

Charmonia from the lattice

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QWG, Beijing, 24th April 2013



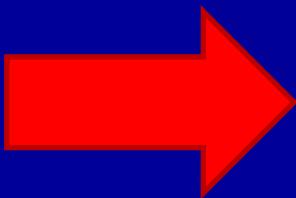
Hadron Spectrum Collaboration

Outline

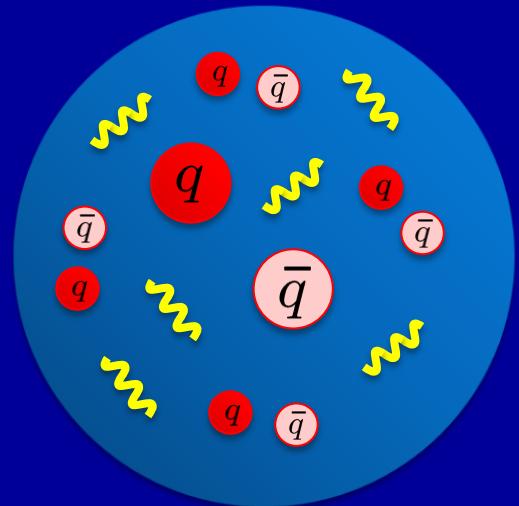
- Introduction
- Charmonia from the lattice
- Excited charmonium spectrum [JHEP 07 (2012) 126]
- Summary and outlook

Charmonium Spectroscopy

Quantum
Chromodynamics

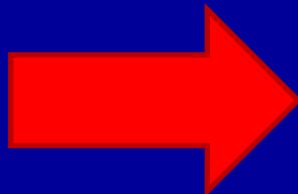


Probe low energy d.o.f. of QCD. Gluons?

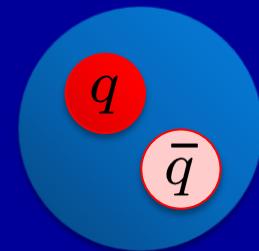


Charmonium Spectroscopy

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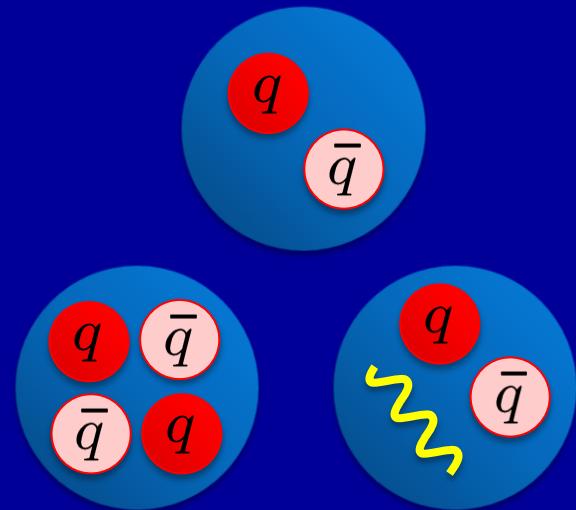
Charmonium Spectroscopy

Quantum Chromodynamics

?

Probe low energy d.o.f. of QCD. Gluons?

Exotic quantum numbers ($J^{PC} = \mathbf{0}^{+-}, \mathbf{1}^{-+}, \mathbf{2}^{+-}, \dots$)
– can't just be a $q\bar{q}$ pair

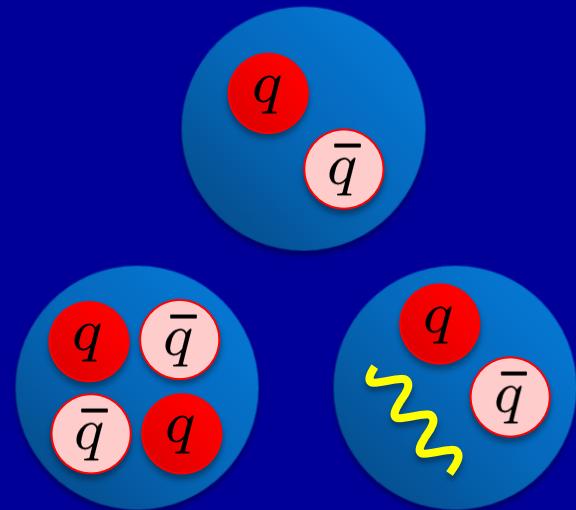


Charmonium Spectroscopy

Quantum Chromodynamics



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X(3872), Y(4260), Z⁺(4430), ... ?

Exotic 1⁻⁺ in charmonium?

BESIII

LHC



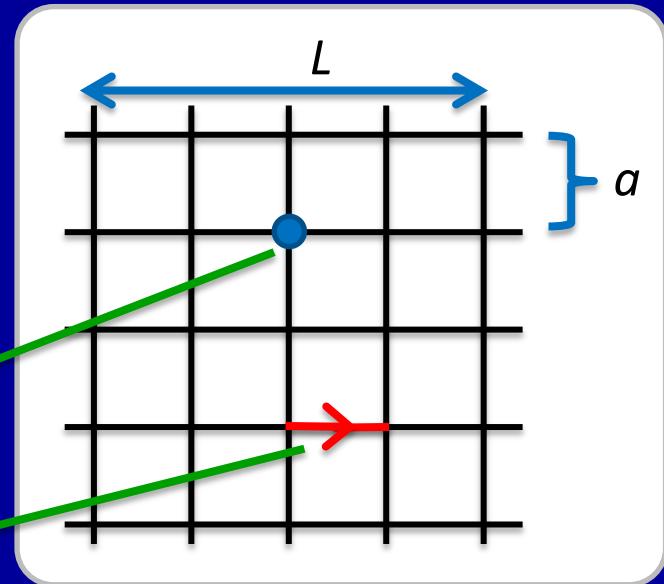
...

Need first-principles calculations in QCD

QCD on a lattice

Discretise (spacing = a) – regulator
Finite volume \rightarrow finite no. of d.o.f.

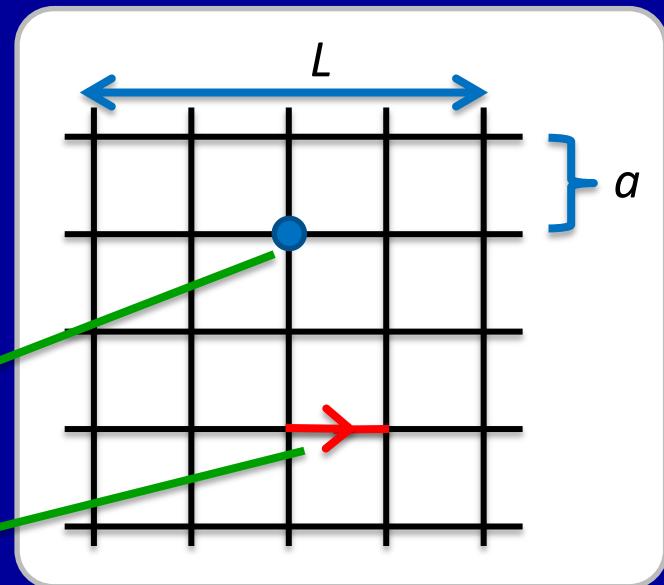
Quarks fields
Gauge fields (gluons)



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Quarks fields
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Path integral formulation

$$\int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U f(\psi, \bar{\psi}, U) e^{-\tilde{S}[\psi, \bar{\psi}, U]}$$

Euclidean time: $t \rightarrow i t$

Numerical methods (Monte Carlo)
 \rightarrow high performance computing

- Finite a and L
(and reduced sym.)
- Unphysical m_π

Spectroscopy on the lattice

Energy eigenstates of
Hamiltonian from 2-pt corr. fns.

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle$$

Operators (interpolating fields)

$$C_{ij}(t) = \sum_n \frac{e^{-E_n t}}{2} \langle 0 | \mathcal{O}_i(0) | n \rangle \langle n | \mathcal{O}_j^\dagger(0) | 0 \rangle$$

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Large basis of ops → matrix of corrs.

Eigenvalue problem

$$C_{ij}(t) v_j^{(n)} = \lambda^{(n)}(t) C_{ij}(t_0) v_j^{(n)}$$

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$$C_{ij}(t) v_j^{(n)} = \lambda^{(n)}(t) C_{ij}(t_0) v_j^{(n)}$$

$$Z_i^{(n)} \equiv \langle 0 | \mathcal{O}_i | n \rangle$$

$$\lambda^{(n)}(t) \rightarrow e^{-E_n(t-t_0)}$$

Eigenvectors → $Z^{(n)}$

$(t \gg t_0)$

Charmonia from the lattice

Recent lattice charmonia results

Recent review: Daniel Mohler, Charm 2012 [arXiv:1209.5790]

Lower lying states, hyperfine splittings, etc – Craig McNeile's talk

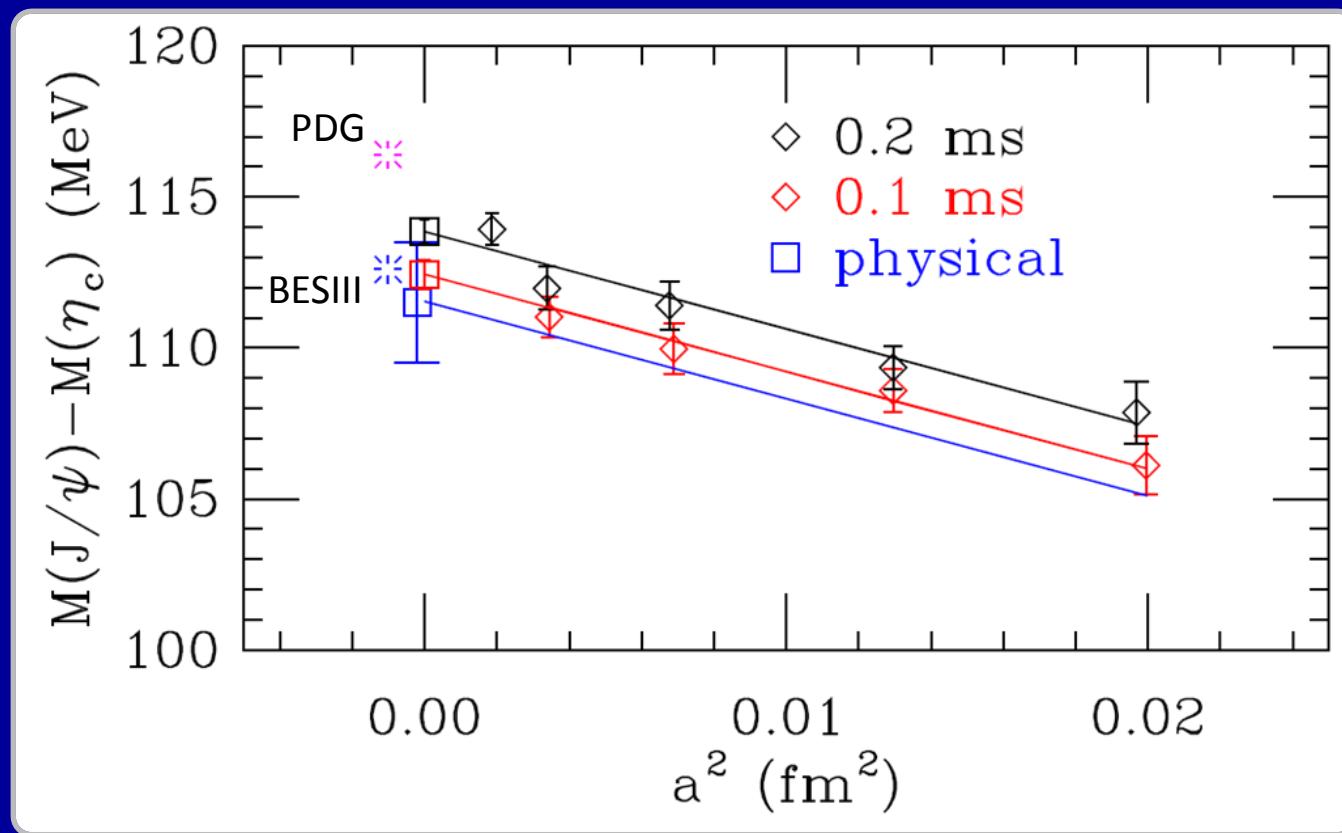
Higher lying states

- Bali, Collins, Ehmann, PR D84, 094506 (2011)
- Bali et al (BMW & QCDSF), Lattice 2011 [arXiv:1108.6147]
- Mohler, Woloshyn, PR D84, 054505 (2011)
- Hadron Spectrum Collab, JHEP 07 (2012) 126
- Mohler, Prelovsek, Woloshyn, PR D87, 034501 (2013)

Charm/charmonium talks at QWG

- Gunnar Bali – Multiply charmed states [Weds am]
- Craig McNeile – J/ψ , η_c , α_s , m_c , m_b [Thrs am, two talks]
- Francesco Sanfilippo – J/ψ and h_c radiative decays to η_c [Thrs pm]

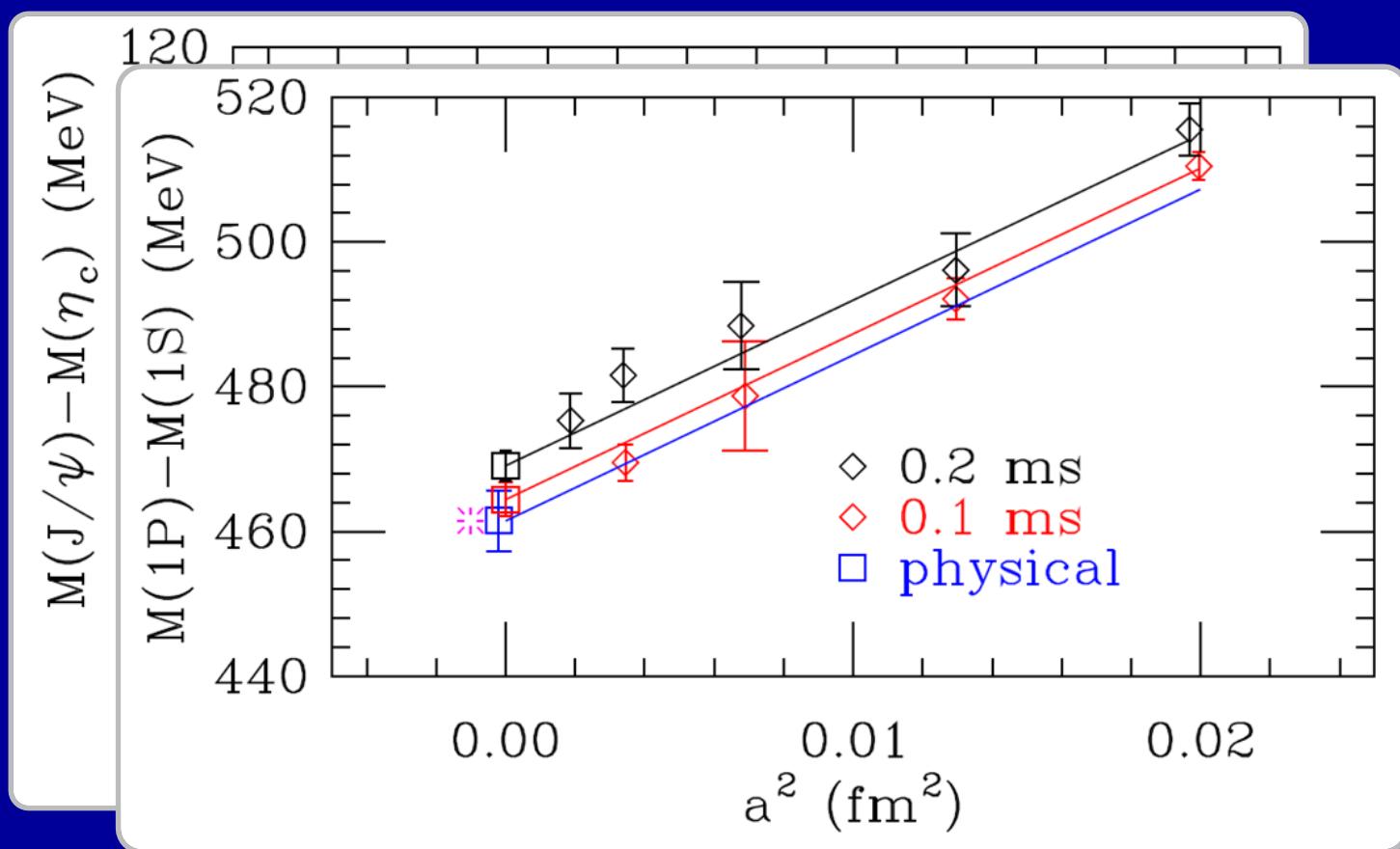
Charmonia from the lattice – other work (lower states)



$N_f = 2+1$, extrapolate to continuum limit and physical m_π

DeTar et al (Fermilab and MILC), Lattice 2012 [arXiv:1211.2253]

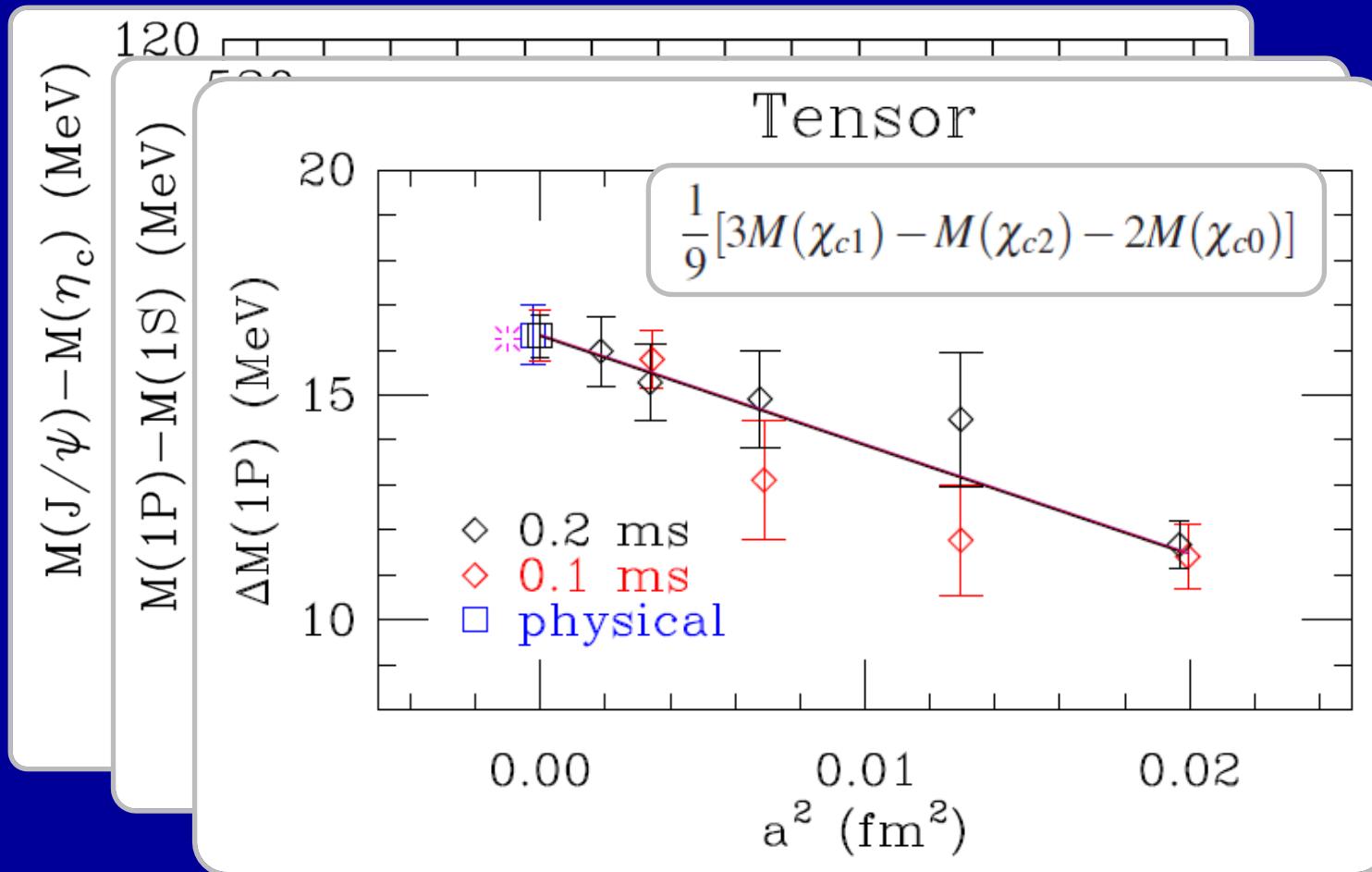
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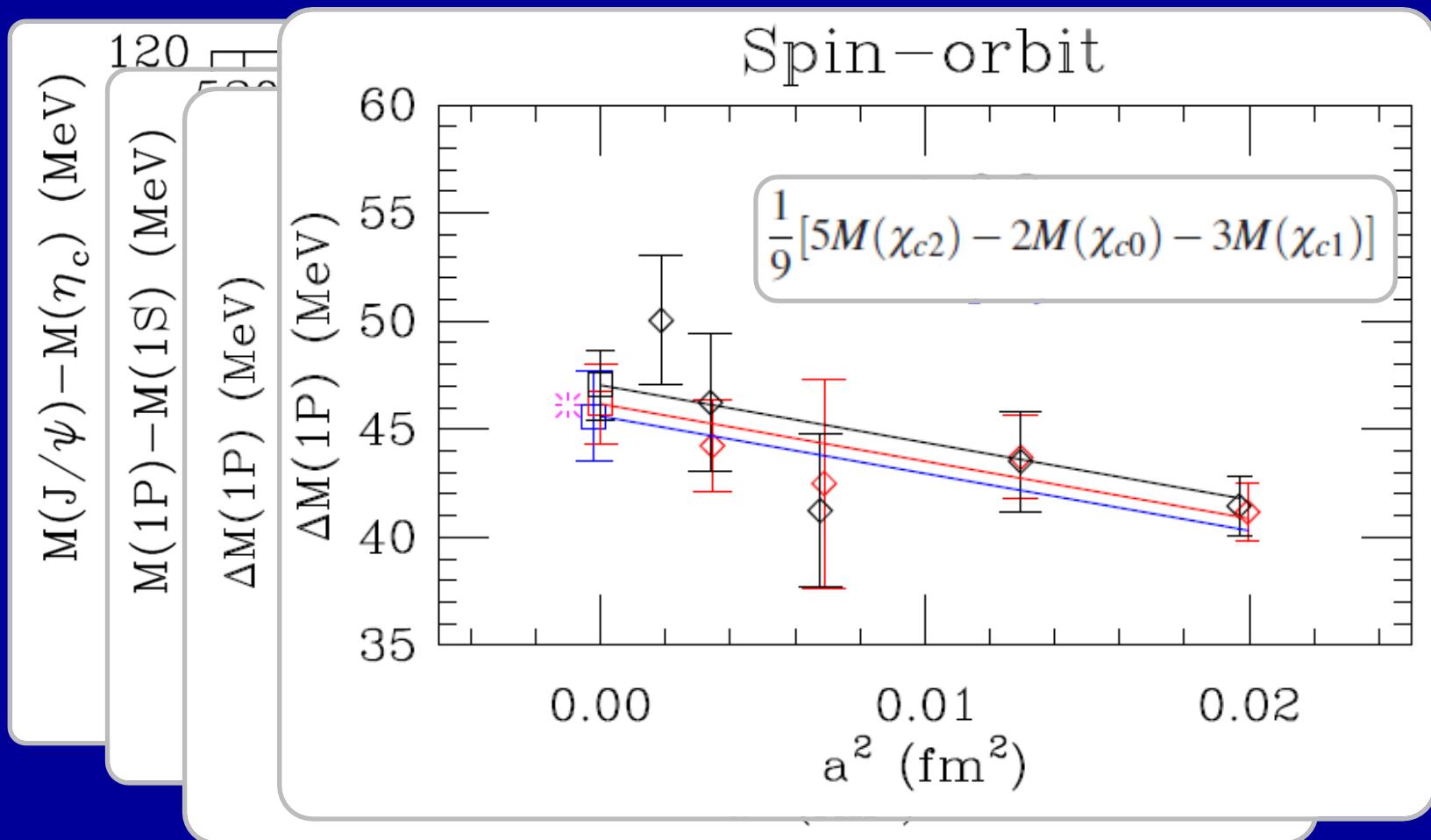
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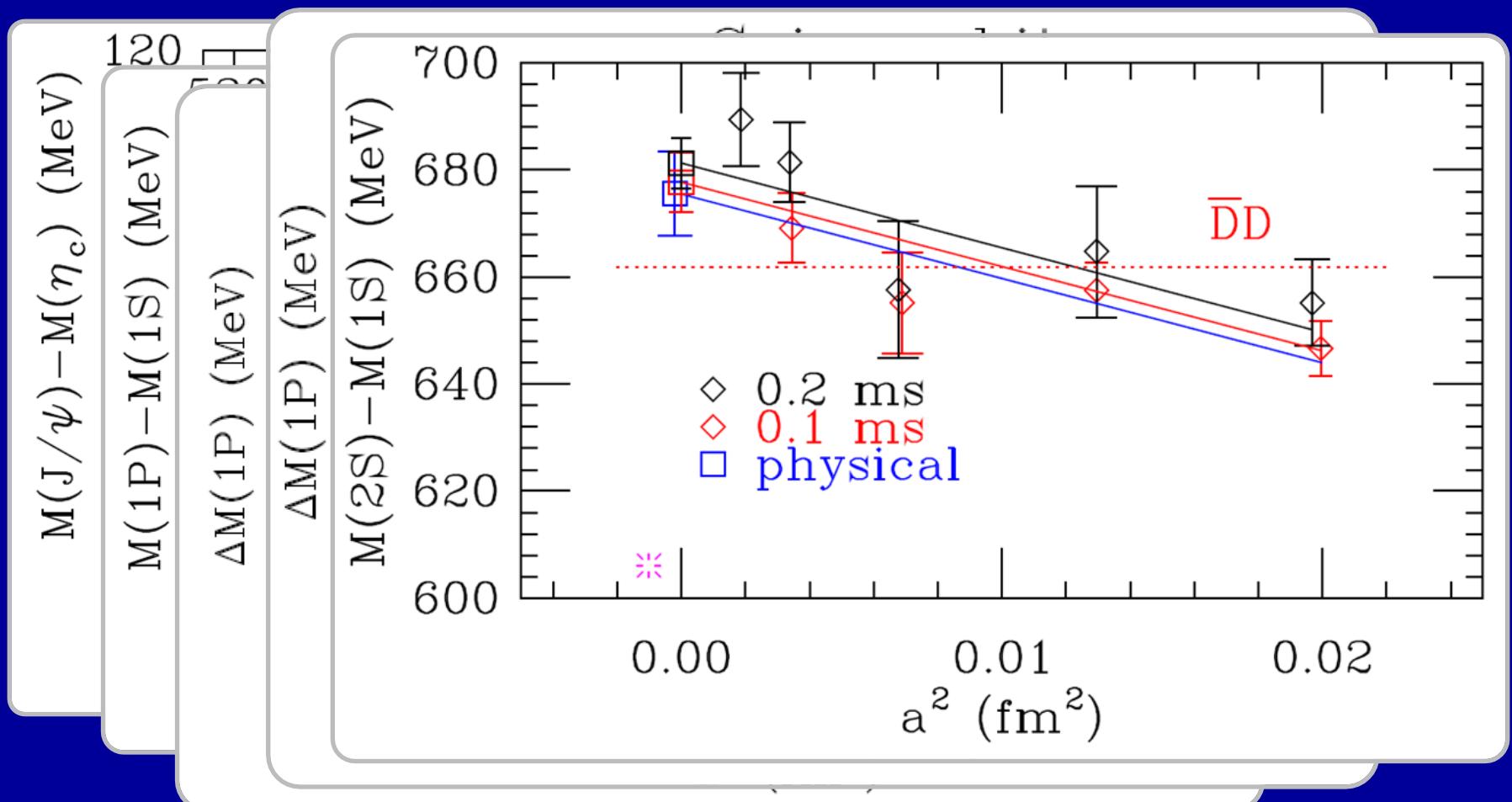
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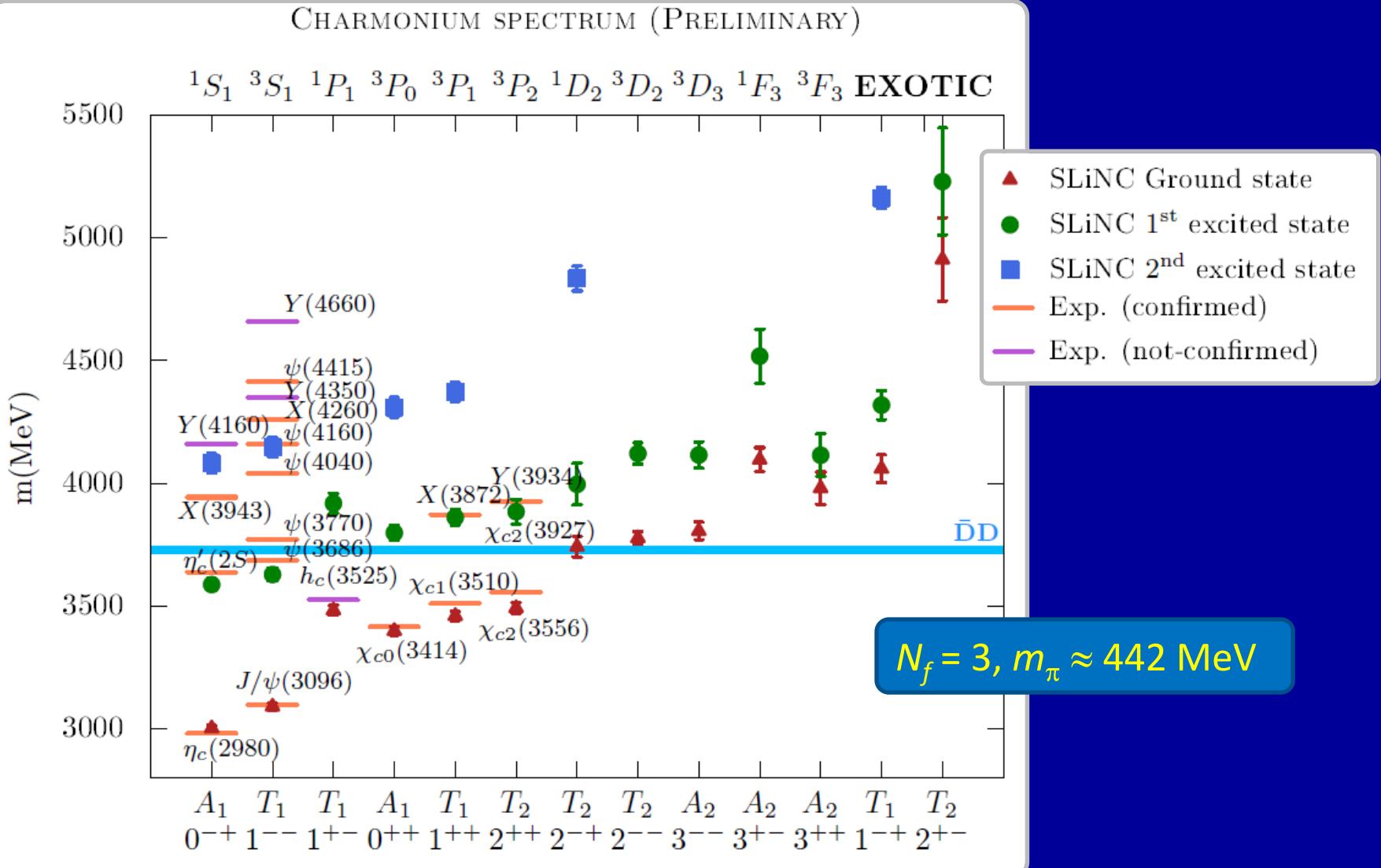
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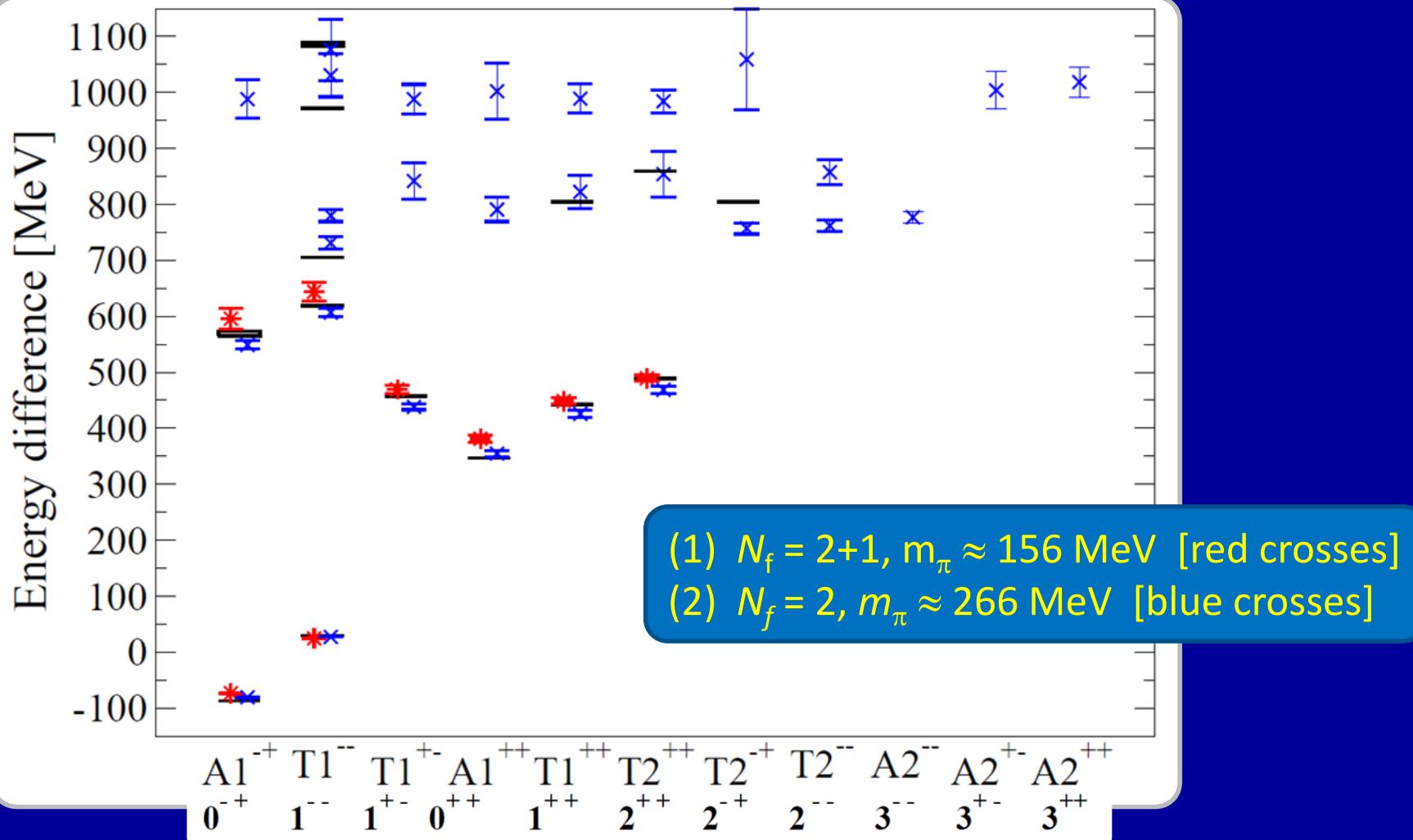
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Charmonia from the lattice – other work (higher states)



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Charmonia from the lattice – other work (higher states)



(1) Mohler, Woloshyn, PR D84, 054505 (2011)

(2) Mohler, Prelovsek, Woloshyn, PR D87, 034501 (2013)

Excited Charmonium Spectrum

Hadron Spectrum Collaboration

JHEP 07 (2012) 126 – Liuming Liu, Graham Moir, Mike Peardon, Sinéad Ryan, CT, Pol Vilaseca;
Jo Dudek, Robert Edwards, Bálint Joó, David Richards

Excited Charmonium Spectrum

Hadron Spectrum Collaboration

Dynamical (unquenched) u , d and s quarks [$N_f = 2+1$]

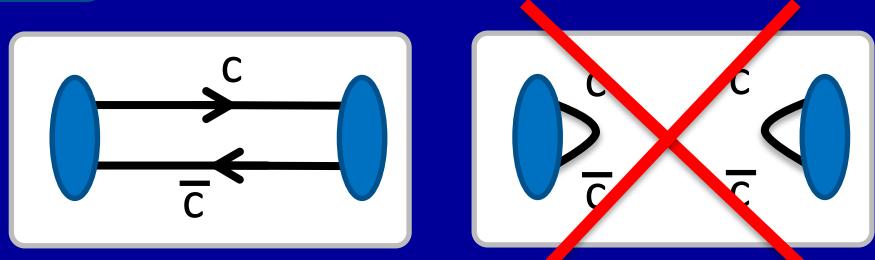
Relativistic charm

Anisotropic – finer in temporal dir ($a_s/a_t \approx 3.5$), $a_s \approx 0.12$ fm

Two volumes: 16^3 , $\mathbf{24}^3$ ($L_s \approx 1.9$, 2.9 fm)

$M_\pi \approx 400$ MeV

Only connected contributions



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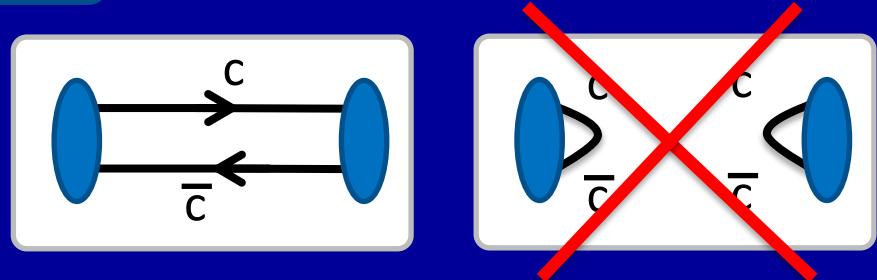
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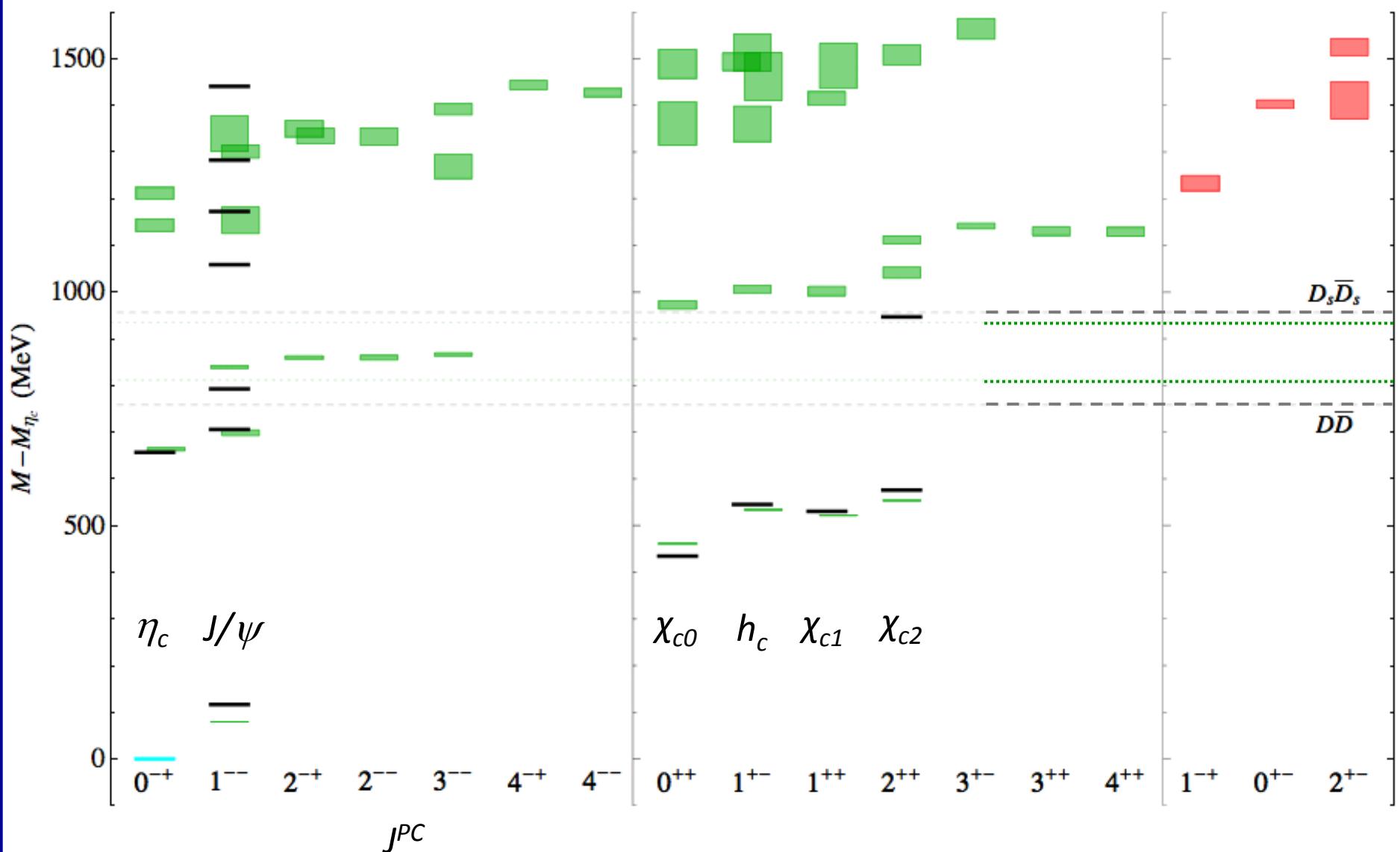
Various techniques

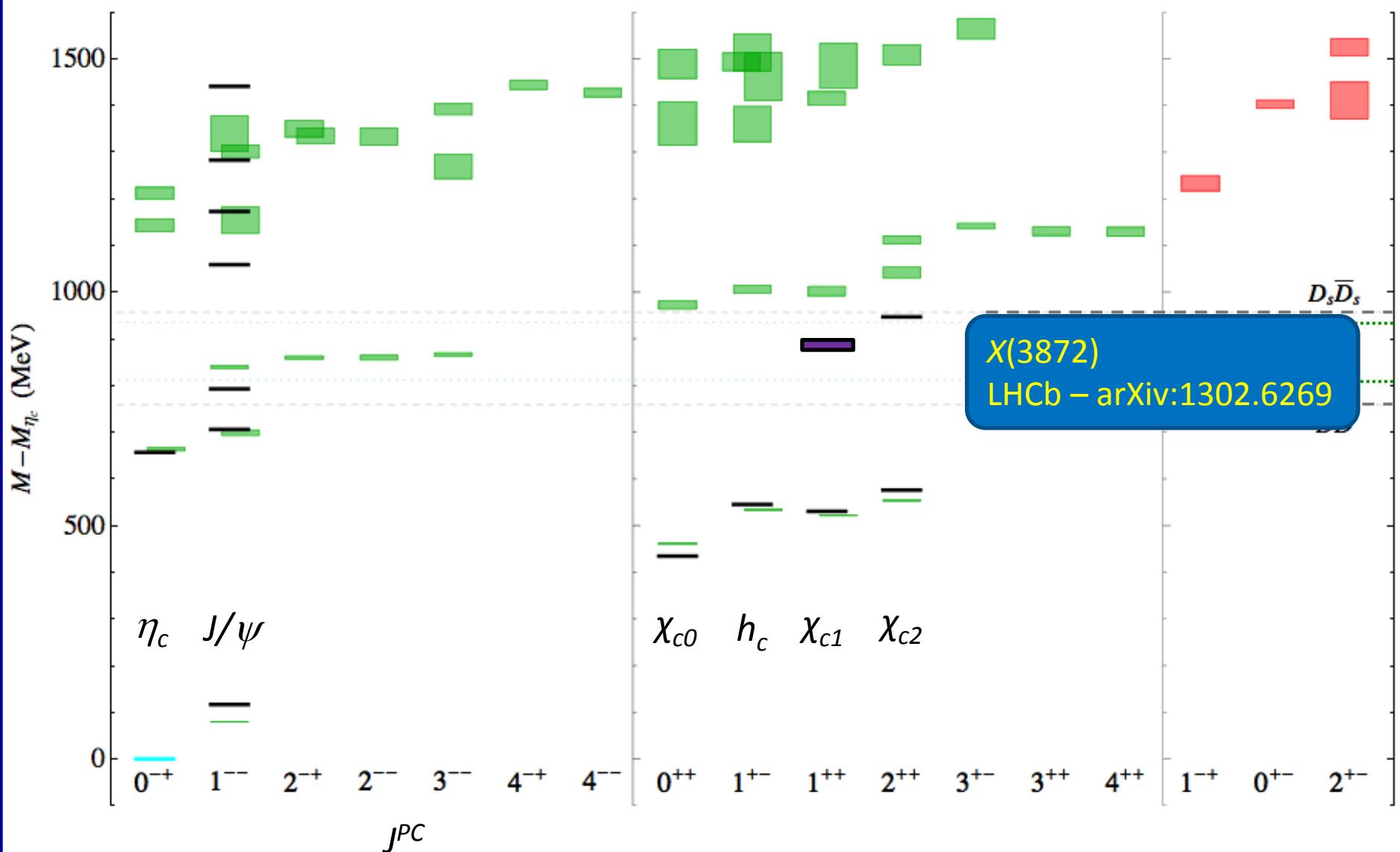
$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle$$

Large no. of ops. of different structures, including gluonic excitations

→ Large number and variety of states

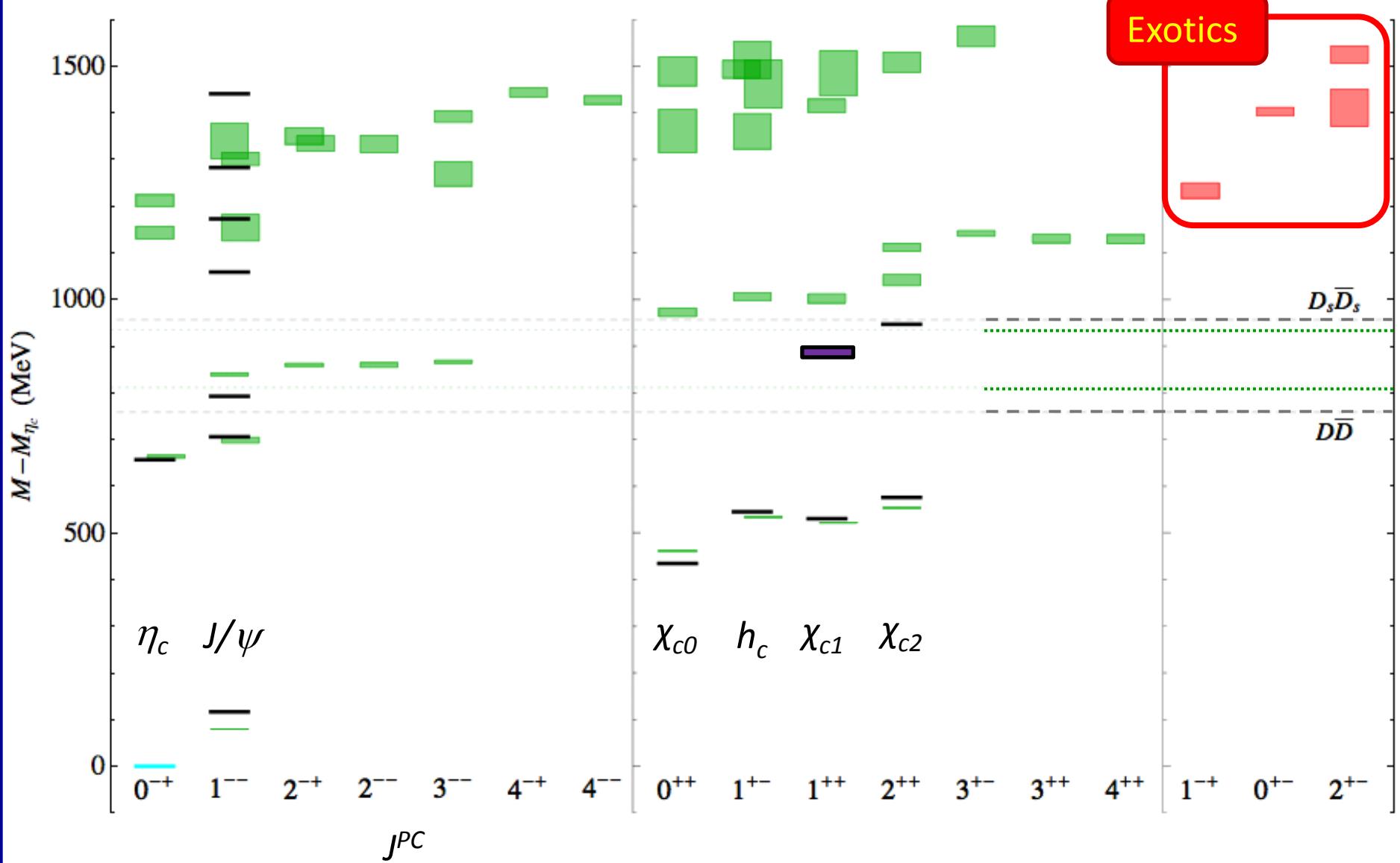
$$\sim \bar{\psi} \Gamma D \dots \psi$$





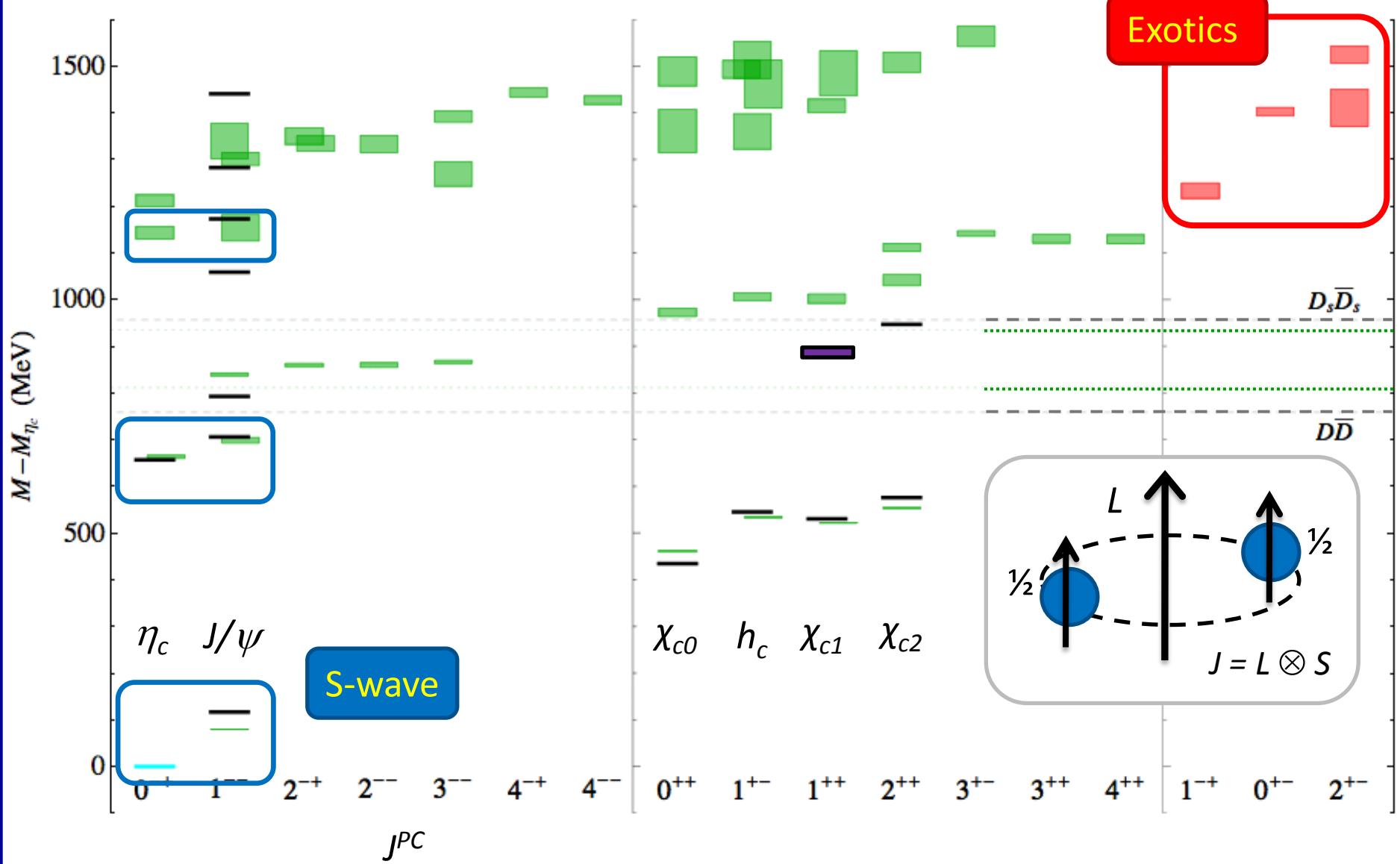
$24^3, M_\pi \approx 400$ MeV

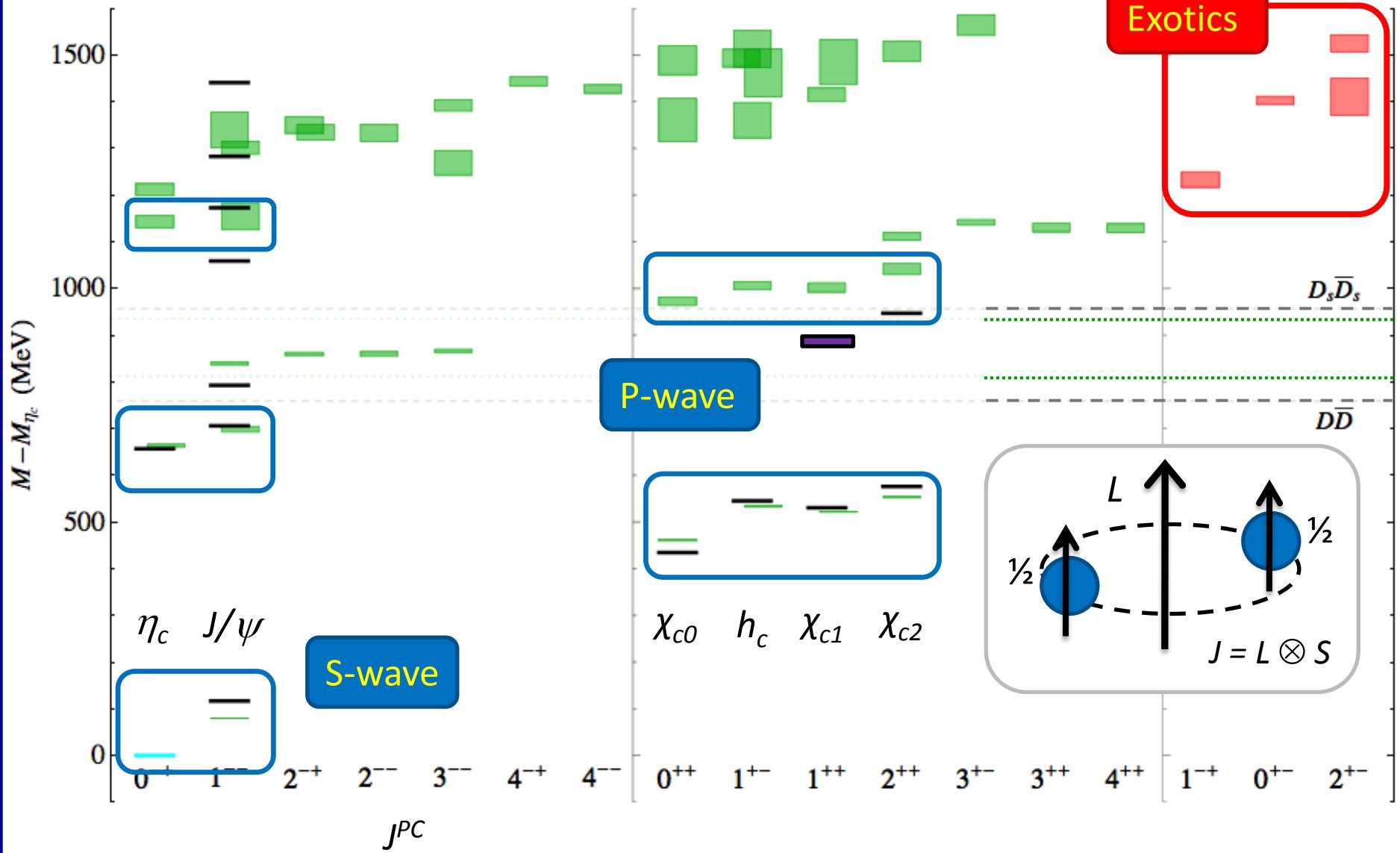
JHEP 07 (2012) 126

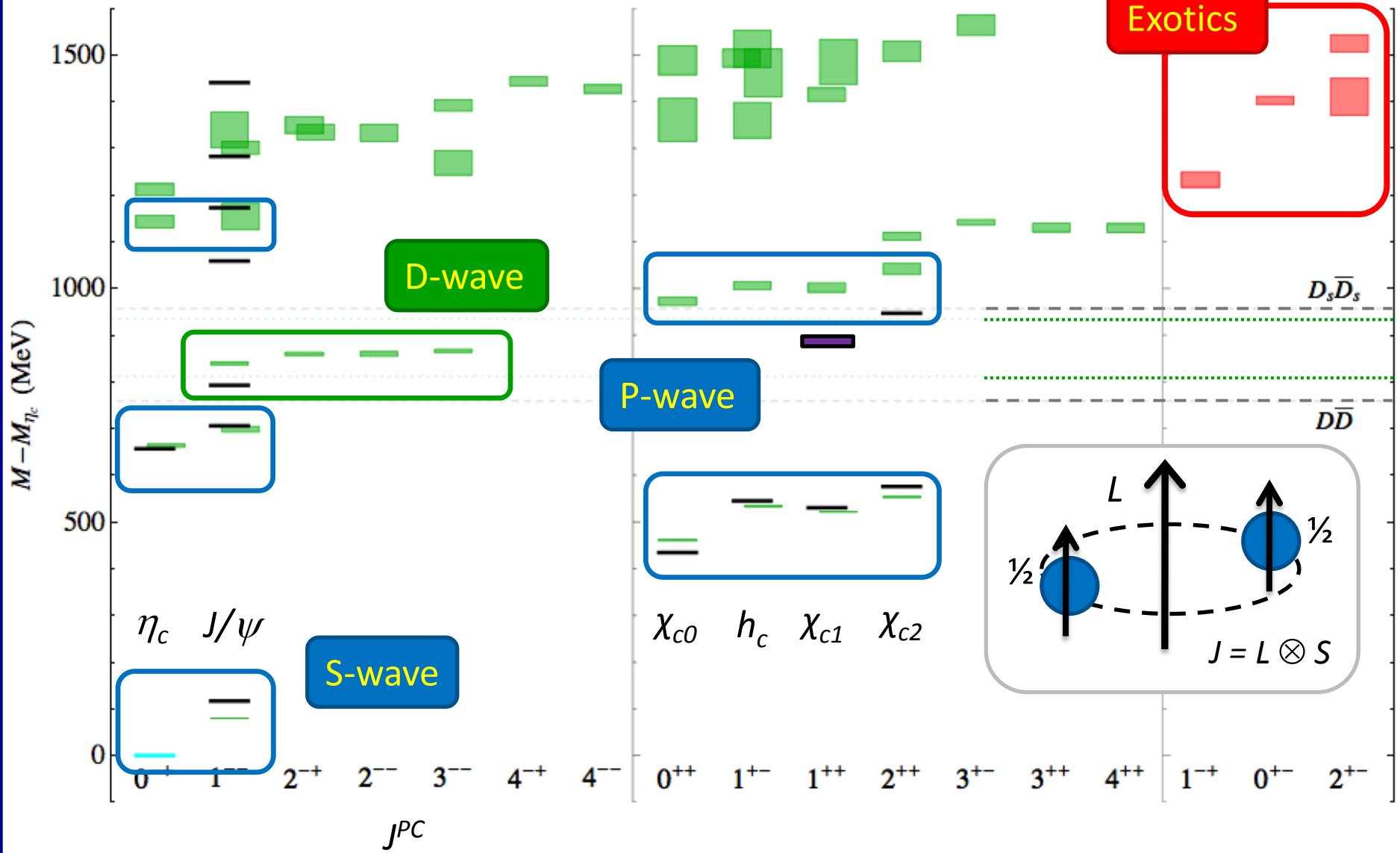


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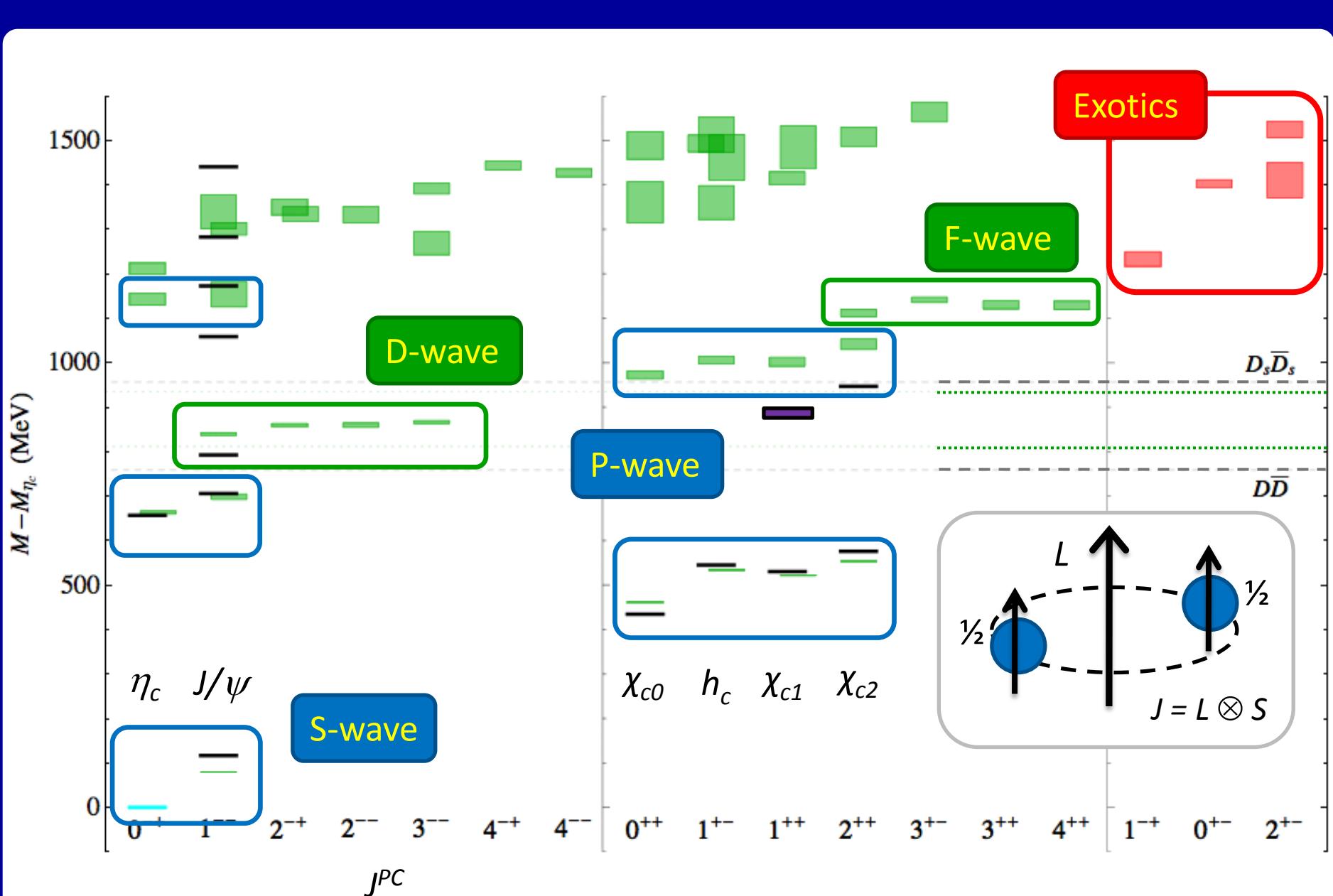






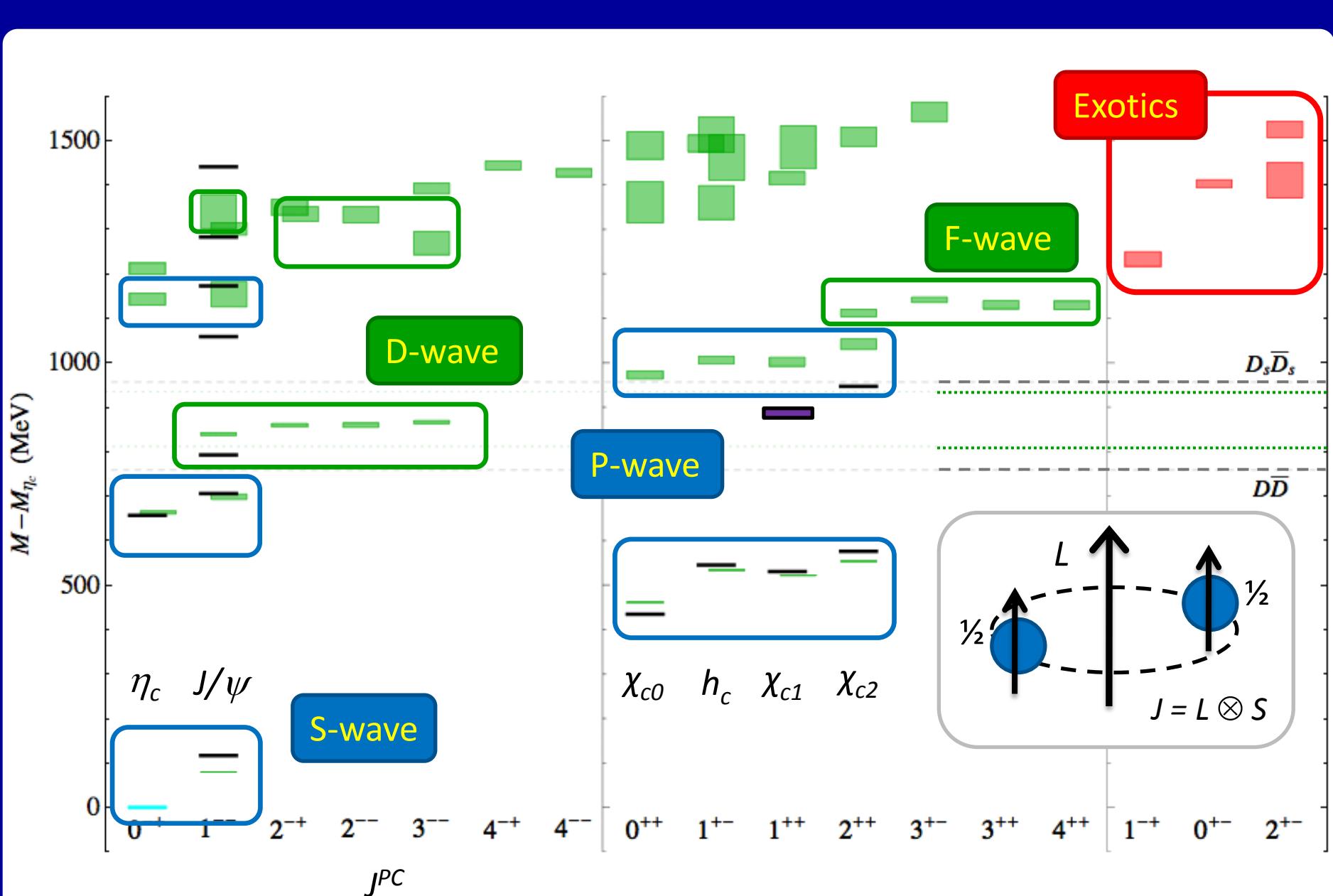
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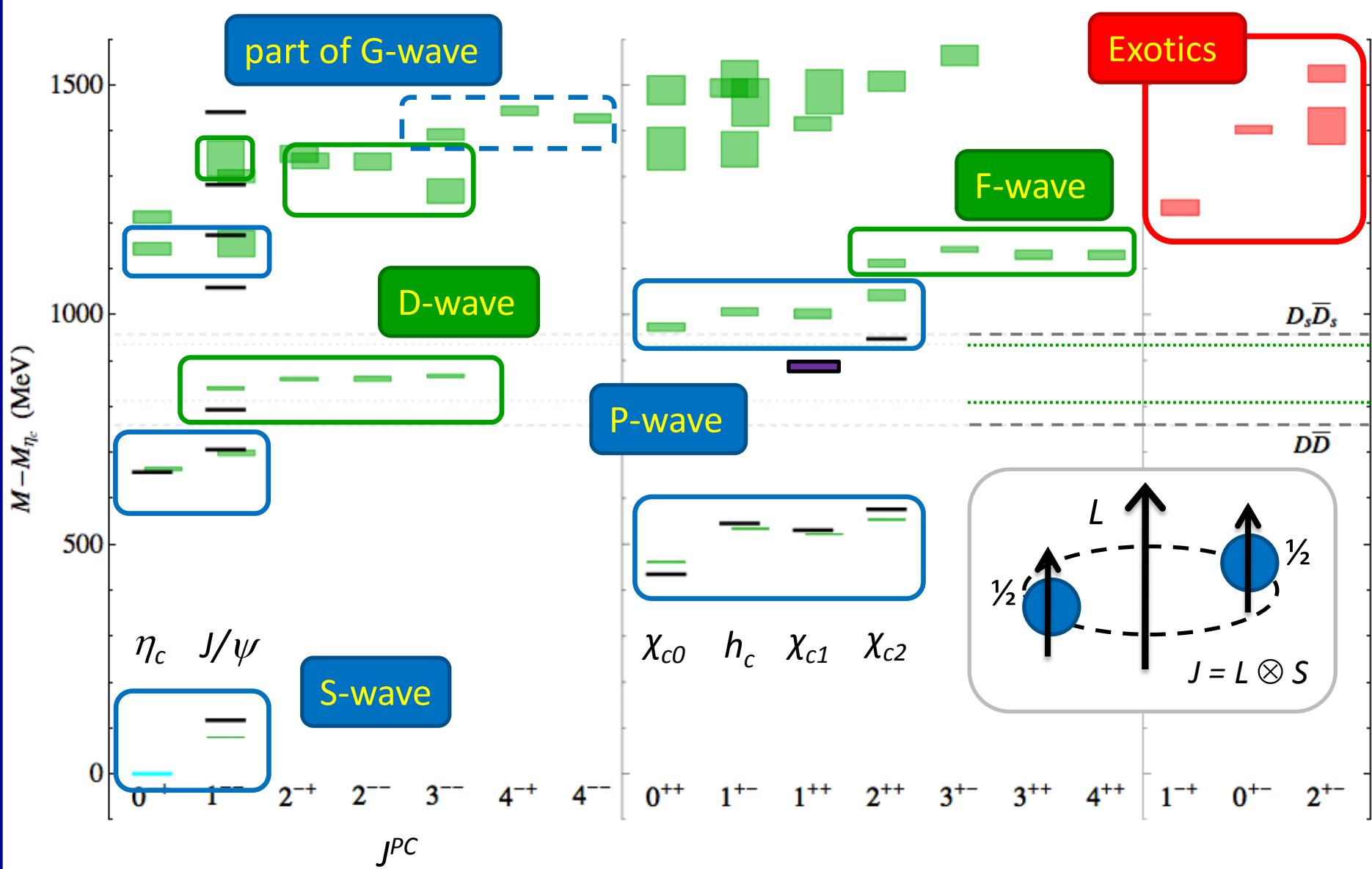
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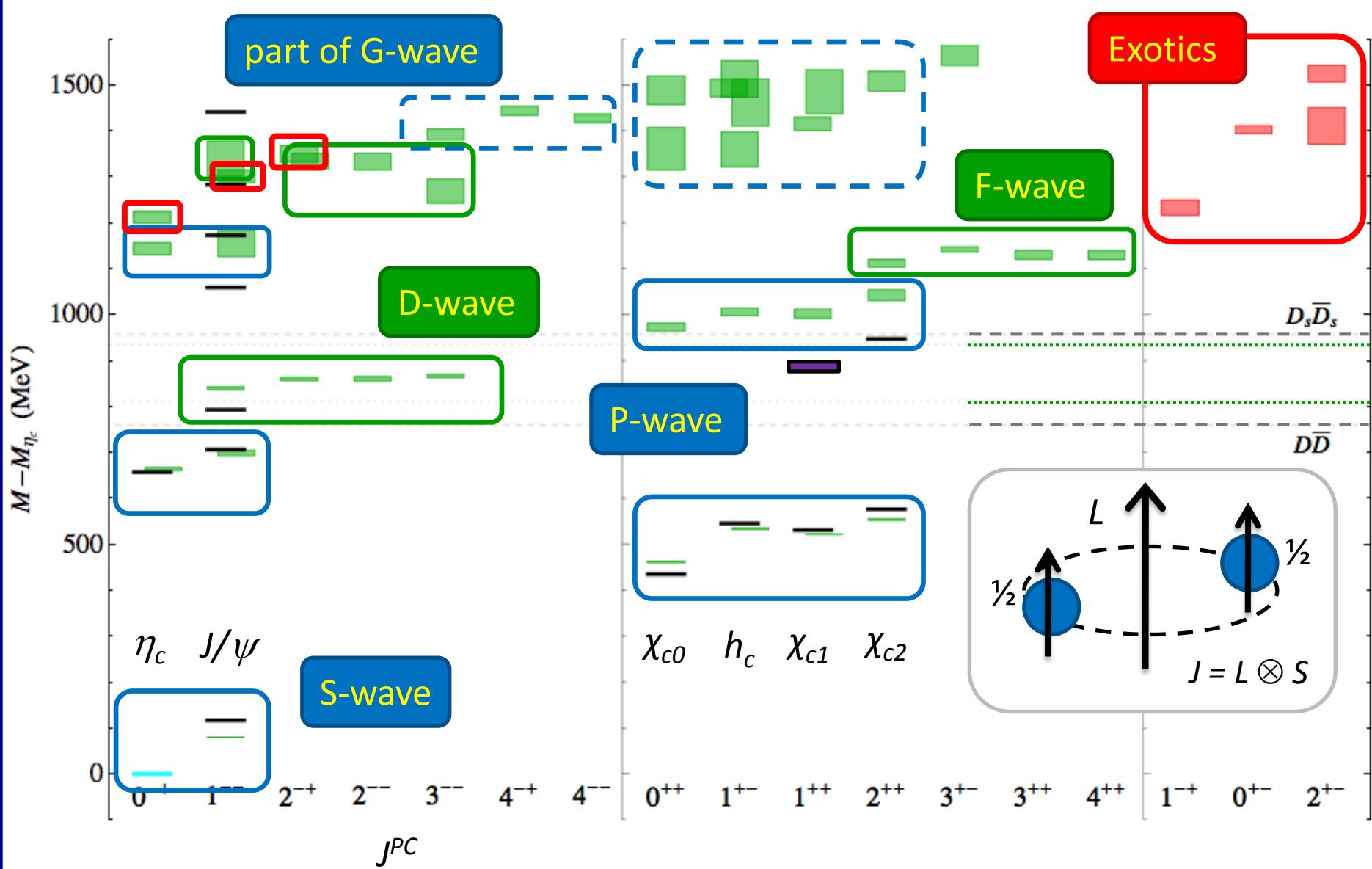
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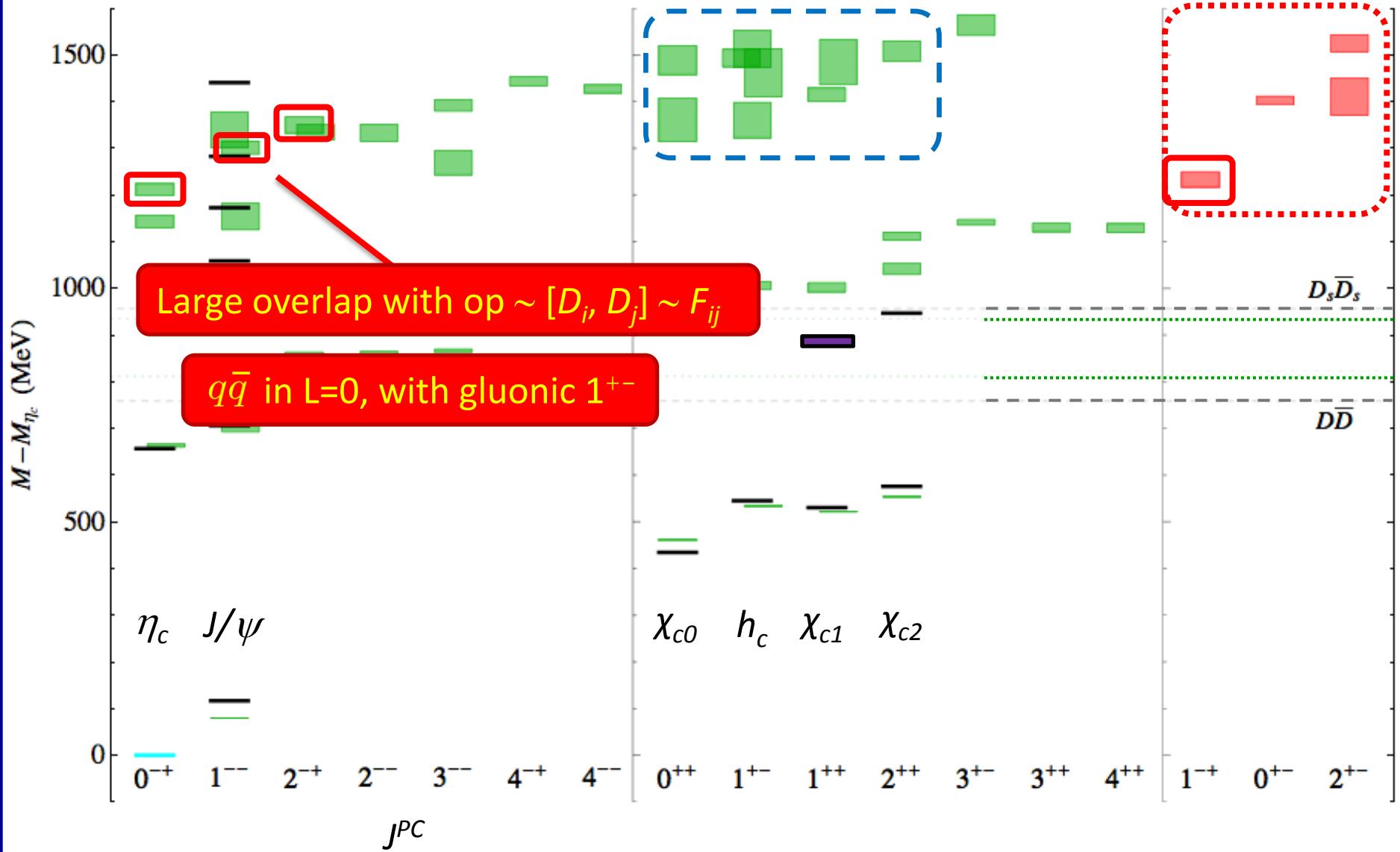




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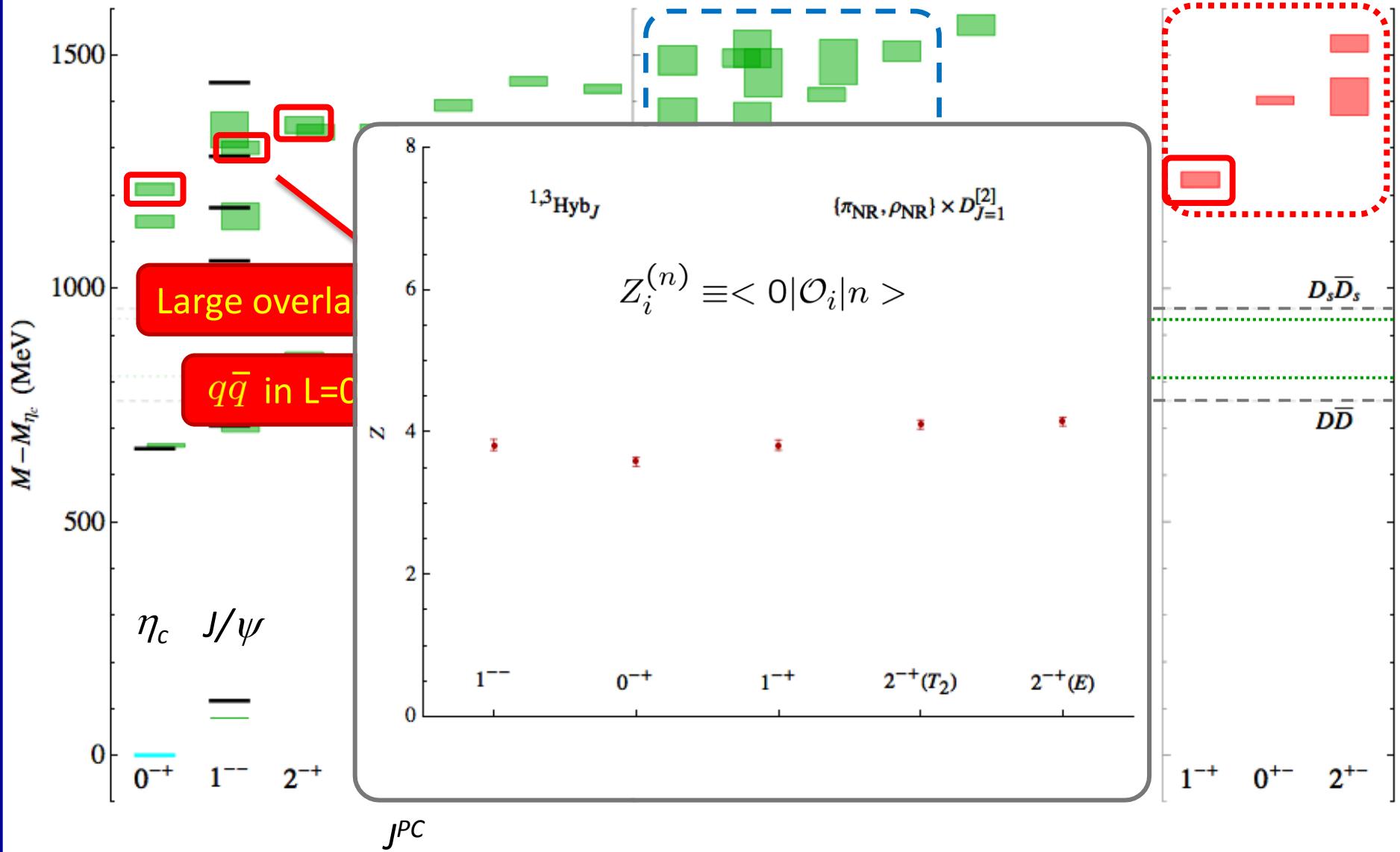
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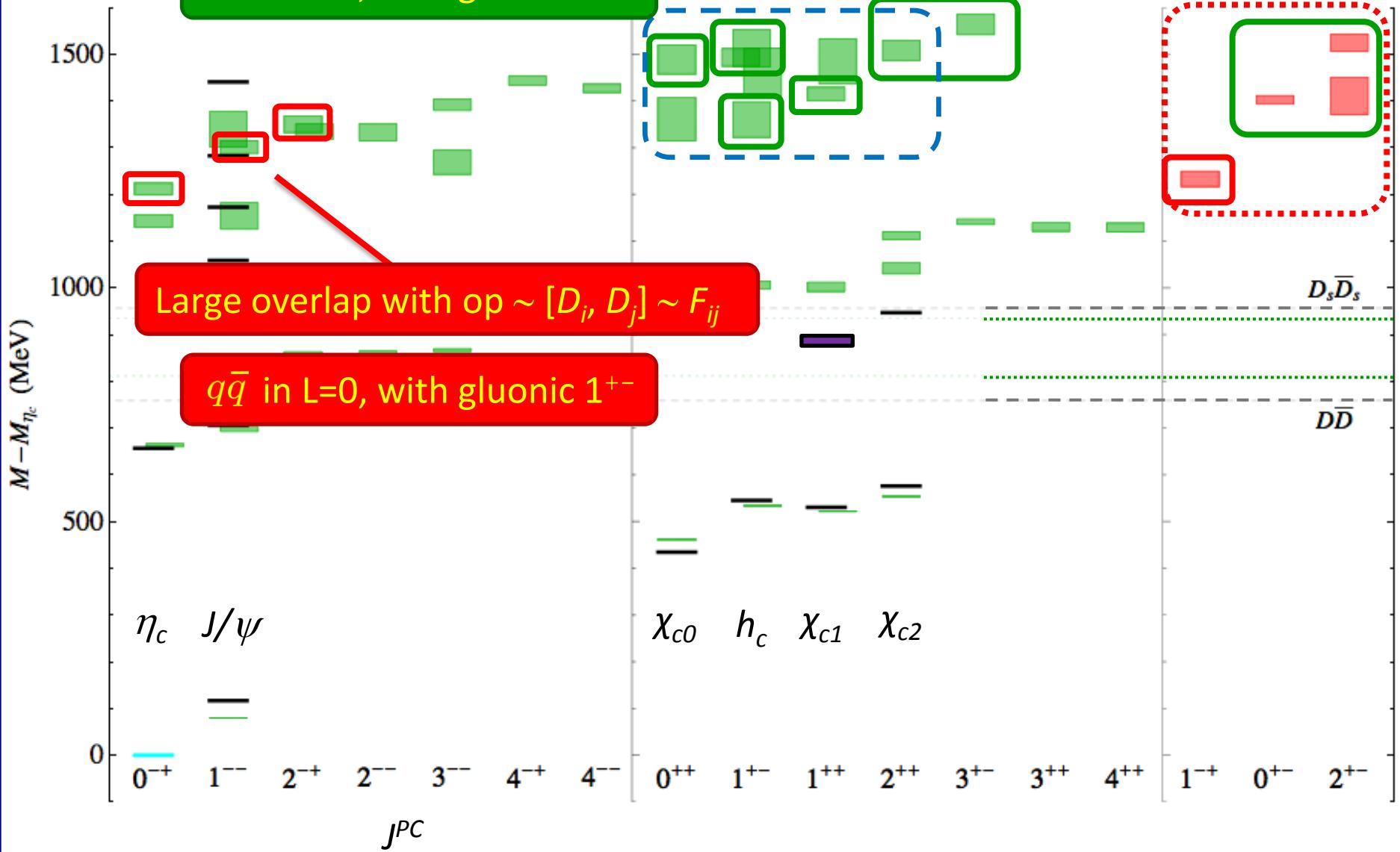
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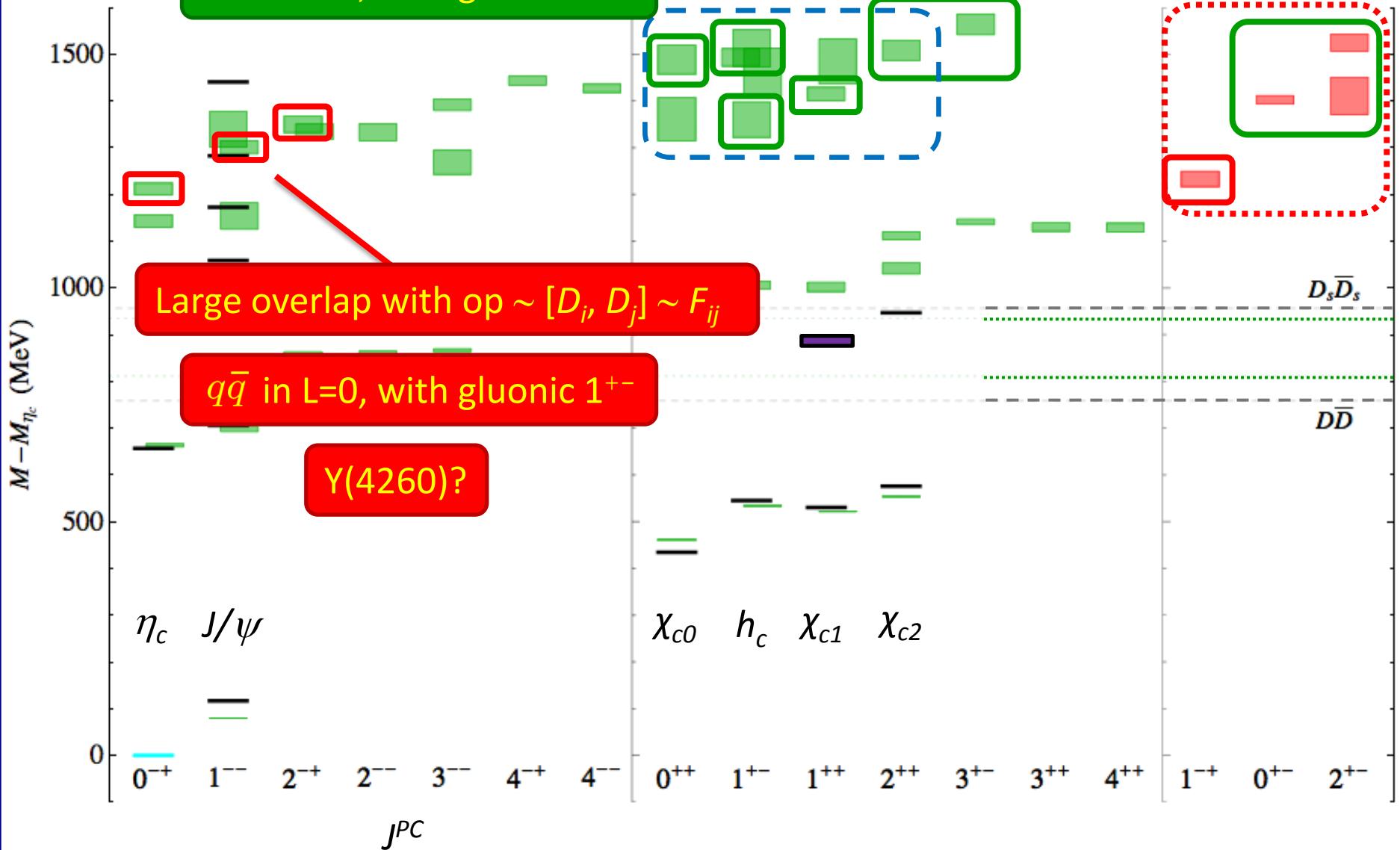
$q\bar{q}$ in L=1, with gluonic 1^{+-}



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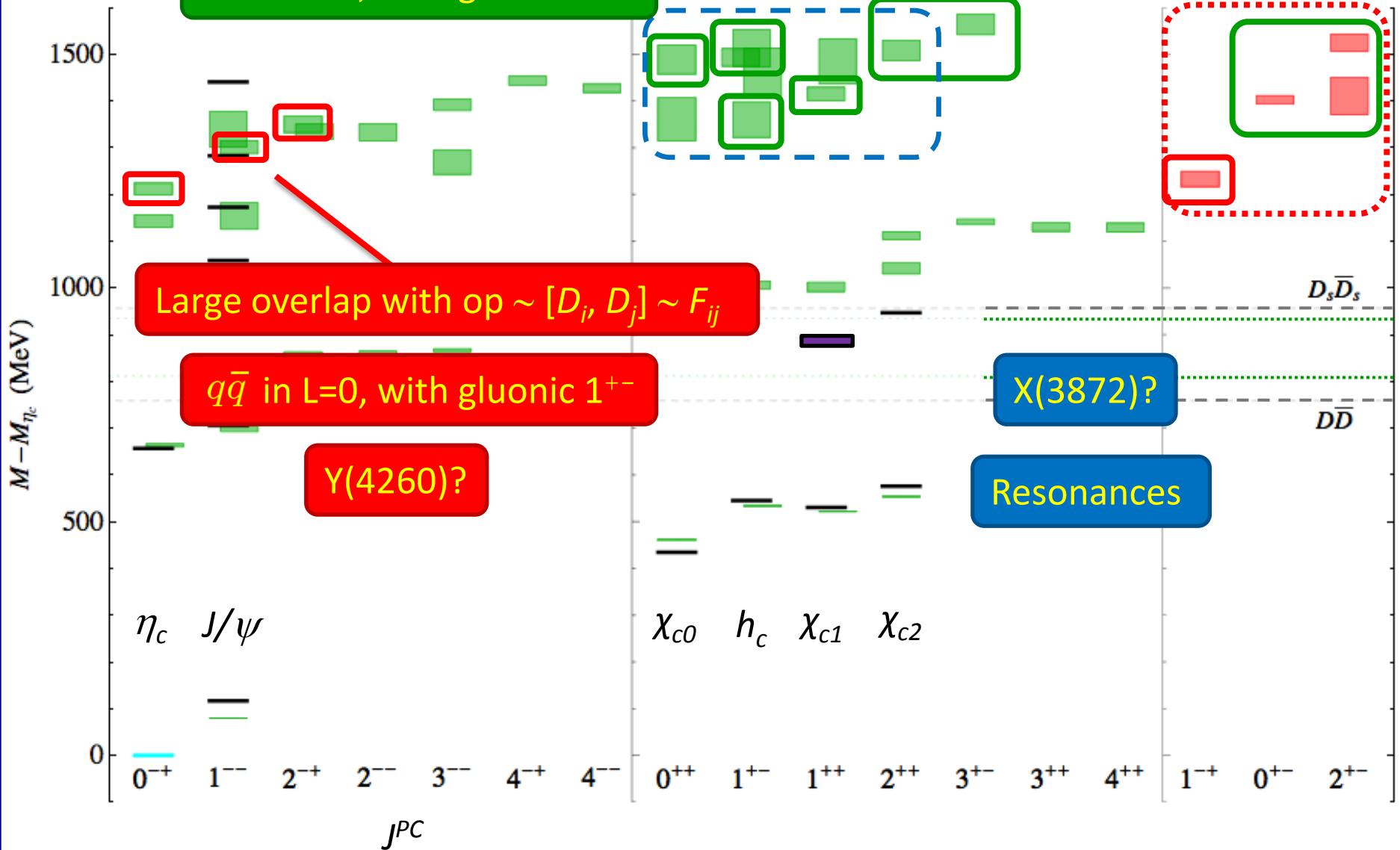
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Summary and Outlook

Summary

- Extensive charmonium spectrum, high spin, **exotics**
- Hybrid supermultiplets – **probe low energy d.o.f.**
- Also: charm mesons (D/D_s), light ($|l|=0,1$) mesons, baryons



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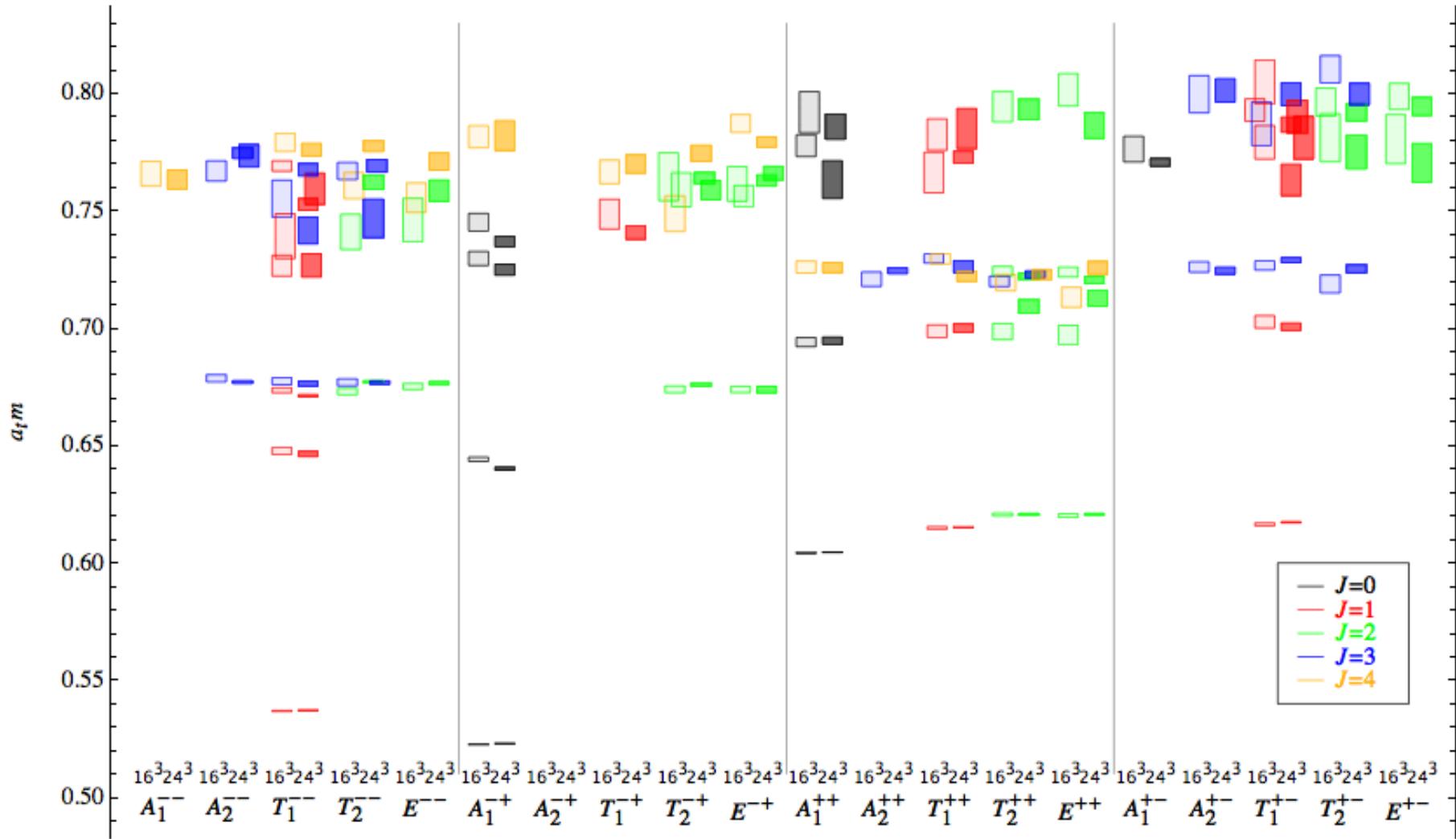
Outlook

- **Scattering** – resonances, decays, ...
[PR D83 071504, D86 034031, D87 034505]
- **Transitions**
- Disconnected contributions, glueball mixing, etc
- Lighter pion masses, larger volumes, ...

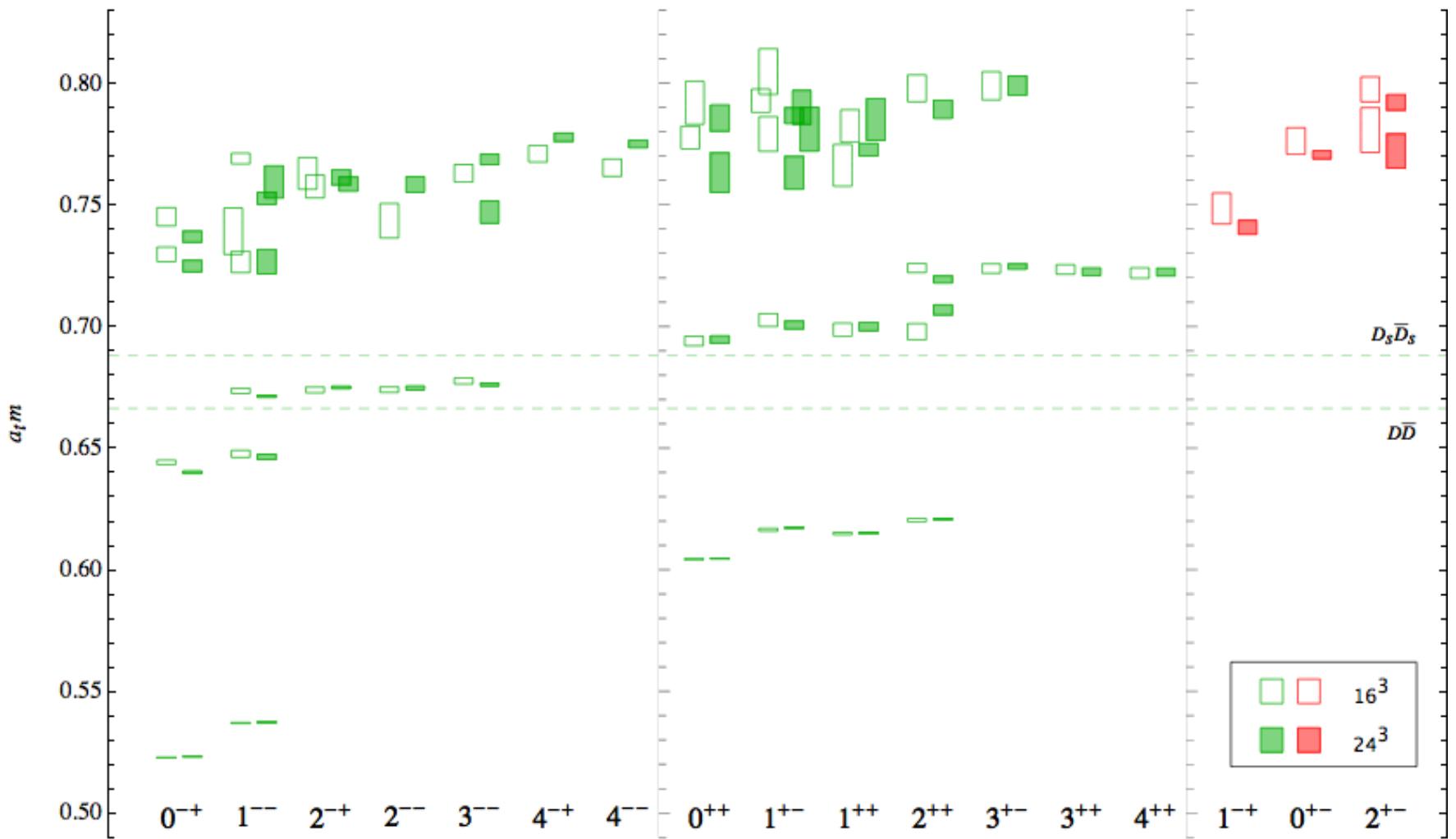


Extra Slides

Charmonium – irreps



Charmonium – volume comparison



Charmonium – hybrid candidates

