

# Properties of the moving quarkonium at high temperature

Miguel A. Escobedo

Physik-Department T30f. Technische Universität München

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Work done in collaboration with Floriana Giannuzzi, Massimo Mannarrelli and Joan Soto. arXiv:1304.4087.

# Outline

1 Introduction

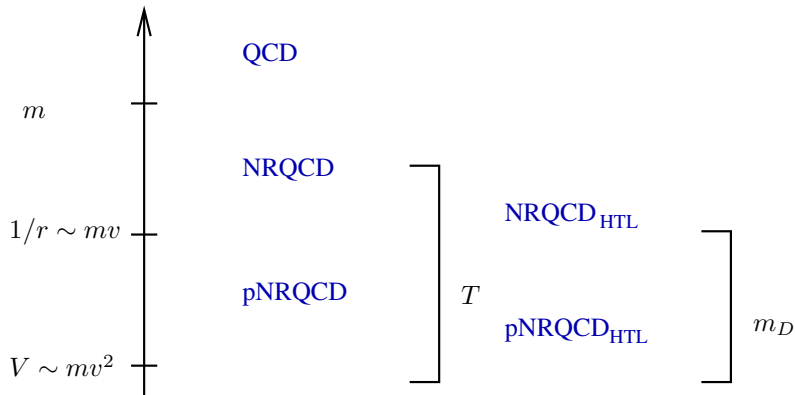
2  $1/r \gg T \gg E \gg m_D$

3  $T \gg 1/r \sim m_D$

4 Conclusions

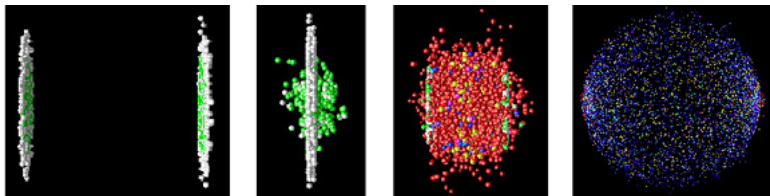
# Introduction

# EFT for bound states at finite temperature



# Ideal conditions

- The EFTs for HQ at finite temperature and the imaginary part of the potential were obtained assuming thermal equilibrium and that the bound state is at rest.
- This is not what happens in heavy-ion collisions.



# Relax this conditions

- Anisotropic plasma
- Quarkonium is moving
- ...

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Burnier, Laine and Vepsäläinen. Dumitru, Guo and Strickland.  
Philipsen and Tassler.
- Quarkonium is moving
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# Medium effects on a moving quarkonium

For a heavy quark in NRQCD

$$P^\mu = m_Q u^\mu + k^\mu$$

- $m_Q u^\mu$  information about the center of mass momentum.  $u^2 = 1$  and  $u = (1, 0, 0, 0)$  in the bound state rest frame.
- $k^\mu$  information about other properties, as for example the binding.



# Medium effects on a moving quarkonium

For a heavy quark in NRQCD

$$P^\mu = m_Q u^\mu + k^\mu$$

- Medium may modify  $m_Q u^\mu$ . Heavy quark energy loss. Only happens when there is a finite momentum.
- Medium may modify  $k^\mu$ . Existence or not of heavy quarkonium states. Happens in the comoving case but it may also be modified when there is a finite momentum.

## General framework

We choose the frame where the bound state is at rest and the thermal bath is moving.

$$f(\beta^\mu k_\mu) = \frac{1}{e^{|\beta^\mu k_\mu|} \pm 1},$$

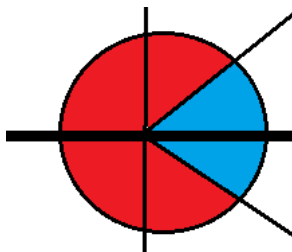
$$\beta^\mu = \frac{\gamma}{T}(1, \mathbf{v}) = \frac{u^\mu}{T},$$

We use a generalization of the real-time formalism called Non-equilibrium field theory (Zhou, Su, Han and Liu). At tree level substitute the equilibrium distribution functions by the non-equilibrium ones in the propagator.

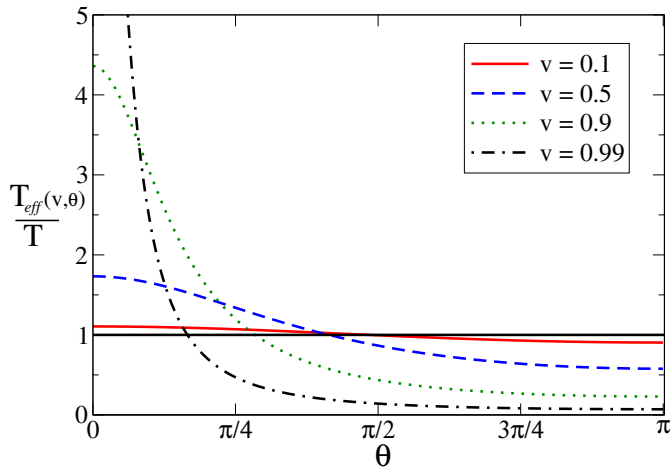
# Massless particles

We can define an *effective temperature* depending on the incidence angle.

$$T_{\text{eff}}(\theta, v) = \frac{T\sqrt{1-v^2}}{1-v\cos\theta}.$$



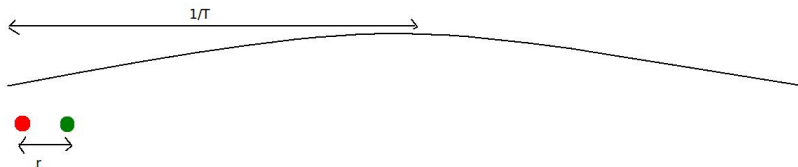
# Effective temperature



$$1/r \gg T \gg E \gg m_D$$

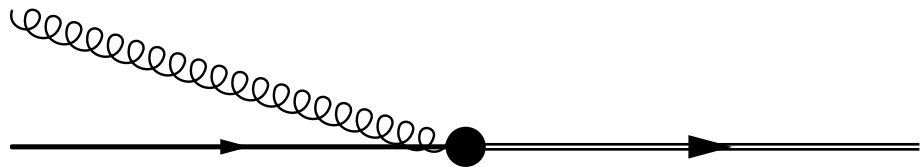
# The $1/r \gg T \gg E \gg m_D$ regime

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- Decay width is dominated by the process  $HQ + g \rightarrow \text{octet}$ .  
Cross-section does not depend on incidence angle. Decay width is expected to decrease with increasing velocity because for most angles  $T_{\text{eff}} < T$ .

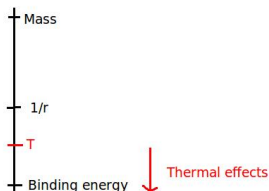


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- A QED study was previously done. M. A. E, Mannarelli and Soto.



# EFT framework



- The starting point can be pNRQCD.
- Matching from pNRQCD to  $pNRQCD_{HTL}$ . Effects of the scale T are encoded in a **modification of the potential**.
- Computation of the scale  $E$  effects in  $pNRQCD_{HTL}$ . Modifications to the decay width and binding energy. **Not necessary potential like**.

## Matching to $pNRQCD_{HTL}$ . Modification of the potential.

All effects encoded in a modification of the potential.

$$\delta V_s = \frac{2\pi C_F \alpha_s T^2}{3m_Q} + \frac{\pi N_c C_F \alpha_s^2 T^2 r}{12} \left( \frac{4}{3} + f(v) - \frac{1}{3} + \frac{(\mathbf{r} \cdot \mathbf{v})^2}{r^2 v^2} (1 - 3f(v)) \right)$$

where

$$f(v) = \frac{1}{v^3} (v(2 - v^2) - 2(1 - v^2) \tanh^{-1}(v))$$

## Computation in $pNRQCD_{HTL}$ . Binding energy

$$\delta E_{nlm} = \frac{2\pi C_F T^2}{3} \left[ \frac{\alpha_s}{m_Q} + \frac{N_c \alpha_s^2}{2} \langle r \rangle_{nlm} + \frac{N_c \alpha_s^2}{2} \langle r \rangle_{nlm} (1 - 3f(v)) \langle 2l00 | l0 \rangle \langle 2l0m | lm \rangle \right]$$

where  $\langle ll' mm' | lm \rangle$  are the Clebsch-Gordan coefficients.

In the **s-wave case**

$$\delta E_n^{s\text{-wave}} = \frac{2\pi C_F \alpha_s T^2}{3m_Q} + \frac{\pi N_c C_F \alpha_s^2 T^2 a_0 n^2}{6}$$

No momentum effects in the s-wave.

## Computation in $pNRQCD_{HTL}$ . Decay width

$$\Gamma_{nlm} = \frac{\alpha_s C_F T \sqrt{1-v^2}}{3v} \left[ 4 \left( -\frac{2E_n^c}{m_Q} + \frac{\alpha_s N_c}{m_Q a_0^2 n^2} + \frac{\alpha_s^2 N_c^2}{8} \right) \log \left( \frac{1+v}{1-v} \right) + \left( -\frac{4E_n^c}{m_Q} - \frac{\alpha_s N_c}{m_Q a_0 n^2} + \frac{\alpha_s^2 N_c^2}{4} \right) h(v) \langle 2l00 | l0 \rangle \langle 2l0m | lm \rangle \right]$$

where

$$h(v) = \left[ \left( 1 - \frac{3}{v^2} \right) \log \left( \frac{1+v}{1-v} \right) + \frac{6}{v} \right]$$

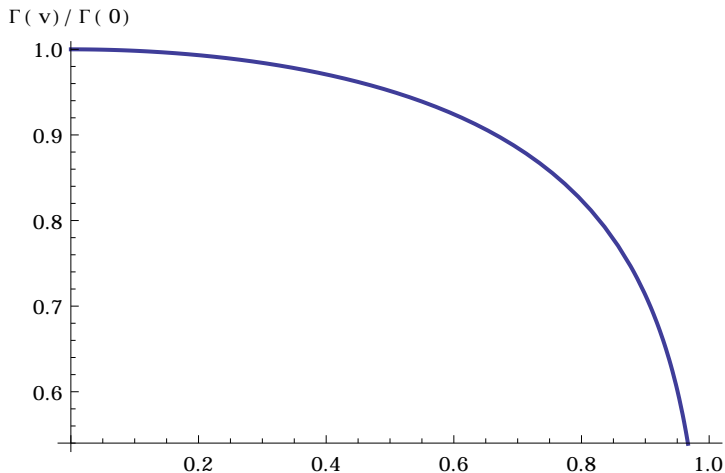
# Computation in $pNRQCD_{HTL}$ . Decay width

In the **s-wave case**

$$\Gamma_n^{s\text{-wave}} = \frac{4\alpha_s C_F T \sqrt{1-v^2}}{3v} \left( -\frac{2E_n^c}{m_Q} + \frac{\alpha_s N_c}{m_Q a_0 n^2} + \frac{\alpha_s^2 N_c^2}{8} \right) \log \left( \frac{1+v}{1-v} \right)$$

- Decreasing function with velocity.
- Goes to 0 as  $v \rightarrow 1$ .

# Computation in $pNRQCD_{HTL}$ . Decay width



## Comparison with lattice computations.

$$1/r \gg T \gg E \gg m_D$$

Aarts, Allton, Kim, Lombardo, Oktay, Ryan, Sinclair, Skullerud (2012).

Plasma rest frame.

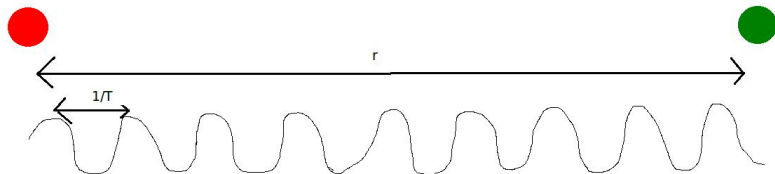
- They observe no additional effects due to the finite momentum.
- Results are still compatible with our predictions because the maximum velocity achieved in the lattice still fulfils  $v \ll 1$ .
- If our prediction is right an important effect should be observed for  $v > 0.5$ .

$$T \gg 1/r \sim m_D$$



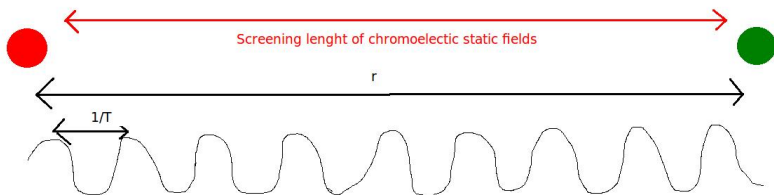
## The $T \gg 1/r \sim m_D$ regime

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- The **screening length** is of the same order as the size of the **bound state**.



## The $T \gg 1/r \sim m_D$ regime

- Effects at the energy **scale  $T$**  are going to see heavy quarks as elements that are **very far away** from each other.
- The **screening length** is of the same order as the size of the **bound state**.
- Decay width is dominated by the process  **$HQ + parton \rightarrow octet + parton$** . The tri-momentum of the interchanged gluon is perpendicular to the one of the incident parton. **No isotropic potential**.



## The real part of the potential, normalization

At  $v = 0$

$$\text{Re } V(r) = -\frac{4\alpha_s e^{-m_D r}}{3r} = -\frac{4\alpha_s C_F m_D g(m_D r)}{3}$$

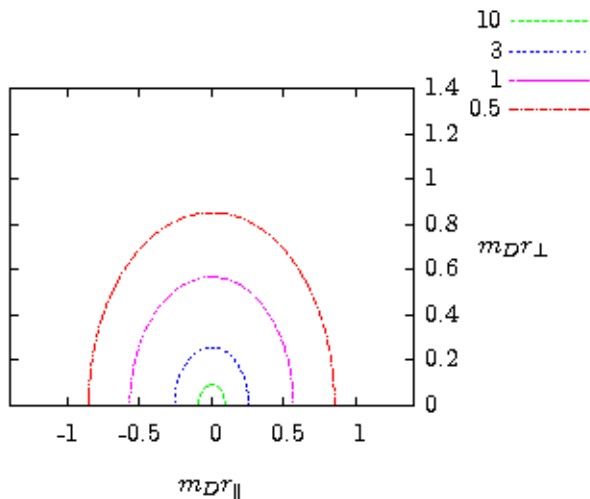
at any velocity we can define

$$g(m_D r) = -\frac{3\text{Re}V(r)}{4\alpha_s m_D}$$

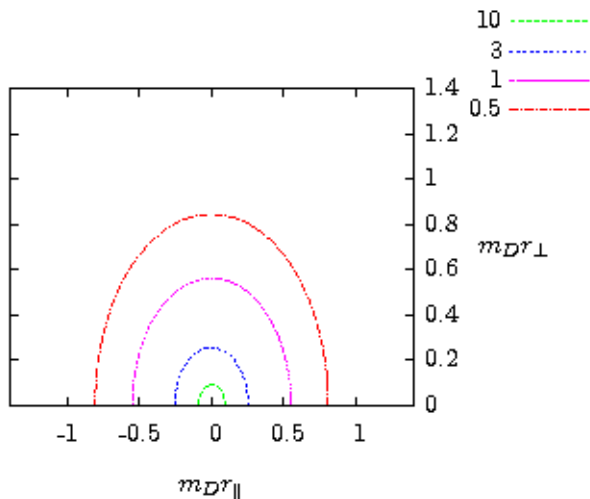
$g(x)$  does not depend on  $T$ , it is useful to compare the same  $T$  with different  $v$ . This is what we are going to plot.

Computed by Matsui and Chu.

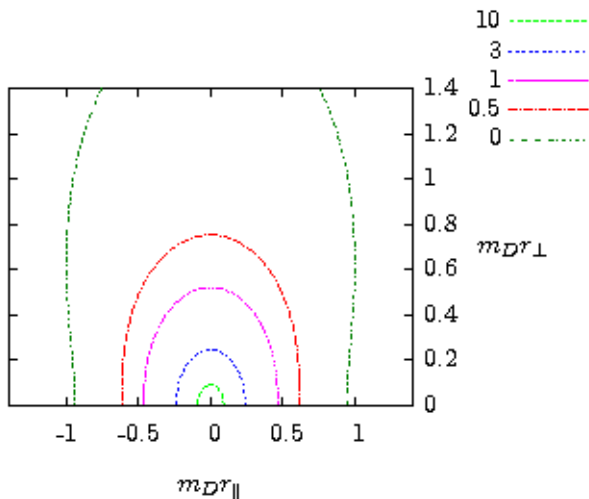
# The real part of the potential at $v = 0$



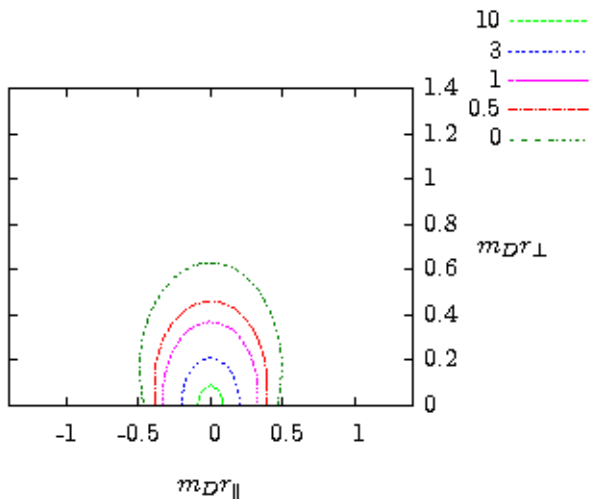
# The real part of the potential at $v = 0.5$



# The real part of the potential at $v = 0.9$



# The real part of the potential at $v = 0.99$





## The imaginary part of the potential at $v = 0$

$$\text{Im } V(r) = V_S(r) = -\frac{4\alpha_s T \phi(m_D r)}{3},$$

with

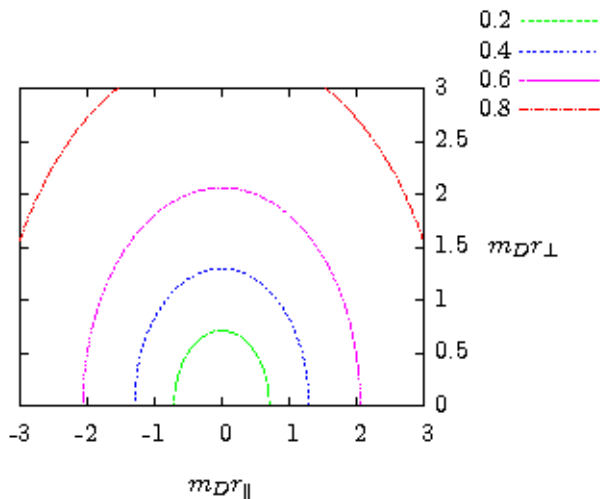
$$\phi(x) = 2 \int_0^\infty \frac{dz z}{(z^2 + 1)^2} \left( 1 - \frac{\sin(zx)}{zx} \right).$$

(Laine, Philipsen, Romatschke and Tassler). At any velocity we can define

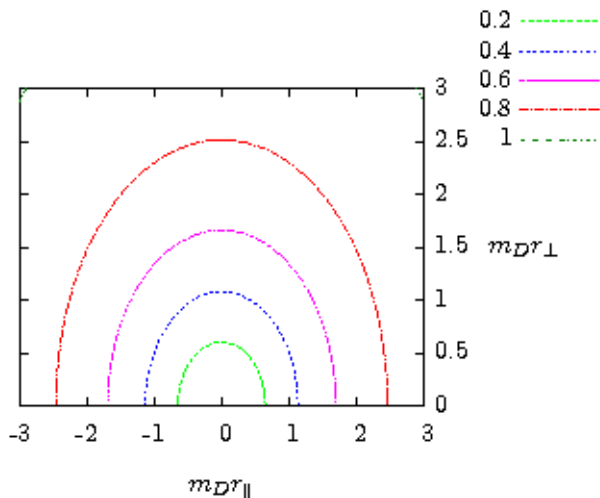
$$\phi(m_D r) = -\frac{3 \text{Im } V(r)}{4\alpha_s T}.$$

This is what we are going to plot. (Computed for muonic hydrogen in M.A.E, Mannarelli and Soto).

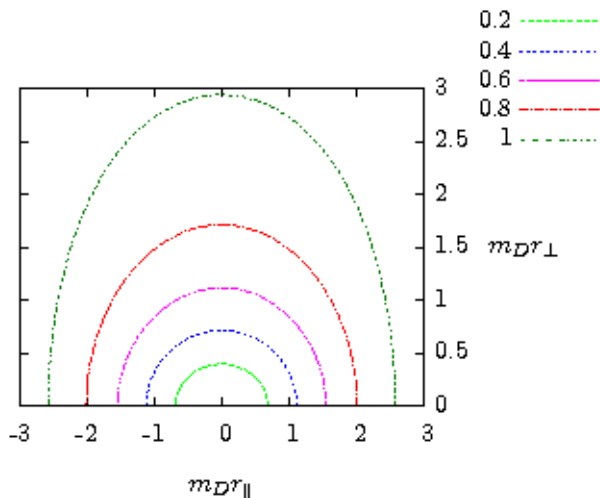
# The imaginary part of the potential at $v = 0$



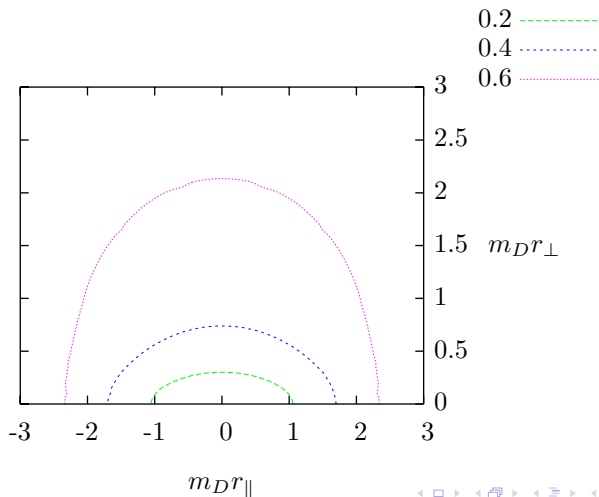
## The imaginary part of the potential at $v = 0.5$



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# The imaginary part of the potential at $v = 0.99$



# Spectral function

- Allow to do more **quantitative** statements about dissociation.
- Directly related with **dilepton production**.
- Allow to compare result with lattice computations.

Computation at  $v = 0$  was already available (Laine (2007)). For finite  $v$  not spherical symmetry, only **cylindrical symmetry**.

# Running coupling constant

The potential can be written as

$$V(r) = \frac{\alpha_s(\mu_1)}{r} f(m_D(\mu_2)r)$$

In the original  $v = 0$  computation  $\mu_1 \sim 2\pi T$  and  $\mu_2 \sim 2\pi T$ .  
We call this choice running 1.

# Running coupling constant

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$$V(r) = \frac{\alpha_s(\mu_1)}{r} f(m_D(\mu_2)r)$$

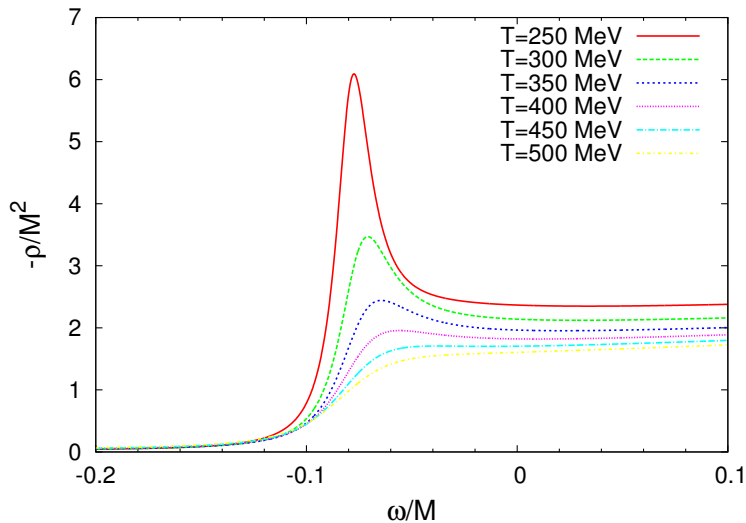
In pNRQCD at  $T = 0$  one uses  $\mu_1 = 1/r$  or  $\mu_1 = 1/a_0$ .

Use  $\mu_1 = 1/a_0$  and  $\mu_2 = 2\pi T$ .

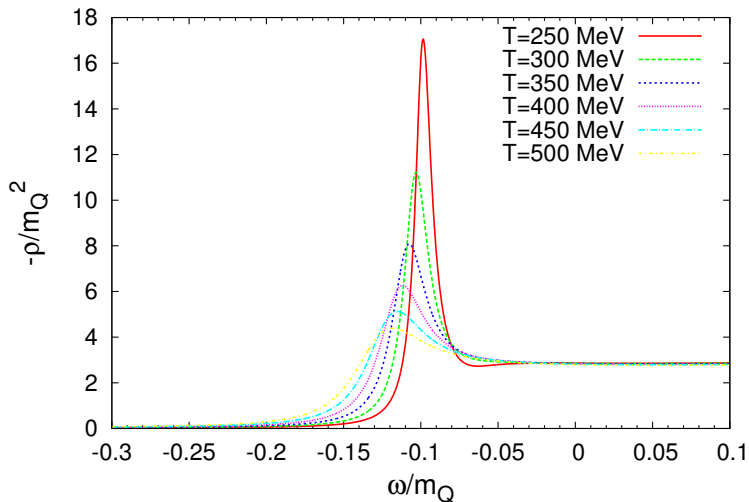
We call this choice running 2. Used at  $v = 0$  to determine dissociation temperature for  $\Upsilon(1S)$ . M.A.E and Soto.



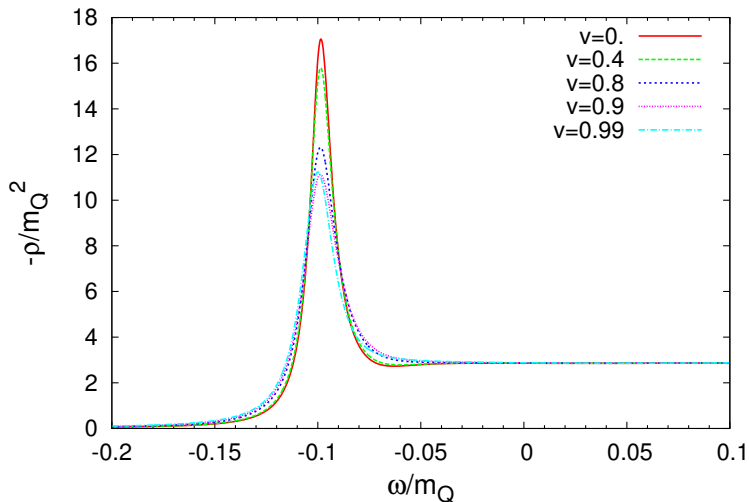
# Spectral function at $\nu = 0$ . Running 1



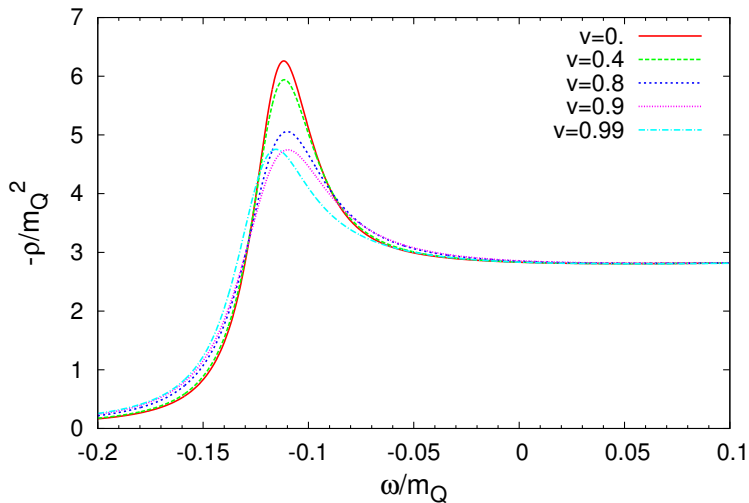
## Spectral function at $\nu = 0$ . Running 2



# Spectral function at $T = 250$ MeV



# Spectral function at $T = 400$ MeV



# Conclusions.

# Conclusions

- In the regime where gluo-dissociation dominates the decay width decreases with velocity.
- In the regime relevant for dissociation and for moderate velocities dissociation increases with velocity.
- In the regime relevant for dissociation and for very large velocities the width decreases with velocity.
- We confirm that for very large velocities modifications of the real part of the potential are very important.