The Bubble Wall Velocity in Local Thermal Equilibrium and Energy Budget with Full Effective Potential

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Outline

I. Motivation

- Definition of the EWPT and bubble wall velocity.
- Why determine the bubble wall velocity precisely.

II. Method

- Determining the bubble wall velocity and fluid profiles.
- Classifying hydrodynamic modes.
- Improving the bag model.

III. Result

IV. Summary

Motivation

Intensity Frontier

flavor BSM、long-lived particle、neutrino BSM、precision BSM effects ……

Energy Frontier

LHC、FCC、CEPC、Muon Collider ……

Cosmic Frontier

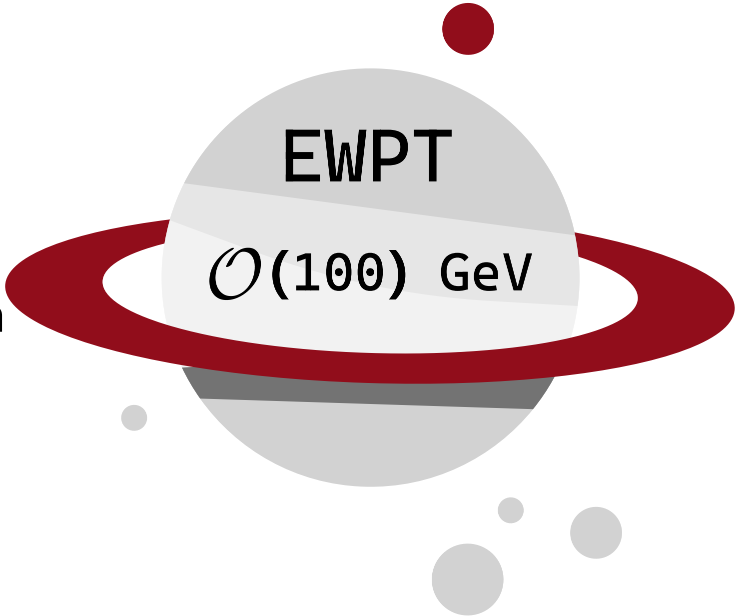
Cosmology, dark matter, dark energy, gravity waves ……

Motivation

➢ What is the electroweak phase transition ?

✓ Theory

- Effective potential
- Vacuum structure
- Phase transition strength
- Critical temperature
- Nucleation temperature
- ……

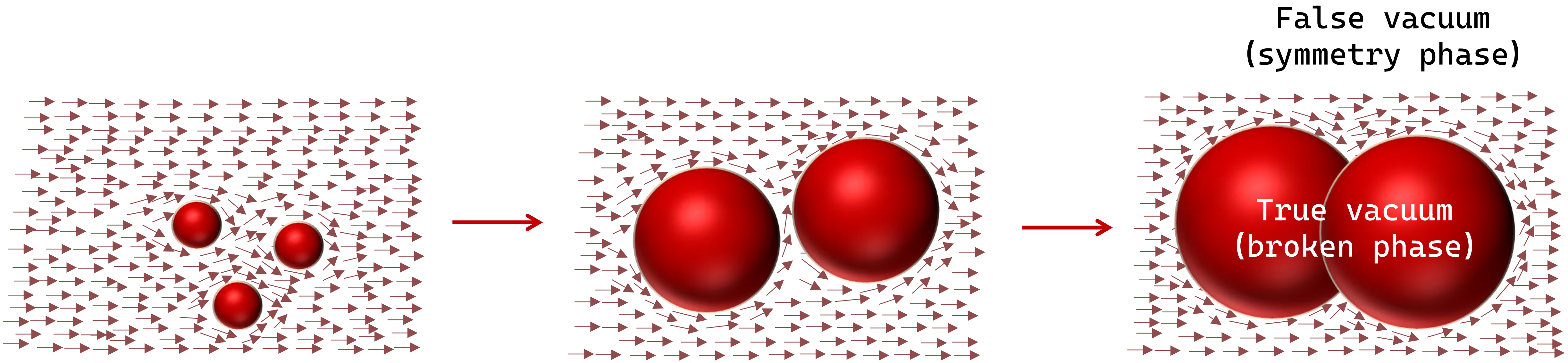


✓ Experiments

- Accessible scale for future high-energy colliders
- Phase transition gravitational wave
- High-energy collider
- Taiji, TianQin and LISA
- ……

Motivation

➢ What is the definition of ξ_w ?



Motivation

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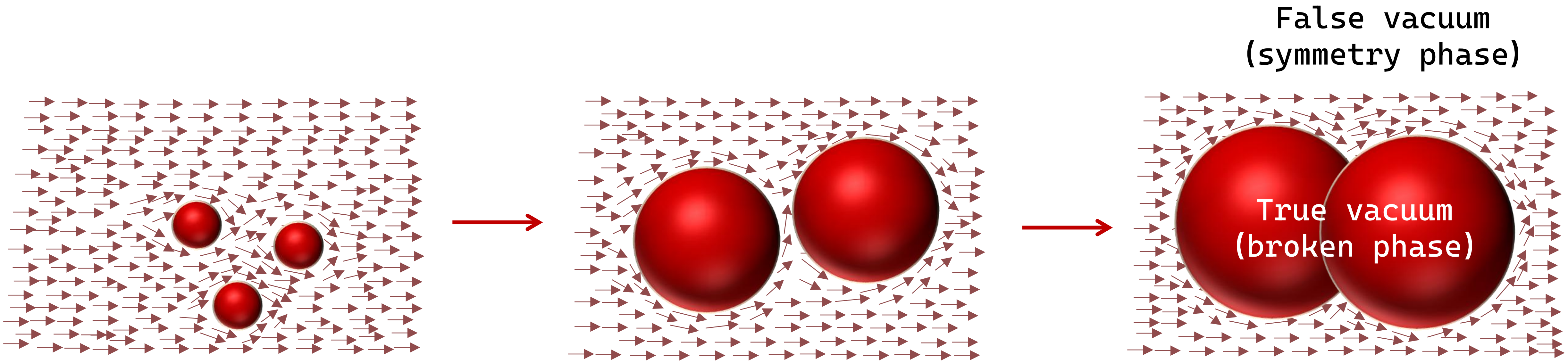

The bubble wall velocity is a key parameter in phase transition dynamics.

Motivation

➢ **What is the definition of ξ_w ?**

$\boxed{\alpha_N, \xi_w}$

$V_{\rm eff} = \epsilon - \frac{1}{3}\mathbf{a}\mathbf{T}^4$

JCAP 06 (2010) 028

Bag Model

$c_s^2 = \frac{1}{3}$

Nucl.Phys.B 891 (2015)

μν model

MLRSM

Phys.Rev.D 110 (2024) 7

$V_{\rm eff} = \epsilon - \frac{1}{3}\mathbf{a}\mathbf{T}^4 - \mathbf{b}\mathbf{T}^2 + \mathbf{c}\mathbf{T}$

JCAP 10 (2022) 047

Beyond Bag model

μν model

JCAP 07 (2023) 002

$V_{\rm eff} = \frac{1}{2}(\mathbf{m}^2 + \mathbf{c}\mathbf{T}^2)\phi^2 - \mathbf{E}\mathbf{T}\phi^3 + \frac{\lambda}{4}\phi^4$

JCAP 07 (2023) 006

Model dependent

Bag model, xSM HT

JCAP 03 (2022) 03, 015

local thermal equilibrium (LTE) approximation

Motivation

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JCAP 07 (2023) 006

Model dependent

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JCAP 03 (2022) 03, 015

local thermal equilibrium (LTE) approximation

`The aim of this work : determining the Bubble Wall Velocity and Fluid Profile for Arbitrary BSM Models.`

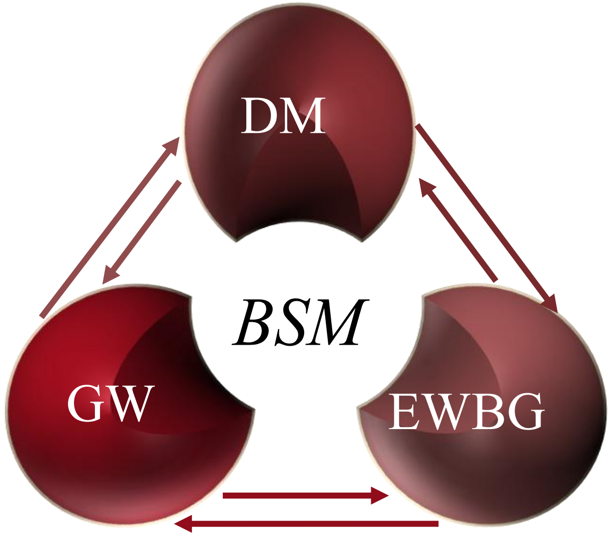
Motivation

➢ Why accurately calculate ξ_w?

- Indirect approaches to probing new physics.

$$\Omega_{\rm sw}h^2 = 2.65\times 10^{-6}\left(\frac{H(T_*)}{\beta}\right) K^2 \left(\frac{100}{g_*(T_*)}\right)^{1/3} \xi_w \Upsilon(\tau_{\rm sw}) \times \left(\frac{f}{f_{\rm sw}}\right)^3 \left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{7/2},$$

$$K = \frac{\kappa_{\rm sw}\alpha_N}{1+\alpha_N} \qquad K = \frac{\rho_{\rm kin}}{\rho_{\rm tot}}$$



Phys.Rev.D 92 (2015) 12

JCAP 07 (2020) 07

Phys.Rev.D 101 (2020) 6

- The bubble wall velocity acts as the dynamical bridge connecting these three aspects.

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Method

➢ How to calculate ξ_w and fluid configuration ?

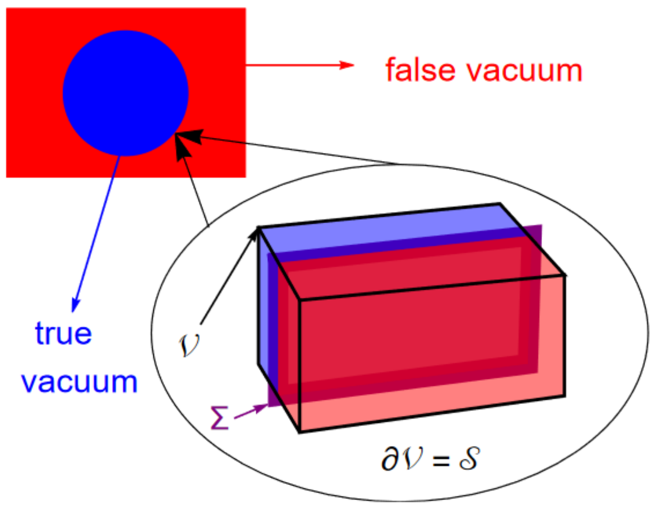
$$T_{\mu\nu} = (e + p)u_\mu u_\nu + p g_{\mu\nu}$$

determined by the effective potential

- At the bubble wall :

Bubble wall frame

$$\boxed{\begin{aligned} w_+ v_+ \gamma_+^2 &= w_- v_- \gamma_-^2 \\ w_+ v_+^2 \gamma_+^2 + p_+ &= w_- v_-^2 \gamma_-^2 + p_- \end{aligned}}$$



Method

➤ How to calculate ξ_w and fluid configuration ?

- The equation of fluid motion

$$\partial_\mu T^{\mu\nu} = 0$$

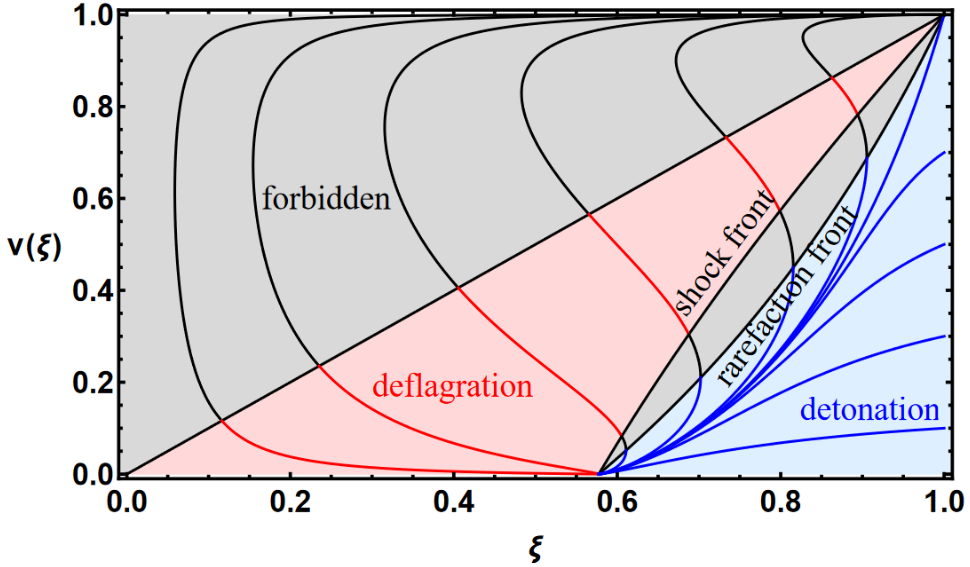
Project along parallel and perpendicular direction

$$\xi = r/t$$

Bubble center frame

$$\boxed{\frac{2\bar{v}}{\xi} = \gamma^2 (1 - \xi\bar{v}) \left(\frac{\mu^2}{c_s^2} - 1 \right) \frac{\partial \bar{v}}{\partial \xi}}$$

Lorentz-boost transformation $\mu(\xi, v) = \frac{\xi - v}{1 - \xi v}$



Method

- **More Constraints ⇒ Fewer Free Variables**

$$\partial_\mu S^\mu \equiv \partial_\mu (s u^\mu) = 0$$

local thermal equilibrium (LTE).

$$\gamma T = \text{const}$$

!!

$$w_+ v_+ \gamma_+^2 = w_- v_- \gamma_-^2$$

$$w_+ v_+^2 \gamma_+^2 + p_+ = w_- v_-^2 \gamma_-^2 + p_- \quad \propto \quad v_+ v_- T_+ , T_-$$

$$\gamma_+ T_+ = \gamma_- T_-$$

$$\frac{2\bar{v}}{\xi} = \gamma^2 (1 - \xi \bar{v}) \left(\frac{\mu^2}{c_s^2} - 1 \right) \frac{\partial \bar{v}}{\partial \xi}$$

	Detonation	Deflagration	Hybrid
two matching condition	$\boldsymbol{T_+}, \boldsymbol{\bar{v}_+}, T_-, \bar{v}_-$	$T_+, \bar{v}_+, \boldsymbol{T_-}, \boldsymbol{\bar{v}_-}$	$T_+, \bar{v}_+, \boldsymbol{T_-}$
three matching condition	$\boldsymbol{T_+}, \bar{v}_+, T_-, \bar{v}_-$	$T_+, \bar{v}_+, \boldsymbol{T_-}, \bar{v}_-$	$T_+, \bar{v}_+, T_-$

Method

➢ **Our work improves the Bag model in three aspects**

Bag Model **VS** Our work

Equation of state (EOS)

CosmoTransitions

Comput.Phys.Commun. 183 (2012) 2006-2013

Matching conditions

$$\gamma T = \mathrm{const}$$

Solve equation

$$\begin{tikzcd} & \bar{v} \arrow[dl] & \\ T \arrow[rr] & & c_s^2 \arrow[ul] \end{tikzcd}$$

$$\frac{2\bar{v}}{\xi} = \gamma^2 (1 - \xi\bar{v}) \left(\frac{\mu^2}{c_s^2(T)} - 1 \right) \frac{\partial \bar{v}}{\partial \xi}$$

Result

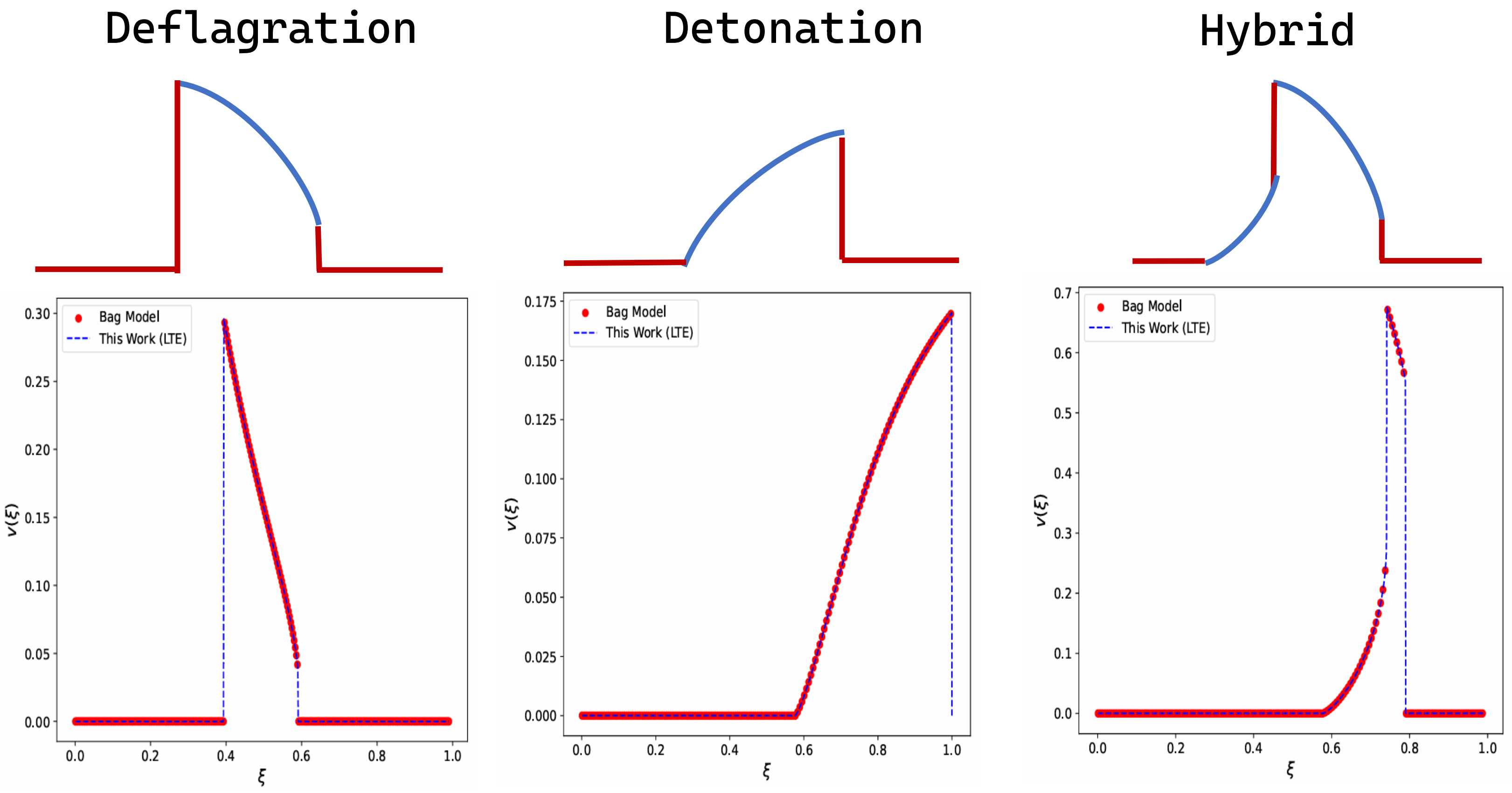
➢ Check our work with Bag Model

Select a bag model-like V_{eff}

↓ our work

↓ Bag Model (α_N, ξ_w)

↓ Calculate profiles of the fluid

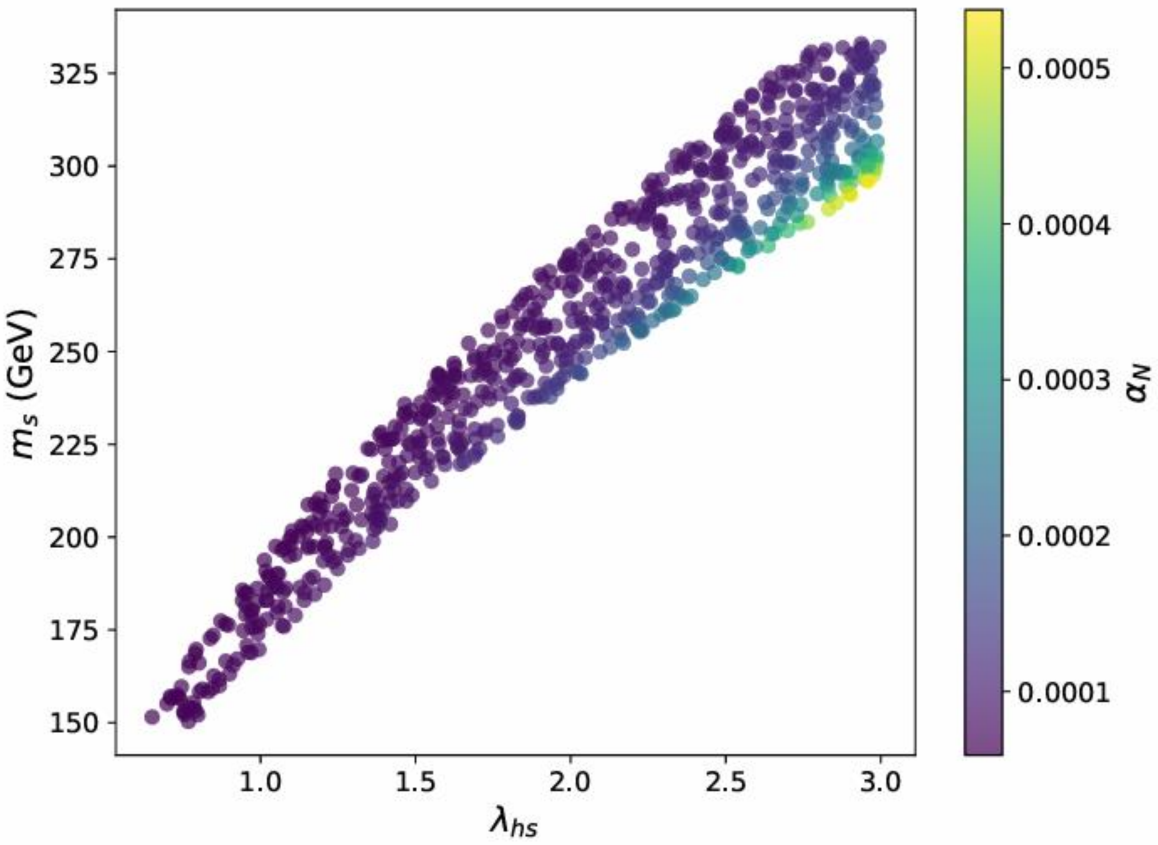


- LTE Results Consistent with Bag Model.

Result

➢ Scan parameters under XSM

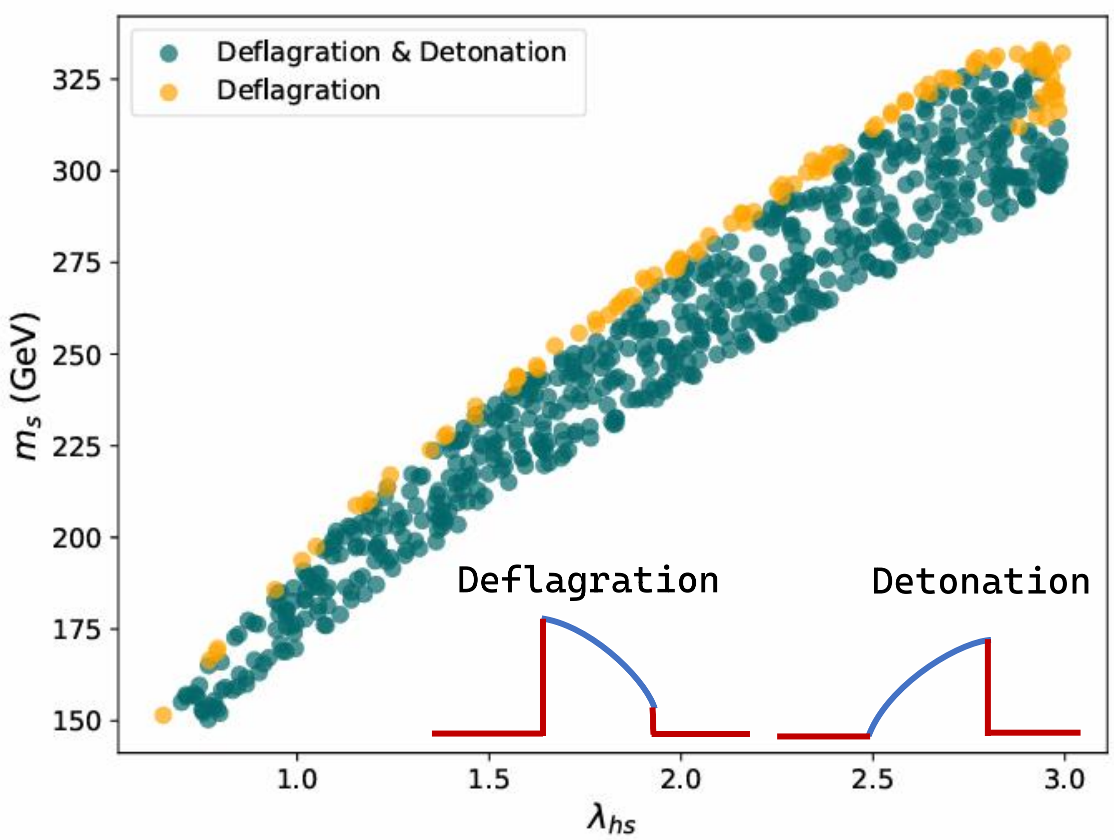
$$V_{\text{eff}}(h,s;T) = V_0(h,s) + V_{\text{CW}}(h,s) + V_{1T}(h,s;T) + V_{\text{ring}}(h,s;T)$$



- The LTE valid only for small α_N.
- Large α_N parameters are excluded.

Result

➢ Scan parameters under XSM



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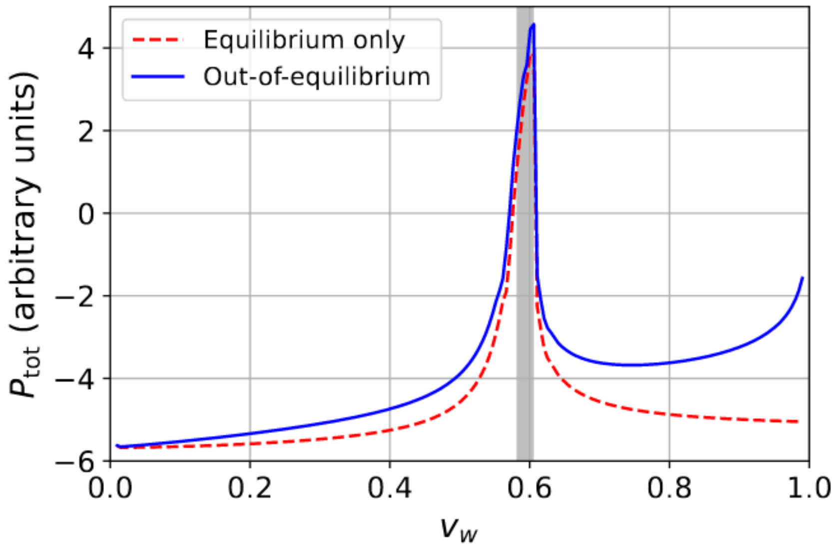


- Deflagration is the most likely fluid motion mode.
- For small α_N, only Deflagration mode can occur.

Result

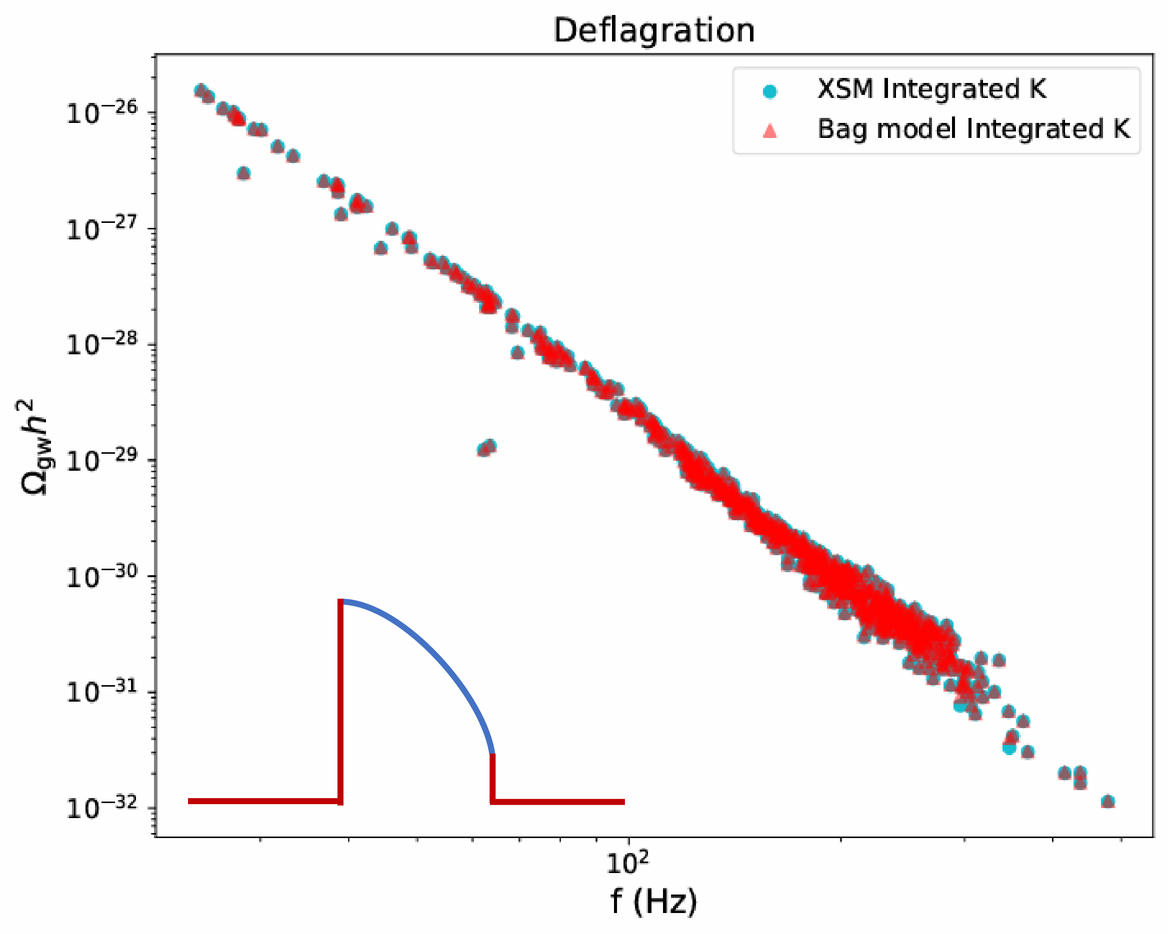
➢ Compare gravitational wave results

- The gravitational spectrum is highly sensitive to the equation of state (EOS), kinetic energy fraction κ , and bubble wall velocity ξ_w .

ξ_w	κ	EOS
Accurate calculation under LTE	Integration over the fluid profile	XSM
or	or	or
Choose a fixed value	fitting formulas	Bag model

Result

➢ Compare gravitational wave results

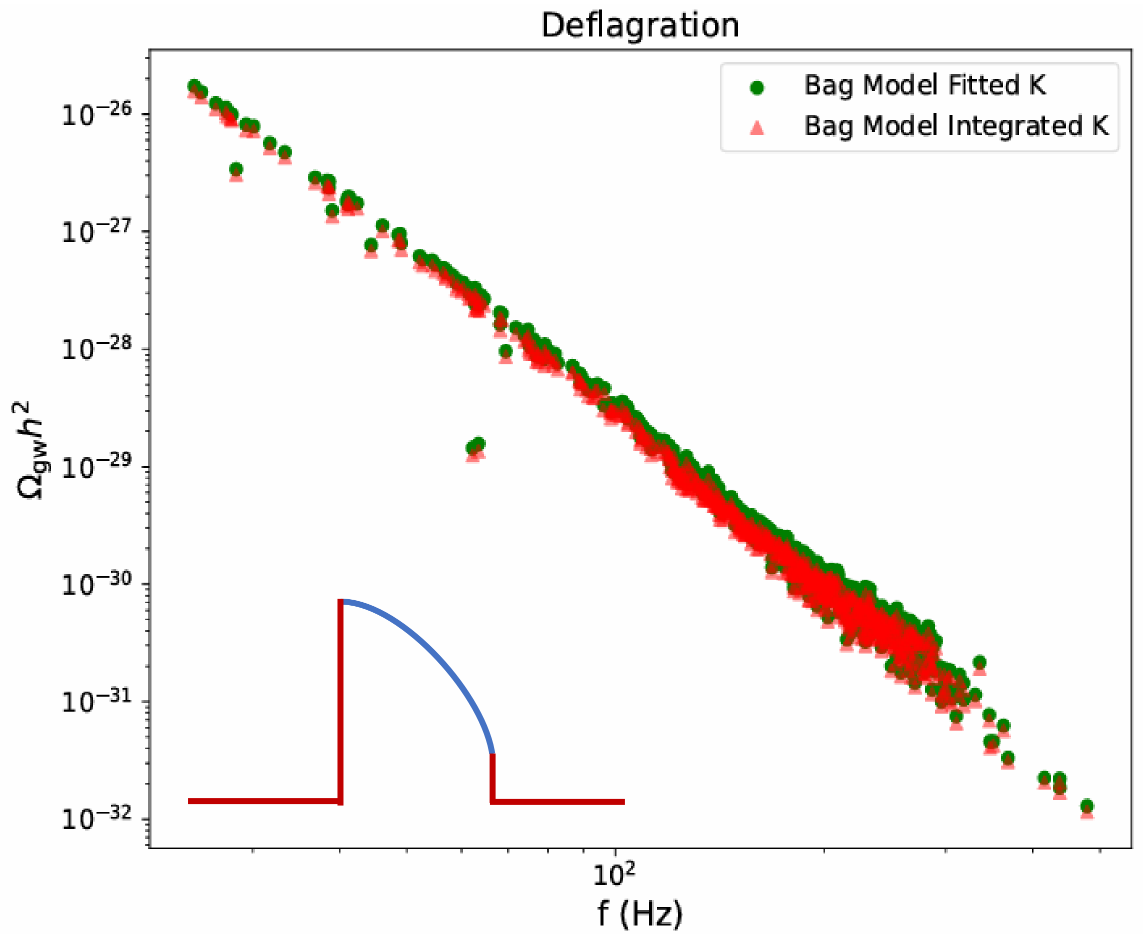


	ξ_w	K	EOS
Bag Model & XSM	√	√	x
Bag Model	√	x	√
XSM	x	√	√

- Bag model serves as a reliable approximation.

Result

➢ Compare gravitational wave results



$$K = \frac{\kappa_{sw}\alpha_N}{1+\alpha_N}$$

$$K = \frac{\rho_{kin}}{\rho_{tot}}$$

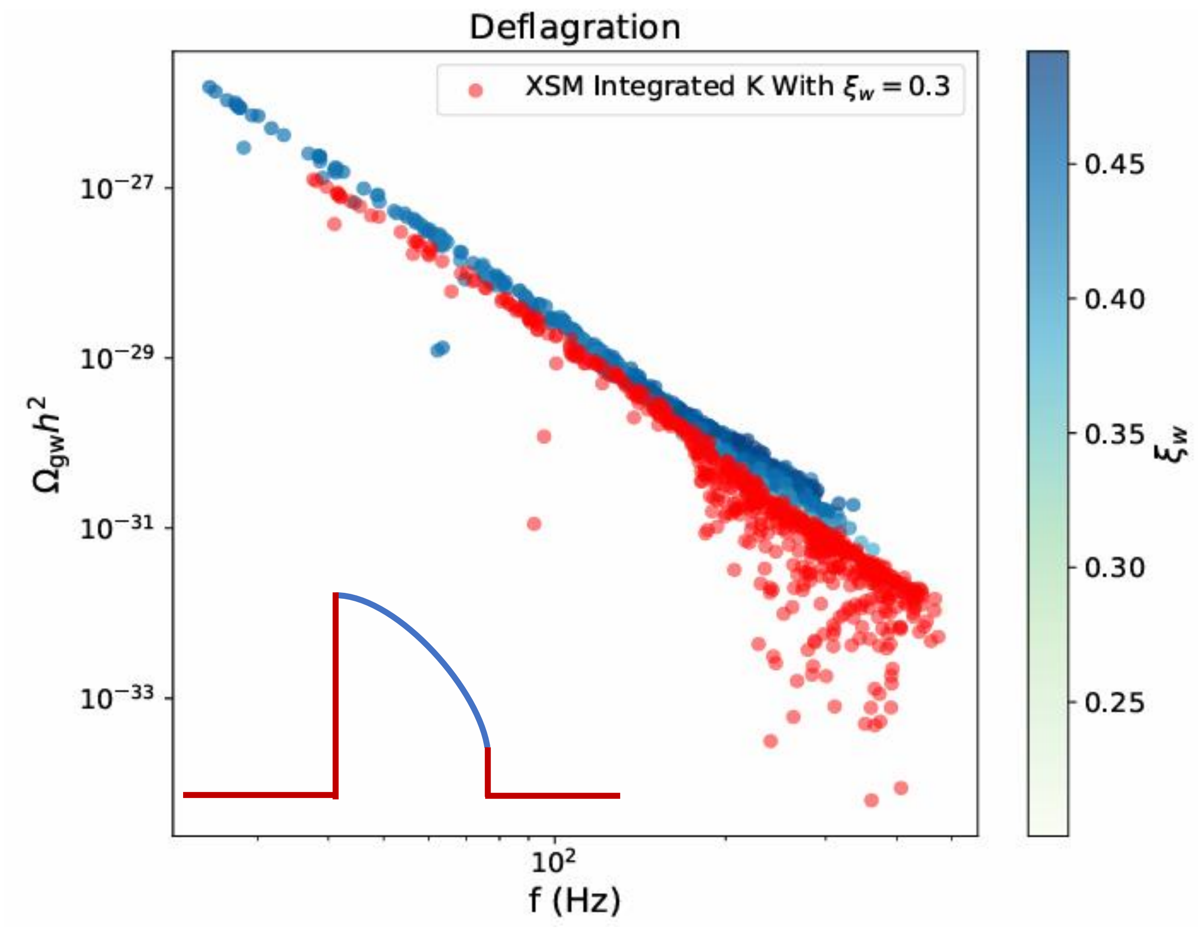
Fitted K **VS** Integrated K

	ξ_w	K	EOS
Bag Model & XSM	√	√	x
Bag Model	√	x	√
XSM	x	√	√

- The fitting formula overestimate the peak amplitude of GW .

Result

➢ Compare gravitational wave results



Fix ξ_w **VS** LTE ξ_w

	ξ_w	K	EOS
Bag Model & XSM	√	√	x
Bag Model	√	x	√
XSM	x	√	√

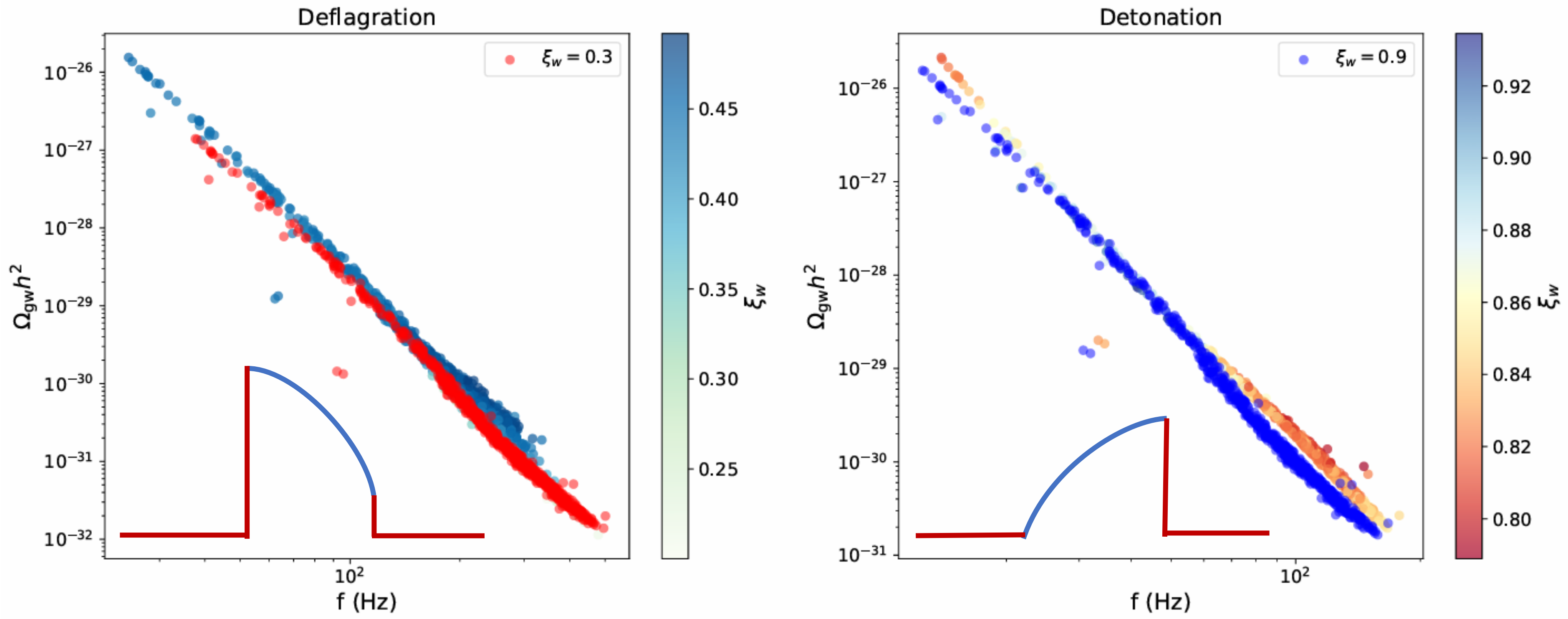
- The fixed-velocity lower than the LET result.

Result

Fix ξ_w
Fitted K

VS

LTE ξ_w
Integrated K



- Deflagration
  48% for the peak frequency and 90% for the gravitational wave amplitude.

- Detonation
  6% for the peak frequency and 18% for the gravitational wave amplitude.

Summary

- We develop a general framework based on the full one-loop finite-temperature effective potential, within which the bubble wall velocity can be evaluated under the local thermal equilibrium approximation for arbitrary BSM scenarios.
- Deflagration is the most prevalent mode of fluid motion.
- Gravitational wave spectra with LTE-based wall velocity and integrated K differ markedly from those obtained with the bag model.

https://github.com/bwlte2025/bubblewall_LTE

Thanks for listening !