Attempt extracting ππ s-wave scattering lengths from cusp effect in heavy quarkonium dipion transitions

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Outline

- Brief review of $\pi\pi$ scattering
- Experimental methods of measuring $\pi\pi$ scattering lengths
- Investigating cusp effect in heavy quarkonium dipion transition according to non-relativistic effective theory (NREFT)
- Explore the availability of experiment
- Summary

$\pi\pi$ scattering

Pions play a special role in strong interactions.

At low energy, the strength of $\pi\pi$ interaction is described by the scattering length

$$a_0^0 = \frac{7M_\pi^2}{32\pi F_\pi^2} + \mathcal{O}(m_q^2) \qquad a_0^2 = -\frac{M_\pi^2}{16\pi F_\pi^2} + \mathcal{O}(m_q^2)$$

Weinberg, PRL17,616(1966)

Vanish in the chiral limit

ChPT combined with Roy equations, to two loops

G. Colangelo et al, PLB488,261(2000), NPB603,125(2001)

$$a_0^0 = 0.220 \pm 0.005, \; a_0^2 = -0.0444 \pm 0.0010$$
$$a_0^0 - a_0^2 = 0.265 \pm 0.004$$

Experimental methods

- K_{e4} decay, $K^+ \rightarrow \pi^+\pi^- e^+ \nu_e$

  sensitive to the $\pi\pi$ phase shifts which are related

  with the scattering lengths

  Rosselet et al, Geneva-Saclay, PRD15,574(1977); BNL-E865,PRL87, 221801(2011);NA48/2, EPJC54,411(2008)

- Lifetime of pionium($\pi^+\pi^-$ bound state)

$$\Gamma_{\pi\pi} \propto (a_0-a_2)^2$$

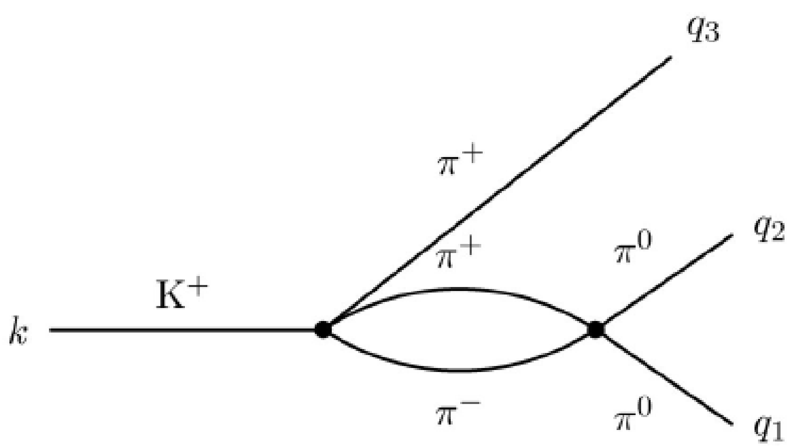
  Uretsky & Palfrey, PR121,1798(1961); DIRAC collab. PLB619,50(2005)

- Cusp effects, $K\rightarrow 3\pi$, $\eta\rightarrow 3\pi$, $\eta'\rightarrow \eta\ \pi\pi$

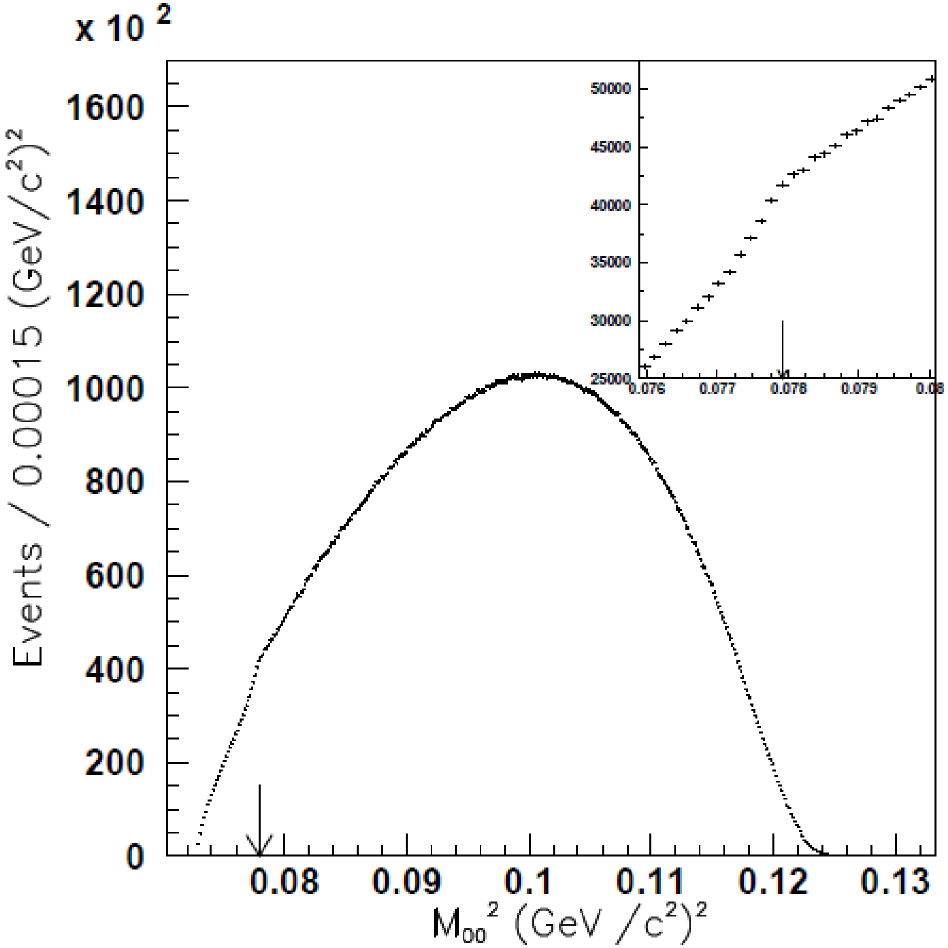
  Budini & Fonda, 1961; Cabibbo, 2004; NA48/2, 2006; Gasser, Kubis & Rusetsky, 2011; Kubis & Schneider, 2009; …….

Cusp effect

Resulted from charge-exchange rescattering $\pi^+\pi^- \rightarrow \pi^0\pi^0$ and the pion mass difference



Budini & Fonda, 1961;
Cabibbo, 2004;



NA48/2, PLB633,173
6 ×10^7 events

Branching ratio

$K^+ \rightarrow \pi^+\pi^-\pi^+$ $((5.59 \pm 0.04)\%)$

much larger than

$K^+ \rightarrow \pi^0\pi^0\pi^+$ $((1.761 \pm 0.022)\%)$

Cusp effect in $\psi' \to J/\psi\pi^0\pi^0$ & $\Upsilon(3S) \to \Upsilon(2S)\pi^0\pi^0$

✓**Branching ratios**

$Br(\psi' \to J/\psi\pi^0\pi^0)$ **~(17.73±0.34) %**

$Br(\psi' \to J/\psi\pi^+\pi^-)/Br(\psi' \to J/\psi\pi^0\pi^0) \approx 2$

$Br(\Upsilon(3S) \to \Upsilon(2S)\pi^0\pi^0) \sim (1.85 \pm 0.14)\%$

$Br(\Upsilon(3S) \to \Upsilon(2S)\pi^+\pi^-) \sim (2.45 \pm 0.23)\%$

$$\boxed{\begin{array}{l} Br(\eta' \to \eta\pi^+\pi^-)/Br(\eta' \to \eta\pi^0\pi^0) \sim 2 \\ Br(\eta \to \pi^+\pi^-\pi^0)/Br(\eta \to 3\pi^0) \sim 0.7 \\ Br(K_L \to \pi^+\pi^-\pi^0)/Br(K_L \to 3\pi^0) \sim 0.64 \end{array}}$$

✓**Huge data sample of heavy quarkonium**

BESIII, Belle, BarBar, CLEOc, LHCb ……

✓**Interaction between pion and heavy quarkonium is highly OZI suppressed, which may simplifies the problem compared with other processes**

NREFT method

The process is similar to η'→ηππ

Near the energy region of ππ threshold, follow the NREFT method adapted in

[Kubis&Schneider,EPJC62,511(2009)] and references therein

Power counting scheme:

Introduce a formal small parameter ε, in the initial state rest frame,

✓**three momenta** $|\vec{p}_i| \sim O(\epsilon)$

✓**The mass M ~ O(1)**

✓**Kinetic energy** $T_i = p_i^0 - M \sim O(\epsilon^2)$

✓**Combined with another small parameter** $a_{\pi\pi}$

NREFT method

Effective Lagrangian

$$\mathcal{L} = \mathcal{L}_\psi + \mathcal{L}_{\pi\pi},$$

Up to $\mathcal{O}(\epsilon^2)$

$$\begin{aligned}\mathcal{L}_\psi &= \frac{1}{2}\sum_{n=0}^{1} G_n \left(\psi'^\dagger_i (W_{J/\psi} - M_{J/\psi})^n J_i \Phi_0 \Phi_0 + h.c.\right) \\ &+ \sum_{n=0}^{1} H_n \left(\psi'^\dagger_i (W_{J/\psi} - M_{J/\psi})^n J_i \Phi_+ \Phi_- + h.c.\right) + \cdots ,\end{aligned}$$

$$W_{J/\psi} = \sqrt{M_{J/\psi}^2 - \triangle}$$

Laplacian

NREFT method

$$\begin{aligned}
\mathcal{L}_{\pi\pi} &= 2\sum_{k=0,\pm} \Phi_k^\dagger W_k(i\partial_t - W_k)\Phi_k \\
&+ C_x(\Phi_0^\dagger\Phi_0^\dagger\Phi_+\Phi_- + h.c.) + \frac{1}{4}C_{00}(\Phi_0^\dagger\Phi_0^\dagger\Phi_0\Phi_0 + h.c.) \\
&+ D_x\left[(\Phi_0^\dagger)_\mu(\Phi_0^\dagger)^\mu\Phi_+\Phi_- + \Phi_0^\dagger\Phi_0^\dagger(\Phi_+^\dagger)_\mu(\Phi_-^\dagger)^\mu + h.c.\right] \\
&+ \frac{1}{4}D_{00}\left[(\Phi_0^\dagger)_\mu(\Phi_0^\dagger)^\mu\Phi_0\Phi_0 + \Phi_0^\dagger\Phi_0^\dagger(\Phi_0^\dagger)_\mu(\Phi_0^\dagger)^\mu + h.c.\right] + \cdots .
\end{aligned}$$

$$\begin{aligned}
(\Phi_k)_\mu &= (\mathcal{P}_k)_\mu\Phi_k,\ (\mathcal{P}_k)_\mu = (W_k, -i\nabla) \\
(\Phi_k^\dagger)_\mu &= (\mathcal{P}_k^\dagger)_\mu\Phi_k^\dagger,\ (\mathcal{P}_k^\dagger)_\mu = (W_k, i\nabla)
\end{aligned}$$

$$W_k = \sqrt{M_k^2 - \triangle}$$

Couplings

Effective range expansion

$$T^I(s,t) = 32\pi \sum_{l=0}^{\infty} (2l+1) t_l^I(s) P_l(z)$$

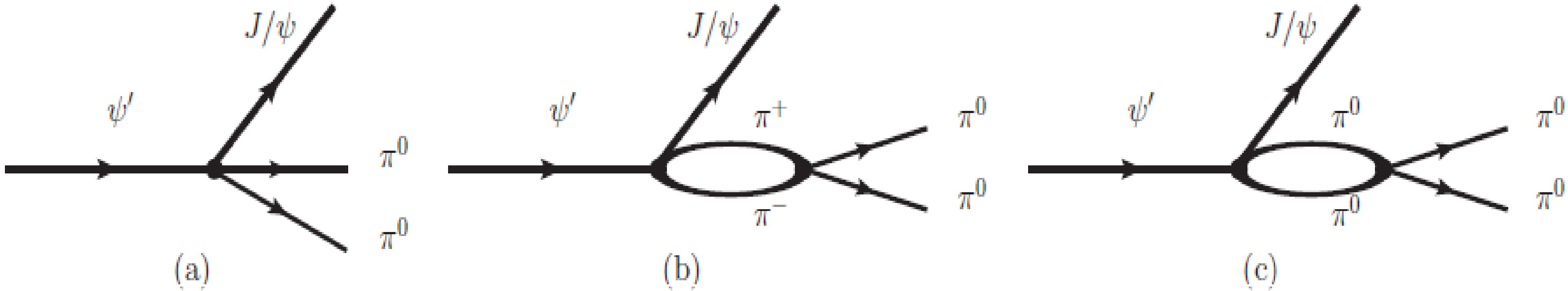
$$\mathrm{Re}\ t_l^I(s) = q_{ab}^{2l}\left[a_l^I + b_l^I q_{ab}^2 + \mathcal{O}(q_{ab}^4)\right]$$

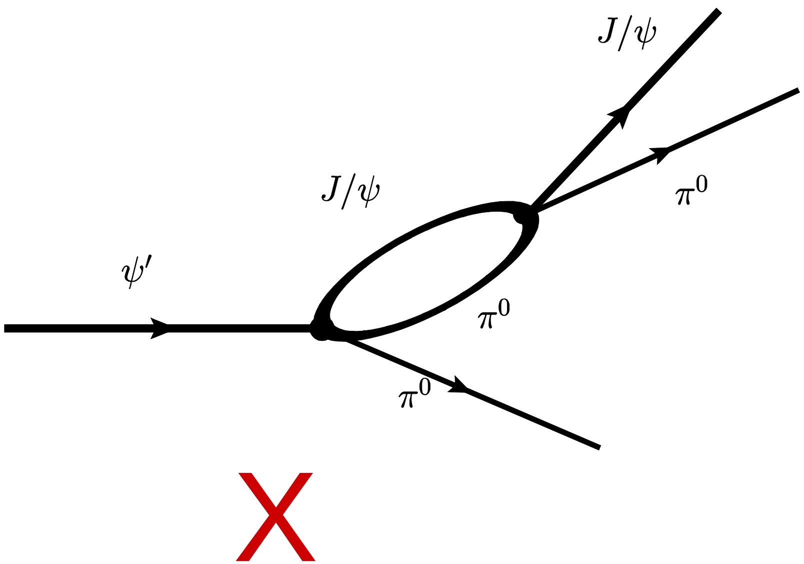
Ananthanarayan et al, Phys.Rep.353, 207(2001)

t_l^I is the partial wave amplitude with the angular momentum l and isospin I, via mathching to the amplitude obtained by NREFT

$$\begin{aligned}
C_x &= \frac{16\pi}{3} M_{\pi^+}(a_2 - a_0), \\
C_{00} &= \frac{16\pi}{3} M_{\pi^+}(a_0 + 2a_2), \\
D_x &= \frac{4\pi}{3} M_{\pi^+}(b_2 - b_0), \\
D_{00} &= \frac{4\pi}{3} M_{\pi^+}(b_0 + 2b_2),
\end{aligned}$$

Feynman Diagrams





highly OZI suppressed

Utilizing the data of heavy quarkonium dipon transitions

$$|a_{J/\psi\pi}| < 0.02\text{fm}$$
$$|a_{\Upsilon(2S)\pi}| < 0.01\text{fm}$$

Liu et al, EPJC73,2284(2013)

$$a_{J/\psi\pi} = (-0.01 \pm 0.01)\text{fm}$$

Liu et al, Pos LATTICE2008,112

Transition Amplitude

$$\psi'(P_{\psi'}) \to \pi^0(p_1)\pi^0(p_2)J/\psi(p_3), \qquad s_i = (P_{\psi'} - p_i)^2$$

$$\begin{aligned} T^{tree} &= \left[G_0 + G_1(p_3^0 - M_J)\right] \vec{\epsilon}_{\psi'} \cdot \vec{\epsilon}_J, \\ T^{1-loop} &= 2\left[C_x + D_x(s_3 - 4M_{\pi^+}^2)\right]\left[H_0 + H_1(p_3^0 - M_J)\right] J_{+-}(s_3)\vec{\epsilon}_{\psi'} \cdot \vec{\epsilon}_J \\ &+ \left[C_{00} + D_{00}(s_3 - 4M_{\pi^0}^2)\right]\left[G_0 + G_1(p_3^0 - M_J)\right] J_{00}(s_3)\vec{\epsilon}_{\psi'} \cdot \vec{\epsilon}_J, \end{aligned}$$

loop intergral

$$J_{ab}(P^2) = \int \frac{d^D l}{i(2\pi)^D} \frac{1}{2w_a(\vec{l})(w_a(\vec{l}) - l_0)} \frac{1}{2w_a(\vec{P} - \vec{l})(w_a(\vec{P} - \vec{l}) - P_0 + l_0)}$$

$$w(\vec{l}) = \sqrt{M^2 + \vec{l}^2}$$

Transition Amplitude

$$J_{+-}(s_3) = \frac{1}{16\pi}\sqrt{\frac{4M^2_{\pi^+} - s_3}{s_3}}, \text{ when } s_3 \le 4M^2_{\pi^+},$$

$$J_{+-}(s_3) = \frac{i}{16\pi}\sqrt{\frac{s_3 - 4M^2_{\pi^+}}{s_3}}, \text{ when } s_3 > 4M^2_{\pi^+},$$

$$J_{00}(s_3) = \frac{i}{16\pi}\sqrt{\frac{s_3 - 4M^2_{\pi^0}}{s_3}},$$

~ O(ε)

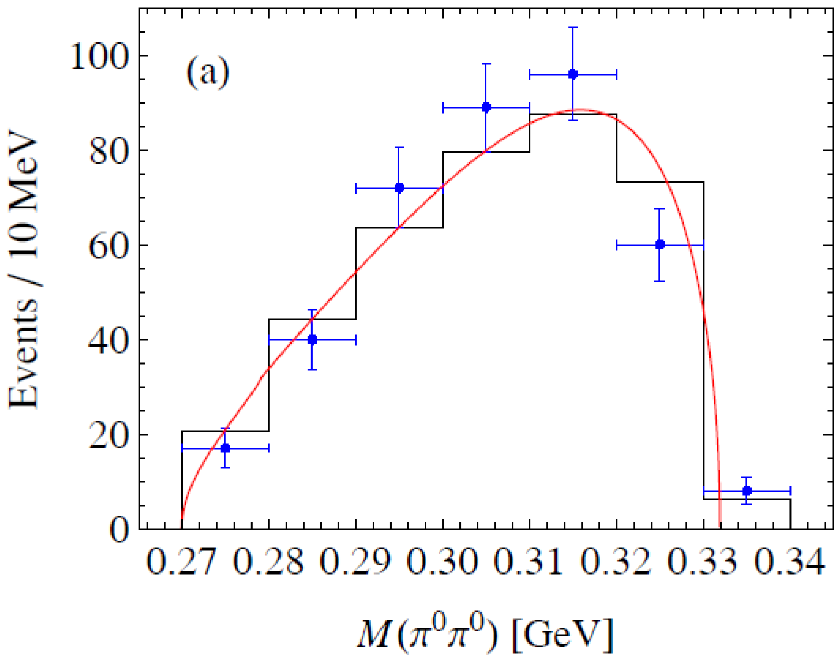
To one loop, the final result is derived up to $O(a_{\pi\pi}\varepsilon^2)$

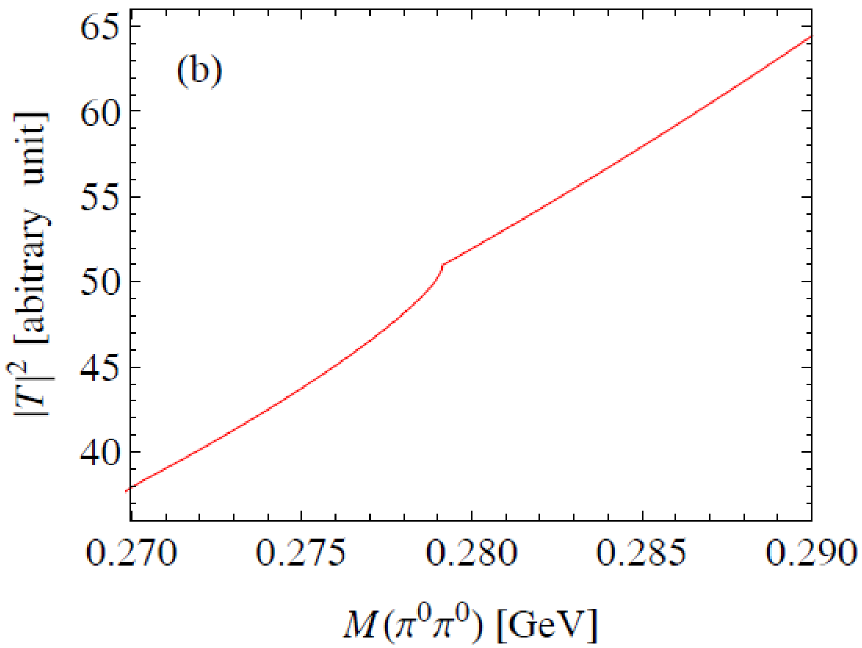
$$\boxed{T = \left[G_0 + G_1(p_3^0 - M_J) + 2C_x H_0 J_{+-}(s_3) + C_{00}G_0 J_{00}(s_3)\right]\vec{\epsilon}_{\psi'} \cdot \vec{\epsilon}_J}$$

Explore the availability of experiment

- ✓The kinematical region around $\pi\pi$ threshold is suppressed in $\psi' \rightarrow J/\psi\pi^0\pi^0$, $\Upsilon(3S) \rightarrow \Upsilon(2S)\pi^0\pi^0$ is better
- ✓To make a MC simulation, it is necessary to determine the values of G_0 and G_1, or the ratio G_0/G_1
- ✓Since NREFT will only work in the kinematical region around $\pi\pi$ threshold, the Chiral Unitary Approach (CHUA) is adopted to estimate G_0/G_1, where $\pi\pi$ FSI will be taken into account.

Estimation on G_0/G_1



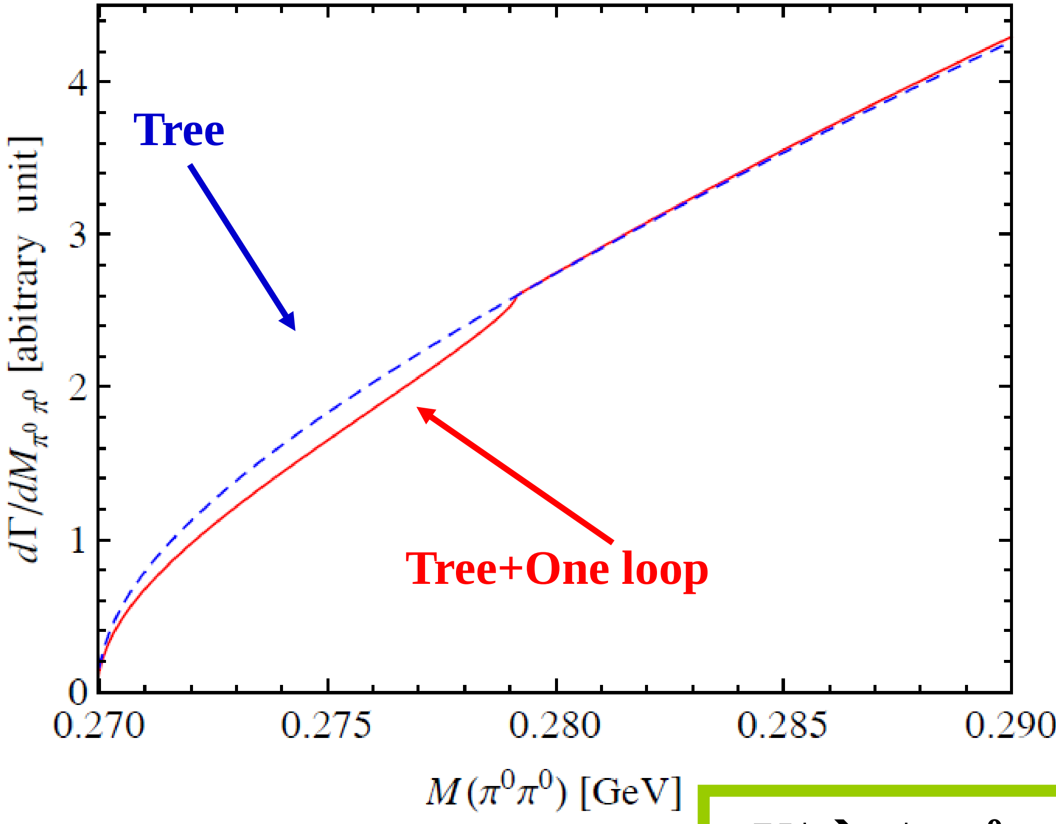


Fit to the $\pi^0\pi^0$ invariant mass spectrum of $\Upsilon(3S) \to \Upsilon(2S)\pi^0\pi^0$ measured by CLEOc [PRD76, 072001] with CHUA

By matching NREFT to CHUA, a rough estimation obtained

$$\frac{G_0}{G_1} = -4.37^{+0.81}_{-0.56}\ \text{MeV}$$

Cusp effect at the $\pi^+\pi^-$ threshold in the reaction $\Upsilon(3S) \to \Upsilon(2S)\pi^0\pi^0$ calculated in the NREFT framework



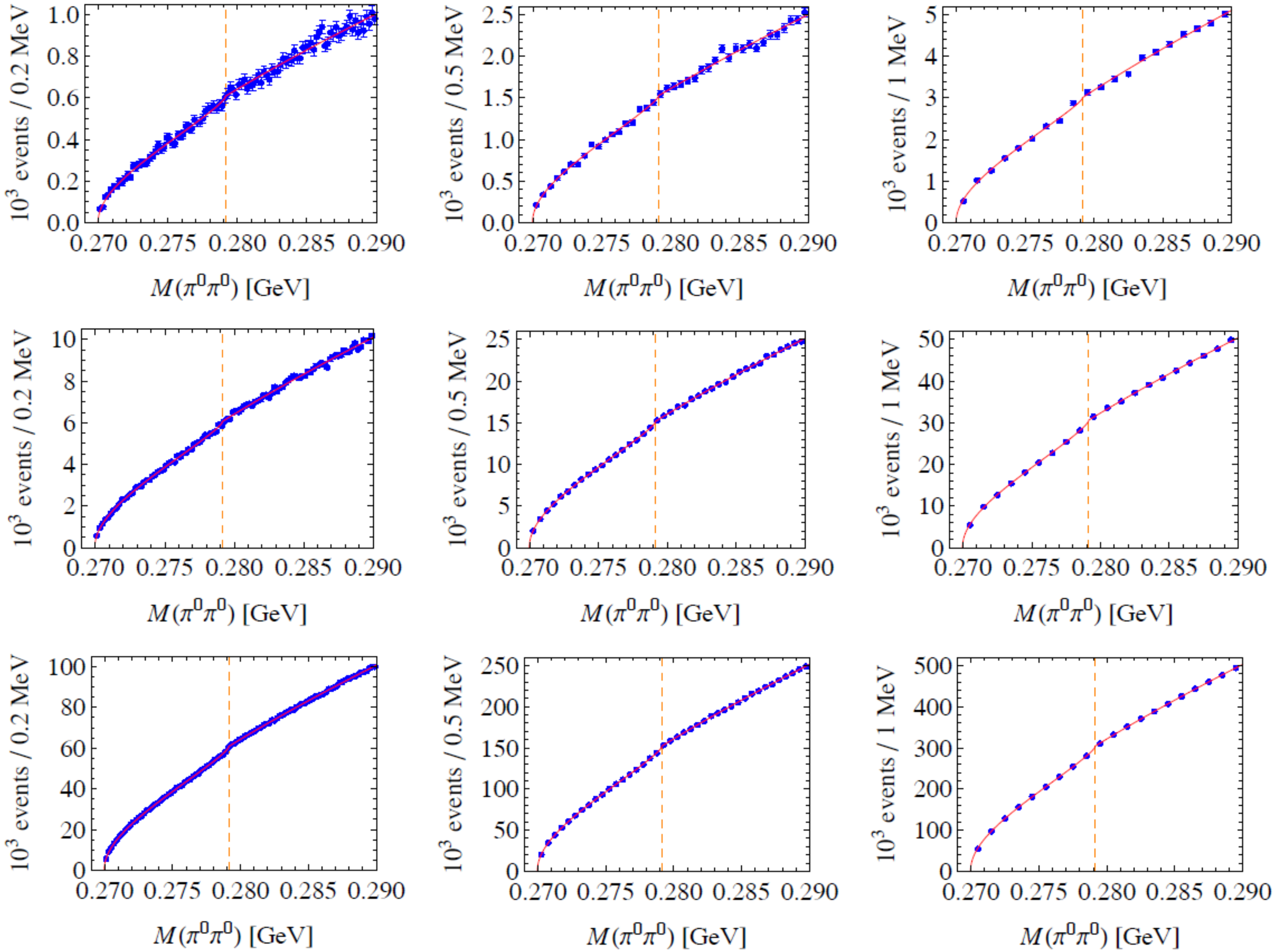
Reduce the number of events about 9%

$K^+ \to \pi^+\pi^-\pi^0$	13%
$\eta' \to \eta\pi\pi$	8%
$\eta \to 3\pi$	less than 2%

Kubis&Schneider, (2009)

Monte Carlo simulations

Number of events



6×10^6

6×10^5

6×10^4

Monte Carlo simulations

Results of fitting to various sets of MC data (in units of $M_{\pi^+}^{-1}$)

Bin width	Events	6×10^4	6×10^5	3×10^6	6×10^6
0.1 MeV	χ^2/dof	1.21	1.09	1.16	0.88
	a_0-a_2	0.293 ± 0.036	0.260 ± 0.012	0.2717 ± 0.0048	0.2661 ± 0.0036
0.2 MeV	χ^2/dof	0.72	1.15	1.05	1.12
	a_0-a_2	0.286 ± 0.035	0.251 ± 0.014	0.2722 ± 0.0048	0.2621 ± 0.0038
0.5 MeV	χ^2/dof	0.93	0.54	1.27	1.30
	a_0-a_2	0.262 ± 0.026	0.256 ± 0.012	0.2659 ± 0.0051	0.2693 ± 0.0035
1 MeV	χ^2/dof	1.05	0.78	1.17	0.69
	a_0-a_2	0.221 ± 0.054	0.291 ± 0.010	0.2658 ± 0.0054	0.2661 ± 0.0037
2 MeV	χ^2/dof	0.59	1.06	1.05	1.37
	a_0-a_2	0.260 ± 0.040	0.262 ± 0.012	0.2592 ± 0.0055	0.2632 ± 0.0037

Input 0.264

~10-20%

Compared with input value, statistical precision

~1.5-2%

6×10^6 events in the range of [270,290]MeV corresopnd to about 2 billion Y(3S) events

- **The precision seems to be insensitive to the bin widths!**

Summary

- **Propose to extract the $\pi\pi$ scattering lengths from heavy quarkonium dipion transitions, especially** $\Upsilon(3S) \rightarrow \Upsilon(2S)\pi^0\pi^0$
- **Weak $J/\psi\pi$ ($\Upsilon\pi$) interaction will simplify the model, and larger branching ratio rate will lead to obvious cusp effect**
- **NREFT method is taken to calculate the amplitude, which is derived up to $O(a_{\pi\pi}\epsilon^2)$**
- **By utilizing MC simulations, we explore the availability of experiment. It seems that the results will be insensitive to the bin widths.**

Thanks!