In-medium QCD forces at high temperatures

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References:
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Y.A., PRD87,045016(2013).
Short introduction:
Y.A., arXiv:1303.2976 [nucl-th].

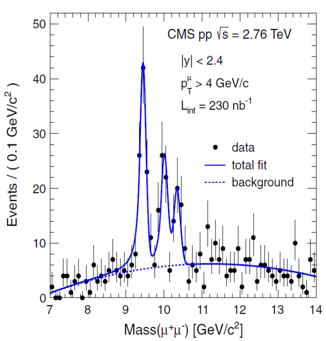
Contents

1. INTRODUCTION

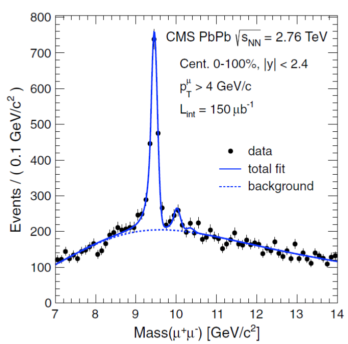
Taking advantage of the last talk …

- We are well-motivated to study quarkonia in medium
  - by the beautiful experiments at LHC & RHIC
  - by the wonderful scenario by Matsui & Satz

p+p



A+A



CMS

Matsui

Satz

Map

Heavy quark potential
F(r) or E(r)

unknown

Re[V(r)]=E(r) ??

Current correlator
$\langle [J(x),J(y)] \rangle$

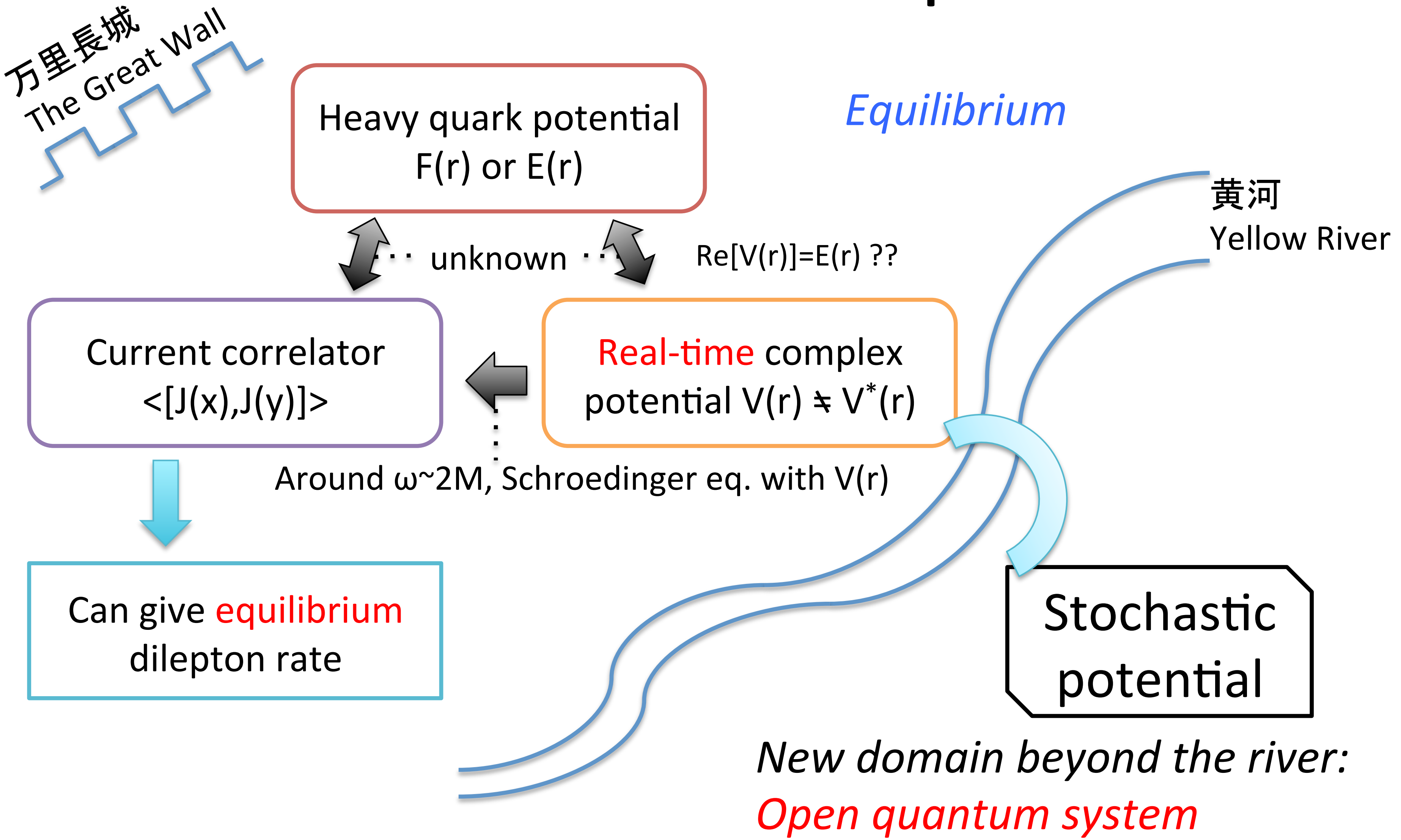
Real-time complex
potential $V(r) \neq V^*(r)$

Around ω~2M, Schroedinger eq. with V(r)

Can give equilibrium
dilepton rate

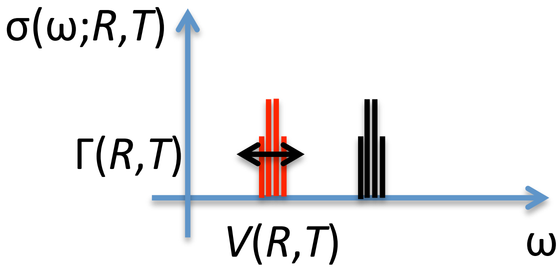
All fine in equilibrium!
How about near equilibrium?

A broader map



Stochastic potential

- Complex potential (M=∞)



$$\left\langle \Psi(t;R) \right\rangle_T \propto \left\langle J(t;R) J^\dagger(0;R) \right\rangle_T$$
$$\propto \sum_{n,m} \left| \left\langle m \right| J^\dagger(0;R) \left| n \right\rangle \right|^2 e^{-\beta E_n(R)} \exp\left[i\{E_n(R) - E_m(R)\}t \right]$$
$$\sim \exp\left[-i\{V(R,T) - i\Gamma(R,T)/2\}t \right]$$

Laine et al (07), Beraudo et al (08), Brambilla et al (10), Rothkopf et al (12)

- Stochastic interpretation

$$\Psi(t+\Delta t, R) = \exp\left[-i\Delta t \left\{ V(R) + \Theta(t,R) \right\} \right] \Psi(t,R),$$
$$\left\langle \Theta(t,R) \right\rangle = 0, \quad \left\langle \Theta(t,R)\Theta(t',R') \right\rangle = \Gamma(R,R')\delta_{tt'} / \Delta t,$$
$$\Rightarrow i\frac{\partial}{\partial t} \left\langle \Psi(t,R) \right\rangle_T = \left\{ V(R) - \frac{i}{2}\Gamma(R,R) \right\} \left\langle \Psi(t,R) \right\rangle_T .$$

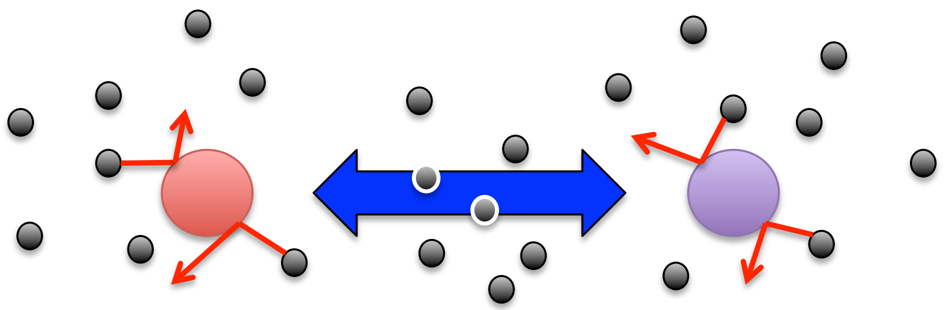
Unitary evolution

Imaginary part from averaging the random phase rotations

Akamatsu & Rothkopf (12)

Classical picture of the open system

- $M=\infty$ or $M<\infty$ matters



$M=\infty$

Debye screened force

+

Fluctuation

(stochastic/complex)
Potential force
→Hamiltonian dynamics

$M<\infty$

Drag force

--------- Non-potential force
→Not Hamiltonian dynamics

Langevin dynamics

Key words: Forces, open quantum system, influence functional

2. INFLUENCE FUNCTIONAL

Open quantum system

- Basics

sys = heavy quarks
env = gluons, light quarks

Hilbert space $$\mathrm{H}_{\mathrm{tot}} = \mathrm{H}_{\mathrm{sys}} \otimes \mathrm{H}_{\mathrm{env}}$$

von Neumann equation $$i\frac{d}{dt}\hat{\rho}_{\mathrm{tot}}(t) = \left[\hat{H}_{\mathrm{tot}}, \hat{\rho}_{\mathrm{tot}}(t)\right]$$

⬇ Trace out the environment

Reduced density matrix

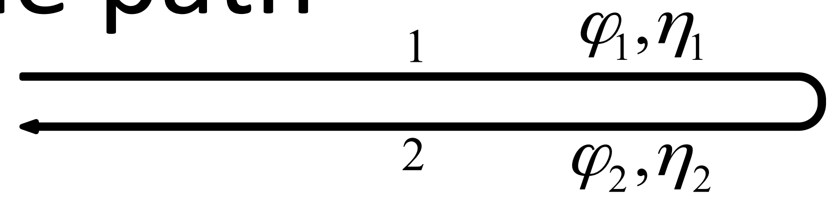
$$\hat{\rho}_{\mathrm{red}}(t) \equiv \mathrm{Tr}_{\mathrm{env}}\left[\hat{\rho}_{\mathrm{tot}}(t)\right] = \sum_{n\in\mathrm{env}} \langle n| \left[\sum_{\alpha\in\mathrm{tot}} w_\alpha |\alpha\rangle\langle\alpha|\right] |n\rangle$$

Master equation

$$i\frac{d}{dt}\hat{\rho}_{\mathrm{red}}(t) = ?$$ (Markovian limit)

Closed-time path

$$\langle \varphi_1 | \hat{\rho}(t_0) | \varphi_2 \rangle$$



- Partition function

$$Z[\eta_1,\eta_2] \sim \int D[\psi_{1,2} q_{1,2} A_{1,2}] \langle \psi_1^* q_1^* A_1(t_0) | \hat{\rho}_{\text{tot}} | \psi_2 q_2 A_2(t_0) \rangle$$
$$\times \exp\left[iS_{\text{kin}}[\psi_1] - iS_{\text{kin}}[\psi_2] + i\int \psi_1 \eta_1 - i\int \psi_2 \eta_2 \right]$$
$$\times \exp\left[iS[q_1 A_1] - iS[q_2 A_2] + ig\int j_1 A_1 - ig\int j_2 A_2 \right]$$

$$\hat{\rho}_{\text{tot}} = \hat{\rho}_{\text{env}}^{\text{eq}} \otimes \hat{\rho}_{\text{sys}}$$ Factorized initial density matrix

$$\rightarrow \langle \psi_1^* q_1^* A_1(t_0) | \hat{\rho}_{\text{tot}} | \psi_2 q_2 A_2(t_0) \rangle = \langle q_1^* A_1(t_0) | \hat{\rho}_{\text{env}}^{\text{eq}} | q_2 A_2(t_0) \rangle \cdot \langle \psi_1^*(t_0) | \hat{\rho}_{\text{sys}} | \psi_2(t_0) \rangle$$

Influence functional   Feynman & Vernon (63)

$$= Z_{qA}[j_1, j_2] \equiv \exp\left[iS_{\text{FV}}[j_1, j_2] \right]$$
$$= \exp\left[-g^2/2 \int j_1 G_A^{\text{F}} j_1 + j_2 G_A^{\tilde{\text{F}}} j_2 - j_1 G_A^{>} j_2 - j_2 G_A^{<} j_1 + \int g^3 G_A^{(3)} jjj + g^4 G_A^{(4)} jjjj + \cdots \right]$$

Influence functional

- LO pQCD, NR limit, slow dynamics

Stochastic potential (finite in M→∞)

$$S_{1+2} = S_{\rm kin}^{\rm NR}[Q_{1(c)}] - S_{\rm kin}^{\rm NR}[Q_{2(c)}] + S_{\rm FV}^{\rm LONR}[j_1, j_2] + \cdots$$

$$S_{\rm FV}^{\rm LONR}[j_1,j_2] = -\frac{1}{2}\int_{t,\vec{x},\vec{y}} (\rho_{1a},\rho_{2a})_{(t,\vec{x})} \begin{bmatrix} V(\vec{x}-\vec{y}) & -iD(\vec{x}-\vec{y}) \\ -iD(\vec{x}-\vec{y}) & -V^*(\vec{x}-\vec{y}) \end{bmatrix} \begin{pmatrix} \rho_{1a} \\ \rho_{2a} \end{pmatrix}_{(t,\vec{y})}$$

$$-\int_{t,\vec{x},\vec{y}} \left\{ \frac{\vec{\nabla} D(\vec{x}-\vec{y})}{4T} \cdot \left(\vec{j}_{1a}(t,\vec{x})\rho_{2a}(t,\vec{y}) + \rho_{1a}(t,\vec{x})\vec{j}_{2a}(t,\vec{y}) \right) \right\}$$

Drag force (vanishes in M→∞)

$$-g^2\left\{\overline{G}_{00,ab}^{R}(\vec{x}-\vec{y}) + i\overline{G}_{00,ab}^{>}(\vec{x}-\vec{y})\right\} \equiv V(\vec{x}-\vec{y})\delta_{ab}$$

$$-g^2\overline{G}_{00,ab}^{>}(\vec{x}-\vec{y}) \equiv D(\vec{x}-\vec{y})\delta_{ab} = {\rm Im}V(\vec{x}-\vec{y})\delta_{ab}$$

3. REAL-TIME DYNAMICS

Functional differential equation

- Path integral → "Schroedinger equation"

$$\left\langle Q^*_{1(c)} \middle| \hat{\rho}_{\rm red}(t) \middle| \tilde{Q}^*_{2(c)} \right\rangle \sim \int_{t_0}^{t, Q^*_{1(c)}, \tilde{Q}^*_{2(c)}} D[Q^{(*)}_{1(c)}, \tilde{Q}^{(*)}_{2(c)}] \left\langle Q^*_{1(c)}(t_0) \middle| \hat{\rho}_{\rm sys} \middle| \tilde{Q}^*_{2(c)}(t_0) \right\rangle$$

$$\times \exp\left[iS_{\rm NR}[Q^{(*)}_{1(c)}] - iS_{\rm NR}[\tilde{Q}^{(*)}_{2(c)}] + iS^{\rm LOFV}[j_1, j_2] + \cdots \right]$$

$$\left\{ \hat{Q}_1(\vec{x}), \hat{Q}^\dagger_1(\vec{y}) \right\} = \left\{ \hat{Q}_{1c}(\vec{x}), \hat{Q}^\dagger_{1c}(\vec{y}) \right\} = \delta(\vec{x} - \vec{y}) \Leftrightarrow Q_{1(c)} = \frac{\delta}{\delta Q^*_{1(c)}}$$

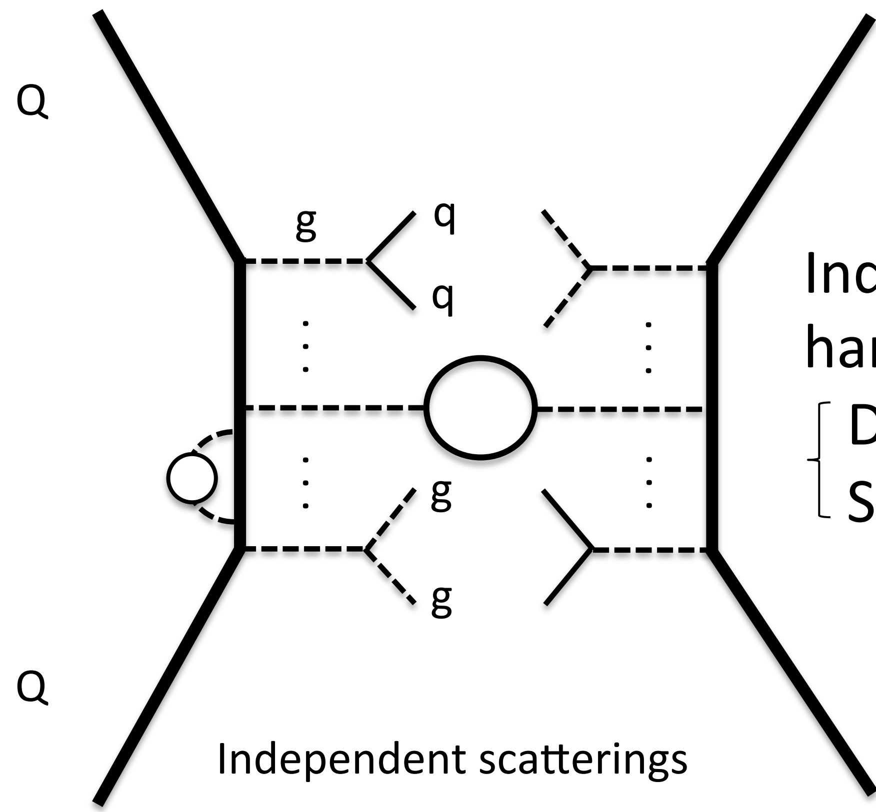
$$\left\{ \hat{\tilde{Q}}_2(\vec{x}), \hat{\tilde{Q}}^\dagger_2(\vec{y}) \right\} = \left\{ \hat{\tilde{Q}}_{2c}(\vec{x}), \hat{\tilde{Q}}^\dagger_{2c}(\vec{y}) \right\} = -\delta(\vec{x} - \vec{y}) \Leftrightarrow \tilde{Q}_{2(c)} = -\frac{\delta}{\delta \tilde{Q}^*_{2(c)}}$$

$$i \frac{\partial}{\partial t} \left\langle Q^*_{1(c)} \middle| \hat{\rho}_{\rm red}(t) \middle| \tilde{Q}^*_{2(c)} \right\rangle = H^{\rm func}_{1+2}[Q^*_{1(c)}, \tilde{Q}^*_{2(c)}] \left\langle Q^*_{1(c)} \middle| \hat{\rho}_{\rm red}(t) \middle| \tilde{Q}^*_{2(c)} \right\rangle$$

*Coherent states are generators of heavy quark quanta.

Physical process

- Scatterings in t-channel



Independent scatterings with hard particles contribute to

- Diffusion
- Screening

Master equation

- Single HQ

$$i\partial_t \rho_Q(t,\vec{x},\vec{y}) = \left\{ \left(a - a^*\right)M + \left(-\frac{\nabla_x^2 - \nabla_y^2}{2M}\right) \right\} \rho_Q(t,\vec{x},\vec{y})$$

(color traced)

$$+ C_F \left\{ -iD(\vec{x}-\vec{y}) + \frac{\vec{\nabla}_x D(\vec{x}-\vec{y})}{4T} \cdot \frac{\vec{\nabla}_x - \vec{\nabla}_y}{iM} \right\} \rho_Q(t,\vec{x},\vec{y})$$

$$a \equiv 1 + \frac{C_F}{2M} \lim_{r\to 0} V^{(T>0)}(r), \quad V^{(T>0)}(r) \equiv V(r) - V^{(T=0)}(r)$$

$$\Rightarrow \begin{cases} \dfrac{d}{dt}\langle \vec{x} \rangle = \dfrac{\langle \vec{p} \rangle}{M}, \quad \dfrac{d}{dt}\langle \vec{p} \rangle = -\dfrac{\gamma}{2MT}\langle \vec{p} \rangle, \\ \dfrac{d}{dt}\langle E \rangle = -\dfrac{\gamma}{MT}\left(\langle E \rangle - \dfrac{3T}{2}\right). \\ \gamma = \dfrac{C_F}{3}\nabla^2 D(x)\Big|_{x=0} = \dfrac{g^2 C_F}{9}\displaystyle\int \dfrac{d^3k}{(2\pi)^3} k^2 \tilde{G}^{>}_{00,aa}(\omega=0,k) \end{cases}$$

Langevin dynamics

Moore & Teaney (05)

Complex potential

- QQbar

$$i\partial_t \left\langle \Psi_{ij}(t;\vec{x},\vec{y}) \right\rangle_T = \left(2M - \frac{\nabla_x^2 + \nabla_y^2}{2M} \right) \left\langle \Psi_{ij}(t;\vec{x},\vec{y}) \right\rangle_T - \frac{1}{2} \left(\delta_{ij}\delta_{kl} - \frac{\delta_{ik}\delta_{jl}}{N_c} \right) V(\vec{x}-\vec{y}) \left\langle \Psi_{kl}(t;\vec{x},\vec{y}) \right\rangle_T$$

⇓ Projection onto singlet state

Debye screening w/ imaginary part
(complex potential)

$$V_{\text{singlet}}(R) = 2(a-1)M - C_F V(R) \ = -\frac{C_F g^2}{4\pi} \left(\omega_D + \frac{e^{-\omega_D R}}{R} + iT\varphi\left(\omega_D R\right) \right)$$

Laine et al (07), Beraudo et al (08), Brambilla et al (10)

4. SUMMARY

So far and beyond

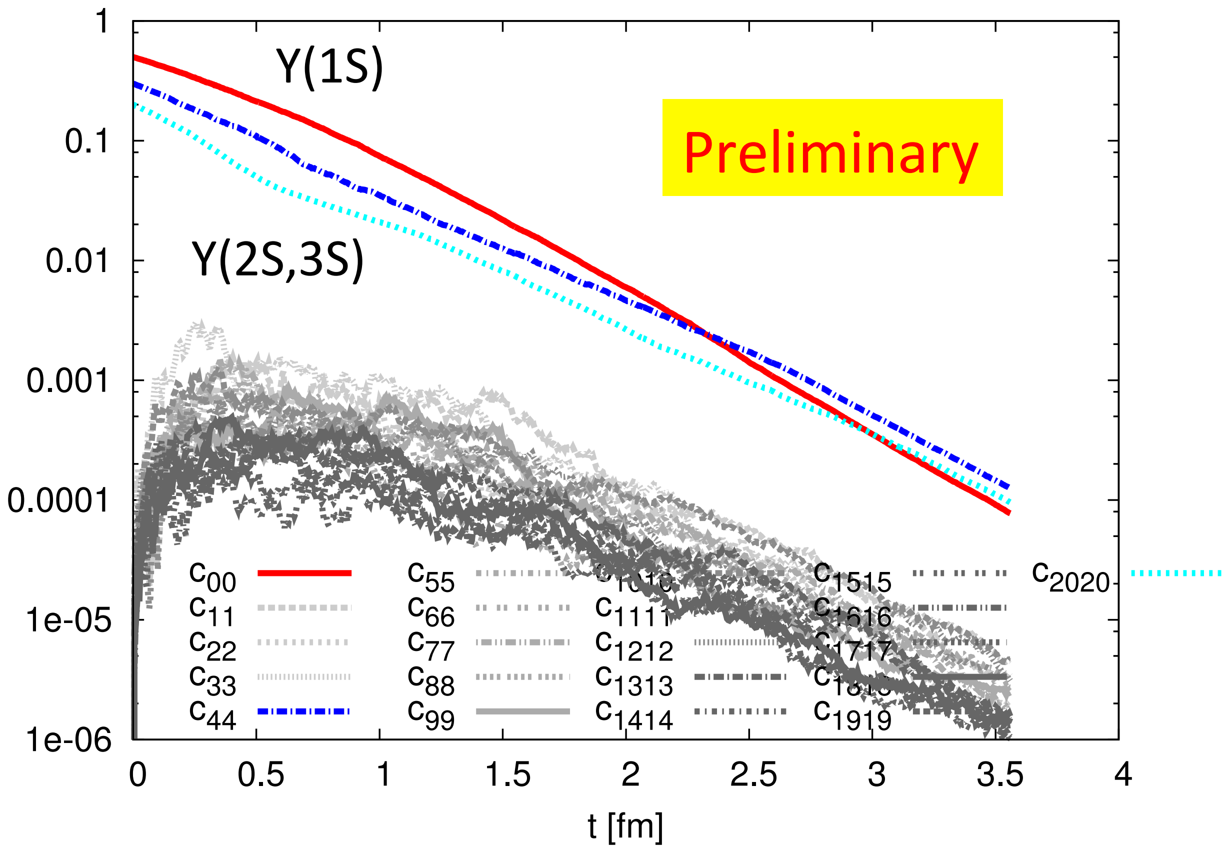
- LO perturbation
  - Heavy quarks in QGP as open quantum system
  - Unified description of forces
  - Scatterings in t-channel

- NLO perturbation
  - Gluo-dissociation, singlet-singlet transitions
- Phenomenology at RHIC/LHC

Backup

Numerical simulation

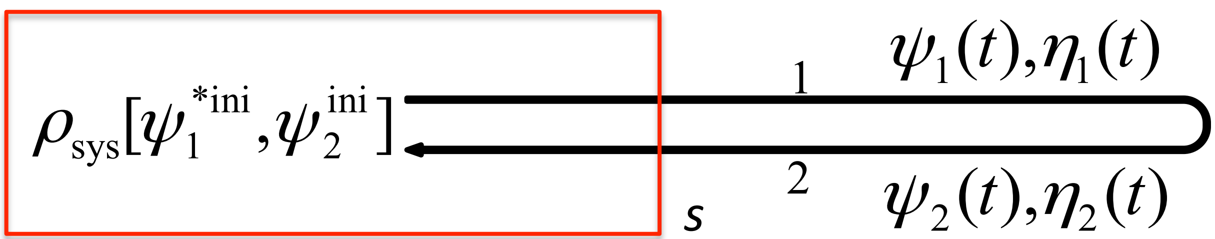
- Stochastic evolution



Influence functional

- Open Quantum System

$$Z[\eta_1,\eta_2] \sim \int D[\psi_{1,2}]\rho_{\rm sys}[\psi_1^{*\rm ini},\psi_2^{\rm ini}]$$
$$\times \exp\left[iS[\psi_1] - iS[\psi_2] + iS^{\rm FV}[j_1,j_2] + i\int\psi_1\eta_1 - i\int\psi_2\eta_2\right]$$



Path integrate until *s*, with boundary condition $\psi_1(s)=\psi_1, \psi_2(s)=\psi_2$

$$\square = \rho_{\rm red}[s,\psi_1^*,\psi_2] = \langle\psi_1^*|\hat{\rho}_{\rm red}(s)|\psi_2\rangle$$

Density matrix for a few HQs

- Remember coherent states

$$\left\langle Q^*_{1(c)} \right| \hat{\rho}_{\rm red}(t) \left| \tilde{Q}^*_{2(c)} \right\rangle :$$

$$\left\langle Q^*_{1(c)} \right| = \left\langle \Omega \right| \exp\left[-\int_{\vec{x}} \left\{ \hat{Q} Q^*_1 + \hat{Q}_c Q^*_{1(c)} \right\} \right]$$

$$\left| \tilde{Q}^*_{2(c)} \right\rangle = \exp\left[-\int_{\vec{x}} \left\{ \tilde{Q}^*_2 \hat{\tilde{Q}}^\dagger + \tilde{Q}^*_{2c} \hat{\tilde{Q}}^\dagger_c \right\} \right] \left| \Omega \right\rangle$$

$$\Rightarrow$$

$$\frac{\delta}{\delta Q^*_1(\vec{x})} \left\langle Q^*_{1(c)} \right| \Big|_{Q^*_{1(c)}=0} = \left\langle \Omega \right| \hat{Q}(\vec{x})$$

$$\frac{\delta}{\delta \tilde{Q}^*_2(\vec{x})} \left| \tilde{Q}^*_{2(c)} \right\rangle \Big|_{\tilde{Q}^*_{2(c)}=0} = -\hat{Q}^\dagger(\vec{x}) \left| \Omega \right\rangle$$

  - Single HQ

$$\rho_Q(t,\vec{x},\vec{y}) = \left\langle \Omega \right| \hat{Q}(\vec{x}) \hat{\rho}_{\rm red}(t) \hat{Q}^\dagger(\vec{y}) \left| \Omega \right\rangle$$

$$= -\frac{\delta}{\delta Q^*_1(\vec{x})} \frac{\delta}{\delta \tilde{Q}^*_2(\vec{y})} \left\langle Q^*_{1(c)} \right| \hat{\rho}_{\rm red}(t) \left| \tilde{Q}^*_{2(c)} \right\rangle \Big|_{Q^*_{1(c)}=\tilde{Q}^*_{2(c)}=0}$$

  - Similar for two HQs, …

$$\rho_{QQ_c}(t,\vec{x}_1,\vec{x}_2,\vec{y}_1,\vec{y}_2), \cdots$$

Stochastic process

- M=∞

$$
\begin{aligned}
&\exp\left[iS_{\mathrm{FV}}^{\mathrm{LONR}}[j_1, j_2]\right] \\
&= \exp\left[-i/2\int_{t,\vec{x},\vec{y}} \mathrm{Re}\,V(\vec{x}-\vec{y})\{\rho_{1a}(t,\vec{x})\rho_{1a}(t,\vec{y}) - \rho_{2a}(t,\vec{x})\rho_{2a}(t,\vec{y})\}\right] \\
&\quad \times \left\langle \exp\left[-i\int_{t,\vec{x},\vec{y}} \xi_a(t,\vec{x})\{\rho_{1a}(t,\vec{x}) - \rho_{2a}(t,\vec{x})\}\right]\right\rangle_{\xi}
\end{aligned}
$$

$$
\langle \xi_a(t,\vec{x})\xi_b(s,\vec{y})\rangle = -\delta_{ab}\delta(t-s)D(\vec{x}-\vec{y})
$$

Debye screening + fluctuation
(stochastic potential)

Akamatsu & Rothkopf (12)