Non-perturbative determination of the heavy quark potential at T>0

Alexander Rothkopf
Albert Einstein Center for Fundamental Physics
University of Bern

with T.Hatsuda, S.Sasaki: **PRL 108 (2012) 162001**
a brief review: A.R. **MPLA 28 (2013) 1330005**
with Y. Burnier: **PRD 86 (2012) 051503, arXiv:1304.4154**

QQbar potential and the Wilson loop

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- Effective field theory guides us to define:

$$\lim_{t\to\infty} \frac{i\partial_t W_\square(\mathbf{R},t)}{W_\square(\mathbf{R},t)} = V^0(\mathbf{R})$$

- Make real-time dependence of the Wilson loop explicit

$$W_\square(R,t) = \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\, \rho_\square(R,\omega) \iff W_\square(R,\tau) = \int_{-\infty}^{\infty} d\omega\, e^{-\omega\tau}\, \rho_\square(R,\omega)$$

Bayesian spectral analysis (e.g. MEM)

$$V^0(R) = \lim_{t\to\infty} \frac{\int_{-\infty}^{\infty} d\omega\, \omega\, e^{-i\omega t}\, \rho_\square(R,\omega)}{\int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\, \rho_\square(R,\omega)}$$

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- How to efficiently extract the values of the potential from spectral information
  - Only need the lowest lying spectral features since t->∞ limit
  - Goal: replace discrete Fourier transform by spectral fitting + analytic FT

Real and Imaginary part from the spectrum

- Wilson Loop is a QCD quantity, contains all scales (i.e. also non-potential physics)

$$i\partial_t W_\square(R,t) = \Phi(R,t) W_\square(R,t)$$

$$\Phi(R,t) = V(R) + \phi(R,t)$$

$$\lim_{t\to\infty} \phi(R,t) = 0 \qquad \sigma_\infty(R) = \int_0^\infty \phi(R,t)\,dt$$

Y. Burnier, A.R. Phys.Rev. D86 (2012) 051503

- Fourier transform from $W_\square(R,t)$ to $\rho_\square(R,\omega)$ mixes time scales even at small ω:

$$\rho_\square(R,\omega) = \frac{1}{\pi} e^{\mathrm{Im}[\sigma_\infty](R)} \frac{|\mathrm{Im}[V](R)|\cos[\mathrm{Re}[\sigma_\infty](R)] - (\mathrm{Re}[V](R)-\omega)\sin[\mathrm{Re}[\sigma_\infty](R)]}{\mathrm{Im}[V](R)^2 + (\mathrm{Re}[V](R)-\omega)^2}$$
$$+ \kappa_0(R) + \kappa_1(R)(\mathrm{Re}[V](R)-\omega) + \kappa_2(R)(\mathrm{Re}[V](R)-\omega)^2 + \cdots$$

- Remnants of early time physics σ_∞ induce **skewing** and a **smooth background**

$$V^0(R) = \lim_{t\to\infty} \frac{\int_{-\infty}^{\infty} d\omega\, \omega\, e^{-i\omega t} \rho_\square(R,\omega)}{\int_{-\infty}^{\infty} d\omega\, e^{-i\omega t} \rho_\square(R,\omega)} = \mathrm{Re}[V](R) - i|\mathrm{Im}[V](R)|$$

The strategy for an extraction of V(R)

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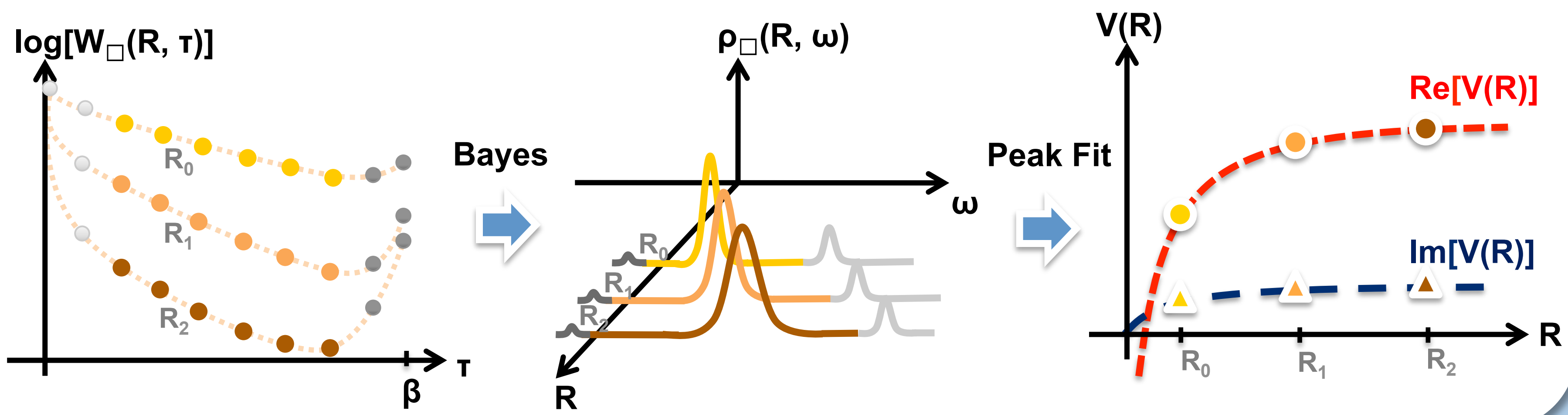

A.R. Mod. Phys. Lett. A, 28, 1330005 (2013)

From lattice QCD etc.

The complex heavy quark V(R)

HTL benchmark of the extraction strategy

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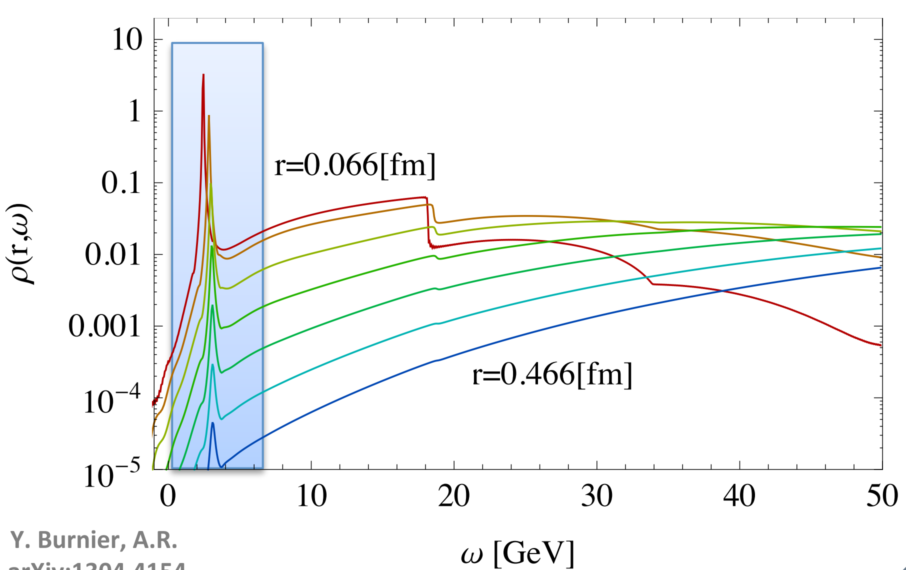
- 1^{st} test: Extract the known HTL potential from HTL Wilson loop spectra

HTL Wilson loop spectrum (cutoff Λ=5πGeV)



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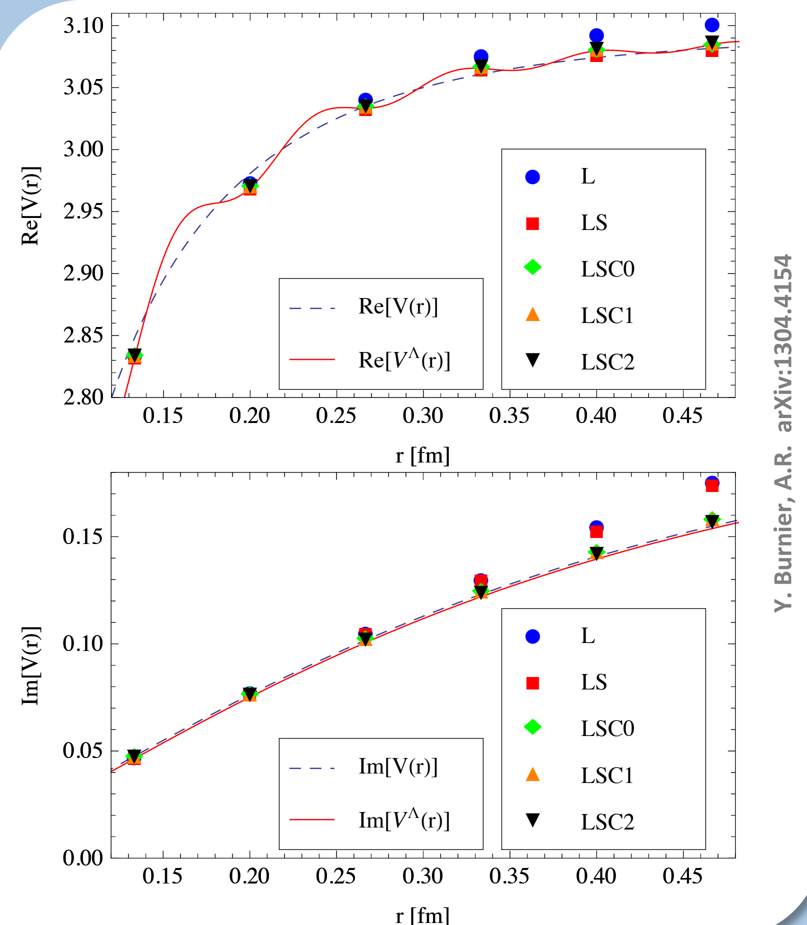

- Simple Lorentzian (L) overetimates true values
- Skewed Lorentzian & background (LSC2) spot on

Reconstructing ρ(ω) from HTL correlators

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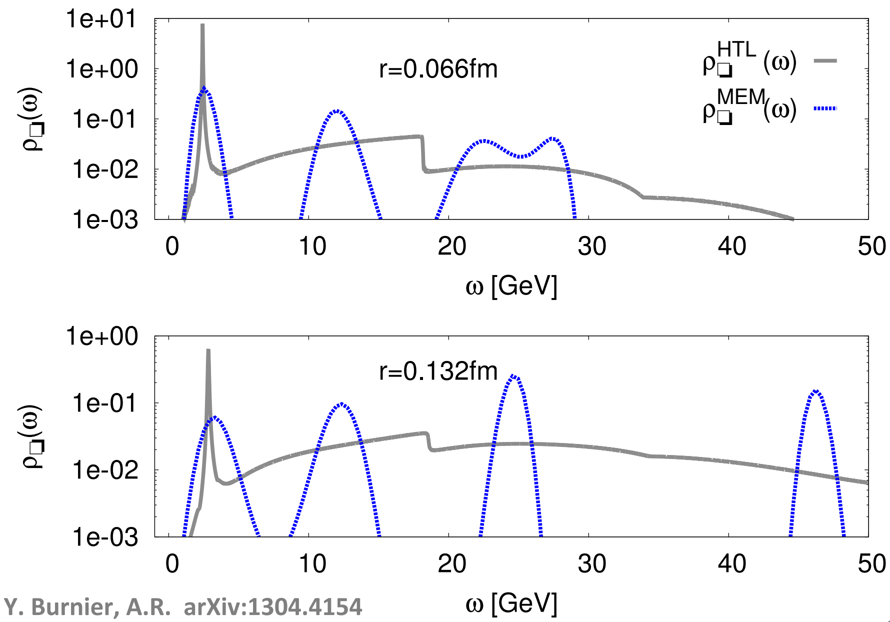
- 2^{nd} test: Reconstruct HTL spectra from the Euclidean time HTL Wilson loop
  - Based on N_τ=32 unperturbed data points with artificial error bars (best case)

MEM with 80 basis functions



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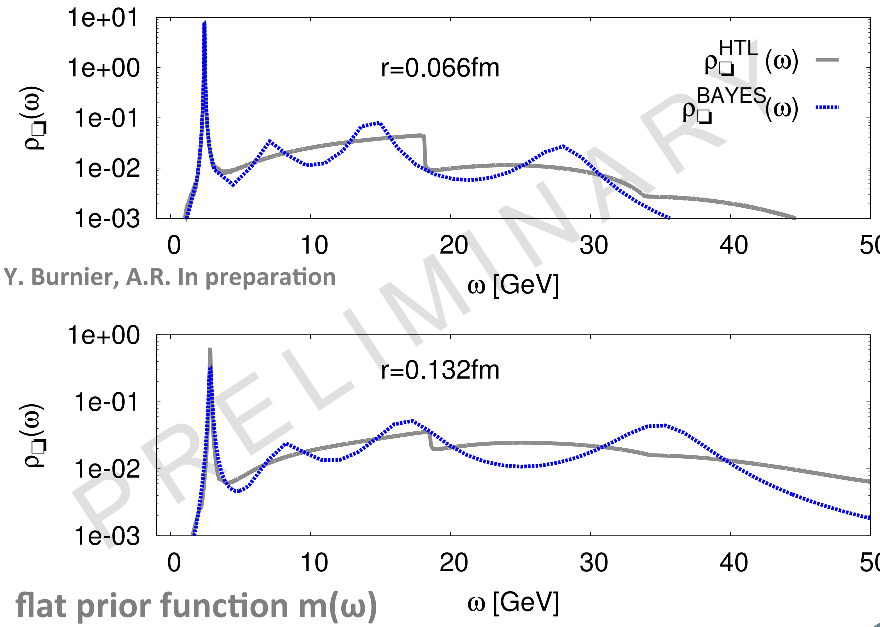
Improved Bayes with full search space



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flat prior function m(ω)

- Extremely challenging with standard MEM
- Improved Bayesian spectral analysis with full search space is promising

Reanalysis of quenched lattice data

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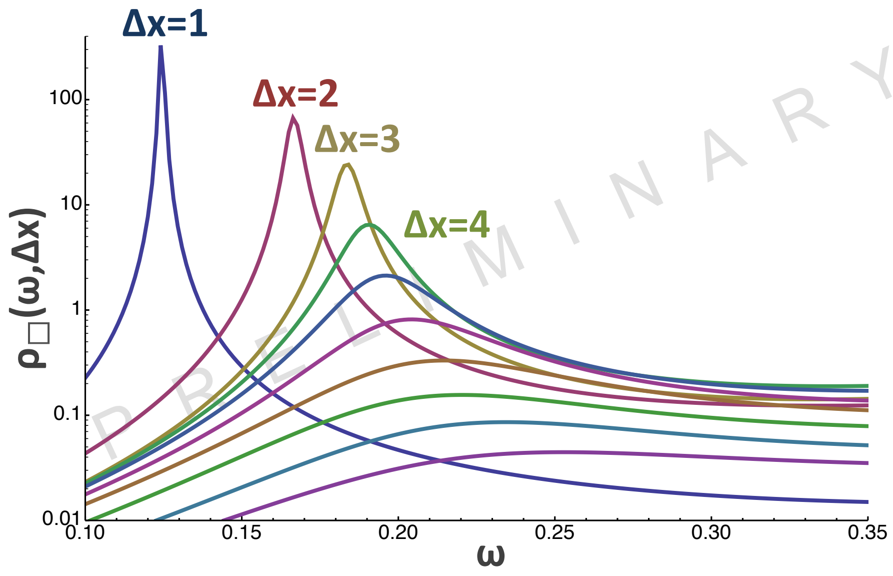
- Use the N_τ=32 lattice data N_x=20 β=7.0 ξ_r=4.0 at T=2.33T_C
  - Most humble use of prior function m(ω)=const (W□(0,R)=1 fixes area to unity)
  - Require the LBFGS minimizer to find an extremum up to tolerance Δ=10^{-30}
  - A large frequency interval is chosen I_ω=[-12:14] β_N=20 N_ω=800



Preliminary V(r) @ T=2.33TC in lattice QCD

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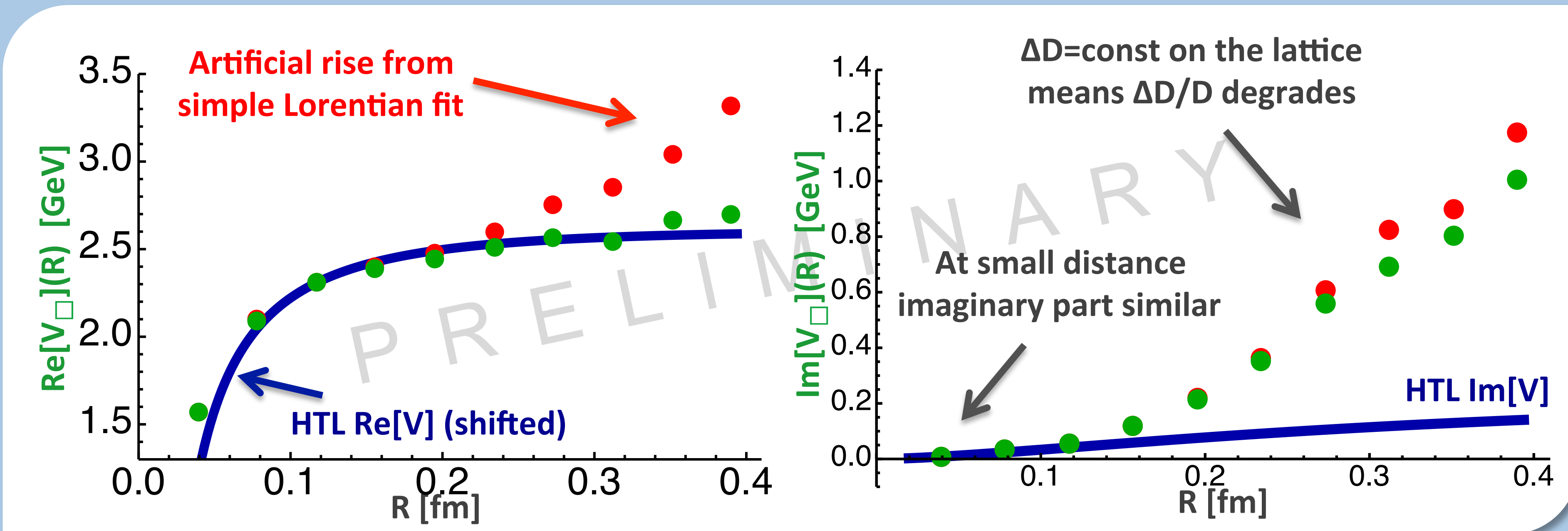

- Results appear to lie closer to the HTL result, compared to previous studies
- Debye screening seems to be visible already at r > 0.3 fm
- Lattice Im[V] is of comparable size to HTL imaginary part for small r

Conclusion & Outlook

- Effective Field theory allows **derivation** of the **complex** heavy quark potential
- Late real-time limit of Wilson loop encodes potential physics
  - **Spectral decomposition** allows connection to Euclidean time domain
  - We can extract Re[V] and Im[V] from lowest lying peak position and width
  - Caveat: coupling of time-scales leads to non-trivial spectral structure: **skewing**
  - An **HTL benchmark** of the peak fitting procedure confirms effectiveness
- Numerical challenge: Reconstructing the spectra from imaginary time Wilson loops
  - Standard MEM with extended search space suffers from broad spectral features
  - Work in progress: Improving the Bayesian spectral analysis with full search space

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Underlying goal: reduce complexity

- Separation of scales:

$$\frac{\Lambda_{QCD}}{m_Q c^2} \ll 1, \quad \frac{T}{m_Q c^2} \ll 1, \quad \frac{\mathbf{p}}{m_Q c} \ll 1$$

- Effective field theory: Systematic expansion in m_Q^{-1}

Brambilla, Ghiglieri, Vairo and Petreczky PRD 78 (2008) 014017

Brambilla et. al. Rev.Mod.Phys. 77 (2005) 1423 PRD 78 (2008) 014017

Relativistic thermal field theory

QCD
Dirac fields
$\bar{Q}(x), Q(x)$

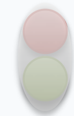
NRQCD
Pauli fields
$\bar{\chi}(x), \chi(x)$

$\bar{q}(x), q(x), A^{\mu}(x)$

pNRQCD
Singlet/Octet

QM path integrals
Heavy quark propagators

Quantum mechanics



Barchielli et. al. Nucl.Phys. B296 (1988) 625

- Matching based on real-time quarkonium forward correlator $D^>$

$$\langle \bar{\psi}(\mathbf{z}_1(0))\Gamma \mathbf{U}\psi(\mathbf{z}_2(0))\bar{\psi}(\mathbf{z}_2(t))\bar{\Gamma}\mathbf{U}^\dagger\psi(\mathbf{z}_1(t))\rangle = \exp[-2imc^2t]\int \mathcal{D}[\mathbf{z}_1,\mathbf{p}_1]\int \mathcal{D}[\mathbf{z}_2,\mathbf{p}_2]\times$$
$$\exp\Big[i\int_0^t ds \sum_i \Big(\mathbf{p}_i(s)\dot{\mathbf{z}}_i(s) - \frac{\mathbf{p}_i^2(s)}{2m}\Big)\Big] \Big\langle \frac{1}{N}\mathrm{Tr}\Big[\mathcal{P}_{\mathcal{C}}\exp\Big[\frac{ig}{c}\oint_{\mathcal{C}} dx^\mu A_\mu(x)\Big]\Big]\Big\rangle$$

Improved fits on previous MEM spectra

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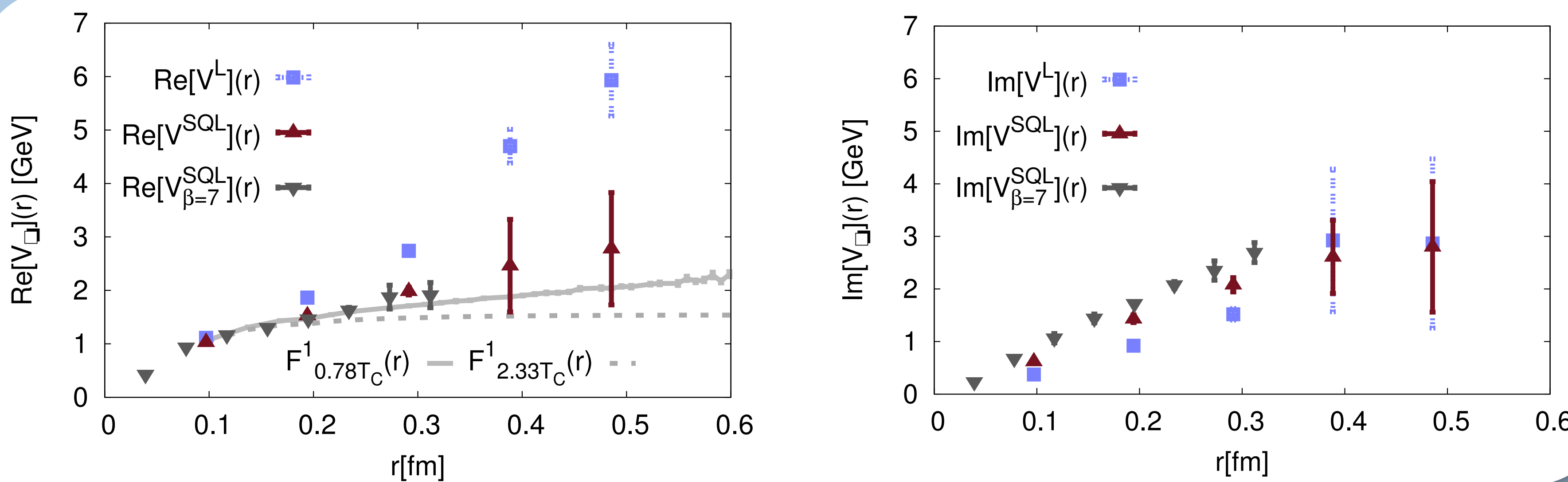
- New fits on previous Maximum Entropy Method spectra yield:



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  - Unphysical rise in the real part is significantly reduced
  - Imaginary part does not see a large change, still of same order as the real part
- This result is still not satisfactory. Can we test the approach more thoroughly?