Heavy quarkonium production at high p_T using pQCD factorization

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Based on works done with: Z.-B. Kang, J. Qiu, G. Sterman, and H. Zhang

The 9th International Workshop on Heavy Quarkonium 2013
IHEP, P. R. China from 22 - 26 April, 2013

Outline

- **Introduction**
  - Heavy quarkonium, historical production models
  - Difficulties in NRQCD model
- **PQCD factorization for heavy quarkonium production**
  - Leading power in $1/p_T$ expansion
  - Next-to-leading power in $1/p_T$ expansion
  - Calculation of hard part
  - Calculation of Evolution equation for FFs
  - Determine initial condition for FFs using NRQCD model
  - Application
- **Summary and outlook**

Heavy quarkonium, produciton models

- **Effectively, a non-relativistic QCD system:**

Charmonium: $v^2 \approx 0.3$ Bottomonium: $v^2 \approx 0.1$

Potentially, could be similar to a QED bound state, like positronium

- **Multiple well-separated scales – ideal for effective theory:**

Quark mass:	M
Momentum:	Mv
Energy:	Mv^2

$M >> Mv >> Mv^2 \sim \Lambda_{QCD}$

- **Historical mostly used production models:**

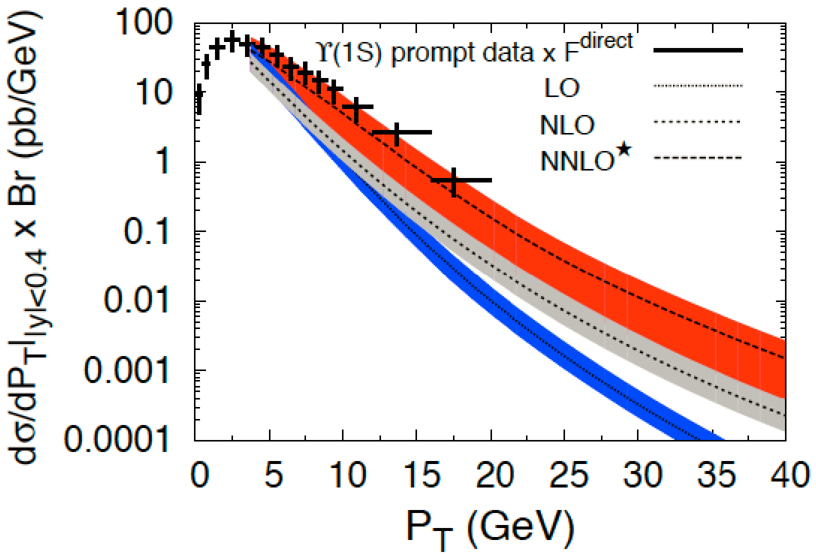
  - Color singlet model (CSM) : 1975 — Einhorn, Ellis (1975), Chang (1980), Berger and Jone (1981), …
  - Color evaporation model (CEM) : 1977 — Fritzsch (1977), Halzen (1977), …
  - NRQCD model : 1986,1994 — Caswell, Lapage (1986) Bodwin, Braaten, Lepage, 9407339

It was proved that both CSM and CEM are just special cases of NRQCD model. Bodwin, Braaten, Lee, 0504014

Large high order corrections in NRQCD

➢ S-wave channel ($^3S_1^{[1]}$):

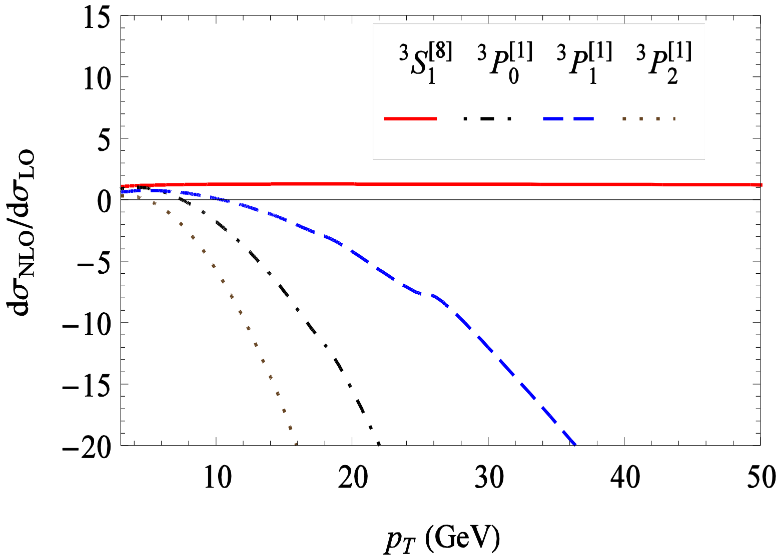
- Large corrections are first found for NLO relative to LO
- The estimated $\mathrm{NNLO}^\star$ contribution is still larger then NLO



Campbell, Maltoni, Tramontano, 0703113,
Artoisenet, Campbell, Lansburg, Maltoni, Tramontano, 0806.3282

➢ P-wave channel ($^3P_{J=0,1,2}^{[1,8]}$):

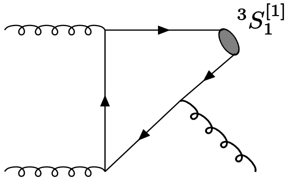
- Large NLO corrections are first found for CS channel, but **with negative sign**
- later CO channels are found to have similar behavior



YQM, Wang, Chao, 1002.3987
YQM, Wang, Chao, 1009.3655
Butensckön, Kniehl, 1009.5662

Explain the large corrections

- **LO in α_s but NNLP in $1/p_T$:**



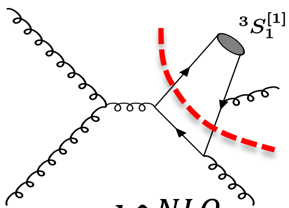
$$\frac{d\hat{\sigma}^{LO}}{dp_T^2} \sim \alpha_s^3(p_T)\frac{m_Q^4}{p_T^8}$$

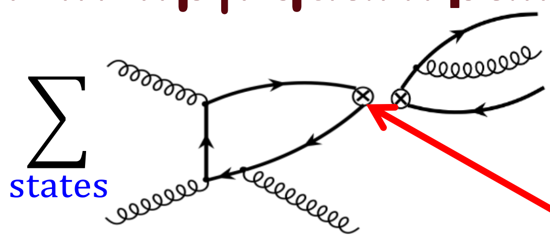
YQM, 1207.3073

The behavior $\alpha_s^3 \frac{m_Q^4}{p_T^8}$ persists even calculated to all order in v^2.

- **NLO in α_s but NLP in $1/p_T$: quark pair fragmentation**

Kang, Qiu and Sterman, 1109.1520



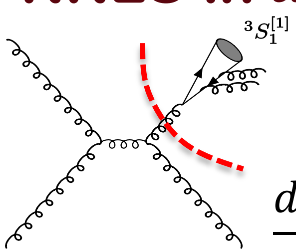


Relativistic Projector to all “spin states”

$$\frac{d\hat{\sigma}^{NLO}}{dp_T^2} \to \frac{\alpha_s^3(p_T)}{p_T^6} \otimes \alpha_s(\mu)\log(\mu^2/\mu_0^2)\, m_Q^2 \qquad \mu_0 \gtrsim 2m_Q$$

- **NNLO in α_s but LP in $1/p_T$: guon fragmaentation**

Braaten, Yuan, 9303205



LP contribution numerically not large, thus NNLO will be still dominated by quark pair fragmentation for a quit large range of p_T, which is NLP.

$$\frac{d\hat{\sigma}^{NNLO}}{dp_T^2} \to \frac{\alpha_s^2(p_T)}{p_T^4} \otimes \alpha_s^3(\mu)\log^m(\mu^2/\mu_0^2)$$

✓ **Similar explaination for large corrections of $^3P_J^{[1,8]}$ channel: LO in α_s gives NLP, while NLO in α_s gives LP.**

New approach is needed!

Conclusion: LO α_s expansion $\neq$ LP in $1/p_T$ expansion!

Question: How reliable is the perturbative expansion?

- **pQCD factorization approach: leading power**

  Braaten, Yuan, 9303205
  Nayak, Qiu, Sterman, 0509021

  - Pick up the LP contribution, resummation
  - Not good enough: e.g. for ${}^3S_1^{[1]}$ and ${}^1S_0^{[8]}$ state, which are dominated by NLP.

- **pQCD factorization approach: up to next power**

  Kang, Qiu, Sterman, 1109.1520
  Kang, YQM, Qiu, Sterman, 1304.xxxx

  - Can be proven to all order in α_s
  - Application range: $p_T \gg M_H \sim 2m_Q$
  - Double expansion: M_H/p_T power expansion + α_s–expansion
  - Take care of both power expansion and resummation of the large logarithms

- **Effectively, SCET factorization approach can give equivalent formula once the pQCD factorization is proven. (next talk)**

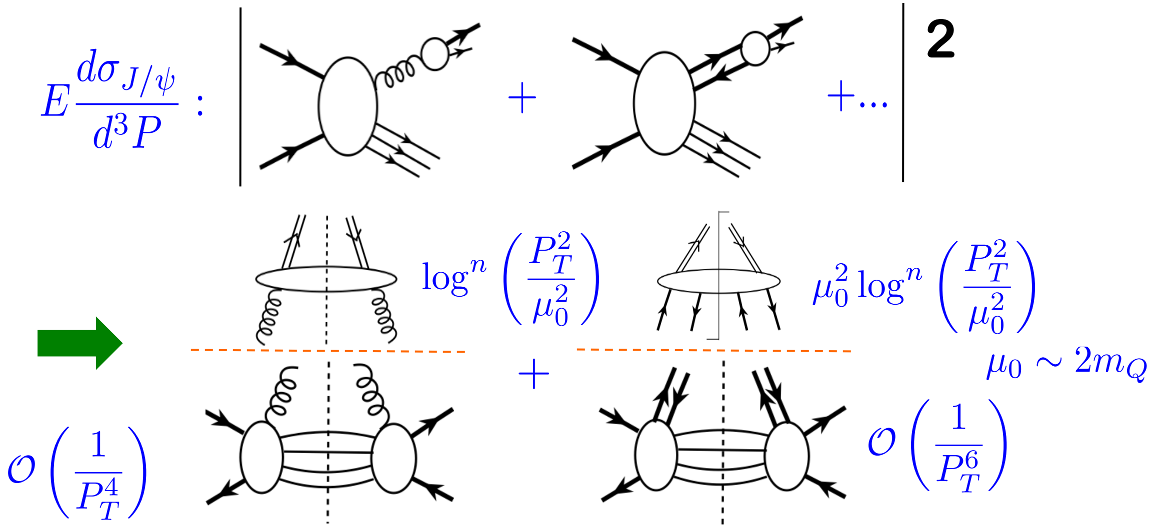
  Fleming, Leibovich, Mehen, Rothstein 1207.2578

  - If SCET factorization is argued to be valid, pQCD factorization may not work.
  - If pQCD factorization is proven to be valid, SCET factorizaiton should work

Perturbative factorization approach

- **Ideas:**

Nayak, Qiu, Sterman, 0509021
Kang, YQM, Qiu, Sterman, 1304.xxxx



At high pT, dominant contributions come from the region of phase space where active partons are close to mass-shell

- **Collinear factorization – an "EFT" of QCD**
  - Integrate out the virtuality of active partons – power expansion in 1/pT
  - Match the factorized form and pQCD at the factorization scale: $\mu_F \sim pT$

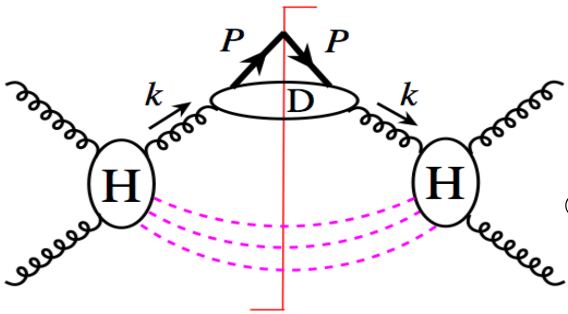
$$\sigma(p_T/\mu, \alpha_s(\mu)) = \hat{\sigma}(p_T/\mu_F, \mu/\mu_F, \alpha_s(\mu)) \otimes D(\mu_F, \alpha_s(\mu)) + \mathcal{O}(1/p_T)$$

  - μ_F – independence: evolution of non-perturbative PDFs or FFs, …
  - Predictive power: Universality of PDFs or FFs, evolution, …

Single parton fragmentation

Nayak, Qiu, and Sterman, 0509021, …

- **Perturbative pinch singularity:**



Dominated by $k^2 \sim 0$ region

$$\propto \int d^4k\, \mathcal{H}_{gg\to g}(Q,k) \frac{1}{k^2+i\epsilon} \frac{1}{k^2-i\epsilon} \mathcal{D}_{g\to \mathrm{J}/\psi}(k,P)$$

$$\approx \int \frac{dz}{z} d^2k_\perp \, \mathcal{H}_{gg\to g}(Q,k^2=0) \int dk^2 \frac{1}{k^2+i\epsilon} \frac{1}{k^2-i\epsilon} \mathcal{D}_{g\to \mathrm{J}/\psi}(k,P)$$

Long-lived parton state

- **Parton model collinear factorization:**

$$k^2, k_\perp^2 << Q \qquad z = P\cdot n/k\cdot n$$

Fragmentation function

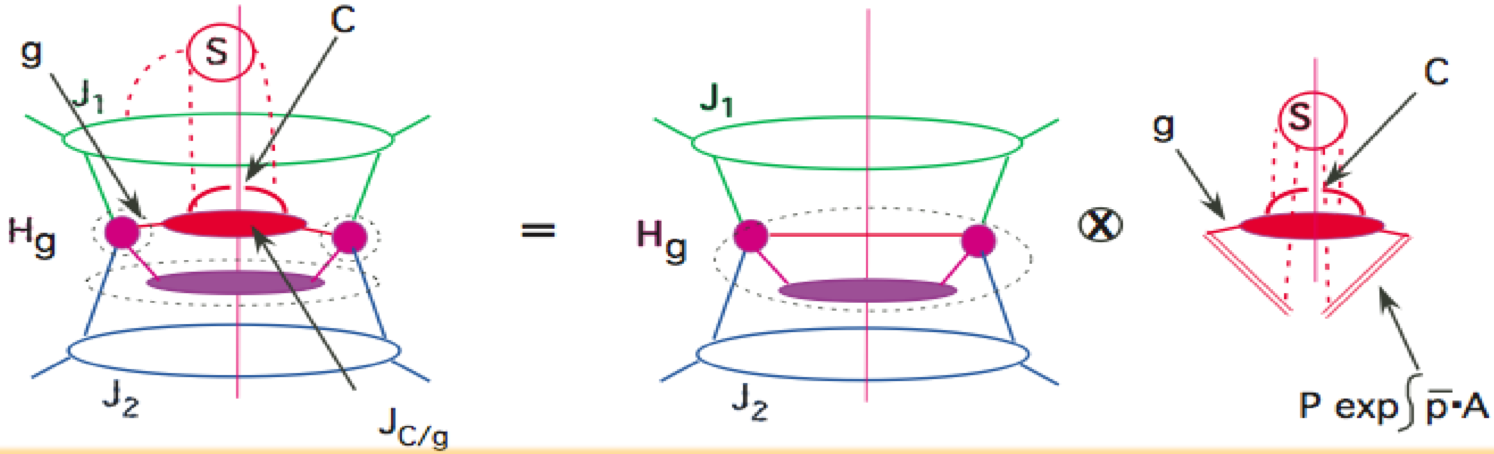
$$\approx \int \frac{dz}{z} \mathcal{H}_{gg\to g}(Q, z=P\cdot n/k\cdot n) \int dk^2\, d^2k_\perp \frac{1}{k^2+i\epsilon} \frac{1}{k^2-i\epsilon} \mathcal{D}_{g\to \mathrm{J}/\psi}(k,P)$$

Short-distance part

Factorization: fragmentation at leading power

Nayak, Qiu, and Sterman, 0509021, …

- **Leading power single-hadron production:**



$$d\sigma_{A+B\to H+X}(p_T) = \sum_i d\tilde{\sigma}_{A+B\to i+X}(p_T/z,\mu) \otimes D_{H/i}(z,m_c,\mu) + \mathcal{O}(m_H^2/p_T^2)$$

- **Fragmentation function – gluon to a hadron H (e.g., J/ψ):**

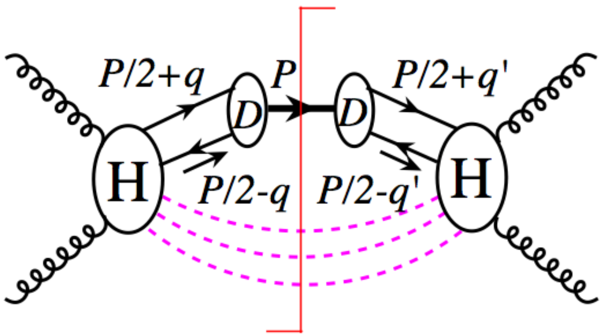
$$D_{H/g}(z,m_c,\mu) \propto \frac{1}{P^+}\mathrm{Tr}_{\mathrm{color}} \int dy^- e^{-ik^+y^-}$$
$$\times\langle 0|F^{+\lambda}(0)[\Phi_-^{(g)}(0)]^\dagger a_H(P^+)a_H^\dagger(P^+)\Phi_-^{(g)}(y^-)F_\lambda{}^+(y^-)|0\rangle$$

Cannot get fragmentation func. from PDFs or decay matrix elements

Production of heavy quark pairs

➢ Perturbative pinch singularity:

Kang, YQM, Qiu, Sterman, 1304.xxxx



$$P^\mu = (P^+, 4m^2/2P^+, 0_\perp)$$

$$q^\mu = (q^+, q^-, q_\perp)$$

$$q \neq q'$$

$$D_{ij}(P,q) \propto \langle \mathrm{J}/\psi | \psi_i^\dagger(0)\, \chi_j(y) | 0 \rangle$$

- **Scattering amplitude:**

$$\mathcal{M} \propto \int \frac{d^4q}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}(P,q,Q)\, \frac{\gamma\cdot(P/2-q)+m}{(P/2-q)^2-m^2+i\epsilon}\, \hat{D}(P,q)\, \frac{\gamma\cdot(P/2+q)+m}{(P/2+q)^2-m^2+i\epsilon}\right]$$

- **Potential poles:**

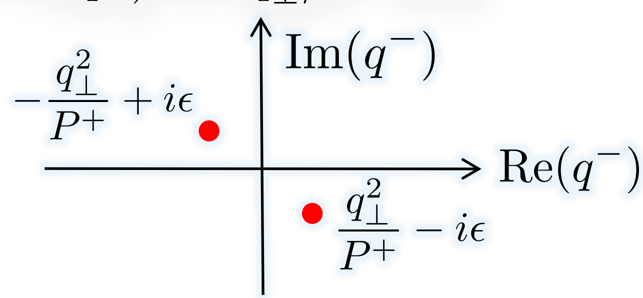
$$q^- = [q_\perp^2 - 2m^2(q^+/P^+)]/(P^+ + 2q^+) - i\epsilon\theta(P^+ + 2q^+) \rightarrow q_\perp^2/P^+ - i\epsilon$$

$$q^- = -[q_\perp^2 + 2m^2(q^+/P^+)]/(P^+ - 2q^+) + i\epsilon\theta(P^+ - 2q^+) \rightarrow -q_\perp^2/P^+ + i\epsilon$$

- **Condition for pinched poles:**

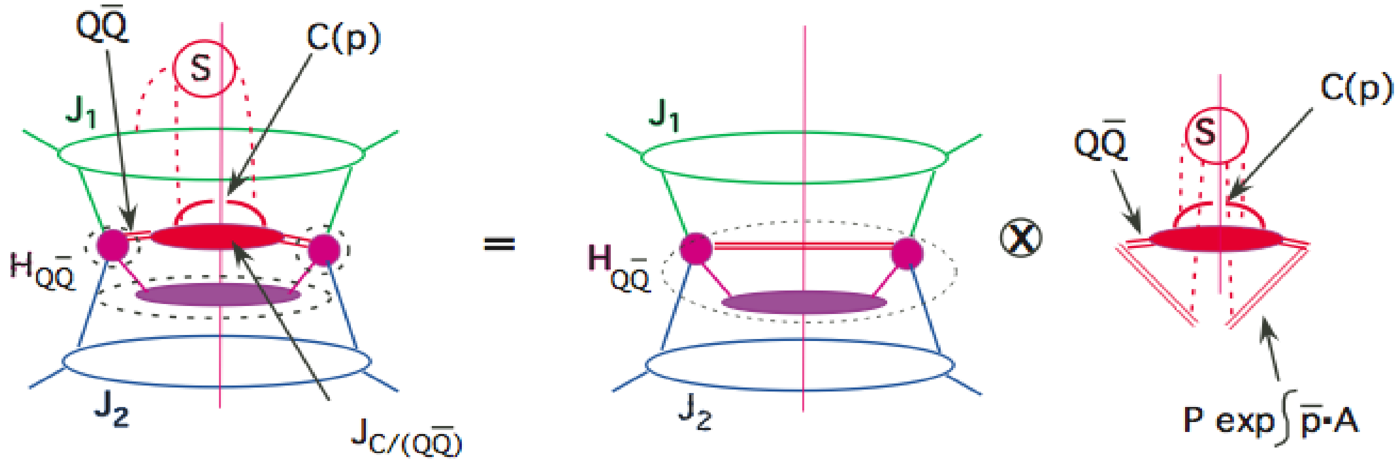
$$P^+ \gg q^+(2m^2/q_\perp^2)$$
$$P^+ \gg 2m$$

At High P_T →



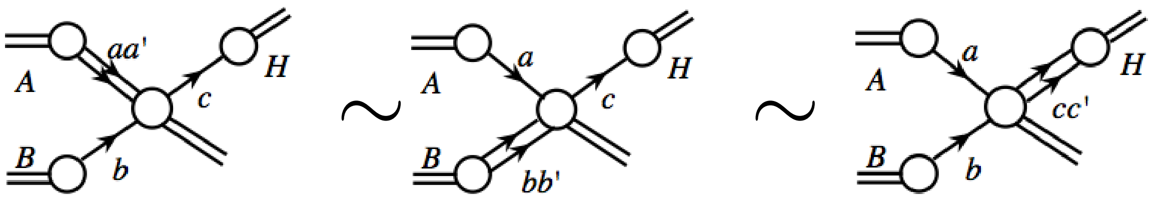
➢ Heavy quark pair fragmentation:

Qiu, Sterman (1991)
Kang, YQM, Qiu, Sterman, 1304.xxxx



$$\sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(p(1\pm\zeta)/2z, p(1\pm\zeta')/2z) \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q)$$

➢ Other channels of power corrections:



$$\sim \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{P_T^2}\right) \otimes D_{c\to H}$$

or $\mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{P_T^2}\right) \otimes \mathcal{D}_{[ff]\to H}$

Factorization formalism and evolution

- **Factorization formalism:**

Kang, YQM, Qiu, Sterman, 1304.xxxx

$$d\sigma_{A+B\to H+X}(p_T) = \sum_f d\hat{\sigma}_{A+B\to f+X}(p_f = p/z) \otimes D_{H/f}(z, m_Q)$$
$$+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(p(1\pm\zeta)/2z, p(1\pm\zeta')/2z)$$
$$\otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q)$$
$$+ \mathcal{O}(m_Q^4/p_T^4)$$

produce pair at $1/m_Q$

produce pair at $1/P_T$

$\kappa = v, a, t$ for spin, and $1, 8$ for color.

- **Independence of the factorization scale:** $\frac{d}{d\ln(\mu)}\sigma_{A+B\to HX}(P_T) = 0$

- **Evolution equations at NLP:**

produce pair between [$1/m_Q$, $1/P_T$]

$$\frac{d}{d\ln\mu^2} D_{H/f}(z, m_Q, \mu) = \sum_j \frac{\alpha_s}{2\pi}\gamma_{f\to j}(z) \otimes D_{H/j}(z, m_Q, \mu)$$
$$+ \frac{1}{\mu^2}\sum_{[Q\bar{Q}(\kappa)]} \frac{\alpha_s^2}{(2\pi)^2}\Gamma_{f\to[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta') \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q,\mu)$$

$$\frac{d}{d\ln\mu^2}\mathcal{D}_{H/[Q\bar{Q}(c)]}(z,\zeta,\zeta',m_Q,\mu) = \sum_{[Q\bar{Q}(\kappa)]}\frac{\alpha_s}{2\pi}K_{[Q\bar{Q}(c)]\to[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta')$$
$$\otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q,\mu)$$

Predictive power

- **Calculation of short-distance hard parts in pQCD:**

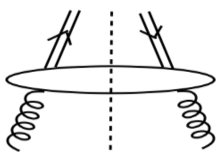
  Power series in α_s , without large logarithms

- **Calculation of evolution kernels in pQCD:**

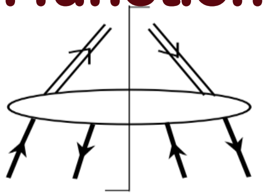
  Power series in α_s , scheme in choosing factorization scale μ

  Could affect the term with mixing powers

- **Universality of input fragmentation functions at μ_0:**



$D_{H/f}(z, m_Q, \mu_0)$



$\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z, \zeta, \zeta', m_Q, \mu_0)$

- **Physics of $\mu_0 \sim 2m_Q$ – a parameter:**

  Evolution stops when $\log\left[\frac{\mu_0^2}{(4m_Q^2)}\right] \sim \left[\frac{4m_Q^2}{\mu_0^2}\right]$

Different quarkonium states require different input distributions!

Short-distance hard parts

- **Calculation of hard parts at NLP level is similar as the calculation in NRQCD factorization, but with two differences:**
  - Set heavy quark mass m_Q as zero
  - Using the relativistic spin projectors

Kang, Qiu, Sterman, 1109.1520

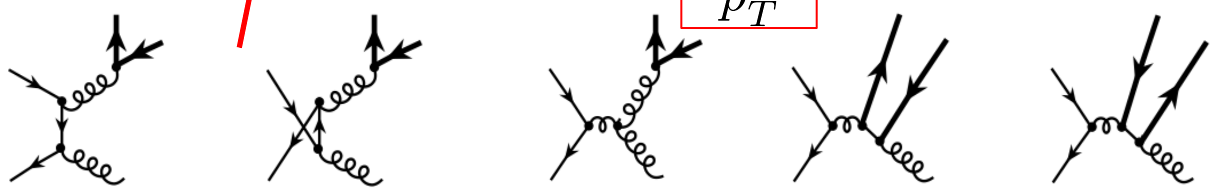
$$\mathcal{P}_a = \frac{\hat{\not{p}}_{\bar{c}}\gamma^+\gamma^5\hat{\not{p}}_c}{2\hat{P}^+}\,,\quad \mathcal{P}_v = \frac{\hat{\not{p}}_{\bar{c}}\gamma^+\hat{\not{p}}_c}{2\hat{P}^+}\,,\quad \mathcal{P}_t^\mu = \frac{\hat{\not{p}}_{\bar{c}}\gamma^+\gamma_\perp^\mu\hat{\not{p}}_c}{2\hat{P}^+}\,,$$

$$\sigma^{(3)}_{q\bar{q}\to[Q\bar{Q}(c)]g} = \hat{\sigma}^{(3)}_{q\bar{q}\to[Q\bar{Q}(\kappa)]g} \otimes D^{(0)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(c)]} + \hat{\sigma}^{(2)}_{q\bar{q}\to gg} \otimes D^{(1)}_{g\to[Q\bar{Q}(c)]}$$

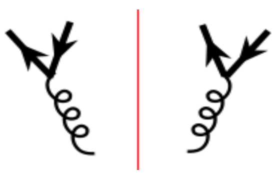
$$\Rightarrow \quad \hat{\sigma}^{(3)}_{q\bar{q}\to[Q\bar{Q}]g} = \sigma^{(3)}_{q\bar{q}\to[Q\bar{Q}]g} - \sigma^{(2)}_{q\bar{q}\to g} \otimes D^{(1)}_{g\to[Q\bar{Q}]}$$

$\frac{\alpha_s^3(\mu)}{p_T^6}$ $\qquad$ $\frac{\alpha_s^2(\mu)}{p_T^4}$ $\qquad$ $\frac{\alpha_s(2m_Q)}{(2m_Q)^2}$

$\sigma^{(3)}_{q\bar{q}\to[Q\bar{Q}(c)]g}$:



$D^{(1)}_{g\to[Q\bar{Q}]}$:



$$\widetilde{\mathcal{P}}_{\mu\nu}(p) = \frac{1}{2}\left[-g_{\mu\nu} + \frac{p_\mu n_\nu + n_\mu p_\nu}{p\cdot n} - \frac{p^2}{(p\cdot n)^2}n_\mu n_\nu\right]$$

- **Calculate to NLP, even tree-level needs subtraction!**

Set $m_Q = 0$ with care!

Evolution kernels: 1 → 2

- **Expand evolution equation to $O(\alpha_s^2)$:**

Kang, YQM, Qiu, Sterman, 1304.xxxx

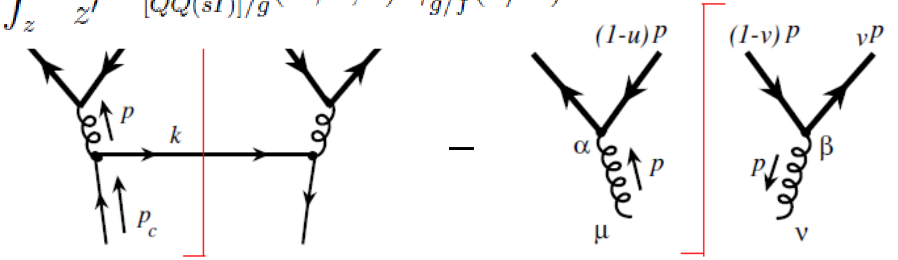
$$\frac{\partial}{\partial \ln\mu^2} D^{(2)}_{[Q\bar{Q}(sI)]/f}(z,\mu^2;u,v) = \int_z^1 \frac{dz'}{z'} D^{(1)}_{[Q\bar{Q}(sI)]/g}(z';u,v)\,\gamma^{(1)}_{g/f}(z/z') + \frac{1}{\mu^2}\int_z^1 \frac{dz'}{z'}\int_0^1 du' \int_0^1 dv' \times \mathcal{D}^{(0)}_{[Q\bar{Q}(sI)]/[Q\bar{Q}(s'I')]}(z',u',v';u,v)\,\gamma^{(2)}_{[Q\bar{Q}(s'I')]/f}(z/z',u',v')$$

$$\mathcal{D}^{(0)}_{[Q\bar{Q}(sI)]/[Q\bar{Q}(s'I')]}(z',u',v';u,v) = \delta^{ss'}\,\delta^{II'}\,\delta(1-z/z')\,\delta(u-u')\,\delta(v-v')$$

- **Using the form of zeroth order fragmentation function:**

$$\frac{1}{\mu^2}\,\gamma^{(2)}_{[Q\bar{Q}(sI)]/f}(z,u,v) = \frac{\partial}{\partial \ln\mu^2} D^{(2)}_{[Q\bar{Q}(sI)]/f}(z,\mu^2;u,v) - \int_z^1 \frac{dz'}{z'} D^{(1)}_{[Q\bar{Q}(sI)]/g}(z';u,v)\,\gamma^{(1)}_{g/f}(z/z')$$

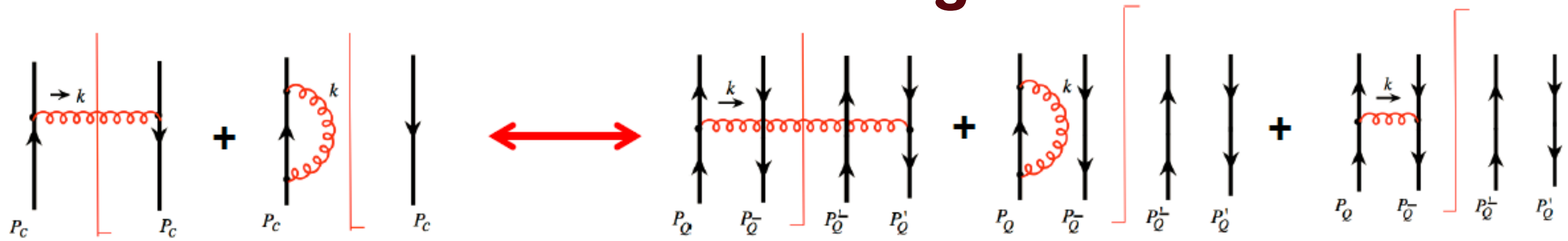
Example: “$q \to Q\bar{Q}$” =



$$D^{(2)}_{[Q\bar{Q}(v8)]/q}(z';u,v,\mu^2) = \int^{\mu^2} \frac{dp_c^2}{(p_c^2)^2}\left[\alpha_s^2\left(\frac{N_c^2-1}{N_c}\frac{8(1-z)}{z^2}\right)\right] \qquad D^{(1)}_{[Q\bar{Q}(v8)]/g}(z';u,v) = 0$$

Evolution kernels: 2 → 2

- **Mismatch of "+" momentum integration range between the real and virtual diagrams :** Kang, YQM, Qiu, Sterman, 1304.xxxx



$k^+ \in (0, \frac{(1-z)P^+}{z})$ $k^+ \in (0, P^+)$ ⟷ $k^+ \in (0, \frac{(1-z)P^+}{z})$ $k^+ \in (0, P_Q{}^+)$ $k^+ \in (0, P_Q^+/P_{\bar{Q}}^+)$

- **Consequence of mismatch: our results have uncanceled *logarithmic terms***

$$\int_0^{\mu_2} \frac{dk^+}{k^+} - \int_0^{\mu_1} \frac{dk^+}{k^+} = \int_{\mu_1}^{\mu_2} \frac{dk^+}{k^+} = \ln(\mu_2/\mu_1)$$

Logarithmic terms depend on the definition of "plus functions", one may eliminate the logarithm by choosing different definitions. Our definition:

$$\begin{cases} \int \langle f(v,v')\rangle_{+0}\, g(v)dv \equiv \int_0^1 \left[f(v,v')\theta(v-v') + f(\bar{v},\bar{v}')\theta(\bar{v}-\bar{v}')\right] g(v)dv \\ \int \langle f(v,v')\rangle_{+0}\, g(v')dv' \equiv \int_0^1 \left[f(v,v')\theta(v-v') + f(\bar{v},\bar{v}')\theta(\bar{v}-\bar{v}')\right] g(v')dv' \end{cases}$$

$$\begin{cases} \int \langle f(v,v')\rangle_{+1}\, g(v)dv \equiv \int_0^1 \left[f(v,v')\theta(v-v') + f(\bar{v},\bar{v}')\theta(\bar{v}-\bar{v}')\right] \left[g(v) - g(v') - \log\frac{1}{v'\bar{v}'}\right] dv \\ \int \langle f(v,v')\rangle_{+1}\, g(v')dv' \equiv \int_0^1 \left[f(v,v')\theta(v-v') + f(\bar{v},\bar{v}')\theta(\bar{v}-\bar{v}')\right] \left[g(v') - g(v) - \log\frac{1}{v\bar{v}}\right] dv' \end{cases}$$

$$\int \frac{1}{(1-z)_+} g(z)dz \equiv \int_{z_0}^1 \frac{\left[g(z) - g(1) - \log\frac{1}{1-z_0}\right]}{1-z} dz$$

Evolution kernels

Two → two

$$P_{v1\to v1} = P_{a1\to a1} = (3 - S_0) C_F \Delta_0 + C_F \Delta_v,$$
$$P_{t1\to t1} = (3 - S_0) C_F \Delta_0 + C_F \tilde{\Delta}_v,$$
$$P_{v8\to v8} = P_{a8\to a8} = (3 - S_0) C_F \Delta_0 - \frac{1}{2N_c}\Delta_v + \frac{1}{2N_c}\frac{z}{2(1-z)_+} S_+ \Delta_+^{[8]},$$
$$P_{t8\to t8} = (3 - S_0) C_F \Delta_0 - \frac{1}{2N_c}\tilde{\Delta}_v + \frac{1}{2N_c}\frac{z}{2(1-z)_+}\left(S_+ \Delta_+^{[8]} + S_- \Delta_-^{[8]}\right)$$

$$P_{v8\to v1} = P_{a8\to a1} = \frac{z}{2(1-z)} S_+ \Delta_-^{[1]},$$
$$P_{t8\to t1} = \frac{z}{2(1-z)}\left(S_+ \Delta_-^{[1]} + S_- \Delta_+^{[1]}\right),$$
$$P_{v8\to a1} = P_{a8\to v1} = \frac{z}{2(1-z)} S_- \Delta_-^{[1]},$$
$$P_{v8\to a8} = P_{a8\to v8} = \frac{1}{2N_c}\frac{z}{2(1-z)} S_- \Delta_+^{[8]},$$
$$P_{X1\to Y8} = \frac{N_c^2-1}{4N_c^2} P_{X8\to Y1}, \quad \text{for} \quad X, Y = v, a, t.$$

$$S_0 = \ln(u\bar{u}v\bar{v}), \quad S_\pm = \left(\frac{u}{u'} \pm \frac{\bar{u}}{\bar{u}'}\right)\left(\frac{v}{v'} \pm \frac{\bar{v}}{\bar{v}'}\right),$$
$$\Delta_0 = \delta(1-z)\delta(u-u')\delta(v-v')$$
$$\Delta_\pm^{[1]} = [\delta(u - zu') \pm \delta(\bar{u} - z\bar{u}')]\,[\delta(v - zv') \pm \delta(\bar{v} - z\bar{v}')],$$
$$\Delta_\pm^{[8]} = \{(N_c^2 - 2)\,[\delta(u - zu')\delta(v - zv') + \delta(\bar{u} - z\bar{u}')\delta(\bar{v} - z\bar{v}')]$$
$$\pm 2\,[\delta(u - zu')\delta(\bar{v} - z\bar{v}') + \delta(\bar{u} - z\bar{u}')\delta(v - zv')]\},$$
$$\Delta_v = \delta(1-z)\delta(u-u')\left[\left\langle\frac{1}{v-v'}\right\rangle_{+1} - \left\langle\frac{\bar{v}'}{v}\right\rangle_{+0}\right] + u \leftrightarrow v,$$
$$\tilde{\Delta}_v = \delta(1-z)\delta(u-u')\left[\left\langle\frac{1}{v-v'}\right\rangle_{+1} - \left\langle\frac{1}{v}\right\rangle_{+0}\right] + u \leftrightarrow v,$$

Kang, YQM, Qiu, Sterman, 1304.xxxx

One → two

Light quark case:

$$\gamma^{(2)}_{[Q\bar{Q}(v8)]/q} = \alpha_s^2 \frac{N_c^2-1}{N_c}\frac{8(1-z)}{z^2}$$
$$\gamma^{(2)}_{[Q\bar{Q}(v1)]/q} = \gamma^{(2)}_{[Q\bar{Q}(a8)]/q} = \gamma^{(2)}_{[Q\bar{Q}(a1)]/q} = \gamma^{(2)}_{[Q\bar{Q}(t8)]/q} = \gamma^{(2)}_{[Q\bar{Q}(t1)]/q} = 0$$

Heavy quark case:

$$\gamma^{(2)}_{[Q\bar{Q}(v8)]/Q} = \alpha_s^2 \frac{N_c^2-1}{2N_c^3}\frac{1}{\bar{u}\bar{v}}\frac{1-z}{z^2}\frac{4N_c\bar{u}(1-zu) + z(1+z\bar{u})}{1-zu} \times \frac{4N_c\bar{v}(1-zv) + z(1+z\bar{v})}{1-zv}$$
$$\gamma^{(2)}_{[Q\bar{Q}(v1)]/Q} = \alpha_s^2 \left(\frac{N_c^2-1}{N_c}\right)^2 \frac{1-z}{\bar{u}\bar{v}}\frac{(1+z\bar{u})(1+z\bar{v})}{(1-zu)(1-zv)}$$
$$\gamma^{(2)}_{[Q\bar{Q}(a8)]/Q} = \alpha_s^2 \frac{N_c^2-1}{2N_c^3}\frac{1-z}{\bar{u}\bar{v}}\frac{(1+z\bar{u})(1+z\bar{v})}{(1-zu)(1-zv)}$$
$$\gamma^{(2)}_{[Q\bar{Q}(a1)]/Q} = \alpha_s^2 \left(\frac{N_c^2-1}{N_c}\right)^2 \frac{1-z}{\bar{u}\bar{v}}\frac{(1+z\bar{u})(1+z\bar{v})}{(1-zu)(1-zv)}$$
$$\gamma^{(2)}_{[Q\bar{Q}(t8)]/Q} = \gamma^{(2)}_{[Q\bar{Q}(t1)]/Q} = 0$$

Gluon case:

$$\gamma^{(2)}_{[Q\bar{Q}(v8)]/g} = \alpha_s^2 \frac{1}{4u\bar{u}v\bar{v}}\left\{\frac{N_c}{z^2}\left[4(1-z)^2 - 4(1-2u\bar{u}-2v\bar{v})(1-z)^2(z+2)\right.\right.$$
$$\left. + (u-\bar{u})^2(v-\bar{v})^2(2z^4 + 2z^3 - 3z^2 - 4z + 4)\right]$$
$$\left. + \frac{N_c^2-4}{N_c}(u-\bar{u})(v-\bar{v})[z^2 + (1-z)^2]\right\}$$
$$\gamma^{(2)}_{[Q\bar{Q}(v1)]/Q} = \alpha_s^2 \frac{(u-\bar{u})(v-\bar{v})}{u\bar{u}v\bar{v}}[z^2 + (1-z)^2]$$
$$\gamma^{(2)}_{[Q\bar{Q}(a8)]/Q} = \alpha_s^2 \frac{1}{u\bar{u}v\bar{v}}\left[\frac{N_c}{2}(\bar{u}\bar{v} + uv) - \frac{1}{N_c}\right][z^2 + (1-z)^2]$$
$$\gamma^{(2)}_{[Q\bar{Q}(a1)]/Q} = \alpha_s^2 \frac{1}{u\bar{u}v\bar{v}}[z^2 + (1-z)^2]$$
$$\gamma^{(2)}_{[[Q\bar{Q}(t8)]/Q} = \gamma^{(2)}_{[[Q\bar{Q}(t1)]Q} = 0$$

Apply NRQCD to FFs

- **Input distributions are universal, non-perturbative:**

  Should, in principle, be extracted from experimental data

- **If NRQCD is valid – only proof to NNLO!** Nayak, Qiu and Sterman, 0509021

  Replace unknown functions by a few unknown numbers - matrix elements!

- **Apply NRQCD to the input distributions:**

  - All possible single parton FFs – up to NLO in α_s Braaten, Yuan, 9303205, Braaten, Lee , 0004228,……

$$D_{g\to J/\psi}(z,\mu_0,m_Q) \to \sum_{[Q\bar{Q}(c)]} \hat{d}_{g\to[Q\bar{Q}(c)]}(z,\mu_0,m_Q)\langle\mathcal{O}_{[Q\bar{Q}(c)]}(0)\rangle|_{\rm NRQCD}$$

  - All possible heavy quark pair FFs – up to NLO in α_s Kang, Qiu, Sterman, 1109.1520 YQM, Qiu, Zhang, 13xx.xxxx
  - Divergences at NLO also confirm the correctness of evolution kernels

$$\mathcal{D}_{[Q\bar{Q}(\kappa)]\to J/\psi}(z,\zeta,\zeta',\mu_0,m_Q) \to \sum_{[Q\bar{Q}(c)]} \hat{d}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(c)]}(z,\zeta,\zeta',\mu_0,m_Q)\langle\mathcal{O}_{[Q\bar{Q}(c)]}(0)\rangle_{\rm NRQCD}$$

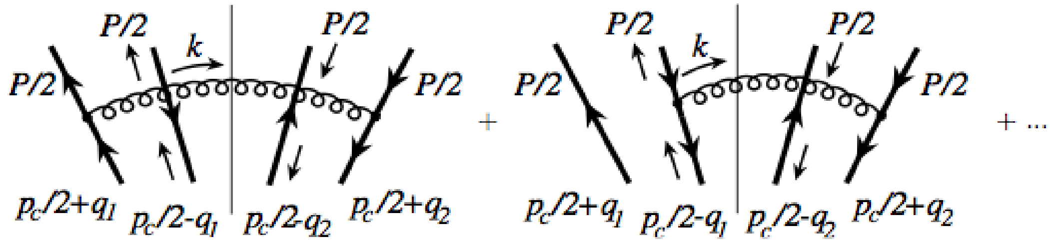
Polarization of ${}^3S_1^{[1]}$ channel

Kang, Qiu and Sterman, 1109.1520

- **Fragmentation functions determine the polarization**

  Short-distance dynamics at $r \sim 1/p_T$ is NOT sensitive to the details taken place at the scale of hadron wave function ~ 1 fm

- **Heavy quark pair fragmentation functions at LO:**



NRQCD to a singlet pair:

$$\mathcal{D}_{[Q\bar{Q}(\kappa)]\to J/\psi} = 2\,\mathcal{D}^T_{[Q\bar{Q}(\kappa)]\to J/\psi} + \mathcal{D}^L_{[Q\bar{Q}(\kappa)]\to J/\psi}$$

$$\mathcal{D}^L_{[Q\bar{Q}(a8)]\to J/\psi}(z,\zeta,\zeta',m_Q,\mu) = \frac{1}{2N^2}\frac{\langle O^{J/\psi}_{1(^3S_1)}\rangle}{3m_c}\Delta(\zeta,\zeta')\;\frac{\alpha_s}{2\pi}z(1-z)\left[\ln\left(r(z)+1\right)-\left(1-\frac{1}{1+r(z)}\right)\right]$$

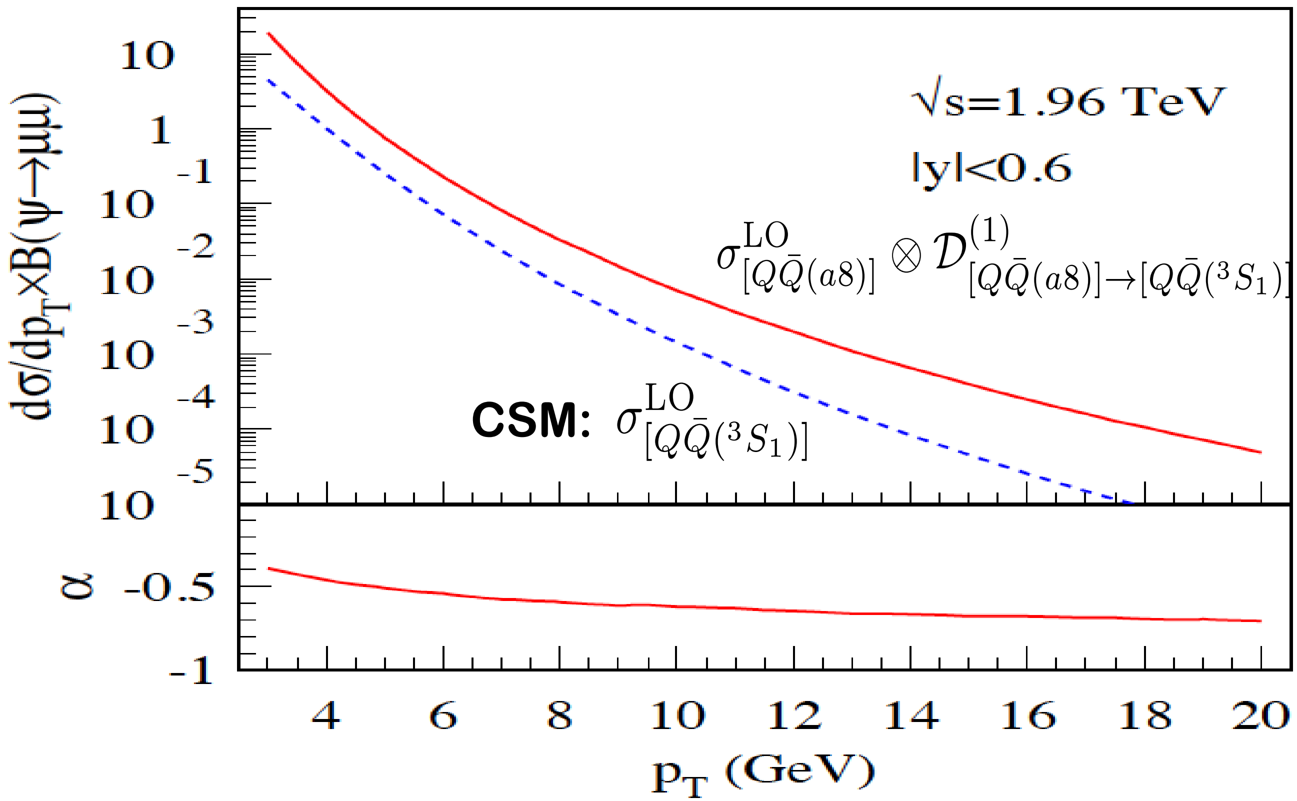
$$\mathcal{D}^T_{[Q\bar{Q}(a8)]\to J/\psi}(z,\zeta,\zeta',m_Q,\mu) = \frac{1}{2N^2}\frac{\langle O^{J/\psi}_{1(^3S_1)}\rangle}{3m_c}\Delta(\zeta,\zeta')\,\frac{\alpha_s}{2\pi}z(1-z)\left[1-\frac{1}{1+r(z)}\right]$$

where $$\Delta(\zeta,\zeta') = \frac{1}{4}\sum_{a,b}\delta(\zeta-a(1-z))\delta(\zeta'-b(1-z)) \quad , \quad r(z) \equiv \frac{z^2\mu^2}{4m_c^2(1-z)^2}$$

Polarization of $^3S_1^{[1]}$ channel

Kang, Qiu and Sterman, 1109.1520

- **LO hard parts + LO fragmentation contributions:**

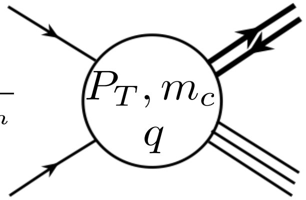


LO heavy quark pair fragmentation contribution reproduces the bulk of NLO color singlet contribution, and the polarization!

Relativistic corrections

- **Leading v² relativistic correction in NRQCD:**

$$\frac{d^n}{dq^{\alpha_1}\ldots dq^{\alpha_n}}$$



$$P_c = P/2 + q$$
$$P_{\bar{c}} = P/2 - q$$

Fan, YQM, Chao, 0904.4025
Xu, Li, Liu, Zhang,1203.0207

Large p_T behavior:

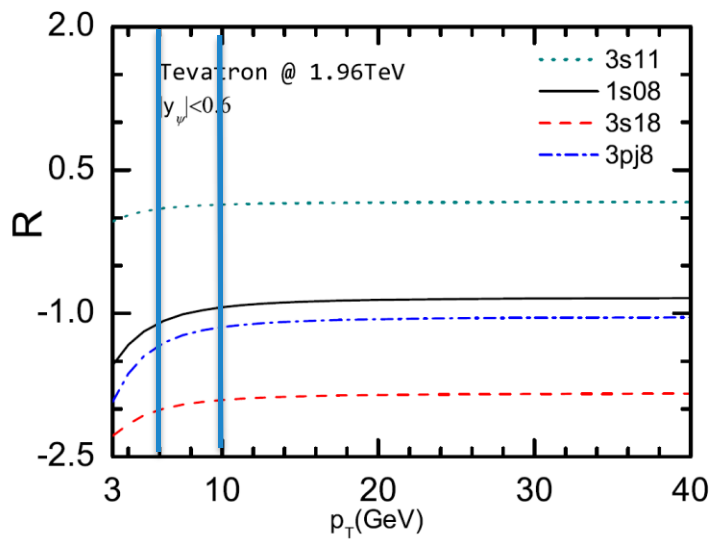
$$R^{(1)}({}^3S_1^{[1]}) = \frac{G({}^3S_1^{[1]})}{F({}^3S_1^{[1]})}\Big|_{p_T\gg m} = \frac{1}{6},$$

$$R^{(1)}({}^1S_0^{[8]}) = \frac{G({}^1S_0^{[8]})}{F({}^1S_0^{[8]})}\Big|_{p_T\gg m} = -\frac{5}{6},$$

$$R^{(1)}({}^3S_1^{[8]}) = \frac{G({}^3S_1^{[8]})}{F({}^3S_1^{[8]})}\Big|_{p_T\gg m} = -\frac{11}{6},$$

$$R^{(1)}({}^3P^{[8]}) = \frac{G({}^3P^{[8]})}{F({}^3P^{[8]})}\Big|_{p_T\gg m} = -\frac{31}{30},$$

Complete results:



Large p_T approximation: dominant for p_T>10GeV; gives reasonable results for p_T>6GeV.

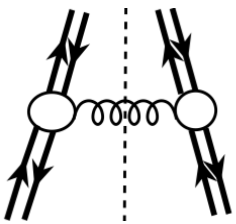
Relativistic corrections

YQM, Qiu, 13xx.xxxx

- **v^2 all order corrections in pQCD factorization:**

$$P_c = P/2 + q$$
$$P_{\bar{c}} = P/2 - q$$

$$\frac{d^n}{dq^{\alpha_1}\ldots dq^{\alpha_n}}$$



$$R({}^1S_0^{[8]}) = 1 - \frac{5}{6}\delta + \frac{259}{360}\delta^2 - \frac{3229}{5040}\delta^3 + \cdots .$$

n	0	1	2	3	$\cdots$	∞
$\sum_{i=0}^n \delta^i R^{(i)}({}^1S_0^{[8]})\big\vert_{\delta=0.3}$	1	0.750	0.815	0.797	$\cdots$	0.801
$\sum_{i=0}^n \delta^i R^{(i)}({}^1S_0^{[8]})\big\vert_{\delta=0.1}$	1	0.917	0.924	0.923	$\cdots$	0.923

$$R({}^3S_1^{[8]}) = 1 - \frac{11}{6}\delta + \frac{191}{72}\delta^2 - \frac{167}{48}\delta^3 + \cdots ,$$

n	0	1	2	3	4	$\cdots$	∞
$\sum_{i=0}^n \delta^i R^{(i)}({}^3S_1^{[8]})\big\vert_{\delta=0.3}$	1	0.450	0.689	0.595	0.630	$\cdots$	0.620
$\sum_{i=0}^n \delta^i R^{(i)}({}^3S_1^{[8]})\big\vert_{\delta=0.1}$	1	0.817	0.843	0.840	0.840	$\cdots$	0.840

$$R({}^3P^{[8]}) = \frac{2R_a({}^3P^{[8]}) + R_v({}^3P^{[8]})}{3} = 1 - \frac{31}{30}\delta + \frac{4111}{4200}\delta^2 - \frac{4631}{5040}\delta^3 + \cdots ;$$

n	0	1	2	3	$\cdots$	∞
$\sum_{i=0}^n \delta^i R^{(i)}({}^3P^{[8]})\big\vert_{\delta=0.3}$	1	0.690	0.778	0.753	$\cdots$	0.759
$\sum_{i=0}^n \delta^i R^{(i)}({}^3P^{[8]})\big\vert_{\delta=0.1}$	1	0.897	0.906	0.906	$\cdots$	0.906

- Using leading p_T approximation
- O(v^2) corrections reproduced
- Convergence of v^2 expansion are found.

Summary

- **When $p_T >> m_Q$ at collider energies, earlier models for calculating the production rate of heavy quarkonia are not perturbatively stable**

  LO inα_s-expansion may not be the LP term in $1/p_T$-expansion

- **When $p_T >> m_Q$, $1/p_T$-power expansion before α_s-expansion**

  pQCD factorization approach takes care of both $1/p_T$-expansion and resummation of the large logarithms

- **pQCD factorization approach and SCET approach seem to be consistent in the region where they both apply.**

- **Preliminary applications already show the power of pQCD factorization. More works, particularly, detailed comparisons with data are needed!**

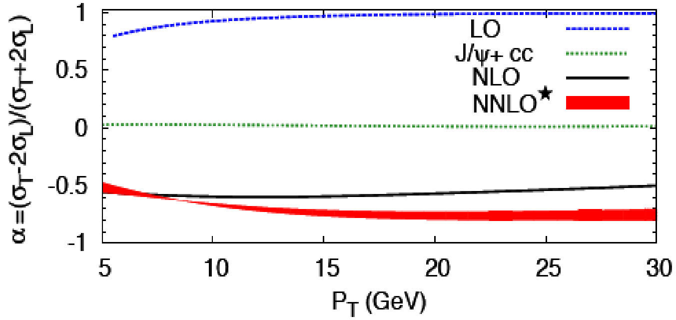
Thank you!

Anomalies from J/ψ polarization in NRQCD

➢ CS: LO V.S. higher order

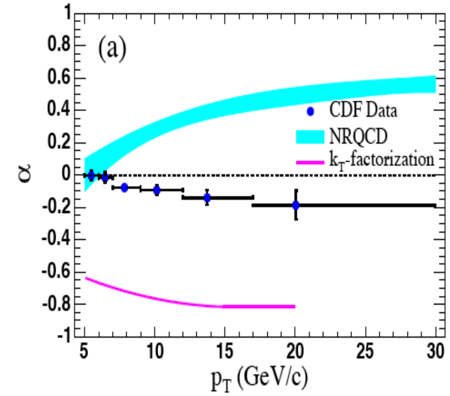
- LO gives transverse polarization
- NLO and NNLO gives longitudinal polarization

Gong, Wang, 0805.2469
Lansberg, 0811.4005



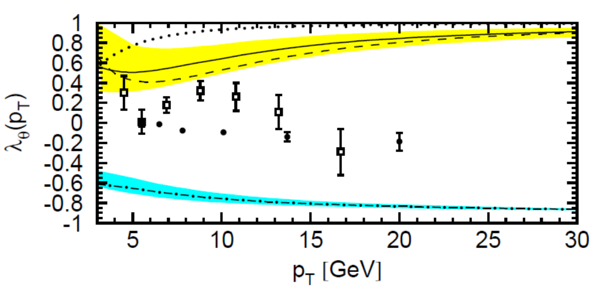
➢ CO: LO V.S. higher order

- NLO corrections for J/psi polarization are worked out by three different groups
- Polarization at NLO can be significantly different from LO, depending on CO LDMEs

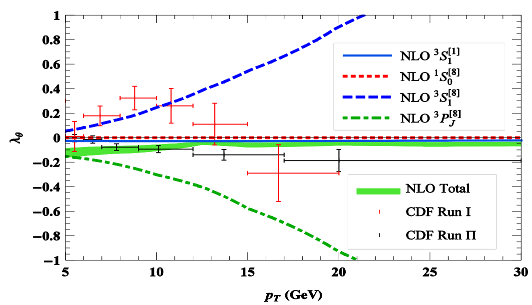


LO prediction:

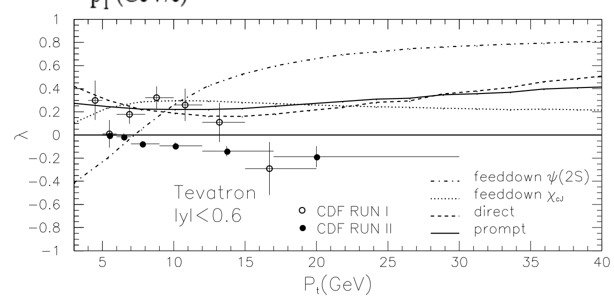
Cho, Wise, 9408352
Beneke, Rothstein, 9509375, …



Butensckön, Kniehl, 1201.1872



Chao,YQM,Shao,Wang,Zhang, 1201.2675



Gong,Wan,Wang,Zhang, 1205.6682