

QCD LCDA of Heavy Mesons from bHQET

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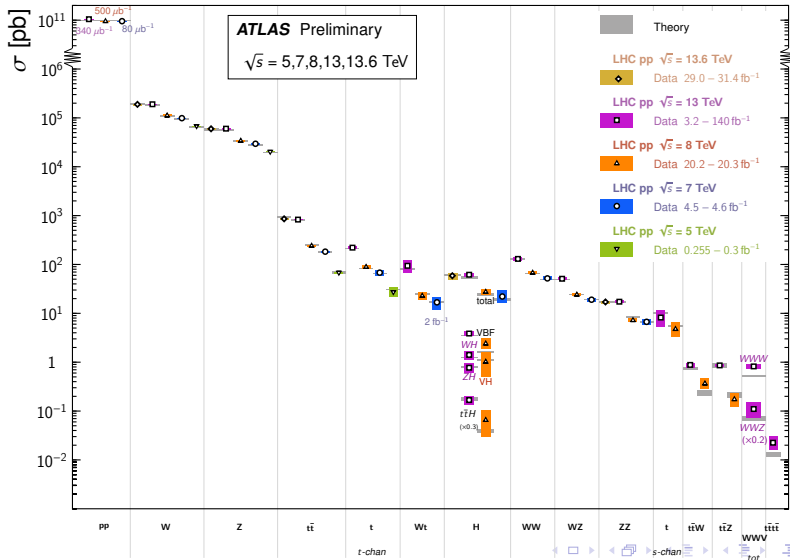
Beneke, Finauri, Vos and **YBW**: 2305.06401

Deng, Wang, **YBW** and Zeng: 2409.00632

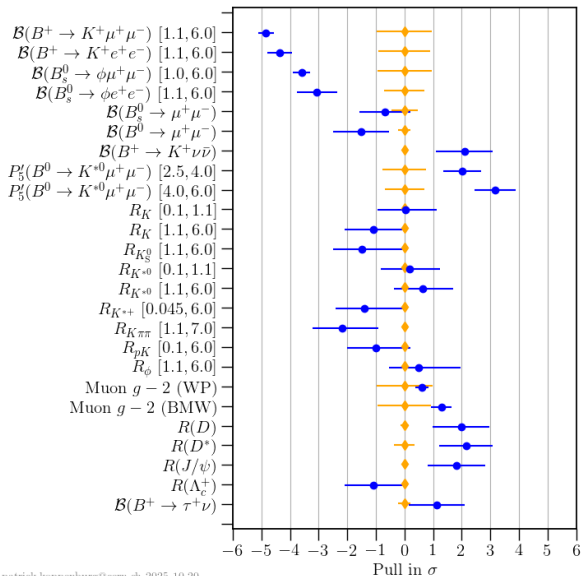


Standard Model Total Production Cross Section Measurements

Status: June 2024



B Physics



- Multi-scale problem:

$$m_b > \sqrt{\Lambda m_b} > \Lambda$$

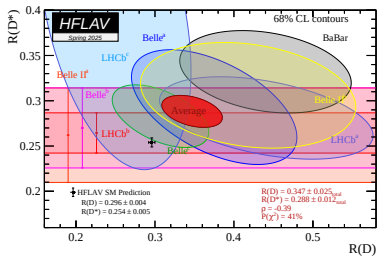
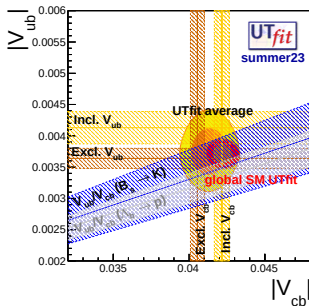
- Power correction:

$$\frac{\Lambda_{\text{QCD}}}{m_b} \sim \mathcal{O}(20\%)$$



New physics beyond the SM

- Indirect search: flavour physics
- LHC, Belle-II
- HL-LHC

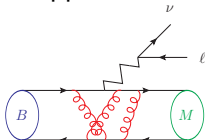


- CPV: $\Lambda_b^0 \rightarrow pK^-\pi^+\pi^-$ [LHCb, 25'], [Wang and Yu, 24']
- $R(D^{(*)})$: [Wang, YBW, Shen and Lü, 17']
- $|V_{ub}/cb|$: [Shen and YBW, 21']



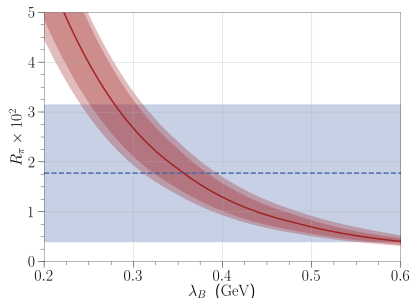
HQET LCDA of heavy meson

HQET LCDA: appears in exclusive B decays



- $B \rightarrow \pi K \dots$
- $B \rightarrow D^{(*)} \ell \nu \dots$
- $B \rightarrow \gamma \ell \nu \dots$

First inverse momentum $\lambda_B \sim \mathcal{O}(\Lambda_{\text{QCD}})$: NOT well determined



[Belle, 18'], [Beneke, Braun, Ji, and YBW, 18']



Direct study of HQET LCDA

HQET LCDA: non-perturbative physics

- Inverse moments: [Wang and Shen, 18'], [Beneke, Braun, Ji, and YBW, 18'], [Belle, 18'], [Galda, Neubert and Wang, 22'], [Han, et al., 24']
- EOM [Kawamura, Kodaira, Qiao and Tanaka, 02'], [Lü, Shen, Wang and YBW, 18']
- RG evolution properties: [Braun, Ji and Manashov, 19'], [Galda and Neubert, 20'], [Cui, Shen, Wang and YBW, 23']
- generalized LCDA [Beneke, Böer, Toelstede, and Vos, 22'], [Qin, Shen, Wang and Wang, 22'], [Huang, Ji, Shen, Wang and Wang, 23']

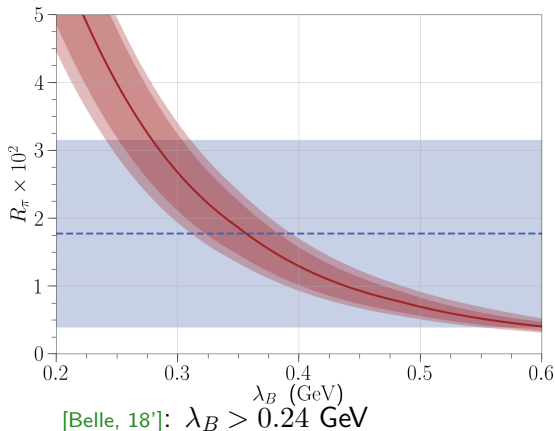
Direct determination: λ_B

- Calculate with LQCD [LPC collaboration, 22']
- Extract from the experiments: clean process $B \rightarrow \gamma \ell \nu$



Direct determination

$B \rightarrow \gamma \ell \nu$: [Wang and Shen, 18'], [Beneke, Braun, Ji, and **YBW**, 18'], [Belle, 18'], [Galda, Neubert and Wang, 22'], [Cui, Shen, Wang, and **YBW**, 23']



Power correction $\Rightarrow$ 50% increase $\Rightarrow \lambda_B > 0.3$ GeV? [Liu, and **YBW**, in progress]



Indirect study of HQET LCDA

Indirect determination: λ_B

From **QCD LCDA** to **HQET LCDA** [Ishaq, Jia, Xiong and Yang, 19'], [Zhao, 19'],
[Beneke, Finauri, Vos and **YBW**, 23'], [Deng, Wang, **YBW** and Zeng, 24']

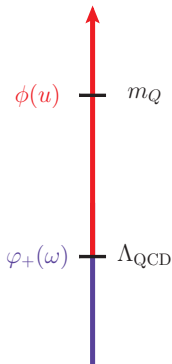
$$\phi(u) = \mathcal{J}(u, \omega) \otimes \varphi_+(\omega)$$

The heavy meson **QCD LCDA** [Braun and Filyanov, 89']

$$\langle H(p_H) | \bar{Q}(0) \not{n}_+ \gamma^5 q(tn_+) | 0 \rangle = -if_H n_+ \cdot p_H \int_0^1 du e^{iutn_+ \cdot p_H} \phi(u)$$

The leading-twist heavy-meson **HQET LCDA** [Grozin and Neubert, 96']

$$\langle H_v | \bar{h}_v(0) \not{n}_+ \gamma^5 q_s(tn_+) | 0 \rangle = -i\tilde{f}_H(\mu) n_+ \cdot v \int_0^\infty d\omega e^{i\omega tn_+ \cdot v} \varphi_+(\omega)$$



Boosted HQET

QCD LCDA $\Rightarrow$ B -meson production: **highly boosted**

The **heavy quark field in bHQET** [Fleming, Hoang, Mantry and Stewart, 07'], [Dai, Kim and Leibovich, 21'], [Finauri: PhD thesis]

$$h_n(x) \equiv \sqrt{\frac{2}{n_+ v}} e^{im_Q v \cdot x} \frac{\not{n}_- \not{n}_+}{4} Q(x)$$

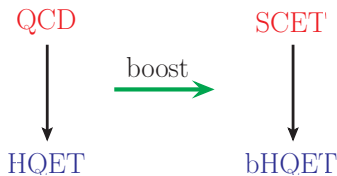
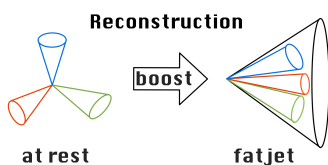
For heavy particle with large energy

- **Top-quark** production [Fleming, Hoang, Mantry and Stewart, 07']
- B -meson leptonic decay with τ **lepton** [Huang, Shen, Zhao and Zhou, 23'], [Shen, Wang, **YBW** and Zhou, in progress]



Systematic power counting

※① **Systematic power counting**: boosted frame



The LP operators

$$\hat{\mathcal{O}}_k = \frac{1}{n_+ v} \sqrt{\frac{n_+ v}{2}} \bar{h}_n \not{p}_+ \left(n_+ v \frac{i \not{D}_\perp}{i n_+ D} \right)^k \gamma^5 \xi_{sc}$$

Only operators $\hat{\mathcal{O}}_0$ and $\hat{\mathcal{O}}_1$ will appear at LP [Beneke, Finauri, Vos and YBW



Factorization of the QCD LCDA

※② **Cut δ** to QCD LCDA: avoid end-point singularity

The factorization formula

$$\phi(u) = \begin{cases} u \sim \delta : & \mathcal{J}_p(u, \omega) \otimes \varphi_+(\omega), & \mathcal{O}_{\text{QCD}} = \mathcal{J}_p \otimes \mathcal{O}_0, \\ u \sim 1 : & \mathcal{J}_{\text{tail}}(u), & \mathcal{O}_{\text{QCD}} = \mathcal{J}_0 \mathcal{O}_0 + \mathcal{J}_1 \mathcal{O}_1, \end{cases}$$

The jet function $\mathcal{J}: \mathcal{O}(m_Q)$, HQET LCDA $\varphi_+: \mathcal{O}(\Lambda_{\text{QCD}})$

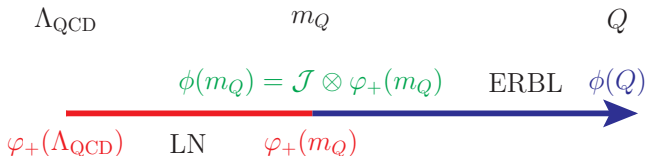
End-point singularity: refactorization-based subtraction (RBS)

- NLP in $h \rightarrow \gamma\gamma$: [Liu and Neubert, 19']
- Off-diagonal parton scattering, NLP in gluon thrust: [Beneke et. al., 20', 22']
- QED correction to $B_s \rightarrow \mu^+ \nu_\mu$: [Cornella, König and Neubert, 22']



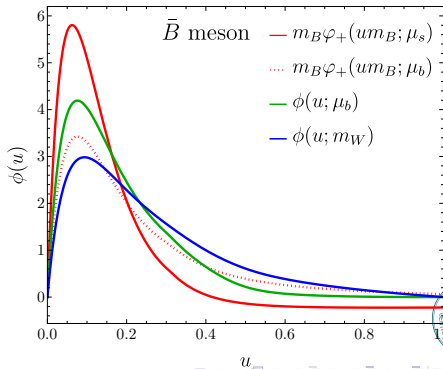
Evolution of the LCDAs: φ_+ and ϕ

Two-step evolutions: $\Lambda_{\text{QCD}} \rightarrow m_Q \rightarrow Q$



$\phi(m_Q) \rightarrow \phi(Q)$:
evolve of the Gegenbauer moments

$$\frac{a_n(\mu_h)}{a_n(\mu)} = \left(\frac{\alpha_s(\mu_h)}{\alpha_s(\mu)} \right)^{\frac{\gamma_n}{2\beta_0}}$$



Heavy-quark spin symmetry

Heavy-quark symmetry [Grozin and Neubert, 97]: relates LCDAs of P and V heavy mesons

Factorization formulas

$$\phi_i(u) = \frac{\tilde{f}_H}{f_i} m_H \mathcal{J}_{\text{peak}}^i(m_H) \varphi_+(um_H), \quad i = P, \parallel, \perp$$

EOM of the quarks, method of region [Deng, Wang, YBW and Zeng, 24']

$$\mathcal{J}_{\text{peak}}^i(u, \omega) = 1 + \frac{\alpha_s C_F}{4\pi} \left(\frac{L^2}{2} + \frac{L}{2} + \frac{\pi^2}{12} + 2 \right) + \mathcal{O}(\alpha_s^2)$$

Estimate NLP corrections



$W \rightarrow B\gamma$: branch ratio

Rare decay $W^- \rightarrow B^- \gamma$ [Grossman, König and Neubert, 15']

[Grossman, König and Neubert, 15']:

QCD LCDA

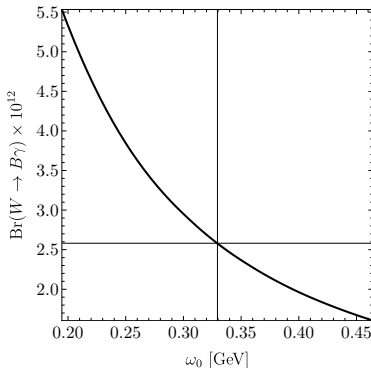
$$\text{Br} = (1.99 \pm 0.17_{\text{in}}^{+0.03}_{-0.06} \mu_h^{+2.48}_{-0.80} \lambda_B) \cdot 10^{-12}$$

Our result: with $\ln \Lambda_{\text{QCD}}/m_Q$ resummation

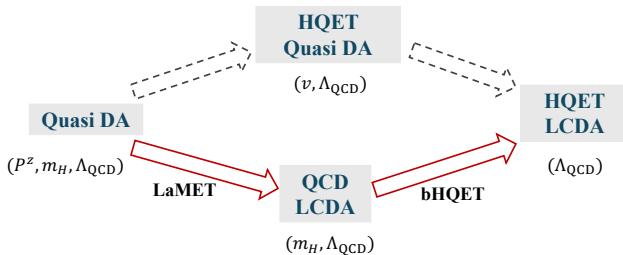
$$\text{Br} = (2.58 \pm 0.21_{\text{in}}^{+0.05}_{-0.08} \mu_h^{+2.95}_{-0.98} \lambda_B) \cdot 10^{-12}$$

30% increase

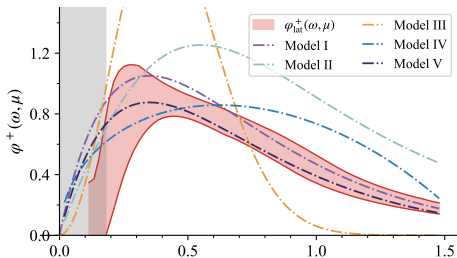
LHC: $H \rightarrow B + \text{jets}$, $Z \rightarrow B + \text{jets}$



Lattice QCD



LaMET [Ji, 13']: quasi DA $\rightarrow$ QCD LCDA $\rightarrow$ HQET LCDA [Han, et al. 24']



First inverse moment
 $\lambda_B = 0.376(63) \text{ GeV}$



Future plans

- ✧ NLP operators in bHQET [Lu, and **YBW**, in progress]
- ✧ From NLL $\rightarrow$ NNLL
- ✧ Endpoint singularity in RBS scheme



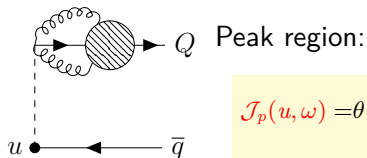
Summary

- ✧ Introduction to the QCD LCDA and HQET LCDAs
- ✧ Match QCD LCDA to the HQET LCDA
 - ① Systematic power counting in bHQET
 - ② Cut δ : avoid endpoint singularity
- ✧ $W^- \rightarrow B^- \gamma$ decay: 30% enhancement

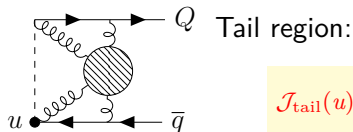
Thank you!



Short distance function



$$\mathcal{J}_p(u, \omega) = \theta(m_Q - \omega) \delta\left(u - \frac{\omega}{m_Q}\right) \left[1 + \frac{\alpha_s C_F}{4\pi} \left(\frac{L^2}{2} + \frac{L}{2} + \frac{\pi^2}{12} + 2 \right) \right], \quad L = \ln \frac{\mu^2}{m_Q^2}$$



$$\mathcal{J}_{\text{tail}}(u) = \frac{\alpha_s C_F}{4\pi} \frac{2\bar{u}}{u} \left((1+u)[L - 2\ln u] - u + 1 \right)$$

