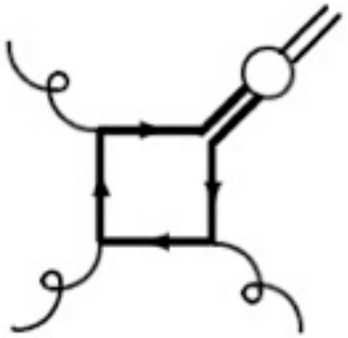
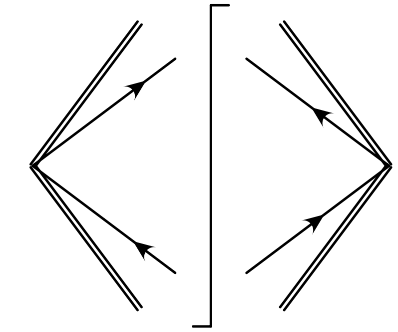




Double-Parton Fragmentation in Quarkonium Production

Adam Leibovich
PITT PACC, University of Pittsburgh

Work done with:
Sean Fleming, Tom Mehen, Ira Rothstein
1207.2578, 1301.3822

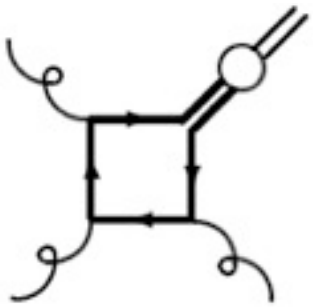
Quarkonium 2013, IHEP, Beijing, April 22, 2013

Outline

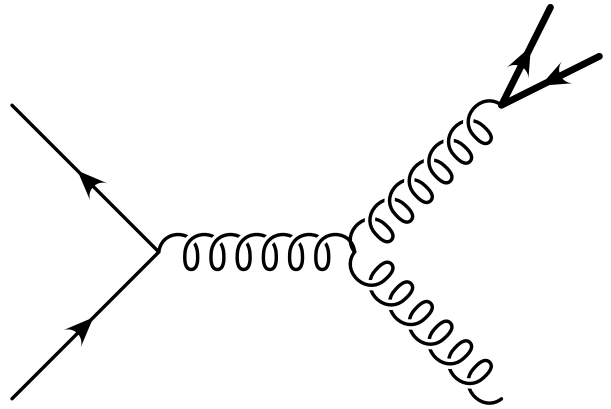
- The different expansions
- Fragmentation: single and double
- Matching, running, matching...
- Conclude

Expansions

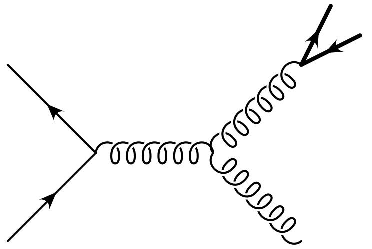
- QCD - expansion in α_s

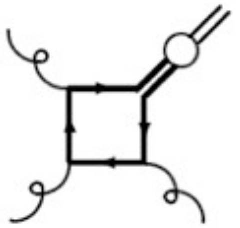


- NRQCD - expansion in v



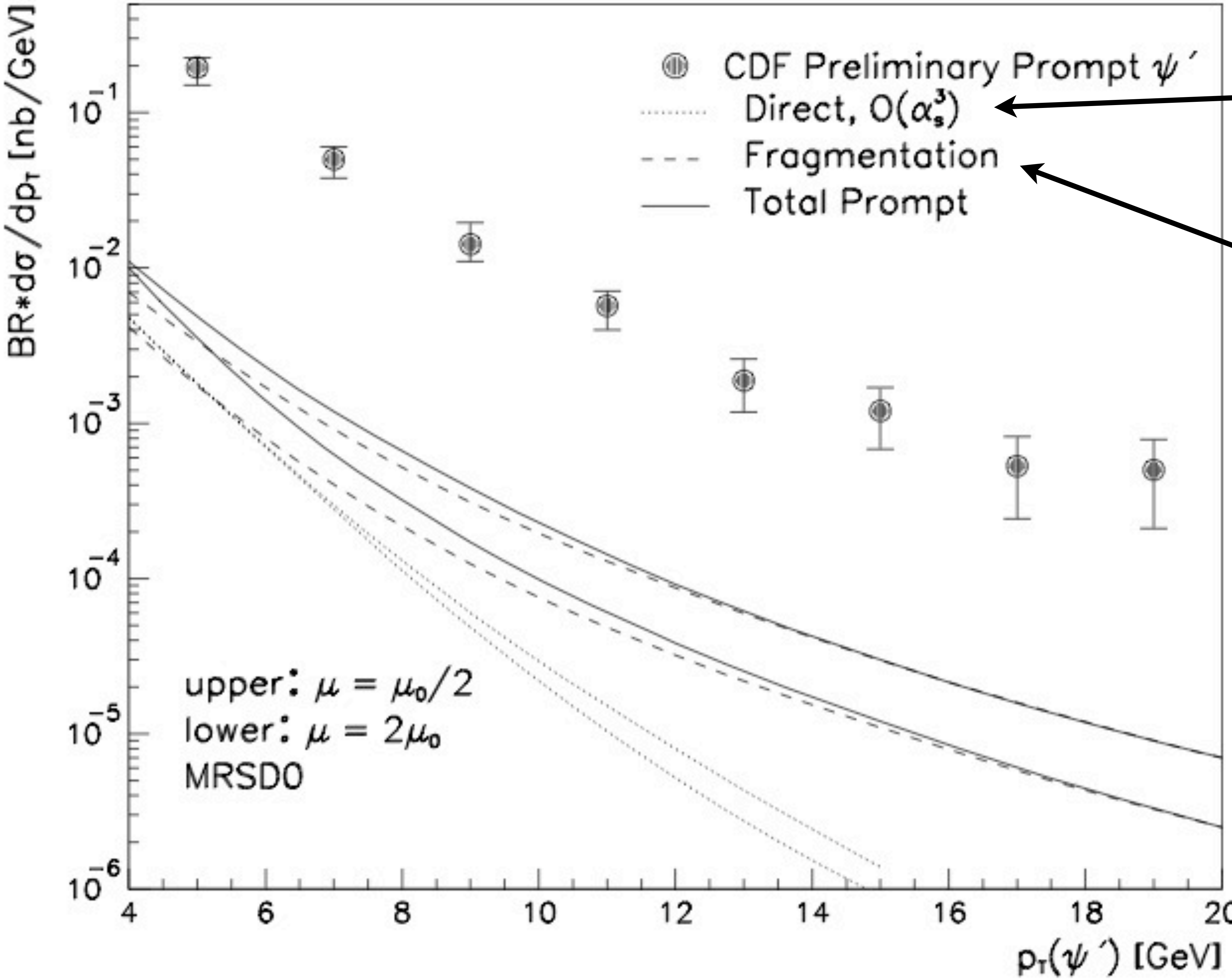
- SCET - expansion in $\frac{m_Q}{p_\perp}$

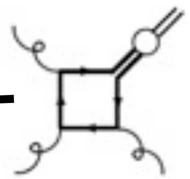


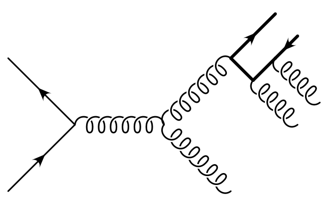


Problem with CSM

(expansion in α_s)

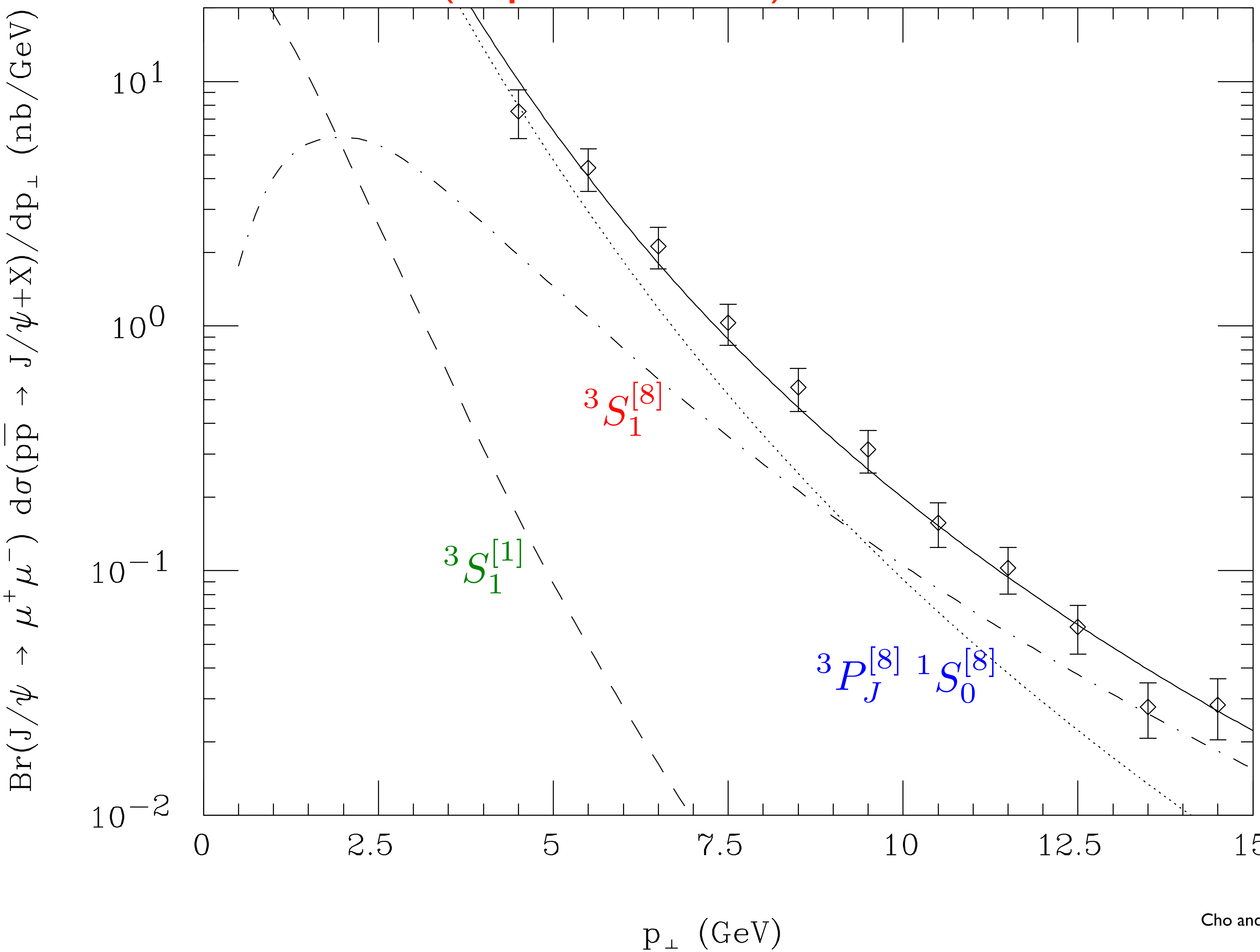






Problem fixed?

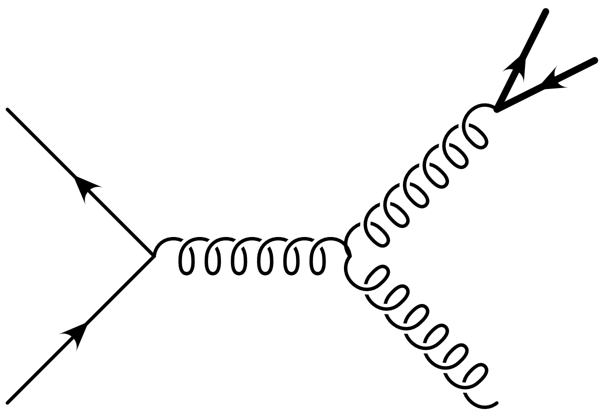
(expansion in v)

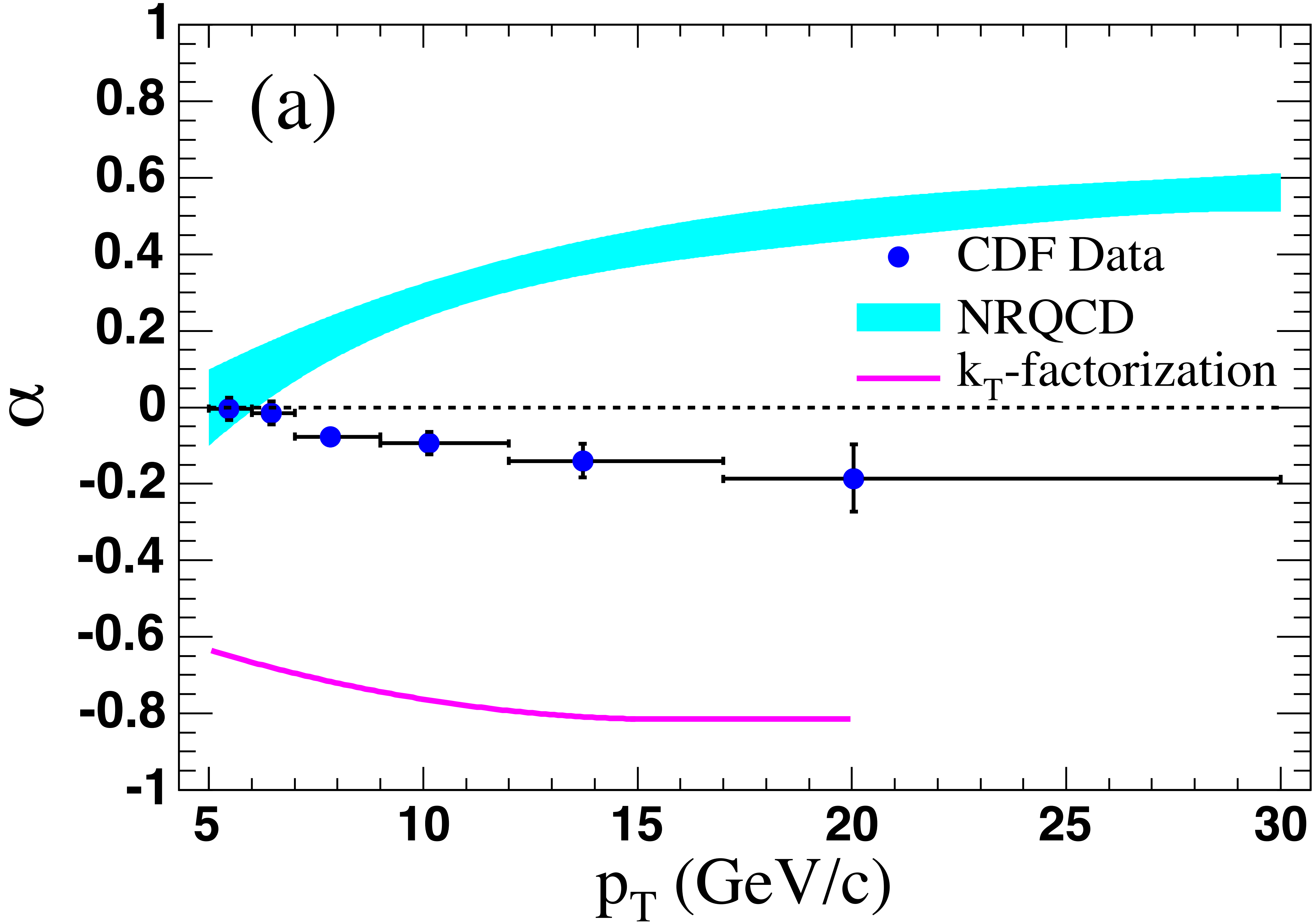


Cho and AKL

Problem fixed?

(expansion in v)





Quarkonium Fragmentation

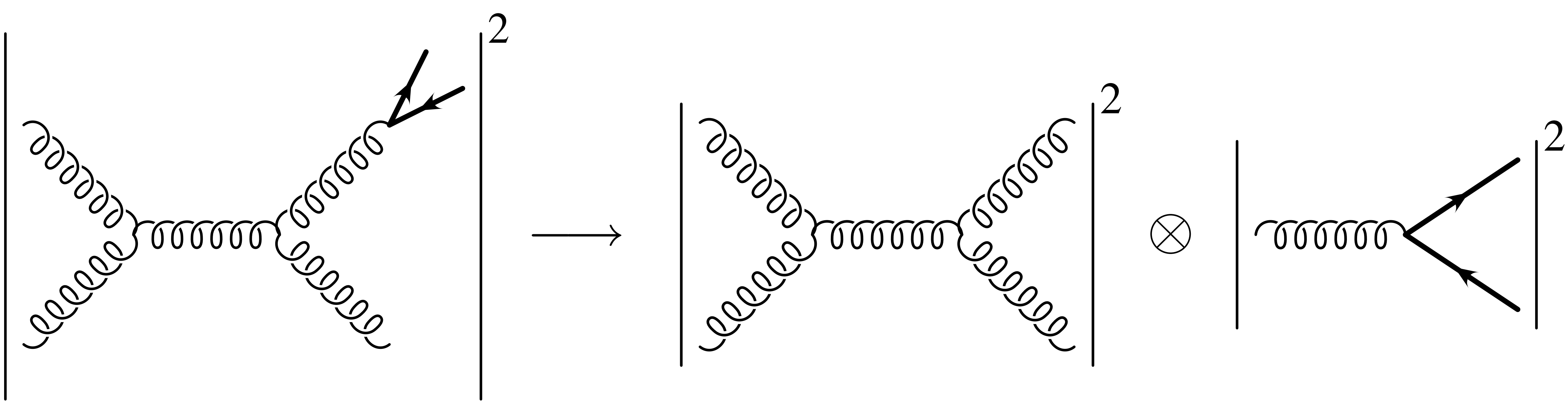
Braaten & Fleming

- J/ψ Production at large $p_\perp$ in hadronic collisions

$\hat{\sigma}(a+b \to c\bar{c}({}^3S_1^{[8]})+X)\langle 0|\mathcal{O}^{J/\psi}({}^3S_1^{[8]})|0\rangle$ is special

Fragmentation: $\dfrac{d\hat{\sigma}}{dp_\perp}(ij \to J/\psi + X)_{\mathrm{octet}}$

$$\xrightarrow{p_\perp \to \infty} \int dz \frac{d\hat{\sigma}}{dp_\perp}(ij \to g(p_\perp/z)+X) D_{g\to J/\psi}(z)$$



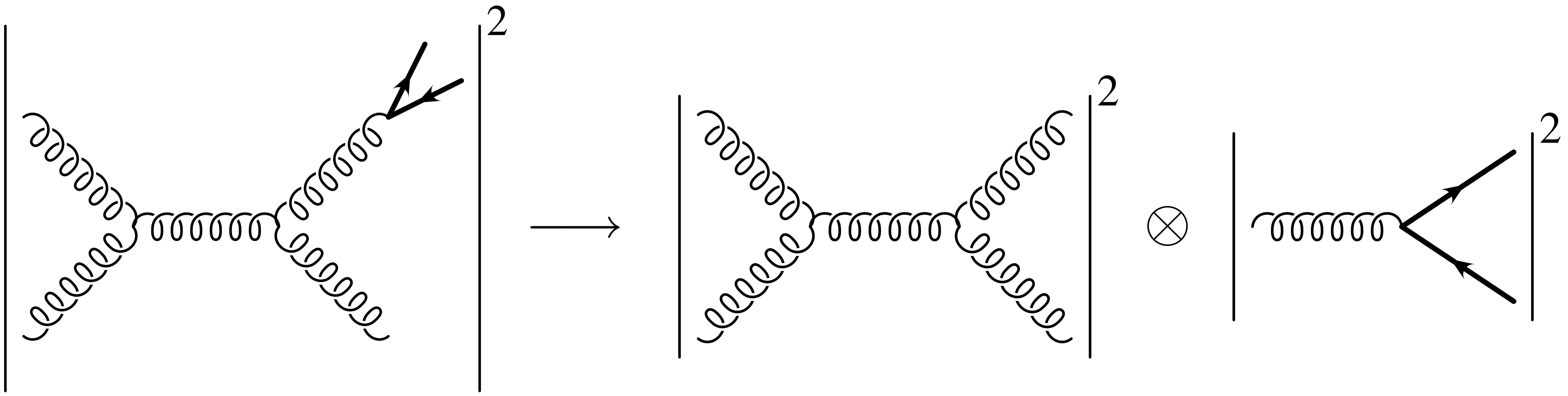
Quarkonium Fragmentation

Sum logs: run from $p_\perp$ to $2m_c$

$$\mu\frac{dD_{g\to\psi_Q}}{d\mu}(z,\mu)=\frac{\alpha_s(\mu)}{\pi}\int_z^1\frac{dy}{y}P_{gg}(y)D_{g\to\psi_Q}(\frac{z}{y},\mu)$$

$$P_{gg}(y)=6\Big[\frac{y}{(1-y)_+}+\frac{1-y}{y}+y(1-y)+\frac{33-2n_f}{36}\delta(1-y)\Big]$$

$$D_{g\to\psi'}(z,2m_c)=\frac{\pi\alpha_s(2m_c)}{24m_c^3}\delta(1-z)\langle 0|\mathcal{O}_8^{\psi'}(^3S_1)|0\rangle$$



Improved Factorization Approach

Kang, Qiu & Sterman; Fleming, Leibovich, Mehen & Rothstein

Improved NRQCD factorization:

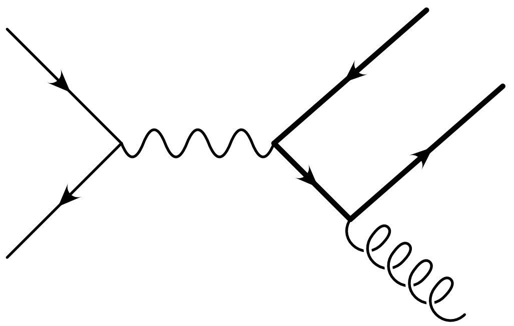
First organizes $\frac{d\sigma}{dp_\perp}$ in powers of $\frac{(2m_c)}{p_\perp}$

Then organizes $\frac{d\sigma}{dp_\perp}$ in powers of v

Systematic perturbative expansion in $\alpha_s(p_\perp)$ and $\alpha_s(2m_c)$

Then we have to consider to what order we work in for *each* parameter

Need for SCET



+ *crossed diagram*

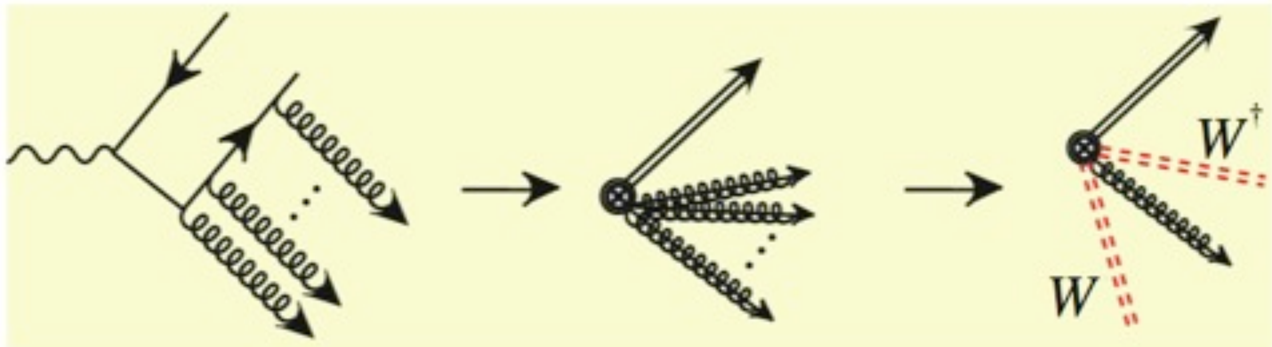
- NRQCD doesn't contain correct d.o.f.
  - At endpoint need both soft and collinear modes
  - NRQCD only has soft d.o.f
- SCET couples collinear and soft d.o.f.
  - Was created to sum Sudakov logarithms
  - Expansion in α_s and $\lambda \sim p_\perp / p^-$
- Couple NRQCD and SCET for corners of phase space

SCET Intro

$p = (p^+, p^-, p_\perp)$

- Systematic expansion in $\lambda \sim p_\perp / p^-$
- Degrees of freedom:
  - –Collinear particles with $p = Q(\lambda^2, 1, \lambda)$
  - –Soft particles with $p = Q(\lambda, \lambda, \lambda)$
  - –Ultrasoft particles with $p = Q(\lambda^2, \lambda^2, \lambda^2)$
- By using gauge invariance, operators constrained
- Field redefinition allows leading order factorization theorems

Gauge invariance restrictions



$$W = \mathrm{P}\,\exp\left(ig\int_{-\infty}^{x} ds\,\bar{n}\cdot A_n(s\bar{n})\right)$$

$$\chi^\dagger D_\perp \psi \to \chi^\dagger W D_\perp W^\dagger \psi$$

Factorization

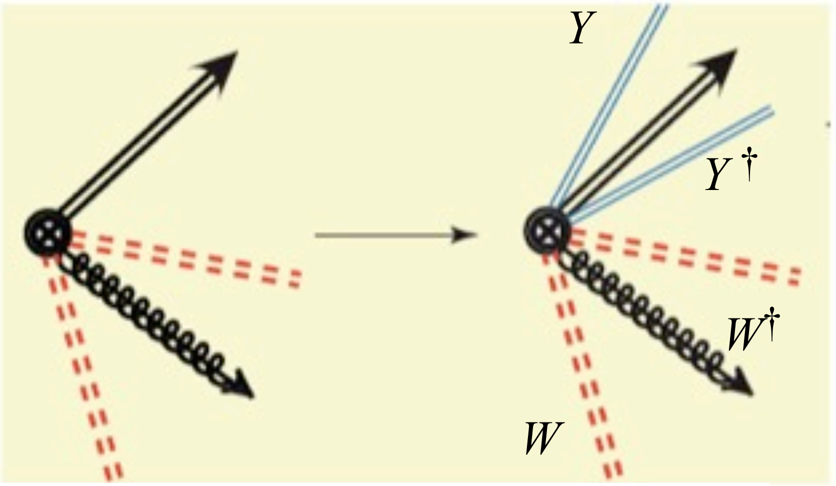
$$\mathcal{L} = \bar{\xi}_{n,p'}\left\{in\cdot D_c + \underbrace{gn\cdot A_u}_{\text{Only coupling to ultrasoft sector}} + i\not{D}_c^\perp \frac{1}{i\bar{n}\cdot D_c} i\not{D}_c^\perp\right\}\frac{\bar{n}}{2}\xi_{n,p}$$

Introduce usoft Wilson line: $Y_n(x) = \mathrm{P}\exp\left[\int_{-\infty}^{x} ds\; n\cdot A_{us}(sn)\right]$

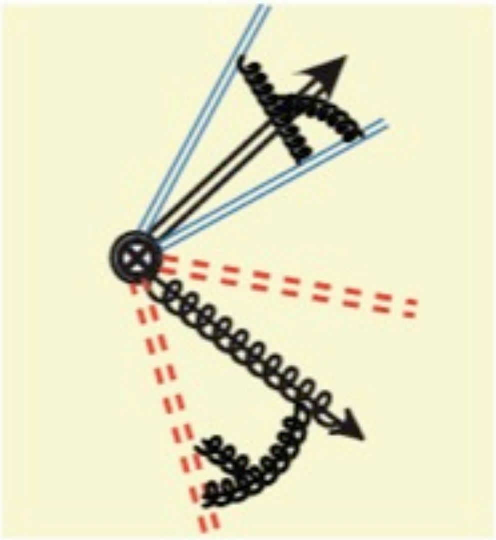
Field redef: $\xi_{n,p} = Y_n \xi_{n,p}^{(0)}, \quad A_{n,p}^\mu = Y_n A_{n,p}^{(0)\mu} Y_n^\dagger, \quad W_n = Y_n W_n^{(0)} Y_n^\dagger$

Ultrasoft decouples: $\mathcal{L} \to \bar{\xi}_{n,p'}\left\{in\cdot D_c + 0 + i\not{D}_c^\perp \frac{1}{i\bar{n}\cdot D_c} i\not{D}_c^\perp\right\}\frac{\bar{n}}{2}\xi_{n,p}$

In Pictures



$$\chi^\dagger W D_\perp W^\dagger \psi \to \chi^\dagger Y W D_\perp W^\dagger Y^\dagger \psi$$



Heavy/soft modes do not interact with collinear modes
⇒ Rate factors!

Quarkonium Factorization in SCET

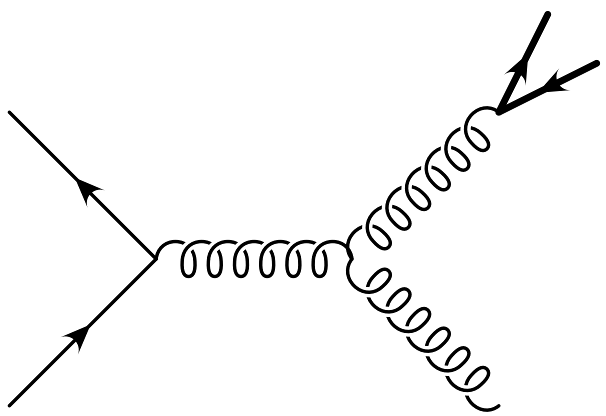
Quarkonium production at large $p_\perp$ in hadron collisions

$$p_\perp \sim \sqrt{\hat{s}} \gg m_Q$$

1. At $\mu \sim p_\perp$ match QCD onto massive SCET

   Expansion in $\alpha_s(p_\perp)$

   Power counting in $\lambda \sim (2m_Q)/p_\perp$



2. Factor rate
3. Run to $\mu \sim m_Q$
4. Match onto NRQCD

   Expansion in $\alpha_s(2m_Q)$

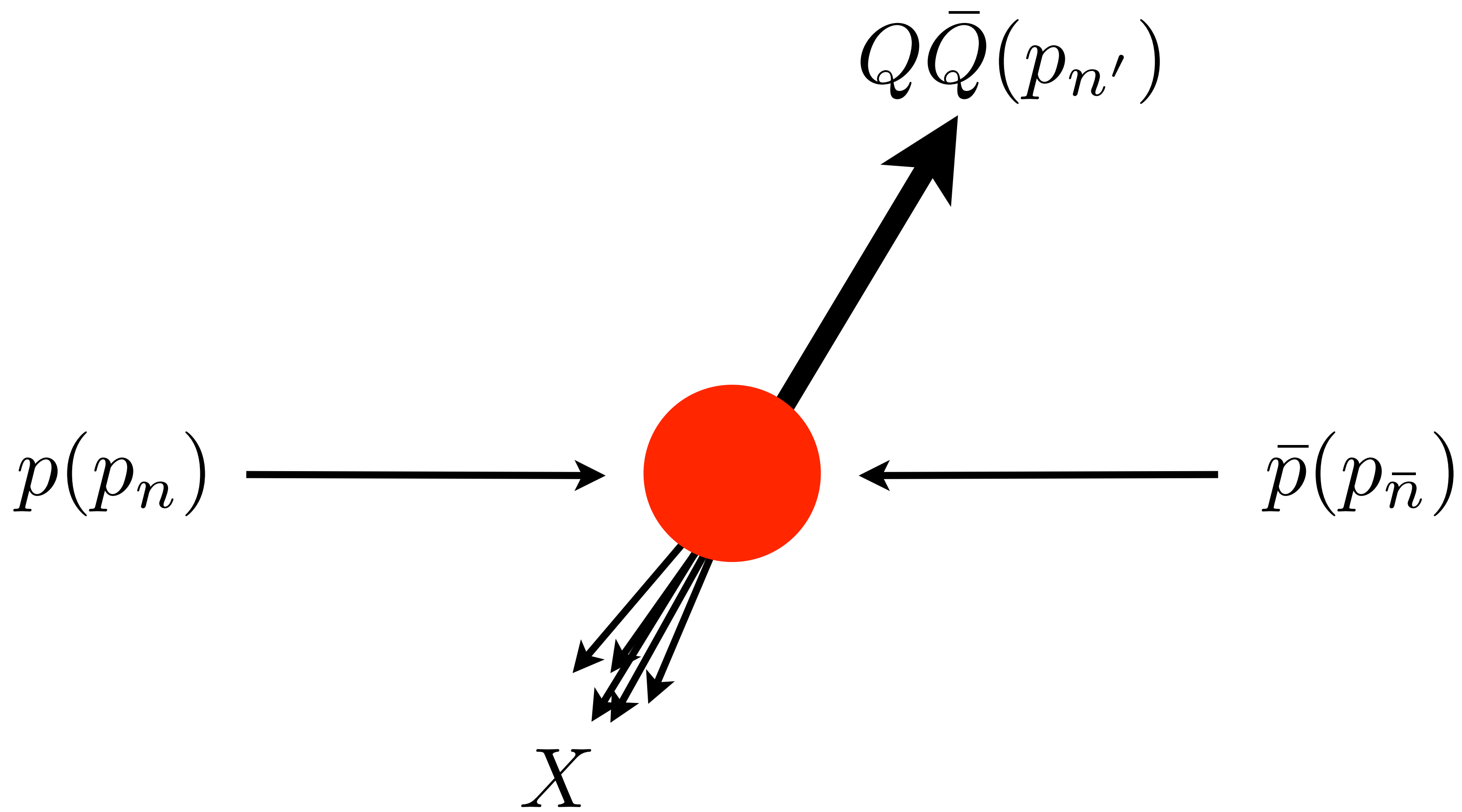
   Power counting in v

Quarkonium Factorization in SCET

We have multiple directions

$$n^\mu = (1,0,0,1), \bar{n}^\mu = (1,0,0,-1), n'^\mu = (\cosh y, 1, 0, \sinh y)$$

$$p_n \approx \frac{Q}{2} n^\mu \qquad p_{\bar{n}} \approx \frac{Q}{2} \bar{n}^\mu \qquad p_{n'} \approx \frac{p_\perp}{2} n'^\mu$$



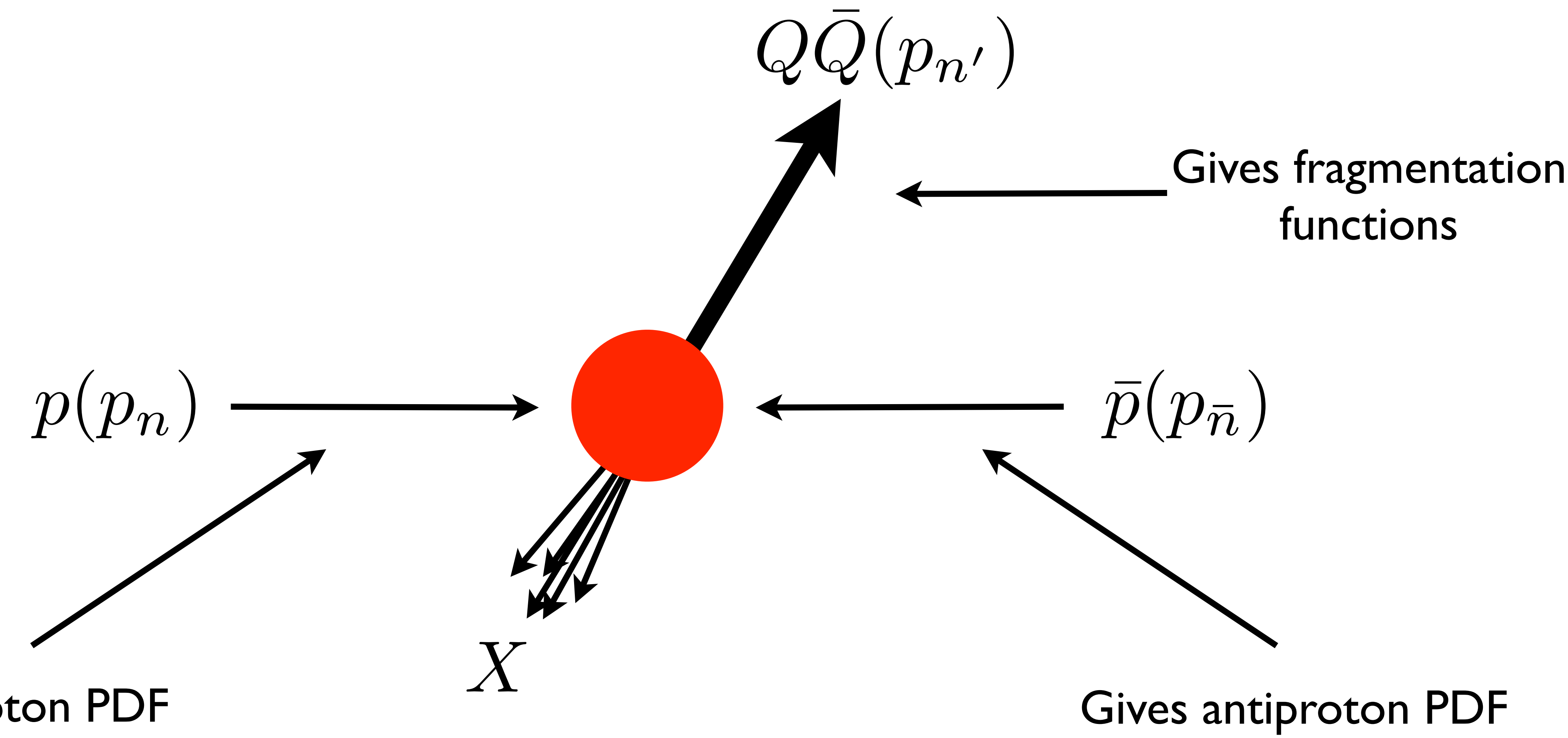
Integrate out X

Quarkonium Factorization in SCET

We have multiple directions

$$n^\mu = (1,0,0,1), \bar{n}^\mu = (1,0,0,-1), n'^\mu = (\cosh y, 1, 0, \sinh y)$$

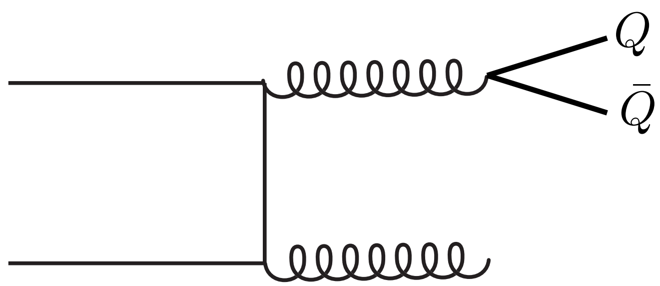
$$p_n \approx \frac{Q}{2} n^\mu \qquad p_{\bar{n}} \approx \frac{Q}{2} \bar{n}^\mu \qquad p_{n'} \approx \frac{p_\perp}{2} n'^\mu$$

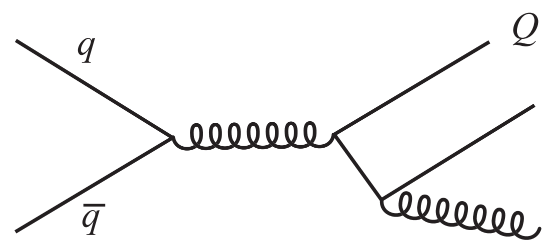


Integrate out X

Matching

Example: $q\bar{q} \to Q\bar{Q}$

$\Big|$ 

$+ \cdots +$ 

$+ \cdots + \Big|^2$

$O(1)$ $\qquad$ $O(\lambda^2)$ $\qquad$ $\lambda \sim m_Q/p_\perp$

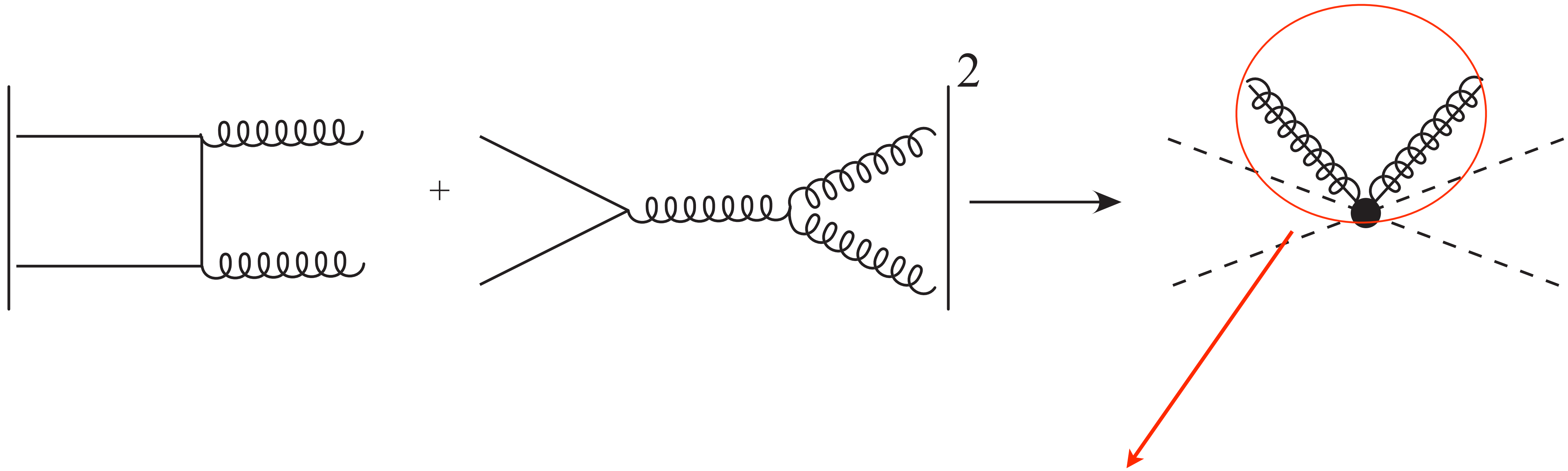
Give different fragmentation functions

$$\frac{d^2\sigma}{dp_\perp^2\, dy} \propto \int dx_1 dx_2 f_{q/p}(x_1) f_{\bar{q}/\bar{p}}(x_2) \left[\hat{\sigma}^\beta(x_1, x_2, p_\perp, y) \otimes D_{Q\bar{Q}/\beta}\right]$$

$$D_{Q\bar{Q}/g}(z) \qquad\qquad D_{Q\bar{Q}/Q\bar{Q}}(u, v, z)$$

Matching

Example: $q\bar{q} \to g$ at $O(1)$



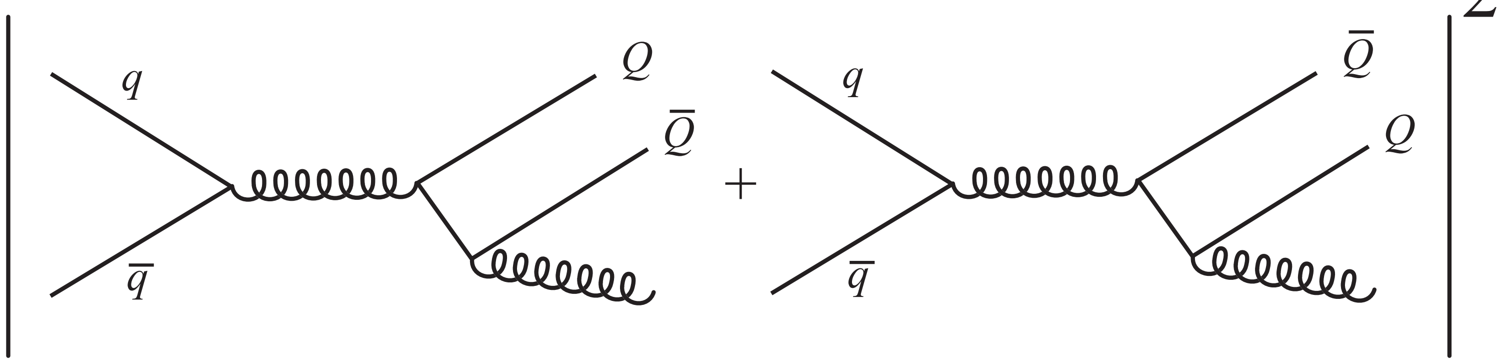
$$\langle 0|\mathrm{Tr}\Big\{(B^{a\nu}_{n',\omega'_1})\mathcal{P}^\dagger_H(p_\perp,y)\mathcal{P}_H(p_\perp,y)(B^{a\rho}_{n',\omega'_2})\Big\}|0\rangle$$

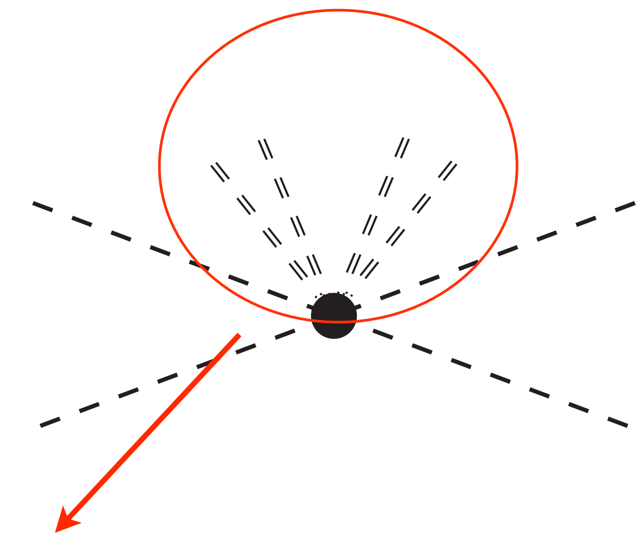
$$= -\frac{\omega'^2_+}{2}\int_0^1 \frac{dz}{z}\delta(\omega'_-)\delta(\omega'_+ - \frac{2\bar{n}'\cdot p}{z})D_{H/g}(z)$$

Matching

Example: $q\bar{q} \longrightarrow Q\bar{Q}$ at $O(\lambda^2)$

(DPFF)

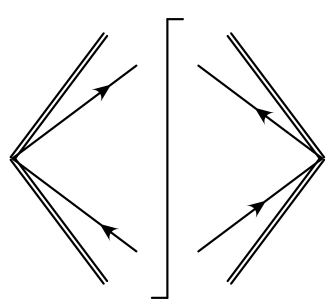




$$\langle 0|\bar{\chi}_{n',\omega_2'}\Gamma^{i(\mu)}\{1,T^A\}\chi_{n',\omega_1'}\mathcal{P}^H_{n',Q}\bar{\chi}_{n',\omega_4'}\Gamma^i_{(\mu)}\{1,T^A\}\chi_{n',\omega_3'}|0\rangle$$

$$= \delta(\omega_1'-\omega_2'+\omega_3'-\omega_4')\int\frac{dz}{z}\,du\,dv\,\delta\Big(z-\frac{\bar{n}'\cdot p}{\omega_1'-\omega_2'}\Big)$$

$$\times\delta\Big(v-1-z\frac{\omega_2'}{\bar{n}'\cdot p}\Big)\delta\Big(u-z\frac{\omega_4'}{\bar{n}'\cdot p}\Big)D^{Q\bar{Q}}_{i\{1,8\}}(u,v,z)$$

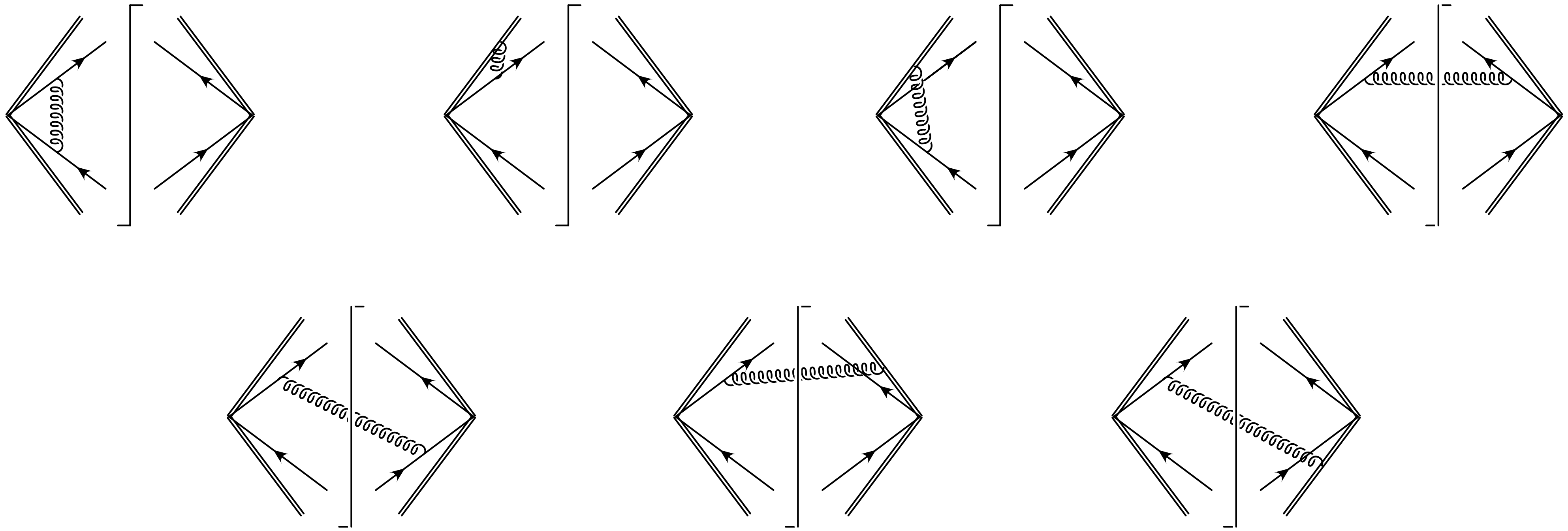


$$\Gamma^{i(\mu)} = \frac{1}{2}\,\{\bar{n}\!\!\!/',\bar{n}\!\!\!/'\gamma^5,\bar{n}\!\!\!/'\gamma^\mu_\perp\}$$

Note: Usoft Wilson lines cancel

$\bar{n}'\cdot p_1 = u\,\bar{n}'\cdot p_{Q\bar{Q}}$ $\qquad$ $\bar{n}'\cdot p_4 = v\,\bar{n}'\cdot p_{Q\bar{Q}}$ $\qquad$ $\bar{n}'\cdot p_H = z\bar{n}'\cdot p_{Q\bar{Q}}$

Running



Essentially Efremov-Radyushkin-Brodsky-Lepage evolution in u, v

$$\mu^2 \frac{d}{d\mu^2} D^{[1]}_{Q\bar{Q}\to H}(u, v, z; \mu) = \int_0^1 dw V(u, w; \mu) D^{[1]}_{Q\bar{Q}\to H}(w, v, z; \mu)$$

DGLAP in z

$$\mu^2 \frac{d}{d\mu^2} D^{[1]}_{Q\bar{Q}\to H}(u, v, z; \mu) = \int_0^1 dx P_{Q\bar{Q}[1]\to Q\bar{Q}[8]}(x; \mu) D^{[8]}_{Q\bar{Q}\to H}(w, v, z/x; \mu)$$

$$\bar{n}'\cdot p_1 = u\,\bar{n}'\cdot p_{Q\bar{Q}} \qquad \bar{n}'\cdot p_4 = v\,\bar{n}'\cdot p_{Q\bar{Q}} \qquad \bar{n}'\cdot p_H = z\bar{n}'\cdot p_{Q\bar{Q}}$$

Matching onto NRQCD

At scale $\sim 2m_Q$ integrate out heavy quark mass

- Use boosted NRQCD

Decompose momentum in quarkonium frame

$$p^\mu = m_Q v^\mu + k^\mu$$

$$v^\mu = \frac{1}{2}\frac{Q}{2m_Q}n'^\mu + \frac{1}{2}\frac{m_Q}{Q}\bar{n}'^\mu \qquad Q = 2p_\perp \cosh y$$

SCET scaling

$$p^\mu = \tilde{p}^\mu + r^\mu \Longrightarrow \bar{n}' \cdot \tilde{p} \rightarrow m_Q \bar{n}' \cdot v$$

$$r^\mu = \tilde{r}^\mu + r_s^\mu = \tilde{r}^\mu + k^\mu$$

Matching onto NRQCD

At scale $\sim 2m_Q$ integrate out heavy quark mass

- Use boosted NRQCD
- NRQCD inherits Wilson lines (cancels for color-singlet)

$$\bar{\chi}_{n',\omega_2}\Gamma^a\{1,T^A\}\chi_{n',\omega_1} = \mathcal{C}_a\delta(m_Q\bar{n}'\cdot v-\omega_1)\delta(m_Q\bar{n}'\cdot v-\omega_2)\chi_v^\dagger W_v\tilde{\Gamma}^a\{1,T^A\}W_v^\dagger\psi_v + \mathrm{h.c.}$$

Matching onto NRQCD

At scale $\sim 2m_Q$ integrate out heavy quark mass

- Use boosted NRQCD
- NRQCD inherits Wilson lines (cancels for color-singlet)

$$\bar{\chi}_{n',\omega_2}\Gamma^a\{1,T^A\}\chi_{n',\omega_1} = \mathcal{C}_a\delta(m_Q\bar{n}'\cdot v-\omega_1)\delta(m_Q\bar{n}'\cdot v-\omega_2)\chi_v^\dagger W_v\tilde{\Gamma}^a\{1,T^A\}W_v^\dagger\psi_v + \mathrm{h.c.}$$

NRQCD fields

Matching onto NRQCD

At scale $\sim 2m_Q$ integrate out heavy quark mass

- Use boosted NRQCD
- NRQCD inherits Wilson lines (cancels for color-singlet)

$$\bar{\chi}_{n',\omega_2}\Gamma^a\{1,T^A\}\chi_{n',\omega_1} = \mathcal{C}_a\delta(m_Q\bar{n}'\cdot v-\omega_1)\delta(m_Q\bar{n}'\cdot v-\omega_2)\chi_v^\dagger W_v\tilde{\Gamma}^a\{1,T^A\}W_v^\dagger\psi_v + \mathrm{h.c.}$$

Wilson lines with
NRQCD gluons

Matching onto NRQCD

At scale $\sim 2m_Q$ integrate out heavy quark mass

- Use boosted NRQCD
- NRQCD inherits Wilson lines (cancels for color-singlet)

$$D^{Q\bar{Q}}_{i,\{1,8\}}(u,v,z) \to C_i \delta(1-z)\delta\left(u-\frac{1}{2}\right)\delta\left(v-\frac{1}{2}\right)$$
$$\times \langle 0|\chi_v^\dagger W \Gamma^i \{1,T^A\} W^\dagger \psi_v \mathcal{P}^{H_v} \psi_v^\dagger W \Gamma^i \{1,T^A\} W^\dagger \chi_v |0\rangle$$

- If ignore running, recover NRQCD results

Conclusions

- Can prove factorization using SCET
- Power suppressed fragmentation functions
- Interesting running
- Maybe important phenomenologically

謝謝

Backup slides

Anomalous Dimensions

$$\gamma_{11,11} = -\frac{\alpha_s C_F}{\pi}\delta(1-z/z')\Bigg(3\delta(u-u')\delta(v-v')$$
$$+\delta(v-v')\left\{\theta(u'-u)\frac{u}{u'}\left[\frac{1}{(u'-u)_+}+1\right]+\theta(u-u')\frac{\bar{u}}{\bar{u}'}\left[\frac{1}{(u-u')_+}+1\right]\right\}$$
$$+\delta(u-u')\left\{\theta(v'-v)\frac{v}{v'}\left[\frac{1}{(v'-v)_+}+1\right]+\theta(v-v')\frac{\bar{v}}{\bar{v}'}\left[\frac{1}{(v-v')_+}+1\right]\right\}\Bigg),$$

$$\gamma_{21,21} = \gamma_{11,11}\,,$$

$$\gamma_{18,11} = -\frac{\alpha_s}{\pi}\theta(1-z/z')\left(\frac{z}{z'}\right)^2\Bigg\{\left[\frac{uv'+vu'}{(1-z/z')_+}+\frac{1-z/z'}{2z/z'}\right]\frac{1}{u'v'}\delta(\bar{v}-\frac{z}{z'}\bar{v}')\delta(\bar{u}-\frac{z}{z'}\bar{u}')$$
$$+\left[\frac{\bar{u}\bar{v}'+\bar{v}\bar{u}'}{(1-z/z')_+}+\frac{1-z/z'}{2z/z'}\right]\frac{1}{\bar{u}'\bar{v}'}\delta(v-\frac{z}{z'}v')\delta(u-\frac{z}{z'}u')$$
$$-\left[\frac{u\bar{v}'+\bar{v}u'}{(1-z/z')_+}+\frac{1-z/z'}{2z/z'}\right]\frac{1}{u'\bar{v}'}\delta(v-\frac{z}{z'}v')\delta(\bar{u}-\frac{z}{z'}\bar{u}')$$
$$-\left[\frac{\bar{u}v'+v\bar{u}'}{(1-z/z')_+}+\frac{1-z/z'}{2z/z'}\right]\frac{1}{\bar{u}'v'}\delta(\bar{v}-\frac{z}{z'}\bar{v}')\delta(u-\frac{z}{z'}u')\Bigg\},$$

$$\gamma_{18,21} = -\frac{\alpha_s}{\pi}\theta(1-z/z')\left(\frac{z}{z'}\right)\frac{1-z/z'}{2}\Bigg[\frac{1}{u'v'}\delta(\bar{v}-\frac{z}{z'}\bar{v}')\delta(\bar{u}-\frac{z}{z'}\bar{u}')$$
$$+\frac{1}{\bar{u}'\bar{v}'}\delta(v-\frac{z}{z'}v')\delta(u-\frac{z}{z'}u')+\frac{1}{u'\bar{v}'}\delta(v-\frac{z}{z'}v')\delta(\bar{u}-\frac{z}{z'}\bar{u}')$$
$$+\frac{1}{\bar{u}'v'}\delta(\bar{v}-\frac{z}{z'}\bar{v}')\delta(u-\frac{z}{z'}u')\Bigg],$$

$$\gamma_{28,11} = \gamma_{18,21}, \qquad \gamma_{28,18} = \gamma_{18,28}$$
$$\gamma_{28,21} = \gamma_{18,11}. \qquad \gamma_{28,28} = \gamma_{18,18}.$$

Anomalous Dimensions

$$
\begin{aligned}
\gamma_{18,18} =\; & -3\frac{\alpha_s C_F}{\pi}\delta(u-u')\delta(v-v')\delta(1-z/z') \\
& +\frac{\alpha}{\pi}\frac{1}{2N_c}\delta(v-v')\delta(1-\frac{z}{z'})\left[\theta(u'-u)\frac{u}{u'}\left(\frac{1}{(u'-u)_+}+1\right)+(u\leftrightarrow\bar{u},u'\leftrightarrow\bar{u}')\right] \\
& +\frac{\alpha_s}{\pi}\frac{1}{2N_c}\delta(u-u')\delta(1-\frac{z}{z'})\left[\theta(v'-v)\frac{v}{v'}\left(\frac{1}{(v'-v)_+}+1\right)+(v\leftrightarrow\bar{v},v'\leftrightarrow\bar{v}')\right] \\
& -\frac{\alpha_s}{\pi}\left(\frac{z}{z'}\right)^2\left\{\frac{N_c^2-2}{2N_c}\left[\frac{uv'+vu'}{(1-z/z')_+}+\frac{1-z/z'}{2z/z'}\right]\frac{1}{u'v'}\delta(\bar{v}-\frac{z}{z'}\bar{v}')\delta(\bar{u}-\frac{z}{z'}\bar{u}')\right. \\
& \qquad +\frac{N_c^2-2}{2N_c}\left[\frac{\bar{u}\bar{v}'+\bar{v}\bar{u}'}{(1-z/z')_+}+\frac{1-z/z'}{2z/z'}\right]\frac{1}{\bar{u}'\bar{v}'}\delta(v-\frac{z}{z'}v')\delta(u-\frac{z}{z'}u') \\
& \qquad +\frac{1}{N_c}\left[\frac{u\bar{v}'+\bar{v}u'}{(1-z/z')_+}+\frac{1-z/z'}{2z/z'}\right]\frac{1}{u'\bar{v}'}\delta(v-\frac{z}{z'}v')\delta(\bar{u}-\frac{z}{z'}\bar{u}') \\
& \qquad \left.+\frac{1}{N_c}\left[\frac{\bar{u}v'+v\bar{u}'}{(1-z/z')_+}+\frac{1-z/z'}{2z/z'}\right]\frac{1}{\bar{u}'v'}\delta(\bar{v}-\frac{z}{z'}\bar{v}')\delta(u-\frac{z}{z'}u')\right\}\theta(1-z/z'), \\
\gamma_{18,28} =\; & -\frac{\alpha_s}{\pi}\left(\frac{z}{z'}\right)\frac{1-z/z'}{2}\left[\frac{N_c^2-2}{2N_c}\frac{1}{u'v'}\delta(\bar{v}-\frac{z}{z'}\bar{v}')\delta(\bar{u}-\frac{z}{z'}\bar{u}')\right. \\
& \qquad +\frac{N_c^2-2}{2N_c}\frac{1}{\bar{u}'\bar{v}'}\delta(v-\frac{z}{z'}v')\delta(u-\frac{z}{z'}u') \\
& \qquad -\frac{1}{N_c}\frac{1}{u'\bar{v}'}\delta(v-\frac{z}{z'}v')\delta(\bar{u}-\frac{z}{z'}\bar{u}') \\
& \qquad \left.-\frac{1}{N_c}\frac{1}{\bar{u}'v'}\delta(\bar{v}-\frac{z}{z'}\bar{v}')\delta(u-\frac{z}{z'}u')\right]\theta(1-z/z'),
\end{aligned}
$$

Anomalous Dimensions

$$\begin{aligned}
\gamma_{31,31} &= -3\frac{\alpha_s C_F}{\pi}\delta(u-u')\delta(v-v')\delta(1-z/z') \\
&\quad -\frac{\alpha_s C_F}{\pi}\delta(v-v')\delta(1-\frac{z}{z'})\left[\theta(u'-u)\frac{u}{u'}\frac{1}{(u'-u)_+}+\theta(u-u')\frac{\bar{u}}{\bar{u}'}\frac{1}{(u-u')_+}\right] \\
&\quad -\frac{\alpha_s C_F}{\pi}\delta(u-u')\delta(1-\frac{z}{z'})\left[\theta(v'-v)\frac{v}{v'}\frac{1}{(v'-v)_+}+\theta(v-v')\frac{\bar{v}}{\bar{v}'}\frac{1}{(v-v')_+}\right],
\end{aligned}$$

$$\begin{aligned}
\gamma_{38,31} &= -\frac{\alpha_s}{\pi}\theta(1-z/z')\left(\frac{z}{z'}\right)^2\left\{\left[\frac{uv'+vu'}{(1-z/z')_+}+\frac{1-z/z'}{z/z'}\right]\frac{1}{u'v'}\delta(\bar{v}-\frac{z}{z'}\bar{v}')\delta(\bar{u}-\frac{z}{z'}\bar{u}')\right. \\
&\quad +\left[\frac{\bar{u}\bar{v}'+\bar{v}\bar{u}'}{(1-z/z')_+}+\frac{1-z/z'}{z/z'}\right]\frac{1}{\bar{u}'\bar{v}'}\delta(v-\frac{z}{z'}v')\delta(u-\frac{z}{z'}u') \\
&\quad -\left[\frac{u\bar{v}'+\bar{v}u'}{(1-z/z')_+}+\frac{1-z/z'}{z/z'}\right]\frac{1}{u'\bar{v}'}\delta(v-\frac{z}{z'}v')\delta(\bar{u}-\frac{z}{z'}\bar{u}') \\
&\quad \left.-\left[\frac{\bar{u}v'+v\bar{u}'}{(1-z/z')_+}+\frac{1-z/z'}{z/z'}\right]\frac{1}{\bar{u}'v'}\delta(\bar{v}-\frac{z}{z'}\bar{v}')\delta(u-\frac{z}{z'}u')\right\}.
\end{aligned}$$

$$\gamma_{i1,j8} = \frac{C_F}{2N_c}\gamma_{i8,j1}$$

Anomalous Dimensions

$$
\begin{aligned}
\gamma_{38,38} =\ & -3\frac{\alpha_s C_F}{\pi}\delta(u-u')\delta(v-v')\delta(1-z/z') \\
& +\frac{\alpha}{\pi}\frac{1}{2N_c}\delta(v-v')\delta(1-\frac{z}{z'})\left[\theta(u'-u)\frac{u}{u'}\frac{1}{(u'-u)_+}+\theta(u-u')\frac{\bar{u}}{\bar{u}'}\frac{1}{(u-u')_+}\right] \\
& +\frac{\alpha_s}{\pi}\frac{1}{2N_c}\delta(u-u')\delta(1-\frac{z}{z'})\left[\theta(v'-v)\frac{v}{v'}\frac{1}{(v'-v)_+}+\theta(v-v')\frac{\bar{v}}{\bar{v}'}\frac{1}{(v-v')_+}\right] \\
& -\frac{\alpha_s}{\pi}\theta(1-z/z')\left(\frac{z}{z'}\right)^2\left\{\frac{N_c^2-2}{2N_c}\left[\frac{uv'+vu'}{(1-z/z')_+}+\frac{1-z/z'}{z/z'}\right]\frac{1}{u'v'}\delta(\bar{v}-\frac{z}{z'}\bar{v}')\delta(\bar{u}-\frac{z}{z'}\bar{u}')\right. \\
& \qquad +\frac{N_c^2-2}{2N_c}\left[\frac{\bar{u}\bar{v}'+\bar{v}\bar{u}'}{(1-z/z')_+}+\frac{1-z/z'}{z/z'}\right]\frac{1}{\bar{u}'\bar{v}'}\delta(v-\frac{z}{z'}v')\delta(u-\frac{z}{z'}u') \\
& \qquad +\frac{1}{N_c}\left[\frac{u\bar{v}'+\bar{v}u'}{(1-z/z')_+}+\frac{1-z/z'}{z/z'}\right]\frac{1}{u'\bar{v}'}\delta(v-\frac{z}{z'}\xi)\delta(\bar{u}-\frac{z}{z'}\bar{u}') \\
& \qquad \left.+\frac{1}{N_c}\left[\frac{\bar{u}v'+v\bar{u}'}{(1-z/z')_+}+\frac{1-z/z'}{z/z'}\right]\frac{1}{\bar{u}'v'}\delta(\bar{v}-\frac{z}{z'}\bar{v}')\delta(u-\frac{z}{z'}u')\right\}.
\end{aligned}
$$