$\mathcal{O}(\alpha_s)$ correction to $e^+e^- \longrightarrow J/\psi + \chi_{c0,1,2}$ production at B factories

Feng Feng

Center for High Energy Physics, Peking University

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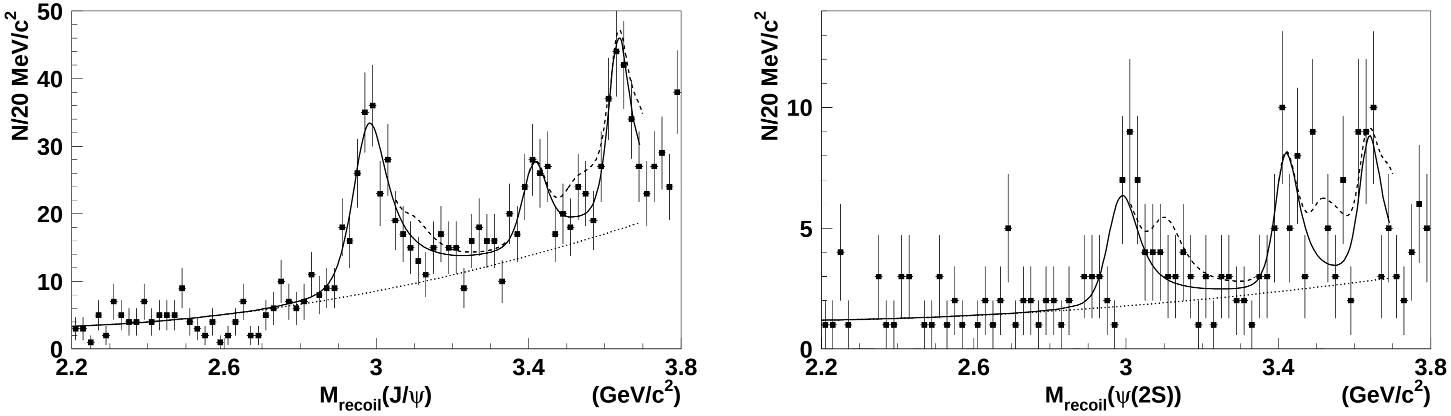
Outline of this talk

- Introduction
  - ➜ Motivation of studying exclusive double charmonium production at B factories
  - ➜ Light-cone vs. NRQCD factorization approaches (refactorization)
  - ➜ Single vs double logarithm in $\mathcal{O}(\alpha_s)$ NRQCD short-distance coefficients
- $\mathcal{O}(\alpha_s)$ correction to $e^+e^- \to J/\psi + \chi_{c0,1,2}$ within the NRQCD factorization framework
  - ➜ Brief description of the calculation
  - ➜ Phenomenological impact of this correction
  - ➜ Some theoretical issues
- Outlook

Why double charmonium production at B factory is interesting

- Phenomenological: New window for discovering the X, Y, Z states by fitting the recoiling mass spectrum against J/ψ (ψ').



Famous examples: $X(3940)$ and $X(4160)$ discovered this way. (And $X(3872)$?)

- Theoretical: Novel arena to enrich our understanding of pQCD in hard exclusive reactions: light-cone vs. NRQCD approaches

Comparing two first-principle pQCD schemes: light-cone vs NRQCD approaches (I)

- Light-cone approach is based on the twist (collinear) expansion

$$A[\gamma^* \to H_1 + H_2] \sim \phi_{H_1}(x) \underbrace{\otimes \quad T(x,y) \quad \otimes}_{\rm hard-scattering\ kernel} \phi_{H_2}(y) + O(1/s),$$

Convolution integral, involving nonperturbative light-cone distribution amplitude (LCDA) for charmonium.

➜ Pros

Applicable to both light meson production as well as heavy quarkonium production;

Using evolution equation to resum large logarithm $(\alpha_s \ln s/m_c^2)^n$

➜ Cons

Charmonia LCDA are unknown functions, the way of parametrization is model-dependent. The scale m_c not yet disentangled

✓Endpoint singularity problem in higher-twist channel: hindered our ability to do NLO correction

Double charmonium production from *light-cone* approaches (incomplete list)

- $e^+e^- \to J/\psi + \eta_c$
  - ➜ Ma and Si, PRD **70**, 074007 (2004);
  - ➜ Bondar and Chernyak, PLB **612**, 215 (2005);
  - ➜ Braguta, Likhoded and Luchinsky, PRD **72**, 074019 (2005);
  - ➜ Bodwin, Kang and Lee, PRD **74**, 114028 (2006);
  - ➜ ......
  - ➜ A general symptom is that, the light-cone calculation tends to predict very large cross sections;
  - ➜ Recall $e^+e^- \to J/\psi + \eta_c$ is helicity-flipped (higher twist); All these LC calculations are done only at LO in α_s!
- Some phenomenological work to deduce charmonium LCDA
  - ➜ Braguta, Likhoded and Luchinsky, PLB **646**, 80 (2007)
  - ➜ Braguta, PRD **75**, 094016 (2007)
  - ➜ Hwang, EPJC **62**, 499 (2009)

Comparing two first-principle pQCD schemes: light-cone vs NRQCD approaches (II)

- NRQCD approach is based on the velocity (local) expansion

$$A[\gamma^* \to H_1 + H_2] \sim \sum_{n,m} \underbrace{C_{n,m}(s, m_c^2)}_{\text{NRQCD hard coeff.}} \langle H_1|\mathcal{O}_n|0\rangle\langle H_2|\mathcal{O}_m|0\rangle$$

Sum of products of NRQCD short-distance coefficients and the vacuum-to-charmonium matrix elements;

➜ Pros

Exploits nonrelativistic nature of quarkonium, v expansion explicit;
Nonperturbative inputs are numbers rather than functions, predictions more restrictive;
Works for high-twist process; can readily go beyond LO in α_s

➜ Cons

Short-distance coefficients $C_{n,m}(s, m_c^2)$ contain two disparate scales; ambiguity in setting $\alpha_s(\mu)$

Double charmonium production from NRQCD approaches (incomplete list: < 2009)

- $e^+e^- \to J/\psi + \eta_c$
  - ➜ LO in α_s and v

    Braaten and Lee, PRD **67**, 054007 (2003);

    Liu, He and Chao, PLB **557**, 45 (2003);

    Hagiwara, Kou, Qiao, PLB **570**, 39 (2003)

    Almost one order-of-magnitude smaller than BELLE data
  - ➜ NLO in α_s but LO in v

    Zhang, Gao and Chao, PRL **96**, 092001 (2006);

    Gong and Wang, PRD **77**, 054028 (2008)

    Positive and substantial correction (K=1.96)
  - ➜ NLO in v but LO in α_s

    Braaten and Lee, PRD **67**, 054007 (2003);

    He, Fan and Chao, PRD **75**, 074011 (2007);

    Bodwin, Lee and Yu, PRD **77**, 094018 (2008)

The relativistic correction appears to be modest

The work of He et al. and Bodwin et al. assume different values of the order-v^2 NRQCD matrix elements for J/ψ and η_c

♣ Both groups claim that including the $\mathcal{O}(v^2)$ corrections largely solve the discrepancy between the data and NRQCD.

- $e^+e^- \to J/\psi + \chi_{c0}$
  - ➜ LO in α_s and v

    Braaten and Lee, PRD **67**, 054007 (2003);

    Liu, He and Chao, PLB **557**, 45 (2003);

    Considerably smaller than the BELLE measurement
  - ➜ NLO in α_s but LO in v

    Ma, Zhang and Chao, PRD **78**, 054006 (2008)

    K factor is found to be as large as 2.8. Including $\mathcal{O}(\alpha_s)$ correction helps to resolve the discrepancy
  - ➜ NLO in v but LO in α_s: Not available yet

Some recent (≥ 2011) progress in double charmonium production in NRQCD approach

Listed in chronological order

- $\mathcal{O}(\alpha_s)$ correction to $e^+e^- \to J/\psi + \chi_{c0,1,2}$:

  K. Wang, Y. -Q. Ma, K. -T. Chao, Phys. Rev. **D84**, 034022 (2011).

  ✓ H. -R. Dong, F. Feng and Y. Jia, JHEP **1110**, 141 (2011).

- $\mathcal{O}(\alpha_s v^2)$ correction to $e^+e^- \to J/\psi + \eta_c$:

  H. -R. Dong, F. Feng and Y. Jia, Phys. Rev. D **85**, 114018 (2012).

  X. -H. Li and J. -X. Wang, arXiv:1301.0376 [hep-ph].

- $\mathcal{O}(\alpha_s)$ correction to $e^+e^- \to J/\psi + \eta_{c2}(\chi'_{c1})$

  H. -R. Dong, F. Feng and Y. Jia, arXiv:1301.1946 [hep-ph].

Combining the light-cone and NRQCD approaches through refactorization

- Refactorizing quarkonium LCDA

  ➜ Ma and Si, PLB **647**, 419 (2007)

  ➜ Bell and Feldmann, JHEP **0804**, 061 (2008) ✓

  ♣ Disentangle the perturbatively calculable jet function from NRQCD matrix elements

  ♢ Analogous to inclusive onium production: refactorizing the (double-parton) fragmentation functions into quarkonium.

- Taking the twsit-2 LCDA of the B_c meson as example:

$$\Phi_{B_c}(x,\mu_F^2) = \frac{f_{B_c}}{2\sqrt{2N_c}}\,\hat{\phi}(x,\mu_F^2),$$

$$f_{B_c} = f_{B_c}^{(0)}\left(1 + \frac{\alpha_s(M_{B_c}^2)}{\pi}\mathfrak{f}_{B_c}^{(1)} + \cdots\right), \qquad f_{B_c}^{(0)} = \sqrt{\frac{2}{M_{B_c}}}\langle 0|\chi_b^\dagger \psi_c|B_c^+\rangle = \sqrt{\frac{4N_c}{M_{B_c}}}\,\psi_{B_c}(0).$$

$$\hat{\phi}(x,\mu_F^2) = \hat{\phi}^{(0)}(x) + \frac{\alpha_s(\mu_F^2)}{\pi}\hat{\phi}^{(1)}(x,\mu_F^2) + \cdots, \qquad \hat{\phi}^{(0)}(x) = \delta(x - x_0).$$

$$\begin{aligned}\hat{\phi}^{(1)}(x,\mu_F^2) \quad &= \quad \frac{C_F}{2}\left\{\left(\ln\frac{\mu_F^2}{M_{B_c}^2(x_0-x)^2} - 1\right)\left[\frac{x_0+\bar{x}}{x_0-x}\frac{x}{x_0}\theta(x_0-x) + \left(\begin{array}{c} x \leftrightarrow \bar{x} \\ x_0 \leftrightarrow \bar{x}_0 \end{array}\right)\right]\right\}_+ \\ &+ \quad C_F\left\{\left(\frac{x\bar{x}}{(x_0-x)^2}\right)_{++} + \frac{1}{2}\delta'(x-x_0)\left(2x_0\bar{x}_0\ln\frac{x_0}{\bar{x}_0} + x_0 - \bar{x}_0\right)\right\}.\end{aligned}$$

- B_c electromagnetic form factor through NLO in α_s
  - ➜ Jia, Yang and Wang, JHEP **1110**, 105 (2011)

$$F_{\rm LC} = F_{\rm LC}^{(0)} + \frac{\alpha_s}{\pi}F_{\rm LC}^{(1)} + \cdots,$$

  - ➜ The LC predictions implementing refactorization:

$$F_{\rm LC}^{(0)} \sim \hat{\phi}^{(0)} \otimes T_H^{(0)} \otimes \hat{\phi}^{(0)},$$

$$F_{\rm LC}^{(1)} \sim \hat{\phi}^{(0)}\otimes T_H^{(1)}\otimes\hat{\phi}^{(0)}, \quad \hat{\phi}^{(1)}\otimes T_H^{(0)}\otimes\hat{\phi}^{(0)}, \quad \hat{\phi}^{(0)}\otimes T_H^{(0)}\otimes\hat{\phi}^{(1)}, \quad \mathfrak{f}_{B_c}^{(1)}\,\hat{\phi}^{(0)}\otimes T_H^{(0)}\otimes\hat{\phi}^{(0)}.$$

➜ The compact expressions of $\mathcal{O}(\alpha_s)$ light-cone predictions:

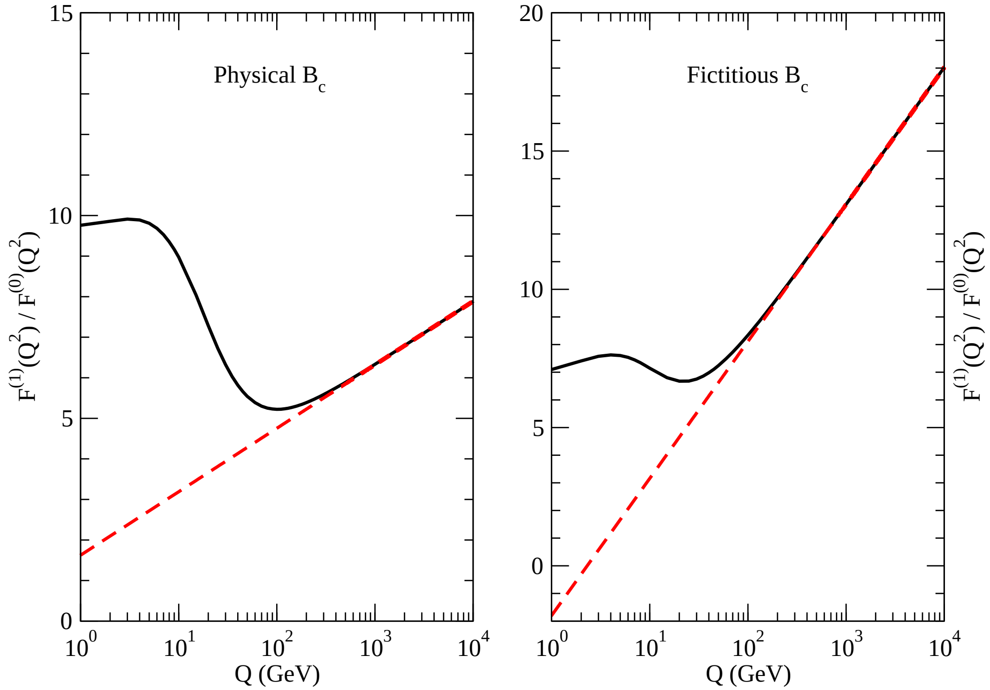
$$F^{(1)}_{\rm LC}(Q^2) = \frac{2\pi C_F\alpha_s(\mu_R^2)}{N_c}\frac{f^{(0)\,2}_{B_c}}{Q^2}\left\{\frac{e_c}{\bar{x}_0^2}\left[\frac{\beta_0}{4}\left(\frac{5}{3}-2\ln\bar{x}_0+\ln\frac{\mu_R^2}{Q^2}\right)+\overbrace{\frac{C_F}{2}(3+2\ln\bar{x}_0)\ln\frac{Q^2}{M^2_{B_c}}}^{\text{single collinear log}}\right.\right.$$
$$+3\,{\rm Li}_2(\bar{x}_0)+\frac{1}{6}\ln^2 x_0+\frac{1-32x_0+37x_0^2}{36x_0^2}\ln\bar{x}_0+\frac{4-85\bar{x}_0}{36\bar{x}_0}\ln x_0$$
$$\left.\left.+\frac{1-102x_0}{36x_0}-\frac{17\,\pi^2}{36}\right]-(e_c\to e_b,\,x_0\leftrightarrow\bar{x}_0)\right\}.$$

➜ By contrast, the $\mathcal{O}(\alpha_s)$ NRQCD expressions are extremely cumbersome. Only its asymptotic expression (expanding in powers of $M^2_{B_c}/Q^2$) coincides with $F^{(1)}_{\rm LC}(Q^2)$

- For this helicity-conserving process, only single collinear logarithm appears at $\mathcal{O}(\alpha_s)$

One can employ Efremov-Radyushkin-Brodsky-Lepage (ERBL) equation to resum them. Jia and Yang, NPB **814**, 217 (2009)

- Comparison between NRQCD (black curve) and light-cone (red line) predictions for B_c EM form factor over a wide range of Q^2.



The ratio $F^{(1)}_{B_c}(Q^2)/F^{(0)}_{B_c}(Q^2)$ as a function of Q with $M_{B_c} = 6.3$ GeV, n_f=5 ($\beta_0 = \frac{23}{3}$), and $\mu_R = Q$. The left panel is for the EM form factor of the physical B_c state with $m_c = 1.5$ GeV and $m_b = 4.8$ GeV, while the right panel for a fictitious B_c state with $m_c = m_b = 3.15$ GeV.

Arising of double logarithm in $e^+e^- \to J/\psi + \eta_c$ at $\mathcal{O}(\alpha_s)$

- First specified in Jia, Yang and Wang, JHEP **1110**, 147 (2011)

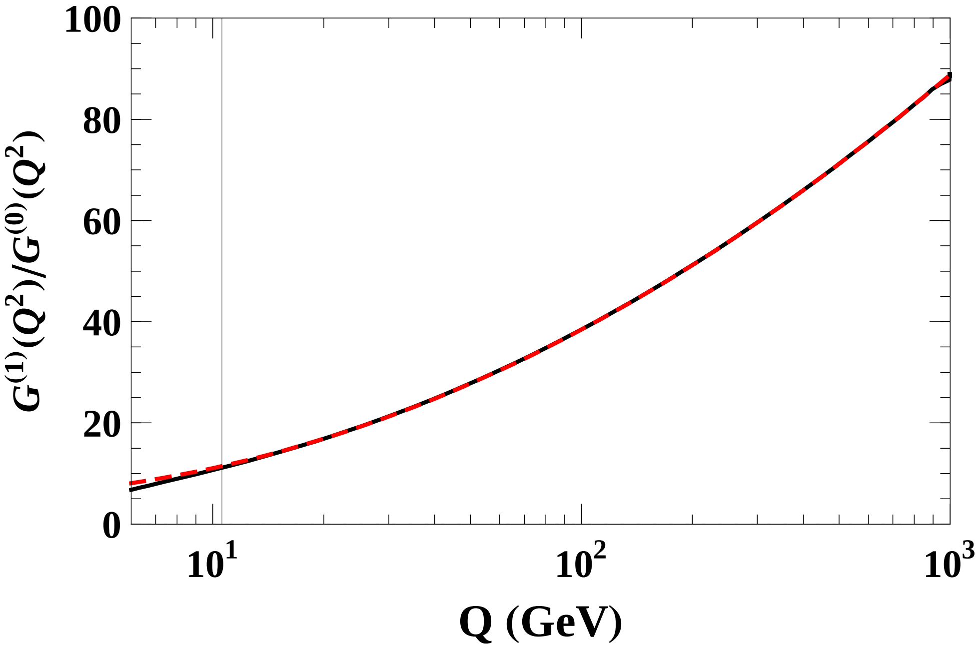
$$\frac{\mathrm{Re}[C^{(1)}_{\mathrm{asym}}(Q)]}{C^{(0)}_{\mathrm{asym}}(Q)} = \frac{13}{24}\ln^2\frac{Q^2}{m_c^2} - \frac{41}{24}(2\ln 2 - 1)\ln\frac{Q^2}{m_c^2} + \frac{\beta_0}{4}\ln\frac{\mu_R^2}{Q^2} + \frac{71}{8}\ln 2 + \frac{59}{24}\ln^2 2 - \frac{23}{18} - \frac{\pi^2}{36}.$$

♣ It is not clear how to compute the $\mathcal{O}(\alpha_s)$ correction to this helicity-suppressed process in the light-cone approach

♢ The endpoint singularity problem may pose difficulty

- It was conjectured that the double-logarithm is always associated with the helicity-flipped (higher-twist) process

- Comparison between the $\mathcal{O}(\alpha_s)$ NRQCD exact (black) and asymptotic (red) predictions for $J/\psi + \eta_c$ timelike EM form factor.

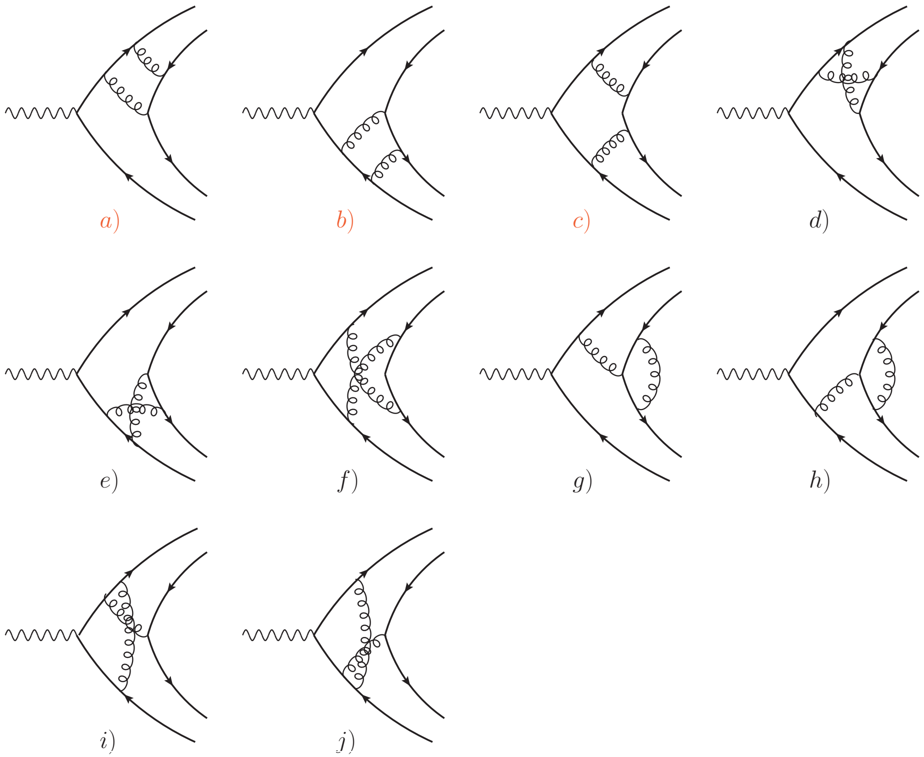


- Double logarithm $\alpha_s \ln^2(s/m_c^2)$ is numerically dominant for K factor

  ♢ Should them first be resummed to all orders in α_s before any reliable prediction is claimed?

  ♢ If unable, do we really claim to understand $e^+e^- \to J/\psi + \eta_c$?

Tracing the origin of double logarithm



The NLO diagrams responsible for the double logarithm $\alpha_s \ln^2(s/m_c^2)$ (in Feynman gauge) for the processes $\gamma^* \to J/\psi + \eta_{c2}\ (\eta_c, \chi_{c0,1,2})$. [Omit charge-conjugated diagrams]

- Distinguish harmless Sudakov double logarithm and troublesome endpoint double logarithm $\Rrightarrow$ Chung's talk

Motivation to study $\mathcal{O}(\alpha_s)$ correction to $e^+e^- \to J/\psi + \chi_{c0,1,2}$ in NRQCD factorization

- Phenomenological interest: to confront the B factory data

♣ Will work at the level of the helicity amplitude/polarized cross sections; informative for exclusive reactions

♢ Helicity selection rule (HSR) serves the guideline

- To verify Ma, Zhang and Chao (2008)'s result on $e^+e^- \to J/\psi + \chi_{c0}$, and further address $e^+e^- \to J/\psi + \chi_{c1,2}$

- For theoretical curiosity

Numerous distinct helicity configurations

$\Rrightarrow$ An ideal theoretical laboratory to examine the pattern of double logarithms

Polarized cross sections and HSR

- Differential polarized cross section

$$\frac{d\sigma[e^+e^- \to J/\psi(\lambda_1) + \chi_{cJ}(\lambda_2)]}{d\cos\theta} = \frac{2\pi\alpha}{s^{3/2}} \sum_{S_z=\pm 1} \frac{d\Gamma[\gamma^*(S_z) \to J/\psi(\lambda_1) + \chi_{cJ}(\lambda_2)]}{d\cos\theta}$$

$$= \frac{\alpha}{8s^2}\left(\frac{|\mathbf{P}|}{\sqrt{s}}\right) \underbrace{|\mathcal{A}^J_{\lambda_1,\lambda_2}|^2}_{\text{helicity amplitude}} \times \begin{cases} \frac{1+\cos^2\theta}{2}, & (\lambda_1-\lambda_2=\pm 1) \\ \sin^2\theta, & (\lambda_1-\lambda_2=0) \end{cases}$$

- Parity conservation constraint on helicity amplitude

$$\mathcal{A}^J_{\lambda_1,\lambda_2} = (-1)^J \mathcal{A}^J_{-\lambda_1,-\lambda_2}.$$

Consequently, $\underline{\gamma^* \to J/\psi(0) + \chi_{c1}(0)}$ is strictly forbidden.

- There are 2, 3 and 5 independent helicity amplitudes for $\gamma^* \to J/\psi + \chi_{c0,1,2}$, respectively.

- Integrated cross sections

$$\sigma[J/\psi+\chi_{c0}] = \frac{\alpha}{6s^2}\left(\frac{|\mathbf{P}|}{\sqrt{s}}\right)\left(\left|\mathcal{A}^0_{0,0}\right|^2 + 2\left|\mathcal{A}^0_{1,0}\right|^2\right),$$

$$\sigma[J/\psi+\chi_{c1}] = \frac{\alpha}{6s^2}\left(\frac{|\mathbf{P}|}{\sqrt{s}}\right)\left(2\left|\mathcal{A}^1_{1,0}\right|^2 + 2\left|\mathcal{A}^1_{0,1}\right|^2 + 2\left|\mathcal{A}^1_{1,1}\right|^2\right),$$

$$\sigma[J/\psi+\chi_{c2}] = \frac{\alpha}{6s^2}\left(\frac{|\mathbf{P}|}{\sqrt{s}}\right)\left(\left|\mathcal{A}^2_{0,0}\right|^2 + 2\left|\mathcal{A}^2_{1,0}\right|^2 + 2\left|\mathcal{A}^2_{0,1}\right|^2 + 2\left|\mathcal{A}^2_{1,1}\right|^2 + 2\left|\mathcal{A}^2_{1,2}\right|^2\right).$$

- Helicity selection rule (HSR) Brodsky, Lepage PRD (1981)

$$\sigma[J/\psi(\lambda_1)+\chi_{cJ}(\lambda_2)] \sim \alpha^2 v^8 \left(\frac{m_c^2}{s}\right)^{2+|\lambda_1+\lambda_2|},$$

- Leading twist vs higher twist
  - ➜ leading-twist (helicity conserved): $(\lambda_1,\lambda_2)=(0,0)$, $\sigma\sim 1/s^2$.
  - ➜ higher-twist (helicity flipped): $|\lambda_1+\lambda_2|\geq 1$

    $e^+e^-\to J/\psi+\eta_c(\chi_{c1},\eta_{c2})$: parity invariance forbids the $(0,0)$ configuration, therefore of “higher twist" nature.

Brief description of the calculation

- Threshold expansion — Beneke and Smirnov, NPB **522**, 321 (1998)

  When projecting each $c\bar{c}$ pair onto the intended orbital angular momentum state, one first expands the amplitude in powers of the relative quark momentum q BEFORE performing the loop integration

  $\sharp$ Much simpler than standard matching method

- Using covariant spin projectors to compute on-shell parton amplitude $\gamma^* \to c\bar{c}(P_1, {}^3S_1^{(1)}) + c\bar{c}(P_2, {}^3P_J^{(1)})$

  Bodwin and Petrelli, PRD **66**, 094011 (2002)

  Spin-triplet projector used in this work:

$$v(\bar{p})\bar{u}(p) \to \frac{1}{4\sqrt{2}E(E+m_c)}(\not{\bar{p}} - m_c)\left[\not{\epsilon}^*\right](\not{P} + 2E)(\not{p} + m_c) \otimes \frac{\mathbf{1}}{\sqrt{N_c}}$$

10 Helicity projectors are handy to use

Define $g_{\perp\,\mu\nu} = g_{\mu\nu} + \frac{P_\mu P_\nu}{|\mathbf{P}|^2} - \frac{Q\cdot P}{M_\Upsilon^2|\mathbf{P}|^2}(P_\mu Q_\nu + Q_\mu P_\nu) + \frac{M_{J/\psi}^2}{M_\Upsilon^2}\frac{Q_\mu Q_\nu}{|\mathbf{P}|^2}$

- 2 helicity projectors for $\gamma^* \to J/\psi + \chi_{c0}$:

$$
\begin{aligned}
\mathbb{P}_{0,0}^{\mu\nu} &= \frac{1}{|\mathbf{P}|^2}\left(P_\mu - \frac{Q\cdot P}{M_\Upsilon^2}Q_\mu\right)\left(\frac{Q\cdot P}{M_{J/\psi}M_\Upsilon}P_\nu - \frac{M_{J/\psi}}{M_\Upsilon}Q_\nu\right), \\
\mathbb{P}_{1,0}^{\mu\nu} &= -\frac{1}{2}\,g_{\perp\,\mu\nu},
\end{aligned}
$$

- 3 helicity projectors for $\gamma^* \to J/\psi + \chi_{c1}$:

$$
\begin{aligned}
\mathbb{P}_{1,0}^{\mu\nu\alpha} &= \frac{i}{2M_\Upsilon|\mathbf{P}|^2}\,\epsilon_{\mu\nu\rho\sigma}Q^\rho \widetilde{P}^\sigma\left(\frac{Q\cdot\widetilde{P}}{M_{\chi_{c1}}M_\Upsilon}\tilde{P}_\alpha - \frac{M_{\chi_{c1}}}{M_\Upsilon}Q_\alpha\right), \\
\mathbb{P}_{0,1}^{\mu\nu\alpha} &= -\frac{i}{2M_\Upsilon|\mathbf{P}|^2}\,\epsilon_{\mu\alpha\rho\sigma}Q^\rho P^\sigma\left(\frac{Q\cdot P}{M_{J/\psi}M_\Upsilon}P_\nu - \frac{M_{J/\psi}}{M_\Upsilon}Q_\nu\right), \\
\mathbb{P}_{1,1}^{\mu\nu\alpha} &= \frac{i}{2M_\Upsilon|\mathbf{P}|^2}\,\epsilon_{\nu\alpha\rho\sigma}Q^\rho P^\sigma\left(P_\mu - \frac{Q\cdot P}{M_\Upsilon^2}Q_\mu\right),
\end{aligned}
$$

- 5 helicity projectors for $\gamma^* \to J/\psi + \chi_{c2}$:

$$\mathbb{P}_{0,0}^{\mu\nu\alpha\beta} = \frac{1}{\sqrt{6}|\mathbf{P}|^2}\left(\frac{Q\cdot P}{M_{J/\psi}M_\Upsilon}P_\nu - \frac{M_{J/\psi}}{M_\Upsilon}Q_\nu\right)\left(P_\mu - \frac{Q\cdot P}{M_\Upsilon^2}Q_\mu\right)\Bigg[g_{\perp\,\alpha\beta} + \frac{2}{|\mathbf{P}|^2} \times\left(\frac{Q\cdot\tilde{P}}{M_{\chi_{c2}}M_\Upsilon}\tilde{P}_\alpha - \frac{M_{\chi_{c2}}}{M_\Upsilon}Q_\alpha\right)\left(\frac{Q\cdot\tilde{P}}{M_{\chi_{c2}}M_\Upsilon}\tilde{P}_\beta - \frac{M_{\chi_{c2}}}{M_\Upsilon}Q_\beta\right)\Bigg],$$

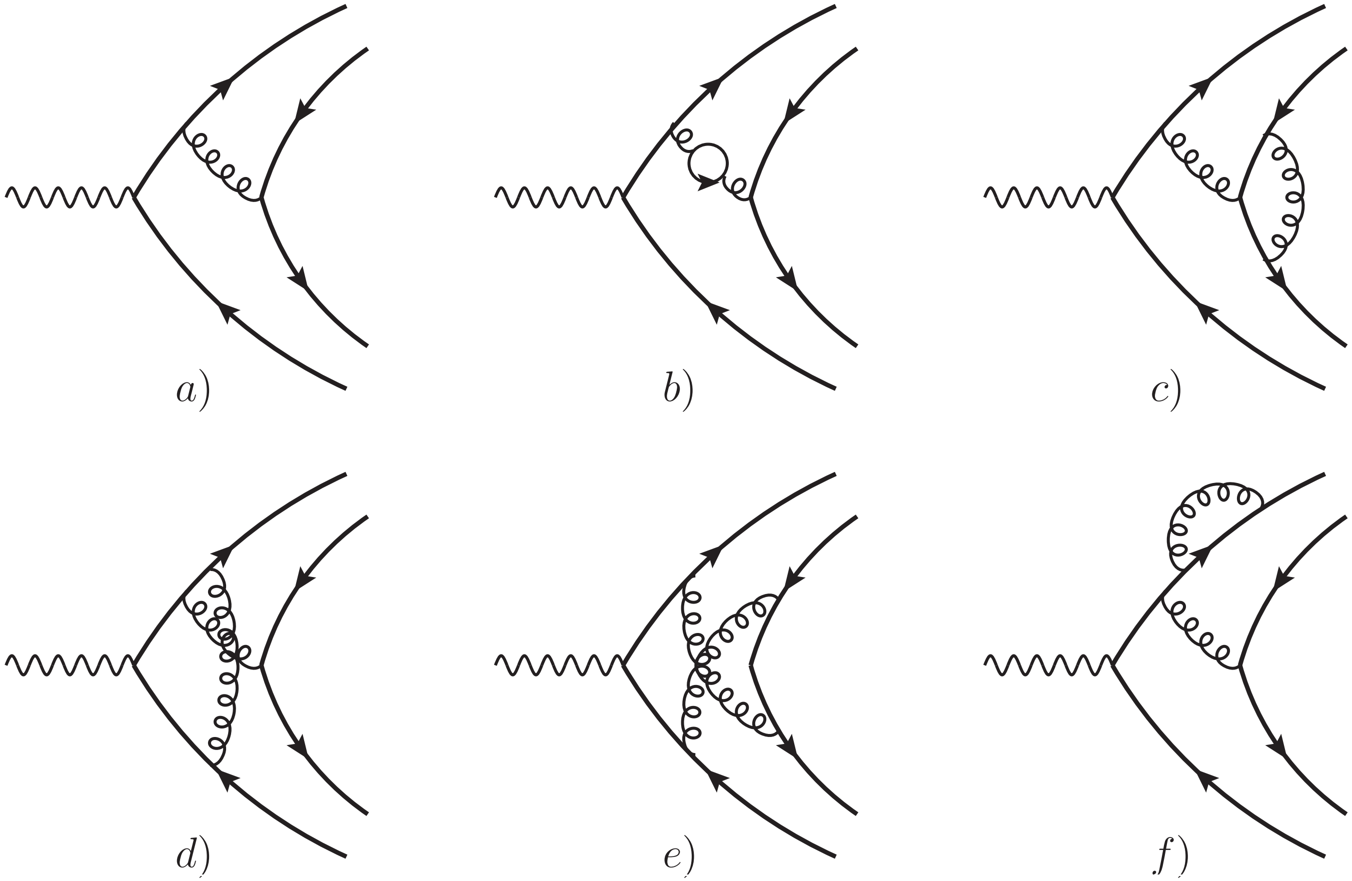
$$\mathbb{P}_{1,0}^{\mu\nu\alpha\beta} = -\frac{1}{2\sqrt{6}}\,g_{\perp\,\mu\nu}\Bigg[g_{\perp\,\alpha\beta} + \frac{2}{|\mathbf{P}|^2}\left(\frac{Q\cdot\tilde{P}}{M_{\chi_{c2}}M_\Upsilon}\tilde{P}_\alpha - \frac{M_{\chi_{c2}}}{M_\Upsilon}Q_\alpha\right) \times\left(\frac{Q\cdot\tilde{P}}{M_{\chi_{c2}}M_\Upsilon}\tilde{P}_\beta - \frac{M_{\chi_{c2}}}{M_\Upsilon}Q_\beta\right)\Bigg],$$

$$\mathbb{P}_{0,1}^{\mu\nu\alpha\beta} = \frac{1}{2\sqrt{2}|\mathbf{P}|^2}\left(\frac{Q\cdot P}{M_{J/\psi}M_\Upsilon}P_\nu - \frac{M_{J/\psi}}{M_\Upsilon}Q_\nu\right)\Bigg[g_{\perp\,\mu\alpha}\left(\frac{Q\cdot\tilde{P}}{M_{\chi_{c2}}M_\Upsilon}\tilde{P}_\beta - \frac{M_{\chi_{c2}}}{M_\Upsilon}Q_\beta\right) + (\alpha\leftrightarrow\beta)\Bigg],$$

$$\mathbb{P}_{1,1}^{\mu\nu\alpha\beta} = -\frac{1}{2\sqrt{2}|\mathbf{P}|^2}\left(P_\mu - \frac{Q\cdot P}{M_\Upsilon^2}Q_\mu\right)\Bigg[g_{\perp\,\nu\alpha}\left(\frac{Q\cdot\tilde{P}}{M_{\chi_{c2}}M_\Upsilon}\tilde{P}_\beta - \frac{M_{\chi_{c2}}}{M_\Upsilon}Q_\beta\right) + (\alpha\leftrightarrow\beta)\Bigg],$$

$$\mathbb{P}_{1,2}^{\mu\nu\alpha\beta} = \frac{1}{4}(g_{\perp\,\mu\nu}\,g_{\perp\,\alpha\beta} - g_{\perp\,\mu\alpha}\,g_{\perp\,\nu\beta} - g_{\perp\,\mu\beta}\,g_{\perp\,\nu\alpha}).$$

Some representative NLO Feynman diagrams



20 two-point, **20** three-point, **18** four-point, and **6** five-point one-loop diagrams

Tools for analytic NLO calculation

- MATHEMATICA 9
  - ➜ Computation Environment: http://www.wolfram.com/
- FEYNARTS
  - ➜ Feynman Diagrams: http://www.feynarts.de/
- FEYNCALC/FEYNCALCFORMLINK
  - ➜ DiracTrace & Contraction: http://www.feyncalc.org/
  - ➜ FORM Embedded: http://www.feyncalc.org/formlink/
- APART
  - ➜ Partial Fraction:

    F. Feng, Comput. Phys. Commun.183, 2158(2012)
- FIRE
  - ➜ Feynman Integral Reduction: http://science.sander.su/FIRE.html

Peculiarity encountered in NLO calculation involving P-wave charmonium

- Integration-by-part (IBP) reduction & Master Integrals (MI)

  ➜ One encounters some unusual one-loop integrals that contain the propagators of (up to) quadratic power, due to taking the derivative over relative momentum q.

  ➜ The MATHEMATICA packages FIRE and the code APART are utilized to reduce these unconventional higher-point one-loop tensor integrals into a minimal set of masters integrals.

  ➜ All the encountered master integrals are nothing but the ordinary 2-point and 3-point one-loop scalar integrals, whose analytic expressions can be readily found in literature.

Cancelation of IR divergences in $\mathcal{O}(\alpha_s)$ short-distance coefficients

- A factorization theorem states that NRQCD works for exclusive production of J/ψ plus a charmonium H carrying arbitrary orbital angular momentum $L \geqq 0$

  Bodwin, Garcia i Tormo and Lee, PRL **101**, 102002 (2008).

  ➜ IR divergence was first shown to cancel in Ma, Zhang and Chao, PRD (2008) in NLO perturbative correction to $e^+e^- \to J/\psi + \chi_{c0}$

  ➜ After summing up all the diagrams, and renormalizing the charm quark field mass and the QCD coupling constant, we end up with UV and IR finite expressions for the $O(\alpha_s)$ short-distance coefficients associated with $\gamma^* \to J/\psi + \chi_{c0,1,2}$.

  ➜ Recently we also verified the cancelation of IR divergence in S-wave+D-wave charmonium production: $\gamma^* \to J/\psi + \eta_{c2}$.

LO NRQCD predictions

- Pull out the factor $r^{\frac{1}{2}(1+|\lambda_1+\lambda_2|)}$ dictated by HSR ($r \equiv 4m_c^2/s$)

$$\mathcal{A}^{J\,(0)}_{\lambda_1,\lambda_2} = \frac{4ee_c\alpha_s C_F R_{J/\psi}(0)R'_{\chi_{cJ}}(0)}{m_c^3}\, r^{\frac{1}{2}(1+|\lambda_1+\lambda_2|)}\, c^J_{\lambda_1,\lambda_2}(r),$$

- The LO polarized cross sections

$$\sigma^{(0)}[J/\psi(\lambda_1)+\chi_{cJ}(\lambda_2)] = \frac{32\pi e_c^2\alpha^2 C_F^2\alpha_s^2}{3s^2m_c^6}\left(\frac{|\mathbf{P}|}{\sqrt{s}}\right) R^2_{J/\psi}(0)R'^2_{\chi_{cJ}}(0) r^{1+|\lambda_1+\lambda_2|}\left|c^J_{\lambda_1,\lambda_2}(r)\right|^2.$$

- The 10 tree-level short-distance coefficients

$$c^0_{0,0}(r) = 1+10r-12r^2 \qquad c^0_{1,0}(r) = 9-14r,$$
$$c^1_{0,1}(r) = -\sqrt{6}(2-7r) \qquad c^1_{1,0}(r) = -\sqrt{6}r \qquad c^1_{1,1}(r) = -2\sqrt{6}(1-3r),$$
$$c^2_{0,0}(r) = \sqrt{2}(1-2r-12r^2) \qquad c^2_{0,1}(r) = \sqrt{6}(1-5r) \qquad c^2_{1,0}(r) = \sqrt{2}(3-11r),$$
$$c^2_{1,1}(r) = 2\sqrt{6}(1-3r) \qquad c^2_{1,2}(r) = 2\sqrt{3}.$$

$\mathcal{O}(\alpha_s)$ reduced helicity amplitudes

- $\mathcal{O}(\alpha_s)$ corrections to helicity amplitudes and cross sections

$$\mathcal{A}^{J\,(1)}_{\lambda_1,\lambda_2} \quad = \quad \frac{\alpha_s}{\pi}\, K^J_{\lambda_1,\lambda_2}\left(r, \frac{\mu^2}{s}\right) \mathcal{A}^{J\,(0)}_{\lambda_1,\lambda_2},$$

$$\sigma^{(1)}[J/\psi(\lambda_1)+\chi_{cJ}(\lambda_2)] = 2\left(\frac{\alpha_s}{\pi}\right) \mathrm{Re}\left\{ K^J_{\lambda_1,\lambda_2}\left(r, \frac{\mu^2}{s}\right)\right\} \sigma^{(0)}[J/\psi(\lambda_1)+\chi_{cJ}(\lambda_2)],$$

- The cross section is decomposed into

$$\sigma_{\mathrm{NLO}} = \sigma^{(0)} + \sigma^{(1)}$$

- The analytic expressions of the $K^J_{\lambda_1,\lambda_2}$ functions are lengthy and cumbersome.

  $\natural$ Knowing their asymptotic expressions are much more illuminating!

Asymptotic expressions of the $\mathcal{O}(\alpha_s)$ coefficients

- for $J/\psi + \chi_{c0}$

$$K^{0}_{0,0}\left(r, \frac{\mu^2}{s}\right)_{\text{asym}} = -\frac{1}{3}(4 - \ln 2)\ln r + \frac{\beta_0}{4}\left(\ln\frac{4\mu^2}{s} + \frac{8}{3}\right)$$
$$-\frac{1}{18}(46 + \pi^2 - 40\ln 2 + 33\ln^2 2) + \frac{i\pi}{4}\left(\beta_0 - \frac{16}{3} + \frac{4}{3}\ln 2\right),$$

$$K^{0}_{1,0}\left(r, \frac{\mu^2}{s}\right)_{\text{asym}} = \frac{1}{3}\ln^2 r - \frac{1}{108}(139 - 104\ln 2)\ln r + \frac{\beta_0}{4}\left(\ln\frac{4\mu^2}{s} + \frac{17}{9}\right)$$
$$-\frac{1}{54}\left(161 + \frac{8\pi^2}{3} - \frac{495}{2}\ln 2 + 100\ln^2 2\right) + \frac{i\pi}{4}\left(\frac{8}{3}\ln r + \beta_0 - \frac{1}{27}(139 - 104\ln 2)\right).$$

- for $J/\psi + \chi_{c1}$

$$r\,K^1_{1,0}\left(r,\frac{\mu^2}{s}\right)_{\rm asym} = -\frac{1}{12}\left\{5\ln^2 r + (7-2\ln 2)\ln r - 19 + 2\pi^2 + 75\ln 2 - 21\ln^2 2\right.$$
$$\left. + i\pi(10\ln r + 7 - 2\ln 2)\right\},$$

$$K^1_{0,1}\left(r,\frac{\mu^2}{s}\right)_{\rm asym} = \frac{1}{24}\left\{\frac{25}{2}\ln^2 r - (46-99\ln 2)\ln r + 6\beta_0\left(\ln\frac{4\mu^2}{s} + \frac{13}{6}\right)\right.$$
$$\left. - \frac{1}{6}(616 + 74\pi^2 - 1696\ln 2 + 303\ln^2 2) + i\pi(25\ln r + 6\beta_0 - 46 + 99\ln 2)\right\},$$

$$K^1_{1,1}\left(r,\frac{\mu^2}{s}\right)_{\rm asym} = \frac{1}{24}\left\{10\ln^2 r + 2(1+17\ln 2)\ln r + 6\beta_0\left(\ln\frac{4\mu^2}{s} + \frac{13}{6}\right)\right.$$
$$\left. - \frac{1}{3}(266 + 7\pi^2 - 128\ln 2 + 147\ln^2 2) + 2i\pi(10\ln r + 3\beta_0 + 1 + 17\ln 2)\right\}.$$

- for $J/\psi + \chi_{c2}$

$$K^2_{0,0}\left(r, \frac{\mu^2}{s}\right)_{\text{asym}} = -\frac{1}{3}(4 - \ln 2)\ln r + \frac{\beta_0}{4}\left(\ln\frac{4\mu^2}{s} + \frac{8}{3}\right)$$

$$-\frac{1}{18}(64 + \pi^2 + 104\ln 2 + 33\ln^2 2) + \frac{i\pi}{4}\left(\beta_0 - \frac{10}{3} + \frac{4}{3}\ln 2\right),$$

$$K^2_{0,1}\left(r, \frac{\mu^2}{s}\right)_{\text{asym}} = \frac{1}{12}\left\{\frac{13}{2}\ln^2 r - (22 - 43\ln 2)\ln r + 3\beta_0\left(\ln\frac{4\mu^2}{s} + \frac{8}{3}\right)\right.$$

$$\left.-\frac{1}{6}(284 + 30\pi^2 - 380\ln 2 + 159\ln^2 2) + i\pi(13\ln r + 3\beta_0 - 14 + 43\ln 2)\right\},$$

$$K^2_{1,0}\left(r, \frac{\mu^2}{s}\right)_{\text{asym}} = \frac{1}{6}\left\{2\ln^2 r + \frac{1}{6}(5 + 8\ln 2)\ln r + \frac{3}{2}\beta_0\left(\ln\frac{4\mu^2}{s} + \frac{7}{3}\right)\right.$$

$$\left.-\frac{1}{18}(291 - 8\pi^2 + 171\ln 2 + 312\ln^2 2) + i\pi\left(4\ln r + \frac{3}{2}\beta_0 + \frac{11}{6} + \frac{4}{3}\ln 2\right)\right\},$$

$$K^2_{1,1}\left(r, \frac{\mu^2}{s}\right)_{\text{asym}} = \frac{1}{24}\left\{4\ln^2 r - (46 - 62\ln 2)\ln r + 6\beta_0\left(\ln\frac{4\mu^2}{s} + \frac{13}{6}\right)\right.$$

$$\left.-\frac{1}{3}(274 + 27\pi^2 - 316\ln 2 + 9\ln^2 2) + i\pi(8\ln r + 6\beta_0 - 46 + 62\ln 2)\right\},$$

$$K^2_{1,2}\left(r, \frac{\mu^2}{s}\right)_{\rm asym} = -\frac{1}{4}\left\{2\ln^2 r + \frac{2}{3}(1+13\ln 2)\ln r - \beta_0\left(\ln\frac{4\mu^2}{s} + \frac{5}{3}\right)\right.$$
$$\left. + \frac{1}{9}(-7\pi^2 + 140 - 104\ln 2 + 237\ln^2 2) - i\pi\left(\beta_0 + 3 - \frac{26}{3}\ln 2\right)\right\}.$$

- The leading-twist processes $\gamma^* \to J/\psi(0) + \chi_{c0,2}(0)$ indeed only contains single collinear logarithm: amenable to ERBL evolution equation for resummation.

- The above results convincingly confirm the early conjecture by Jia, Yang and Wang (2011)

  $\sharp$ Double logarithms can solely arise from the helicity-suppressed double charmonium production channels

- *No clear pattern: channel-dependent double logarithm.*

Anatomy of double logarithm for $e^+e^- \to J/\psi + \chi_{cJ}$

Dong, Feng and Jia, arXiv:1301.1946[hep-ph]

Diagrams		$a)$	$b)$	$c)$	$d)$	$e)$	$f)$	$g)$	$h)$	$i)$	$j)$
Color Factor		C_1	C_1	C_1	C_2	C_2	C_2	C_2	C_2	C_2	C_2
χ_{c0}	(1,0)	$\frac{7}{384}$	$\frac{1}{192}$	$\frac{1}{768}$	$\frac{1}{96}$	$\frac{7}{384}$	$\frac{11}{384}$	$-\frac{1}{192}$	$-\frac{7}{384}$	$-\frac{1}{192}$	$-\frac{7}{384}$
χ_{c1}	(1,0)	–	$-\frac{3}{128}$	$-\frac{3}{256}$	$-\frac{3}{64}$	$\frac{3}{128}$	$-\frac{3}{128}$	$\frac{3}{128}$	$-\frac{3}{128}$	$\frac{3}{128}$	$-\frac{3}{128}$
	(0,1)	$\frac{9}{512}$	$\frac{3}{256}$	$\frac{9}{1024}$	$\frac{3}{512}$	$\frac{9}{512}$	$\frac{9}{256}$	$-\frac{3}{512}$	$-\frac{9}{512}$	$-\frac{3}{512}$	$-\frac{9}{512}$
	(1,1)	$\frac{27}{1024}$	$\frac{3}{512}$	$-\frac{3}{1024}$	$-\frac{3}{512}$	$\frac{15}{512}$	$\frac{3}{128}$	$\frac{3}{512}$	$-\frac{15}{512}$	$\frac{3}{512}$	$-\frac{15}{512}$
χ_{c2}	(0,1)	$-\frac{3}{256}$	$-\frac{3}{128}$	$-\frac{3}{512}$	$-\frac{3}{256}$	$-\frac{3}{256}$	$-\frac{3}{64}$	$\frac{3}{256}$	$\frac{3}{256}$	$\frac{3}{256}$	$\frac{3}{256}$
	(1,0)	$-\frac{3}{1408}$	$-\frac{3}{704}$	$-\frac{3}{2816}$	$-\frac{3}{352}$	$-\frac{3}{1408}$	$-\frac{15}{1408}$	$\frac{3}{704}$	$\frac{3}{1408}$	$\frac{3}{704}$	$\frac{3}{1408}$
	(1,1)	$\frac{9}{1024}$	$\frac{9}{512}$	$-\frac{9}{1024}$	$\frac{15}{512}$	$\frac{3}{512}$	$\frac{15}{256}$	$-\frac{9}{512}$	$-\frac{3}{512}$	$-\frac{9}{512}$	$-\frac{3}{512}$
	(1,2)	–	$\frac{3}{256}$	$\frac{3}{256}$	$\frac{3}{128}$	–	$\frac{9}{128}$	$-\frac{3}{128}$	–	$-\frac{3}{128}$	–

The coefficients of the double logarithm associated with each diagram from the various helicity states (λ_1, λ_2) in the processes $\gamma^* \to J/\psi + \chi_{c0,1,2}$. The two types of color factors are represented by $C_1 = C_F^2$ and $C_2 = C_F(C_F - \frac{1}{2}C_A)$

Anatomy of double logarithm for $e^+e^- \to J/\psi + \eta_{c2}(\eta_c)$

Diagrams		$a)$	$b)$	$c)$	$d)$	$e)$	$f)$	$g)$	$h)$	$i)$	$j)$
Color Factor		C_1	C_1	C_1	C_2	C_2	C_2	C_2	C_2	C_2	C_2
η_{c2}	(1,0)	$\frac{3}{64}$	$\frac{9}{128}$	$\frac{3}{256}$	$\frac{9}{64}$	$\frac{3}{128}$	$\frac{21}{128}$	$-\frac{9}{128}$	$-\frac{3}{128}$	$-\frac{9}{128}$	$-\frac{3}{128}$
	(0,1)	–	$-\frac{3\sqrt{3}}{128}$	–	–	–	$-\frac{3\sqrt{3}}{128}$	–	–	–	–
	(1,1)	–	$-\frac{3\sqrt{3}}{256}$	$\frac{3\sqrt{3}}{256}$	–	–	–	–	–	–	–
	(1,2)	–	$-\frac{3\sqrt{6}}{256}$	–	–	–	–	–	–	–	–
η_c	(1,0)	$\frac{3}{64}$	$\frac{3}{32}$	$\frac{3}{128}$	$\frac{3}{32}$	$\frac{3}{64}$	$\frac{9}{64}$	$-\frac{3}{64}$	$-\frac{3}{64}$	$-\frac{3}{64}$	$-\frac{3}{64}$

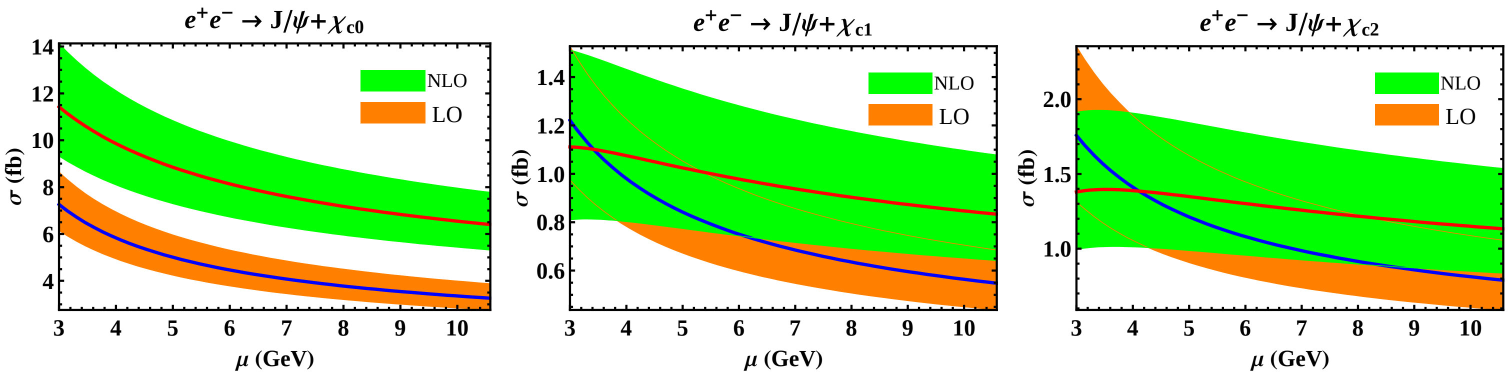
The coefficients of the double logarithm associated with each diagram from the various helicity states (λ_1, λ_2) in the processes $\gamma^* \to J/\psi + \eta_{c2}(\eta_c)$. The two types of color factors are represented by $C_1 = C_F^2$ and $C_2 = C_F(C_F - \frac{1}{2}C_A)$

Phenomenology for $e^+e^- \to J/\psi + \chi_{c0,1,2}$

Input parameters: $|R_{J/\psi}(0)|^2 = 0.81\ \mathrm{GeV}^3$, $|R_{\psi'}(0)|^2 = 0.529\ \mathrm{GeV}^3$, and $|R'_{\chi_{cJ}}(0)|^2 = 0.075\ \mathrm{GeV}^5$ Buchmüller-Tye potential model, $m_c = 1.5$ GeV, $\sqrt{s} = 10.58$ GeV, $\alpha(\sqrt{s}) = 1/130.9$.

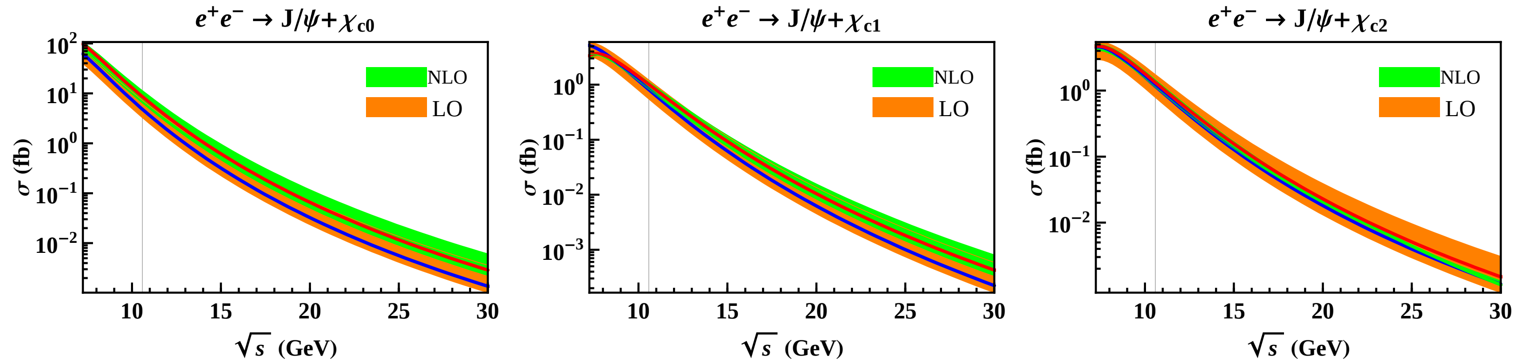
- A weakness of NRQCD approach is the ambiguity in choosing the optimal renormalization scale
- The μ-dependence of LO and NLO cross sections

  Error band due to varying m_c from 1.4 to 1.6 GeV.



- The $\sqrt{s}$-dependence of LO and NLO cross sections

  Error band due to varying μ from $2m_c$ to $\sqrt{s}$



- $\natural$ The cross section for $J/\psi + \chi_{c1}$ falls off faster as $\sqrt{s}$ increases, because of lacking of the leading-twist contribution

Phenomenology for $e^+e^- \to J/\psi + \chi_{c0,1,2}$

$\alpha_s(\sqrt{s}/2) = 0.211$

		$\sigma_{(0,0)}$	$\sigma_{(1,0)}$	$\sigma_{(0,1)}$	$\sigma_{(1,1)}$	$\sigma_{(1,2)}$	σ_{tot}	K
$J/\psi + \chi_{c0}$	LO	1.11	1.86	–	–	–	4.83	1.79
	NLO	1.92	3.35	–	–	–	8.62	
$J/\psi + \chi_{c1}$	LO	–	0.0012	0.37	0.033	–	0.81	1.24
	NLO	–	−0.0078	0.49	0.028	–	1.01	
$J/\psi + \chi_{c2}$	LO	0.43	0.27	0.064	0.033	0.0023	1.17	1.14
	NLO	0.44	0.33	0.081	0.039	0.0026	1.33	

$\alpha_s(2m_c) = 0.259$

		$\sigma_{(0,0)}$	$\sigma_{(1,0)}$	$\sigma_{(0,1)}$	$\sigma_{(1,1)}$	$\sigma_{(1,2)}$	σ_{tot}	K
$J/\psi + \chi_{c0}$	LO	1.67	2.80	–	–	–	7.26	1.57
	NLO	2.50	4.46	–	–	–	11.43	
$J/\psi + \chi_{c1}$	LO	–	0.0017	0.56	0.050	–	1.22	0.91
	NLO	–	−0.016	0.55	0.020	–	1.11	
$J/\psi + \chi_{c2}$	LO	0.65	0.40	0.097	0.050	0.0035	1.76	0.78
	NLO	0.40	0.35	0.089	0.041	0.0026	1.38	

Confronting the old B factory data

We choose $|R_{J/\psi}(0)|^2 = 0.81\ \mathrm{GeV}^3$, $|R_{\psi'}(0)|^2 = 0.529\ \mathrm{GeV}^3$, and $|R'_{\chi_{cJ}}(0)|^2 = 0.075\ \mathrm{GeV}^5$ given by the Buchmüller-Tye potential model, $m_c = 1.5$ GeV, $\sqrt{s} = 10.58$ GeV, $\alpha(\sqrt{s}) = 1/130.9$. The error is estimated by varying μ from $2m_c$ to $\sqrt{s}$, where the central value refers to $\mu = \sqrt{s}/2$.

	BELLE $\sigma \times \mathcal{B}_{>2(0)}$	BABAR $\sigma \times \mathcal{B}_{>2}$	LO prediction	NLO prediction
$\sigma(J/\psi + \chi_{c0})$	$6.4 \pm 1.7 \pm 1.0$	$10.3 \pm 2.5^{+1.4}_{-1.8}$	$4.83^{+2.43}_{-1.57}$	$8.62^{+2.80}_{-2.22}$
$\sigma(J/\psi + \chi_{c1})$	PRD **79**,071101 (2009)	-	$0.81^{+0.41}_{-0.26}$	$1.01^{+0.10}_{-0.18}$
$\sigma(J/\psi + \chi_{c2})$		-	$1.17^{+0.59}_{-0.38}$	$1.33^{+0.04}_{-0.20}$
$\sigma(J/\psi + \chi_{c1}) + \sigma(J/\psi + \chi_{c2})$	< 5.3 at 90% CL	-	$1.98^{+1.00}_{-0.64}$	$2.35^{+0.14}_{-0.38}$
$\sigma(\psi' + \chi_{c0})$	$12.5 \pm 3.8 \pm 3.1$	-	$2.79^{+1.41}_{-0.91}$	$4.98^{+1.62}_{-1.28}$
$\sigma(\psi' + \chi_{c1})$	PRD **79**,071101 (2009)	-	$0.47^{+0.24}_{-0.15}$	$0.58^{+0.06}_{-0.10}$
$\sigma(\psi' + \chi_{c2})$		-	$0.68^{+0.34}_{-0.22}$	$0.77^{+0.03}_{-0.12}$
$\sigma(\psi' + \chi_{c1}) + \sigma(\psi' + \chi_{c2})$	< 8.6 at 90% CL	-	$1.14^{+0.58}_{-0.37}$	$1.36^{+0.08}_{-0.22}$

Comparing with other work

- Disagree with Ma, Zhang and Chao, PRD (2008)
  - ➜ With the same input parameter, we obtain K factor equal to 1.57 for $e^+e^- \to J/\psi + \chi_{c0}$, while theirs is 2.8

- There was an independent calculation on the $\mathcal{O}(\alpha_s)$ corrections to $e^+e^- \to J/\psi + \chi_{c0,1,2}$
  - ➜ Wang, Ma, Chao, PRD **84**, 034022 (2011)
  - ➜ They computed the $\mathcal{O}(\alpha_s)$ corrections to the unpolarized cross sections $\sigma_{\rm tot}[e^+e^- \to J/\psi + \chi_{c0,1,2}]$
  - ➜ Taking the same input parameters, we find exact agreement

Some observations made from our calculation

- The $\mathcal{O}(\alpha_s)$ correction is positive and substantial for $e^+e^- \to J/\psi + \chi_{c0}$
- Including this correction helps the NRQCD prediction in agreement with the measured $\sigma[e^+e^- \to J/\psi + \chi_{c0}]$.
- $e^+e^- \to \psi' + \chi_{c0}$ remains problematic even after incorporating the $\mathcal{O}(\alpha_s)$ correction: measured Xsection still much larger than theory
- The $\mathcal{O}(\alpha_s)$ corrections to $e^+e^- \to J/\psi + \chi_{c1,2}$ has mild impact, even with the sign uncertain.
  - ➜ K factor could be larger or smaller than 1, depending on the choice of the renormalization scale μ
  - ➜ Large ambiguity in setting the optimal scale: intrinsic weakness of the NRQCD factorization approach

- The hierarchy of various polarized cross sections

HSR seems to be largely violated: $\sqrt{s} = 10.6$ GeV is still too low to trust the asymptotic scaling rule?

➜ For $J/\psi + \chi_{c0}$ production, both the $(0,0)$ and $(\pm 1, 0)$ helicity channels have comparable magnitude, and the latter is even somewhat greater.

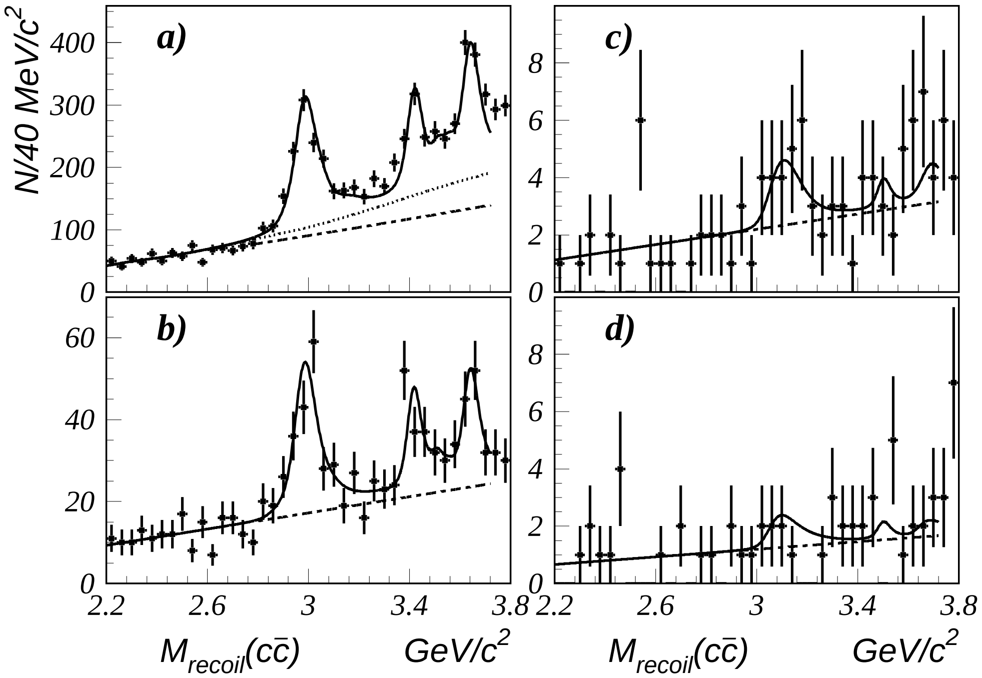
➜ For $J/\psi + \chi_{c1}$ production, the contribution from the $(0, \pm 1)$ channel is far more significant than the other two. Super B experiments may verify that the angular distribution of J/ψ (or χ_{c1}) is proportional to $1 + \cos^2\theta$.

➜ For $J/\psi + \chi_{c2}$ production, the bulk of Xsections comes from the $(0,0)$ and $(\pm 1, 0)$ helicity states. The produced χ_{c2} is predominantly longitudinally-polarized

Confront the updated BELLE measurements

- Recently BELLE collaboration updated their analysis on exclusive double charmonium production

BELLE Collaboration, PRD **79**,071101 (2009)



The mass of the system recoiling against the reconstructed a) J/ψ, b) ψ', c) χ_{c1} and d) χ_{c2}.

$e^+e^- \to (c\bar{c})_{\rm tag}(c\bar{c})_{\rm res}$ signal yields (significances) from a simultaneous fit to $M_{\rm recoil}((c\bar{c})_{\rm tag})$ spectra.

$(c\bar{c})_{\rm res}$	$(c\bar{c})_{\rm tag}$: J/ψ	ψ'	χ_{c1}	χ_{c2}
η_c	$1032 \pm 62\,(19)$	$161 \pm 22\,(8.2)$	—	—
J/ψ	—	—	$16 \pm 5\,(3.2)$	$9 \pm 4\,(2.1)$
χ_{c0}	$525 \pm 54\,(9.6)$	$75 \pm 19\,(4.3)$	—	—
χ_{c1}	$119 \pm 39\,(3.2)$	12 ± 12	—	—
h_c	—	—	4 ± 6	1 ± 5
χ_{c2}	$99 \pm 43\,(2.1)$	7 ± 16	—	—
η_c'	$679 \pm 63\,(10)$	$81 \pm 19\,(4.5)$	—	—
ψ'	—	—	6 ± 6	2 ± 5

- Our predictions for $e^+e^- \to J/\psi + \chi_{c1}$ are qualitatively compatible with the data
- Needs more statistics to establish $e^+e^- \to J/\psi + \chi_{c2}$ signals

Outlook

- Relativistic correction to $e^+e^- \to J/\psi(\psi') + P,\ D-\mathrm{wave\ charmonium}$
  - ➜ Useful to resolve the discrepancy for $e^+e^- \to \psi' + \chi_{c0}$?
  - ➜ More $\mathcal{O}(v^2)$ NRQCD matrix elements needed for P-wave states.
  - ➜ $\diamondsuit$ Can lattice NRQCD be of help to determine the nonperturbative matrix elements of P-wave onium ?
  - ➜ Alternatively, they may be fitted by confronting NRQCD prediction with the latest $\chi_{c0,2} \to \gamma\gamma$ data $\Rrightarrow$ Wen-Long Sang's talk
- Tough challenge: How to tame the double logarithm $\alpha_s \ln^2(s/m_c^2)$?
  - ➜ Correlation between the double logarithms and endpoint singularity problem in light-cone approach?
  - ➜ Starting from the light-cone approach, can we reproduce the asymptotic expression of $\mathcal{O}(\alpha_s)$ NRQCD short-distance coefficients for any helicity-suppressed process such as $e^+e^- \to J/\psi + \eta_c$?
  - ➜ $\sharp$ Is it possible to resum these double logarithms to all orders in α_s for $e^+e^- \to J/\psi + \eta_c$?

Thanks for your attention!

Backup slides: $e^+e^- \to J/\psi + \eta_c$

- Large discrepancy between experimental data and Leading-Order NRQCD predictions

  ➜ Experiment

  Belle (2004) : $\sigma[e^+e^- \to J/\psi + \eta_c] \times B_{>2} = 25.6 \pm 2.8 \pm 3.4$ fb.

  BABAR (2005) : $\sigma[e^+e^- \to J/\psi + \eta_c] \times B_{>2} = 17.6 \pm 2.8^{+1.5}_{-2.1}$ fb.

  ➜ NRQCD at LO in α_s and v

  Braaten, Lee (2003) : $\sigma[e^+e^- \to J/\psi + \eta_c] = 3.7 \pm 1.26$ fb.

  Liu, He, Chao (2003) : $\sigma[e^+e^- \to J/\psi + \eta_c] = 5.5$ fb.

  The two calculations employ different choices of m_c, NRQCD matrix elements, and α_s.

  Braaten and Lee also include QED effects.

$e^+e^- \to J/\psi + \eta_c$

- Some Possible Explanations
  - Some of the $J/\psi + \eta_c$ data sample may consist of $J/\psi + J/\psi$ events.
  - Some of the data sample may be from $e^+e^- \to J/\psi + \text{glueball}$.
- α_s Corrections to $e^+e^- \to J/\psi + \eta_c$
  - An important step in resolving the discrepancy.
    - Zhang, Gao, Chao (2005) found that corrections at NLO in α_s yield a K factor of about 1.96.
    - Confirmed by Gong and Wang (2007).
  - Not enough by itself to bring theory into agreement with experiment.
- Relativistic Corrections to $e^+e^- \to J/\psi + \eta_c$
  - Direct and Indirec Corrections.
  - Corrections at NLO in α_s plus relativistic corrections may bring theory into agreement with experiment.
    - Confirmed by He, Fan, Chao (2007).

$e^+e^- \to J/\psi + \eta_c$

- $\sigma_{\text{total}}(e^+e^- \to J/\psi + \eta_c)$ consists of:

5.4 fb	Leading order in α_s and v^2 (including indir. rel. corr., but without QED contribution)
1.0 fb	QED contribution
2.9 fb	Direct relativistic corrections
6.9 fb	Corrections of NLO in α_s
1.4 fb	Interference between rel. corr. and corr. of NLO in α_s
17.6 fb	Total

- "The uncalculated correction to $\sigma(e^+e^- \to J/\psi + \eta_c)$ of relative order $\alpha_s v^2$ is potentially large, as is the uncalculated correction of relative order α_s^4. While the calculation of the former correction may be feasible, the calculation of the latter correction is probably beyond the current state of the art."
arXiv:1010.5827v3 [hep-ph] 11 Feb 2011

$e^+e^- \to J/\psi + \eta_c$

- Time-like electromagnetic form factor

$$\langle J/\psi(P_1,\lambda) + \eta_c(P_2)|J^\mu_{\rm em}|0\rangle = i\,G(s)\,\epsilon^{\mu\nu\rho\sigma}P_{1\nu}P_{2\rho}\varepsilon^*_\sigma(\lambda)$$

$$\sigma[e^+e^- \to J/\psi + \eta_c] = \frac{4\pi\alpha^2}{3}\left(\frac{|\mathbf{P}|}{\sqrt{s}}\right)^3 |G(s)|^2$$

- NRQCD factorization formula

$$\begin{aligned} G(s) \quad &= \quad \sqrt{4M_{J/\psi}M_{\eta_c}}\langle J/\psi|\psi^\dagger\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}\chi|0\rangle\langle\eta_c|\psi^\dagger\chi|0\rangle \\ &\quad\left[c_0 + c_{2,1}\langle v^2\rangle_{J/\psi} + c_{2,2}\langle v^2\rangle_{\eta_c} + \cdots\right]\end{aligned}$$

$$\langle v^2\rangle_{J/\psi} = \frac{\langle J/\psi(\lambda)|\psi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})^2\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}(\lambda)\chi|0\rangle}{m_c^2\,\langle J/\psi(\lambda)|\psi^\dagger\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}(\lambda)\chi|0\rangle} \quad \langle v^2\rangle_{\eta_c} = \frac{\langle\eta_c|\psi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})^2\chi|0\rangle}{m_c^2\,\langle\eta_c|\psi^\dagger\chi|0\rangle}$$

$$e^+e^- \to J/\psi + \eta_c$$

- Total cross section

$$\sigma[e^+e^- \to J/\psi + \eta_c] = \sigma_0 + \sigma_2 + \mathcal{O}(\sigma_0 v^4)$$

$$\sigma_0 = \frac{8\pi\alpha^2 m_c^2 (1-4r)^{3/2}}{3} \langle \mathcal{O}_1 \rangle_{J/\psi} \langle \mathcal{O}_1 \rangle_{\eta_c} |c_0|^2,$$

$$\sigma_2 = \frac{4\pi\alpha^2 m_c^2 (1-4r)^{3/2}}{3} \langle \mathcal{O}_1 \rangle_{J/\psi} \langle \mathcal{O}_1 \rangle_{\eta_c} \left\{ \left(\frac{1-10r}{1-4r} |c_0|^2 + 4\,\mathrm{Re}[c_0 c_{2,1}^*] \right) \langle v^2 \rangle_{J/\psi} + \left(\frac{1-10r}{1-4r} |c_0|^2 + 4\,\mathrm{Re}[c_0 c_{2,2}^*] \right) \langle v^2 \rangle_{\eta_c} \right\}$$

- LO coefficients

$$c_0^{(0)} = \frac{32\pi C_F e_c \alpha_s}{N_c\, m_c s^2}, \quad c_{2,1}^{(0)} = c_0^{(0)} \left[\frac{3-10r}{6} + \left(1 - \frac{16}{9} r\right) \epsilon + \mathcal{O}\left(\epsilon^2\right) \right]$$

$$c_{2,2}^{(0)} = c_0^{(0)} \left[\frac{2-5r}{3} + \left(\frac{10}{9} - \frac{16}{9} r \right) \epsilon + \mathcal{O}\left(\epsilon^2\right) \right], \qquad r \equiv \frac{4m_c^2}{s}$$

$$e^+e^- \longrightarrow J/\psi + \eta_c$$

- Next-to-leading order coefficients

$$c_0^{(1)}\left(r, \frac{\mu_r^2}{s}\right)\underset{\rm asym}{=} c_0^{(0)} \times \Bigg\{ \beta_0\left(-\frac{1}{4}\ln\frac{s}{4\mu_r^2}+\frac{5}{12}\right)+\left(\frac{13}{24}\ln^2 r+\frac{5}{4}\ln 2\ln r-\frac{41}{24}\ln r -\frac{53}{24}\ln^2 2+\frac{65}{8}\ln 2-\frac{1}{36}\pi^2-\frac{19}{4}\right)+i\pi\left(\frac{1}{4}\beta_0+\frac{13}{12}\ln r+\frac{5}{4}\ln 2-\frac{41}{24}\right)\Bigg\},$$

$$c_{2,1}^{(1)}\left(r, \frac{\mu_r^2}{s}, \frac{\mu_f^2}{m_c^2}\right)\underset{\rm asym}{=} \frac{1}{2}c_0^{(0)} \times \Bigg\{ \frac{16}{9}\ln\frac{\mu_f^2}{m_c^2}+\beta_0\left(-\frac{1}{4}\ln\frac{s}{4\mu_r^2}+\frac{11}{12}\right)+\left(\frac{3}{8}\ln^2 r+\frac{19}{12}\ln 2\ln r +\frac{31}{24}\ln r-\frac{1}{24}\ln^2 2+\frac{893}{216}\ln 2-\frac{5}{36}\pi^2-\frac{497}{72}\right)+i\pi\left(\frac{1}{4}\beta_0+\frac{3}{4}\ln r+\frac{19}{12}\ln 2+\frac{9}{8}\right)\Bigg\},$$

$$c_{2,2}^{(1)}\left(r, \frac{\mu_r^2}{s}, \frac{\mu_f^2}{m_c^2}\right)\underset{\rm asym}{=} \frac{2}{3}c_0^{(0)} \times \Bigg\{ \frac{4}{3}\ln\frac{\mu_f^2}{m_c^2}+\beta_0\left(-\frac{1}{4}\ln\frac{s}{4\mu_r^2}+\frac{2}{3}\right)+\left(\frac{1}{12}\ln^2 r+\frac{11}{12}\ln 2\ln r -\frac{1}{24}\ln r-\frac{11}{8}\ln^2 2+\frac{241}{144}\ln 2-\frac{1}{8}\pi^2-\frac{99}{16}\right)+i\pi\left(\frac{1}{4}\beta_0+\frac{1}{6}\ln r+\frac{11}{12}\ln 2-\frac{1}{24}\right)\Bigg\}.$$

- Phenomenology

  ➜ Numeric parameters

$$\sqrt{s} = 10.58 \text{ GeV}, \quad \alpha(\sqrt{s}) = 1/130.9,$$
$$\langle \mathcal{O}_1 \rangle_{J/\psi} \approx \langle \mathcal{O}_1 \rangle_{\eta_c} = 0.387 \text{ GeV}^3,$$
$$\langle v^2 \rangle_{J/\psi} = 0.223, \quad \langle v^2 \rangle_{\eta_c} = 0.133, \quad \mu_f = m_c$$

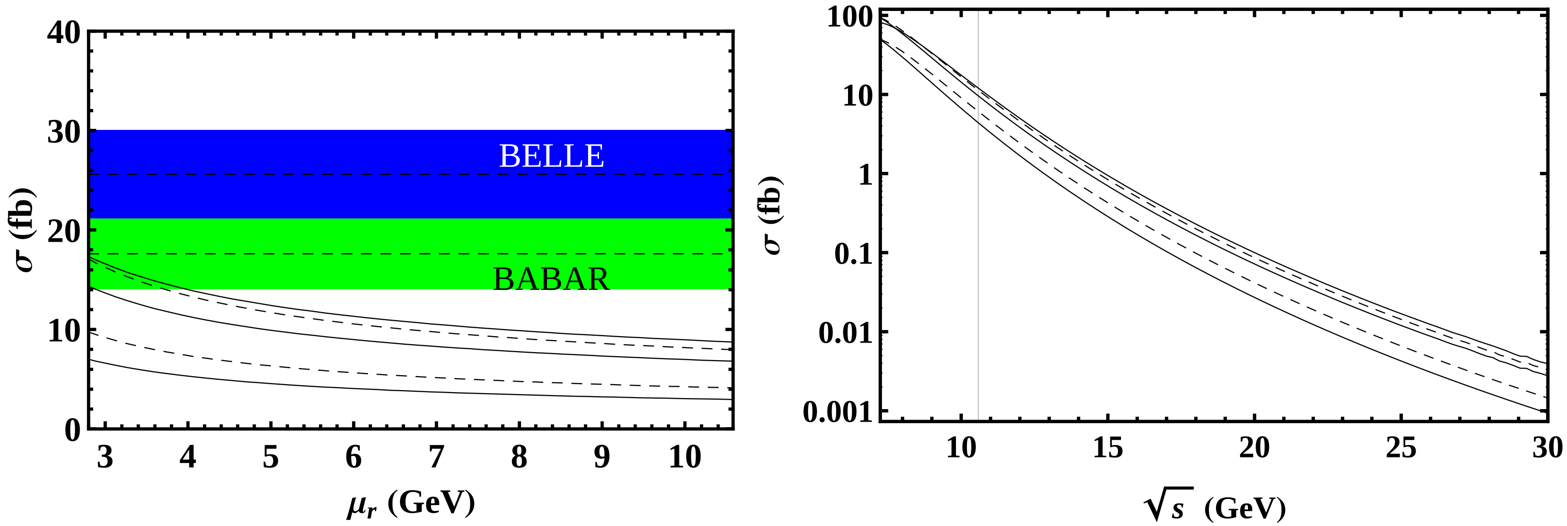
  ➜ Contributions from different parts

	$\sigma_0^{(0)}$	$\sigma_0^{(1)}$	$\sigma_2^{(0)}$	$\sigma_2^{(1)}$
$\alpha_s(\frac{\sqrt{s}}{2}) = 0.211$	4.40	5.22	1.72	0.73
$\alpha_s(2m_c) = 0.267$	7.00	7.34	2.73	0.24

Individual contributions to the predicted $\sigma[e^+e^- \to J/\psi + \eta_c]$ at $\sqrt{s} = 10.58$ GeV, labeled by powers of α_s and v.

The cross sections are in units of fb.

$$e^+e^- \to J/\psi + \eta_c$$



The μ_r and $\sqrt{s}$- dependence of the cross section for $e^+e^- \to J/\psi + \eta_c$. The 5 curves from bottom to top are $\sigma_0^{(0)}$ (solid line), $\sigma_0^{(0)} + \sigma_2^{(0)}$ (dashed line), $\sigma_0^{(0)} + \sigma_0^{(1)}$ (solid line), $\sigma_0^{(0)} + \sigma_2^{(0)} + \sigma_0^{(1)}$ (dashed line), and $\sigma_0^{(0)} + \sigma_2^{(0)} + \sigma_0^{(1)} + \sigma_2^{(1)}$ (solid line), respectively.