Exclusive charmonium production processes and charmonium wave functions

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Outline:

- Introduction
- Study of charmonia distribution amplitudes
  - Models for distribution amplitudes
  - Properties of distribution amplitudes
- Application
- Conclusion

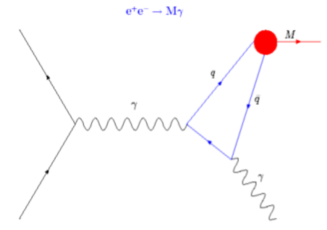
Hard exclusive processes

- Decays: $\Upsilon \to \rho\pi, \eta_b \to J/\psi J/\psi, \chi_{b0} \to J/\psi\psi' \dots$
- Annihilations: $e^+e^- \to J/\psi\eta_c, J/\psi J/\psi, \chi_{c0}\gamma, \dots$
- Different formfactors: $F(Q^2)$

- General property: $E_h \gg \Lambda_{QCD}, M$
- Expansion parameter $\sim \frac{M^2}{E_h^2} \sim \frac{1}{10}$
- $\sigma = \frac{a_n(E_h{=}\infty)}{E_h^n} + \frac{a_{n+1}(E_h{=}\infty)}{E_h^{n+1}} + \dots$

Light cone expansion formalism (LCF)

Factorization



Factorization formula :

$$T = \sum_n \overbrace{C_n}^{Short\ Dist.} \times \underbrace{< M \mid O_n(0) \mid 0 >}_{Large\ Dist.}$$

Different contributions

- Short distance contribution: C_n (perturbative QCD)
- Large distance contribution: $\langle M | O_n | 0 \rangle$ (nonperturbative effects)

Operators that contribute to pseudoscalar meson production

- $\bar{Q}\gamma_\mu\gamma_5 Q,\ \bar{Q}\gamma_\mu\gamma_5 D_{\mu_1} Q,\ \bar{Q}\gamma_\mu\gamma_5 D_{\mu_1} D_{\mu_2} Q,\ \dots$
- $\bar{Q}\sigma_{\alpha\beta}\gamma_5 Q,\ \bar{Q}\sigma_{\alpha\beta}\gamma_5 D_{\mu_1} Q,\ \bar{Q}\sigma_{\alpha\beta}\gamma_5 D_{\mu_1} D_{\mu_2} Q,\ \dots$
- $\bar{Q}\gamma_\mu\gamma_5 G_{\alpha\beta} Q,\ \bar{Q}\gamma_\mu\gamma_5 G_{\alpha\beta} D_{\mu_1} Q,\ \bar{Q}\gamma_\mu\gamma_5 G_{\alpha\beta} D_{\mu_1} D_{\mu_2} Q,\ \dots$

...

$$\sigma = \frac{\sigma_0}{s^n} + \frac{\sigma_1}{s^{n+1}} + \frac{\sigma_1}{s^{n+1}} + \dots$$

At a given accuracy some operators can be omitted

The leading twist distribution amplitudes

Operators that contribute at the leading twist approximation:

$$\langle M(p)|\bar{Q}\gamma_+\gamma_5(D_+)^n Q|0\rangle_\mu = p_+^{(n+1)} \int d\xi \xi^n \phi(\xi,\mu)$$

$v_+ = v_0 + v_z$, $\xi = x_1 - x_2$,

The distribution amplitude (DA) $\phi(\xi)$ can be considered as a meson's wave function

Exclusive processes at the leading twist

$$T = \int d\xi H(\xi,\mu) \times \phi(\xi,\mu), \quad \mu \sim E_h$$

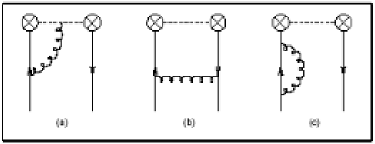
- Resume infinite series of operators
- No double logarithmic corrections ($\sim \alpha_s(E_h) \cdot \log^2 E_h$)
- Resume leading logarithmic corrections ($\sim \alpha_s(E_h) \cdot \log E_h$) in all loops

A.V. Efremov, A.V. Radyushkin, Phys.Lett. B94 (1980) 245,
G.P. Lepage, S. J. Brodsky, Phys.Rev. D22 (1980) 2157

Evolution of the leading twist distribution amplitudes (ERBL equation)

$$\langle P|\bar{Q}(z)\,\gamma_\mu\gamma_5[z,-z]Q(-z)|0\rangle = i\,f_P\int_{-1}^{1}d\xi\,e^{i(pz)\xi}\varphi(\xi,\mu)$$

$$[z,-z] = \mathrm{P}\exp\left(i\,g\int_{-z}^{z}dx^\mu A_\mu\right)$$



Renormalization group evolution equation:

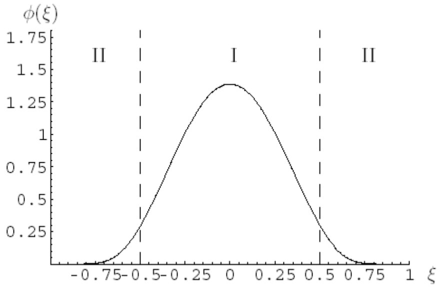
$$\frac{\partial}{\partial\log\mu}\varphi(\xi,\mu) = \int_{-1}^{1}d\eta\,V(\xi,\eta,\mu)\,\varphi(\xi,\mu)$$

The solution of RG equation:

$$\varphi(\xi,\mu) = \frac{3}{4}(1-\xi^2)\left(\sum_n a_n(\mu)\,G_n^{3/2}(\xi)\right)$$

$$a_n(\mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{\gamma_n}{b_0}}\times a_n(\mu_0)$$

Distribution amplitudes of nonrelativistic mesons



Properties

- The width of DA is $< \xi^2 > \sim v^2$
- The motion in the region **I** ($\xi^2 \leq v^2$) is nonrelativistic
- The motion in the region **II** ($\xi^2 \sim 1$) is relativistic

Papers devoted to the study of charmonia distribution amplitudes

- G. T. Bodwin, D. Kang, J. Lee, Phys.Rev. D74 (2006) 114028
- V.V. Braguta, A.K. Likhoded, A.V. Luchinsky, Phys.Lett. B646 (2007) 80
- V.V. Braguta, Phys.Rev. D75 (2007) 094016
- J.P. Ma, Z.G. Si, Phys.Lett. B647 (2007) 419
- G. Bell, T. Feldmann, JHEP 0804 (2008) 061
- V.V. Braguta, Phys.Rev. D77 (2008) 034026
- V.V. Braguta, A.K. Likhoded, A.V. Luchinsky, Phys.Rev. D79 (2009) 074004
- Chien-Wen Hwang, JHEP 0910 (2009) 074
- Chien-Wen Hwang, Phys.Rev. D81 (2010) 114024
- Chien-Wen Hwang, Phys.Rev. D86 (2012) 094031
- V.V. Braguta, A.K. Likhoded, A.V. Luchinsky, Phys.Atom.Nucl. 75 (2012) 97

Leading twist distribution amplitudes of S-wave mesons

$$\langle 0|\bar{Q}(z)\gamma_\alpha\gamma_5[z,-z]Q(-z)|\eta_c\rangle_\mu = if_{\eta_c} p_\alpha \int_{-1}^{1} d\xi\, e^{i(pz)\xi}\phi_P(\xi,\mu)$$

$$\langle 0|\bar{Q}(z)\gamma_\alpha[z,-z]Q(-z)|J/\psi(\epsilon_{\lambda=0},p)\rangle_\mu = f_L p_\alpha \int_{-1}^{1} d\xi\, e^{i(pz)\xi}\phi_L(\xi,\mu)$$

$$\langle 0|\bar{Q}(z)\sigma_{\alpha\beta}[z,-z]Q(-z)|J/\psi(\epsilon_{\lambda=\pm1},p)\rangle_\mu = f_T(\mu)(\epsilon_\alpha p_\beta - \epsilon_\beta p_\alpha)\int_{-1}^{1} d\xi\, e^{i(pz)\xi}\phi_T(\xi,\mu)$$

$$[z,-z] = P\exp[ig\int_{-z}^{z} dx^\mu A_\mu(x)]$$

At leading order approximation in relative velocity

$$\phi_P(\xi) = \phi_L(\xi) = \phi_T(\xi) = \phi(\xi)$$

Leading twist distribution amplitudes of P-wave mesons

$$\langle \chi_{c0}(p)|\bar{Q}(z)\gamma_\mu[z,-z]Q(-z)|0\rangle = f_{\chi_0} p_\mu \int_{-1}^{1} d\xi\, e^{i(pz)\xi} \phi_{\chi_0}(\xi,\mu)$$

$$\langle \chi_{c1}(p,\epsilon_{\lambda=0})|\bar{Q}(z)\gamma_\mu\gamma_5[z,-z]Q(-z)|0\rangle = f'_{\chi_1} p_\mu \int_{-1}^{1} d\xi\, e^{i(pz)\xi} \phi'_{\chi_1}(\xi,\mu)$$

$$\langle \chi_{c1}(p,\epsilon_{\lambda=\pm1})|\bar{Q}(z)\sigma_{\mu\nu}[z,-z]Q(-z)|0\rangle = f_{\chi_1} e_{\mu\nu\alpha\beta}\epsilon^\alpha p^\beta \int_{-1}^{1} d\xi\, e^{i(pz)\xi} \phi_{\chi_1}(\xi,\mu)$$

$$\langle h_c(p,\epsilon_{\lambda=0})|\bar{Q}(z)\gamma_\mu\gamma_5[z,-z]Q(-z)|0\rangle = f_h p_\mu \int_{-1}^{1} d\xi\, e^{i(pz)\xi} \phi_h(\xi,\mu)$$

$$\langle h_c(p,\epsilon_{\lambda=\pm1})|\bar{Q}(z)\sigma_{\mu\nu}[z,-z]Q(-z)|0\rangle = f'_h e_{\mu\nu\alpha\beta}\epsilon^\alpha p^\beta \int_{-1}^{1} d\xi\, e^{i(pz)\xi} \phi'_h(\xi,\mu)$$

$$\langle \chi_2(p,\epsilon_{\lambda=0})|\bar{Q}(z)\gamma_\mu[z,-z]Q(-z)|0\rangle = f_{\chi_2} p_\mu \int_{-1}^{1} d\xi\, e^{i(pz)\xi} \phi_{\chi_2}(\xi,\mu)$$

$$\langle \chi_2(p,\epsilon_{\lambda=\pm1})|\bar{Q}(z)\sigma_{\mu\nu}[z,-z]Q(-z)|0\rangle = \tilde{f}_{\chi_2} M_{\chi_2}(\rho_\mu p_\nu - \rho_\nu p_\mu) \int d\xi\, e^{i(pz)\xi} \tilde{\phi}_{\chi_2}(\xi),\ \rho_\mu = \frac{\epsilon_{\mu\sigma} z^\sigma}{pz}$$

At leading order approximation in relative velocity

$$\Phi(\xi) = \phi_{\chi_0}(\xi) = \phi_{\chi_1}(\xi) = \phi_h(\xi) = \phi_{\chi_2}(\xi) = \tilde{\phi}_{\chi_2}(\xi)$$

$$\phi'_{\chi_1}(\xi) = \phi'_h(\xi) = -\int_{-1}^{\xi} dt\, \Phi(t)$$

Parametrization through the expansion coefficients C_n

$$\phi(\xi,\mu) = \frac{3}{2}(1-\xi^2)\left(1+\sum_n C_n(\mu) G_n^{3/2}(\xi)\right)$$

- $G_n^{3/2}(\xi)$ is Gegenbauer polynomials
- $C_n(\mu)$ is multiplicatively renormalized

Not applicable for nonrelativistic mesons

Parametrization through the moments $\langle \xi^n \rangle$

$$\langle \xi^n \rangle_\mu = \int_{-1}^{+1} d\xi \xi^n \phi(\xi,\mu) \sim \langle M|\bar{Q}\gamma_+\gamma_5 (D_+)^n Q|0\rangle_\mu$$

Applicable for nonrelativistic mesons

Functional approach

Based on Schrodinger or Bethe-Salpeter equations

Operator approach

- NRQCD
- QCD sum rules
- Lattice QCD

Functional approach

Brodsky-Huang-Lepage procedure:

- Solve Schodinger (Bethe-Salpeter) equation and obtain $\psi(\vec{k})$
- Make the substitution

$$k_\perp \to k_\perp, \quad k_z = \xi \frac{M_0}{2}, \quad M_0 = 4\frac{m_c^2+k_\perp^2}{1-\xi^2}$$

- Integrate out transverse momentum

$$\phi(\xi,\mu) \sim \int^{k_\perp^2<\mu^2} d^2k_\perp \psi(\xi,k_\perp)$$

Moments within NRQCD

At leading order approximation in relative velocity v

$$if_{\eta_c} p_+^{2k+1}\langle\xi^n\rangle = \langle 0|\bar{Q}\gamma_+\gamma_5(iD_+)^{2k}Q|\eta_c(p)\rangle \to \langle 0|\chi^+(i\vec{D})^2)^k\psi|\eta_c(p)\rangle$$

$$\langle\xi^n\rangle = \frac{\langle v^n\rangle}{n+1} + O(v^{n+2}), \quad \langle v^{2k}\rangle = \frac{\langle 0|\chi^+(i\vec{D})^2)^k\psi|\eta_c(p)\rangle}{\langle 0|\chi^+\psi|\eta_c(p)\rangle}$$

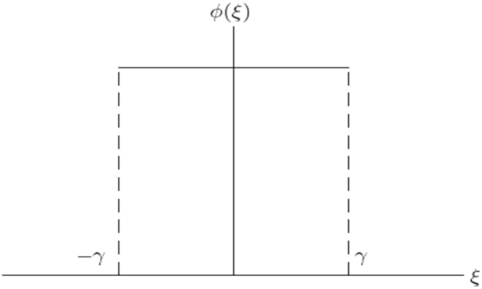
The meaning of the second moment

$$\langle\xi^2\rangle = \frac{\langle v^2\rangle}{3}$$

- $\langle\xi^2\rangle$ is one-dimensional velocity
- The width of distribution amplitude is determined by relative velocity $\langle v^2\rangle$

Distribution amplitudes at leading order approximation in v^2

- $\langle v^n \rangle = \gamma^n$ for $n = 2k$ (G. T. Bodwin, D. Kang, J. Lee, Phys.Rev. D74 (2006) 014014)
- $\gamma^2 = \langle v^2 \rangle = 0.25 \pm 0.08$ for 1S states and $\gamma^2 = \langle v^2 \rangle = 0.65 \pm 0.42$ for 2S states
- One gets the equations $\langle \xi^n \rangle = \frac{\gamma^n}{n+1}$
- The solution is $\phi(\xi) = \frac{1}{\sqrt{\gamma}} \theta(\gamma - \xi^2)$



Moments within QCD sum rules

Advantage: Free from relativistic corrections

Disadvantage: Sensitivity to QCD sum rules parameters

QCD sum rules are the most accurate approach to study charmonia distribution amplitudes

The model of distribution amplitudes from QCD sum rules

1S states: $\phi_{1s}(\xi) \sim (1-\xi^2)\exp\left(-\frac{\beta}{1-\xi^2}\right)$

2S states: $\phi_{2s}(\xi) \sim (1-\xi^2)(\alpha+\xi^2)\exp\left(-\frac{\beta}{1-\xi^2}\right)$

1P states: $\phi_{1p}(\xi) \sim (1-\xi^2)\xi\exp\left(-\frac{\beta}{1-\xi^2}\right)$

Moments of 1S charmonia

$\langle \xi^n \rangle$	Buchmuller-Tye potential	Cornell potential	NRQCD	QCD sum rules
$n=2$	0.086	0.071	0.075 ± 0.011	0.070 ± 0.007
$n=4$	0.020	0.014	0.010 ± 0.003	0.012 ± 0.002
$n=6$	0.0067	0.0044	0.0017 ± 0.0007	0.0032 ± 0.0009

Moments of 2S charmonia

$\langle \xi^n \rangle$	Buchmuller-Tye potential	Cornell potential	NRQCD	QCD sum rules
$n=2$	0.16	0.16	0.22 ± 0.14	$0.18\ ^{+0.05}_{-0.07}$
$n=4$	0.042	0.046	0.085 ± 0.110	$0.051\ ^{+0.031}_{-0.031}$
$n=6$	0.015	0.016	0.039 ± 0.077	$0.017\ ^{+0.016}_{-0.014}$

Moments of 1P charmonia

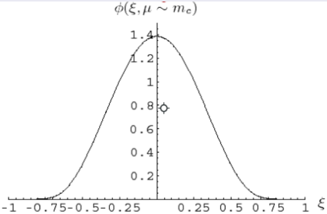
$\langle \xi^n \rangle$	Buchmuller-Tye potential	Cornell potential	NRQCD	QCD sum rules
$n=3$	0.18	0.16	$--$	0.18 ± 0.03
$n=5$	0.047	0.040	$--$	0.005 ± 0.001

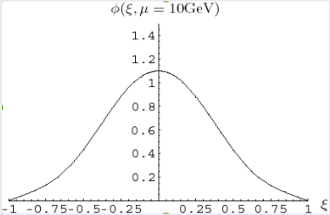
NRQCD matrix elements

$\langle v^2 \rangle$ **parameterizes relativistic corrections**

- $\langle v^2 \rangle_{1S} = 0.21 \pm 0.07$
- $\langle v^2 \rangle_{1P} = 0.30 \pm 0.10$
- $\langle v^2 \rangle_{2S} = 0.54 \pm 0.35$

Distribution amplitude before/after RG evolution

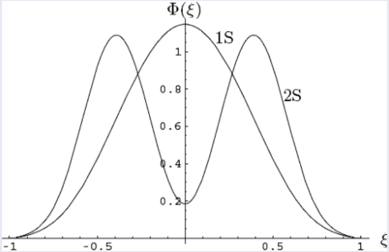




- Before evolution the DA in the end point region $\xi \sim \pm 1$ is suppressed (narrow function)
- After evolution relativistic tail appeared $\phi(\xi \sim \pm 1, \mu \sim Q) \sim \alpha_s(Q) \log\left(\frac{Q}{m_c}\right)(1-\xi^2)$
- Relativistic tail is due to the radiative corrections (in agreement with NRQCD)
- RG evolution of distribution amplitudes of nonrelativistic meson is very rapid

Distribution amplitudes of 2S states

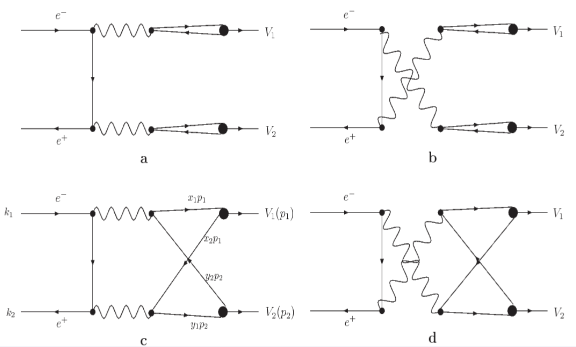
$$\phi(\xi, \mu \sim m_c) \sim (1 - \xi^2)\Phi(\xi, \mu \sim m_c)$$



Distribution amplitude of 2S state has 3 extremums

NRQCD expansion near to the $v^2 \sim 0$ $(\xi \sim 0)$
is badly convergent

$e^+e^- \to V_1 V_2$



NRQCD

G. T. Bodwin, J. Lee and E. Braaten, Phys. Rev. D 67, 054023 (2003);
G. T. Bodwin, J. Lee and E. Braaten, Phys. Rev. Lett. 90, 162001 (2003);
A. V. Luchinsky, arXiv:hep-ph/0301190;
B. Gong and J. X. Wang, Phys. Rev. Lett. 100, 181803 (2008)
G. T. Bodwin, E. Braaten, J. Lee and C. Yu, Phys. Rev. D 74, 074014 (2006)

$\sigma(e^+e^- \to J/\psi J/\psi) = 1.69 \pm 0.35 fb$
$\sigma(e^+e^- \to J/\psi\psi') = 0.96 \pm 0.36 fb$
$\sigma(e^+e^- \to \psi'\psi') = 0.11 \pm 0.09 fb$

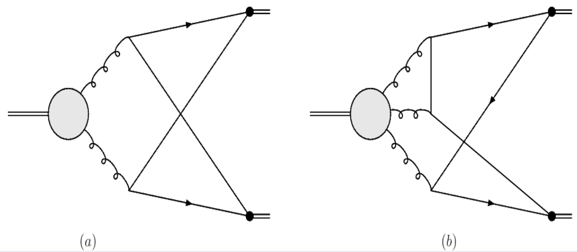
Light cone expansion formalism

$\sigma(e^+e^- \to J/\psi J/\psi) = 2.02 \pm 0.25 fb$
$\sigma(e^+e^- \to J/\psi\psi') = 1.32 \pm 0.16 fb$
$\sigma(e^+e^- \to \psi'\psi') = 0.20 \pm 0.06 fb$

Experiment (Belle collaboration, Phys. Rev. D 70, 071102 (2004))

$\sigma(e^+e^- \to J/\psi J/\psi) \times Br_{>2}(J/\psi) < 9.1$ fb   90 % CL,
$\sigma(e^+e^- \to J/\psi\psi') \times Br_{>2}(\psi') < 13.3$ fb   90 % CL,
$\sigma(e^+e^- \to J/\psi\psi') \times Br_{>0}(J/\psi) < 16.9$ fb   90 % CL,
$\sigma(e^+e^- \to \psi'\psi') \times Br_{>0}(\psi') < 5.2$ fb   90 % CL,

Bottomonia decays to double charmonia



All leading twist C-even bottomonia decays were considered in paper
V.V. Braguta, A.K. Likhoded, A.V. Luchinsky, Phys.Rev. D80 (2009) 094008, Erratum-ibid. D85 (2012) 119901

Leading twist decays of the η_b meson

reaction	$\Gamma_{\rm NRQCD}$, eV	$\Gamma_{\rm LC}$, eV	$\mathrm{Br}_{\rm LC}, 10^{-5}$
$\eta_b \to h_c \psi$	$16.^{+2.3}_{-1.5} \pm 8.4 \pm 8.1$	$32. \pm 2.6 \pm 6.1 \pm 8.2$	0.33
$\eta_b \to h_c \psi(2S)$	$7.8^{+1.1}_{-0.72} \pm 6.5 \pm 3.9$	$16. \pm 1.4 \pm 3.1 \pm 4.2$	0.17
$\eta_b \to \eta_c \chi_{c0}$	$13.^{+3.5}_{-2.7} \pm 6.8 \pm 6.5$	$9.1 \pm 0.73 \pm 4.6 \pm 2.3$	0.092
$\eta_b \to \eta_c(2S) \chi_{c0}$	$6.3^{+1.7}_{-1.3} \pm 5.2 \pm 3.1$	$4.3 \pm 0.36 \pm 3. \pm 1.1$	0.043
$\eta_b \to \eta_c \chi_{c2}$	$3.6^{+1.1}_{-1.1} \pm 8.4 \pm 1.8$	$18. \pm 1.4 \pm 8.7 \pm 4.5$	0.18
$\eta_b \to \eta_c(2S) \chi_{c2}$	$1.7^{+0.54}_{-0.53} \pm 4.1 \pm 0.86$	$8.2 \pm 0.7 \pm 5.6 \pm 2.1$	0.083
$\eta_b \to \chi_{c0} \chi_{c1}$	$2.3^{+0.21}_{-0.29} \pm 2.2 \pm 1.2$	$4.4 \pm 0.38 \pm 2.3 \pm 1.1$	0.045
$\eta_b \to \chi_{c1} \chi_{c2}$	$0.93^{+0.22}_{-0.21} \pm 2.9 \pm 0.46$	$8.6 \pm 0.73 \pm 4.3 \pm 2.2$	0.087

Leading twist decays of the χ_{b0} meson

$\chi_{b0} \to \eta_c \chi_{c1}$	$1.9^{+0.23}_{-0.27} \pm 1.9 \pm 0.93$	$9.8 \pm 0.25 \pm 4.8 \pm 2.5$	1.2
$\chi_{b0} \to \eta_c(2S) \chi_{c1}$	$0.9^{+0.11}_{-0.13} \pm 1.1 \pm 0.45$	$5.9 \pm 1. \pm 4. \pm 1.5$	0.73
$\chi_{b0} \to \chi_{c0} \chi_{c2}$	$0.00015^{+0.0007}_{-0.00014} \pm 0.038 \pm 7.6 \times 10^{-5}$	$0.14 \pm 0.034 \pm 0.07 \pm 0.034$	0.017
$\chi_{b0} \to \eta_c \eta_c$	$7.9^{+0.69}_{-0.57} \pm 5.6 \pm 4.$	$10. \pm 0.45 \pm 4.9 \pm 2.5$	1.3
$\chi_{b0} \to \eta_c \eta_c(2S)$	$7.8^{+0.68}_{-0.56} \pm 7.5 \pm 3.9$	$12. \pm 2.1 \pm 8.3 \pm 3.$	1.5
$\chi_{b0} \to \eta_c(2S) \eta_c(2S)$	$1.9^{+0.16}_{-0.14} \pm 2.8 \pm 0.94$	$3.6 \pm 1.4 \pm 3. \pm 0.91$	0.45
$\chi_{b0} \to \psi \psi$	$4.3^{+0.28}_{-0.25} \pm 5.7 \pm 2.2$	$15. \pm 0.68 \pm 0.51 \pm 3.8$	1.9
$\chi_{b0} \to \psi \psi(2S)$	$4.3^{+0.28}_{-0.25} \pm 6.3 \pm 2.1$	$20. \pm 3.5 \pm 0.62 \pm 5.$	2.5
$\chi_{b0} \to \psi(2S) \psi(2S)$	$1.^{+0.068}_{-0.06} \pm 1.9 \pm 0.52$	$6.5 \pm 2.5 \pm 0.18 \pm 1.6$	0.81
$\chi_{b0} \to h_c h_c$	$0.014^{+0.0025}_{-0.0035} \pm 0.021 \pm 0.0071$	$0.3 \pm 0.074 \pm 0.079 \pm 0.075$	0.037
$\chi_{b0} \to \chi_{c0} \chi_{c0}$	$0.006^{+0.0076}_{-0.0041} \pm 0.022 \pm 0.003$	$0.035 \pm 0.0087 \pm 0.018 \pm 0.0088$	0.0044
$\chi_{b0} \to \chi_{c1} \chi_{c1}$	$0.087^{+0.037}_{-0.025} \pm 0.63 \pm 0.043$	$2.4 \pm 0.12 \pm 1.2 \pm 0.6$	0.3
$\chi_{b0} \to \chi_{c2} \chi_{c2}$	$0.0032^{+0.0038}_{-0.0012} \pm 0.035 \pm 0.0016$	$0.13 \pm 0.033 \pm 0.066 \pm 0.033$	0.017

Leading twist decays of the χ_{b1} meson

$\chi_{b1} \to h_c \psi$	$0.18^{+0.0016}_{-0.0077} \pm 0.13 \pm 0.091$	$0.88 \pm 0.078 \pm 0.17 \pm 0.22$	0.68
$\chi_{b1} \to h_c \psi(2S)$	$0.089^{+0.00076}_{-0.0037} \pm 0.086 \pm 0.045$	$0.67 \pm 0.18 \pm 0.13 \pm 0.17$	0.52
$\chi_{b1} \to \eta_c \chi_{c0}$	$0.038^{+0.0048}_{-0.0055} \pm 0.038 \pm 0.019$	$0.25 \pm 0.022 \pm 0.12 \pm 0.061$	0.19
$\chi_{b1} \to \eta_c(2S) \chi_{c0}$	$0.019^{+0.0023}_{-0.0027} \pm 0.022 \pm 0.0093$	$0.17 \pm 0.046 \pm 0.12 \pm 0.043$	0.13
$\chi_{b1} \to \eta_c \chi_{c2}$	$0.11^{+0.0031}_{-0.0066} \pm 0.075 \pm 0.055$	$0.48 \pm 0.042 \pm 0.24 \pm 0.12$	0.37
$\chi_{b1} \to \eta_c(2S) \chi_{c2}$	$0.054^{+0.0015}_{-0.0032} \pm 0.051 \pm 0.027$	$0.33 \pm 0.089 \pm 0.23 \pm 0.083$	0.26
$\chi_{b1} \to \chi_{c0} \chi_{c1}$	$0.08^{+0.022}_{-0.018} \pm 0.061 \pm 0.04$	$0.12 \pm 0.015 \pm 0.06 \pm 0.03$	0.091
$\chi_{b1} \to \chi_{c1} \chi_{c2}$	$0.018^{+0.0015}_{-0.00087} \pm 0.028 \pm 0.0091$	$0.23 \pm 0.03 \pm 0.11 \pm 0.057$	0.18

Leading twist decays of the χ_{b2} meson

$\chi_{b2} \to \eta_c \chi_{c1}$	$0.26^{+0.0073}_{-0.015} \pm 0.18 \pm 0.13$	$0.63 \pm 0.011 \pm 0.31 \pm 0.16$	0.31
$\chi_{b2} \to \eta_c(2S) \chi_{c1}$	$0.13^{+0.0036}_{-0.0075} \pm 0.12 \pm 0.064$	$0.35 \pm 0.044 \pm 0.24 \pm 0.086$	0.17
$\chi_{b2} \to \chi_{c0} \chi_{c2}$	$0.076^{+0.02}_{-0.017} \pm 0.058 \pm 0.038$	$0.049 \pm 0.0075 \pm 0.025 \pm 0.012$	0.025
$\chi_{b2} \to \eta_c \eta_c$	$0.26^{+0.069}_{-0.069} \pm 0.69 \pm 0.13$	$0.64 \pm 0.02 \pm 0.31 \pm 0.16$	0.32
$\chi_{b2} \to \eta_c(2S) \eta_c$	$0.26^{+0.068}_{-0.068} \pm 0.7 \pm 0.13$	$0.71 \pm 0.092 \pm 0.48 \pm 0.18$	0.36
$\chi_{b2} \to \eta_c(2S) \eta_c(2S)$	$0.062^{+0.016}_{-0.017} \pm 0.18 \pm 0.031$	$0.2 \pm 0.068 \pm 0.17 \pm 0.051$	0.1
$\chi_{b2} \to \psi \psi$	$9.7^{+0.87}_{-0.73} \pm 6.9 \pm 4.9$	$9.6 \pm 0.42 \pm 0.33 \pm 2.4$	4.8
$\chi_{b2} \to \psi(2S) \psi$	$9.6^{+0.86}_{-0.72} \pm 9.3 \pm 4.8$	$11. \pm 1.9 \pm 0.35 \pm 2.8$	5.7
$\chi_{b2} \to \psi(2S) \psi(2S)$	$2.3^{+0.21}_{-0.17} \pm 3.5 \pm 1.2$	$3.4 \pm 1.4 \pm 0.094 \pm 0.84$	1.7
$\chi_{b2} \to h_c h_c$	$0.061^{+0.012}_{-0.012} \pm 0.17 \pm 0.031$	$0.48 \pm 0.034 \pm 0.13 \pm 0.12$	0.24
$\chi_{b2} \to \chi_{c0} \chi_{c0}$	$0.0021^{+0.00037}_{-0.00044} \pm 0.0037 \pm 0.0011$	$0.013 \pm 0.0019 \pm 0.0065 \pm 0.0032$	0.0063
$\chi_{b2} \to \chi_{c1} \chi_{c1}$	$0.026^{+0.0069}_{-0.0074} \pm 0.063 \pm 0.013$	$0.28 \pm 0.03 \pm 0.14 \pm 0.069$	0.14
$\chi_{b2} \to \chi_{c2} \chi_{c2}$	$0.028^{+0.0038}_{-0.0052} \pm 0.042 \pm 0.014$	$0.54 \pm 0.11 \pm 0.27 \pm 0.13$	0.27
$\chi_{b2} \to h_c \psi$	$1.1^{+0.12}_{-0.14} \pm 1. \pm 0.57$	$3.6 \pm 0.09 \pm 0.68 \pm 0.9$	1.8
$\chi_{b2} \to h_c \psi(2S)$	$0.56^{+0.057}_{-0.069} \pm 0.62 \pm 0.28$	$2.1 \pm 0.36 \pm 0.39 \pm 0.52$	1.
$\chi_{b2} \to \chi_{c1} \chi_{c2}$	$0.044^{+0.0008}_{-0.0015} \pm 0.036 \pm 0.022$	$0.49 \pm 0.1 \pm 0.24 \pm 0.12$	0.25

Comparison of different results

	$Br \cdot 10^{-5}$ NRQCD [1]	$Br \cdot 10^{-5}$ NRQCD [2]	$Br \cdot 10^{-5}$ LCF [3]	$Br \cdot 10^{-5}$ Exp. [4]
$\chi_{b0} \to 2J/\psi$	0.5	1.9	1.9 ± 0.5	< 7.1
$\chi_{b2} \to 2J/\psi$	3.4	17.5	4.8 ± 1.2	< 4.5
$\chi_{b0} \to J/\psi\ \psi(2S)$	—	—	2.5 ± 0.7	< 12
$\chi_{b2} \to J/\psi\ \psi(2S)$	—	—	5.7 ± 1.8	< 4.9
$\chi_{b0} \to 2\psi(2S)$	—	—	0.8 ± 0.4	< 3.1
$\chi_{b2} \to 2\psi(2S)$	—	—	1.7 ± 0.8	< 1.6

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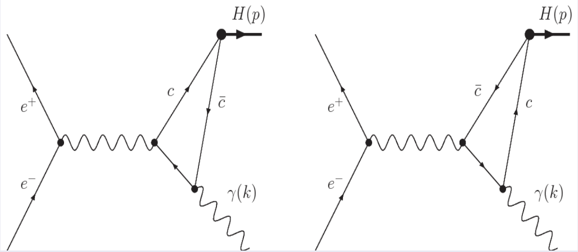
NRQCD calculation:
[1] Juan Zhang, Hairong Dong, Feng Feng, Phys.Rev. D84 (2011) 094031
[2] Wen-Long Sang, Reyima Rashidin, U-Rae Kim, Jungil Lee, Phys.Rev. D84 (2011) 074026

Light cone formalism calculation:
[3] V.V. Braguta, A.K. Likhoded, A.V. Luchinsky, Phys.Rev. D80 (2009) 094008
Erratum-ibid. D85 (2012) 119901

Belle experiment:
[4] Phys.Rev. D85 (2012) 071102

$e^+e^- \to H+\gamma, H=\eta_c, \chi_{c0}, \chi_{c1}, \chi_{c2},$



$e^+e^- \to H + \gamma$

H	$\sigma(H)$(fb) LCF [1]	$\sigma(H)$(fb) NRQCD [2]	$\sigma(H)$(fb) NRQCD [3]	$\sigma(H)$(fb) NRQCD [4]
η_c	41.6 ± 14.1	$82.0^{+21.4}_{-19.8}$	$42.5 - 53.7$	$68.0^{+22.2}_{-20.3}$
η_c'	24.2 ± 14.5	$49.2^{+9.4}_{-7.4}$	$27.7 - 35.1$	$42.6^{+10.9}_{-8.8}$
χ_{c0}	6.1 ± 3.9	$1.3^{+0.2}_{-0.2}$	$1.53 - 2.48$	$1.36^{+0.26}_{-0.26}$
χ_{c1}	24.2 ± 13.3	$13.7^{+3.4}_{-3.1}$	$11.1 - 17.7$	$10.9^{+3.7}_{-3.4}$
χ_{c2}	12.0 ± 17.4	$5.3^{+1.6}_{-1.3}$	$1.65 - 3.53$	$1.95^{+1.85}_{-1.56}$

Light cone formalism calculation:
[1] V.V. Braguta, Phys.Rev. D82 (2010) 074009

NRQCD calculation:
[2] H. S. Chung, J. Lee and C. Yu, Phys. Rev. D 78, 074022 (2008)
[3] D. Li, Z. G. He and K. T. Chao, Phys. Rev. D 80, 114014 (2009)
[4] W. L. Sang and Y. Q. Chen, Phys. Rev. D 81, 034028 (2010)

From LCF to NRQCD

- Take $\phi(\xi,\mu)=\delta(\xi)$ at scale $\mu\sim m_c$
- Evolve $\phi(\xi,\mu)$ from $\mu\sim m_c$ to $\mu\sim\sqrt{s}$ (ERBL equation)
- Substitute to the expression for LCF

$$\langle\eta_c(p)\gamma(k)|J_\mu(0)|0\rangle = F\; e_{\mu\nu\alpha\beta}\epsilon^\nu p^\alpha k^\beta, \quad F=\frac{16\pi\alpha Q_c^2 f_{\eta_c}}{s}\int_{-1}^{1} d\xi\,\frac{\phi(\xi,\mu)}{(1-\xi^2)}$$

- LCF prediction for NRQCD

$$F=\frac{16\pi\alpha Q_c^2}{s}\sqrt{\frac{\langle O\rangle_S}{m_c}}\left(1+C_f\frac{\alpha_s(s)}{4\pi}\log\left(\frac{s}{m_c^2}\right)(3-2\log 2)\right)$$

- NRQCD result (W. L. Sang and Y. Q. Chen, Phys. Rev. D 81, 034028 (2010))

$$F=\frac{16\pi\alpha Q_c^2}{s}\sqrt{\frac{\langle O\rangle_S}{m_c}}\left(1+C_f\frac{\alpha_s(s)}{4\pi}\log\left(\frac{s}{m_c^2}\right)(3-2\log 2)\right)$$

- The same program can be repeated for any leading process with any desirable accuracy in α_s, v^2:
  Yu Jia, Deshan Yang, Nucl.Phys. B814 (2009) 217-230
  Yu Jia, Jian-Xiong Wang, Deshan Yang, JHEP 1110 (2011) 105
- **Equivalence principle for NRQCD and LCF?**

For leading twist processes radiative corrections are under control

Conclusion

- The models of charmonia distribution amplitudes are built
- These models resume relativistic and leading logarithmic corrections to production amplitude
- For leading twist processes radiative corrections are under control
- The ψ', η_c' mesons are not nonrelativistic. One should resume relativisitic corrections!
- Equivalence principle for NRQCD and LCF?

THANK YOU