# Extracting the *b* quark mass from lattice NRQCD

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HPQCD Collaboration

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# Outline

- Extracting $m_b$ from lattice NRQCD
  - Nonperturbative calculation
  - Two loop perturbation theory – a mixed approach:
    - Gauge contributions – weak coupling simulations
    - Femionic – automated lattice perturbation theory
- Results
- Summary

# Extracting $m_b$

Relate lattice quantities to $\overline{MS}$ mass in two stages via pole mass:

$$\overline{m}_b(\mu) \Leftrightarrow m_{\text{pole}} \Leftrightarrow aE^{\text{sim}}$$

Stage 1: lattice to pole mass

$$2m_{\text{pole}} = M_{\Upsilon}^{\text{expt}} - a^{-1}\left(aE_{\Upsilon}^{\text{sim}} - 2aE_0\right)$$
$$m_{\text{pole}} = M_{B_s}^{\text{expt}} - a^{-1}\left(aE_{B_s}^{\text{sim}} - aE_0\right)$$

Stage 2: pole mass to $\overline{MS}$ mass

$$\overline{m}_b(\mu) = Z_M^{-1}(\mu) m_{\text{pole}}$$

[Melnikov and van Ritbergen, Phys.Lett. B482 (2000) 99]

Previous HPQCD result: $\overline{m}_b(\overline{m}_b, n_f = 5) = 4.4(1)$ GeV

- ⋆ uncertainty dominated by two loop contribution to $aE_0$

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# Determining $aE_{\mathrm{sim}}$: Lattice Details

Nonperturbative $aE^{\mathrm{sim}}$:

- ⋆ ground state energy
- ⋆ determined from multi-exponential fits to meson correlators

Bare $b$ quark mass tuned via spin-averaged kinetic mass

$$\overline{M_{b\bar{b}}} = \left(3M_{\Upsilon}^{\mathrm{kin}} + M_{\eta_b}^{\mathrm{kin}}\right)/4 \qquad aM^{\mathrm{kin}} = \frac{a^2P^2 - (a\Delta E)^2}{2a\Delta E}$$

[Dowdall *et al.*, Phys.Rev. D85 (2012) 054509]

Account for missing EM, sea $c$ quarks and gluon annihilation

- ⋆ tune to adjusted "experimental" mass $M_{b\bar{b}}^{\mathrm{expt}} = 9.450(4)$ GeV

[Gregory *et al.*, Phys.Rev. D83 (2011) 014506]

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# DETERMINING $aE_0$

Why "a mixed approach" to lattice perturbation theory?

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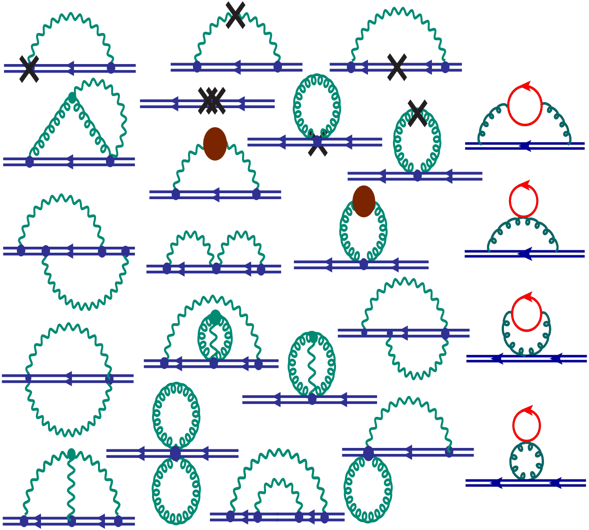
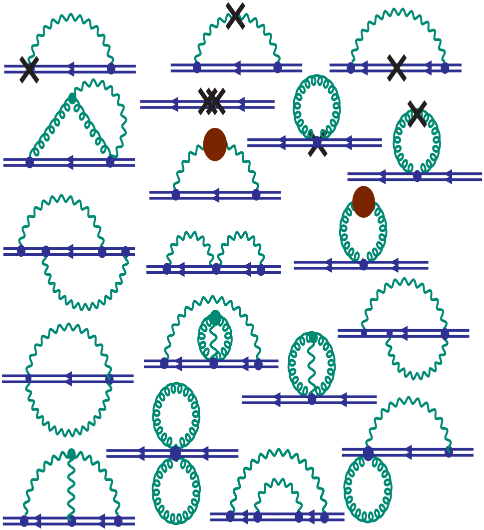


# Weak Coupling Quenched Simulations



# Weak Coupling Quenched Simulations

- ⋆ Extract $aE_0$ from quark propagator at large enough $t \geq t_{\rm min}$

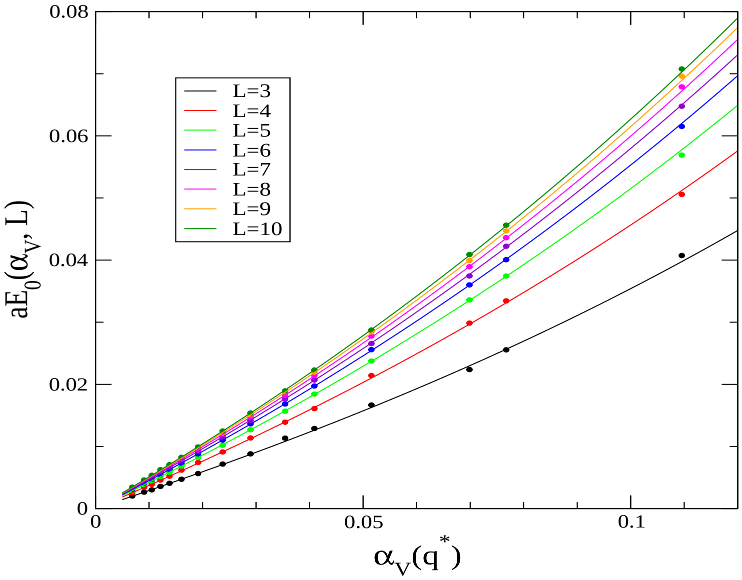
$$G(\mathbf{p}, t, \beta, L) = Z_\psi e^{(E_0 + \mathbf{p}^2/2M_{\rm pole} + \cdots)t}$$

- ⋆ Simultaneous fit of $aE_0$ to $\alpha_V^{n_f}(q^\star)$ and $L$

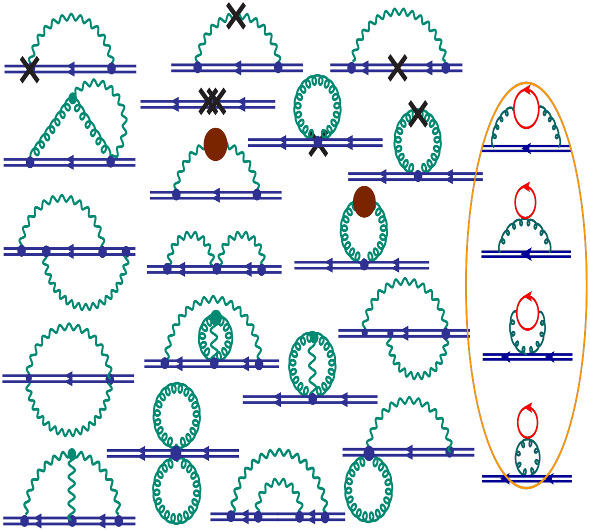
⇒ $L \to \infty$ limit for expansion of $aE_0$ as a power series in $\alpha_V$

- ⋆ For final fit, one loop coefficient constrained by exact one loop value from finite volume perturbation theory
- ⋆ $L^3 \times T$ lattices, with $T = 3L$ and $L$ in range $[3, 10]$
- ⋆ 15 values of $\beta$ in range $[12, 120]$
- ⋆ Impose twisted boundary conditions

# Weak Coupling Quenched Extrapolations



# Automated Perturbation Theory

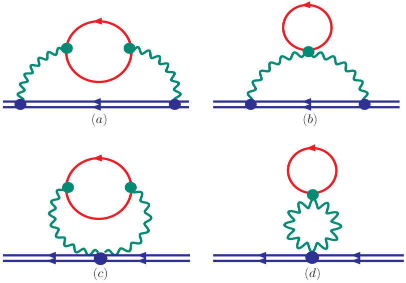


# Automated Perturbation Theory

Fermionic diagrams calculated via two automated routines:

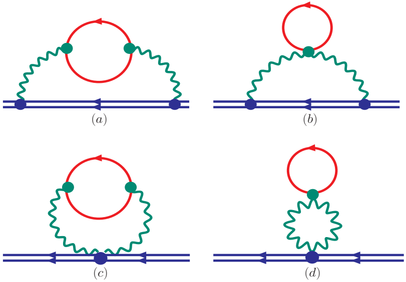
- ⋆ `HiPPy`:
  - ⋆ generates Feynman rules, encoded as "vertex files"
- ⋆ `HPsrc`:
  1. reads "vertex files"
  2. constructs diagrams
  3. evaluates diagrams numerically with `VEGAS`

[Hart *et al.*, Comput.Phys.Commun. 180 (2009) 2698]



# Automated Perturbation Theory

$$
\begin{aligned}
\mathtt{rainbow\_bubble}(\mathtt{p}, -\mathtt{p}) = {} & \mathtt{QQg\_vertex}(\mathtt{p}, -\mathtt{p}-\mathtt{k}, \mathtt{k}) \\
& \times \mathtt{nrqcd\_prop}(\mathtt{p}+\mathtt{k}) \\
& \times \mathtt{QQg\_vertex}(\mathtt{p}, -\mathtt{p}-\mathtt{k}, \mathtt{k}) \\
& \times \mathtt{gluon\_prop\_1loop}(\mathtt{k})
\end{aligned}
$$



# Nonperturbative And Perturbative RESULTS

Nonperturbative results:

| $am_0$ | $am_s^{\mathrm{val}}$ | $aE_{\mathrm{sim}}(\Upsilon)$ | $aE_{\mathrm{sim}}(\eta_b)$ | $aE_{\mathrm{sim}}(B_s)$ | $aE_{\mathrm{sim}}(B_s^\star)$ |
|---|---|---|---|---|---|
| 2.50 | 0.0496 | 0.46591(6) | 0.42579(3) | 0.6278(5) | 0.6595(6) |
| 1.72 | 0.0337 | 0.41385(4) | 0.38124(2) | 0.4812(5) | 0.5027(7) |

Perturbative results:

| $am_0$ | $aE_0^{(1)}$ | $aE_0^{(2)}$ | $aE_0^{u_0,f}$ | $aE_0^{(3),q}$ |
|---|---|---|---|---|
| 2.50 | 0.6788(1) | 1.16(4)-0.2823(6)$n_f$ | 0.158531(16)$n_f$ | 2.3(3) |
| 1.72 | 0.5752(1) | 1.30(4)-0.3041(6)$n_f$ | 0.186607(19)$n_f$ | 2.3(3) |

# Putting The Results Together

Recall: we extract $m_b$ from

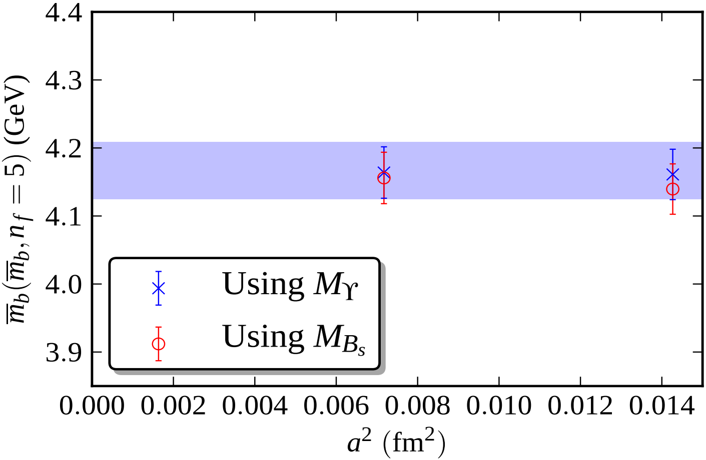
$$\overline{m}_b(\mu) = \frac{1}{2} Z_M^{-1}(\mu) \left[ M_\Upsilon^{\text{expt}} - a^{-1} \left( aE_\Upsilon^{\text{sim}} - 2aE_0 \right) \right]$$

Here $aE_\Upsilon^{\text{sim}}$ obtained with $n_f = 3$; $aE_0$ a series in $\alpha_V(q^\star)$.

$\Rightarrow$ we have $\overline{m}_b(q^\star, n_f = 3)$, but desire $\overline{m}_b(\overline{m}_b, n_f = 5)$

1. reexpress $aE_0$ as series in $\alpha_{\overline{\text{MS}}}(q^\star)$
2. run $\alpha$ to $\alpha_{\overline{\text{MS}}}^{n_f=3}(\overline{m}_b) \Rightarrow \overline{m}_b(\overline{m}_b, n_f = 3))$
3. run $\overline{m}_b(\overline{m}_b, n_f = 3)$ down to $\overline{m}_b(\overline{m}_s, n_f = 3)$
4. match to $\overline{m}_b(\overline{m}_s, n_f = 4)$ and run up to $\overline{m}_b(\overline{m}_b, n_f = 4)$
5. match to $\overline{m}_b(\overline{m}_b, n_f = 5)$
6. repeat for $B_s$ and account for $a$-dependence via Bayesian fit

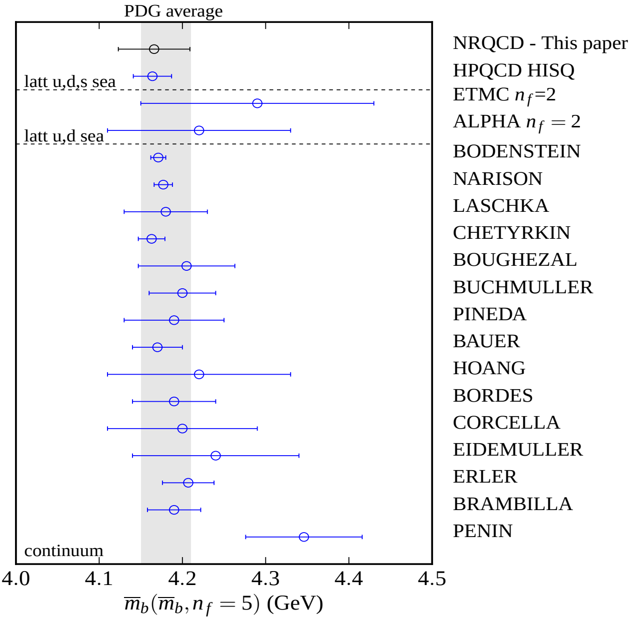
# Mass Results




$$\overline{m}_b(\overline{m}_b, n_f = 5) = 4.166(43)\ \text{GeV}$$

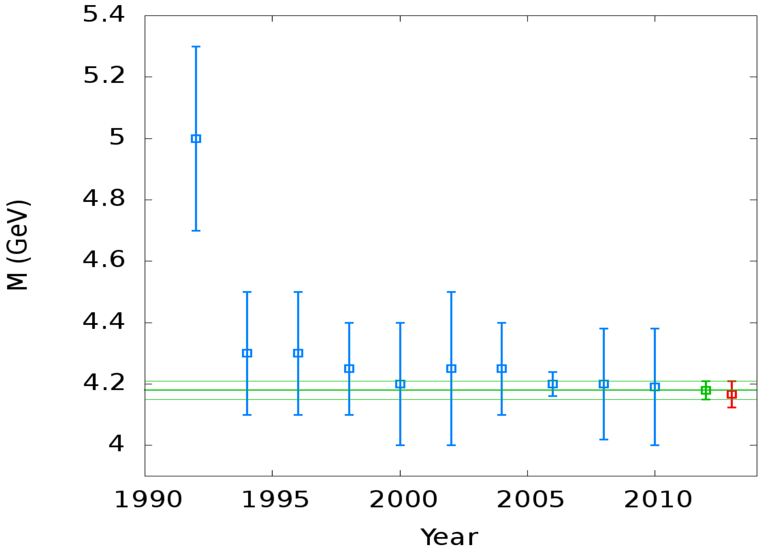
# Error Budget

| **Source** | **Error (MeV)** | **Error (%)** |
|---|---|---|
| Perturbative | 36 | 0.9 |
| Experimental | $< 0.1$ | $< 0.01$ |
| $E_{\text{sim}}$ | $< 0.1$ | $< 0.01$ |
| Bare mass tuning | 6 | 0.14 |
| `VEGAS` integration | $< 0.1$ | $< 0.01$ |
| Weak coupling simulation statistics | 14 | 0.35 |
| Lattice spacing dependence | 16 | 0.38 |
| Scale uncertainity | 4.4 | 0.10 |
| Coupling constant uncertainty | 0.2 | 0.01 |
| Relativistic corrections: $\mathcal{O}(v^6)$ | 5 | 0.12 |
| Radiative corrections: $\mathcal{O}(\alpha_s v^4)$ | 2.5 | 0.06 |
| EM, $c$ sea quarks, annihilation | 1.9 | 0.05 |
| **Total** | **43** | **1.0** |



PDG average
latt u,d,s sea
latt u,d sea
continuum
4.0
4.1
4.2
4.3
4.4
4.5
$\overline{m}_b(\overline{m}_b, n_f = 5)$ (GeV)
NRQCD - This paper
HPQCD HISQ
ETMC $n_f$=2
ALPHA $n_f = 2$
BODENSTEIN
NARISON
LASCHKA
CHETYRKIN
BOUGHEZAL
BUCHMULLER
PINEDA
BAUER
HOANG
BORDES
CORCELLA
EIDEMULLER
ERLER
BRAMBILLA
PENIN

# PDG values of $m_b$ through the ages



# Summary

- Determined $m_b$ from lattice NRQCD calculations of $\Upsilon$ and $B_s$
- $m_b$ extracted via two loop $aE_0$
- Mixed approach to higher order lattice perturbation theory
  - quenched contributions: weak coupling computation
  - fermionic contributions: automated lattice perturbation theory
- Two-loop $aE_0$ significantly improves dominant uncertainty

Thank You

Parameterise discretisation effects via Bayesian fit

$$\overline{m}_b(\overline{m}_b)(a, \delta x_m) = \overline{m}_b(\overline{m}_b)\left[1 + \sum_{j=1}^{2} d_j \left(\Lambda a\right)^{2j}(1 + d_{jb}\delta x_m + d_{jbb}(\delta x_m)^2)\right]$$

Here

- ⋆ $\Lambda = 0.5$ GeV
- ⋆ $\delta x_m$ given by

$$\delta x_m = \frac{am_0 - 2.1}{2.5 - 1.7}$$

  with values between $\pm 0.5$
- ⋆ Priors are
  - ⋆ 4.2(5) GeV for the mass
  - ⋆ 0.0(3) for the $a^2$ term, since action one-loop improved
  - ⋆ 0(1) for everything else

# Renormalon Cancellation

Pole mass

- ⋆ purely perturbative concept
  - ⇒ plagued by "renormalon" ambiguities

Renormalons cancel in matching to $\overline{MS}$ mass

$$2\left(Z_{\mathrm{cont}}(\mu)\overline{m}_b(\mu) - E_0\right) = M_{\Upsilon}^{\mathrm{expt}} - E_{\Upsilon}^{\mathrm{sim}}$$

$$Z_{\mathrm{cont}}(\mu)\overline{m}_b(\mu) - E_0 = M_{B_s}^{\mathrm{expt}} - E_{B_s}^{\mathrm{sim}}$$

# NRQCD Hamiltonian

Symanzik-improved, tadpole-improved $\mathcal{O}(v^4)$ NRQCD Hamiltonian

$$aH = aH_0 + a\delta H$$

with leading order kinetic contribution,

$$aH_0 = -\frac{\Delta^{(2)}}{2am_0},$$

and higher order corrections:

$$\begin{aligned} a\delta H = & -c_1 \frac{(\Delta^{(2)})^2}{8(am_0)^3} + c_2 \frac{ig}{8(am_0)^2} \left(\nabla \cdot \widetilde{\mathbf{E}} - \widetilde{\mathbf{E}} \cdot \nabla\right) \\ & - c_3 \frac{g}{8(am_0)^2} \sigma \cdot \left(\widetilde{\nabla} \times \widetilde{\mathbf{E}} - \widetilde{\mathbf{E}} \times \widetilde{\nabla}\right) - c_4 \frac{g}{2am_0} \sigma \cdot \widetilde{\mathbf{B}} \\ & + c_5 \frac{a^2 \Delta^{(4)}}{24am_0} - c_6 \frac{a(\Delta^{(2)})^2}{16n(am_0)^2}. \end{aligned}$$

Writing coefficients as $c_i = 1.0 + \alpha_V(q^\star)c_i^{(1)}$, we use:

| **Coefficient** | $\mathbf{am_0 = 2.5}$ | $\mathbf{am_0 = 1.72}$ | $\mathbf{q^\star}$ |
|---|---|---|---|
| $c_1^{(1)}$ | 0.95 | 0.766 | $1.8/a$ |
| $c_2^{(1)}$ | 0.78 | 0.691 | $\pi/a$ |
| $c_4^{(1)}$ | 0.41 | 0.392 | $1.4/a$ |
| $c_5^{(1)}$ | 0.95 | 0.766 | $1.8/a$ |

[Dowdall *et al.*, Phys.Rev. D 85 (2012) 054509]

| **Set** | $\mathbf{u_0^P}$ | $\mathbf{c_1}$ | $\mathbf{c_2}$ | $\mathbf{c_3}$ | $\mathbf{c_4}$ | $\mathbf{c_5}$ | $\mathbf{c_6}$ |
|---|---|---|---|---|---|---|---|
| Coarse | 0.86879 | 1.31 | 1.0 | 1.0 | 1.2 | 1.16 | 1.31 |
| Fine | 0.87821 | 1.21 | 1.0 | 1.0 | 1.16 | 1.12 | 1.21 |

# ENSEMBLES

Gauge configurations generated by MILC collaboration

| Set | $\beta$ | $a^{-1}$ (GeV) | $am_l$ | $am_s$ | $L \times T$ | $n_{cfg}$ |
|---|---|---|---|---|---|---|
| coarse | 6.76 | 1.652(14) | 0.01 | 0.05 | 20×64 | 1380 |
| fine | 7.09 | 2.330(17) | 0.0062 | 0.0310 | 28×96 | 904 |

[Bazavov *et al.*, Rev. Mod. Phys. 82 (2010) 1349]