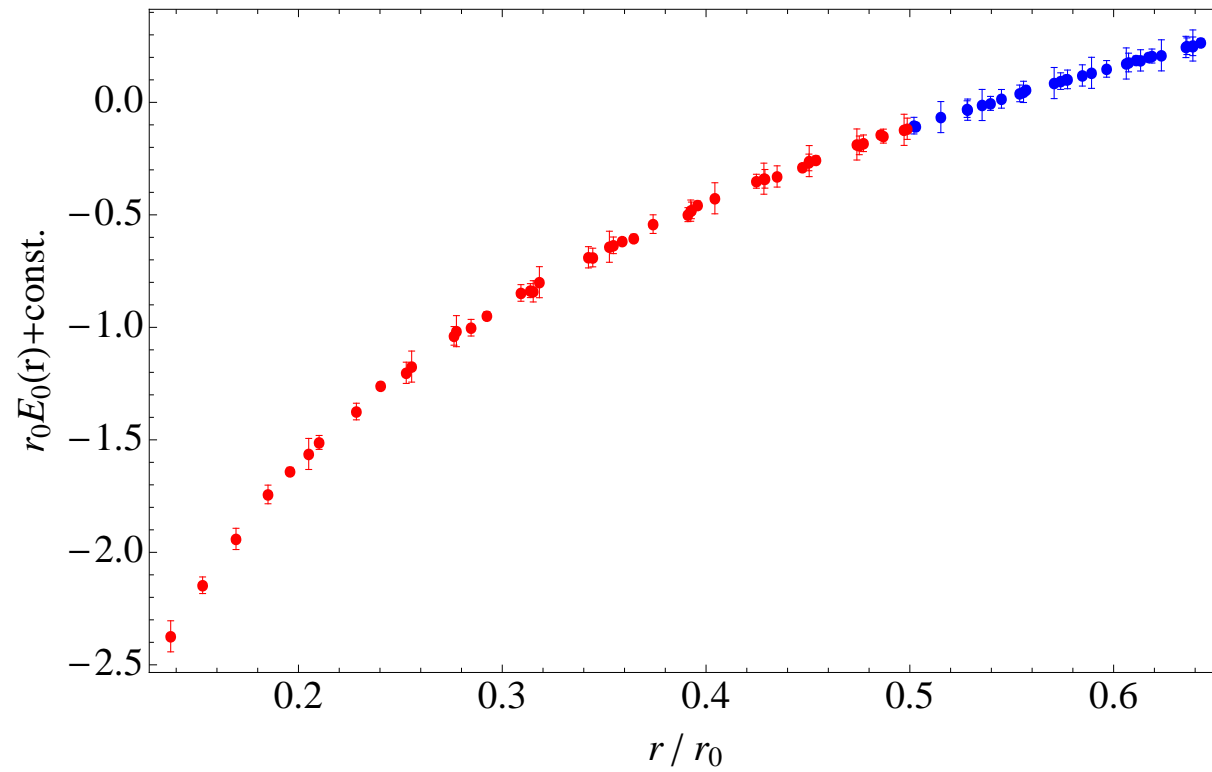
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Both methods lead to the same results, within errors



Static energy in perturbation theory

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$$E_0^{\text{N}^3\text{LO}}(r) = -\frac{C_F \alpha_s(1/r)}{r} \left\{ 1 + 17.4 \frac{\alpha_s(1/r)}{4\pi} + 863 \left(\frac{\alpha_s(1/r)}{4\pi} \right)^2 + \left[1421 \ln \frac{3\alpha_s(1/r)}{2} + 53945 \right] \left(\frac{\alpha_s(1/r)}{4\pi} \right)^3 \right\}$$

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E_0 known at 3 loop +sub-leading ultrasoft log res. (N^3LL)

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Using $r_0 = 0.468 \pm 0.004$ fm Bazavov *et al.* (HotQCD Coll.)'11 obtain
 $r_0 \Lambda_{\overline{\text{MS}}} \rightarrow \Lambda_{\overline{\text{MS}}}$

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Agreement with lattice improves when perturbative order is increased

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In that ϱ range, use reduced χ^2 as weight of fit values $r_0\Lambda_{\overline{\text{MS}}}$ and take the average. This gives our central value for $r_0\Lambda_{\overline{\text{MS}}}$

Accuracy	$r_0 \Lambda_{\overline{\text{MS}}}$
tree level	0.395
1 loop	0.848
2 loop	0.636
N ² LL	0.756
3 loop	0.690
3 loop + lead. us. res.	0.702

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N³⁽²⁾LL accuracy: $\alpha_s^{1+[3(2)+n]} \ln^n \alpha_s$ with $n \geq 0$

N³LL (3loop +sub-lead. us. res.) also known, but depends on additional constant $K_2 \sim \Lambda_{\overline{\text{MS}}}$ (to be fit to the data).

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Take 3 loop + lead. us. res. as our best result

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Cross-check: redo full analysis with energy normalized in units of r_1

$$r^2 \frac{dE_0(r)}{dr} \Big|_{r=r_1} = 1$$

Gives compatible results

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Cross-check: redo full analysis with energy normalized in units of r_1

$$r^2 \frac{dE_0(r)}{dr} \Big|_{r=r_1} = 1$$

Gives compatible results (comes from the same lattice data set in terms of r/a . But error analysis for the normalization of the energy for each lattice spacing is different in the two cases)

Final result:

$$r_0 \Lambda_{\overline{\text{MS}}} = 0.70 \pm 0.07,$$

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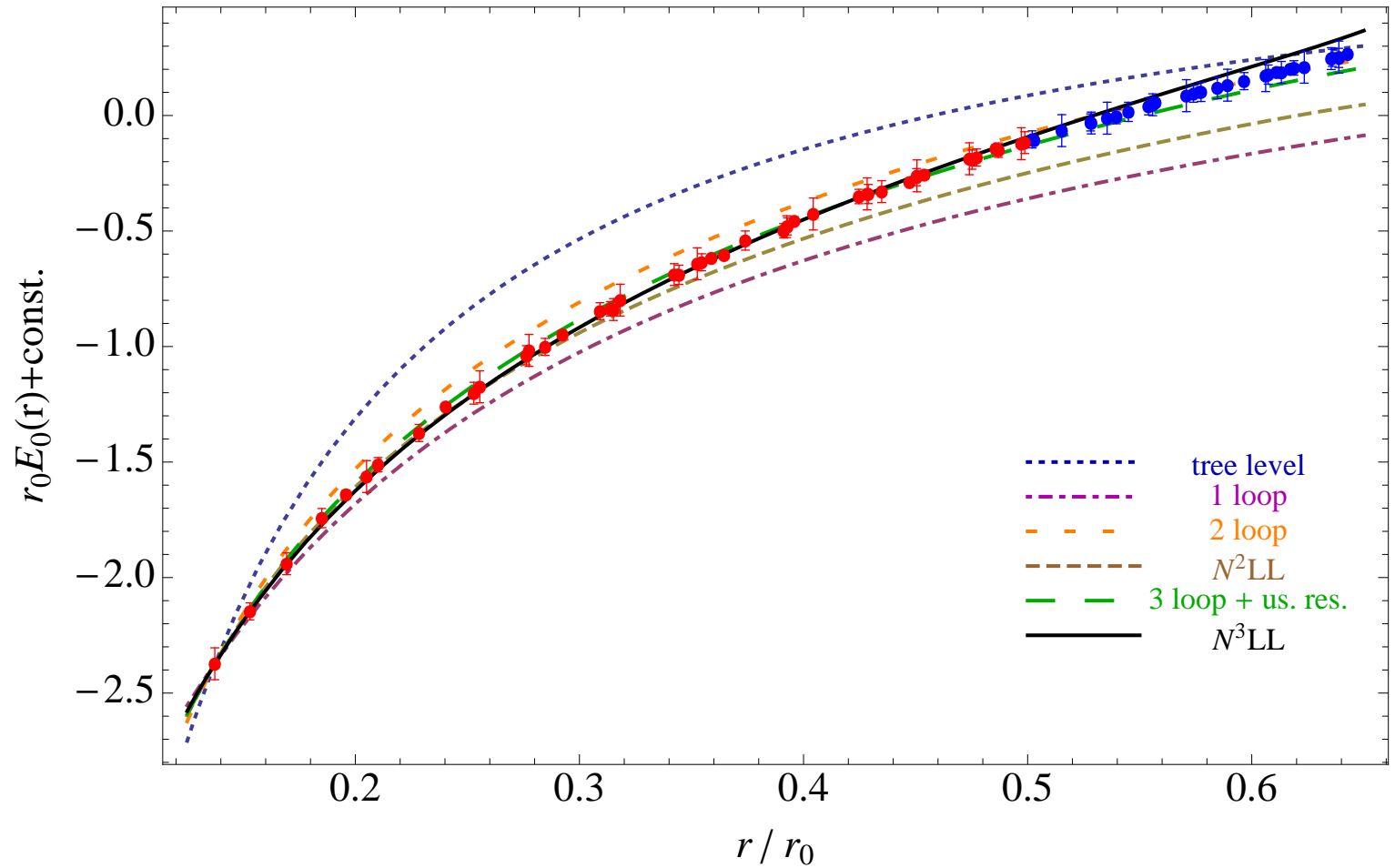
which corresponds to

$$\alpha_s(1.5\text{GeV}, n_f = 3) = 0.326 \pm 0.019$$

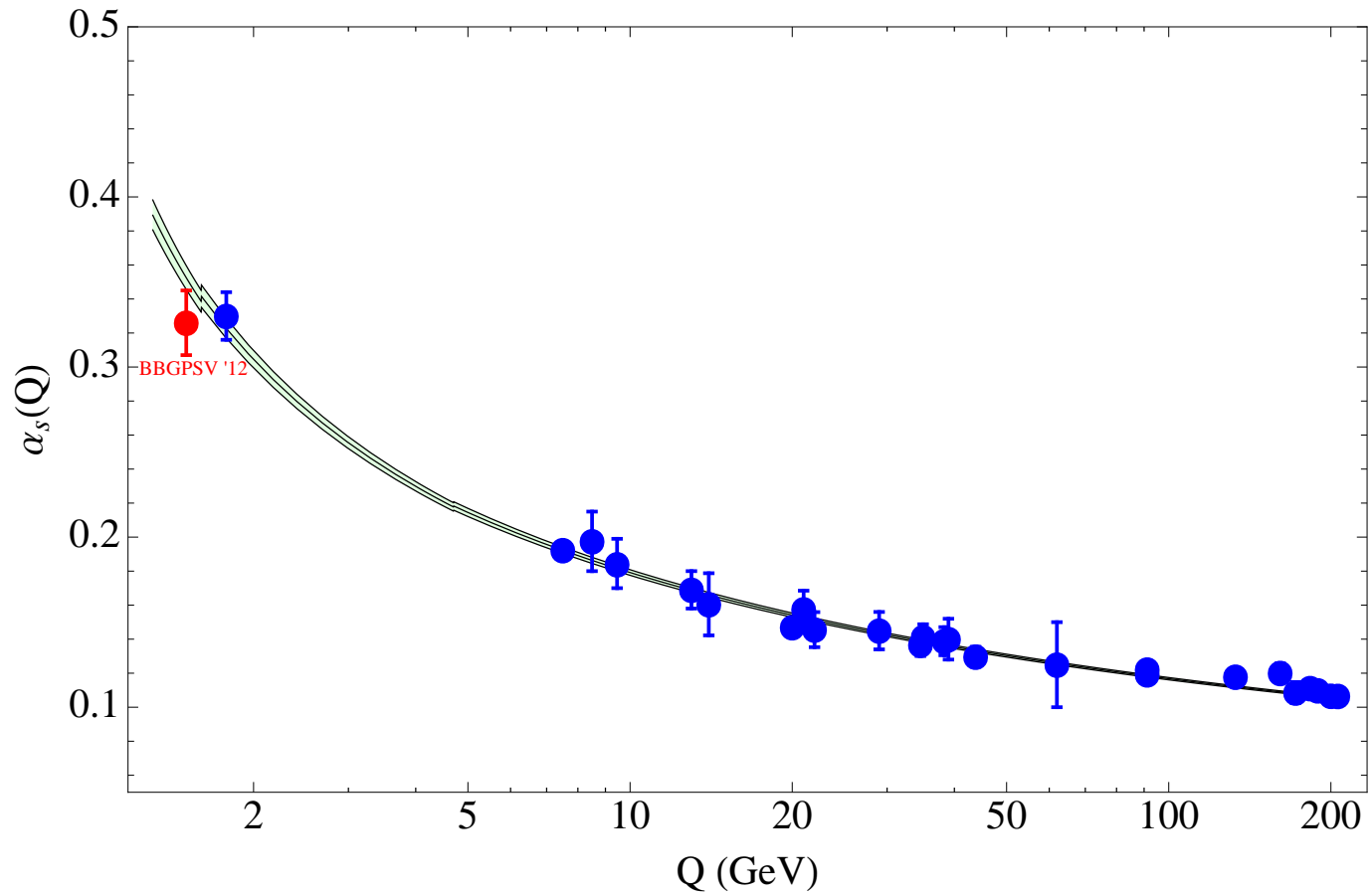
$$\rightarrow \alpha_s(M_Z, n_f = 5) = 0.1156^{+0.0021}_{-0.0022}$$

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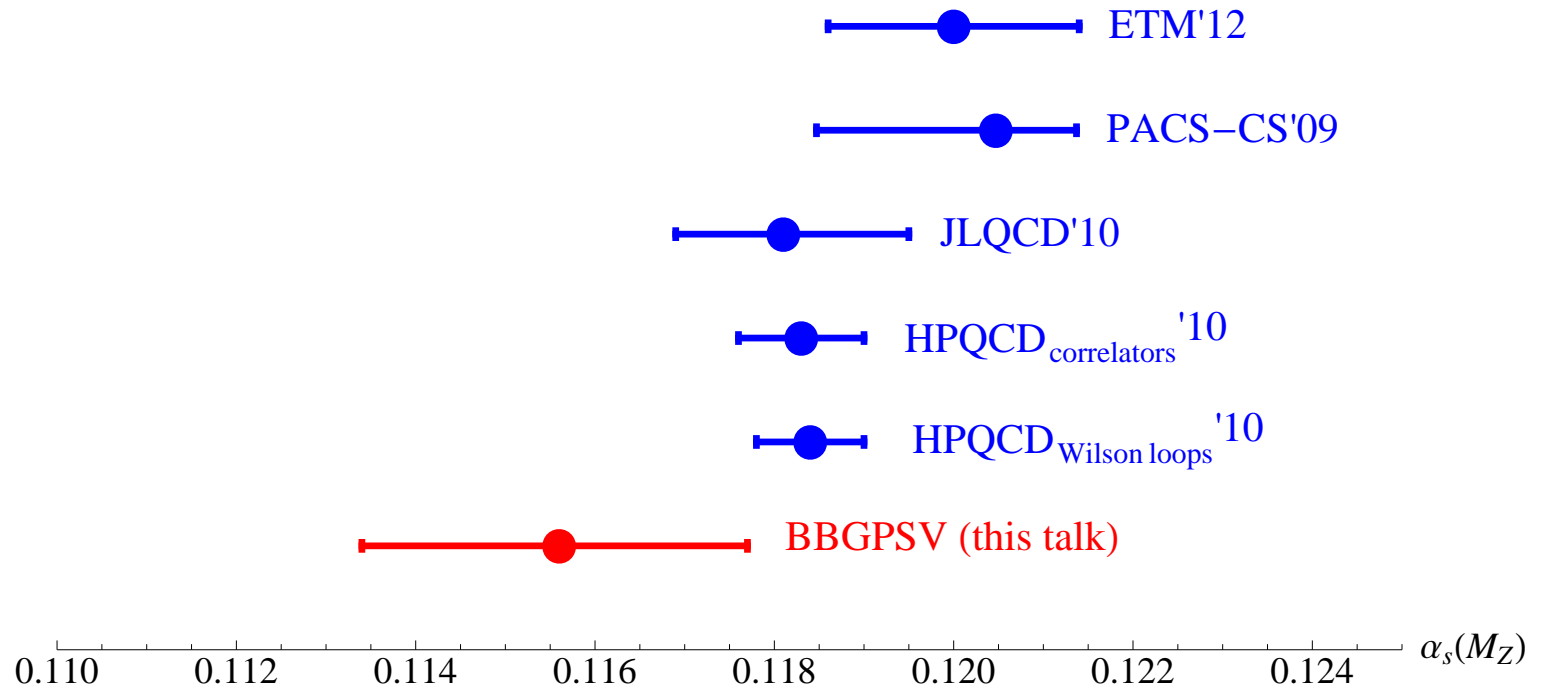


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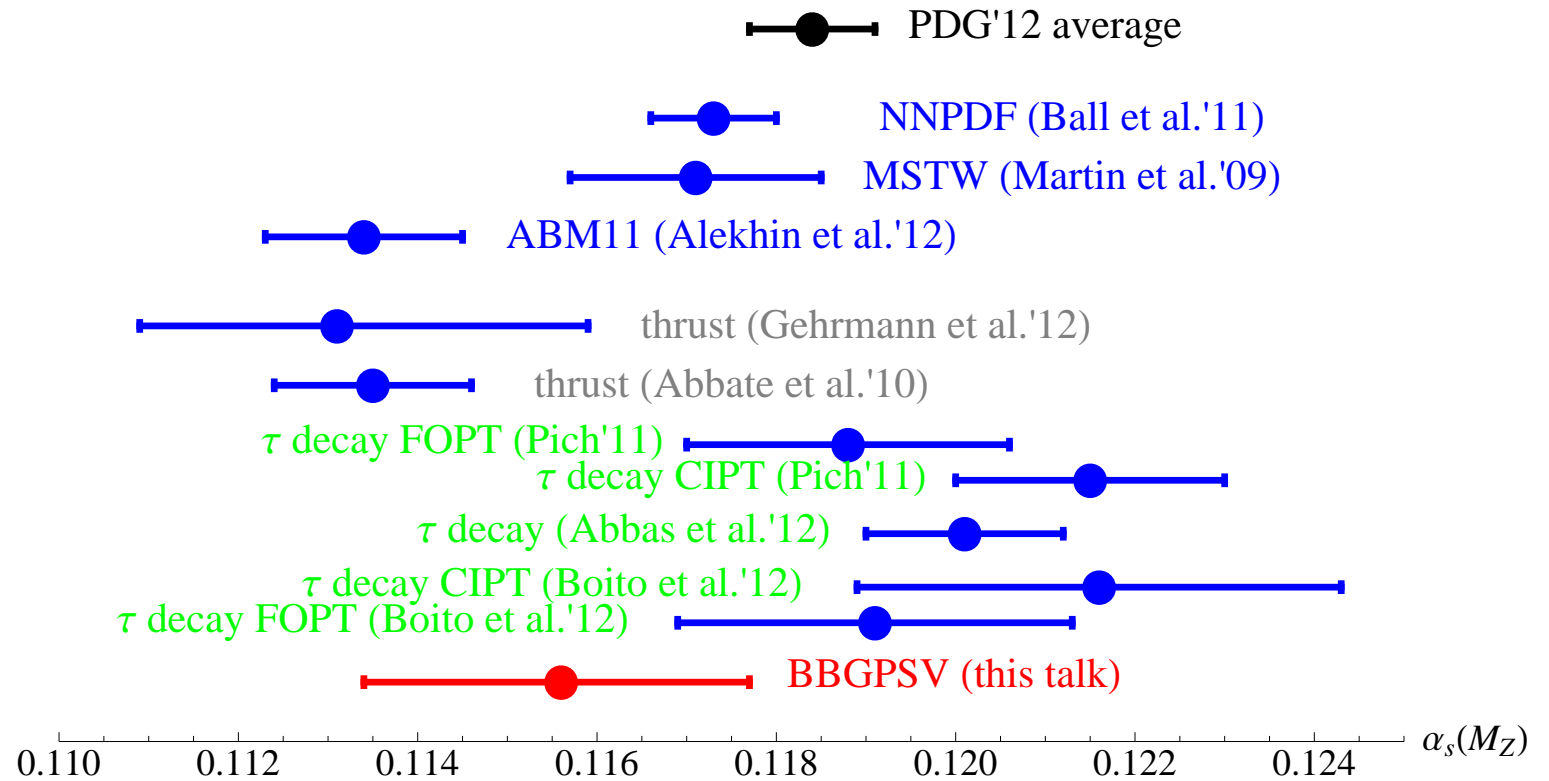


Determinations entering the world average Bethke'09 **This result**

Comparison with other recent lattice determinations



Comparison with a few recent determinations using other techniques



Conclusions

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- Updated analysis (in the near future), also including lattice data at shorter distances. Reduced error for our α_s extraction is conceivable