

Y(4008)

Yuan Changzheng (苑 长 征)

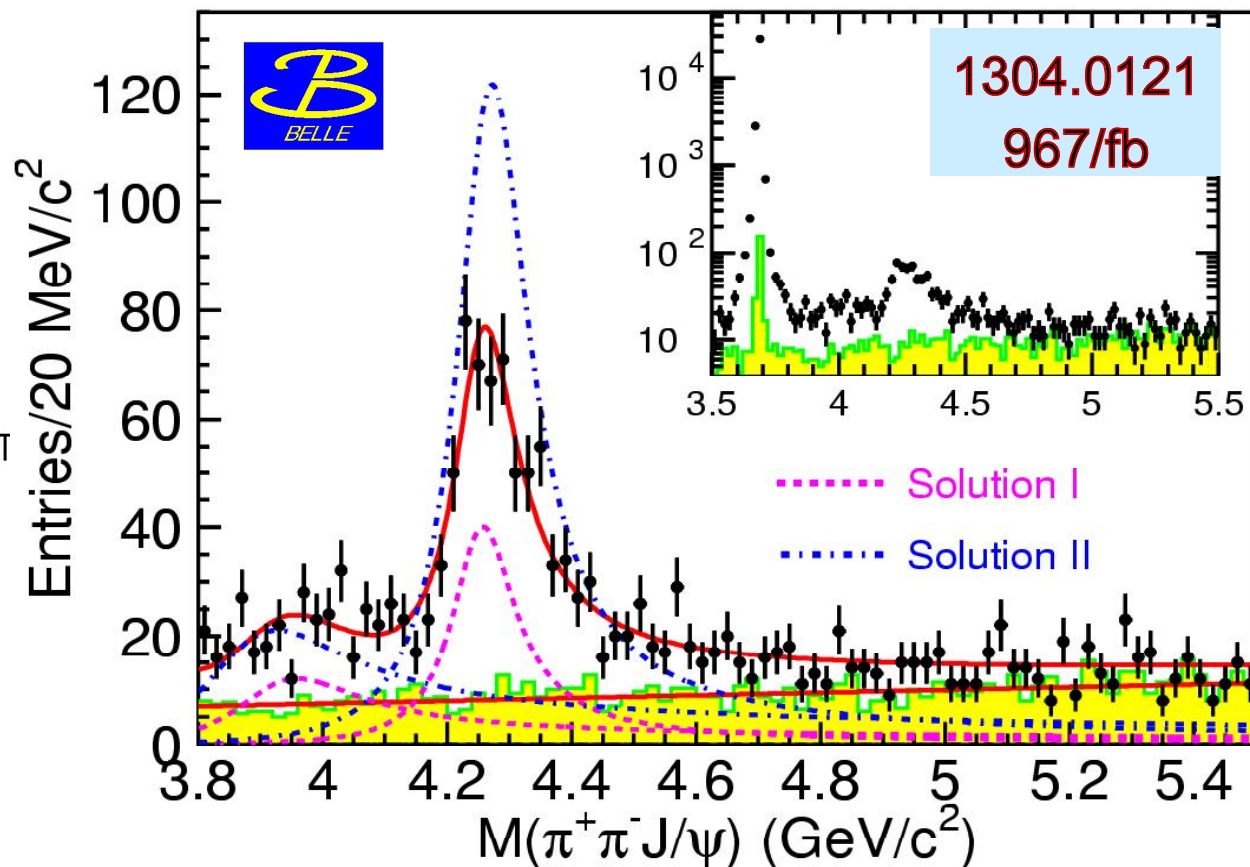
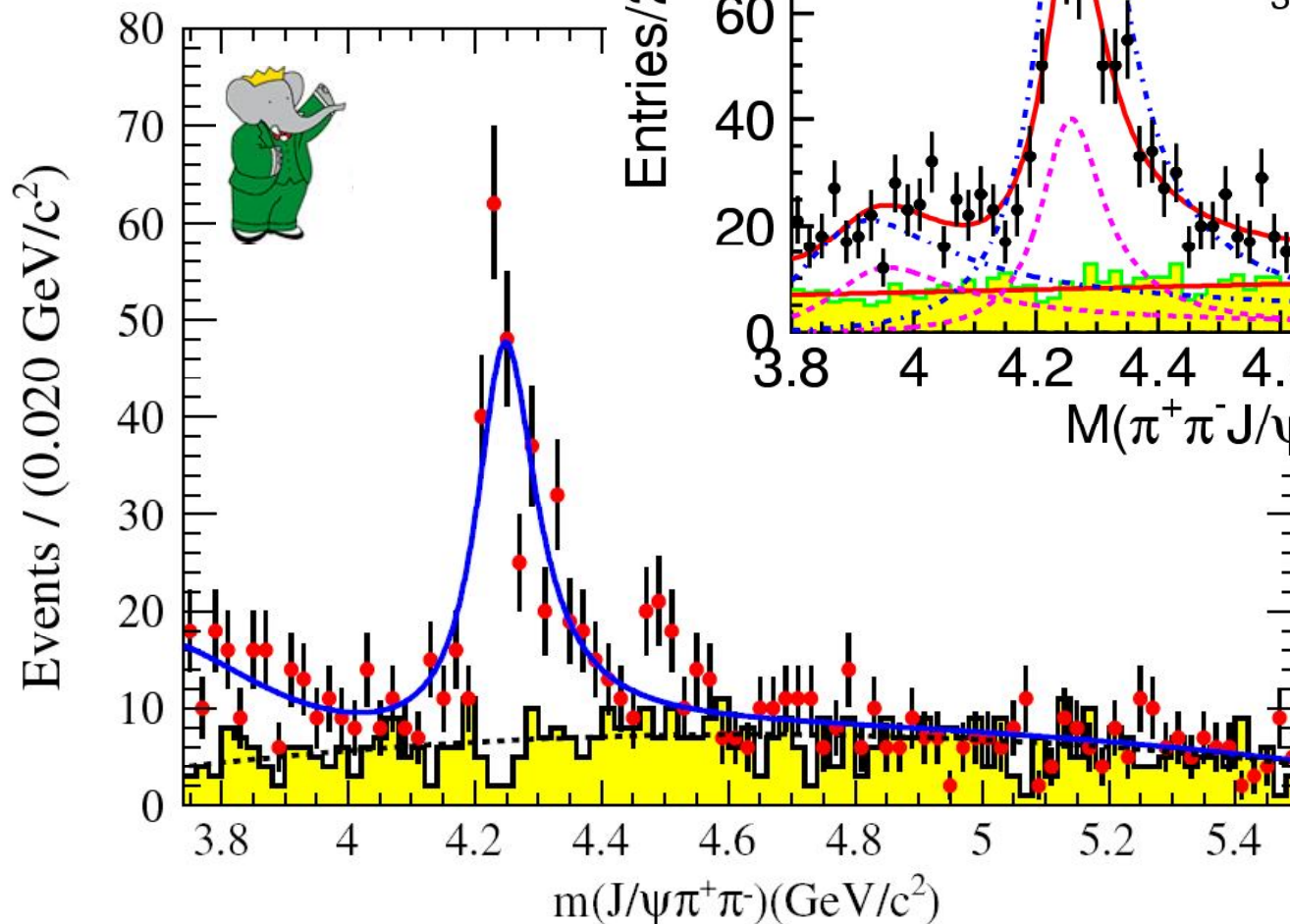
IHEP, Beijing

QWG 2013

# Raw data

PRD86, 051102

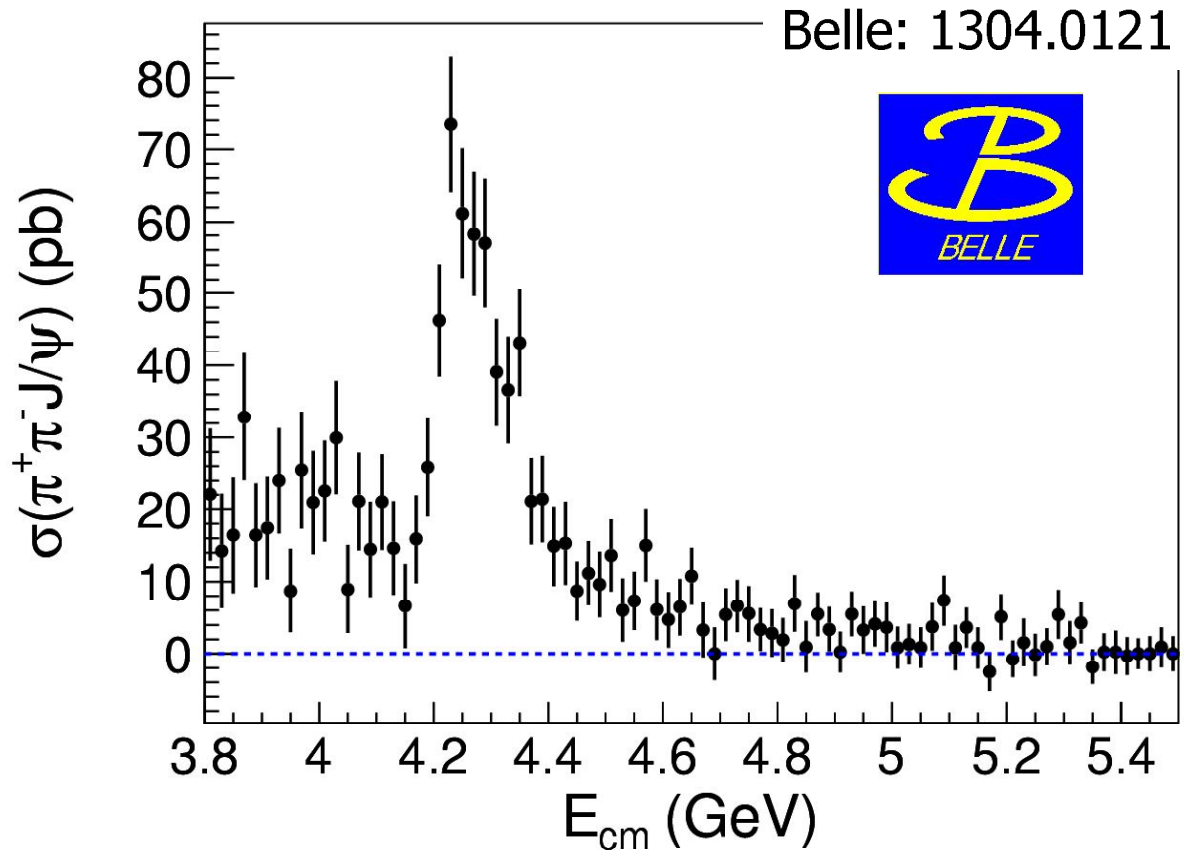
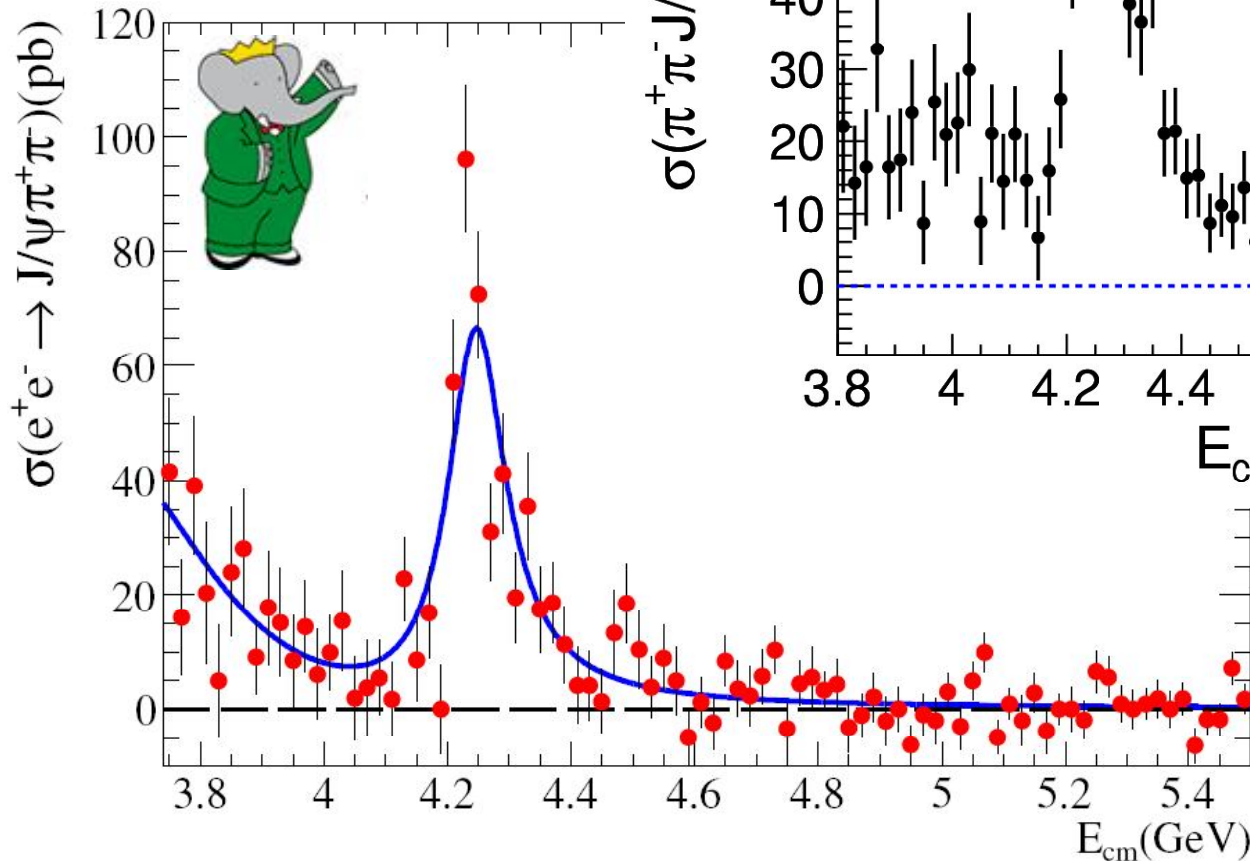
454/fb



Lum ratio = 1:2  
Same bin width  
 $\psi'$  tail and  $\psi''$ ?  
Bkg estimation?

# Cross section of $e^+e^- \rightarrow \pi^+\pi^-J/\psi$

BaBar:  
PRD86, 051102 (2012)



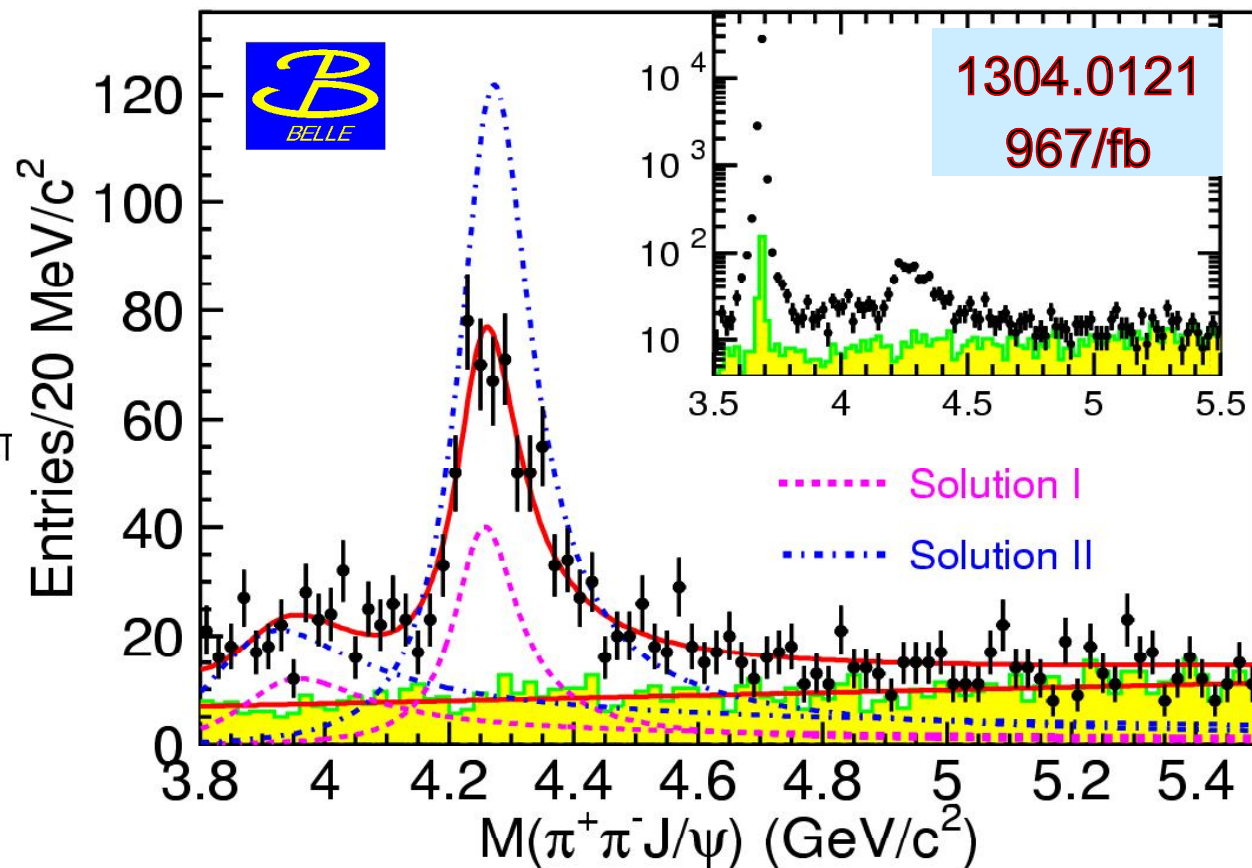
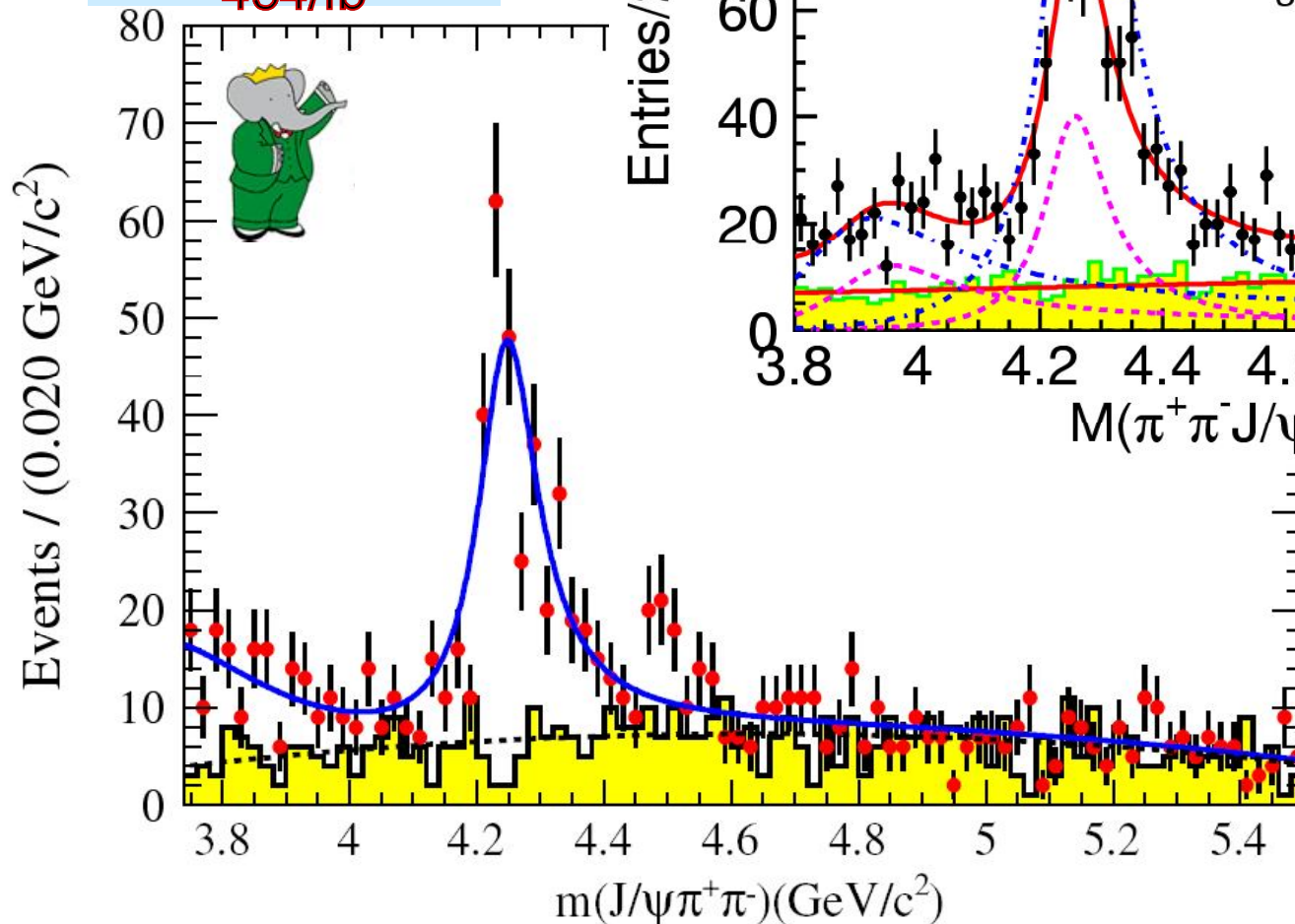
BESIII could be  
able to judge!

# Fits with

$$|A_1 + A_2|^2$$

PRD86, 051102

454/fb



Could be >1  
amplitudes  
at around 4  
GeV!

# Conclusions?

- BESIII scans the cross section between 3.8 and 4.2 GeV with at least 50/pb per point  
$$n^{\text{obs}} = 20 \text{ pb} * 50/\text{pb} * 12\% * 0.5 = 60 !$$
- KEDR scan?
- BaBar+Belle  $\sim 25 \text{ signals} + 15 \text{ bkgs}/[20 \text{ MeV}]$



# ***Y(4140) Status***

Round table: XYZ: where do we stand

*Kai Yi*

*University of Iowa*



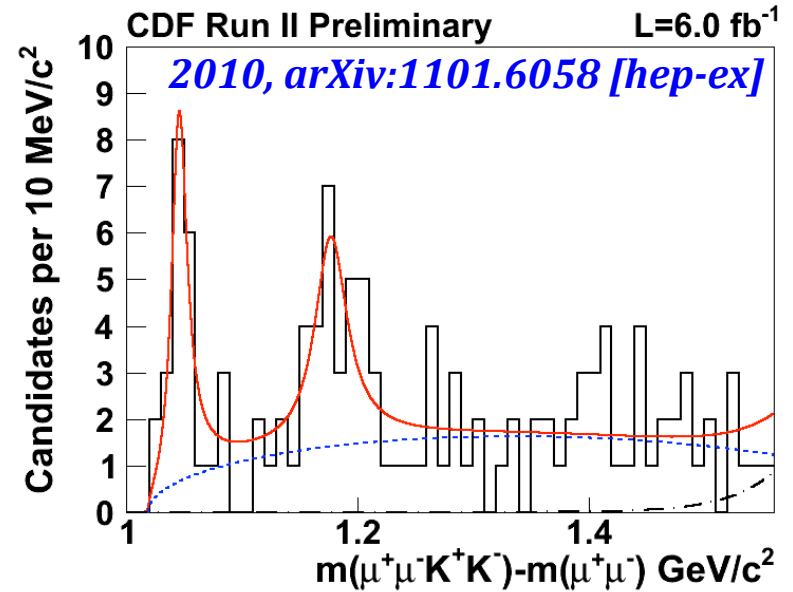
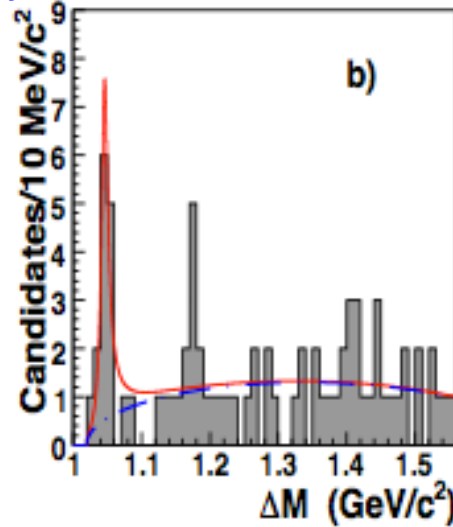
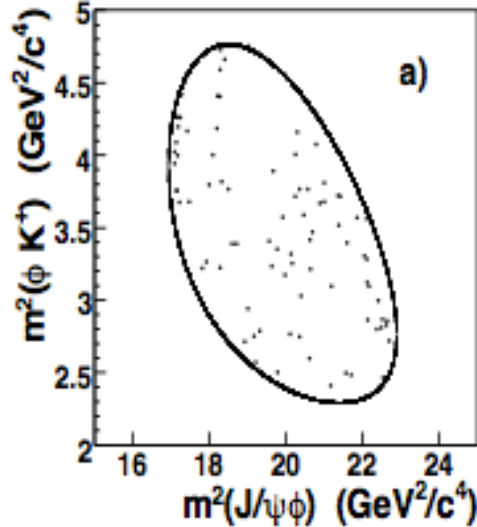
**Quarkonium 2013**

**The 9th International Workshop on Heavy Quarkonium**

**April 22- 26, 2013, IHEP, Beijing**

# Structures in $J/\psi\phi$ Spectrum (CDF)

PRL 102:242002, 2009,  $2.7 \text{ fb}^{-1}$



$$\Delta M = m(\mu^+ \mu^- K^+ K^-) - m(\mu^+ \mu^-)$$

$$\text{Yield}_1 = 19 \pm 6; > 5\sigma$$

$$\text{Yield}_2 = 22 \pm 8; 3.1\sigma$$

$$M_1 = 4143.4^{+2.9}_{-3.0} (\text{stat}) \pm 0.6 (\text{syst}) \text{ MeV}$$

$$M_2 = 4277.4^{+8.4}_{-6.7} (\text{stat}) \pm 1.9 (\text{syst}) \text{ MeV}$$

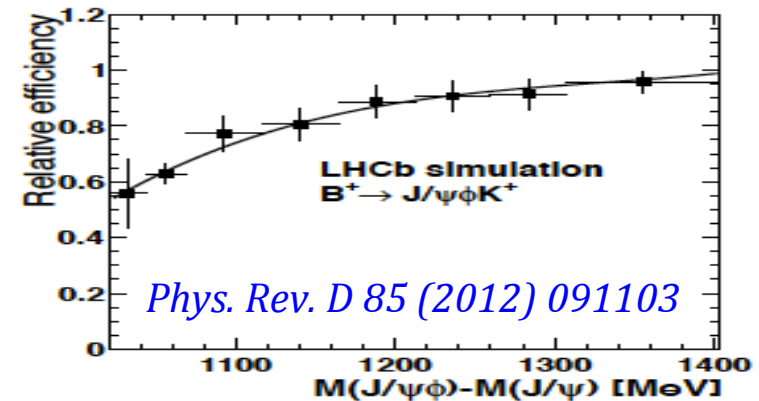
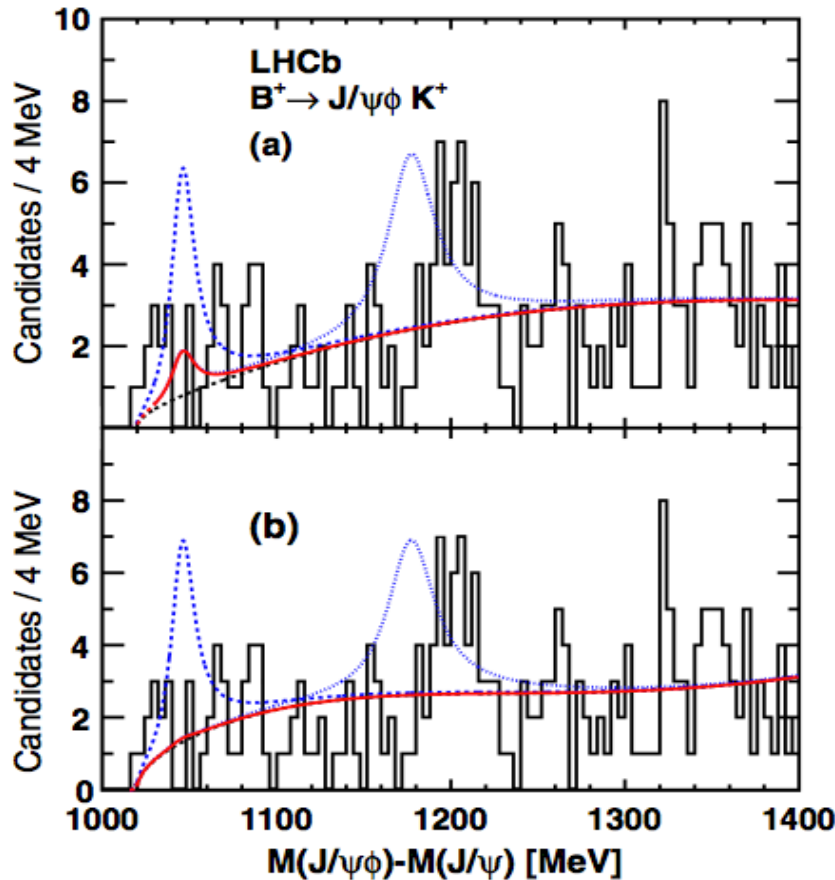
$$\Gamma_1 = 15.3^{+10.4}_{-6.1} (\text{stat}) \pm 2.5 (\text{syst}) \text{ MeV}$$

$$\Gamma_2 = 32.3.7^{+21.9}_{-15.3} (\text{stat}) \pm 7.6 (\text{syst}) \text{ MeV}$$

Observed in exclusive  $B^+ \rightarrow J/\psi \phi K^+$  decays

$$\frac{B(B^+ \rightarrow Y(4140) K^+, Y(4140) \rightarrow J/\psi \phi)}{B(B^+ \rightarrow J/\psi \phi K^+)} = 0.149 \pm 0.039 (\text{stat}) \pm 0.034 (\text{syst})$$

# *LHCb: does not confirm w/ 0.37 fb<sup>-1</sup>*



*LHC<sub>b</sub> confirms neither structure(s)  
2.4σ disagreement with CDF measurement  
@90% CL:*

$$\frac{\mathcal{B}(B^+ \rightarrow X(4140)K^+) \times \mathcal{B}(X(4140) \rightarrow J/\psi\phi)}{\mathcal{B}(B^+ \rightarrow J/\psi\phi K^+)} < 0.07.$$

$$\frac{\mathcal{B}(B^+ \rightarrow X(4274)K^+) \times \mathcal{B}(X(4274) \rightarrow J/\psi\phi)}{\mathcal{B}(B^+ \rightarrow J/\psi\phi K^+)} < 0.08$$

*There was an unofficial LHCb result (1 fb<sup>-1</sup>) in Workshop for New Results on Charmonium Production and Decays, March 6-8, 2013, Orsay France.*

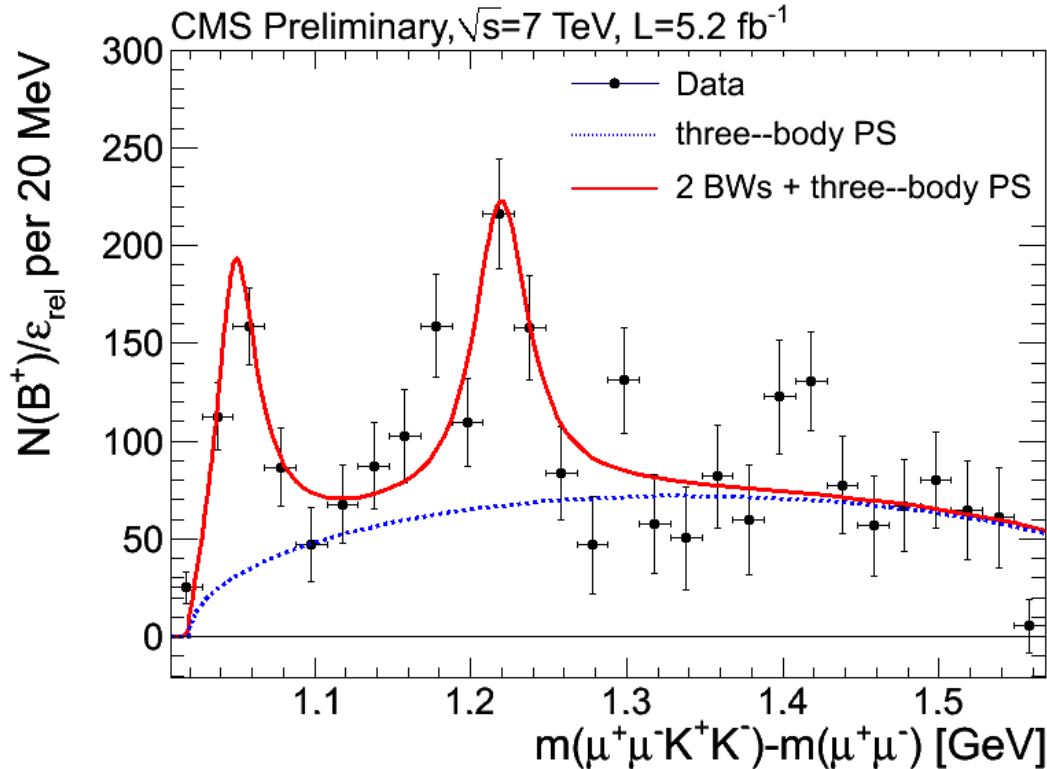
*Waiting for LHCb official result w/ full dataset.*

*???*



# CMS Results on $J/\psi\phi$ Structures

- The efficiency-corrected  $\Delta m = m(\mu^+\mu^-K^+K^-) - m(\mu^+\mu^-)$



*~20 times CDF statistics ( $115 \pm 12$ );*

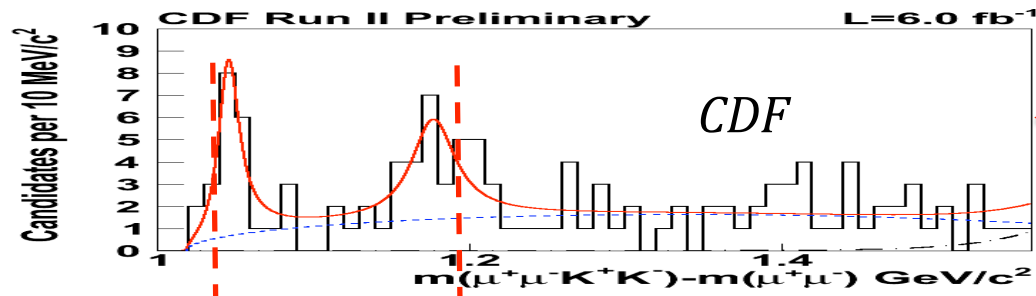
*~7.2X LHCb statistics ( $346 \pm 20$ )*

	Mass (MeV)	Signal Yield
First Peak	$1051.5 \pm 2.0$	$355 \pm 46$
Second Peak	$1220.0 \pm 3.0$	$445 \pm 83$

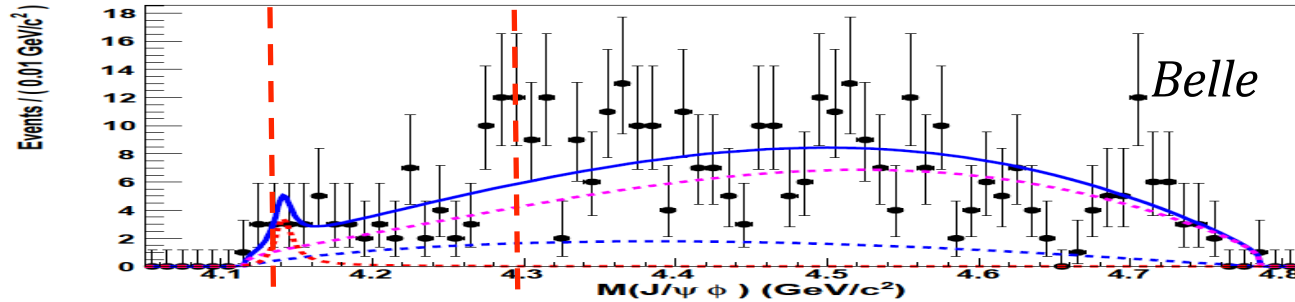
$$m_1 = 4148.2 \pm 2.0(\text{stat.}) \pm 4.6(\text{syst.}) \text{ MeV}$$

$$m_2 = 4316.7 \pm 3.0(\text{stat.}) \pm 7.3(\text{syst.}) \text{ MeV}$$

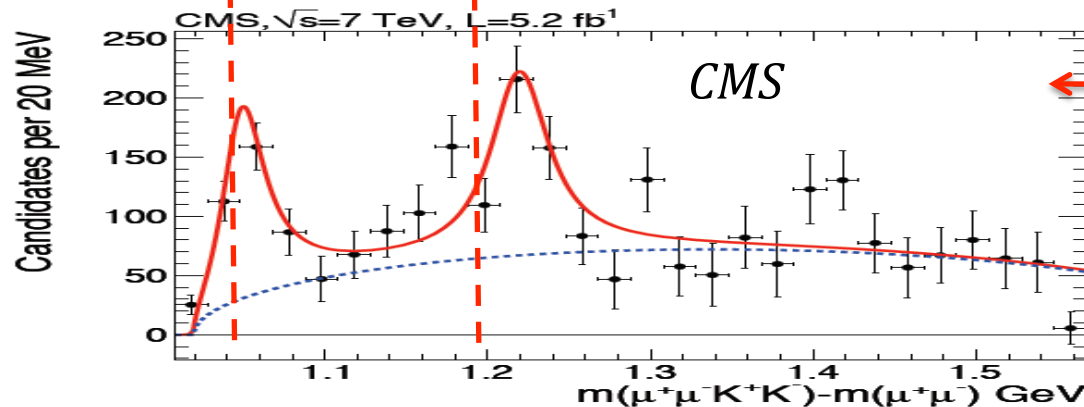
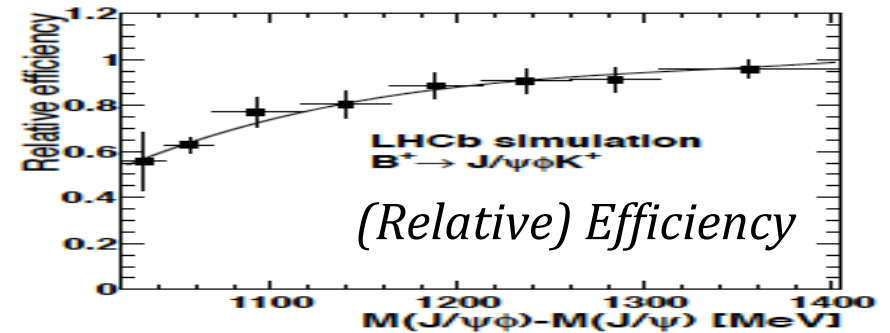
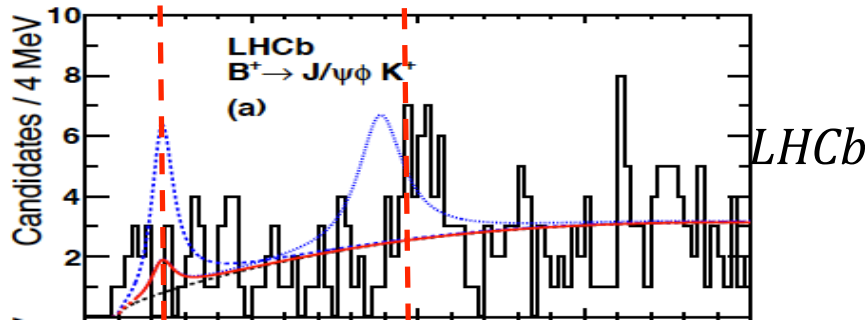
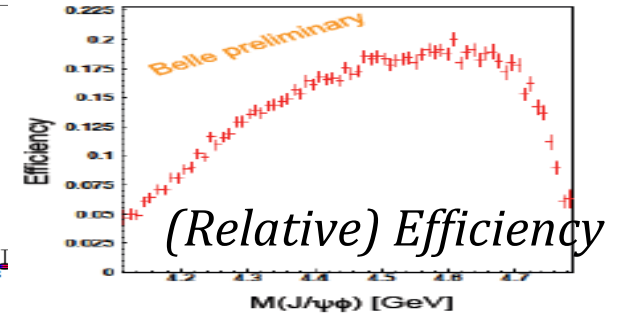
- CMS observed a  $J/\psi\phi$  structure at 4148 MeV with a significance greater than  $5\sigma$  consistent with CDF results on  $Y(4140)$
- CMS finds evidence for a second structure at  $\sim 4317$  MeV



← flat relative efficiency



K. Miyabayashi, QWG 2010



← relative efficiency corrected

CDF/BELLE/LHCb are raw distributions w/o efficiency correction

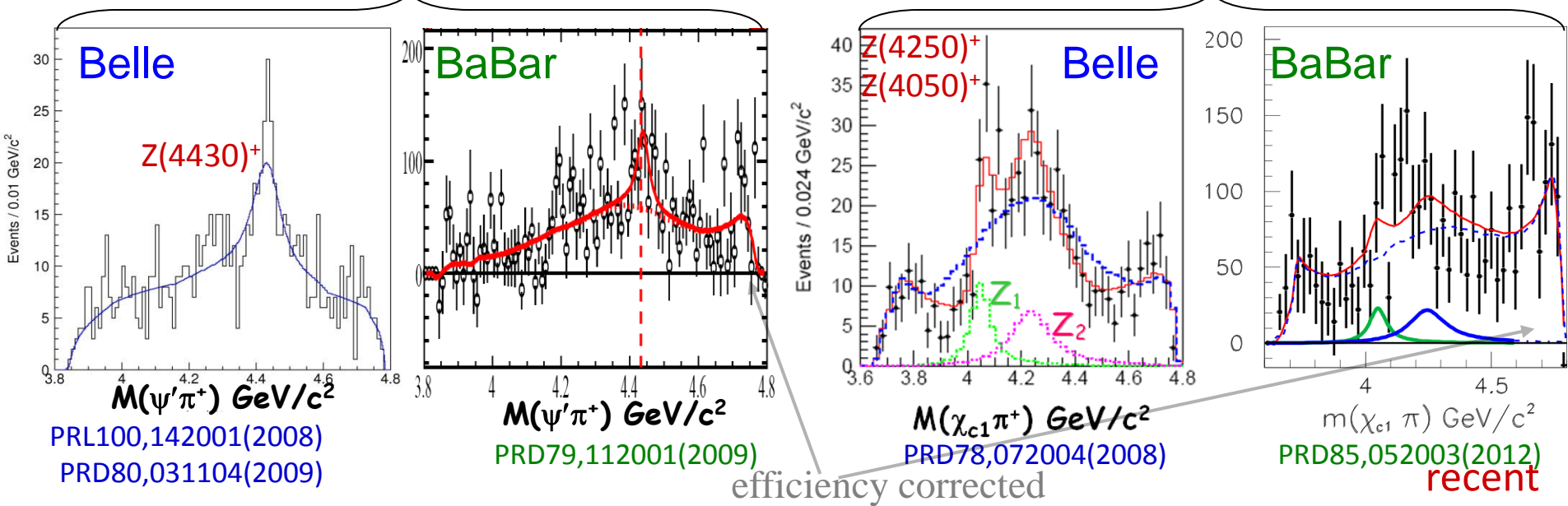
# ***Summary of $Y(4140)$ Status***

- *CDF positive report*
- *Belle cannot confirm or deny it*
- *LHCb did not confirm w/  $0.37 \text{ fb}^{-1}$  data*  
*(no official update, only unofficial plot w/  $1 \text{ fb}^{-1}$ )*
- *CMS confirms it w/ large statistics (7X LHCb)*
- *Apparent activity going on near the threshold*
- *ATLAS/CMS/LHCb can investigate its nature with more data*
- *A possible second structure, full amplitude analysis?*

# Charged $Z^+$ in $\pi^+\chi_{c1}$ and $\pi^+\psi'$ systems ?

$B \rightarrow \psi' \pi K$  candidates for multiquark states

$B \rightarrow \chi_{c1} \pi K$



**Belle** : 3 charged states, **BaBar** : data are consistent with Belle, but no significant  $Z^+$

Peaking structures are present in Belle and BaBar data

Different conclusions are due to different analyses

Belle – amplitude analysis of Dalitz plot, BaBar – moments of Legendre polynomials  
 (same for  $Z(4430)$  and  $Z_1, Z_2$  searches)

To settle Belle/BaBar controversy we need input from LHCb [ $Z(4430)$ ] and Belle-II.

# Measurement of $Z(4430)^+$ quantum numbers

## ★ Amplitude analysis in 4D phase:

$(M^2_{K\pi}, M^2_{\psi'\pi}, \phi_{\psi'K^*}, \theta_{\psi'})$ .

$\phi_{\psi'K^*}$ : angle between  $\Psi'$  and  $K^*$  decay planes

$\theta_{\psi'}$ :  $\Psi'$  helicity angle

TABLE I: Fit results:  $Z^+ \rightarrow \psi' \pi^+$ .

$J^P$	$0^-$	$1^-$	$1^+$	$2^-$	$2^+$
Mass, MeV	$4470 \pm 20$	$4482 \pm 4$	$4500 \pm 12$	$4545 \pm 2$	$4367 \pm 2$
Width, MeV	$139 \pm 36$	$10.9 \pm 0.3$	$126 \pm 20$	$11.2 \pm 0.6$	$9.1 \pm 0.6$
Significance	$4.4\sigma$	$1.2\sigma$	$6.1\sigma$	$2.3\sigma$	$2.6\sigma$

## ★ The $1^+$ hypothesis is preferred

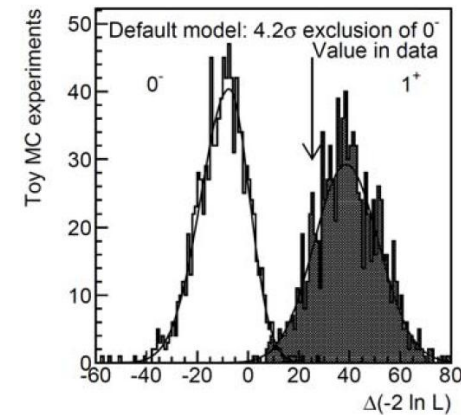
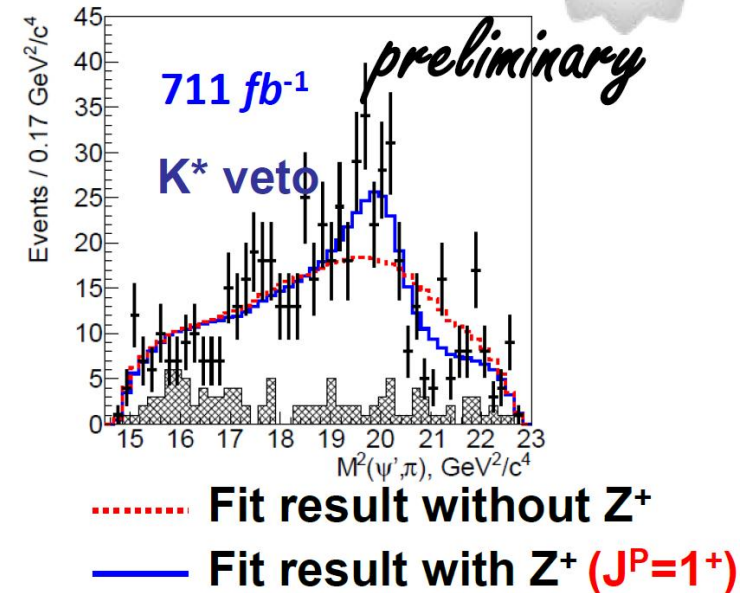
## ★ Exclusion levels are calculated from toy MC

★  $0^-$  is not excluded; significance of  $1^+$  over  $0^-$  is  $2.9\sigma$ .  $1^-$ ,  $2^-$  and  $2^+$  are excluded at levels of  $5.5\sigma$ ,  $4.3\sigma$ , and  $5.4\sigma$

## ★ We also calculated some Brs:

$$\mathcal{B}(\bar{B}^0 \rightarrow \psi' K^- \pi^+) = (5.50 \pm 0.15 \pm 0.42) \times 10^{-4},$$

$$\mathcal{B}(\bar{B}^0 \rightarrow \psi' K^*(892)) = (4.93^{+0.30+1.40}_{-0.23-0.43}) \times 10^{-4},$$



$$\begin{aligned} \mathcal{B}(\bar{B}^0 \rightarrow Z(4430)^+ K^-) \times \mathcal{B}(Z(4430)^+ \rightarrow \psi' \pi^+) = \\ (3.4^{+1.1+0.4}_{-0.7-1.3}) \times 10^{-5} \quad \text{for } J^P = 1^+ \text{ or} \\ (1.4^{+0.8+0.6}_{-0.7-0.2}) \times 10^{-5} \quad \text{for } J^P = 0^-. \end{aligned}$$

## Round table: XYZ: where do we stand

The 9th International Workshop on Heavy Quarkonium.

Beijing, April 26 2013



## Theory

Gunnar Bali (*University of Regensburg*)

Estia Eichten (*FNAL*)

Christoph Hanhart (*IAS, Forschungszentrum Juelich*)

Bernd Kniehl (*II. ITP Univ. Hamburg*)

Wei Wang (*HISKP Universitaet Bonn*)

## Experimental

Ryan Mitchell (*Indiana University*)

Roman Mizyuk (*ITEP Moscow*)

Kai Yi (*University of Iowa*)

Changzheng Yuan (*IHEP Beijing*)

## Questions related to $Z_c(3900)$

- 1 Why does the  $Z_c(3900)$  appear in  $Y(4260)$  decays?
- 2 Why is there a single  $Z_c(3900)$  state in charmonium and two ( $Z_b(10610)$  and  $Z_b(10650)$ ) states in bottomonium?
- 3 Can we understand why the width of  $Z_c$  is larger than the widths of  $Z_b$ 's?
- 4 Do we expect  $Z_c(3900)$  to decay to  $h_c\pi^+$ ?
- 5 The mass of  $Z_c(3900)$  is some 10MeV higher than the  $DD^*$  threshold. How this can be understood in the molecular picture?
- 6 If  $Z_c^\pm$  and  $Z_c^0$  form a triplet, can we expect an associated octet once strangeness is considered. From what we know about the triplet can we make any predictions for the masses of such states.

# Round table discussion

Christoph Hanhart

Forschungszentrum Jülich

**Why does the  $Z_c(3900)$  appear in  $Y(4260)$  decays?**

IF  $Y(4260)$  has a large  $\bar{D}_1 D$  component (referring to previous talk)  
→ low energy  $\bar{D} D^*$  pairs copiously produced  
→ perfect environment for formation of  $\bar{D} D^*$  resonance

**Why is there a single  $Z_c(3900)$  state in charmonium and two ( $Z_b(10610)$  and  $Z_b(10650)$ ) states in bottomonium?**

There should be another  $Z_c$ , but above mechanism not operative  
→ measurement at higher energies necessary

**Can we understand why the width of  $Z_c$  is larger than the widths of  $Z_b$ 's?**

A resonance couples strongly to  $\bar{D} D^*$   
→ width strongly correlated with pole position

**Do we expect  $Z_c(3900)$  to decay to  $h_c \pi^+$ ?**

Yes

## Questions directed to experimentalist

- What are the final states that are preferable/accessible for the experimental analysis, especially for  $Z_b$  or  $Z_c$  decays; what are the conditions?[Ryan Mitchell slide]
- What prevents a combined analysis of different channels? Maybe one can measure various final states at once to ease this? Only a combined analysis of distinct final states offers the hope to disentangle the composition of states.  
[Roman Mizyuk]

# Future XYZ plans at BESIII

Using  $\sim 2/\text{fb}$  data sample around 4.26 GeV:

- $Z_c(3900)$
- PWA analysis to determine  $J^P$
- Precise mass, width and branching ratio measurements.
- potential topics ongoing:
  - $\pi^+\pi^-h_c$
  - $DD^*\pi$
  - $D^*D^*\pi$
  - $\pi^+\pi^-\psi(2S)$
  - Search for  $h_c(2P)$
  - ...

BESIII also has  $\sim 500\text{pb}^{-1}$  at 4.36 GeV and smaller sets at a number of scan points...

The center of mass system is at rest, which allows us to access a wide variety of final states...

More ideas are welcome...



### Questions related to the $Z_b$

- $Z_b(10650)$  decays mainly to  $B^*\bar{B}^*$  and with small probability (if any) to  $B\bar{B}^*$ , despite much larger phase space. Can this be considered a "smoking gun" for the  $B^*\bar{B}^*$  wave-function of the  $Z_b(10650)$ ? Can other approaches (tetraquark?) explain this? [Wei Wang slides]

### Questions related to $X(3872)$

- What are the implication of  $X(3872)$  production cross-section measurement for its interpretation?

$Z_b(10650)$  decays mainly to  $B^*B^*\text{bar}$  and with small probability (if any) to  $BB^*\text{bar}$ , despite much larger phase space.

Can this be considered a "smoking gun" for the  $B^*B^*\text{bar}$  wave-function of the  $Z_b(10650)$ ? Can other approaches (tetraquark?) explain this?

# Can tetraquark (diquark-antidiquark) explain the decay pattern of $Z_b(10650)$ ?

Molecule



$$|Z_{b(10610)}\rangle = (0_{b\bar{q}} \otimes 1_{\bar{b}q} + 1_{b\bar{q}} \otimes 0_{\bar{b}q}) / \sqrt{2}$$

$$|Z_{b(10650)}\rangle = 1_{b\bar{q}} \otimes 1_{\bar{b}q}$$

Tetraquark



$$|Z_{b(10610)}\rangle = (0_{[bq]} \otimes 1_{[\bar{b}\bar{q}]} - 1_{[bq]} \otimes 0_{[\bar{b}\bar{q}]}) / \sqrt{2}$$

$$|Z_{b(10650)}\rangle = 1_{[bq]} \otimes 1_{[\bar{b}\bar{q}]}$$

↓ Fierz ↓

$$|Z_{b(10610)}\rangle = 1_{b\bar{q}}^- \otimes 1_{q\bar{b}}^-$$

$$|Z_{b(10650)}\rangle = (1_{b\bar{q}}^- \otimes 0_{q\bar{b}}^- + 0_{b\bar{q}}^- \otimes 1_{q\bar{b}}^-) / \sqrt{2}$$

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$$Z_{b(10610)} \rightarrow B\bar{B}^* + B^*\bar{B}$$

$$Z_{b(10650)} \rightarrow B^*\bar{B}^*$$

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$$Z_{b(10610)} \rightarrow B^*\bar{B}^*$$

$$Z_{b(10650)} \rightarrow B\bar{B}^* + B^*\bar{B}$$

# Can tetraquark(diquark-antidiquark) explain the decay pattern of $Z_b(10650)$ ?

[Belle Collaboration (2012)]

Molecule

Channel	Fraction, %	
	$Z_b(10610)$	$Z_b(10650)$
$\Upsilon(1S)\pi^+$	$0.32 \pm 0.09$	$0.24 \pm 0.07$
$\Upsilon(2S)\pi^+$	$4.38 \pm 1.21$	$2.40 \pm 0.63$
$\Upsilon(3S)\pi^+$	$2.15 \pm 0.56$	$1.64 \pm 0.40$
$h_b(1P)\pi^+$	$2.81 \pm 1.10$	$7.43 \pm 2.70$
$h_b(2P)\pi^+$	$4.34 \pm 2.07$	$14.8 \pm 6.22$
$B \bar{B}^{*0} \mid \bar{B}^{*0} B^*$	$86.0 \pm 3.6$	$\downarrow \text{Fierz} \downarrow$
$B^{*+} \bar{B}^{*0}$		$73.4 \pm 7.0$

$$|Z_{b(10610)}\rangle = (0_{bq} \otimes 1_{\bar{q}b} + 1_{bq} \otimes 0_{\bar{q}b}) / \sqrt{2}$$

$$|Z_{b(10650)}\rangle = 1_{bq} \otimes 1_{\bar{q}b}$$

$$|Z_{b(10610)}\rangle = (0_{bq} \otimes 1_{\bar{q}b} - 1_{bq} \otimes 0_{\bar{q}b}) / \sqrt{2}$$

$$|Z_{b(10650)}\rangle = 1_{bq} \otimes 1_{\bar{q}b}$$

$$|Z_{b(10650)}\rangle = (1_{b\bar{q}}^- \otimes 0_{q\bar{b}}^- + 0_{b\bar{q}}^- \otimes 1_{q\bar{b}}^-) / \sqrt{2}$$

$$Z_{b(10610)} \rightarrow B\bar{B}^* + B^*\bar{B}$$

$$Z_{b(10650)} \rightarrow B^*\bar{B}^*$$

$$Z_{b(10610)} \rightarrow B^*\bar{B}^*$$

$$Z_{b(10650)} \rightarrow B\bar{B}^* + B^*\bar{B}$$

$c\bar{c}$

$b\bar{b}$

show similar pattern  
but  
related?

$\Upsilon_c(4260)$

$\Upsilon_b(10890)$

$\leftarrow Y(5S)?$

$\pi Z_c(3900)$

$Z_b^{(')}(106XX)\pi$

related?

molecule or  
tetraquark

molecule reasonable  
tetraquark doubtful

# What are the implications of $X(3872)$ production cross section measurements for its interpretation?

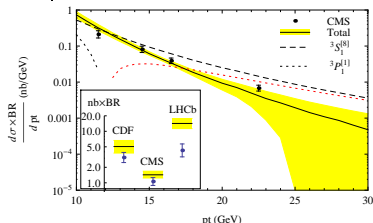
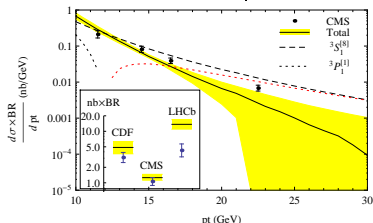
M. Butenschoen, Z. He, BK, 1303.3524 [hep-ph]

Test hypothesis  $X(3872) \equiv \chi_{c1}(2P)$  in prompt hardproduction at NLO in NRQCD

$$d\sigma(pp \rightarrow \chi_{c1}(2P) + X) = \sum_{i,j,n} \int dx dy f_{i/A}(x) f_{j/B}(y) \langle \mathcal{O}^{\chi_{c1}(2P)}[n] \rangle d\sigma(ij \rightarrow c\bar{c}[n] + X)$$

$$\text{Fix } \langle \mathcal{O}^{\chi_{c1}(2P)}(3P_1^{[1]}) \rangle = (2J+1) \frac{3C_A}{2\pi} |R'_{2P}(0)|^2 = 0.438 \text{ GeV}^5.$$

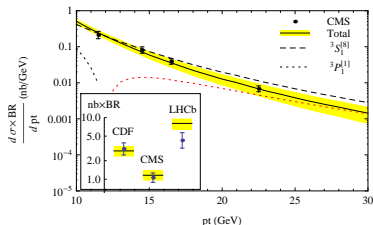
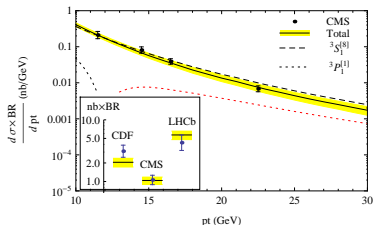
	w/ LHCb	w/o LHCb
$\langle \mathcal{O}^{\chi_{c1}(2P)}(3S_1^{[8]}) \rangle [\text{GeV}^3]$	$(3.84_{-0.24}^{+0.28}) \times 10^{-3}$	$(4.30_{-0.26}^{+0.30}) \times 10^{-3}$
$\chi^2/\text{d.o.f.}$	79.1/5 = 15.8	16.7/4 = 4.18





# Two-parameter fit

	w/ LHCb	w/o LHCb	1304.6710 [hep-ph] <sup>1</sup>
$\langle \mathcal{O}_{\chi c1}(2P)(3P_1^{[1]}) \rangle [\text{GeV}^5]$	$0.10^{+0.050}_{-0.050}$	$0.19^{+0.092}_{-0.094}$	$0.17 \pm 0.7$
$\langle \mathcal{O}_{\chi c1}(2P)(3S_1^{[8]}) \rangle [\text{GeV}^3]$	$(3.84^{+0.28}_{-0.24}) \times 10^{-3}$	$(4.30^{+0.30}_{-0.26}) \times 10^{-3}$	$(3.34 \pm 1.69) \times 10^{-3}$
$\chi^2/\text{d.o.f.}$	$4.26/4 = 1.07$	$0.63/3 = 0.21$	$0.52/2 = 0.26$



<sup>1</sup> C. Meng, H. Han, K.-T. Chao, 1304.6710 [hep-ph]

Fix  $|R'_{2P}(0)|^2 = 0.075 \text{ GeV}^5$  and fit  $\langle \mathcal{O}_{\chi c1}(2P)(3S_1^{[8]}) \rangle$  and overall factor

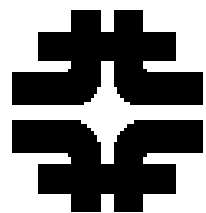
$\frac{d\sigma(pp \rightarrow X(3872) + X)}{d\sigma(pp \rightarrow \chi_{c1}(2P) + X)}$  to CMS  $p_T$  distribution.

## Related to other exotic states

- 1 There are many hadronic transitions from  $\Upsilon(5S)$  to lower bottomonia with anomalously high partial widths  $\Gamma \sim 1\text{MeV}$ . Also transitions with emission of eta are not suppressed relative to  $\pi^+\pi^-$ . How all these can be understood? What is  $\Upsilon(5S)$ ? (bottomonium with admixture of molecule?)
- 2 All charmonium(-like) states above open flavor thresholds like to decay to lower charmonia with emission of light hadrons, with anomalous partial width  $\Gamma > 1\text{MeV}$ . Is this of the same origin as hadronic transitions from the  $\Upsilon(5S)$ ? Why charmonium(-like) states couple to one channel only (e.g.  $Y(4360)$  and  $Y(4660)$  decay to  $\psi(2S)\pi^+\pi^-$ , but not to  $J/\psi\pi^+\pi^-$  etc).

## Question about Large $N_c$ arguments

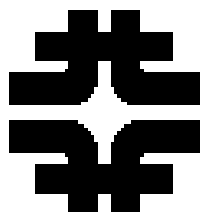
- 3 Do large  $N_c$  arguments favor the molecular picture?



# Comment for panel discussion

Estia Eichten  
Fermilab





# Comment

- Large hadronic transitions for states above threshold only occur when the decay of the initial quarkonium-like state can produce a ground state pair of heavy-light mesons that;
  - In a relative S-wave
  - Very low momentum in CM frame.
- The initial state decays into kinematically allowed associated pair of ground state and P-wave excited heavy light meson according to HQS.

For  $1^{--}$  quarkonium state  $\Phi$  allowed decays are:

$\Phi \rightarrow H(1^-) (j^P = 1/2^-) + H(0^+) (j^P = 1/2^+) \text{ S-wave}$   
 $\Phi \rightarrow H(0^-) (j^P = 1/2^-) + H(1^+) (j^P = 1/2^+) \text{ S-wave}$   
 $\Phi \rightarrow H(1^-) (j^P = 1/2^-) + H(1^+) (j^P = 1/2^+) \text{ S-wave}$   
 $\Phi \rightarrow H(1^-) (j^P = 1/2^-) + H(1^+) (j^P = 3/2^+) \text{ D-wave}$   
 $\Phi \rightarrow H(0^-) (j^P = 1/2^-) + H(2^+) (j^P = 3/2^+) \text{ D-wave}$   
 $\Phi \rightarrow H(1^-) (j^P = 1/2^-) + H(2^+) (j^P = 3/2^+) \text{ D-wave}$

$H(0^+) (j^P = 1/2^+) \rightarrow \pi H(0^-) \text{ (S-wave)}$

$H(1^+) (j^P = 1/2^+) \rightarrow \pi H(1^-) \text{ (S-wave)}$

$H(1^+) (j^P = 3/2^+) \rightarrow \pi H(1^-) \text{ (D-wave)}$

$H(2^+) (j^P = 3/2^+) \rightarrow \pi H(0^-) \text{ (D-wave)}$   
 $\rightarrow \pi H(1^-) \text{ (D-wave)}$

- For hybrid states ( $Y(4260)$ ) or molecular states, I assume the decays are some subset of the decay channels above. eg. hybrid  $\rightarrow H(0^-) (j^P = 1/2^-) + H(1^+) (j^P = 1/2^+, 3/2^+)$  often used.
- Then it is simple kinematics to get the pattern of observed charged states in the bottom and charm system transitions.

# Tetraquarks in large- $N$ QCD (G Bali)

Wick contractions for mixing  $c\bar{c}$  with  $c\bar{q}q\bar{c}$  ( $n_F = 2$ ):

$$\left( \begin{array}{cc} \text{diagram 1} & \text{diagram 2} \\ \text{diagram 3} & \text{diagram 4} \\ \text{diagram 5} & \text{diagram 6} \\ \text{diagram 7} & \text{diagram 8} \end{array} \right) = \begin{pmatrix} \# + \#N & \frac{1}{\sqrt{N}} (\# + \#N) \\ \frac{1}{\sqrt{N}} (\# + \#N) & \frac{1}{N} (\# + \#N) \end{pmatrix}$$

Each closed line obtains a factor  $N$ . Usually each  $M = \bar{q}q$  is divided by  $\sqrt{N}$  so that the normalization of the propagator is  $N$ -independent.

Mixing is governed by

$$\frac{C_{12}^2}{C_{11}C_{22}} = \frac{(\# + \#N)^2}{(\# + \#N)(\# + \#N)} \xrightarrow{N \rightarrow \infty} \text{const?}$$

At large  $N$  the disconnected contributions of  $C_{11}$  and  $C_{22}$  dominate. Their mixing with glueballs is  $\mathcal{O}(1)$  and glueball propagators are also of  $\mathcal{O}(N)$ .

We have ignored that the different  $N$ -sectors decouple. If we look at the connected quarkonium alone (which is of  $\mathcal{O}(1)$  in the  $N$ -counting) then we encounter the situation discussed recently by Weinberg:

$$\begin{pmatrix} \text{diagram 1} & \text{diagram 2} \\ \text{diagram 3} & \text{diagram 4} \end{pmatrix} = \begin{pmatrix} \# & \frac{1}{\sqrt{N}} \\ \frac{1}{\sqrt{N}} & \frac{1}{N} (\# + \#N) \end{pmatrix}$$

The diagrams in the matrix are:

- Top-left: A red loop with two red dots, labeled with a red '2'.
- Top-right: A red loop with two red dots and a blue loop with two blue dots, labeled with a red '2'.
- Bottom-left: A red loop with two red dots and a blue loop with two blue dots, labeled with a red '2'.
- Bottom-right: A red loop with two red dots and a blue loop with two blue dots, labeled with a red '-4'.

Now

$$\frac{C_{12}^2}{C_{11}C_{22}} = \mathcal{O}(1/N) \quad \text{or} \quad \mathcal{O}(1).$$

Mixing with disconnected  $D\bar{D}$  is suppressed by  $1/N$  but it can mix with a connected tetraquark (which is of the same order in  $N$ ).  $\Rightarrow$  Kinematical question: which one is lighter? If the tetraquark is lighter it will dominate.  $\Rightarrow$  Calculation of  $N = \infty$  charmonium and charmed tetraquark spectra could be interesting.