you dao zhen lun

Baryoniums in the perspective of QCD sum rules

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Peking University

A draft review collaborated with Prof. Congfeng Qiao

第三届"有道真论"理论物理前沿研究与教学研讨会

2025.12.07 @济南

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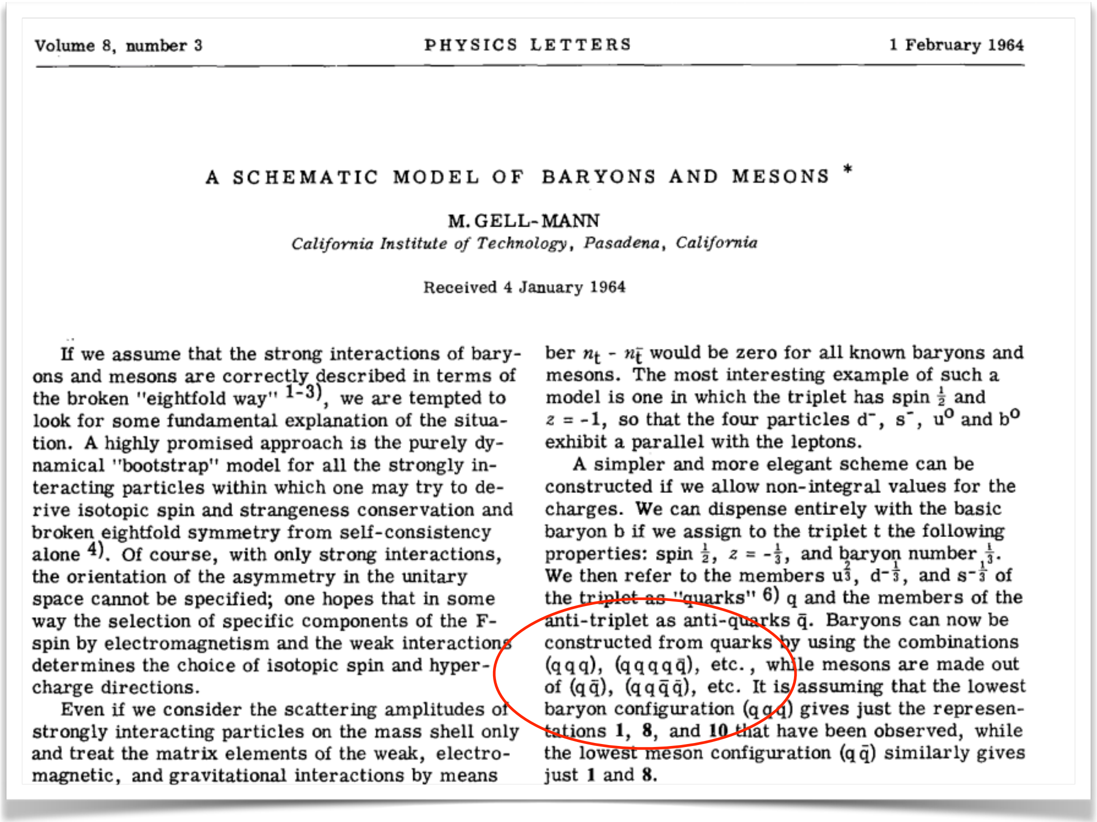
Quark model


Volume 8, number 3 PHYSICS LETTERS 1 February 1964

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN
California Institute of Technology, Pasadena, California

Received 4 January 1964

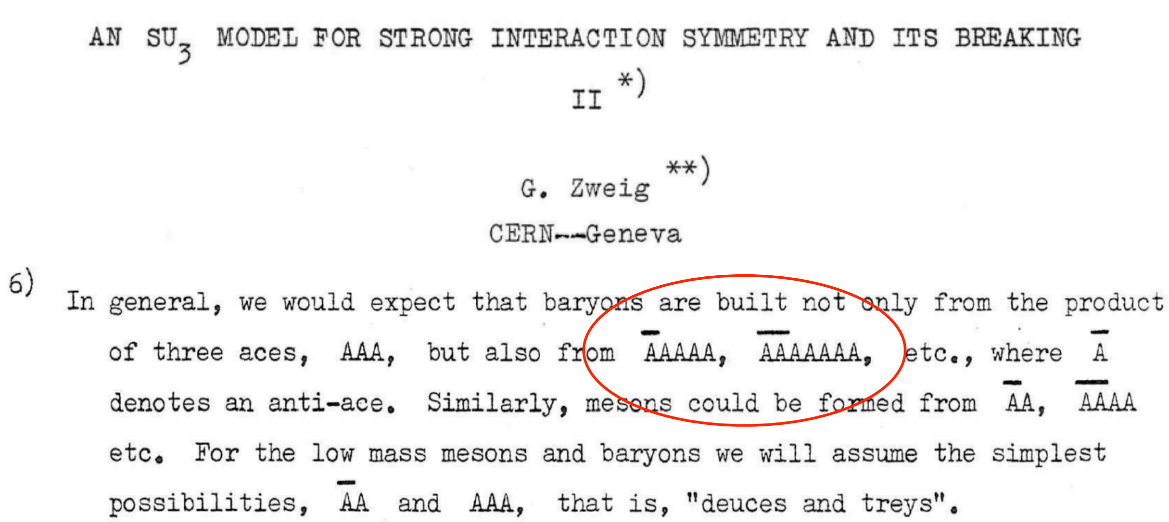
If we assume that the strong interactions of baryons and mesons are correctly described in terms of the broken "eightfold way" [1-3], we are tempted to look for some fundamental explanation of the situation. A highly promised approach is the purely dynamical "bootstrap" model for all the strongly interacting particles within which one may try to derive isotopic spin and strangeness conservation and broken eightfold symmetry from self-consistency alone [4]. Of course, with only strong interactions, the orientation of the asymmetry in the unitary space cannot be specified; one hopes that in some way the selection of specific components of the F-spin by electromagnetism and the weak interactions determines the choice of isotopic spin and hypercharge directions.

Even if we consider the scattering amplitudes of strongly interacting particles on the mass shell only and treat the matrix elements of the weak, electromagnetic, and gravitational interactions by means

ber $n_t - n_{\bar{t}}$ would be zero for all known baryons and mesons. The most interesting example of such a model is one in which the triplet has spin $\frac{1}{2}$ and $z = -1$, so that the four particles d^-, s^-, u^0 and b^0 exhibit a parallel with the leptons.

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" [6] q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations (qqq), $(qqqq\bar{q})$, etc., while mesons are made out of $(q\bar{q})$, $(qq\bar{q}\bar{q})$, etc. It is assuming that the lowest baryon configuration (qqq) gives just the representations **1**, **8**, and **10** that have been observed, while the lowest meson configuration $(q\bar{q})$ similarly gives just **1** and **8**.

Gell-Mann, 1964


AN SU_3 MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING

II *)

G. Zweig **)
CERN---Geneva

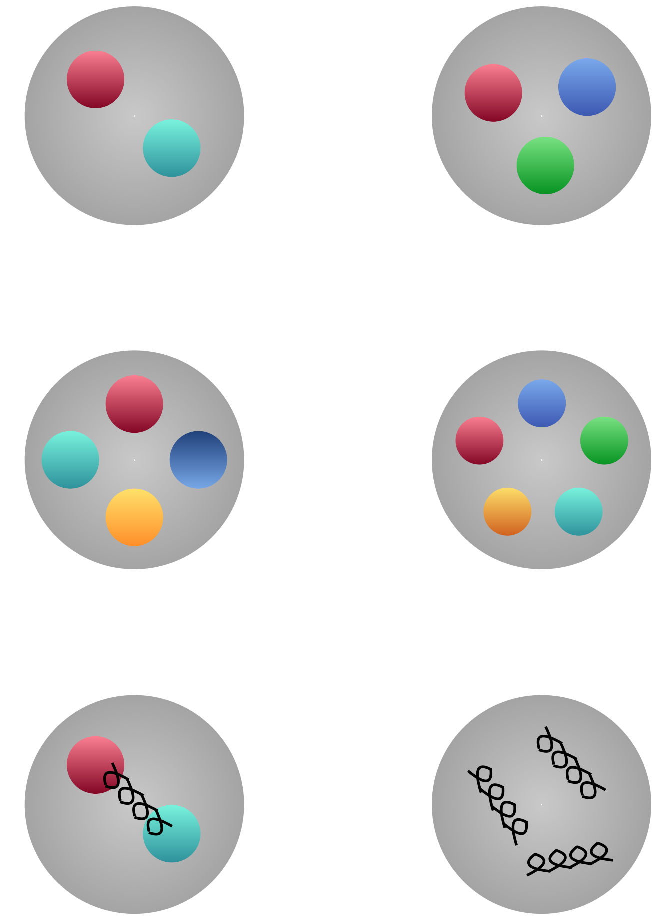
6) In general, we would expect that baryons are built not only from the product of three aces, AAA, but also from $\bar{A}AAAA$, $\bar{A}\bar{A}AAAAA$, etc., where $\bar{A}$ denotes an anti-ace. Similarly, mesons could be formed from $\bar{A}A$, $\bar{A}\bar{A}AA$ etc. For the low mass mesons and baryons we will assume the simplest possibilities, $\bar{A}A$ and AAA, that is, "deuces and treys".

Zweig, 1964

Conventional

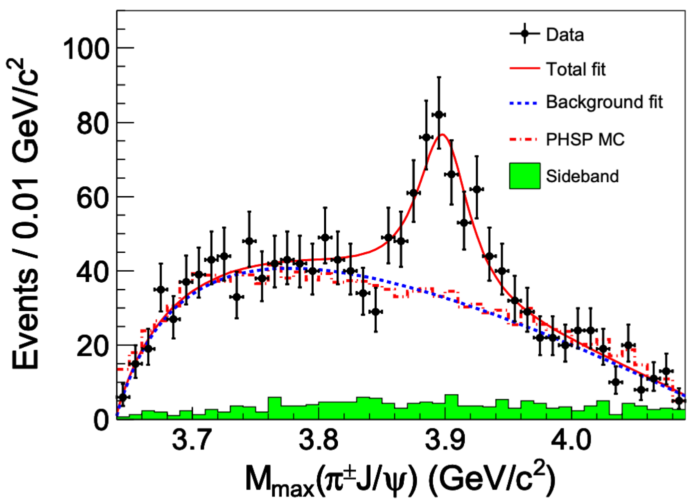
Multi-quark

Hybrid， Glueball



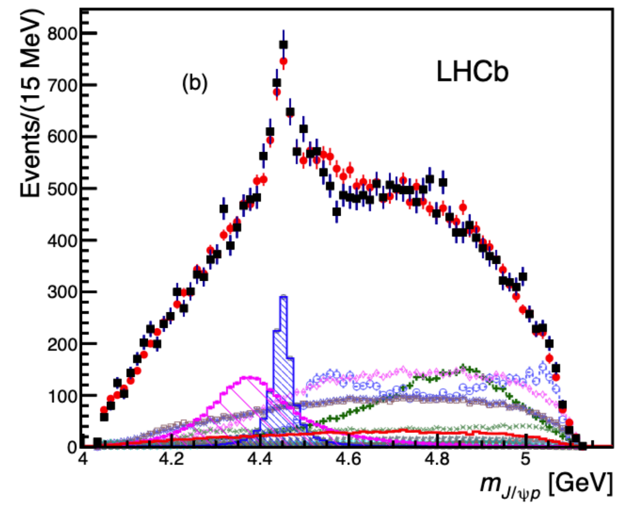
- Hadrons are composed of quarks and gluon
- Probe hadron structures
- Nonperturbative QCD

Hadrons



Tetraquark — $Z_c(3900)^+$

BESIII, 2013



Pentaquark — $P_c(4450)^+$

LHCb, 2015

State	m (MeV)	Γ (MeV)	J^{PC}	Process (mode)	References
$X(3872)$	3871.69 ± 0.17	<1.2	1^{++}	$B\to K(\pi^+\pi^- J/\psi)$	Belle [10,32], BaBar [36], LHCb [34,73]
				$p\bar{p}\to(\pi^+\pi^- J/\psi)+\ldots$	CDF [31,74,75], D0 [76]
				$e^+e^-\to\gamma(\pi^+\pi^- J/\psi)$	BES III [77]
				$B\to K(\omega J/\psi)$	Belle [78], BaBar [33]
				$B\to K(D^{*0}\bar{D}^0)$	Belle [38,79], BaBar [37]
				$B\to K(\gamma J/\psi)$ and $B\to K(\gamma\psi(2S))$	Belle [29], BaBar [30], LHCb [40]
$Z_c(3900)^+$	3888.7 ± 3.4	35 ± 7	1^+	$e^+e^-\to(J/\psi\,\pi^+)\pi^-$	Belle [43], BES III [55]
				$e^+e^-\to(D\bar{D}^*)^+\pi^-$	BES III [56]
$X(3915)$	3915.6 ± 3.1	28 ± 10	$0/2^{?+}$	$B\to K(\omega J/\psi)$	Belle [80], BaBar [33]
				$e^+e^-\to e^+e^-(\omega J/\psi)$	Belle [81], BaBar [82]
$X(3940)$	3942^{+9}_{-8}	37^{+27}_{-17}	$?^{?+}$	$e^+e^-\to J/\psi(DD^*)$	Belle [83]
				$e^+e^-\to J/\psi\,(\ldots)$	Belle [84]
$Y(4008)$	3891 ± 42	255 ± 42	1^{--}	$e^+e^-\to\gamma(\pi^+\pi^- J/\psi)$	Belle [42,43]
$Z_c(4050)^+$	4051^{+24}_{-43}	82^{+51}_{-55}	?	$B\to K(\pi^+\chi_{c1}(1P))$	Belle [53], BaBar [54]
$X(4050)^+$	4054 ± 3	45	?	$e^+e^-\to(\pi^+\psi(2S))\pi^-$	Belle [85]
$Y(4140)$	4143.4 ± 3.0	15^{+11}_{-7}	$?^{?+}$	$B\to K(\phi J/\psi)$	CDF [72],D0 [86]
$X(4160)$	4156^{+29}_{-25}	139^{+113}_{-65}	$?^{?+}$	$e^+e^-\to J/\psi(D\bar{D}^*)$	Belle [83]
$Z_c(4200)^+$	4196^{+35}_{-32}	370^{+99}_{-149}	?	$B\to K(\pi^+J/\psi)$	Belle [87]
$Z_c(4250)^+$	4248^{+185}_{-45}	177^{+321}_{-72}	?	$B\to K(\pi^+\chi_{c1}(1P))$	Belle [53], BaBar [54]
$Y(4260)$	4263 ± 5	108 ± 14	1^{--}	$e^+e^-\to\gamma(\pi^+\pi^- J/\psi)$	BaBar [41,88], CLEO [89], Belle [42,43]
				$e^+e^-\to(\pi^+\pi^- J/\psi)$	CLEO [47], BES III [56]
				$e^+e^-\to(\pi^0\pi^0 J/\psi)$	CLEO [47]
$X(4350)$	$4350.6^{+4.6}_{-5.1}$	$13.3^{+18.4}_{-10.0}$	$?^{?+}$	$e^+e^-\to e^+e^-(\phi J/\psi)$	Belle [90]
$Y(4360)$	4361 ± 13	74 ± 18	1^{--}	$e^+e^-\to\gamma(\pi^+\pi^-\psi(2S))$	BaBar [44], Belle [45,85]
$Z_c(4430)^+$	4485^{+36}_{-25}	200^{+49}_{-58}	1^+	$B\to K(\pi^+\psi(2S))$	Belle [49,51,52], BaBar [50], LHCb [21]
				$B\to K(\pi^+J/\psi)$	Belle [87], BaBar [50]
$X(4630)$	4634^{+9}_{-11}	92^{+41}_{-32}	1^{--}	$e^+e^-\to\gamma(\Lambda_c^+\Lambda_c^-)$	Belle [91]
$Y(4660)$	4664 ± 12	48 ± 15	1^{--}	$e^+e^-\to\gamma(\pi^+\pi^-\psi(2S))$	Belle [45]
$Z_b(10610)^+$	10607.2 ± 2.0	18.4 ± 2.4	1^+	$e^+e^-\to(b\bar{b}\,\pi^+)\pi^-$	Belle [20]
$Z_b(10610)^0$	$10609\pm4\pm4$	NA	1^{+-}	$e^+e^-\to(\Upsilon(2,3S)\pi^0)\pi^0$	Belle [23]
$Z_b(10650)^+$	10652.2 ± 1.5	11.5 ± 2.2	1^+	$e^+e^-\to(b\bar{b}\,\pi^+)\pi^-$	Belle [20]
$Y_b(10888)$	10888.4 ± 3.0	$30.7^{+8.9}_{-7.7}$	1^{--}	$e^+e^-\to(\pi^+\pi^-\Upsilon(nS))$	Belle [60,62]

Hosaka, et al., 2016

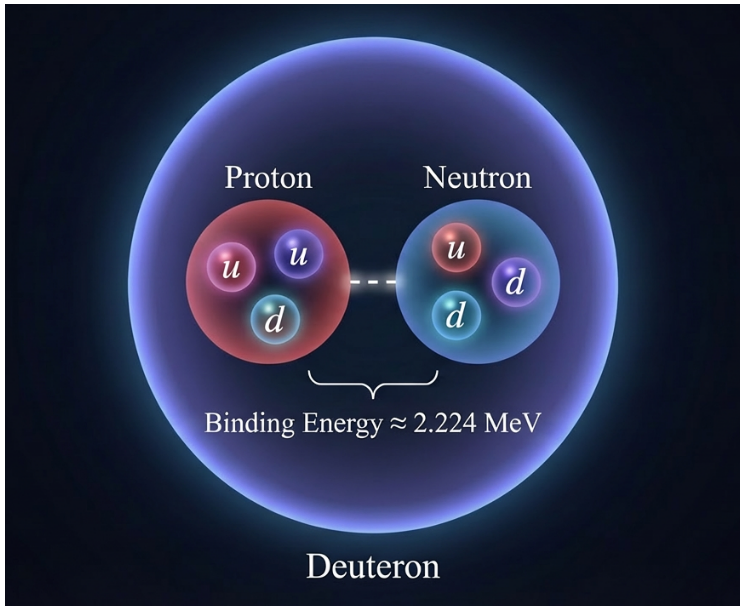
QCD sum rules

- *C.F. Qiao, L. Tang, 2014*
- *H.X. Chen, W. Chen, X. Liu, Steele, S.L. Zhu, 2015*
- *Z.G. Wang, 2021*
- *R.H. Wu, C.Y. Wang, C. Meng, Y.Q. Ma, K.T. Chao, 2023*
- ……

Lattice QCD

- *Y. Chen, 2014*
- *C. Liu, L. Liu, K.L. Zhang, 2020*
- ……

Hexaquark?



Deuteron: $J^P = 1^+$, $E_B = 2.225\,\mathrm{MeV}$

A typical and well-established dibaryon molecular states

PERHAPS A STABLE DIHYPERON*

R. L. Jaffe**
Stanford Linear Accelerator Center
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and

Department of Physics and Laboratory of Nuclear Science†
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

ABSTRACT

In the quark bag model the same gluon exchange forces which make the proton lighter than the $\Delta(1236)$ bind 6 quarks to form a stable, flavor singlet (strangeness −2) $J^P = 0^+$ dihyperon (H) at 2150 MeV. Another isosinglet dihyperon (H*) with $J^P = 1^+$ at 2335 MeV should appear as a bump in $\Lambda\Lambda$ invariant mass plots. Production and decay systematics of the H are discussed.

Jaffe, 1976

H-dihyperon: $uuddss$, $J^P = 0^+$,

$E_B \approx 80\,\mathrm{MeV}$: A deep bound state

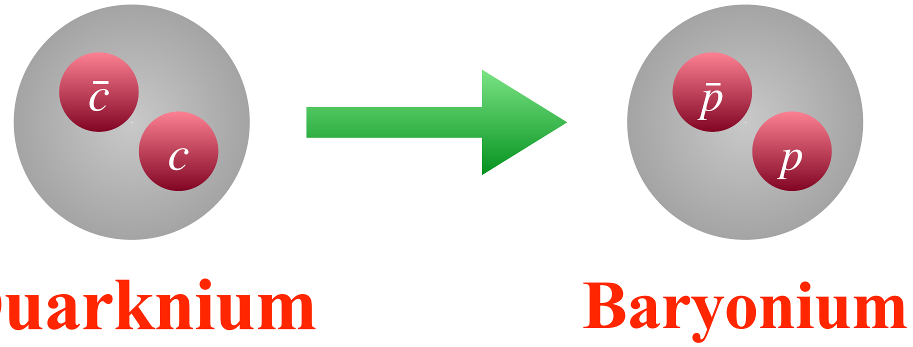
Lattice results for H-dibaryon:

HAL QCD Collaboration: 30-40 MeV, 2011;

NPL QCD Collaboration: 16 MeV, 2011;

HAL QCD Collaboration: Unbound, 2019;

Baryonium



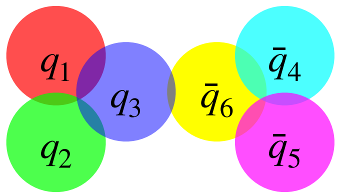
Di-baryon vs Baryon-antibaryon

$$V_{B\text{-}\bar{B}} = (-1)^G \times V_{B\text{-}B}$$

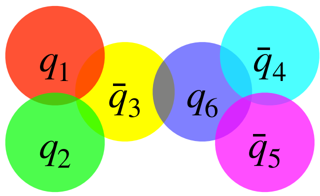
- $V_{B\text{-}B}$: short-range repulsion
- $G(\omega) = -1 \rightarrow V_{B\text{-}\bar{B}}$: strong attraction
- Baryon-antibaryon is more likely to form a deep bound state than dibaryon

Baryonium:

- ✦ Bound states or resonance formed by baryon-antibaryon pair
- ✦ Constituents may be either color singlets (molecular) or color multiplets (compact).

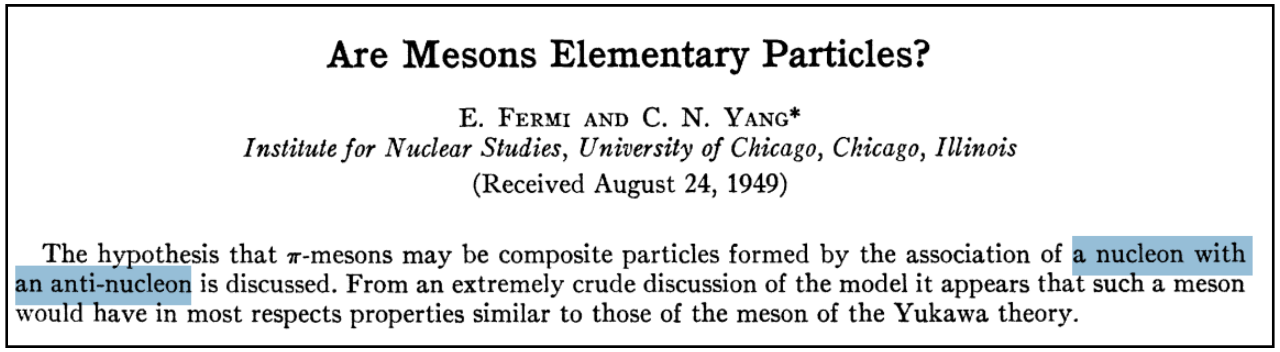


baryon-antibaryon



triquark-antitriquark

N-$\bar{N}$ system

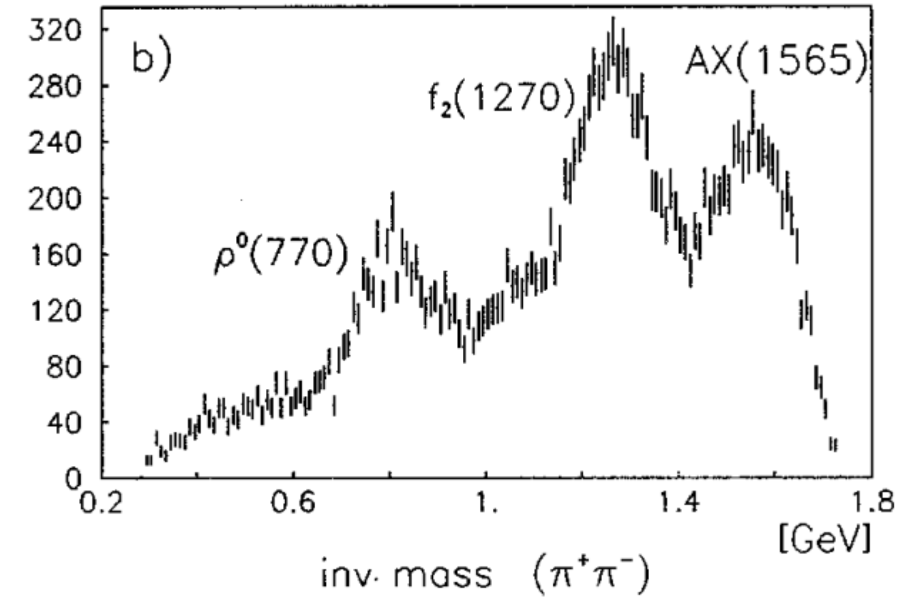


Are Mesons Elementary Particles?

E. Fermi and C. N. Yang*
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois
(Received August 24, 1949)

The hypothesis that π-mesons may be composite particles formed by the association of a nucleon with an anti-nucleon is discussed. From an extremely crude discussion of the model it appears that such a meson would have in most respects properties similar to those of the meson of the Yukawa theory.

Fermi, Yang, 1949



ASTERIX Collaboration,1990

$p\bar{p} \rightarrow \pi^+\pi^-\pi^0$

Potential model

N-$\bar{N}$ baryonium studies:
Dalkarov, Mandelzweig and Shapiro, 1970; Buck, Dover, and Richard, 1979; Dover and Richard, 1979; Amsler, 1987; Dover, Gutsche, and Faessler, 1991

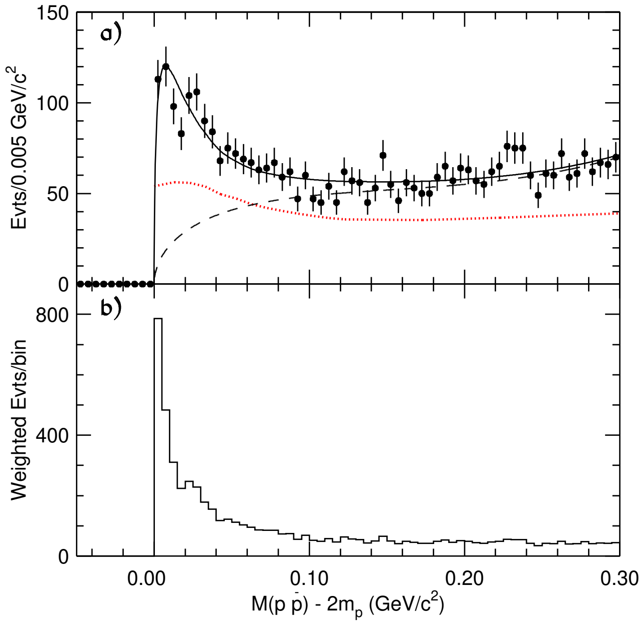
Deeply bound baryoniums with $J^{PC} = 2^{++}, 1^{--}, 0^{++}$ are predicted

$f_2(1565)$: mostly observed in $p\bar{p}$ annihilation

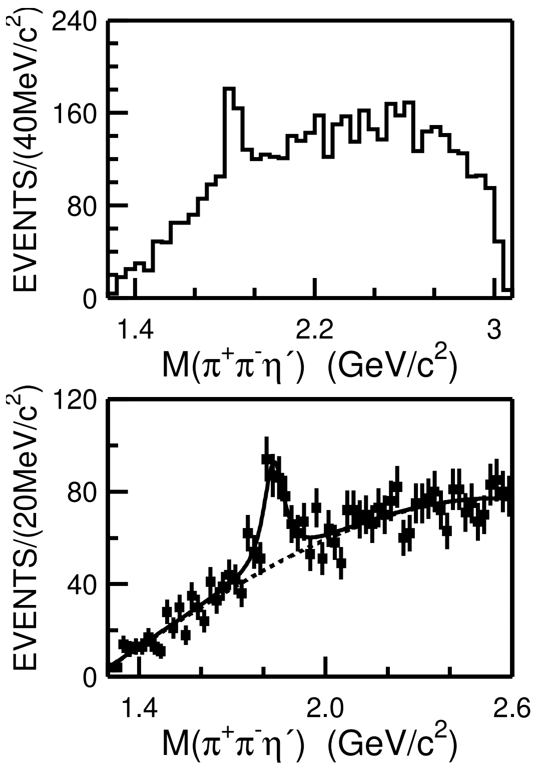
- Considered as candidate for 2^{++} $p\bar{p}$ bound states

$X(1860)$ and $X(1835)$

Two resonances observed near $p\bar{p}$ threshold



BES, 2003



BES, 2005

$X(1860)$: observed in $J/\psi \to \gamma p\bar{p}$

- $M = 1859^{+3}_{-10}(stat)^{+5}_{-25}(sys)\,\text{MeV}$
- $\Gamma < 30\,\text{MeV}$

$X(1835)$: observed in $J/\psi \to \gamma\pi^+\pi^-\eta'$

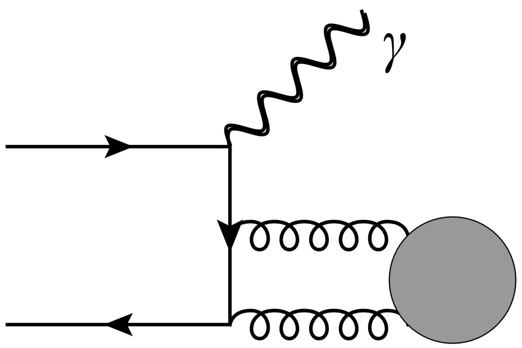
- $M = 1833.7 \pm 6.1(stat) \pm 2.7(sys)\,\text{MeV}$
- $\Gamma = 67.7 \pm 20.3(stat) \pm 7.7(sys)\,\text{MeV}$

○ p-$\bar{p}$ baryonium interpretation

Datta, Donnelll, 2003;

Shilin Zhu, Chongshou Gao, 2006;

Zhigang Wang, Shaolong Wan, 2007;



○ gluonium mixing

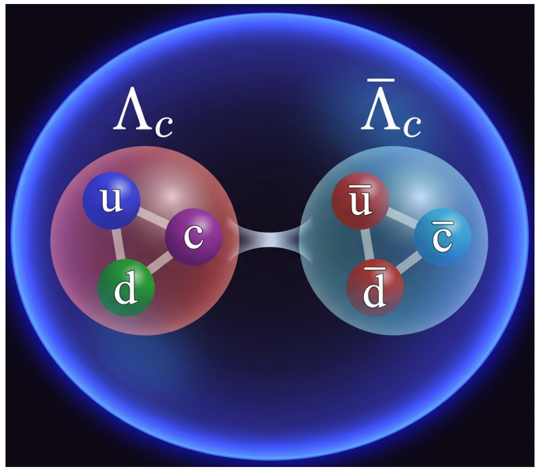
Gang Hao, Congfeng Qiao,

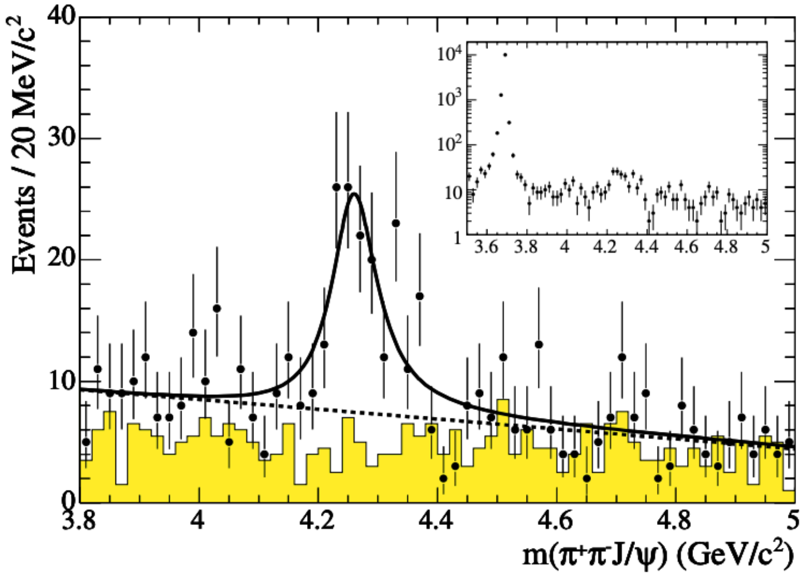
Ailin Zhang 2003;

Kochelev, D.P. Min, 2005;

Bingan Li, 2006;

Hidden charm baryoniums

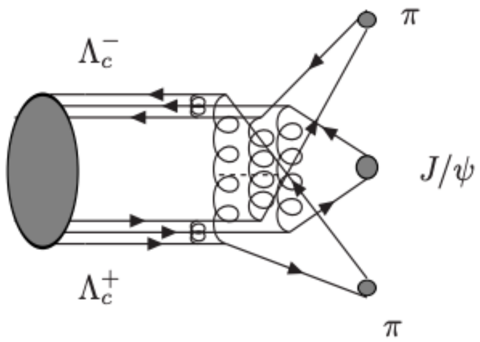


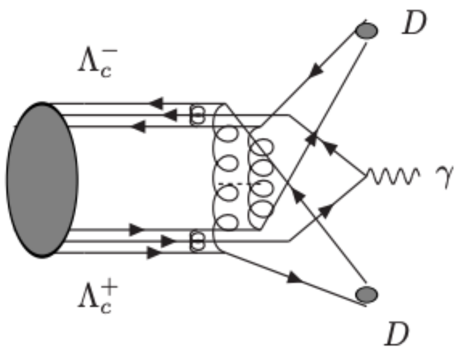


$Y(4260)$:

- Observed in $e^+e^- \rightarrow \gamma_{\text{ISR}} + J/\psi + \pi^+ + \pi^-$
- Mass: 4259 ± 8 (stat) $^{+2}_{-6}$ (syst) MeV
- Width: 88 ± 23 (stat) $^{+6}_{-4}$ (syst) MeV

BaBar, 2005





$Y(4260)$ as Λ_c-$\bar{\Lambda}_c$ baryonium:

- Dominant decay: $J/\psi\pi^+\pi^-$
- Suppressed channels: $D\bar{D}$, $D_s\bar{D}_s$, $J/\psi K^+K^-$, $\psi'\pi^+\pi^-$
- No resonance or bound state in $e^+e^- \rightarrow \gamma_{ISR} + p + \bar{p}$

Congfeng Qiao, 2004

QCD sum rules

Nuclear Physics B147 (1979) 385–447

QCD AND RESONANCE PHYSICS. THEORETICAL FOUNDATIONS

M.A. SHIFMAN, A.I. VAINSHTEIN * and V.I. ZAKHAROV
Institute of Theoretical and Experimental Physics, Moscow, 117259, USSR

Received 24 July 1978

M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, 1979

- Correlation functions: first principle
- Analytical
- Well-defined for J/ψ: $\bar{c}\gamma_\mu c$
- Hadron mass and decay

Observables: mass, decay constant, form factor...

as one works in this approach with spectral functions. The 2nd reason is that *QSSR* is near the abbreviation of the former Soviet Union state (USSR) from which originally come the three inventors of this method. To my opinion, the SVZ sum rule [1] is one of the most

$$QSSR \leftrightarrow USSR$$

Narison, 2010

QCD and Resonance Physics. Theoretical Foundations #1
Mikhail A. Shifman (Moscow, ITEP), A.I. Vainshtein (Moscow, ITEP), Valentin I. Zakharov (Moscow, ITEP) (1978)
Published in: *Nucl.Phys.B* 147 (1979) 385-447
DOI cite claim reference search 6,004 citations

QCD and Resonance Physics: Applications #2
Mikhail A. Shifman (Moscow, ITEP), A.I. Vainshtein (Moscow, ITEP), Valentin I. Zakharov (Moscow, ITEP) (1978)
Published in: *Nucl.Phys.B* 147 (1979) 448-518
DOI cite claim reference search 3,301 citations

Correlation functions

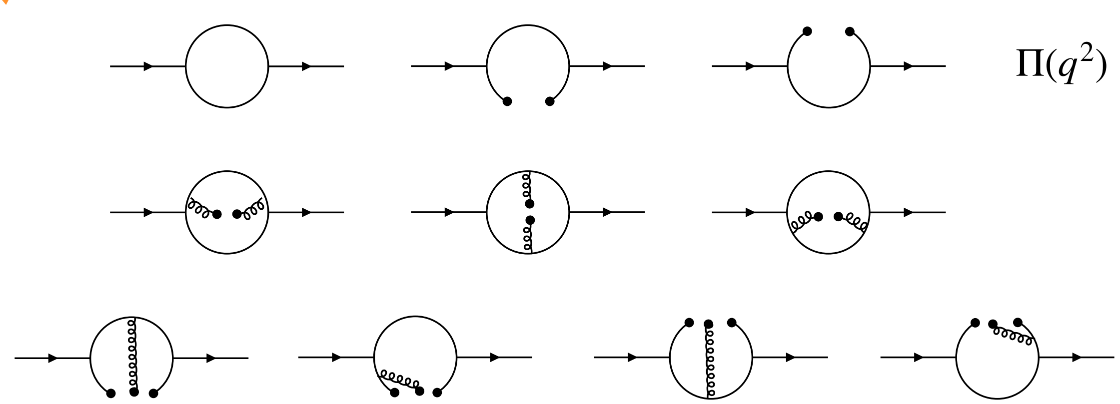
$$\Pi(q^2) = i\int d^4x e^{iq\cdot x}\langle 0|T\{j(x), j^\dagger(0)\}|0\rangle$$

$j(x)$: hadron current, describing the properties of hadrons （J^{PC}, quark constituents）

$\Pi(q^2)$

quark-gluon level：operator product expansion

$$\Pi^{\rm QCD}(q^2) = \int_{s_{min}}^{+\infty} ds\, \frac{\rho^{\rm QCD}(s)}{s-q^2} ds$$



$$\Pi(q^2) = \sum_d C_d(q^2)\langle 0|\hat{O}_d(0)|0\rangle$$

$$= C_0(q^2) + C_3(q^2)\langle\bar{q}q\rangle + C_4(q^2)\langle g_s^2 G^2\rangle + C_5(q^2)\langle\bar{q}Gq\rangle + \ldots$$

hadron level：observables

$$\Pi^{\rm phe}(q^2) = \frac{\lambda_H^2}{q^2 - M_H^2} + \int_{s_{min}}^{+\infty} \frac{\rho^{\rm cont}(s)}{s-q^2} ds + \cdots$$

Quark-hadron duality

PHYSICAL REVIEW D VOLUME 13, NUMBER 7 1 APRIL 1976

Smearing method in the quark model*

E. C. Poggio, H. R. Quinn,† and S. Weinberg
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 8 December 1975)

We propose that perturbation theory in a gauge field theory of quarks and gluons can be used to calculate physical mass-shell processes, provided that both the data and the theory are smeared over a suitable energy range. This procedure is explored in detail for the process of electron-positron annihilation into hadrons. We show that a smearing range of 3 GeV^2 in the squared center-of-mass energy should be adequate to allow the use of lowest-order perturbation theory. The smeared data are compared with theory for a variety of models. It appears to be difficult to fit the present data with theory, unless there is a new charged lepton or quark with mass in the region 2 to 3 GeV.

Poggio, Quinn, Weinberg, 1976

Shifman, 2000; 2002

e^+e^- annihilation， DIS……

Quark-hadron duality: $\Pi^{\rm QCD}(q^2) \simeq \Pi^{\rm phe}(q^2)$

$$\Pi^{\rm QCD}(q^2) = C_0(q^2) + C_3(q^2)\langle\bar{q}q\rangle + C_4(q^2)\langle g_s^2 G^2\rangle + C_5(q^2)\langle\bar{q}Gq\rangle + \ldots$$

- Convergence of OPE

$$\Pi^{\rm phe}(q^2) = \frac{\lambda_H^2}{q^2 - M_H^2} + \int_{s_{min}}^{+\infty} \frac{\rho^{\rm cont}(s)}{s - q^2} ds + \cdots$$

- Higher excited state should be reduced
- Subtraction term should be excluded

Borel transformation

$$\mathscr{B}[\Pi(q^2)] \equiv \Pi(\tau^2) = \lim_{\substack{-q^2, n \to \infty \\ -q^2/n = \tau^2}} \frac{(-q^2)^{n+1}}{n!} \left(\frac{\partial}{\partial q^2}\right)^n \Pi(q^2)$$

- Suppress the OPE and excited states contributions
- Cancel the subtract terms
- Minimal dependence of τ^2

Mass equation: $M_H^2 = \dfrac{\int_{s_{min}}^{s_0} s\,\rho^{\mathrm{QCD}}(s)\, e^{-s/\tau^2}\, ds}{\int_{s_{min}}^{s_0} \rho^{\mathrm{QCD}}(s)\, e^{-s/\tau^2}\, ds}$

Gaussian sum rules Bertlmann, Launer, and E. de Rafael, 1985

$$G(\hat{s}, \sigma) = \frac{1}{\sqrt{4\pi\sigma}} \int_0^\infty \mathrm{d}s \exp\left(-\frac{(s-\hat{s})^2}{4\sigma}\right) \frac{1}{\pi} \mathrm{Im}\,\Pi(s)$$

Orlandini, Steele, and Harnett, 2001; Harnett, and Steele, 2001; Shuiguo Wen, Zhenyu Zhang, and Jueping Liu, 2010; Chunfeng Xian, Feng Wang, and Jueping Liu, 2014; Ho, Berg, Steele, Wei Chen, and Harnett, 2019

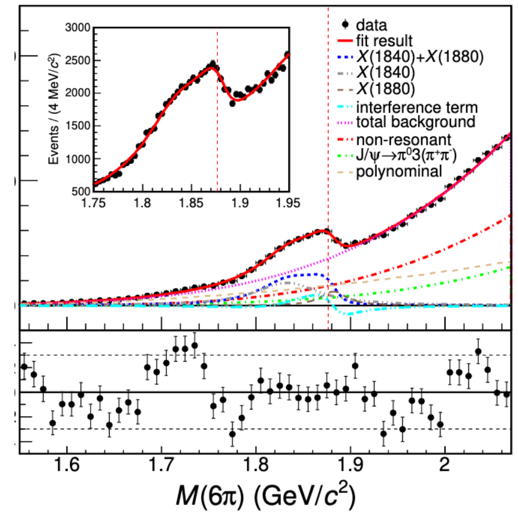
- *Scan the hadronic spectrum, including resonances and excited states*
- *Control the resolution of the spectral probe by σ*

Light baryoniums in QCDSR

TABLE I. The continuum thresholds, Borel parameters, and predicted masses of hexaquark molecular states.

$\mathcal{B}-\bar{\mathcal{B}}$	J^{PC}	$\sqrt{s_0}$ (GeV)	$M_B^2(\text{GeV}^2)$	M^X (GeV)
$N-\bar{N}$	0^{-+}	2.5 ± 0.1	1.6–2.2	1.81 ± 0.09
	1^{--}	2.5 ± 0.1	1.6–2.2	1.82 ± 0.10
	0^{++}	…	…	…
	1^{++}	…	…	…
$\Lambda-\bar{\Lambda}$	0^{-+}	2.9 ± 0.1	1.9–2.5	2.27 ± 0.13
	1^{--}	2.9 ± 0.1	1.9–2.5	2.34 ± 0.12
	0^{++}	…	…	…
	1^{++}	…	…	…
$\Sigma-\bar{\Sigma}$	0^{-+}	3.0 ± 0.1	2.0–2.6	2.39 ± 0.13
	1^{--}	3.1 ± 0.1	2.0–2.6	2.48 ± 0.12
	0^{++}	…	…	…
	1^{++}	…	…	…
$\Xi-\bar{\Xi}$	0^{-+}	3.2 ± 0.1	2.1–2.8	2.67 ± 0.12
	1^{--}	3.3 ± 0.1	2.1–2.9	2.79 ± 0.11
	0^{++}	…	…	…
	1^{++}	…	…	…

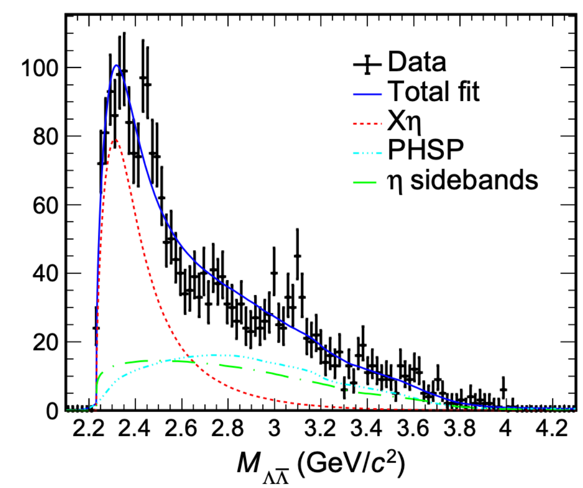
Bingdong Wan, ZSQ, Congfeng Qiao, 2021



$X(1880): M = 1882.1 \pm 1.7 \pm 0.7\,\text{MeV}$

BESIII, 2024

"Protonium: Discovery and Prediction", Boqiang Ma, 2024



Λ-$\bar{\Lambda}$ enhancement:

$M = 2356 \pm 7 \pm 17\,\text{MeV}$

BESIII, 2023

Hidden charm and bottom baryoniums

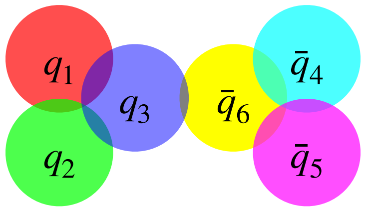
State	J^{PC}	QCDSR	Large N_c [484]	HBCPT [74]	CMI model [489]
Λ_c-$\bar{\Lambda}_c$	0^+	5.00 [540], 5.19±0.24 [541], $5.11^{+0.15}_{-0.12}$ [542]		4.41~4.57	
	0^-	$4.66^{+0.10}_{-0.06}$ [542]			4.65~4.77
	1^+	4.89 [540], $4.99^{+0.10}_{-0.09}$ [542]		4.29~4.56	
	1^-	4.78±0.23 [541], $4.68^{+0.08}_{-0.08}$ [542]	4.78±0.15		4.58~4.94
	2^+	5.15 [540]			
	2^-	4.83 [540]			
	3^+	5.68 [540]			
	3^-	5.04 [540]			
Σ_c-$\bar{\Sigma}_c$	0^+	$5.23^{+0.07}_{-0.07}$ [543]		4.88~4.90	
	0^-	$4.88^{+0.08}_{-0.08}$ [543]			
	1^+	$5.31^{+0.07}_{-0.07}$ [543]		4.87~4.90	
	1^-	$4.88^{+0.09}_{-0.08}$ [543]			
Λ_b-$\bar{\Lambda}_b$	0^+	11.84±0.22 [541]		11.21~11.23	
	0^-				
	1^+			11.20~11.23	
	1^-	11.72±0.26 [541]			

Huaxing Chen, Dan Zhou, Wei Chen, Xiang Liu, Shilin Zhu, 2016; Bingdong Wan, Liang Tang, Congfeng Qiao, 2019;
Xiuwu Wang, Zhigang Wang, Guoliang Yu, 2021; Chun Liu, 2007; Yuede Chen, Congfeng Qiao, Pengnian Shen, Zhuoquan Zeng, 2013; Zhe Liu, Hongtao An, Zhanwei Liu, Xiang Liu, 2021

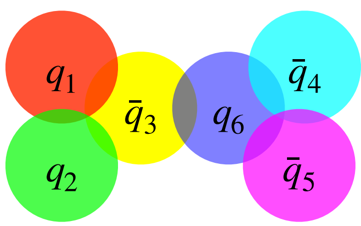
$J^P = 3^-$ channel:

- Inaccessible by S-wave $[\bar{c}c + \pi\pi]$ and $[\bar{c}c\bar{q}q + \pi]$ structures based on QCD sum rules results, search in D-wave $J/\psi\pi\pi$ and P-wave $J/\psi\rho(\omega)$ modes
- $M \sim 5$ GeV, higher than most XYZ states

Triquark currents



baryon-antibaryon



triquark-antitriquark

$$q_1 q_2 \bar{q}_3 = (q_1 q_2) + \bar{q}_3 = (3_c \otimes 3_c) \otimes \bar{3}_c = 3_c \oplus \bar{6}_c \oplus 3_c \oplus 15_c$$

$$\boxed{\begin{aligned} j^{\text{triquark}}_{\bar{3}_c \otimes \bar{3}_c} &\simeq \varepsilon_{\text{dec}} \varepsilon_{eab} \left[q^T_{1a} \Gamma_1 q_{2b} \right] \Gamma_2 \bar{q}_{3c} \\ j^{\text{triquark}}_{6_c \otimes \bar{3}_c} &\simeq \left[q^T_{1a} \Gamma_1 q_{2b} \right] \Gamma_2 \bar{q}_{3b} \,. \end{aligned}}$$

$$q_1 q_2 \bar{q}_3 = (q_1 \bar{q}_2) + q_3 = 3_c \otimes \bar{3}_c \otimes 3_c = 3_c \oplus 3_c \oplus \bar{6}_c \oplus 15_c$$

$$\boxed{\begin{aligned} j^{\text{triquark}}_{1_c \otimes 3_c} &\simeq \left[q_{1a} \Gamma_1 \bar{q}_{2a} \right] \Gamma_2 \bar{q}_{3b} \\ j^{\text{triquark}}_{8_c \otimes 3_c} &\simeq \left[q_{1a} \Gamma_1 (t^A_{ab}) \bar{q}_{2b} \right] \Gamma_2 q_{3b} \end{aligned}}$$

Triquark-antitriquark in QCDSR

A distinct class of baryoniums--p-$\bar{\Lambda}$ system

Experimental evidence: $X(2075)$ and $X(2085)$ observed in $p\bar{\Lambda}$ final state, both near the threshold BES, 2004; BESIII, 2023

States	J^{PC}	Baryon-antibaryon	Triquark-antitriquark
p-$\bar{\Lambda}$	0^+	1.96 ± 0.03	1.99 ± 0.02
	0^-	2.00 ± 0.20	1.84 ± 0.21
	1^-	2.05 ± 0.18	2.01 ± 0.20
	1^+		1.97 ± 0.03
p-$\bar{\Sigma}$	0^+	1.98 ± 0.04	1.99 ± 0.02
	0^-	1.99 ± 0.18	1.84 ± 0.21
	1^-	2.06 ± 0.18	2.02 ± 0.20
	1^+		1.97 ± 0.02

1^+: probe for molecular or compact

Xuanheng Zhang, ZSQ, Congfeng Qiao, 2024

Outlook for QCDSR

Accuracy

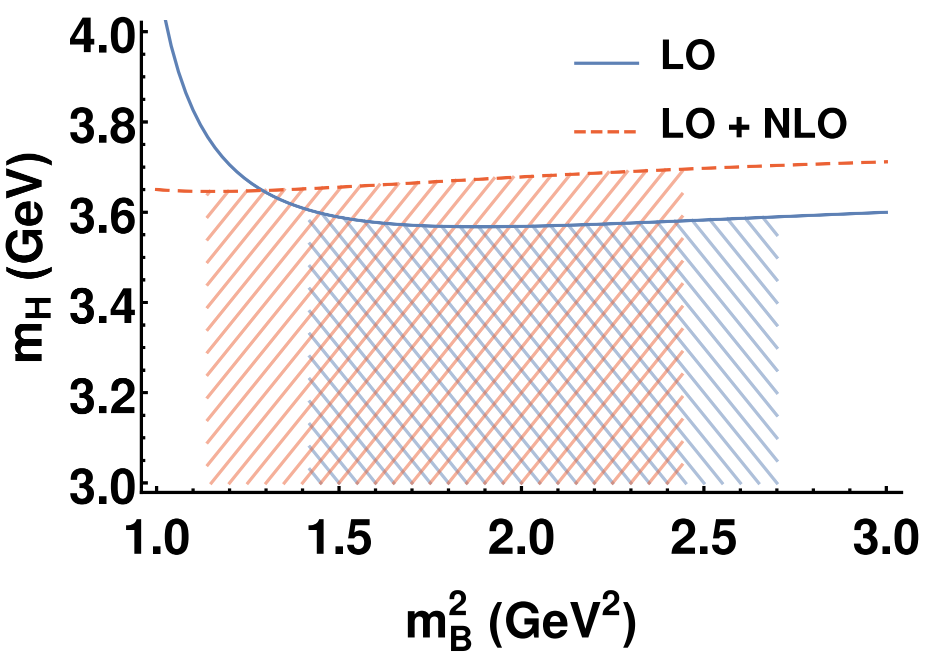
Molecular

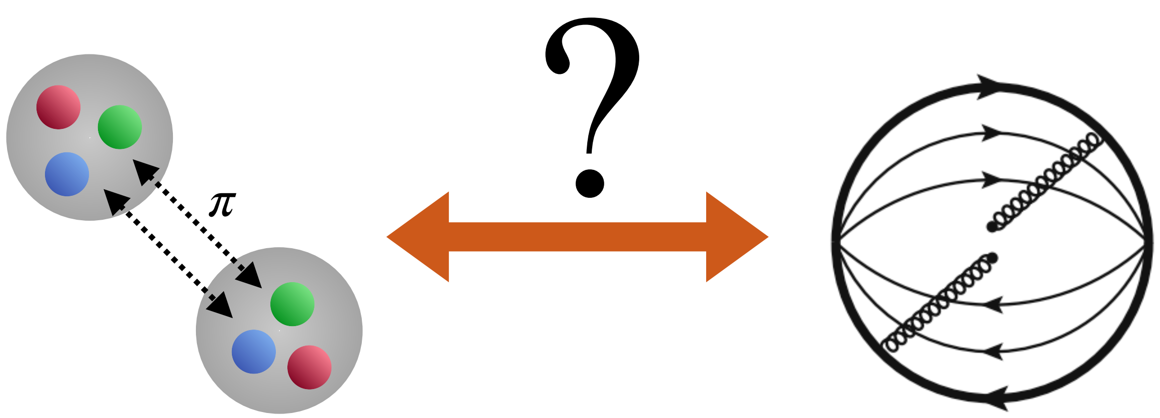
QCD corrections:

Albuquerque, Narison, Rabetiarivony, 2021

Chenyu Wang, Ce Meng, Yanqing Ma,

Kuangta Chao, 2019





Meson-change

Gluon-condensate

Thanks