Factorization in Inclusive Quarkonium Production

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NRQCD Factorization Conjecture

The Conjecture

(GTB, Braaten, Lepage (1995))

- The inclusive cross section for producing a quarkonium at large momentum transfer (p_T) can be written as a sum of "short-distance" coefficients times NRQCD long-distance matrix elements (LDMEs).

$$\sigma(H) = \sum_n F_n(\Lambda)\langle 0|\mathcal{O}_n^H(\Lambda)|0\rangle .$$

- NRQCD factorization separates the perturbative physics at high-momentum scales (m and p_T) from the low-momentum, nonperturbative effects in the heavy-quarkonium bound state.
- The "short-distance" coefficients $F_n(\Lambda)$ are essentially the process-dependent partonic cross sections to make a $Q\bar{Q}$ pair convolved with the parton distributions.
  - They have an expansion in powers of α_s.

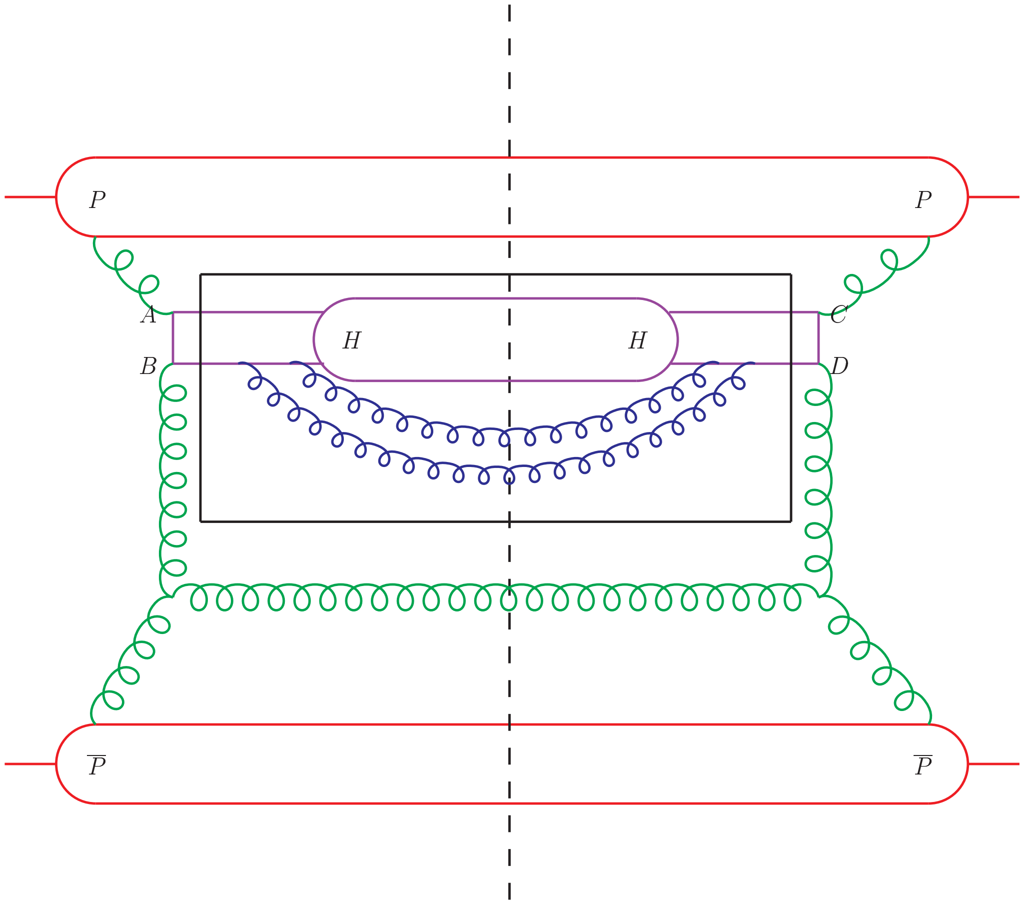
- The LDMEs are the probability for a $Q\bar{Q}$ pair in a particular color and angular-momentum state to evolve into a heavy quarkonium:

$$\mathcal{O}_n^H(\Lambda) = \langle 0|\chi^\dagger \kappa_n \psi \Big(\sum_X |H+X\rangle\langle H+X| \Big) \psi^\dagger \kappa'_n \chi|0\rangle.$$

- An LDME contains a four-fermion operator, but with a projection onto an intermediate state of the quarkonium H plus anything.
  - κ_n and κ'_n are combinations of Pauli and Color matrices.
- The LDMEs are supposed to be universal (process independent).
  - This is what gives NRQCD factorization its predictive power.
- The LDMEs have a known scaling with v.
  $v^2 \approx 0.23$ for the J/ψ. $v^2 \approx 0.1$ for the Υ.
- The current phenomenology of J/ψ, $\psi(2S)$, and Υ production uses LDMEs through relative order v^4:

$$\begin{aligned}
&\langle \mathcal{O}^H({}^3S_1^{[1]})\rangle \quad (O(v^0)),\\
&\langle \mathcal{O}^H({}^1S_0^{[8]})\rangle \quad (O(v^3)),\\
&\langle \mathcal{O}^H({}^3S_1^{[8]})\rangle \quad (O(v^4)),\\
&\langle \mathcal{O}^H({}^3P_J^{[8]})\rangle \quad (O(v^4)).
\end{aligned}$$

Diagrammatic Meaning of NRQCD Factorization



- The points $A(C)$ and $B(D)$ are within $\sim 1/m_Q$ of each other.
  - Kinematics implies that the virtual Q is off shell by order m.
- The points $A(B)$ and $C(D)$ are within $1/p_T$ of each other.
  - The part of the diagram outside the box is insensitive to changes of momentum flow from $A(B)$ to $C(D)$ of order p_T.

- The part of the diagram inside the box corresponds to an LDME.
- The remainder of the diagram is the "short-distance" coefficient.

Modification of the LDMEs

Nayak, Qiu, Sterman (2005, 2006): The color-octet NRQCD matrix elements must be modified by the inclusion of Wilson (eikonal) lines to make them gauge invariant:

$$\langle 0|\mathcal{O}_n^H(\Lambda)|0\rangle = \langle 0|\chi^\dagger(0)\kappa_{n,c}\psi(0)\Phi^\dagger_{cb}(0)\left(\sum_X |H+X\rangle\langle H+X|\right)\Phi_{ba}(0)\psi^\dagger(0)\kappa'_{n,a}\chi(0)|0\rangle,$$

$$\Phi(0) = \mathcal{P}\exp\left[-ig\int_0^\infty d\lambda\, l\cdot A(l\lambda)\right].$$

- The Wilson lines $\Phi(0)$ are path integrals of the gauge field running from the $Q\bar{Q}$ creation and annihilation points to infinity.
- Essential at two-loop order to allow certain soft contributions to be absorbed into the NRQCD LDMEs.
- Does not affect existing phenomenology, which is at tree order or one-loop order in the color-octet contributions.

Possible Ingredients of a Factorization Proof

- A proof is complicated because gluons can dress the basic production process in ways that apparently violate factorization.
- A proof of factorization would involve a demonstration that diagrams in each order in α_s can be re-organized so that
  - All soft singularities cancel or can be absorbed into NRQCD LDMEs.
  - All collinear singularities and spectator interactions can be absorbed into parton distributions.

Proposal for a Two-Step Proof

- Nayak, Qiu, Sterman (2005, 2006): Prove factorization in two steps.
  - Fragmentation-Function Factorization:
    Prove that the inclusive cross section can be written as convolutions of quarkonium fragmentation functions with the short-distance cross sections that produce the fragmenting parton(s).
  - NRQCD Factorization:
    Prove that the fragmentation functions can be written as a sum of short-distance coefficients times NRQCD LDMEs.

Step One: Fragmentation-Function Factorization

(Kang, Qiu, Sterman (2010))

- Write the cross section in terms of
  - single-parton production cross sections convolved with the fragmentation functions for a single parton into a quarkonium

$$d\hat{\sigma}_{A+B\to i+X} \otimes D_{i\to H}$$

  - $Q\bar{Q}$ production cross sections convolved with fragmentation functions for a $Q\bar{Q}$ pair into a quarkonium

$$d\hat{\sigma}_{A+B\to Q\bar{Q}+X} \otimes D_{Q\bar{Q}\to H}$$

- Re-organizes the perturbation expansion as an expansion in powers of $1/p_T$.
- Holds to all orders in perturbation theory up to corrections of order m_Q^4/p_T^4.
- If this step is to be valid, p_T must be much greater than m_Q.

Example of a Single-Particle Fragmentation Function

Gluon Fragmentation into a Quarkonium

$$D_{H/g}(z) = \frac{-z^2}{32\pi P^+} \int dy^- \, e^{(-iP^+/z)y^-} \times \sum_{a,b,c} \langle 0 | F_a^{+\lambda}(0) [\Phi(0)]^\dagger_{ab} \left(\sum_X |H(P) + X\rangle \langle H(P) + X| \right) \times [\Phi(y^-)]_{bc} F^+_{c\lambda}(y^-) |0\rangle .$$

Example of a Two-Particle Fragmentation Function

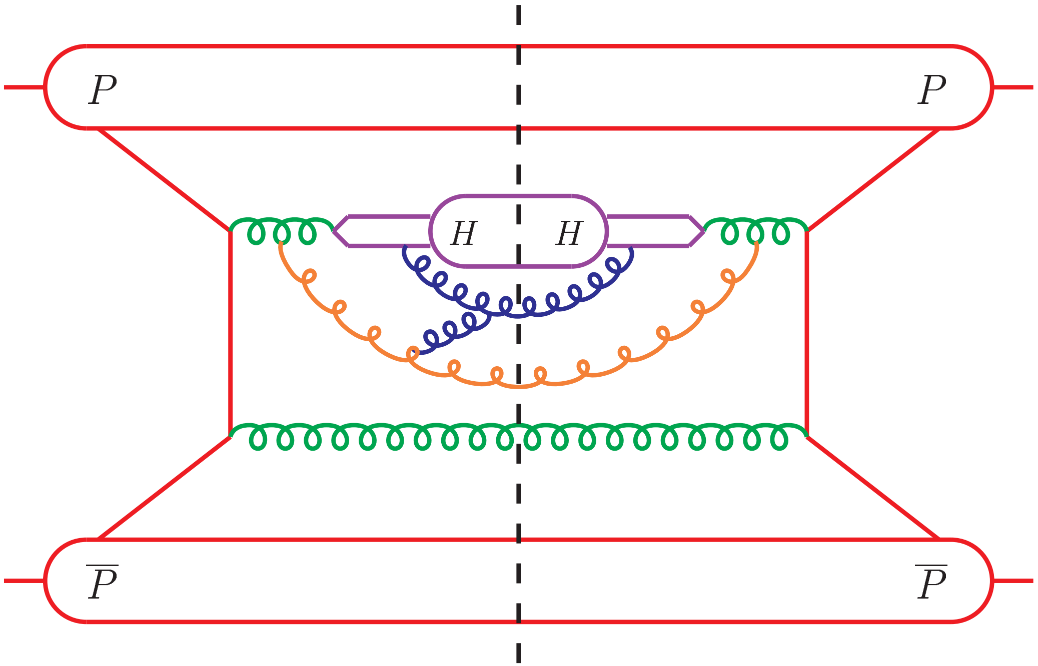
Color-Octet $Q\bar{Q}$ Fragmentation into a Quarkonium

$$D_{H/[Q\bar{Q}(8)]}(z) = \frac{z^2}{4\pi(N_c^2 - 1)P^+} \int dy^- \, e^{(-iP^+/z)y^-} \times \sum_{a,b,c} \langle 0 | \psi(0) \gamma^+ t^a \bar{\psi}(0) [\Phi(0)]^\dagger_{ab} \left(\sum_X |H(P) + X\rangle \langle H(P) + X| \right) \times [\Phi(y^-)]_{bc} \psi(y^-) \gamma^+ t^c \bar{\psi}(y^-) |0\rangle .$$

A Key Difficulty in Proving Step Two

(Nayak, Qiu, Sterman (2005, 2006))

- How do we treat gluons with momenta of order m_Q in the quarkonium rest frame?



- If the orange gluon has momentum of order m_Q, it can't be absorbed into the NRQCD LDME.
- But the orange gluon can have non-vanishing soft exchanges with the quarkonium constituents.
- The orange gluon produces a "minijet" in the fragmentation function.

- The orange gluon can be treated as a soft eikonal (Wilson) line.
- (Nayak, Qiu, Sterman (2006)): If the soft divergences don't depend on the direction of the soft eikonal line, then the effect of the soft divergences is just a color rotation on the Q and $\bar{Q}$ lines.
- The soft divergences associated with the eikonal line in the LDME produce this same color rotation on the Q and $\bar{Q}$ lines in the LDME, provided that the divergences are independent of the direction of the eikonal line in the LDME.

Generalization to More Than One Soft Eikonal Line

- If the soft contributions are independent of all of the soft eikonal line directions, then they also amount to a color rotation on the Q and $\bar{Q}$ lines.
  Includes eikonal lines in the definitions of the fragmentation functions.
- The color rotation is the same in the LDMEs and the fragmentation functions.
- The question of factorization hinges on the (in)dependence of the soft contributions on the eikonal-line directions.

What Do We Know About the Dependence on the Eikonal-Line Direction?

- Nayak, Qiu, Sterman (2005, 2006): At two-loop order (one eikonal line) the dependence on the direction of the eikonal line cancels.
- It is not known if this result generalize to higher orders.
  - There is no known counterexample in the existing calculations of soft functions.
  - The issue is related to the "dipole conjecture" for the soft anomalous dimension (Becher, Neubert (2009)).
- This does not mean that NRQCD factorization holds to NNLO accuracy.
  - α_s is not a small factor for soft gluons.
  - Soft gluons at high loop orders could dress a production process of low order in $\alpha_s(p_T)$ or $\alpha_s(m_c)$ in a way that violates NRQCD factorization.

Presence or Absence of Minijets

- If there are no minijets in the fragmentation functions, then the situation is simpler.
- If we choose the eikonal lines in the LDMEs to have the same directions as the eikonal lines in the fragmentation functions, then the LDMEs have the same soft interactions as the fragmentation functions.
- Corrections to NRQCD factorization involve at least one minijet.
  - Suppressed as $\alpha_s(m_Q)$.
  - For the ${}^3S_1$ color-octet channel, NRQCD Factorization may hold at the 25–30% level.
  - In NLO, the ${}^3P_J$ color-octet channel is kinematically enhanced by a mechanism involving a minijet.
    Corrections to NRQCD factorization may be of order 100% in that channel.

Breakdown of NRQCD Factorization When There Are Co-Moving Heavy Quarks

Nayak, Qiu, Sterman (2007, 2008): If additional heavy quark(s) are approximately co-moving with the $Q\bar{Q}$ pair that forms the quarkonium, there are soft color exchanges between the heavy quark(s) and the $Q\bar{Q}$ pair.

- This process does not fit into the NRQCD factorization picture.
  It requires production matrix elements that contain additional heavy quarks beyond the $Q\bar{Q}$ pair.
- The process is nonperturbative: It can't be calculated reliably.
- Can search for the process experimentally:
  The signature is additional heavy-meson production in a narrow cone ($\sim m_Q v/p_T$) around the quarkonium.
- This effect might be eliminated from the measured cross section through the use of an isolation cut.

Summary

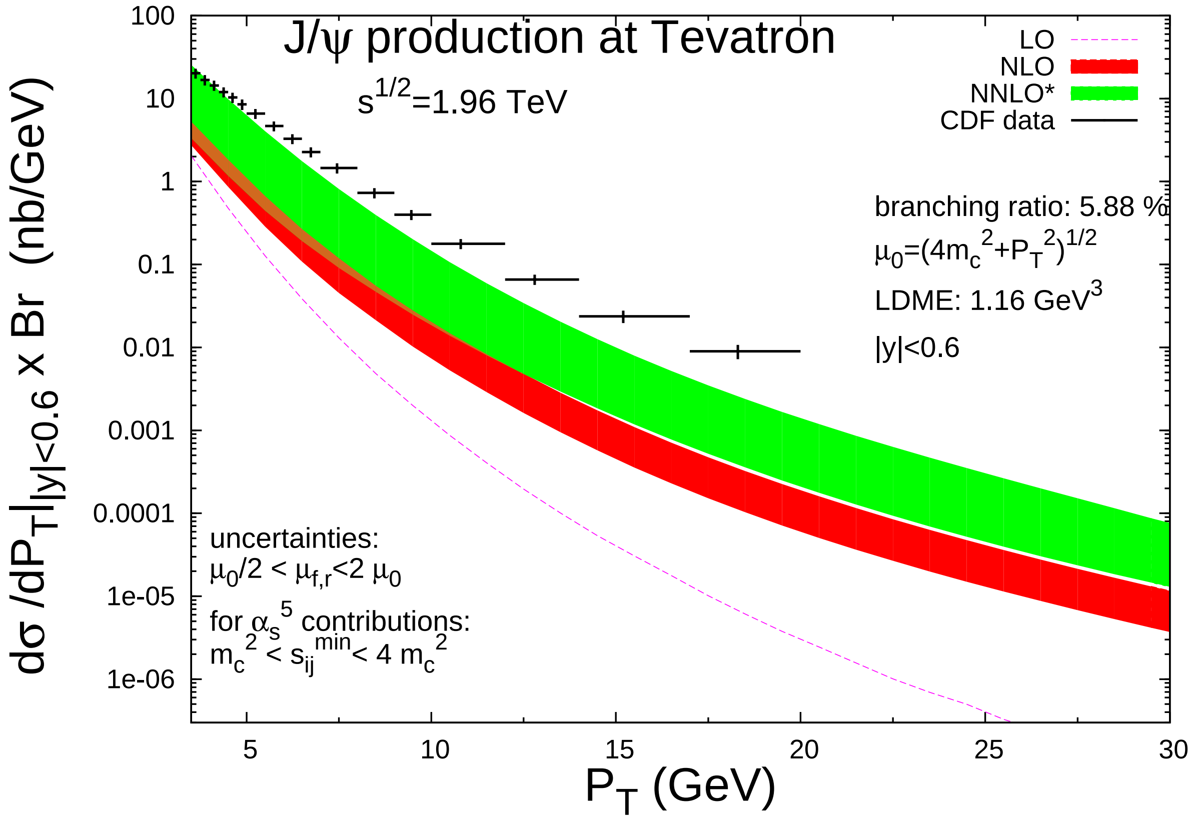
- An all-orders proof of NRQCD factorization for inclusive quarkonium production is still lacking.
- In spite of the two-loop demonstration of NRQCD factorization by Nayak, Qiu, and Sterman (2005, 2006), there could be soft-gluon violations of NRQCD factorization that spoil the accuracy in lower orders.
- Violations of factorization are suppressed at least as $\alpha_s(m_Q)$.
- In the absence of further theoretical progress, experiment will decide the extent to which NRQCD factorization is correct.
- If NRQCD factorization holds to all orders in perturbation theory, then the expectation is that corrections to it would be of relative order m_Q^4/p_T^4.
  - NRQCD factorization would fail when p_T is not large compared to m.
  - The question "How large is large?" must be answered by experiment.
- NRQCD factorization does not hold when heavy quarks (d mesons) are co-moving with the quarkonium.
  - It is important for experiments to look for this process.
  - If such events are a significant part of the rate, then they should be cut from the data before comparisons with NRQCD factorization predictions are made.

Back-Up Slides

The Problem of Large k Factors

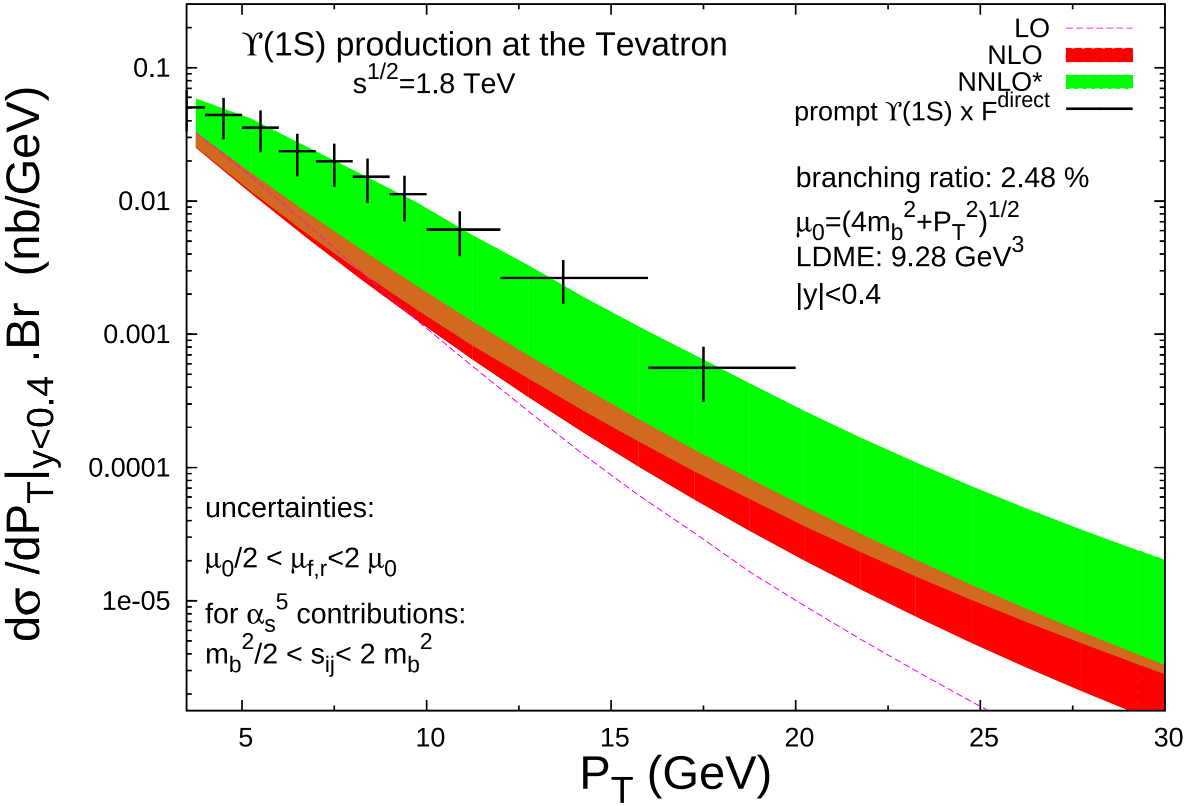
- Higher-order corrections to color-singlet quarkonium production at the Tevatron are unexpectedly large. (Campbell, Maltoni, Tramontano(2007); Artoisenet, Lansberg, Maltoni (2007))

NLO and NNLO* Color-Singlet J/ψ Production



- Plot from Pierre Artoisenet, based on work by Artoisenet, Campbell, Lansberg, Maltoni, Tramontano.
- The NNLO* calculation is an estimate based on real-emission contributions only.
- The data still seem to require a color-octet contribution.

NLO and NNLO* Color-Singlet Υ Production



- Plot from Pierre Artoisenet, based on work by Artoisenet, Campbell, Lansberg, Maltoni, Tramontano (2008)
- NLO results confirmed by Gong and Wang (2007).
- The data could be explained by color-singlet production alone.
- There is still room for a substantial amount of color-octet production.

- A large k factor ~ -10 is also seen in the ${}^3P_J$ color-octet channel.
  (Ma, Wang, and Chao (2010); Butenschön and Kniehl (2010))

- NLO corrections to the S-wave channels are small.
  (Gong, Li, and Wang (2008, 2010))

  – k factors at the Tevatron are about 1.235 for the ${}^1S_0$ channel and 1.139 for the ${}^3S_1$ channel.

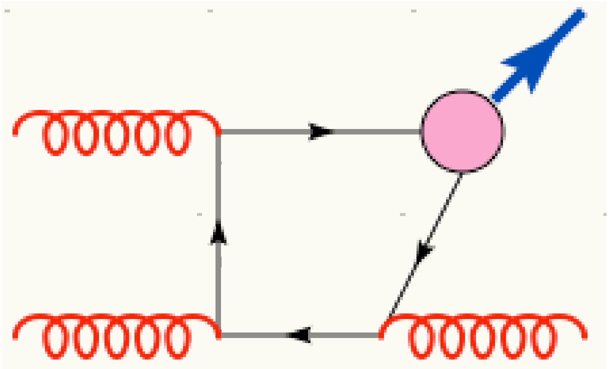
- Does the perturbation series converge?

- How do we understand the different k factors for different channels?

Explanation of Large k Factors

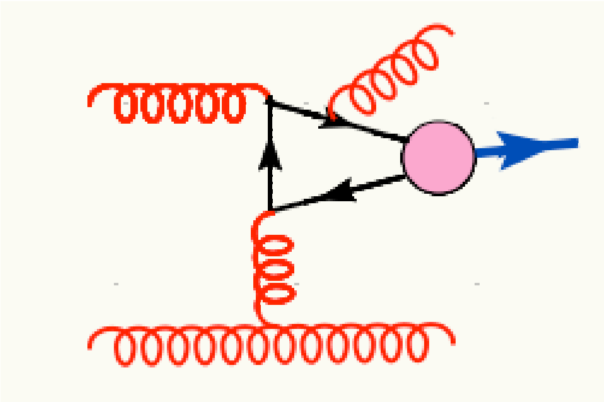
- At high p_T, higher powers of α_s can be offset by a less rapid fall-off with p_T. (Campbell, Maltoni, Tramontano(2007); Artoisenet, Lansberg, Maltoni (2007))

Color-singlet LO:

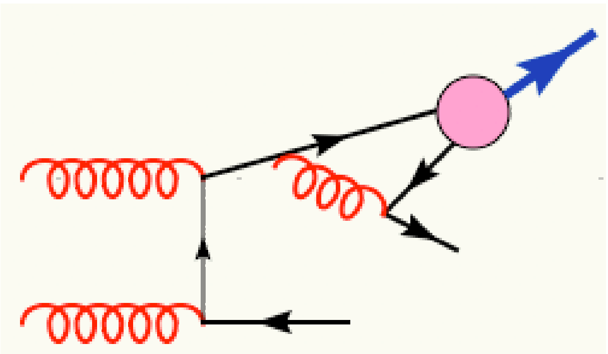


$$\sim \alpha_s^3 \frac{(2m_c)^4}{p_T^8}$$

Color-singlet NLO:

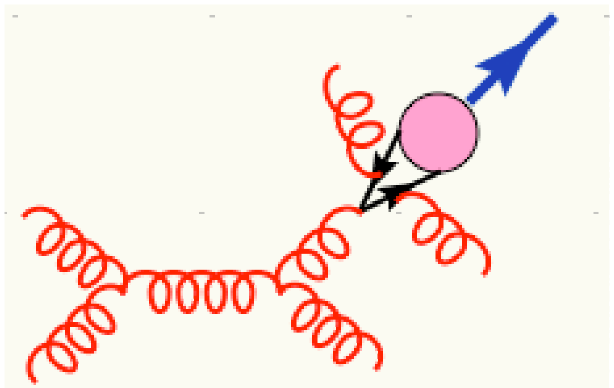


$$\sim \alpha_s^4 \frac{(2m_c)^2}{p_T^6}$$



$$\sim \alpha_s^4 \frac{1}{p_T^4}$$

Color-singlet NNLO:



$$\sim \alpha_s^5 \frac{1}{p_T^4}$$

- Similar explanations account for the k factors in the color-octet channels.

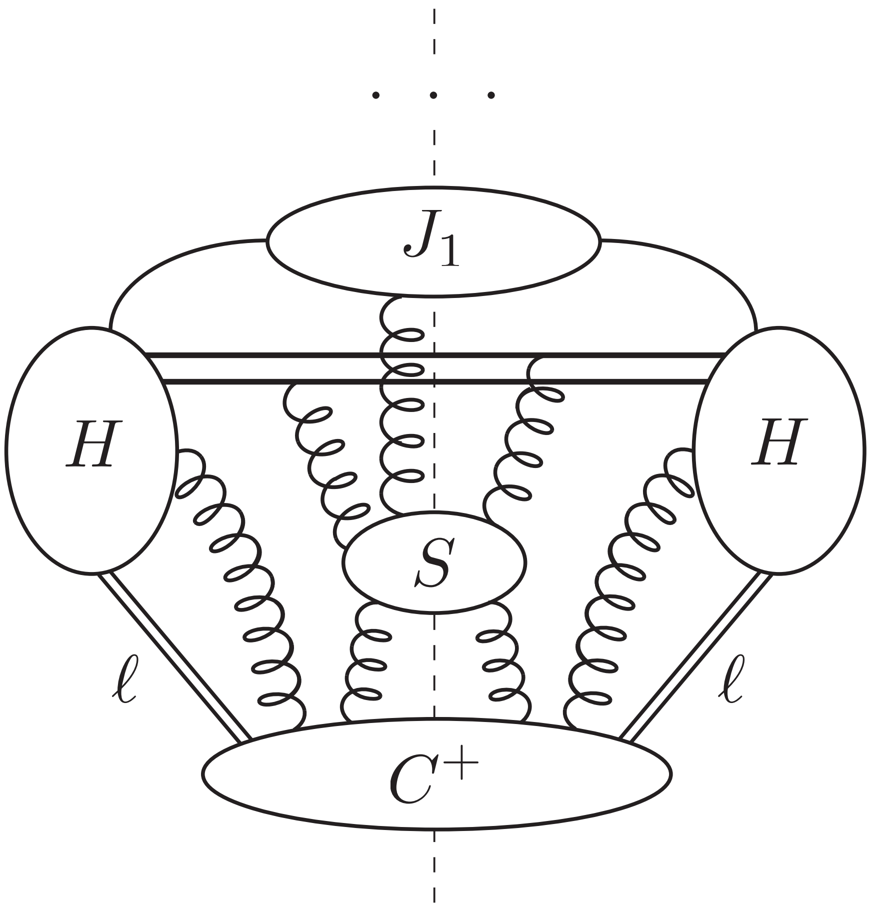
Discussion

- The fragmentation approach of Kang, Qiu, and Sterman allows one to focus on the leading and first subleading power corrections, which seem to account for the large k factors.
  - It should be possible to calculate higher-order corrections and resum logs of p_T^2/m_c^2 for these contributions.
- It is important to check that the fragmentation contributions really do account for all of the large corrections.
  - Confirmed for the color-singlet NLO correction. (Kang, Qiu, Sterman (2011))
  - Preliminary confirmation for the ${}^3P_J$ color-octet NLO correction. (GTB, Jungil Lee)
- The color-singlet NNLO* correction seems be dominated by contributions proportional to $\log^2(p_T^2/p_{T\mathrm{cut}}^2)$. (Ma, Wang, Chao (2011)).
  - These should cancel when virtual corrections are included, making the complete NNLO contribution smaller than the NNLO* contribution.

Singular Contributions in LDMEs and Fragmentation Functions

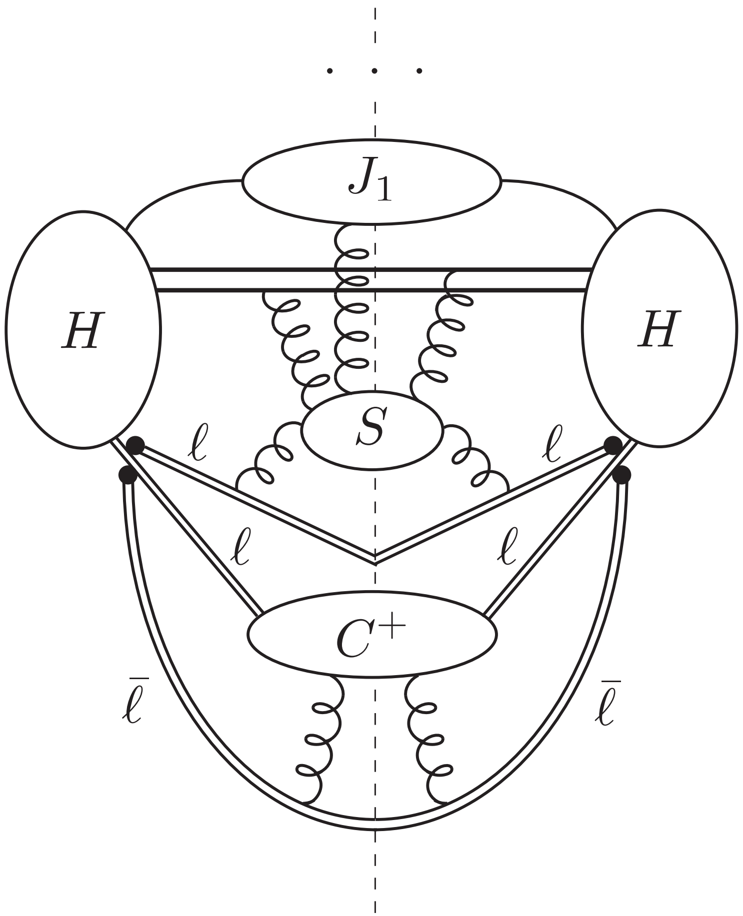
LDMEs

- A Landau analysis of the pinch surfaces in loop integrals in an LDME gives the topology of the leading diagrams that produce soft and collinear singular contributions.



- H is a hard subdiagram (all momentum components of order m.)
- S is a soft subdiagram (all momentum components $\ll m$.)
- C^+ is a collinear to $+$, with momentum components collinear to the light-like eikonal line l which we take to be in the $+$ direction.
- J_1, $\ldots$ are additional light-parton jets.

- Standard techniques and be used to factorize the soft contributions and the contributions that are collinear to the eikonal line.



- The soft and collinear contributions can be factorized from the light-parton jets in a similar way.
- The C^+ (and light-parton) collinear contributions cancel by unitarity in the sum over final-state cuts.
- We are left with a soft function that talks only to soft eikonal lines and the Q and $\bar{Q}$.

Fragmentation Functions

- Evolve the fragmentation functions to a scale $\mu \sim m$.
- A Landau analysis of a fragmentation function in the $Q\bar{Q}$ CM frame gives the same general topology as for the LDME, but the details of the hard parts and the UV cutoff are different.
- Factorization and cancellation of collinear contributions goes through for the fragmentation functions in the same way as for the LDMEs.

Dependence on the Soft-Eikonal-Line Directions

- Kinematics:
  - Q momentum: $P_1 = P/2 - q$, $P_1' = P/2 - q'$
  - $\bar{Q}$ momentum: $P_2 = P/2 - q$, $P_2' = P/2 - q'$
  - In the $Q\bar{Q}$ CM frame, $|\boldsymbol{q}| = |\boldsymbol{q}'|$, but $\boldsymbol{q}$ and $\boldsymbol{q}'$ have independent angles on either side of the final-state cut.
- Take l slightly space-like:

$$l = [l^+, l^-, \boldsymbol{l}_\perp] = [1, -e^{-2y}, 0_\perp]; \qquad l^2 = -2e^{-2y}.$$

We will ultimately take $y \to \infty$.

One Eikonal Line

- Suppose initially that the soft interactions involve only the original Wilson (eikonal) line and the Q and $\bar{Q}$, but do not involve the light-quark jets.
- Then, the Lorentz invariants are

$$\begin{aligned} x_1 &= (l \cdot P_1)^2/l^2, \\ x_2 &= (l \cdot P_2)^2/l^2, \\ x_1' &= (l \cdot P_1')^2/l^2, \\ x_2' &= (l \cdot P_2')^2/l^2. \end{aligned}$$

- Now differentiate the LDME $\mathcal{M}(x_1, x_2, x_1', x_2')$ with respect to y. Each eikonal vertex-propagator factor contributes a term proportional to

$$\frac{\partial}{\partial y}\frac{l}{-l\cdot\tilde{k}+i\epsilon} = -l^2\frac{(\tilde{k}^+, -\tilde{k}^-, 0_\perp)}{(-l\cdot\tilde{k}+i\epsilon)^2},$$

where $\tilde{k}$ is the momentum that flows through the eikonal line.

- The differentiated factor vanishes as $l^2 = -2e^{-2y}$ as $y \to \infty$, unless the denominator is of order e^{-2y}.
  - This can only happen if $\tilde{k}$ is collinear to l.
  - Then one can factor such contributions along with the usual collinear contributions and use unitarity to show that they cancel.

- Therefore,

$$\frac{\partial}{\partial y}\mathcal{M}(x_1, x_2, x_1', x_2') = 0,$$

up to terms of order e^{-2y}.

- Now,

$$\frac{\partial}{\partial y}x_i = -e^{2y}(P_i^-)^2 - e^{-2y}(P_i^+)^2 \approx 2x_i.$$

- Therefore, from the chain rule for differentiation, it follows that

$$2\frac{\partial\mathcal{M}}{\partial\log x_1} + 2\frac{\partial\mathcal{M}}{\partial\log x_2} + 2\frac{\partial\mathcal{M}}{\partial\log x_1'} + 2\frac{\partial\mathcal{M}}{\partial\log x_2'} = 0.$$

- Sterman: The general solution of this equation is any function of ratios of x_1, x_2, x'_1, and x'_2 of degree zero.
  - This enforces the cancellation of factors of l^2, (no collinear-to-l singularities).
- By charge-conjugation invariance, $\mathcal{M}$ must be symmetric under $P_1 \leftrightarrow P_2$ and under $P'_1 \leftrightarrow P'_2$.
- In an orbital-angular-momentum projection, the angular variables for $\boldsymbol{q}$ and $\boldsymbol{q}'$ are just dummy variables of integration, and so we can also symmetrize under the under $c_i \leftrightarrow c'_i$.
- If the functions of x_1, x_2, x'_1, and x'_2 are single logarithms, this implies that the dependence on l cancels.
  - Perhaps “explains” the lack of l dependence in two-loop order.