

Heavy Quarkonium magnetic dipole transitions in pNRQCD

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Motivation

- ▶ 1st principle computation of heavy quarkonium properties from QCD. To give model independent predictions with model independent errors.
- ▶ Determination of Standard Model parameters: m_Q , α_s , ...

Potential Non-Relativistic QCD describes the heavy quarkonium dynamics in the weak coupling situation. $m \gg mv \gg mv^2$

$$\left. \begin{array}{l} \left(i\partial_0 - \frac{\mathbf{p}^2}{2m} - V_s^{(0)}(r) \right) \Phi(\mathbf{r}) = 0 \\ + \text{corrections to the potential} \\ + \text{interaction with other low} \\ \quad \text{energy degrees of freedom} \end{array} \right\} \text{potential NRQCD} \quad E \sim mv^2$$

In the strict weak coupling regime the starting point is

$$V_s^{(0)} = V^C \equiv -C_F \frac{\alpha_s(\mu)}{r}.$$

To which extent is suitable a weak coupling description of heavy quarkonium.

$$V_s^{(0)} = -C_F \frac{\alpha_s(\mu)}{r} \left(1 + a_1 \frac{\alpha_s(\mu)}{4\pi} + a_2 \frac{\alpha_s^2(\mu)}{16\pi^2} + \dots \right)$$

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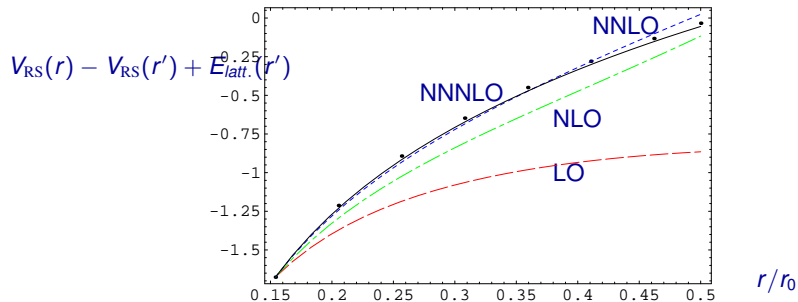
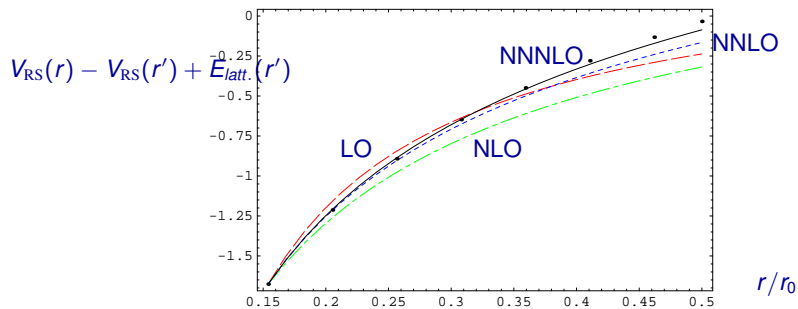
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Proposal: Reorganization of perturbation theory

Strict perturbation theory

$$H^{(0)} = \frac{\mathbf{p}^2}{m} + V^C \longrightarrow E_n^C, \phi_n^C(\mathbf{r})$$

Coulomb potential

$$V_s^C = -C_f \frac{\alpha_s(\mu)}{r}$$

Relativistic + $\delta V^{(0)}$ corrections:

$$\Delta H = \delta V^{(0)} + \frac{V^{(1)}}{m} + \frac{V^{(2)}}{m^2} + \dots$$

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Improved perturbation theory "acceleration of perturbation theory"

$$H^{(0)} = \frac{\mathbf{p}^2}{m} + V^{(0)} \longrightarrow E_n^{(0)}, \phi_n^{(0)}(\mathbf{r})$$

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Applied to inclusive electromagnetic decay ratios (Kiyo, Pineda, Signer).

Apply it to M1 radiative transitions.

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Forbidden transitions. Strict weak coupling: Brambilla, Jia, Vairo

$$\Gamma(n^3 S_1 \rightarrow n'^1 S_0 \gamma) \stackrel{n \neq n'}{=} \frac{4}{3} \alpha e_Q^2 \frac{k_\gamma^3}{m^2} \left[\frac{k_\gamma^2}{24} n' \langle r^2 \rangle_n^C + \frac{5}{6} \frac{n' \langle p^2 \rangle_n^C}{m^2} - \frac{2}{m^2} \frac{n' \langle V_{S^2}^C(\vec{r}) \rangle_n}{E_n^C - E_{n'}^C} \right]^2,$$

$$\Gamma(n^1 S_0 \rightarrow n'^3 S_1 \gamma) \stackrel{n \neq n'}{=} 4 \alpha e_Q^2 \frac{k_\gamma^3}{m^2} \left[\frac{k_\gamma^2}{24} n' \langle r^2 \rangle_n^C + \frac{5}{6} \frac{n' \langle p^2 \rangle_n^C}{m^2} + \frac{2}{m^2} \frac{n' \langle V_{S^2}^C(\vec{r}) \rangle_n^C}{E_n^C - E_{n'}^C} \right]^2,$$

$$V_{S^2}(\vec{r}) = \frac{4}{3} \pi C_f D_{S^2,s}^{(2)}(\nu) \delta^{(3)}(\vec{r})$$

$$D_{S^2,s}^{(2)}(\nu) = \alpha_s(\nu)$$

Forbidden transitions. Improved weak coupling: Pineda, Segovia

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$$V_{S^2}(\vec{r}) = \frac{4}{3} \pi C_f D_{S^2, s}^{(2)}(\nu) \delta^{(3)}(\vec{r})$$

$$D_{S^2, s}^{(2)}(\nu) = \alpha_s(\nu) c_F^2(\nu) - \frac{3}{2\pi C_f} (d_{sv}(\nu) + C_f d_{vv}(\nu))$$

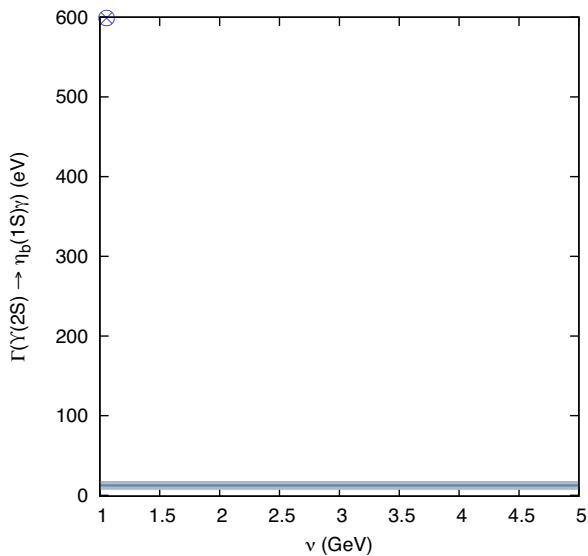
depends on the NRQCD Wilson coefficients. With LL accuracy they read

$$c_F(\nu) = z^{-C_A}, \quad d_{sv}(\nu) = d_{sv}(m),$$

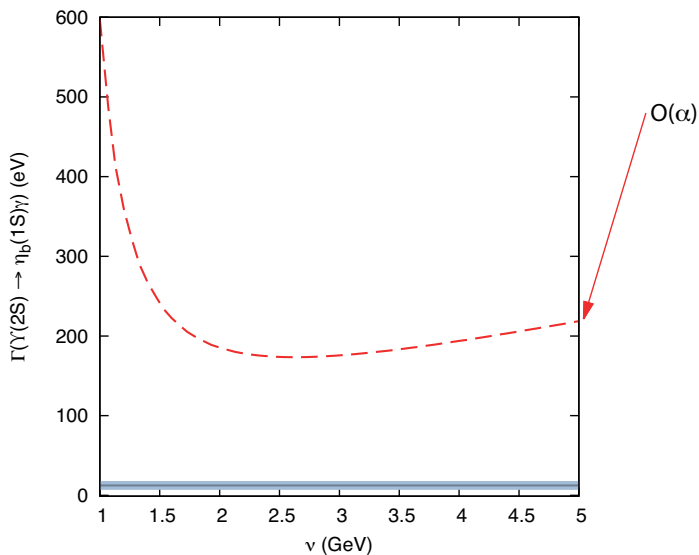
$$d_{vv}(\nu) = d_{vv}(m) + \frac{C_A}{\beta_0 - 2C_A} \pi \alpha_s(m) (z^{\beta_0 - 2C_A} - 1),$$

$$z = \left[\frac{\alpha_s(\nu)}{\alpha_s(m)} \right]^{\frac{1}{\beta_0}} \simeq 1 - \frac{1}{2\pi} \alpha_s(\nu) \ln \left(\frac{\nu}{m} \right),$$

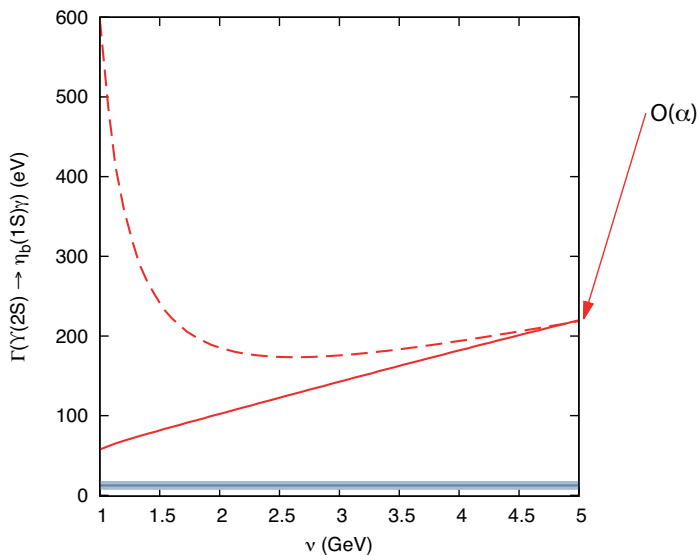
$$d_{sv}(m) = C_f \left(C_f - \frac{C_A}{2} \right) \pi \alpha_s(m), \quad d_{vv}(m) = - \left(C_f - \frac{C_A}{2} \right) \pi \alpha_s(m).$$



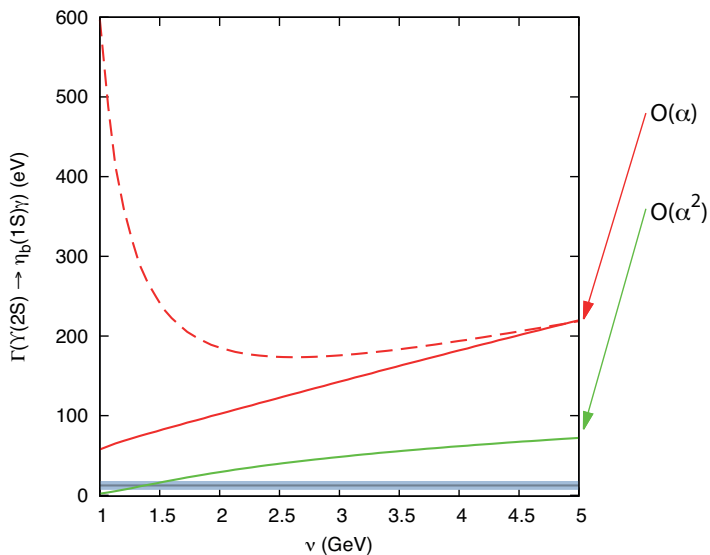
$$\text{no RG, } V_s^{(0)} = -C_F \frac{\alpha_s(\mu)}{r}, \mu = 1 \text{ GeV}$$



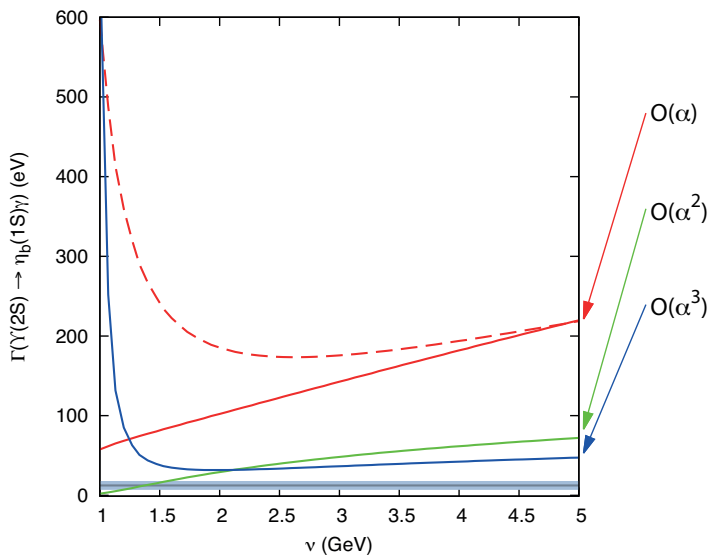
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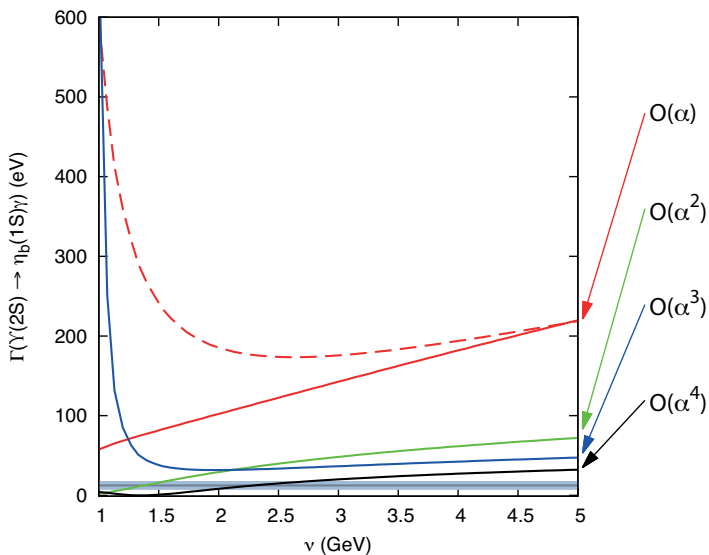
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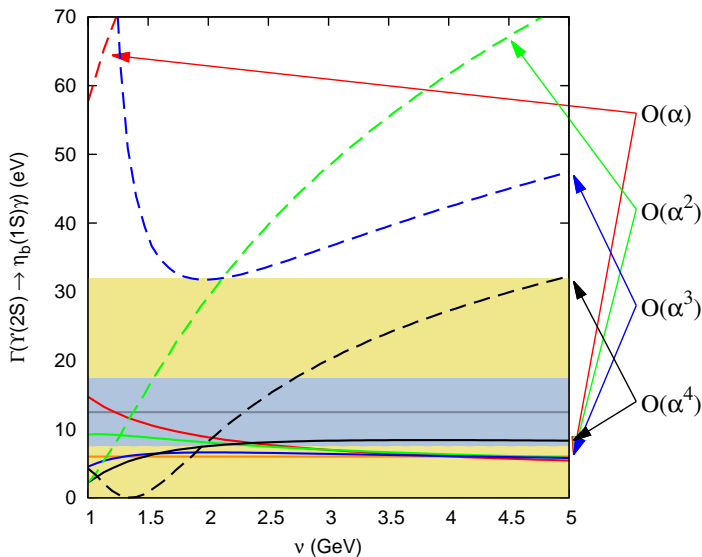
$$RG, V_s^{(0)} = -C_F \frac{\alpha_s(\mu)}{r} \left(1 + a_1 \frac{\alpha_s(\mu)}{4\pi} \right)$$



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Prefactor (keV)	$\mathcal{A}(r^2)$	$\mathcal{A}(\vec{p}^2)$	$\mathcal{A}(V_{S^2})$	Γ (eV)
10.3342	0.022	0.039	-0.042	6.3

Table : *The prefactor, the terms inside the brackets, and the total decay width*

$\Gamma_{\Upsilon(2S) \rightarrow \eta_b(1S)\gamma}$.

$$\Gamma_{\Upsilon(2S) \rightarrow \eta_b(1S)\gamma}^{(\text{th})} = 0.006 \pm 0.006 (\mathcal{O}(v^5))_{-0.006}^{+0.026} (N_m)_{+0.001}^{-0.001} (\alpha_s)_{+0.000}^{-0.000} (m_{\overline{MS}}) \text{ keV}.$$

$$\Gamma_{\Upsilon(2S) \rightarrow \eta_b(1S)\gamma}^{(\text{th})} = 6_{-06}^{+26} \text{ eV}.$$

Matrix element.

Experimental number: 0.035(7)

Ours: $0.025_{-0.025}^{+0.031}$

Lewis, Woloshyn: $\mathcal{O}(v^4) = 0.080(5)$; $\mathcal{O}(v^6) = 0.032(5)$

Both agreement but different physics. To be clarified.

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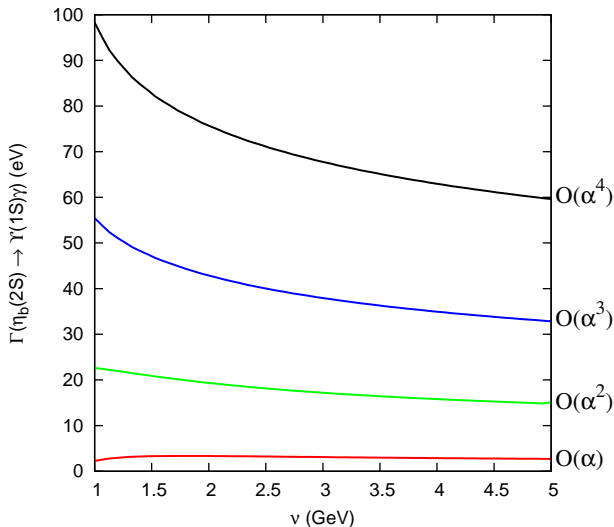


Figure : Plot of $\Gamma_{\eta_b(2S) \rightarrow \Upsilon(1S)\gamma}$ using the static potential $V_{RS'}^{(N)}$ at different orders in perturbation theory: $N = 0, 1, 2, 3$ with $\nu_r = \nu_f = 0.7$ GeV.

Strict weak coupling: Brambilla, Jia, Vairo (leading order= potential models)

$$\Gamma(n^3S_1 \rightarrow n^1S_0\gamma) = \frac{4}{3}\alpha e_Q^2 \frac{k_\gamma^3}{m^2} \left[1 + 2\kappa - \frac{5}{3} \frac{\langle p^2 \rangle_n^C}{m^2} \right],$$

$$\Gamma(n^3P_J \rightarrow n^1P_1\gamma) = \frac{3\Gamma(n^1P_1 \rightarrow n^3P_J\gamma)}{2J+1} = \frac{4}{3}\alpha e_Q^2 \frac{k_\gamma^3}{m^2} \left[1 + 2\kappa - d_J \frac{\langle p^2 \rangle_{n1}^C}{m^2} \right],$$

where $d_0 = 1$, $d_1 = 2$, $d_2 = 8/5$,

$$k_\gamma = |\vec{k}| = \frac{M_H^2 - M_{H'}^2}{2M_H},$$

and the anomalous magnetic moment of the heavy quark reads

$$\kappa = \kappa^{(1)}\alpha_s(m)$$

Improved weak coupling: Pineda, Segovia

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We also implement the renormalon.

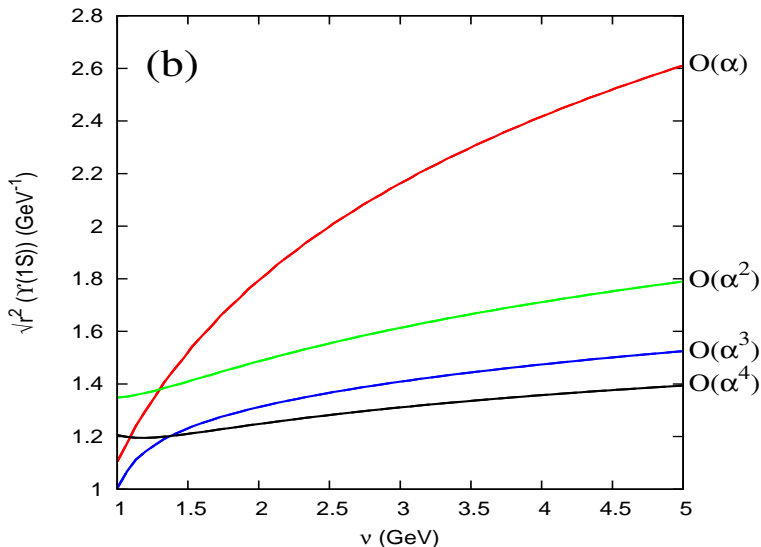


Figure : $\sqrt{\langle r^2 \rangle}_{10}$ using the static potential $V_{RS'}^{(N)}$ at different orders in perturbation theory: $N = 0, 1, 2, 3$. Dashed lines with $\nu_f = 0$. Continuous lines with $\nu_f = 0.7$ GeV. In both cases $\nu_r = \infty$ GeV.

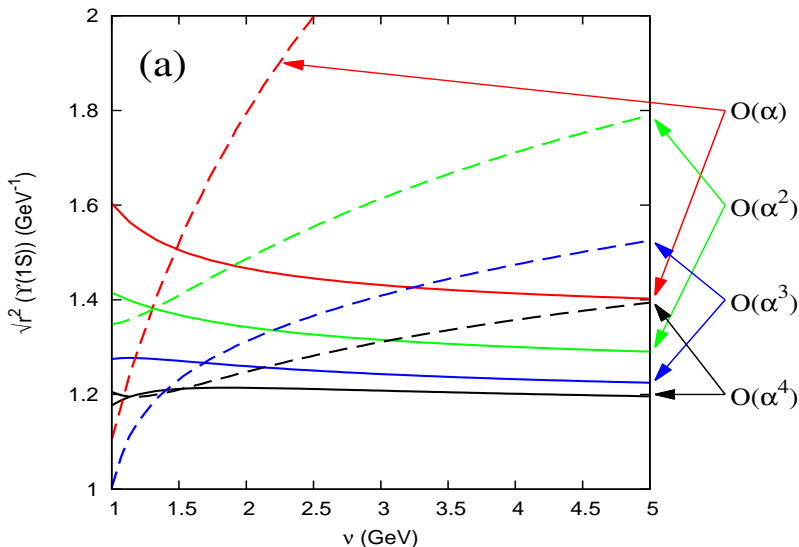


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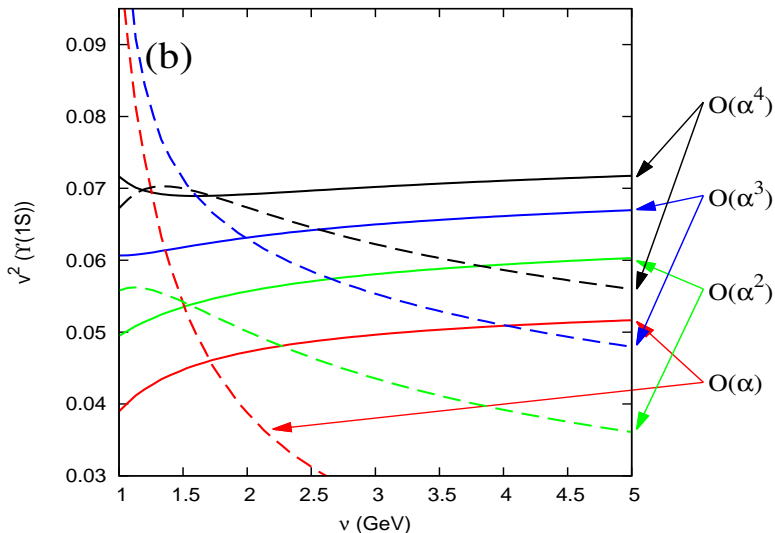


Figure : v_{10}^2 using the static potential $V_{RS'}^{(N)}$ at different orders in perturbation theory: $N = 0, 1, 2, 3$. Dashed lines with $\nu_r = \infty$. Continuous lines with $\nu_r = 0.7$ GeV. In both cases $\nu_f = 0.7$ GeV.

	$b\bar{b}(1S)$	$c\bar{c}(1S)$	$b\bar{b}(1P)$	$b\bar{b}(2S)$
v	0.26	0.43	0.25	0.24
$\sqrt{\langle r^2 \rangle}(\text{GeV}^{-1})$	1.2	2.2	2.1	2.9

Table : Estimates for $v \equiv \sqrt{\langle p^2 \rangle}/m^2$ and $\sqrt{\langle r^2 \rangle}$ for the heavy quarkonium states. For the $b\bar{b}(2S)$ state the number we give for v is quite uncertain.

	LO	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(v^2)$	$\alpha_s \times \mathcal{O}(\alpha_s^2)$	$v \times \mathcal{O}(v^2)$
$\delta\Gamma$ (eV)	14.87	1.29	0.73	-1.71	0.15	-0.45

Table : *The leading and subleading contributions to $\Gamma_{\Upsilon(1S) \rightarrow \eta_b(1S)\gamma}$. The last two numbers are error estimates obtained by multiplying the subleading $\mathcal{O}(\alpha_s^2)$ contribution by α_s and the subleading $\mathcal{O}(v^2)$ contribution by v .*

$$\Gamma_{\Upsilon(1S) \rightarrow \eta_b(1S)\gamma} = 15.18 \pm 0.45 (\mathcal{O}(v^3))_{-0.12}^{-0.05} (N_m)_{+0.04}^{-0.03} (\alpha_s)_{+0.20}^{-0.20} (m_{\overline{MS}}) \text{ eV},$$

which after combining the errors in quadrature reads

$$\Gamma_{\Upsilon(1S) \rightarrow \eta_b(1S)\gamma} = 15.18(51) \text{ eV}.$$

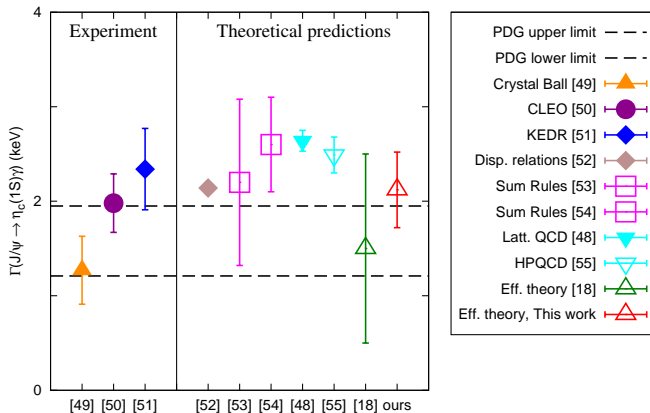


Figure : Comparison of different theoretical and experimental predictions for $\Gamma_{J/\psi \rightarrow \eta_c \gamma}$.

CONCLUSIONS

Precision is $k_\gamma^3/m^2 \times \mathcal{O}(\alpha_s^2, v^2)$ for the allowed transitions. The convergence for the $b\bar{b}$ ground state was quite good, and also quite reasonable for the $c\bar{c}$ ground state and the $b\bar{b}$ $1P$ state.

$$\begin{aligned}\Gamma_{\Upsilon(1S) \rightarrow \eta_b(1S)\gamma} &= 15.18(51) \text{ eV}, \\ \Gamma_{J/\psi(1S) \rightarrow \eta_c(1S)\gamma} &= 2.12(40) \text{ keV}, \\ \Gamma_{h_b(1P) \rightarrow \chi_{b0}(1P)\gamma} &= 0.962(35) \text{ eV}, \\ \Gamma_{h_b(1P) \rightarrow \chi_{b1}(1P)\gamma} &= 8.99(55) \times 10^{-3} \text{ eV}, \\ \Gamma_{\chi_{b2}(1P) \rightarrow h_b(1P)\gamma} &= 0.118(6) \text{ eV}.\end{aligned}$$

Precision is $k_\gamma^3/m^2 \times \mathcal{O}(v^4)$ for the forbidden transitions. Large logarithms associated with the heavy quark mass scale have also been resummed.

$$\Gamma_{\Upsilon(2S) \rightarrow \eta_b(1S)\gamma} = 6_{-06}^{+26} \text{ eV}.$$

This number is perfectly consistent with existing data.

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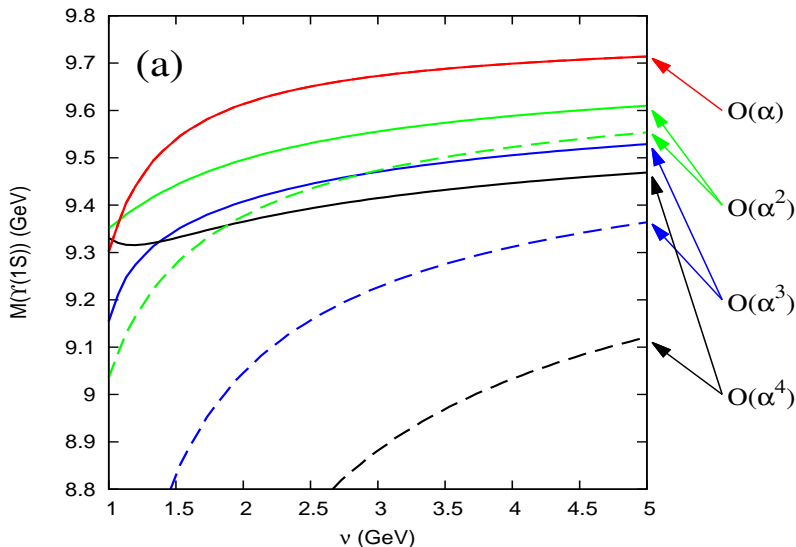


Figure : $M_{10} = 2m_{b,RS'}(0.7 \text{ GeV}) + E_{10}$ using the static potential $V_{RS'}^{(N)}$ at different orders in perturbation theory: $N = 0, 1, 2, 3$. Dashed lines with $\nu_f = 0$. Continuous lines with $\nu_f = 0.7 \text{ GeV}$. In both cases $\nu_r = \infty \text{ GeV}$.

$$V_{RS'}^{(N)}(r) = \begin{cases} (V^{(N)} + 2\delta m_{RS'}^{(N)})|_{\nu=\nu} \equiv \sum_{n=0}^N V_{RS',n} \alpha_s^{n+1}(\nu) & \text{if } r > \nu_r^{-1} \\ (V^{(N)} + 2\delta m_{RS'}^{(N)})|_{\nu=1/r} \equiv \sum_{n=0}^N V_{RS',n} \alpha_s^{n+1}(1/r) & \text{if } r < \nu_r^{-1}. \end{cases}$$

$$\delta m_X^{(N)}(\nu_f) = \nu_f \sum_{n=0}^N \delta m_X^{(n)}\left(\frac{\nu_f}{\nu}\right) \alpha_s^{n+1}(\nu)$$