$\pi\pi$ 谱的一般描述和 f_0 结构分析

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第三届有道真论研讨会 @ 山东大学

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第一届有道真论研讨会

Overview

重矢量介子的弱衰变初探

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Motivation

D_s^* weak decays

- $D_s^* \to \phi$ helicity form factors
- Exclusive D_s^* weak decays

Dimeson LCDAs and $D_s \to [\pi\pi]_S\, e^+ \nu_e$

Conclusion

- Above discussions are all at leading twist level, subleading twist LCDAs are not known yet

Overview

I: Light-cone distribution amplitudes

II: LCDAs of two-pion system and their phenomena

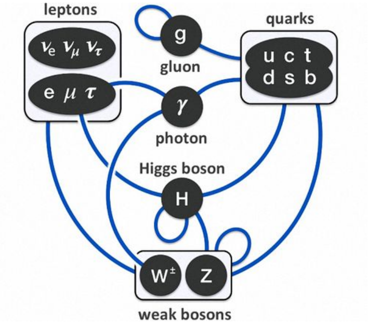
i: LCDAs of two-pion system (2πDAs)

ii: $D_s \to [\pi\pi]_{\rm S}\, l\nu$ decay and $[\pi\pi]_{\rm S}$ configurations

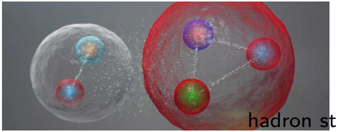
III: Summary and Prospect

Emergent phenomena of QCD

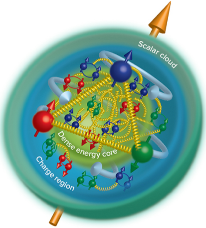
QCD is believed to confine, that is, its physical states are color singlets with internal quark and gluon degrees of freedom



Parts



hadron structure

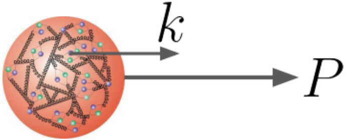


hadron formation

Observed entity

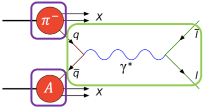
PDF,TMD,GPD

Definitions of pion distribution



One dimension PDF

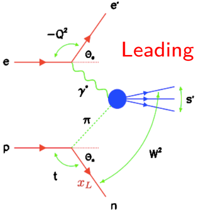
- △ $f_i(\zeta) = \int \frac{dz^-}{4\pi} e^{-i\zeta P^+ z^-} \langle \pi | \bar{\psi}_i(0, z^-, \mathbf{0}_T)\gamma^+ \psi_i(0) | \pi \rangle$
- △ $\zeta = \frac{k^+}{P^+}$, the parton momentum fraction
- △ $f_i(\zeta) \sim \sum_\alpha \int dk_T^2 \langle \pi | b^\dagger_{k,\alpha} b_{k,\alpha}(\zeta P^+, k_T, \alpha) | \pi \rangle$
  number operator
- △ transversal momentum distributions (TMD) $f(\zeta, k_T)$
- △ Generalized parton distributions (GPD) $f(\zeta, b_T)$



Drell-Yan

$$\sigma \propto \sum_{i,j} f_i^\pi(x_\pi, \mu) \otimes f_j^A(x_A, \mu) \otimes C_{i,j}(x_\pi, x_A, Q/\mu)$$

Extracted from **fixed target πA data**



Leading neutrons

Deeply virtuality meson production

- △ TDIS at 12GeV JLab, leading proton observable, fixed target instead of collider (HERA);
- △ EIC, EIcC, great integrated luminosity to reduce the systematics uncertainties;
- △ COMPASS++/AMBER give π-induced DY data.

Light-cone distribution amplitudes (LCDAs)

- **Exclusive QCD processes with larger momentum transfers**
- Define the LCDAs with the Lorentz and gauge invariant ME

$$\langle 0|\bar{u}(x)\gamma_\mu\gamma_5 d(-x)|\pi^-(p)\rangle = f_\pi \int_0^1 du\, e^{i(2u-1)p\cdot x}\left[ip_\mu\left(\phi(u,\mu)+\frac{x^2}{4}\phi_1^4(u,\mu)\right)+\cdots\right]$$

$$\langle 0|\bar{u}(x)\gamma_\mu\gamma_5 d(-x)|\rho^-(p)\rangle = f_\rho m_\rho \int_0^1 du\, e^{i(2u-1)p\cdot x}\left[p_\mu \frac{\epsilon^{(\lambda)}\cdot x}{p\cdot x}\left(\phi_\parallel(u,\mu)-\phi_\perp^3(u,\mu)\right)+\cdots\right]$$

$$\langle 0|\bar{q}(x)\gamma_\mu\gamma_5 q(-x)|f_0(p)\rangle = p_\mu \int_0^1 du\, e^{i(2u-1)p\cdot x}\left[\phi(u,\mu)+\cdots\right]$$

- LCDAs are dimensionless functions of u and renormalization scale μ

△ the probability amplitudes to **find the light meson** in a state with minimal number of constitutes and have small transversal separation of order $1/\mu$

△ nonlocal operators on the lhs are renormalized at scale μ

$$\phi_2(u,\mu) = Z_2(\mu)\int^{|k_\perp|<\mu} d^2k_\perp\, \phi_{\mathrm{BS}}(u,k_\perp)$$

△ decay constant $\langle 0|\bar{u}(0)\gamma_z\gamma_5 d(0)|\pi^-(P)\rangle = if_\pi p_\mu$ △ normalizations such as $\int_0^1 du\,\phi(u) = 1$

- LCDAs of pion achieved great success in describing large Q^2 processes.

  △ the establishment and development of the pQCD factorization $F_\pi(Q^2)$

$\int_0^1 du_i \phi(u_1, \tilde{Q}_1) T_H(u_i, Q) \phi(u_2, \tilde{Q}_2)$	1980s	[Lepage & Brodsky 1980, Efremov & Radyushkin 1980]
$\Downarrow$		
$\int_b^{\bar{b}} du_1 \int_{a/u_1}^{1-a/\bar{u}_1} \phi(u_1, k_{1T}) T_H(u_i, Q) \phi(u_2, k_{2T})$	1990s	[Huang, Shen & Kroll 1991, Huang, Shen & Ma 1994]
$\Downarrow$		
$\psi(u_1, \mu_{r_1})\, T_H(u_i, b_i, Q)\, S_t(u_i)\, e^{-S(u_i, b_i, Q)}\, \psi(u_2, \mu_{r_2})$	2000s	[Botts & Sterman 1989, Li & Sterman 1992, Li 1999]
$\Downarrow$		
$\sum_{t_i} \psi^{t_1}(u_1, \mu_{r_1})\, T_H^{t_i, \mathrm{LO+NLO}}(u_i, b_i, Q) S_t(u_i)\, e^{-S(u_i, b, Q)}\, \psi^{t_2}(u_2, \mu_{r_2})$	2010s	[Li, Shen, Wang & Zou 2011, **SC**, Fan & Xiao 2014]
$\sum_{t_i} \psi^{t_1}(u_1, b_1, \mu_{r_1})\, T_H^{t_i, \mathrm{LO+NLO}}(u_i, b_i, Q)\, S_t(u_i)\, e^{-S(u_i, b, Q)}\, \psi^{t_2}(u_2, b_2, \mu_{r_2})$	2020s	[Chai & **SC** 2025] [Chen^2, Feng & Jia 2024], [Ji, Shi, Wang^3 & Yu 2025]

- LCDAs of proton serves as the fundamental input to explain *ep* scattering
  [Chen^2, Feng, Hu, Jia 2025], [Huang, Shi, Wang, Zhao 2025], [Yu, **SC**, Han, Li, Yu 2025]

- The non-perturbative input for HFP theoretical studies that determines the precision and accuracy predicted the CPVs in the $B \to \pi\pi, K\pi$ decays and et.al.,

- $\cdots$

The limitations of the single particle picture

- LCDAs description of V, S mesons reveal certain limitations in the precision testing era

  $\triangle$ the probability amplitudes to **find the light meson** in a state with $\cdots$

  $\triangle$ the $\pi\pi$ invariant mass spectra in $[0.554, 0.996]$GeV is selected to identify candidates for the ρ resonance, $f_0(980)$

- How to accurately describe the width effects of unstable intermediate particles, the contributions and interference effects of different partial waves, and the QCD backgrounds from non-resonant states ?

- f_0 is a superposition of all possible Fock states

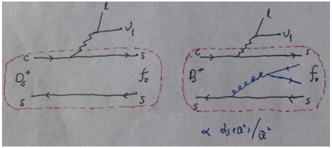
$$|f_0(980)\rangle, \quad |\,[\pi\pi]_{\rm S}\rangle = \psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \psi_{q\bar{q}q\bar{q}}|q\bar{q}q\bar{q}\rangle + \cdots$$

- **Hadron spectroscopy** provides clear evidence for the complex config.

  conventional resonant with possible gluonball component,
  exotic multiparticle state (compact tetraquark state, molecular state)

- **the underlying parton dynamics** can not be extracted directly from spectral analysis

- **Semileptonic** B, D **decays** are powerful probe of the underlying structure
- in terms of LCDAs, scale-dependent functions



color transparency mechanism in $B_{(s)} \to f_0 l^+ \nu_l$ decays

† high Fock states' contribution is doubly suppressed by α_s and $\mathcal{O}(1/Q^2)$, FSI is weak

the mechanism fails in $D_s \to f_0 l^+ \nu_l$ decays

† the produced $q\bar{q}$ is nonrelativistic, likely proceeds through the creation of additional g or dynamical $q\bar{q}$

- while the energy-dependence of partonic configurations is a QCD result
- the cascade decay analyses of $D_s \to (f_0 \to \pi\pi)e\nu$ under $q\bar{q}$ ansatz consists well with data, with $D_s \to f_0$ FFs and Flatté model of f_0 resonant

- QCD calculation have revealed the scale dependence in $D_s \to f_0$ form factor
- the seemingly agreement in D_{l4} decays is attributed to the cascade framework
- the Flatté parameterization of the intermediate resonance disrupts the assessment of color transparency

- a model-independent study directly from the $\pi\pi$ signal state

LCDAs of two-pion system (2πDAs) and their phenomena

- $D_s \to [\pi\pi]_{\mathrm{S}}\, l\nu$ decay and $[\pi\pi]_{\mathrm{S}}$ configurations

2πDAs

- Chiral-even LC expansion with gauge factor $[x, 0]$

$$\langle \pi^a(k_1)\pi^b(k_2)|\bar{q}_f(zn)\gamma_\mu \tau q_{f'}(0)|0\rangle = \kappa_{ab}\, k_\mu \int dx\, e^{iuz(k\cdot n)}\, \Phi_\parallel^{ab,ff'}(u,\zeta,k^2)$$

$$\langle 0|\bar{q}(x)\gamma_\mu\gamma_5 q(-x)|f_0(p)\rangle = p_\mu \int_0^1 du\, e^{i(2u-1)p\cdot x}\phi(u,\mu)$$

- 2πDAs is decomposed in terms of $C_n^{3/2}(2u-1)$ and $C_\ell^{1/2}(2\zeta-1)$

$$\Phi^{I=1}(u,\zeta,k^2,\mu) = 6u(1-u)\sum_{n=0,\mathrm{even}}^{\infty}\ \sum_{l=1,\mathrm{odd}}^{n+1} B_{n\ell}^{I=1}(k^2,\mu)C_n^{3/2}(2u-1)C_\ell^{1/2}(2\zeta-1)$$

$$\Phi^{I=0}(u,\zeta,k^2,\mu) = 6u(1-u)\sum_{n=1,\mathrm{odd}}^{\infty}\ \sum_{l=0,\mathrm{even}}^{n+1} B_{n\ell}^{I=0}(k^2,\mu)C_n^{3/2}(2u-1)C_\ell^{1/2}(2\zeta-1)$$

- How to describe **the evolution from $4m_\pi^2$ to large invariant mass k^2 ?**

‡ Watson theorem of π-π scattering amplitudes

$$B_{n\ell}^I(k^2) = B_{n\ell}^I(0)\,\mathrm{Exp}\left[\sum_{m=1}^{N-1}\frac{k^{2m}}{m!}\frac{d^m}{dk^{2m}}\ln B_{n\ell}^I(0) + \frac{k^{2N}}{\pi}\int_{4m_\pi^2}^{\infty} ds\,\frac{\delta_\ell^I(s)}{s^N(s-k^2-i0)}\right]$$

△ **2πDAs in a wide range of energies is given by δ_ℓ^I and a few subtraction constants**

2πDAs

- The subtraction constants of $B_{n\ell}(k^2)$ at low k^2 (around the threshold)

(nl)	$B^{\parallel}_{n\ell}(0)$	$c_1^{\parallel,(nl)}$	$\frac{d}{dk^2}\ln B^{\parallel}_{n\ell}(0)$	$B^{\perp}_{n\ell}(0)$	$c_1^{\perp,(nl)}$	$\frac{d}{dk^2}\ln B^{\perp}_{n\ell}(0)$
(01)	1	0	1.46 $\to$ 1.80	1	0	0.68 $\to$ 0.60
(21)	-0.113 $\to$ 0.218	-0.340	0.481	0.113 $\to$ 0.185	-0.538	-0.153
(23)	0.147 $\to$ -0.038	0	0.368	0.113 $\to$ 0.185	0	0.153
(10)	-0.556	-	0.413	-	-	-
(12)	0.556	-	0.413	-	-	-

△ firstly studied in the effective low-energy theory based on instanton vacuum [Polyakov 1999]

△ updated with the kinematical constraints and the new a_2^{π}, a_2^{ρ} [**SC** 2019, 2023]

- 2πDAs were introduced at leading twist [Polyakov 1999, Diehl 1998]

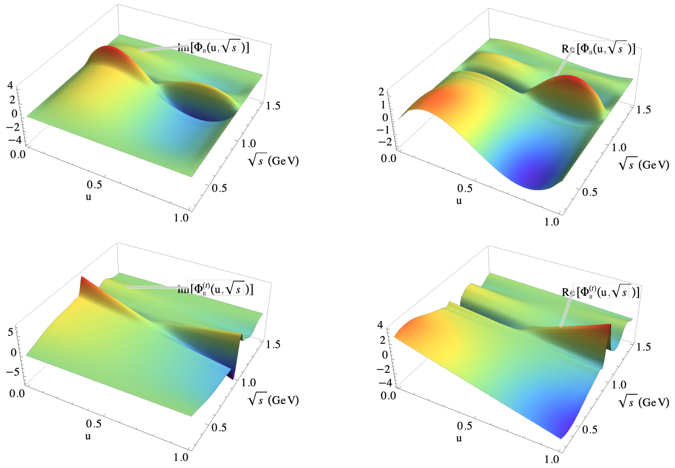
$$\langle \pi^a(k_1)\pi^b(k_2)|\bar{q}_f(zn)\gamma_\mu\tau q_{f'}(0)|0\rangle = \kappa_{ab}\, k_\mu \int dx\, e^{iuz(k\cdot n)}\, \Phi_{\parallel}^{ab,ff'}(u,\zeta,k^2)$$

- improved to twist-three level recently [**SC** 2502.07333]

$$\langle\pi(k_1)\pi(k_2)|\bar{q}(0)q(x)|0\rangle = \int du e^{i\bar{u}k\cdot x}\frac{ik^2(k\cdot x)}{2f_{2\pi}^{\perp}}\Phi_{\parallel}^{(s)},$$

$$\langle\pi(k_1)\pi(k_2)|\bar{q}(0)\sigma^{\mu\nu}q(x)|0\rangle = -\frac{i}{f_{2\pi}^{\perp}}\int du e^{i\bar{u}k\cdot x}\Big[\frac{k_\mu\bar{k}_\nu - k_\nu\bar{k}_\mu}{2\zeta-1}\Phi_{\perp} - k^2\frac{k_\mu x_\nu - k_\nu x_\mu}{k\cdot x}\Phi_{\parallel}^{(t)}\Big].$$

2πDAs $[\pi\pi]_S$



- asymmetry of the twist-3 2πDAs in the parton momentum fraction u
- symmetric for f_0 obtained under the single particle approximation
- where QCD sum rules dictate that the asymmetric component vanishes

$H \to \pi\pi$ from factors

- **QCD dynamics of D_{l4} decays is incorporated in $H \to \pi\pi$ form factors**

$$i\langle \pi^+(k_1)\pi^-(k_2)|\bar{s}\gamma_\nu(1-\gamma_5)c|D_s(p)\rangle = F_\perp(q^2,k^2,\zeta)\,\frac{2}{\sqrt{k^2}\sqrt{\lambda_B}}\,i\epsilon_{\nu\alpha\beta\gamma}\,q^\alpha\,k^\beta\,\bar{k}^\gamma$$

$$+F_t(q^2,k^2,\zeta)\,\frac{q_\nu}{\sqrt{q^2}} + F_0(q^2,k^2,\zeta)\,\frac{2\sqrt{q^2}}{\sqrt{\lambda_B}}\left(k_\nu - \frac{k\cdot q}{q^2}q_\nu\right)$$

$$+F_\parallel(q^2,k^2,\zeta)\,\frac{1}{\sqrt{k^2}}\left(\bar{k}_\nu - \frac{4(q\cdot k)(q\cdot\bar{k})}{\lambda_B}\,k_\nu + \frac{4k^2(q\cdot\bar{k})}{\lambda_B}\,q_\nu\right)$$

† $\lambda = \lambda(m_B^2, k^2, q^2)$ is the Källén function, $q\cdot k = (m_B^2 - q^2 - k^2)/2$,
$q\cdot\bar{k} = \sqrt{\lambda}\beta_\pi(k^2)\cos\theta_\pi/2 = \sqrt{\lambda}\,(2\zeta-1)$, $\beta_\pi(k^2) = \sqrt{1-4m_\pi^2/k^2}$

- **QCDF** (QCD factorization) in the large two-pion mass
  - [P. Böer, T. Feldmann and D. van Dyk, JHEP02, 133(2017)] $T_I \propto F_{Bto\pi} \otimes \phi_\pi$,

- **SU(3) flavor symmetry/breaking** with the intermediate resonance
  - [R.M. Wang, Y.G. Xu, J.H. Sheng, X.D. Cheng and et.al., 2301.00090, PRD 112, 033002 (2025)]

$H \to \pi\pi$ form factors

- **LQCD** (Lattice QCD) in the ρ resonance region with a simple BW model
  - [L. Leskovec and et.al, PRL 134.161901 (2025), **Editors' Suggestion**]

- **HChPT** (Heavy-meson Chiral Perturbative Theory) in the large q^2 by taking dispersive methods in terms of Omnés functions
  - [X.-W. Kang, B. Kubis, C. Hanhart, and U.-G. Meißner, PRD 89. 053015 (2014)]

  in the full phase-space by a novel parameterization with unitarity
  - [F. Herren, B. Kubis and R. van Tonder, PRD 112, 014037 (2025), **Editors' Suggestion**]

- **LCSRs** (Light-cone sum rules) in the small and intermediate q^2
  - [**SC**, A. Khodjamirian and J. Virto, JHEP 05(2017)157] B-meson LCSRs , [S. Descotes-Genon, A. Khodjamirian, J. Virto and K.K. Vos, JHEP 12(2019)083, 06(2023)034] $B \to K\pi$
  - [C. Hambrock and A. Khodjamirian, NPB 905. 379-390(2016)] 2πDAs LCSRS of $F_{\parallel,\perp}$
  - [**SC**, A. Khodjamirian and J. Virto, PRD(R) 96. 051901(2017)] timelike-helicity FF F_t and F_0
  - [**SC**, PRD 99. 053005(2019)] 2πDAs updates and $B \to [\pi\pi]_{\mathrm{S,P}}$ FFs
  - [**SC** and J.M Shen, EPJC 6:554(2020), **SC** and S.L Zhang, EPJC 84:379(2024)] Pheno
  - [**SC**, PRD 112. L111301(2025)] first study of twist-three 2πDAs and $|V_{ub}|$ extraction
  - [**SC**, L.Y. Dai, J.M. Shen and S.L. Zhang, arXiv:2509.15659] $D_s \to [\pi\pi]_{\mathrm{S}}\, e\nu$, minor $q\bar{q}$ contribution

$D_s \to [\pi\pi]\, e^+\nu_e$ decay

- $D_s \to f_0 e^+ \nu$ [CLEO '09], $D_{(s)} \to a_0 e^+ \nu$ [BESIII '18, '21], $D^+ \to f_0/\sigma e^+ \nu$[BESIII '19]
- $D_s \to f_0(\to \pi^0\pi^0, K_s K_s) e^+ \nu$ [BESIII 22], $D_s \to f_0(\to \pi^+\pi^-) e^+ \nu$[BESIII 23]

$$\mathcal{B}(D_s \to f_0(\to \pi^0\pi^0)e^+\nu) = (7.9 \pm 1.4 \pm 0.3) \times 10^{-4}$$
$$\mathcal{B}(D_s \to f_0(\to \pi^+\pi^-)e^+\nu) = (17.2 \pm 1.3 \pm 1.0) \times 10^{-4}$$
$$f_+^{f_0}(0)|V_{cs}| = 0.504 \pm 0.017 \pm 0.035$$

- single particle (narrow width limit) $\quad D_s \to f_0 e^+ \nu$

$$\frac{d\Gamma(D_s^+ \to f_0 l^+ \nu)}{dq^2} = \frac{G_F^2 |V_{cs}|^2 \lambda^{3/2}(m_{D_s}^2, m_{f_0}^2, q^2)}{192\pi^3 m_{D_s}^3} |f_+(q^2)|^2, \; D_s \to f_0 \text{ FF}$$

- improvement with the width effect by resonant models $\quad D_s \to [f_0 \to]\, \pi\pi e^+ \nu$

$$\frac{d\Gamma(D_s^+ \to [\pi\pi]_{\mathrm{S}}\, l^+ \nu)}{ds dq^2} = \frac{1}{\pi} \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_{D_s}^3} |f_+(q^2)|^2 \frac{\lambda^{3/2}(m_{D_s}^2, s, q^2)\, g_1 \beta_\pi(s)}{|m_{\mathrm{S}}^2 - s + i\,(g_1 \beta_\pi(s)) + g_2 \beta_K(s))\,|^2}, \; \text{BESIII}$$

- calculate directly the signal channel $D_s \to [\pi\pi]_{\mathrm{S}}\, e^+ \nu$

$$\frac{d^2\Gamma(D_s^+ \to [\pi\pi]_{\mathrm{S}}\, l^+ \nu)}{dk^2 dq^2} = \frac{G_F^2 |V_{cs}|^2\, \beta_{\pi\pi}(k^2) \sqrt{\lambda_{D_s}} q^2}{3(4\pi)^5 m_{D_s}^3} \sum_{\ell=0}^{\infty} |F_0^{(\ell)}(q^2, k^2)|^2, \; D_s \to \pi\pi \text{ FF}$$

$D_s \to f_0$ FFs and cascade decay $D_s \to (f_0 \to) \pi\pi e^+\nu$

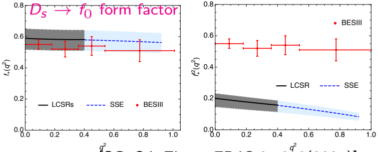
- $\{M^2, s_0\} = \{5.0 \pm 0.5, 6.0 \pm 0.5\}\mathrm{GeV}^2$

this work	3pSRs(07)	LFQM(09)	CLFD/DR(08)	LCSRs(10)
0.63 ± 0.04	0.96	0.87	0.86/0.90	0.30 ± 0.03

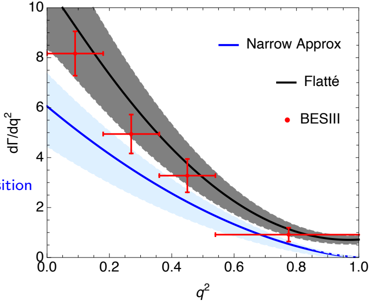
- the BESIII result in the $\pi^+\pi^-$ system $f_+(0) = 0.518 \pm 0.018 \pm 0.036$ [BESIII 23]

different input of the decay constant $\tilde{f}_{f_0} = 335$ MeV, much larger than 180 MeV in LCSRs(10)
we add the first gegenbauer expansion terms in the LCDAs, up-to-date parameters

$s\bar{s}$-$n\bar{n}$ mixing scenario of f_0 with $\theta = 20^\circ \pm 10^\circ$



[**SC**, S.L Zhang EPJC 84:379(2024)]



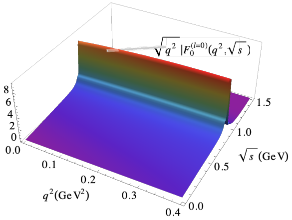
Differential decay width of $D_s^+ \to (f_0 \to) [\pi\pi]_S\, e^+\nu_e$

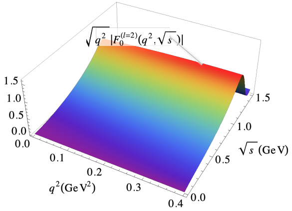
- Twist-3 LCDAs give dominate contribution in $D_s \to f_0$ transition
  - the uncertainty estimation is conservative
  - without NLO correction
  - we need a model independent calculation
  - for the QCD understanding
  - and the future partial-wave measurement

$D_s \to [\pi\pi]_{\rm S}$ FFs and $D_s \to [\pi\pi]_{\rm S}\, e^+\nu$ decay

- The LCSRs ℓ'-wave $D_s \to [\pi\pi]_{\rm S}$ form factors ($\ell' = \text{even}$ & $\ell' \leqslant n+1$)

$$\sqrt{q^2} F_0^{(\ell')}(q^2, k^2) = \frac{m_c(m_c + m_s)\sqrt{q^2}\sqrt{\lambda_{Ds}}}{m_{D_s}^2 f_{D_s}} \sum_{n=1,\text{odd}}^{\infty} \frac{\beta_\pi(k^2)}{\sqrt{2\ell' + 1}} J_n^0(q^2, k^2, M^2, s_0)\, B_{n\ell,\parallel}^{l=0}(k^2)\, I_{\ell\ell'}$$





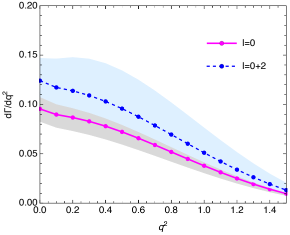
- Twist-2 and twist-3 contributions to $D_s \to \pi\pi, f_0$ form factors at $q^2 = 0$

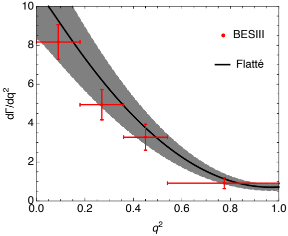
under the $q\bar{q}$ approximation

Form Factors	Twist-2	Twist-3	Total
$\sqrt{q^2}F_0^{(l=0)}(0) = \sqrt{q^2}F_t^{(l=0)}(0)$	$0.20^{+0.02}_{-0.02} - i0.24^{+0.02}_{-0.02}$	$-0.41^{+0.04}_{-0.05} + i0.51^{+0.02}_{-0.04}$	$-0.21^{+0.02}_{-0.01} + i0.27^{+0.03}_{-0.02}$
$\sqrt{q^2}F_0^{(l=2)}(0) = \sqrt{q^2}F_t^{(l=2)}(0)$	$0.27^{+0.03}_{-0.02} + i0.21^{+0.02}_{-0.01}$	$-0.55^{+0.02}_{-0.03} - i0.41^{+0.05}_{-0.04}$	$-0.28^{+0.02}_{-0.02} - i0.20^{+0.02}_{-0.01}$
$f_+(0) = f_0(0)$	$0.20^{+0.03}_{-0.05}$	$0.41^{+0.04}_{-0.06}$	$0.61^{+0.05}_{-0.07}$

$D_s \to [\pi\pi]_S$ FFs and $D_s \to [\pi\pi]_S\, e^+\nu$ decay

- The differential decay width on momentum transfers





$D_s \to [\pi\pi]_S\, e^+\nu_e$	$D_s \to [f_0 \to \pi\pi]\, e^+\nu_e$ [23]	Data [25]
$0.81^{+0.34}_{-0.14}$	$18.8^{+4.5}_{-3.8}$	17.2 ± 1.6

[**SC**, L.Y Dai, J.M Shen and S.L Zhang, 2509.15659]

- Differential widths $d\Gamma/dq^2$ is two-order in magnitude smaller than the data
- non-$q\bar{q}$ Fock states are the dominate component of $[\pi\pi]_S$ in charm decays
- the assessment of color transparency is severe disrupted by the Flatté model
- much different in B decays   leading twist dominated   [**SC**, arXiv: 2502.07333]
- go further to multi-particle DiPion LCDAs in CHARM ($q\bar{q}g$, $q\bar{q}q\bar{q}$)

Summary and Prospect

- 2πDAs 提供了 $\pi\pi$ 质量谱的一般描述

  † 准确厘清不同分波和同一个分波中不同共振态的贡献， † 有效分离、清晰描述非共振态背景的贡献， † 基于交换对称性来理解 SPD/GPD,SPA [Lorcé, Pire & Song PRD 106, 094030 (2022), 109, 074016 (2024)]

- 2πDAs 是重介子四体半轻衰变 (H_{l4}) 研究的关键输入

  † 为 $|V_{ub}|$ 疑难和 FCNC 反常研究提供了新动能， † 为重味强子分布振幅的研究提供了并行检测

- 在三扭度水平研究了 2πDAs，实现了在 B_{l4} 过程抽取 $|V_{ub}|$ [**SC** 2025]
- 理解重介子衰变中的同位旋标量介子 (对) 的部分子结构和多粒子图像
  [**SC**, L.Y Dai, J.M Shen and S.L Zhang 2509.15659]

- 在 $D \to \pi\pi l^+ l^-$ 衰变中深入理解 FCNC 过程可能的反常行为 wishlists
- 在 e^+e^- 湮灭 $\gamma^* \to \pi\pi\gamma, \gamma^*\gamma \to \pi\pi$ 过程中进一步理解 2πDAs
- B 介子多体强子衰变过程 Dalitz 图中非共振态区域的局域 CP 破坏

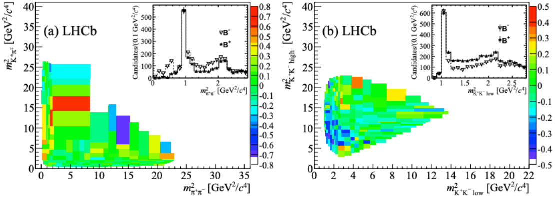
Thank you for your patience.

More opportunities/phenomena of 2πDAs

pion EMFF widely used in the three-body B decays studied from pQCD and QCDF are the asymptotic formula of 2πDAs

[J. Chai, **SC** and A.J Ma PRD 105. 033003 (2022)]

normalized to unit as $\Gamma^{I=1}_{M_1M_2}(0) = 1$. When the invariant mass of dimeson system is small, the higher $\mathcal{O}(s)$ terms in the expansion of coefficient $B_{nl}(s,\mu)$ around the resonance pole can be safely neglected due to the large suppression $\mathcal{O}(s/m_b^2)$ in contrast to the energetic dimeson system in B decay, so the relation $B_{n1}(s,\mu) \to a_n(\mu)\Gamma^{I=1}_{M_1M_2}(s)$ can be obtained in the lowest partial wave approximation. This argument induces the basic assumption in PQCD that the energetic dimeson DAs can be deduced from the DAs of resonant meson by replacing the decay constant by the timelike form factor.



$B \to K\pi\pi, KKK$

[PRL 111.101801(2013) LHCb]

* Non-resonant contributions are small ($< 10\%$) in three-body D decays
* large/dominate in the penguin dominated three-body B decays, the theoretically unresolved problem
* $|V_{cb}|$ in the $B \to D^* l\nu$ processes, B anomalies in $B \to K^* l^+ l^-$ processes