Light Quark Mass Dependence in Heavy Quarkonium Physics

Feng-Kun Guo

Helmholtz-Institut für Strahlen- und Kernphysik, Universität Bonn

QWG 2013, April 22–26, 2013

Based on: Guo, Meißner, PRL**108**(2012)112002; PRL**109**(2012)062001

Introduction

Excited heavy quarkonia on lattice

- Excited heavy quarkonia are under intensive investigations on lattice, e.g.

  Dudek et al, PRD77(2008)034501 (quenched)

  Bali et al, PRD84(2011)094506 ($M_\pi \in [0.28, 1]$ GeV)

  Liu et al (Hadron Spectrum), JHEP07(2012)126; talk by Christopher Thomas ($M_\pi = 396$ MeV)
- For ground states, one may use $a + bM_\pi^2$

  Burch et al (Fermilab Lattice and MILC), PRD81(2010)034508
- Can one still use it for excited states?

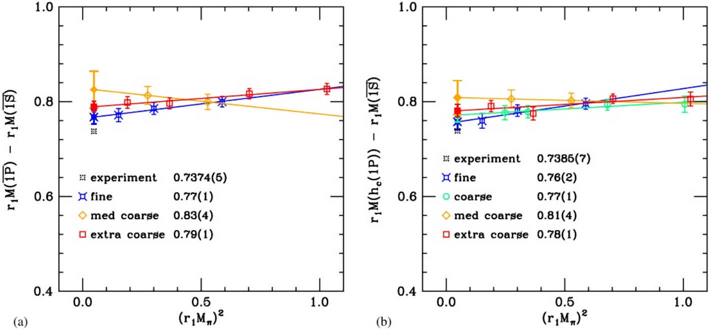
- Excited heavy quarkonia are under intensive investigations on lattice, e.g.

  Dudek et al, PRD77(2008)034501 (quenched)

  Bali et al, PRD84(2011)094506 ($M_\pi \in [0.28, 1]$ GeV)

  Liu et al (Hadron Spectrum), JHEP07(2012)126; talk by Christopher Thomas ($M_\pi = 396$ MeV)

- For ground states, one may use $a + bM_\pi^2$



Burch et al (Fermilab Lattice and MILC), PRD81(2010)034508

- Can one still use it for excited states?

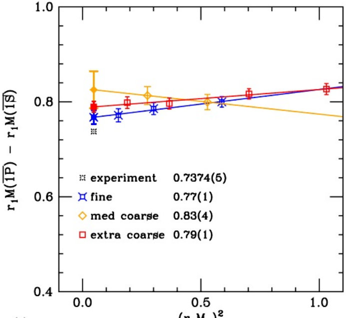
- Excited heavy quarkonia are under intensive investigations on lattice, e.g.

  Dudek et al, PRD77(2008)034501 (quenched)

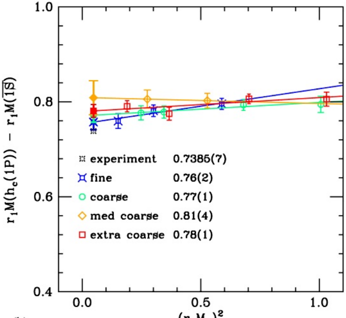
  Bali et al, PRD84(2011)094506 ($M_\pi \in [0.28, 1]$ GeV)

  Liu et al (Hadron Spectrum), JHEP07(2012)126; talk by Christopher Thomas ($M_\pi = 396$ MeV)

- For ground states, one may use $a + bM_\pi^2$



(a)



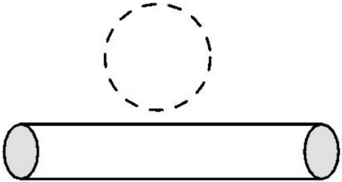
(b)

Burch et al (Fermilab Lattice and MILC), PRD81(2010)034508

- Can one still use it for excited states?

Introduction

Light quark sea in a heavy quarkonium

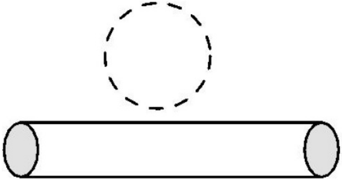


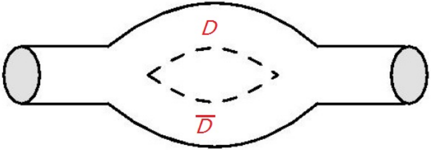
Pion cloud: operators
$\psi^\dagger \psi \langle \chi_+ \rangle,\ \psi^\dagger \psi \langle u_\mu u^\mu \rangle, \ldots$

The contribution is of the form
$c_1 M_\pi^2 \ + \ c_2 M_\pi^4 \log \frac{M_\pi^2}{\mu^2} \ + \ c_3 M_\pi^4 \ + \ \mathcal{O}(M_\pi^6)$

Meson loops:

nonanalytic in M_π^2 due to unitary cut





Pion cloud: operators
$\psi^\dagger\psi\langle\chi_+\rangle,\ \psi^\dagger\psi\langle u_\mu u^\mu\rangle,\ \ldots$

The contribution is of the form
$c_1 M_\pi^2 \ + \ c_2 M_\pi^4 \log\frac{M_\pi^2}{\mu^2} \ + \ c_3 M_\pi^4 \ + \ \mathcal{O}(M_\pi^6)$

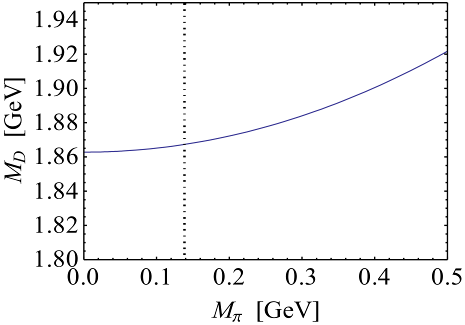
Meson loops:

nonanalytic in M_π^2 due to unitary cut

- D mesons contain light quarks, thus will introduce pion mass dependence into charmonium systems
- Pion-mass dependence of M_D

$$M_D = \mathring{M}_D + h_1 \frac{M_\pi^2}{\mathring{M}_D} + O\left(M_\pi^3\right)$$

$h_1 = 0.44$ determined from the SU(3) mass difference



- Intermediate heavy mesons are nonrelativistic:

$$|2M_D - M_{c\bar{c}}| \ll M_D \Rightarrow v \ll 1$$

- Three-momentum $\vec{p} \sim \mathcal{O}(v)$; NR energy $E \sim \mathcal{O}\left(v^2\right)$
- Propagator $\frac{1}{E-\vec{p}^2/(2M_D)} \sim \mathcal{O}\left(v^{-2}\right)$

Examples

Excited quarkonium masses

Power counting in nonrelativistic effective theory for meson loops

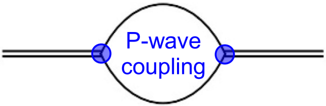
- Intermediate heavy mesons are nonrelativistic:

$$\left|2M_D - M_{c\bar{c}}\right| \ll M_D \Rightarrow v \ll 1$$

- Three-momentum $\vec{p} \sim \mathcal{O}(v)$; NR energy $E \sim \mathcal{O}\left(v^2\right)$
- Propagator $\frac{1}{E-\vec{p}^{\,2}/(2M_D)} \sim \mathcal{O}\left(v^{-2}\right)$

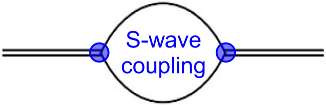
Examples

- *S*-wave charmonium



$$O\left(\frac{v^5}{(v^2)^2} v^2\right) = O\left(v^3\right)$$

- *P*-wave charmonium



$$O\left(\frac{v^5}{(v^2)^2}\right) = O(v)$$

- Regularizing the two-point loop by a three-momentum cutoff λ

$$\Sigma(M^2,\lambda) = \frac{1}{4\pi(m_1+m_2)}\left(-\frac{\lambda}{\pi} + \frac{1}{2}\sqrt{\frac{2m_1m_2}{m_1+m_2}(m_1+m_2-M) - i\varepsilon}\right)$$

the square root may be written as $\sqrt{2\mathring{\mu}_{12}\left(\delta + \frac{h_1}{\mathring{\mu}_{12}}M_\pi^2\right) + \mathcal{O}\left(M_\pi^4\right)}$ where $\mathring{\mu}_{12} = \mathring{m}_1\mathring{m}_2/(\mathring{m}_1+\mathring{m}_2)$ and $\delta = \mathring{m}_1+\mathring{m}_2-M$

- Renormalized mass of a P-wave $Q\bar{Q}$:

$$M(M_\pi) = M_0(\lambda, M_\pi) + g^2 m_1 m_2 \,\mathrm{Re}\Sigma(M^2,\lambda,M_\pi)$$

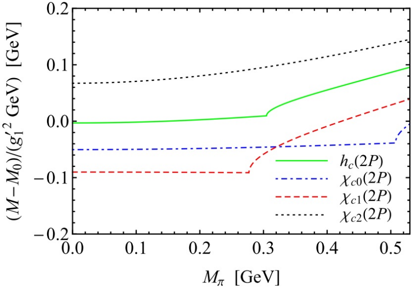
- Regularizing the two-point loop by a three-momentum cutoff λ

$$\Sigma(M^2,\lambda) = \frac{1}{4\pi(m_1+m_2)}\left(-\frac{\lambda}{\pi} + \frac{1}{2}\sqrt{\frac{2m_1m_2}{m_1+m_2}(m_1+m_2-M) - i\varepsilon}\right)$$

the square root may be written as $\sqrt{2\mathring{\mu}_{12}\left(\delta + \frac{h_1}{\mathring{\mu}_{12}}M_\pi^2\right) + \mathcal{O}\left(M_\pi^4\right)}$ where $\mathring{\mu}_{12} = \mathring{m}_1\mathring{m}_2/(\mathring{m}_1+\mathring{m}_2)$ and $\delta = \mathring{m}_1+\mathring{m}_2 - M$

- Renormalized mass of a P-wave $Q\bar{Q}$:

$$M(M_\pi) = M_0(\lambda, M_\pi) + g^2 m_1 m_2 \,\mathrm{Re}\Sigma(M^2,\lambda,M_\pi)$$



Hindered M1 transitions between P-wave states

M_π dependence of hindered M1 transitions

- Hindered M1 transitions: heavy quark spin flip; between two states with different radial excitations
- Highly suppressed in quark models because the overlap of nonrelativistic wave functions vanish

$$\mathscr{A}_{\mathrm{M1}} \propto \frac{v_c}{m_c} E_\gamma$$

- Could be dominated by coupled-channel effects. For P-wave states

- The loop is convergent, and other loops are suppressed by at least $\mathcal{O}(v)$

Hindered M1 transitions between *P*-wave states

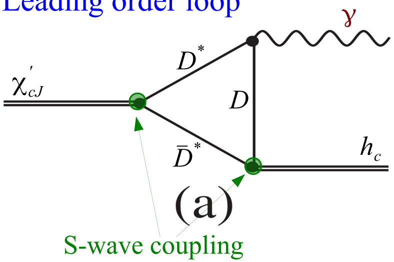
M_π dependence of hindered M1 transitions

- Hindered M1 transitions: heavy quark spin flip; between two states with different radial excitations
- Highly suppressed in quark models because the overlap of nonrelativistic wave functions vanish

$$\mathscr{A}_{\mathrm{M1}} \propto \frac{v_c}{m_c} E_\gamma$$

- Could be dominated by coupled-channel effects. For *P*-wave states

Leading order loop



$$\boxed{\mathcal{A}_{(a)} \sim \frac{v^5}{(v^2)^3}\frac{E_\gamma}{m_c} = \frac{E_\gamma}{m_c v}}$$

Spin-symmetry breaking

- The loop is convergent, and other loops are suppressed by at least $\mathcal{O}(v)$

Considering only the leading order loops

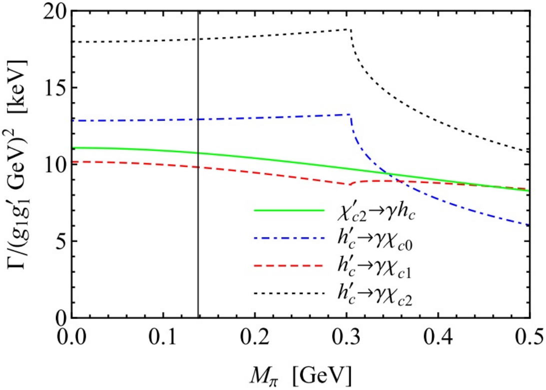
- At physical quark masses, if taking model values: $g_1 = -4\ \mathrm{GeV}^{-1/2}$, $|g_1'| \sim 1\ \mathrm{GeV}^{-1/2}$

  $\Gamma(\chi_{c2}' \to \gamma h_c) \sim 170\ \mathrm{keV} \gg 1.3\ \mathrm{keV}$ (quark model Barnes et al(2005))
- Pion mass dependence

Considering only the leading order loops

- At physical quark masses, if taking model values: $g_1 = -4\ \mathrm{GeV}^{-1/2}$, $|g_1'| \sim 1\ \mathrm{GeV}^{-1/2}$

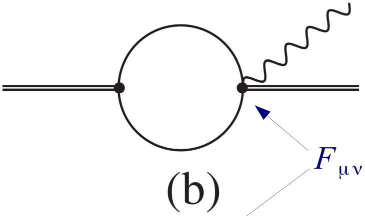
  $\Gamma(\chi_{c2}' \to \gamma h_c) \sim 170\ \mathrm{keV} \gg 1.3\ \mathrm{keV}$ (quark model Barnes et al(2005))
- Pion mass dependence



Summary

- For any excited hadron close to an S-wave coupled hadronic threshold, nonanalyticity should be taken into account in chiral extrapolation
- Pion mass dependence in heavy quarkonium transitions can be strong. Lattice simulations of these transitions would be quite useful in identifying the coupled-channel effects
- Coupled-channel effects may be checked by comparing quenched and unquenched lattice simulations

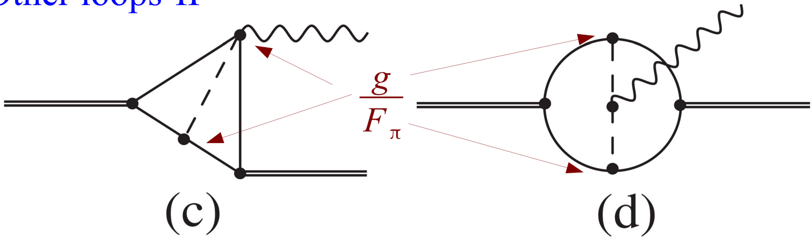
- Other loops-I



- Gauge invariant by itself
- Suppressed by v^2

$$\boxed{\mathcal{A}_{(b)} \sim \frac{v^5}{(v^2)^2}\frac{E_\gamma}{m_c} = v\frac{E_\gamma}{m_c}} \ll \boxed{\mathcal{A}_{(a)} \sim \frac{v^5}{(v^2)^3}\frac{E_\gamma}{m_c} = \frac{E_\gamma}{m_c v}}$$

- Other loops-II



One more loop than (a)

For matching dimensions

$$\mathcal{A}_{(c)} \sim \frac{(v^5)^2}{(v^2)^5} \frac{g^2}{(4\pi)^2 F_\pi^2} \frac{E_\gamma}{m_c} M_D^2 = \frac{E_\gamma}{m_c}\left(\frac{g M_D}{\Lambda_\chi}\right)^2 \sim \frac{E_\gamma}{m_c}$$

- Suppressed compared with (a) by a factor of $v \sim 0.4$