Nonrelativistic QCD as a novel probe of fully-heavy exotic hadrons

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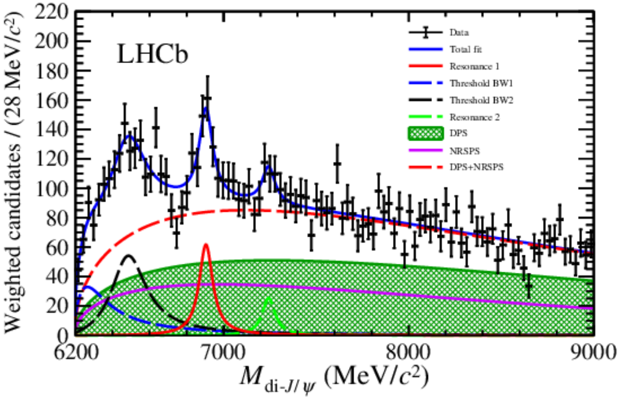
Background

Background

Double J/ψ Resonances

- Double J/ψ resonances observed at the LHC



Possible candidates (S-wave)

$T_{cc\bar{c}\bar{c}}$	$^1S_0(^1S_0, ^1S_0)$	$^1S_0(^3S_1, ^3S_1)$	$^5S_2(^3S_1, ^3S_1)$
$M_{J/\psi J/\psi}$	$^1S_0$	$^5S_2$	

- Mass splitting: $m_c v^4 \approx 100\mathrm{MeV}$

Questions

- How to determine its J^{PC}? (Solved!)
  - From first principle
  - Robust!
- How to explore the structures of these resonances?
  - Compact tetraquarks or molecules?
  - Where is the ground state?
  - Radial excited or orbital excited?

Results for the LHC experiments

Results for the LHC experiments

NRQCD Factorization

- Cross Section Factorization

$$\mathrm{d}\sigma(H) = \sum_n \mathrm{d}\hat{\sigma}(n)\langle\mathcal{O}^H(n)\rangle$$

  - n: intermediate state
  - $\hat{\sigma}(n)$: Short-distance coefficient
  - $\langle\mathcal{O}^H(n)\rangle$: Long-distance Matrix Element
- Difficulty: $\langle\mathcal{O}^H(n)\rangle \sim m_c^9\alpha_s^9$, no available data!

Replacement

- For J/ψ

$$v_i(p_b)\overline{u}_j(p_a) \to \Pi_{J/\psi} \equiv \frac{1}{2\sqrt{2}}\frac{1}{\sqrt{N_c}}\delta_{ij}\not{\epsilon}(\not{p}+m_{J/\psi})$$

$$\mathrm{d}\Phi_a\mathrm{d}\Phi_b \to \mathrm{d}\Phi_{J/\psi} \equiv \frac{\mathrm{d}^3p}{(2\pi)^3 2p_0}\frac{2}{m_{J/\psi}}\left|\psi_{J/\psi}(0)\right|^2$$

Replacement

- For Molecule

$$\epsilon^{\mu} \otimes \epsilon^{\nu} \rightarrow \varepsilon^{\mu\nu}_{ss_z}$$

$$\varepsilon^{\mu\nu}_{00} = \sqrt{\frac{1}{3}}(-g^{\mu\nu} + \frac{P^{\mu}P^{\nu}}{M^2}) \equiv \sqrt{\frac{1}{3}}\Pi^{\mu\nu}$$

$$\sum_{s_z} \varepsilon^{\mu\nu}_{2s_z}\varepsilon^{\alpha\beta *}_{2s_z} = \frac{1}{2}(\Pi^{\mu\alpha}\Pi^{\nu\beta} + \Pi^{\mu\beta}\Pi^{\nu\alpha}) - \frac{1}{3}\Pi^{\mu\nu}\Pi^{\alpha\beta}$$

Replacement

- For Genuine Tetraquark
  - CParity Transformation:

$$\overline{u}_i^{\alpha}(p_b)(\not{k}_1 + m_1)\dots(\not{k}_n + m_n)v_j^{\beta}(p_{(c,d)})$$
$$= \overline{u}_j^{\beta}(p_{(c,d)})(-\not{k}_n + m_n)\dots(-\not{k}_1 + m_1)v_i^{\alpha}(p_b)$$

- Replacement

$$v_{i_b}(p_b)\overline{u}_{i_a}(p_a) \otimes v_{i_{(c,d)}}(p_{(c,d)})\overline{u}_{i_{(d,c)}}(p_{(d,c)})$$
$$\rightarrow \frac{1}{32}\varepsilon^{\mu\nu}_{ss_z}\frac{\delta_{i_a i_c}\delta_{i_b i_d} - \delta_{i_a i_d}\delta_{i_b i_c}}{\sqrt{12}} \times \gamma^{\mu}(\not{P} + M) \otimes \gamma^{\nu}(\not{P} + M)$$
$$v_{i_b}(p_b)\overline{u}_{i_a}(p_a) \otimes v_{i_{(c,d)}}(p_{(c,d)})\overline{u}_{i_{(d,c)}}(p_{(d,c)})$$
$$\rightarrow \frac{1}{32}sgn\frac{\delta_{i_a i_c}\delta_{i_b i_d} + \delta_{i_a i_d}\delta_{i_b i_c}}{\sqrt{24}} \times \gamma^{5}(\not{P} + M) \otimes \gamma^{5}(\not{P} + M)$$

Quantity to explore the spin

- Ratio (R) of cross sections for spin-2 to spin-0

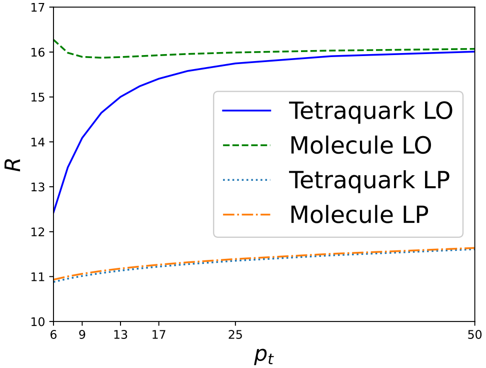
$$R(T) \equiv \frac{\mathrm{d}\sigma(T_{cc\bar{c}\bar{c}}[2^{++}])}{\mathrm{d}\sigma(T_{cc\bar{c}\bar{c}}[0^{++}])} = \frac{5\mathrm{d}\hat{\sigma}(cc\bar{c}\bar{c}[2^{++}])}{\mathrm{d}\hat{\sigma}(cc\bar{c}\bar{c}[0^{++}])}$$

$$R(M) \equiv \frac{\mathrm{d}\sigma(M_{\psi\psi}[2^{++}])}{\mathrm{d}\sigma(M_{\psi\psi}[0^{++}])} = \frac{5\mathrm{d}\hat{\sigma}(\psi\psi[2^{++}])}{\mathrm{d}\hat{\sigma}(\psi\psi[0^{++}])}$$

- Free of nonperturbative parameters!

Ratio versus Transverse Momentum

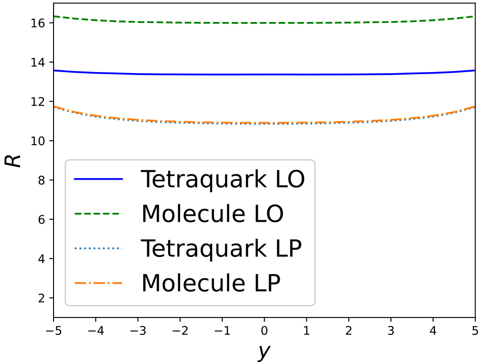
- Ratio of cross sections for spin-2 to spin-0[1]



[1] HFZ, Y.-Q. Ma, W.-L. Sang, Sci. Bull. 70, 1915 (2025)

Ratio versus Rapidity

- Ratio of cross sections for spin-2 to spin-0[2]



[2] HFZ, Y.-Q. Ma, W.-L. Sang, Sci. Bull. 70, 1915 (2025)

CMS Measurement

- Confirmed by the CMS measurement!

Determination of the spin and parity of all-charm tetraquarks

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The traditional quark model[1,2] accounts for the existence of baryons, such as protons and neutrons, which consist of three quarks, as well as mesons, composed of a quark–antiquark pair. Only recently has substantial evidence started to accumulate for exotic states composed of four or five quarks and antiquarks[3]. The exact nature of their internal structure remains uncertain[4–29]. Here we report the first measurement of quantum numbers of the recently discovered family of three all-charm tetraquarks[30–32], using data collected by the CMS experiment at the Large Hadron Collider from 2016 to 2018 (refs. 33,34). The angular analysis techniques developed for the discovery and characterization of the Higgs boson[35–37] have been applied to the new exotic states. Here we show that the quantum numbers for parity P and charge conjugation C symmetries are found to be +1. The spin J of these exotic states is determined to be consistent with $2\hbar$, while $0\hbar$ and $1\hbar$ are excluded at 95% and 99% confidence levels, respectively. The $J^{PC} = 2^{++}$ assignment implies particular configurations of constituent spins and orbital angular momenta, which constrain the possible internal structure of these tetraquarks.

Extraction of LDME by Fitting

- LHCb measured the fraction of the cross section for the resonance over that for double J/ψ in the region $p_t^{\mathrm{di}-J/\psi} > 5.2\mathrm{Gev}$, $2 < y_{J/\psi} < 5$, $p_t^{J/\psi} < 10\mathrm{GeV}$.
- $\left|\Psi_C(0)\right|^2 = (7.0 \pm 2.7) \times 10^{-6}\mathrm{GeV}^9$
- $\left|\Psi_M(0)\right|^2 \left|\psi_{J/\psi}(0)\right|^4 = (2.1 \pm 0.8) \times 10^{-5}\mathrm{GeV}^9$

Ratio to distinguish between compact tetraquarks and molecules

- Ratio $\mathcal{R}$ to distinguish between compact tetraquarks and molecules

$$\mathcal{R} \equiv \frac{\mathcal{B}(T_{cc\bar{c}\bar{c}} \to c\bar{c})}{\mathcal{B}(T_{cc\bar{c}\bar{c}} \to J/\psi\gamma)}$$

- Free of nonperturbative parameters!

Results of $\mathcal{R}$

- Results of $\mathcal{R}$ for compact tetraquarks and molecules:

$$\mathcal{R}(T) \approx 3.4$$
$$\mathcal{R}(M) \approx 780$$

- Perfectly distinguish between the two structures!
- The structures of the resonances can be explored even by measuring the upper or lower limit of $\mathcal{R}$.
- $\mathcal{B}(X(6900) \to J/\psi + \gamma) \approx 10^{-8}$

Results for Belle

Results for Belle

Y states

- Y ($J^{PC} = 1^{--}$) compact tetraquark states:
    - $y_1 = C_{1\lambda}^{S_c L_c} G^+ \chi^+(S_c = 1)\phi(L_c = 1)\chi^-(S_{\bar{c}} = 0)\phi(L_{\bar{c}} = 0)\phi_{cc-\bar{c}\bar{c}}(L = 0)$
    - $y_2 = C_{1\lambda}^{S_c L_{\bar{c}}} G^- \chi^+(S_c = 1)\phi(L_c = 0)\chi^-(S_{\bar{c}} = 0)\phi(L_{\bar{c}} = 1)\phi_{cc-\bar{c}\bar{c}}(L = 0)$
    - $Y_1 = \frac{1}{2}(y_1 - \hat{P}y_1 - \hat{C}y_1 + \hat{P}\hat{C}y_1)$
    - $Y_2 = \frac{1}{2}(y_2 - \hat{P}y_2 - \hat{C}y_2 + \hat{P}\hat{C}y_2)$
    - $Y_3 = C_{1\lambda}^{S_c S_{\bar{c}}} G^- \chi^+(S_c = 1)\phi(L_c = 0)\chi^-(S_{\bar{c}} = 1)\phi(L_{\bar{c}} = 0)\phi_{cc-\bar{c}\bar{c}}(L = 1)$
    - $Y_4 = C_{1\lambda}^{S_c S_{\bar{c}}} G^+ \chi^+(S_c = 0)\phi(L_c = 0)\chi^-(S_{\bar{c}} = 0)\phi(L_{\bar{c}} = 0)\phi_{cc-\bar{c}\bar{c}}(L = 1)$
- Y molecule states:
    - $J/\psi\eta_c$
    - $J/\psi\chi_c$
- It is possible to observe them using initial-state radiation.

Υ Decay to di-J/ψ resonance plus γ

- $\mathcal{B}(\Upsilon \to X(6900) + \gamma) = 2 \times 10^{-8}$
- Number of events at Super B-factories: about 100 events.
- It is possible to be observed in the future.

Ratio to distinguish between compact tetraquarks and molecules for Y states

- Ratio $\mathcal{R}$ to distinguish between compact tetraquarks and molecules for Y states

$$\mathcal{R} \equiv \frac{\mathcal{B}(Y_{cc\bar{c}\bar{c}} \to c\bar{c})}{\mathcal{B}(Y_{cc\bar{c}\bar{c}} \to \eta_c\gamma)}$$

- Free of nonperturbative parameters!

Discussion

- Y states can serve as benchmarks of the fully-charm tetraquarks.
- It is possible that di-J/ψ resonances be observed at B-factories.

Conclusion

Conclusion

Conclusion

- NRQCD as a promising method to access the nature of fully-heavy exotic states.
- Υ states can serve as benchmarks of the fully-charm tetraquarks.
- The structure of a family of exotic states might be determined for the first time!

Thanks

Thanks!

Appendix

Appendix: New Method for Solving Multibody Systems

Hartree-Fock Method

- Wave function ansatz: Slater determinant

$$\begin{vmatrix} \psi_1(x_1) & \psi_2(x_1) & \dots & \psi_n(x_1) \\ \psi_1(x_2) & \psi_2(x_2) & \dots & \psi_n(x_2) \\ \dots & & & \\ \psi_1(x_n) & \psi_2(x_n) & \dots & \psi_n(x_n) \end{vmatrix}$$

- Hartree-Fock Equation

$$-\frac{\hbar^2}{2m_i}\nabla_i\psi_i(\boldsymbol{x_i}) + \int \mathrm{d}^3x_1 \dots \mathrm{d}^3x_{i-1}\mathrm{d}^3x_{i+1} \dots \mathrm{d}^3x_n \\ \times |\psi_1(\boldsymbol{x}_1)|^2 \dots |\psi_{i-1}(\boldsymbol{x_{i-1}})|^2 |\psi_{i+1}(\boldsymbol{x_{i+1}})|^2 \dots |\psi_n(\boldsymbol{x_n})|^2 \\ \times V(\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x_n})\psi_i(\boldsymbol{x_i}) = E_i\psi_i(\boldsymbol{x_i})$$

Drawbacks of the Hartree-Fock Method

- Incapable of strongly correlated systems
  - Slater determinant ansatz
- Low precision
  - fail in solving hydrogen ground-state energy
- Kinetic energy of the center of mass cannot be naturally eliminated
  - each ψ_i has kinetic energy

New Ansatz

- Construct an eigenstate of angular momentum
- Spacial wave function

$$\psi = \sum_{mm' m_1 m_2} C_{LM}^{mm'} Y_{lm}(\theta, \vartheta) R_{nl}(r) C_{l' m'}^{m_1 m_2} \times Y_{l_1 m_1}(\theta_1, \vartheta_1) R_{1 n_1 l_1}(r_1) Y_{l_2 m_2}(\theta_2, \vartheta_2) R_{2 n_2 l_2}(r_2)$$

- Functions to be solved: R_{nl}, $R_{1n_1 l_1}$, $R_{2n_2 l_2}$

Hartree-Fock Equations

- Hartree-Fock Equation via new ansatz

$$\left[-\frac{\hbar^2}{2m_q}\left(\frac{1}{r}\frac{\mathrm{d}^2}{\mathrm{d}r^2}r-\frac{l(l+1)}{r^2}\right)+V_0(r)-\mathcal{E}_0\right]R_{nl}(r)=0,$$
$$\left[-\frac{\hbar^2}{m_q}\left(\frac{1}{r_1}\frac{\mathrm{d}^2}{\mathrm{d}r_1^2}r_1-\frac{l_1(l_1+1)}{r_1^2}\right)+V_{qq}(r_1)+V_1(r_1)-\mathcal{E}_1\right]R_{1n_1l_1}(r_1)=0,$$
$$\left[-\frac{\hbar^2}{m_q}\left(\frac{1}{r_2}\frac{\mathrm{d}^2}{\mathrm{d}r_2^2}r_2-\frac{l_2(l_2+1)}{r_2^2}\right)+V_{qq}(r_2)+V_2(r_2)-\mathcal{E}_2\right]R_{2n_2l_2}(r_2)=0$$

$$V_{q\bar{q}}(\boldsymbol{r},\boldsymbol{r}_1,\boldsymbol{r}_2)=\sum_{lml_1m_1l_2m_2}V_{lml_1m_1l_2m_2}(r,r_1,r_2)Y_{lm}(\theta,\vartheta)Y_{l_1m_1}(\theta_1,\vartheta_1)Y_{l_2m_2}(\theta_2,\vartheta_2)$$

$$V_0(r)=\int \mathrm{d}r_1\mathrm{d}r_2\,\varrho_1(r_1)\varrho_2(r_2)V_{lml_1m_1l_2m_2}(r,r_1,r_2),$$

$$V_1(r_1)=\int \mathrm{d}r\mathrm{d}r_2\,\varrho(r)\varrho_2(r_2)V_{lml_1m_1l_2m_2}(r,r_1,r_2),$$

$$V_2(r_2)=\int \mathrm{d}r\mathrm{d}r_1\,\varrho(r)\varrho_1(r_1)V_{lml_1m_1l_2m_2}(r,r_1,r_2)$$

Thanks!