Endpoint Logarithms in $e^+e^- \to J/\psi + \eta_c$

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G. T. Bodwin, H. S. Chung, J. Lee, PoS CONFINEMENT X 133 (2013) [arXiv:1301.3937 [hep-ph]]
G. T. Bodwin, H. S. Chung, J. Lee, in preparation

The 9th International Workshop on Heavy Quarkonium
April 22-26, 2013, IHEP, Beijing

Outline

- $e^+e^- \to J/\psi + \eta_c$ in B factories
- Endpoint double logarithms in $e^+e^- \to J/\psi + \eta_c$
- Summary

$e^+e^- \to J/\psi + \eta_c$ in B Factories

- Belle (2004) : $\sigma[e^+e^- \to J/\psi + \eta_c] \times B_{>2} = 25.6 \pm 2.8 \pm 3.4\text{fb}$

  Belle, PRL89, 142001 (2002) and PRD70, 071102(R) (2004)

  BABAR (2005) : $\sigma[e^+e^- \to J/\psi + \eta_c] \times B_{>2} = 17.6 \pm 2.8^{+1.5}_{-2.1}\text{fb}$

  BABAR, PRD72, 031101(R) (2005)

- At LO in α_s and v, NRQCD severely underestimates the cross section :

  $\sigma_{\text{LO NRQCD}} = 3.78 \pm 1.26\text{fb}$ Braaten and Lee, PRD67, 054007 (2003)

  5.5fb Liu, He and Chao, PLB557, 45 (2003)

- The relativistic corrections enhance the cross section moderately. Bodwin, Lee and Yu, PRD77, 094018 (2008)

  The QCD NLO corrections give a ***substantial*** enhancement to the cross section.

  Zhang, Gao and Chao, PRL96, 092001 (2006)
  Gong and Wang, PRD77, 054028 (2008)

Double Logarithms in $e^+e^- \to J/\psi + \eta_c$

- A fixed-order calculation is unreliable, because the QCD NLO corrections involve $\alpha_s \log^2(m^2/s)$.
  (In B factories, $\alpha_s \log^2(m^2/s) \approx 3$)

Jia, Wang and Yang, JHEP10, 105 (2011)

- In this work we investigate the ***origin of the double logarithms*** at NLO in α_s.

Finding Double Logarithms in $e^+e^- \to J/\psi + \eta_c$

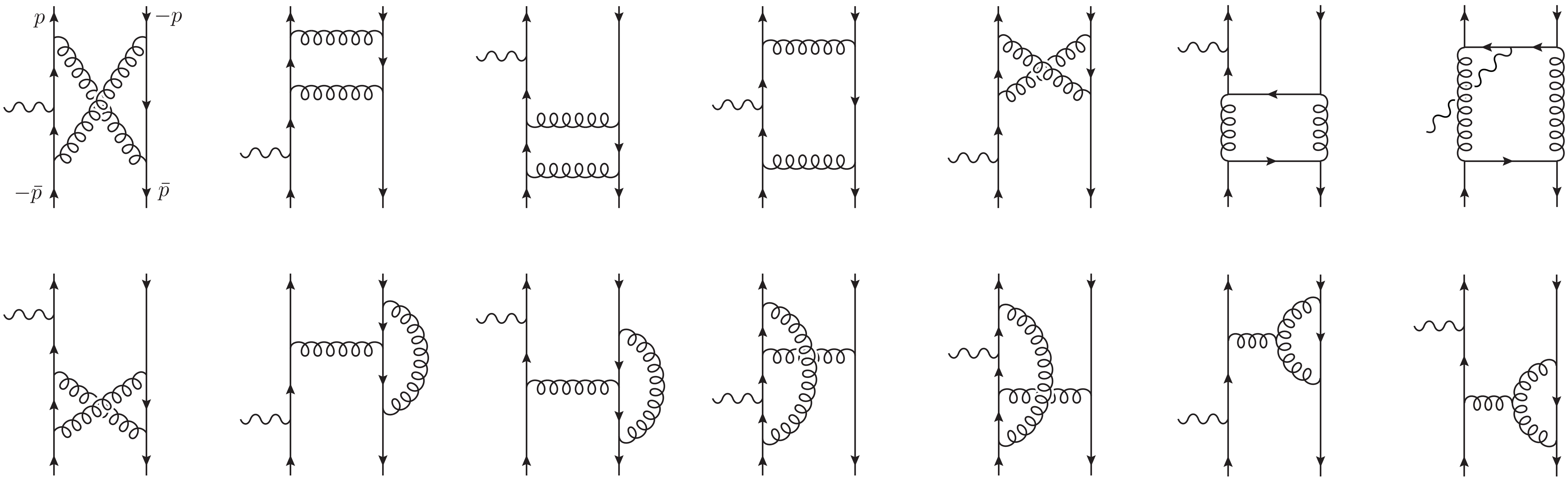
- We work at leading order in the relative velocity of $Q\bar{Q}$ of the charmonia.

$$P_{J/\psi} = 2p, \quad P_{\eta_c} = 2\bar{p}, \quad p^2 = \bar{p}^2 = m^2, \quad Q = P_{J/\psi} + P_{\eta_c}$$

- p is collinear to plus, $\bar{p}$ is collinear to minus.
- The logarithms in m can be identified as would-be soft and collinear singularities regulated by the quark mass m.
- The process $e^+e^- \to J/\psi + \eta_c$ violates the helicity selection rule, so that we cannot ignore the quark masses in the numerators.

Brodsky and Lepage, PRD24, 2848 (1981)

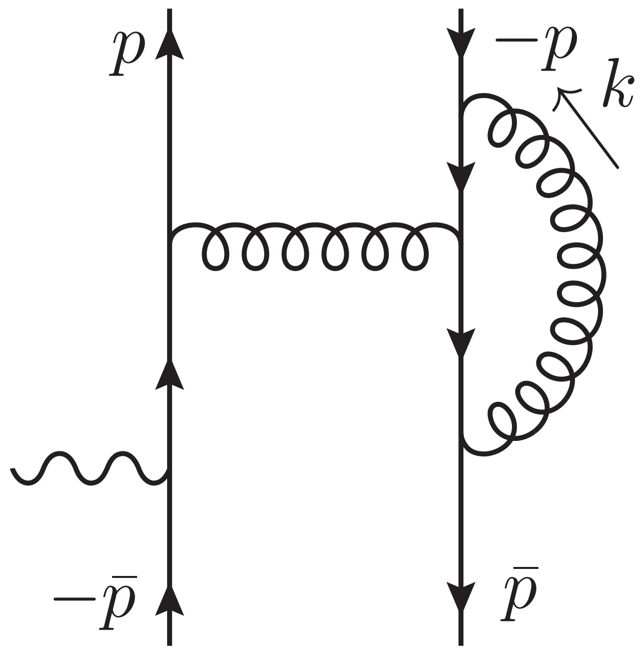
1-Loop Diagrams



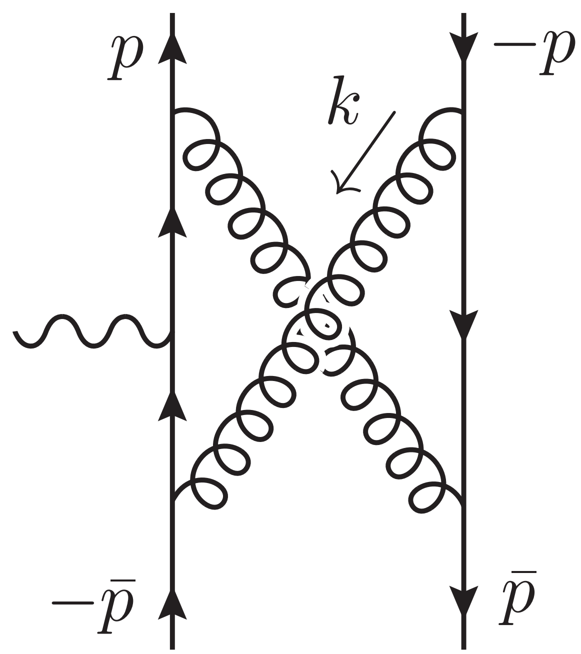
- The $Q\bar{Q}$ pair going up forms the J/ψ, the rest forms η_c.
- There are also charge conjugation diagrams.
- Diagrams that do not allow logarithmic power counting are ignored.

Origins of Double Logarithms

- The double logarithms in each diagram come from the Sudakov and endpoint logarithms.



- Sudakov double logs :
  the momentum of the gluon is simultaneously soft and collinear.



- Endpoint double logs : gluons carry almost all of the collinear momentum from a spectator line to an active line. The double log occurs when the momentum of the spectator line is simultaneously soft and collinear.

Origins of Double Logarithms

- A change of variables makes the endpoint double log look like a Sudakov double log.

Sudakov Double Logs

$$
\begin{aligned}
\mathcal{S} &= \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2+i\varepsilon)[(k-p)^2-m^2+i\varepsilon][(k+\bar{p})^2-m^2+i\varepsilon]} \\
&= \frac{i}{4\pi^2 Q^2}\left[\left(\tfrac{1}{\epsilon_{\mathrm{IR}}}-\log(m^2/\mu^2)\right)\log(m^2/Q^2)+\tfrac{1}{2}\log^2(m^2/Q^2)+\cdots\right]
\end{aligned}
$$

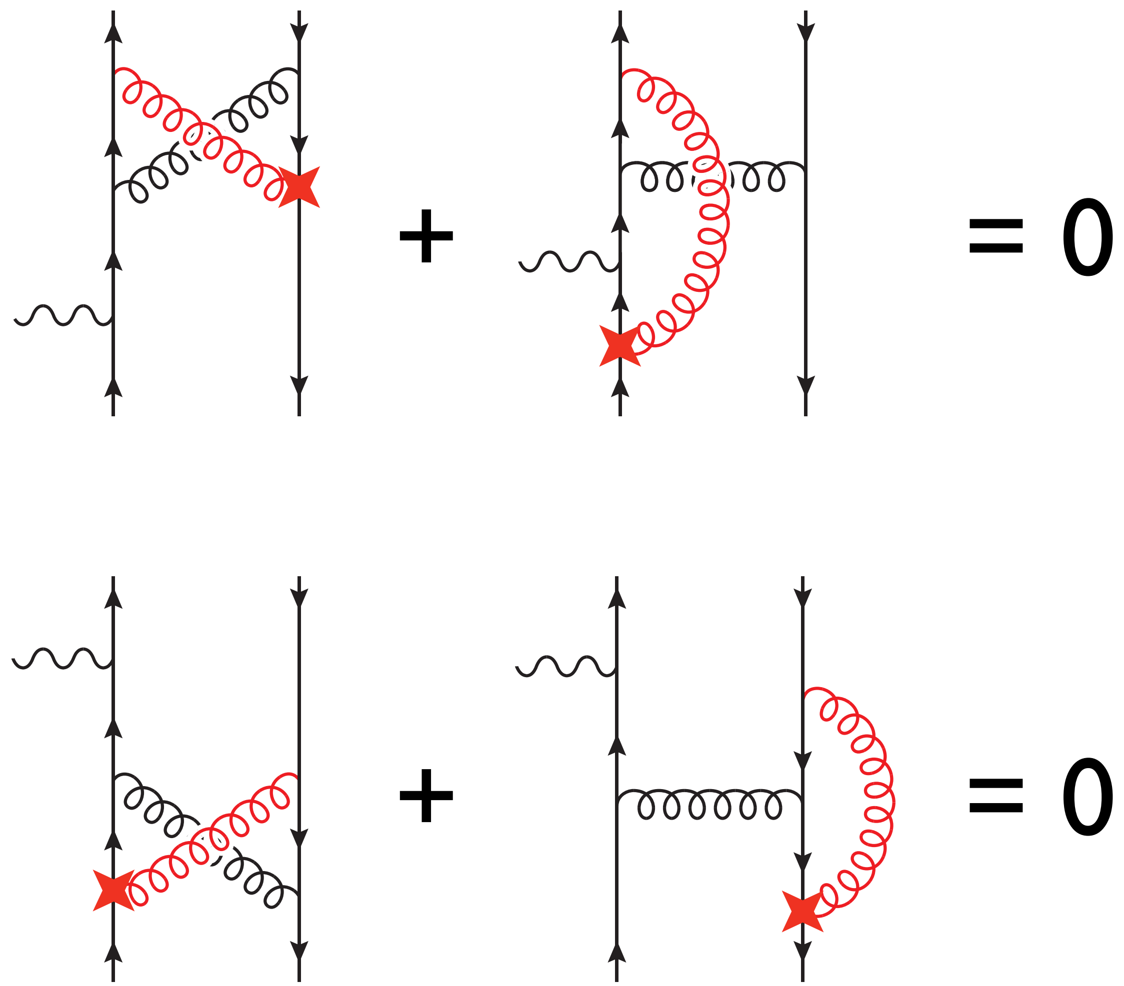
Endpoint Double Logs

$$
\begin{aligned}
\mathcal{E} &= \int \frac{d^d \ell}{(2\pi)^d} \frac{1}{(\ell^2-m^2+i\varepsilon)[(\ell+p)^2+i\varepsilon][(\ell-\bar{p})^2+i\varepsilon]} \\
&= \frac{i}{8\pi^2 Q^2}[\log^2(m^2/Q^2)+\cdots].
\end{aligned}
$$

Sudakov Double Logarithms

- By applying the soft approximation we can prove that the Sudakov logarithms ***cancel*** between soft gluon insertions into quark and antiquark lines.

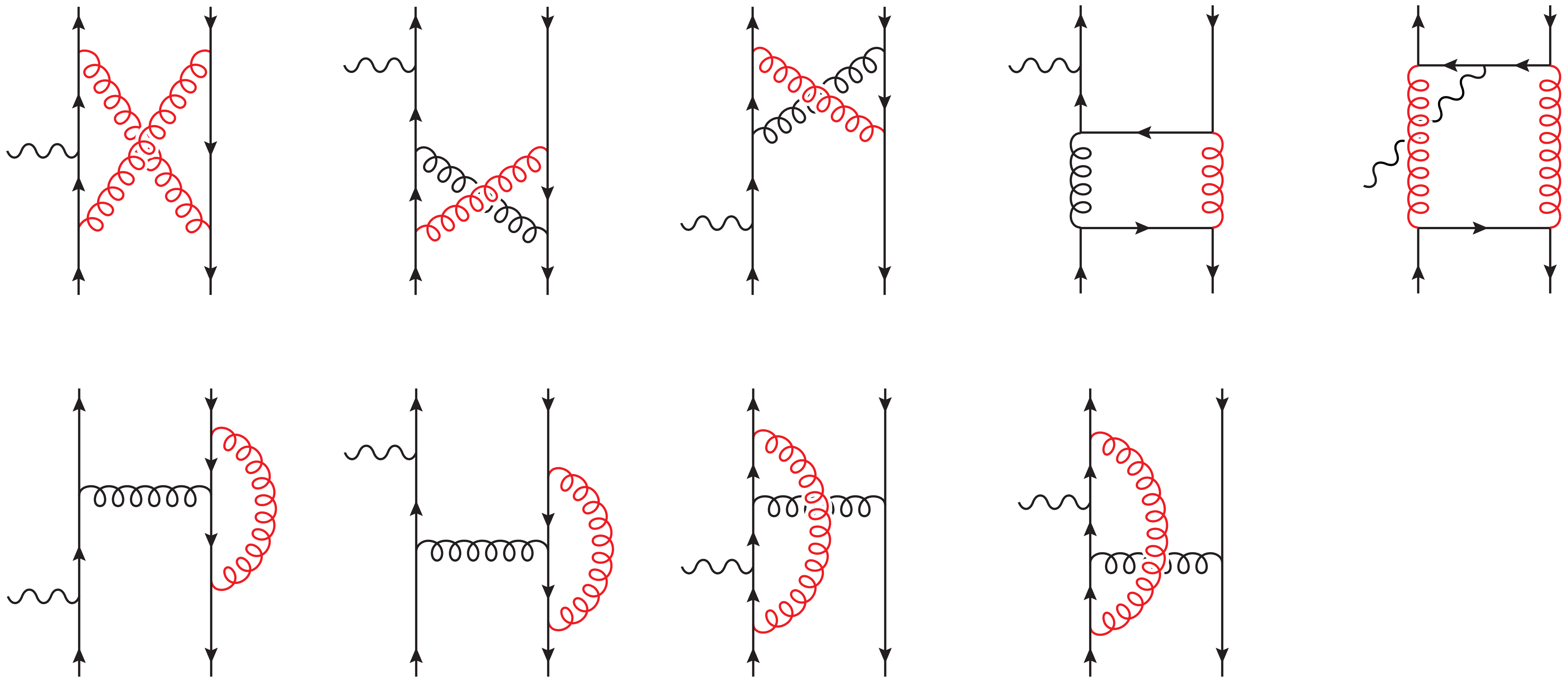
$\mathcal{S}$ cancel in the sum



... and so on.

Sudakov Double Logarithms

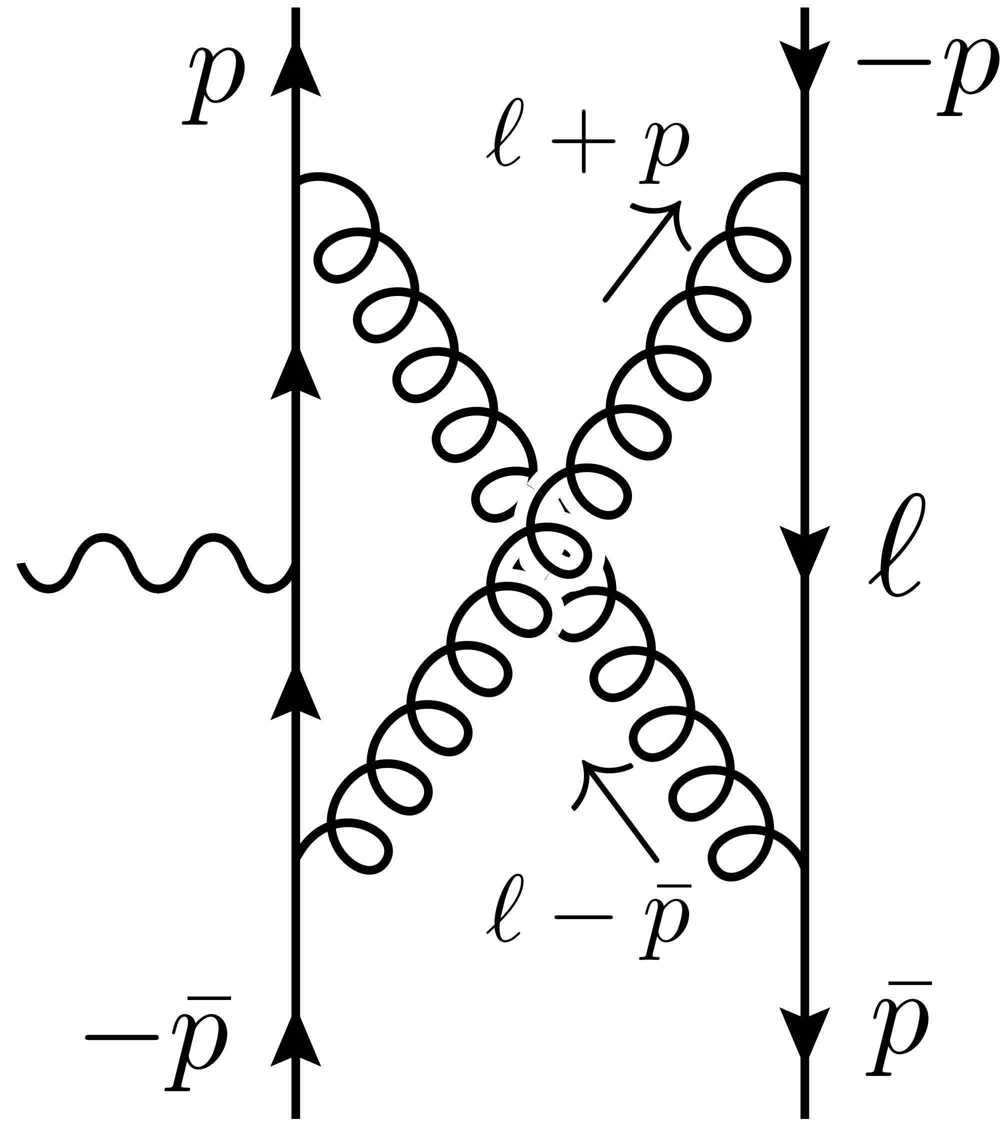
- Therefore the ***Sudakov double logarithms cancel in the sum of all diagrams.***



- The double logarithms in $e^+e^- \to J/\psi + \eta_c$ originate solely from endpoint double logarithms.

Endpoint Double Logarithms

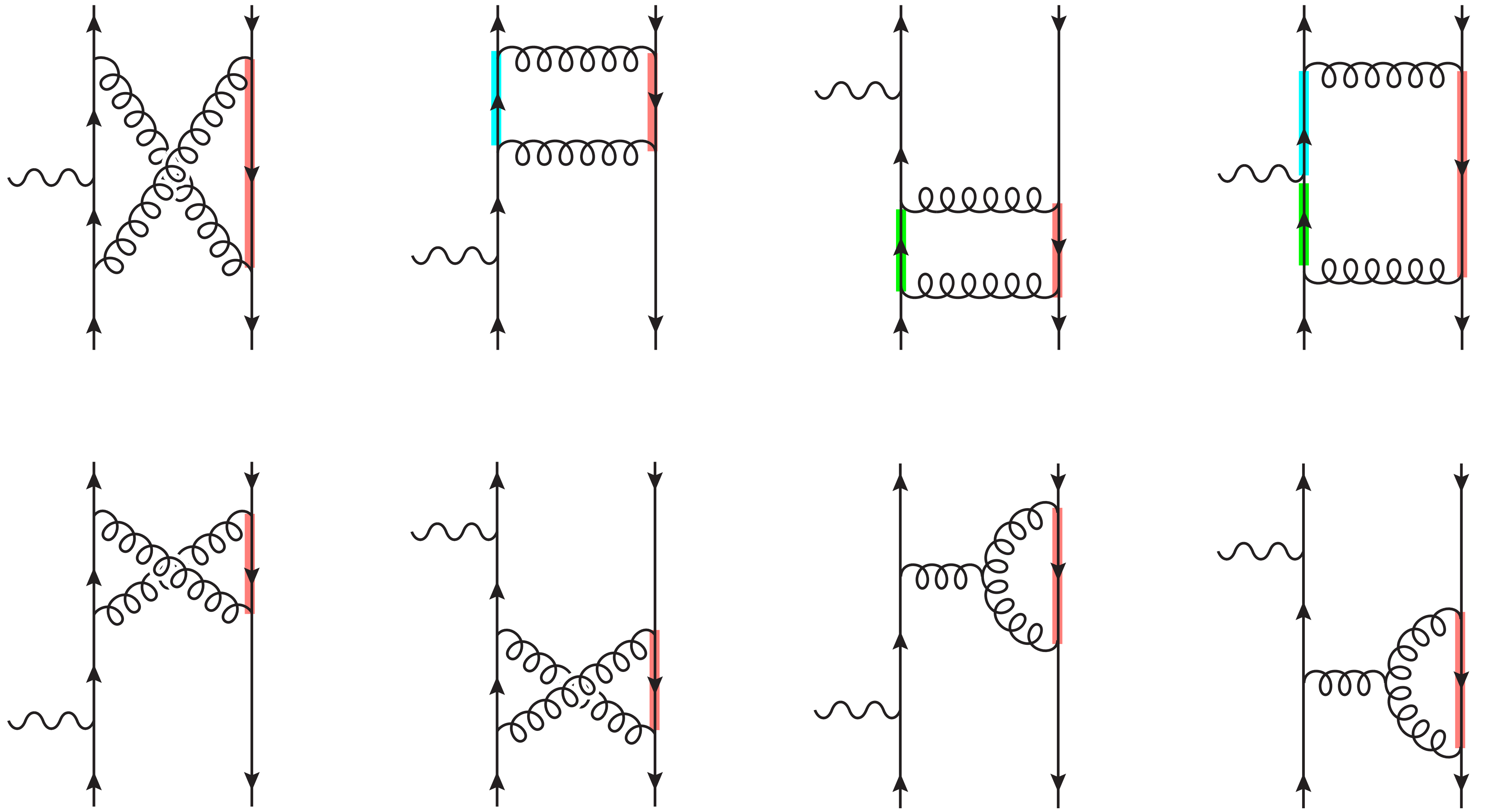
- Endpoint double logs appear when ℓ is soft-collinear.
- Helicity flip in the spectator quark line eliminates a factor of ℓ in the numerator, giving logarithmic power counting for soft-collinear scaling.



$$\mathcal{E} = \int \frac{d^d\ell}{(2\pi)^d} \frac{1}{(\ell^2 - m^2 + i\varepsilon)[(\ell + p)^2 + i\varepsilon][(\ell - \bar{p})^2 + i\varepsilon]}$$

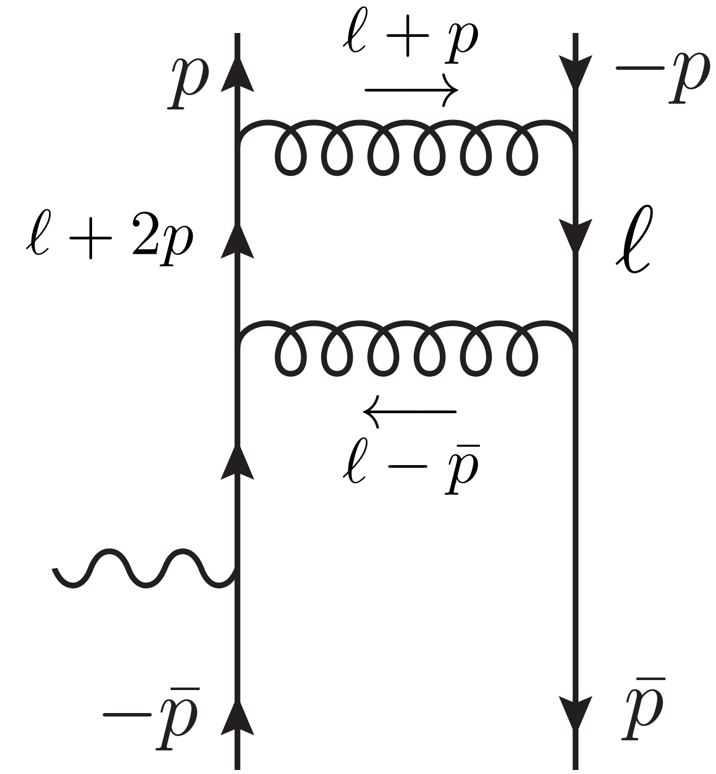
Endpoint Double Logarithms

- The following diagrams potentially contain endpoint logs.



- Soft quark line
- Collinear quark line with momentum containing p, but not $\bar{p}$
- Collinear quark line with momentum containing $\bar{p}$, but not p

Absence of Power Divergences



- The additional collinear lines may give power divergences.
- If ℓ is collinear to +, then all momenta surrounding the upper gluon is collinear to +.
  Then, the numerator vanishes because we can anticommute away any of the momenta so that it is adjacent to another one collinear to the same direction. The numerator must have a factor ℓ^- or $\ell_\perp$.
- Therefore the numerator must have a factor $\ell \cdot p$ or ℓ^2.
  $\ell \cdot p$ produces an endpoint double log, and
  ℓ^2 gives a collinear log, but not a double log.
- The same analysis can be repeated for the remaining diagrams.

Absence of Endpoint Logs without Helicity Flip

- Consider a process satisfying the helicity selection rule, such as $e^+e^- \to h_c + \eta_c$. In the soft approximation,

$$\int \frac{d^d\ell}{(2\pi)^d} \frac{\not{\ell}}{\ell^2 + i\varepsilon} \frac{1}{2\ell\cdot p + i\varepsilon} \frac{1}{-2\ell\cdot\bar{p} + i\varepsilon} \times \text{collinear fermion lines}$$

- The collinear fermion lines give

$$\frac{\not{\ell} + a\not{p}}{2a\ell\cdot p + i\varepsilon}, \quad \frac{\not{\ell} + b\not{\bar{p}}}{2b\ell\cdot\bar{p} + i\varepsilon}$$

  The $\not{p}$ or $\not{\bar{p}}$ is eliminated by the equations of motion; the collinear fermion lines do not change the power of the loop momentum in the integrand, giving $\sim \int_\ell 1/\ell^3$.
  Therefore ***endpoint logs are absent without helicity flip***.

- This even holds when the relative velocity of $Q\bar{Q}$ is retained.

Endpoint Logs with Helicity Flip

- In the helicity-suppressed processes, we have

$$\int \frac{d^d\ell}{(2\pi)^d} \frac{\not{\ell} + m}{\ell^2 + i\varepsilon} \frac{1}{2\ell \cdot p + i\varepsilon} \frac{1}{-2\ell \cdot \bar{p} + i\varepsilon} \times \text{collinear fermion lines}$$

- The collinear fermion lines give

$$\frac{\not{\ell} + a\not{p} + m}{2a\ell \cdot p + i\varepsilon}, \quad \frac{\not{\ell} + b\not{\bar{p}} + m}{2b\ell \cdot \bar{p} + i\varepsilon}$$

- Helicity flip is obtained by either picking up the quark masses or using the equations of motion for the collinear fermion lines. This eliminates a factor of ℓ in the numerator. ***Helicity flip gives the correct logarithmic scaling for endpoint double logarithm.***

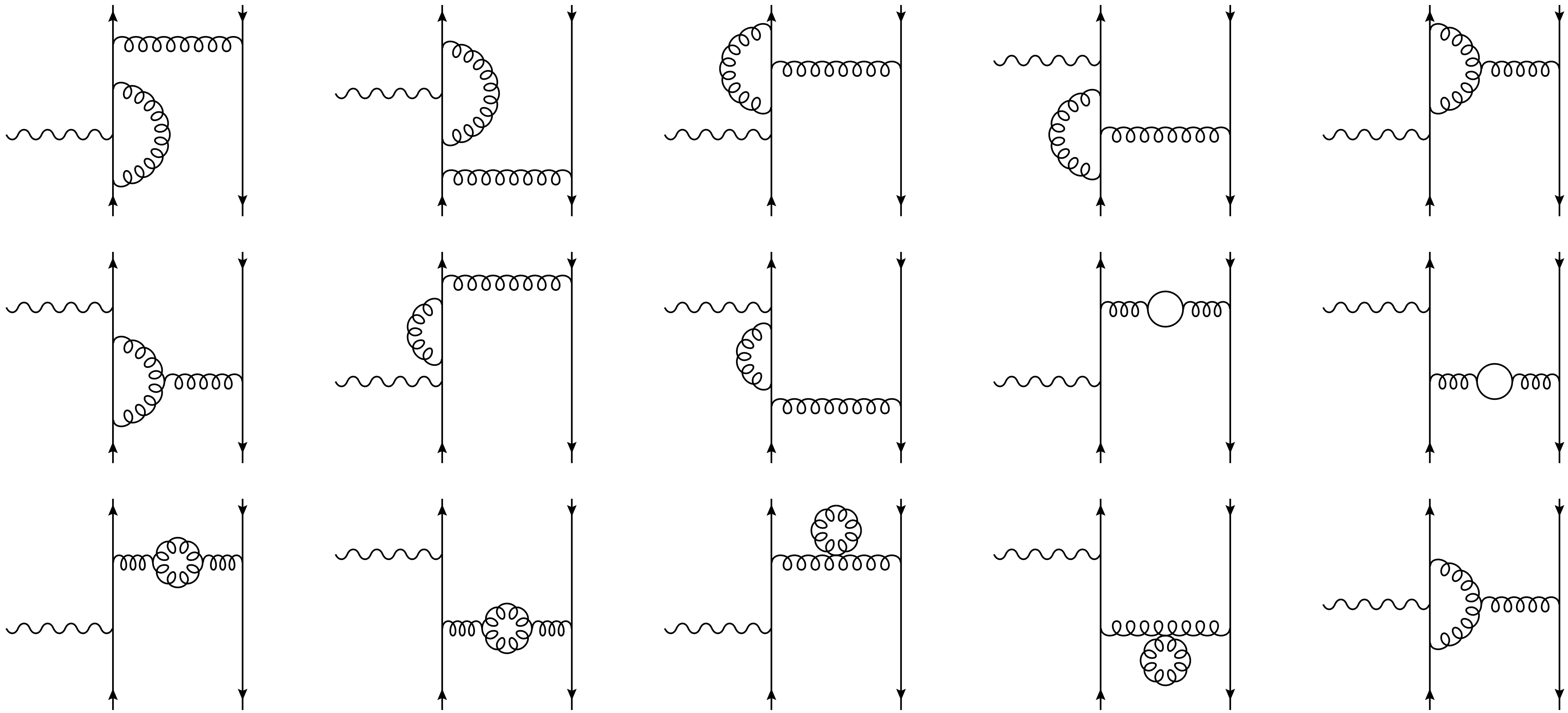
Summary

- We investigated the origin of the double logarithms in $e^+e^- \to J/\psi + \eta_c$.
- Sudakov double logarithms cancel in the sum over all diagrams, while endpoint double logarithms remain.
- The endpoint double logarithms are re-interpreted as a leading region of loop integration in which a spectator fermion line becomes soft-collinear.
- This re-interpretation may simplify the resummation of logarithms in m^2/s.
- The factorization for the helicity-flip process is currently under development.

Supplementary

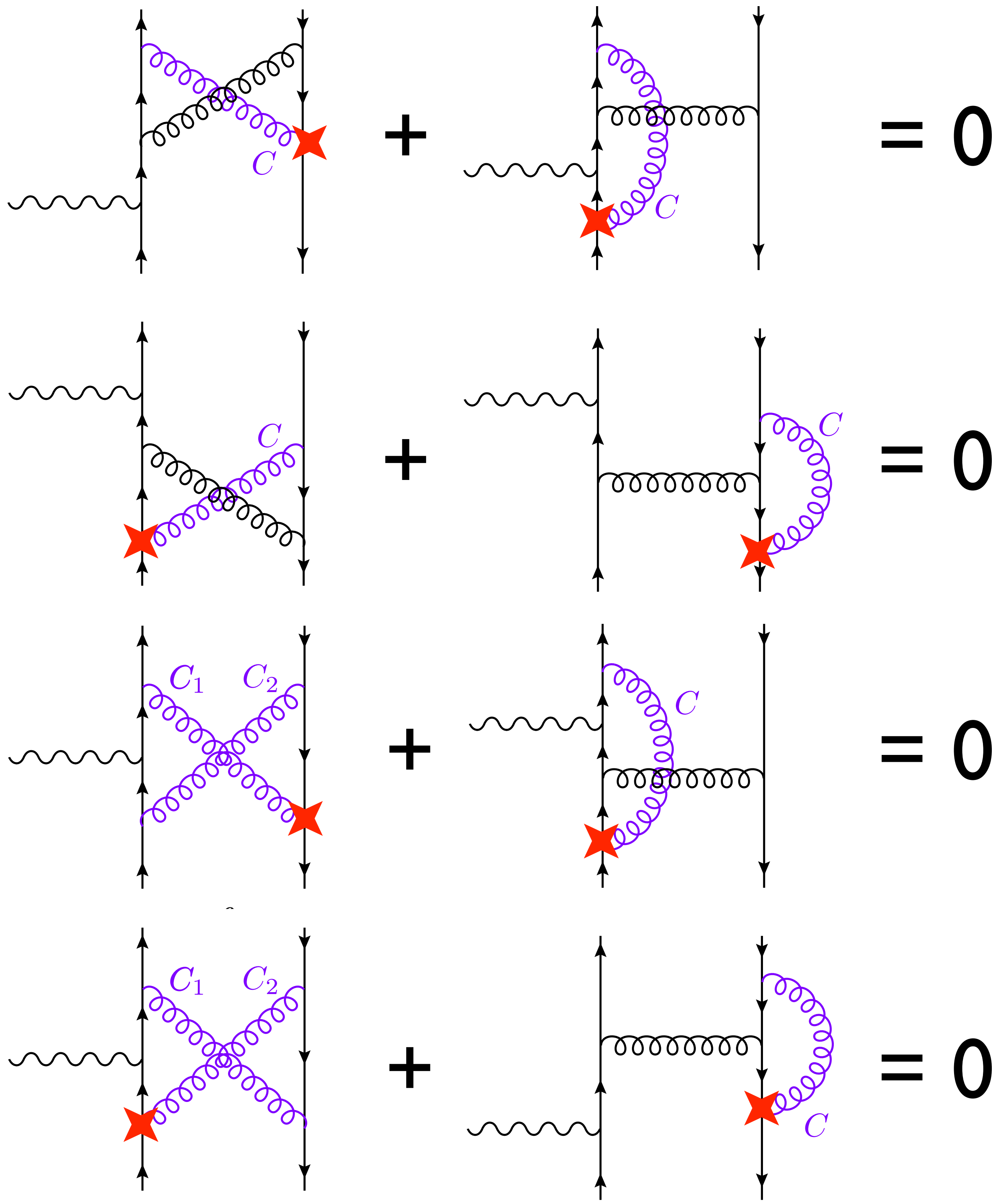
1-Loop Diagrams

- The following diagrams do not have double logs



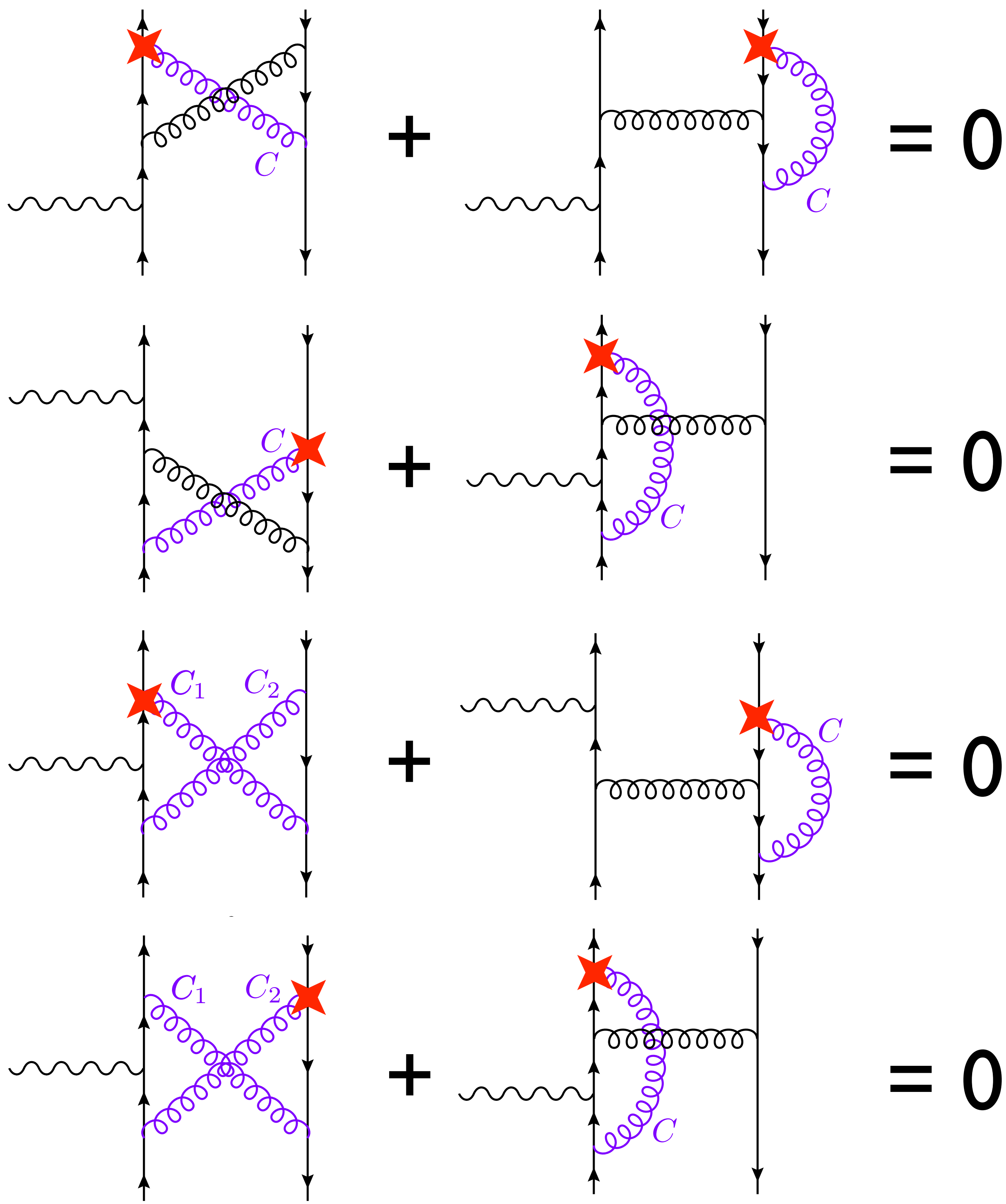
+ charge-conjugation diagrams
+ diagrams with counterterms

Cancellation of Sudakov Logs



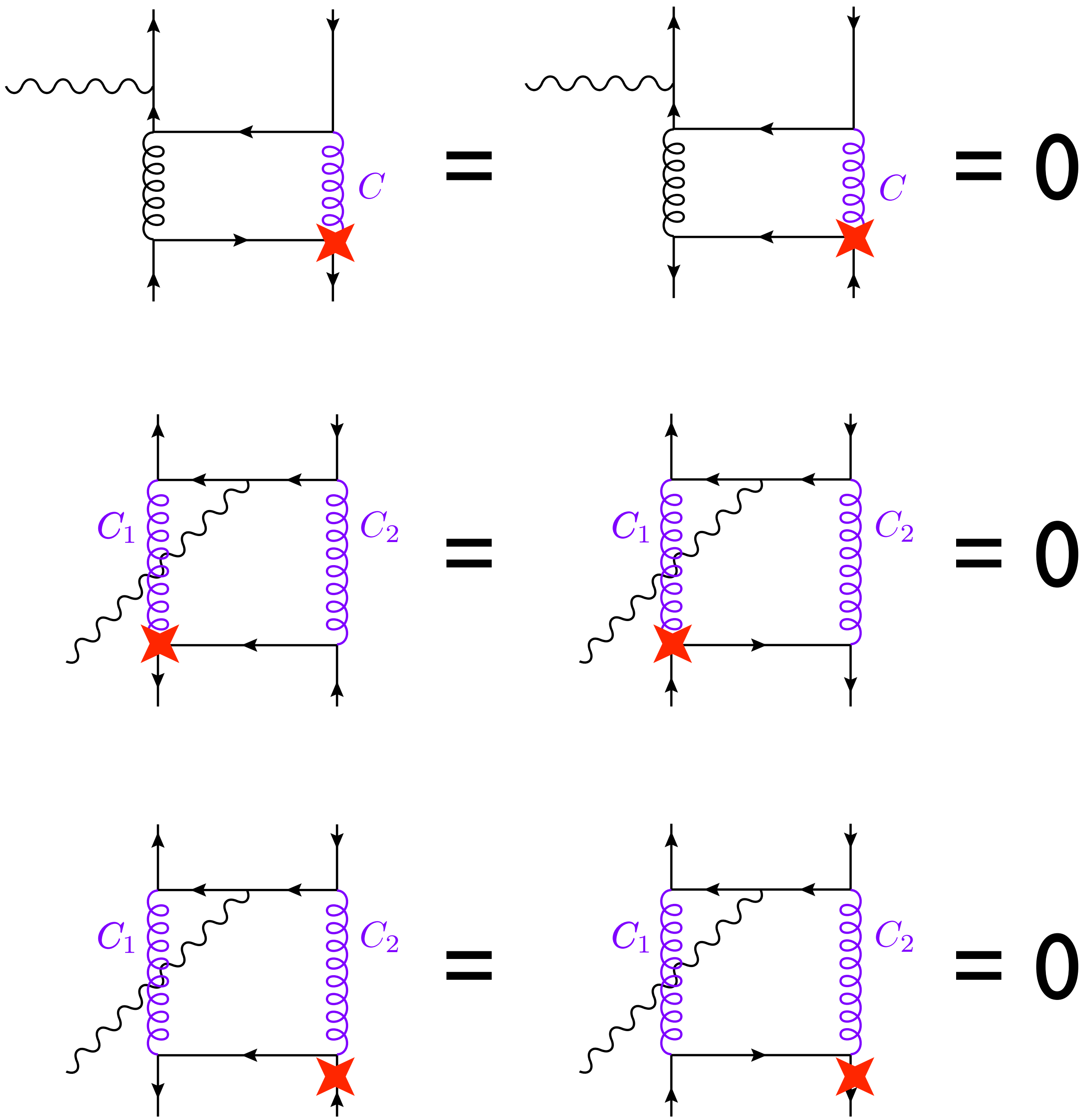
Insertion of soft gluons into collinear-to-minus lines (contribution from collinear-to-plus region). The Sudakov double logs cancel in these combinations.

Cancellation of Sudakov Logs



Insertion of soft gluons into collinear-to-plus lines (contribution from collinear-to-minus region). The Sudakov double logs cancel in these combinations.

Cancellation of Sudakov Logs



Because the lower meson has $C = +1$, the paired diagrams are same.
The Sudakov double logarithms cancel in each diagram.