The mass of the charm quark from unquenched lattice QCD

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Hadron Physics from Lattice QCD

Introduction

- Why compute the masses of the quarks?
- Ratio of masses of the charm and strange quarks.
- Using $m_s(2GeV)$ and m_c/m_s to compute the mass of the charm quark.
- Some basic validation checks with some mass splittings $M_{J/\psi}$ - M_{η_c} and $M_{D_s^\star}$ - M_{D_s}.

This is a joint project between Wuppertal and Regensburg and their collaborations (BMW-c and QCDSF). All the results are preliminary, but there should be a paper soon.

Introduction to quark masses

- The confinement of quarks makes it difficult to experimentally measure the mass of the quarks.
- The mass of the top quark can be measured, because it decays before bound states can form.
- In lattice QCD the quark mass is a parameter in the computer code and there is well defined procedure to renormalise them.
- Here I will talk about current quark masses rather than constituent quark masses. Also Brian Webber tutorial on Herwig on web ($m_d = m_u = 0.32$ GeV and $m_{strange} = 0.5$ GeV.)

My impression is that the heavier quarks: m_c, m_b, and m_t are used more in applications than the light quarks.

Brief introduction to lattice QCD

$$c_{ij}(t) = \frac{1}{Z} \int du \int d\psi \overline{d\psi} O(t)_i O(0)_j^\dagger e^{-S_F - S_G}$$

- The path integral is regulated by the introduction of a space-time lattice. Lattice calculations are now being done at several lattice spacings and volumes.
- The multi-dimensional integral is computed in Euclidean space using Monte Carlo techniques on the computer.
- The dynamics of the gluon and quarks is in the Dirac action (S_F) and gauge action (S_G).

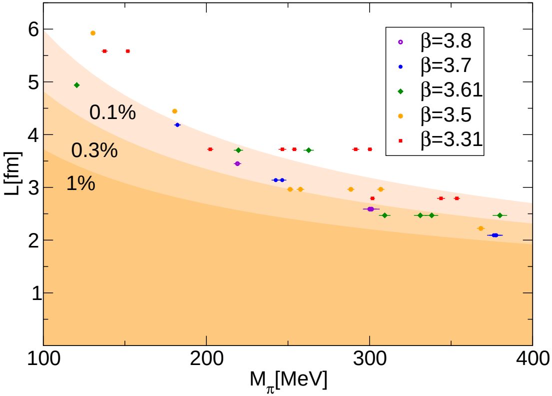
$$c_{ij}(t) \sim a_{ij} e^{-m_a t} + b_{ij} e^{-m_b t} + \ldots$$

Key issue

How physical are the parameters (quark mass, lattice spacing..) ?

Details of lattice calculations

We use the 2 HEX smeared clover action with the tree level Symanzik improved gauge action. 2+1 flavours of sea quarks. Ensembles with 5 different lattice spacings available. A subset of the ensembles below was used.



Lattice QCD and charm physics

- Take the light quark masses and lattice spacing from light quark spectroscopy (m_π, m_K, m_Ω).
- Only valence charm quarks included.
- Charm quark mass from both the η_c mass and the D_s meson, as a systematic check.

Additional complications with lattice QCD calculations that include the charm quark.

- "Main problem" is the mass of the charm quark is $m_c \sim 1.2$ GeV. In lattice units (am_c) is: 0.82, 0.62, 0.47, 0.37, 0.29 for $a^{-1} = 1.7$, 2.1, 2.6, 3.0, 3.7 GeV respectively.
- The leading lattice spacing errors are: $O((am_c)^2)$ or $O(\alpha_s am_c)$ for 2 HEX clover action.
- Which lattice spacings can be included?

From m_c/m_s to $m_c(3GeV)$

- The calculations are done with 3 heavy quarks close to the charm mass.
- An estimate of the mass of the bare charm quark is made by interpolating (or small extrapolation) to get the mass of heavy hadron correct.
- Use the quark mass defined from the PCAC relation.
- The quark mass needs to be renormalized. Study the ratio of the strange to charm quark masses.
- BMW-c has previously computed the mass of the strange quark and renormalized it using the numerical Rome-Southampton method.
- It was too complicated to copy the renormalization method for the strange quark, so $m_c(\mu)$ is extracted from m_c/m_s and $m_s(\mu)$.

Taking the various limits

$$y^{\mathrm{FIT}}(a, m_\pi, m_{\eta_s}, L|\mathbf{A}, \mathbf{B}) = y^{cont} f_X(a) g_Y(a, L, m_\pi)\left((1 + A_2 x_\pi)(1 + A_3 x_s)\right)$$

where y is the physical quantity and $m^2_{\eta_s}$ is the strange-strange pseudo-scalar meson.

$$x_\pi = \frac{(m_\pi^{latt})^2 - (m_\pi^{phys})^2}{(m_\pi^{phys})^2}$$

$$x_s = \frac{(m_{\eta_s}^{latt})^2 - (m_{\eta_s}^{phys})^2}{(m_{\eta_s}^{phys})^2}$$

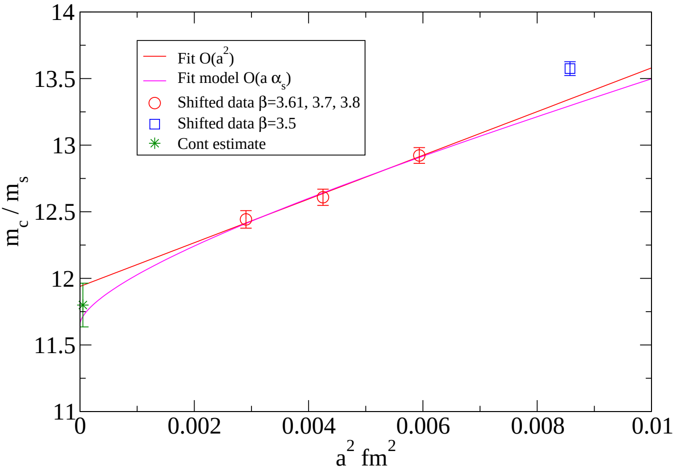
Fit functions for the continuum extrapolation

$$f_1(a) = (1 + A_1 a^2)$$

$$f_2(a) = (1 + A_{1a} a \alpha_s)$$

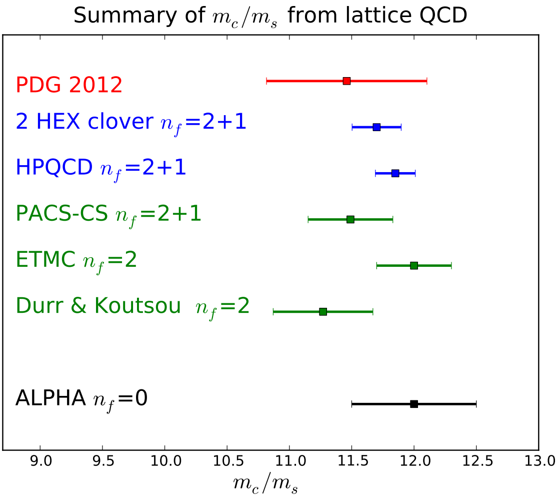
Ratio of masses of the charm and strange quarks

The $\beta = 3.5$ ensemble is cut from the analysis.

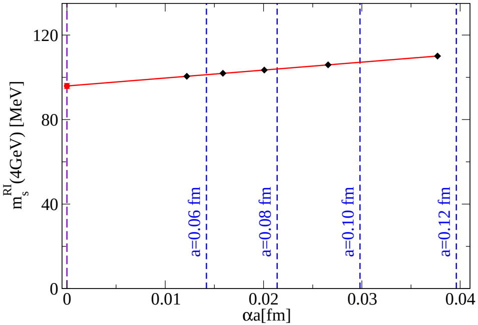


Ratio of masses of the charm and strange quarks

2 HEX Clover number is preliminary.



The mass of the strange quark.

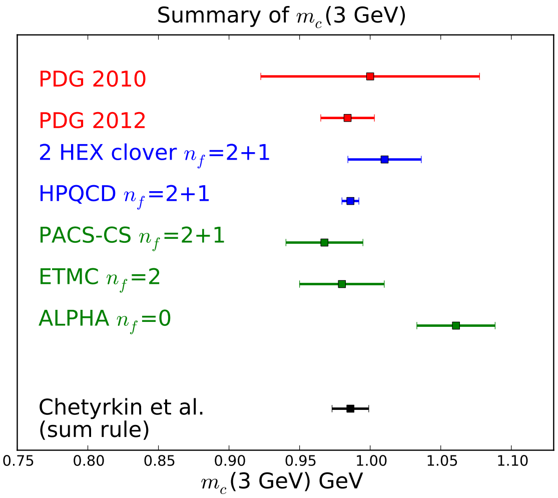


Absolute value of charm mass obtained from m_c/m_s and the mass of the strange quark determined with non-perturbative renormalization (BMW-c arXiv:1011.2711).

$$m_s(2\ \text{GeV}) = 95.5(1.1)(1.5)\text{MeV}$$

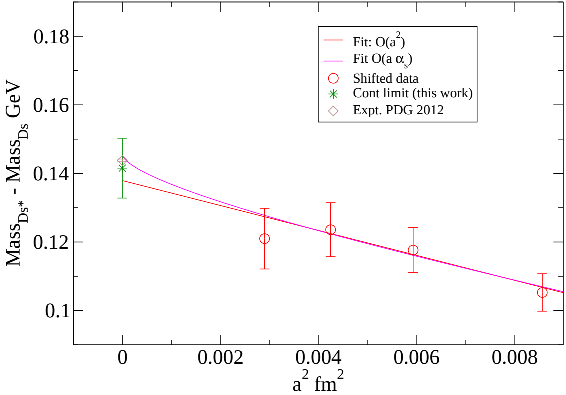
Summary of mass of charm quark

2 HEX Clover number is preliminary.



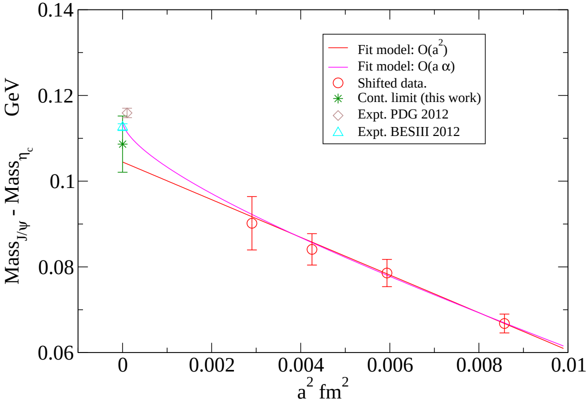
Mass splitting between $D_s^\star$ and D_s mesons.

m_c from η_c. 4 ensembles fitted.

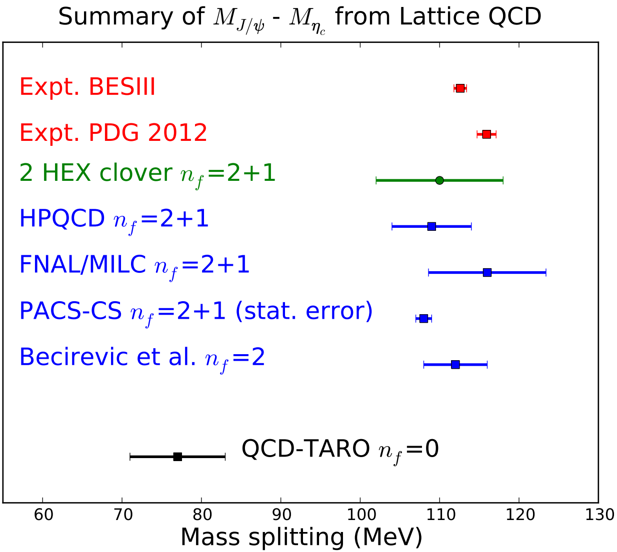


Mass splitting between J/ψ and η_c mesons.

m_c from η_c. 4 ensembles fitted.

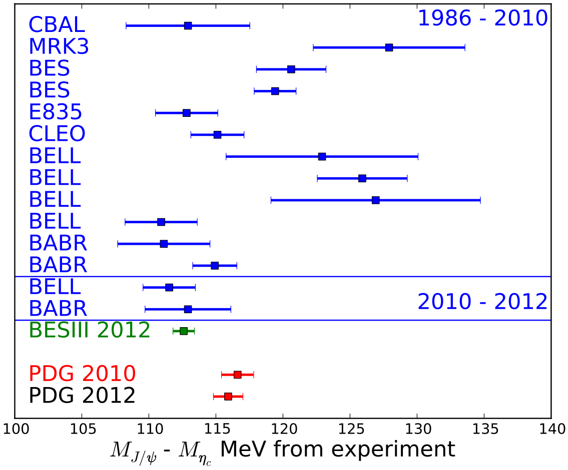


Summary mass splitting between J/ψ and η_c mesons.

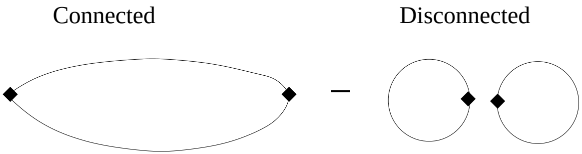


Comment on experimental $M_{J/\psi}$ - M_{η_c}

Take the numbers for mass of η_c from PDG and compute hyperfine splitting (statistical and systematic errors added in quadrature).



Comments on charmonium hyperfine splitting

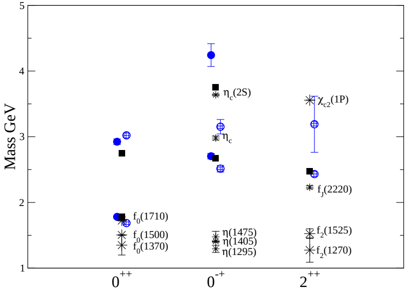


- η_c has a big width (28.6 ± 2.2 MeV) relative to hyperfine splitting
- Disconnected diagrams were not included in this calculation – these are difficult (but not impossible) to estimate.
- Levkova and DeTar (arXiv:1012.1837) compute the disconnected diagrams and estimate the effect of them to be upto $+4$ MeV.
- The HPQCD collaboration (hep-lat/0610092) estimate -2.4 MeV from perturbation theory.

Estimate missing disconnected diagrams ± 2.4 MeV (1208.2855).

Glueball versus expt, 1005.2473, 1208.1858 Richards et al. (plot in quark model review in PDG)

- Unquenching effects seem small – but still need continuum limit
- Solid black are quenched, blue unquenched (solid a=0.09fm, open a=0.12 fm) $n_f = 2 + 1$.
- Disconnected diagrams important for possible $\overline{c}\Gamma c$ heavy glueball mixing.



Conclusions

- I presented results for the m_c/m_s mass ratio in the continuum limit.
- The mass of the charm quark was extracted from m_c/m_s and the mass of the strange quark previously computed.
- The clover 2 HEX action needs to be further improved to reduce the errors.
- I also presented results for the mass splittings: $M_{J/\psi} - M_{\eta_c}$ and $M_{D_s^\star} - M_{D_s}$.