Bottomonium in lattice NRQCD (at $T \neq 0$)

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in FASTSUM collaboration with
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Outline

Lattice NRQCD at $T = 0$

• bottom quark ($\sim$ 4.5 GeV) is too heavy, compared to the inverse lattice spacing currently available for lattice simulation

• Instead of relativistic formulation for bottomon quark, NRQCD formulation is chosen

• NRQCD is an effective field theory: separation of perturbative UV physics ($> M_b$) and non-perturbative IR physics

• inclusive decay rates = partonic decay rate $\times$ the probability for heavy quark to meet anti-heavy quark (cf. Braaten, Bodwin, Lepage, PRD51 (1995) 1125)

• long distance ME can be calculated by lattice method (e.g, Bodwin, Sinclair, Kim, PRL77 (1996) 2376)

Lattice NRQCD at $T = 0$

- lattice NRQCD at $T = 0$ is well established: 2012 PDG summary on QCD

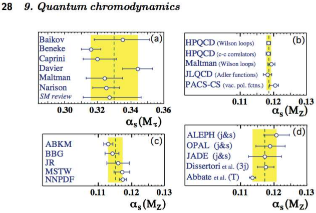


Figure 9.2: Summary of determinations of α_s from hadronic τ-decays (a), from lattice calculations (b), from DIS structure functions (c) and from event shapes and jet production in e^+e^--annihilation (d). The shaded bands indicate the average values chosen to be included in the determination of the new world average of α_s.

- many lattice NRQCD talks in this workshop

What about lattice NRQCD at $T \neq 0$?

• same as $T = 0$, $N_s a$ should be large enough to accommodate hadrons ($\sim$ 2 fm) and a^{-1} should be big enough to accommodate bottom quarks

• for $T \neq 0$, $T = \frac{1}{N_\tau a}$ where a is the lattice spacing

$\rightarrow$ in non-zero T environment, N_τ is limited
(cf. say $a^{-1} \sim 10$ GeV and $T \sim 500$ MeV, $\rightarrow a = 0.0195$ fm, $N_s \sim 100$ and $N_\tau \sim 20$)

• "improvement" which is used for light quark simulation doesn't help for heavy quark case (HotQCD, $48^3 \times 12$, $a \sim 0.066$fm)

What about lattice NRQCD at $T \neq 0$?

- NRQCD at $T \neq 0$should be valid as long as $\frac{T}{M} << 1$
- spectral function encodes thermal effect
- anisotropic lattice (a_s is larger than a_τ) provides more N_τ points
- early work by J. Fingberg (PLB 424 (1998) 343) based on quenched gauge configurations
- FASTSUMers, PRL106 (2011) 061602, JHEP 1111 (2011) 103, JHEP 1013 (2013) 084 based on 2-flavor dynamical gauge configurations

Lattice related technical detail

- $\mathcal{O}(v^4)$ lattice NRQCD lagrangian for bottom quark

- two-plaquette Symanzik improved gauge action, fine-Wilson, coarse-Hamber-Wu fermion action with stout-link smearing

- Anisotropic lattice on $12^3 \times N_t$ (cf. G. Aarts et al, PRD 76 (2007) 094513, $m_\pi/m_\rho \simeq 0.54$)

N_s	N_t	a_τ^{-1}	T(MeV)	T/T_c	No. of Conf.
12	80	7.35GeV	90	0.42	250
12	32	7.35GeV	230	1.05	1000
12	28	7.35GeV	263	1.20	1000
12	24	7.35GeV	306	1.40	500
12	20	7.35GeV	368	1.68	1000
12	18	7.35GeV	408	1.86	1000
12	16	7.35GeV	458	2.09	1000

Spectral function in NRQCD

$$G_O(\tau) = \sum_{\vec{x}} \langle \overline{\psi}(\tau,\vec{x}) O \psi(\tau,\vec{x}) \overline{\psi}(0,\vec{0}) O \psi(0,\vec{0}) \rangle \quad (1)$$

$$= \int_0^\infty \frac{d\omega}{2\pi} K(\tau,\omega) \rho_O(\omega) \quad (2)$$

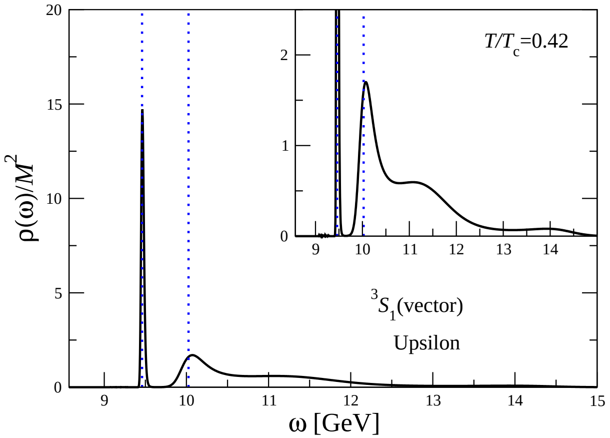
Relativistic formulation:

$$K(\tau,\omega) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}. \quad (3)$$

In NRQCD (with $\omega = 2M + \omega'$ and $T/M << 1$),

$$G(\tau) = \int_{-2M}^\infty \frac{d\omega'}{2\pi} \exp(-\omega'\tau)\rho(\omega') \quad (4)$$

Spectral function in NRQCD



- low temperature behavior
- dotted lines are experimental values for $1S$ and $2S$ of Υ state

Spectral function in NRQCD at $T \neq 0$

- in relativistic formulation

  quarkonia melting is obscured by constant contribution (cf. Umeda PRD75 (2007) 094052, Petreczky et al 07-09)

- in NRQCD,

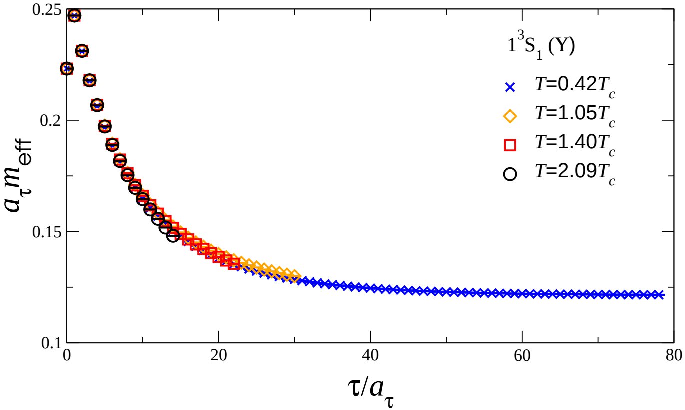
  constant contribution is absent

  no thermal boundary condition for the heavy quark

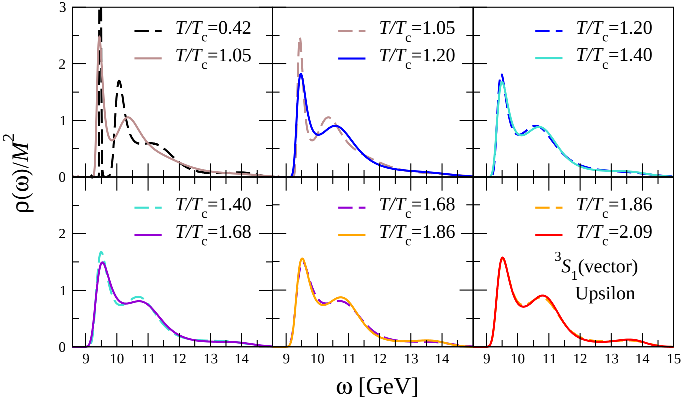
  obtaining spectral function is an inverse Laplace transform problem

S-wave bottomonium at non-zero T

- bound state $\rightarrow$ exponentially falling propagator ($G(\tau) \sim Ae^{-E\tau}$)
- $m_{\text{eff}}(\tau) = -\log[G(\tau)/G(\tau - a_\tau)] \rightarrow$ constant

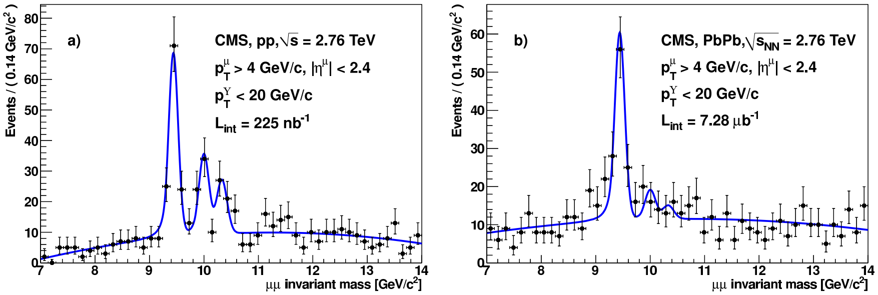


S-wave bottomonium spectral function at non-zero T



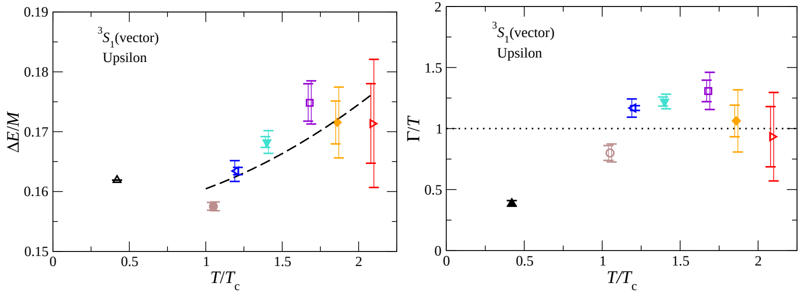
- suppresion of excited states above T_c
- the ground state survives upto $2.09T_c$

S-wave bottomonium spectral function at non-zero T



- CMS collaboration, PRL107 (2011) 052302

T-dependence of the Υ ground state peak

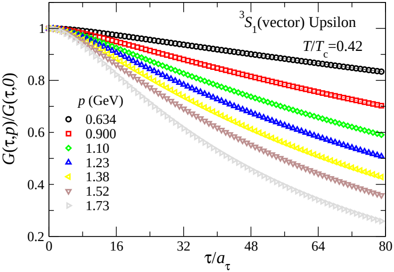


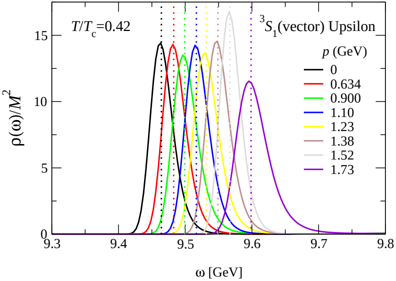
$$\Delta E \sim \alpha_s \frac{T^2}{M}, \qquad \frac{\Gamma}{T} \sim \alpha_s^3 \quad (\alpha_s \sim 0.4) \tag{1}$$

Brambilla, Escobedo, Ghiglieri, Soto, Vairo, JHEP1009 (2010) 038

S-wave moving in thermal bath

- moving with non-relativistic velocity ($\frac{v}{c} \leq 0.2$ at $T \sim 0$)

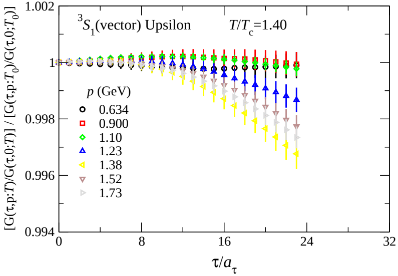


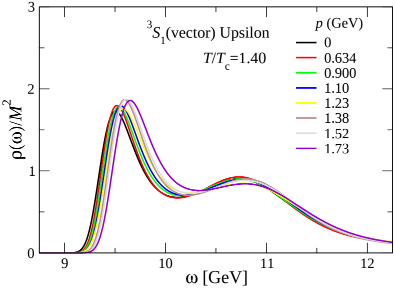


- the ratio, $G(\tau,\vec{p})/G(\tau,\vec{0})$(left) and the spectral function (right) shows momentum dependence

S-wave moving in thermal bath

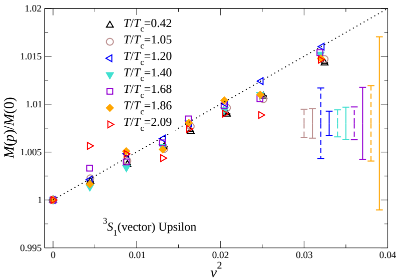
- ratio $[G(\tau,\vec{p};T)/G(\tau,\vec{0};T)]/[G(\tau,\vec{p};T_0)/G(\tau,\vec{0};T_0)]$

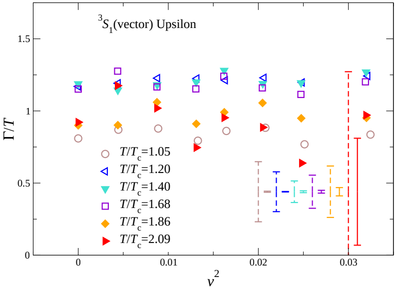




- temperature dependence of the momentum dependence is not noticeable

S-wave moving in thermal bath

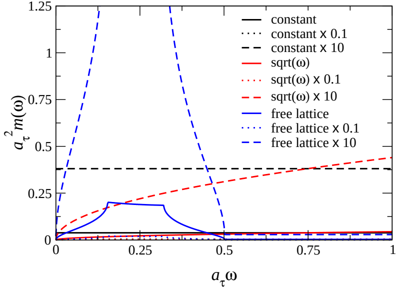


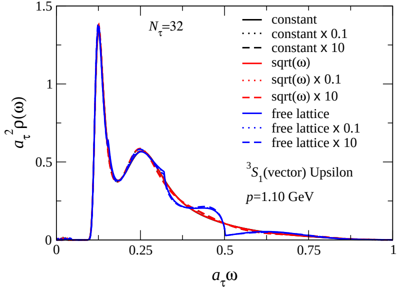


• dispersion relation is satisfied but momentum dependence of thermal width is very small (or negligible) at the range of momenta studied

S-wave moving in thermal bath

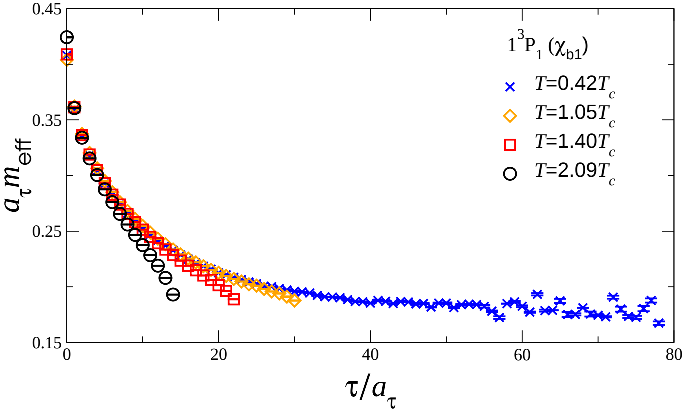
- default model independence



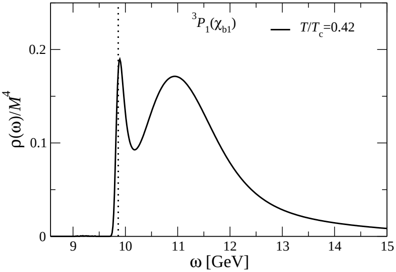


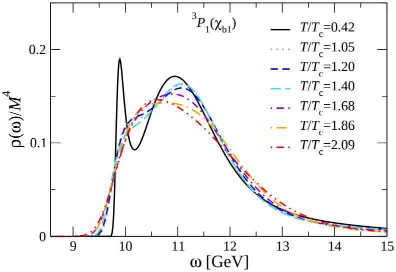
χ_b melts above T_c

- non-bound state $\rightarrow$ power-like propagator ($G(\tau) \sim A'\tau^{-\gamma}$)
- $m_{\text{eff}}(\tau) = -\log[G(\tau)/G(\tau - a_\tau)]$



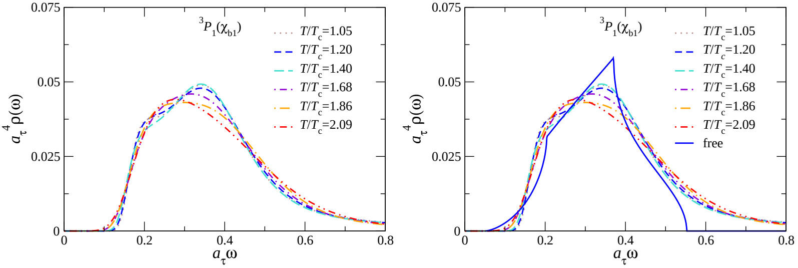
spectral function of χ_b (preliminary)





- dotted line is experimental value
- melts immediately above T_c

spectral function of χ_b (preliminary)



- melts immediately above T_c
- shape is similar to free lattice spectral function

Conclusion

• lattice NRQCD method for bottomonium on anisotropic lattices offers a method which is systematically improvable and is based on the first principel of quantum field theory (not a model)

• lattice NRQCD at zero temperature already produced accurate result and is producing more accurate result

• the ground state of Υ and η_b (S-wave) survives but the excited states are suppressed as the temperature increases above T_c

• 1S peak of S-wave increases with T^2 and 1S width of S-wave has T-dependence

• χ_b (P-wave) melts almost immediately above T_c

• further studies on bottomonium (including systematic error study) are under way