Study $\gamma\gamma \to \tau^+\tau^-$ process including $\tau^+\tau^-$ spin information in Pb-Pb ultraperipheral collision and at Lepton collider

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OUTLINE

Introduction

Theoretical Framework

Numerical results & analysis

Summary

Part 01

Introduction

Background

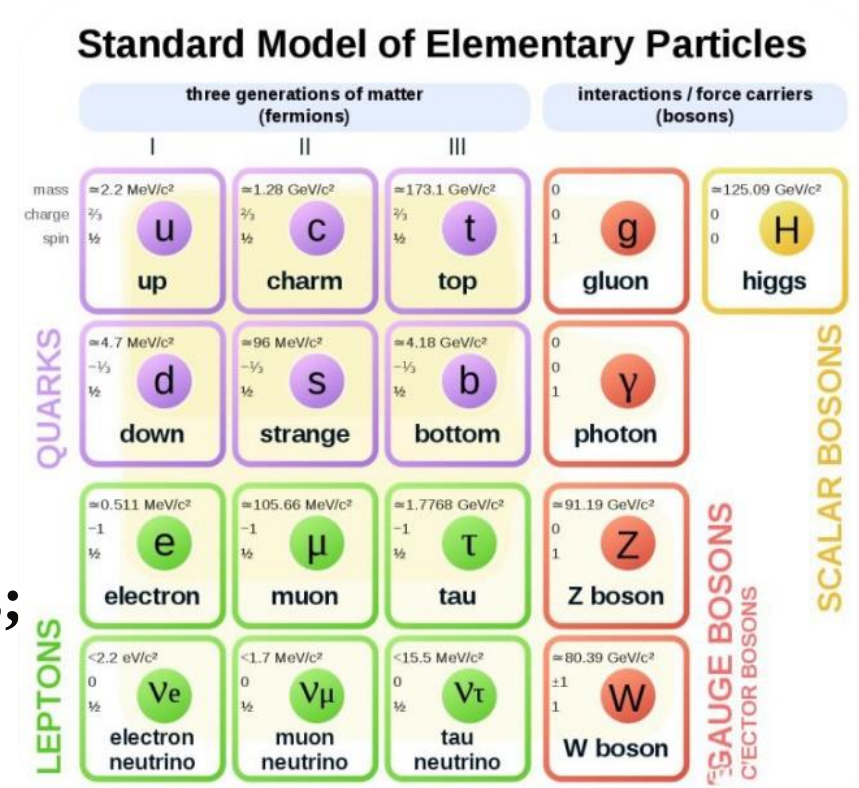
SHANDONG UNIVERSITY

SM has achieved great success;

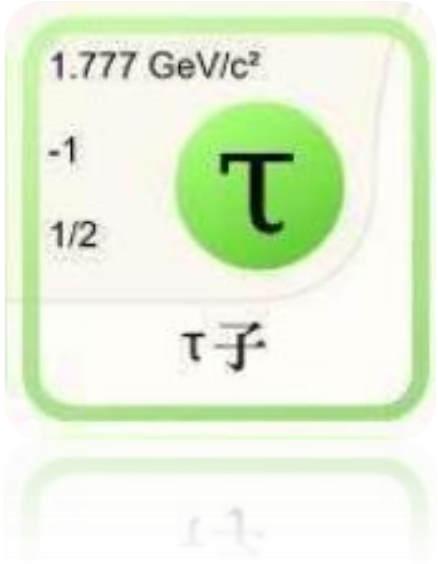
There are still many unresolved issues;

There are currently many studies on quarks;

There are many interesting physics about leptons.



τ physics



- **Mass ~ 1.777GeV; Lifetime ~ 10^{-13} s ;**
- **Connect SM with new physics;**
- **Higher order electroweak corrections enhance the precision of theoretical predictions.**

Experiment

τ physics has been studied on experimental platforms

Decay properties & Production cross sections

Polarization & Spin correlation

$\gamma\gamma \rightarrow \tau^+\tau^-$

Pb-Pb UPC

Z^4 enhanced cross-section

Lepton collider

The environment is clean

Eur. Phys. J. C 20, 401 (2001), Eur. Phys. J. C 36, 283 (2004), Phys. Rev. Lett. 99, 011801 (2007)
Phys. Rev. Lett. 131, 151802 (2023), Phys. Rev. Lett. 131, 151803 (2023)

Research status

Theory

Spin density matrix describes polarization and spin correlation

Bell inequalities & concurrence characterize quantum entanglement

NLO QED &NLO EW with unpolarized τ-pair has been studied

Experiment

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JHEP 10 (2025) 217, JHEP 02, 023 (2025), Phys. Rev. D 111, 036023 (2025)

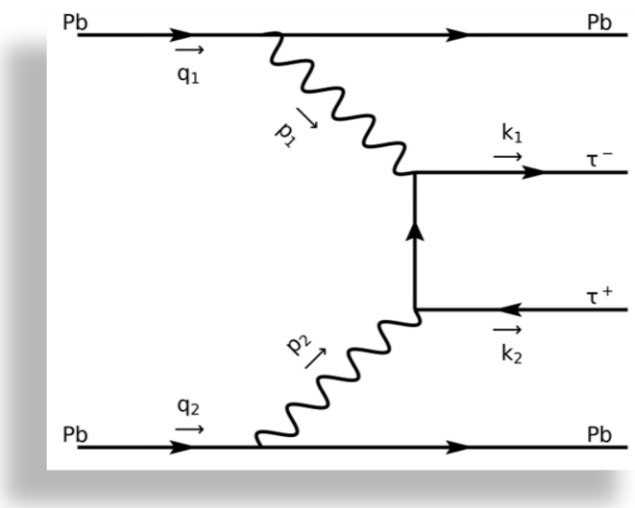
Part 02

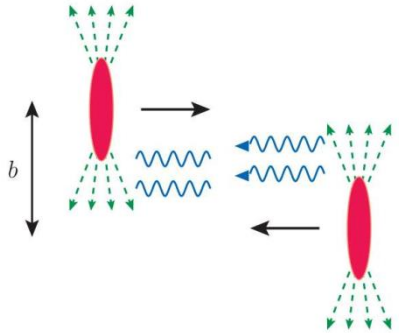
Calculative framework

Calculative framework

$$\sigma_{\mathrm{UPC/LC}} = \int \mathrm{d}x_1 \mathrm{d}x_2 n(x_1) n(x_2)\, \sigma(\gamma\gamma \to \tau^+ \tau^-)$$

For Pb-Pb UPC :





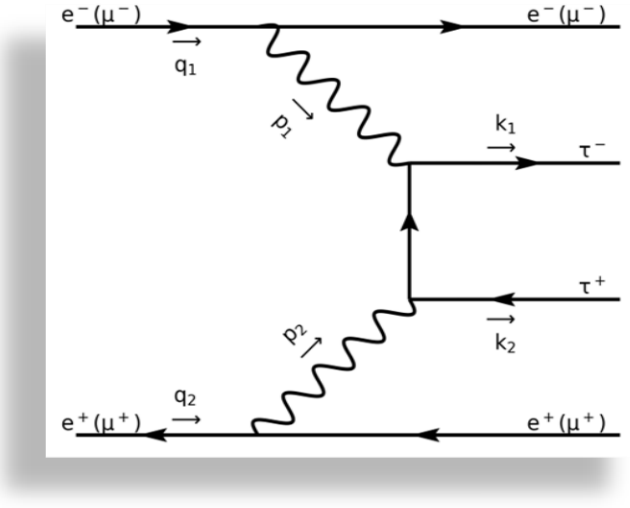
$$n(x_i) = \frac{2Z^2\alpha}{x_i \pi}\{\bar{x}_i K_0(\bar{x}_i) K_1(\bar{x}_i) - \frac{\bar{x}_i^2}{2}[K_1^2(\bar{x}_i) - K_0^2(\bar{x}_i)]\}$$

$$\bar{x}_i = x_i\, m_N\, b_{min}$$

$$\sqrt{s_{\gamma\gamma}} \leq \frac{\gamma_L}{R}$$

JHEP 09, 248 (2022)

For lepton collider:



$$n(x_i) = \frac{\alpha}{2\pi}\left[\frac{1+(1-x_i)^2}{x_i}\ln\left(\frac{Q^2_{max}}{Q^2_{min}}\right) + \frac{2m_f^2\, x_i}{Q^2_{max}} - \frac{2m_f^2\, x_i}{Q^2_{min}}\right]$$

$$Q^2_{min} = \frac{m_f^2 x_i^2}{1-x_i} \qquad Q^2_{max} = \frac{(E_f \theta_c)^2}{1-x_i} + Q^2_{min} \qquad \theta_c = 32\ mrad$$

Phys. Rev. Lett. 89, 032001 (2002),
Phys. Rept. 15, 181 (1975)

Calculative framework

$\gamma(p_1) + \gamma(p_2) \rightarrow \tau^-(k_1, s_1) + \tau^+(k_2, s_2)$

$$|M_{\mathrm{LO}}|^2 = \frac{64\,\alpha^2\pi^2}{\left(1-\beta^2 y^2\right)^2 m^2}\left[A_0 + B_0\,(p_1\cdot s_2)(p_2\cdot s_1) + C_0\,(p_1\cdot s_1)(p_2\cdot s_2) + D_0\,(s_1\cdot s_2)\right]$$

$$A_0 = m^2\left(1-\beta^2\left[2(y^2-1)+\beta^2(2-2y^2+y^4)\right]\right),$$

$$B_0 = \beta^2(\beta^2-1)(y^2-1)(-1+\beta y),$$

$$C_0 = -\beta^2(\beta^2-1)(y^2-1)(1+\beta y),$$

$$D_0 = m^2\left(1-2\beta^2+\beta^4(2-2y^2+y^4)\right),$$

$\beta = \sqrt{1-\frac{4m^2}{s_{\gamma\gamma}}}$, $y = \cos\theta$

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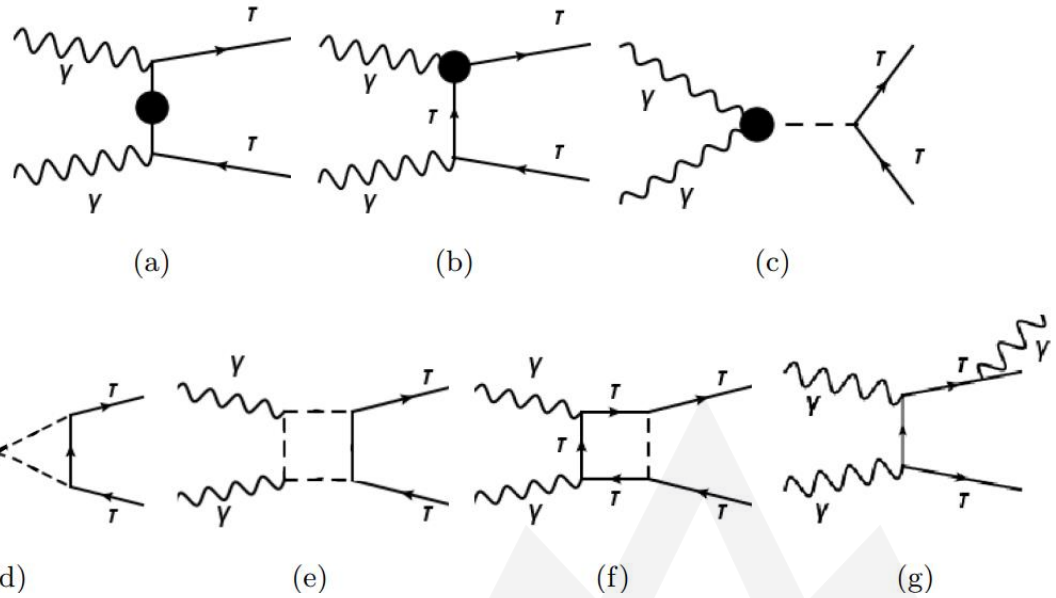
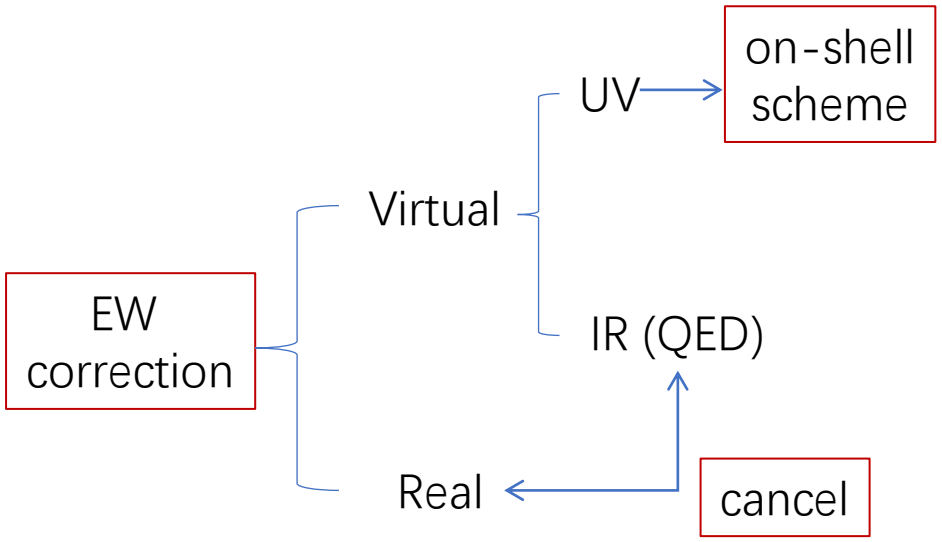
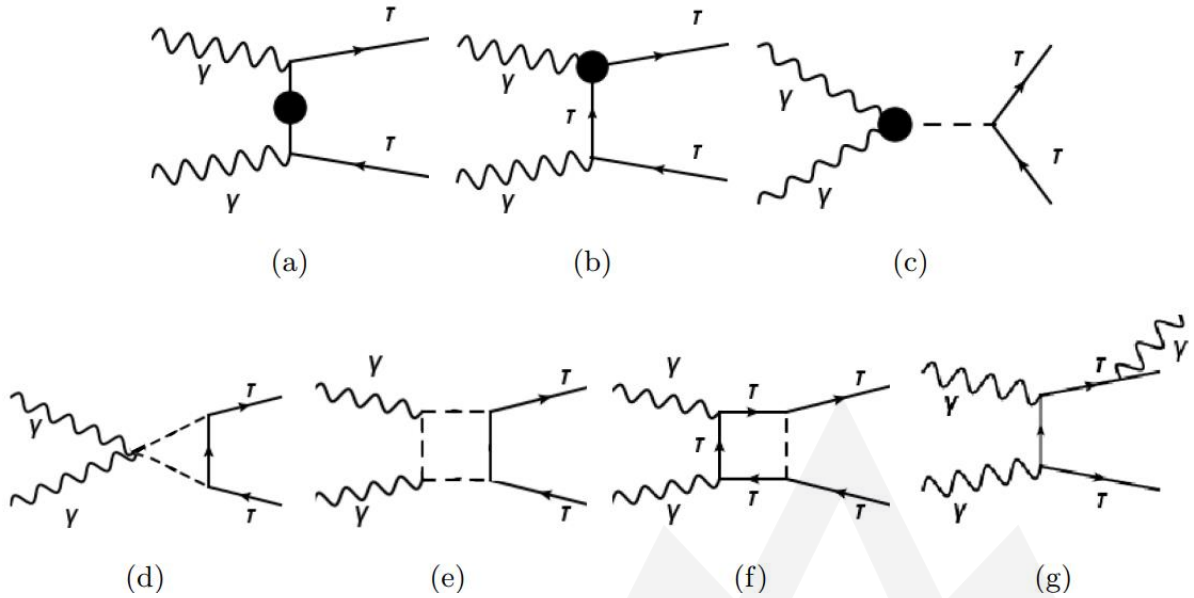


NLO EW corrections

$\gamma(p_1) + \gamma(p_2) \rightarrow \tau^-(k_1, s_1) + \tau^+(k_2, s_2)$

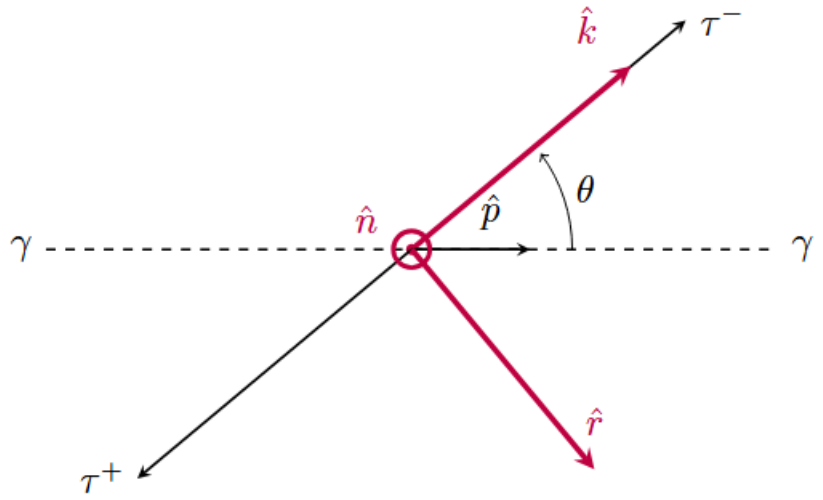
dimentional regularization with D = 4 − 2ε





Phys. Rev. D 111, 036023 (2025)

$$R = \tilde{A}\,\mathbb{1}\otimes\mathbb{1} + \tilde{B}_i^{+}\,\sigma^i\otimes\mathbb{1} + \tilde{B}_i^{-}\,\mathbb{1}\otimes\sigma^i + \tilde{C}_{ij}\,\sigma^i\otimes\sigma^j$$



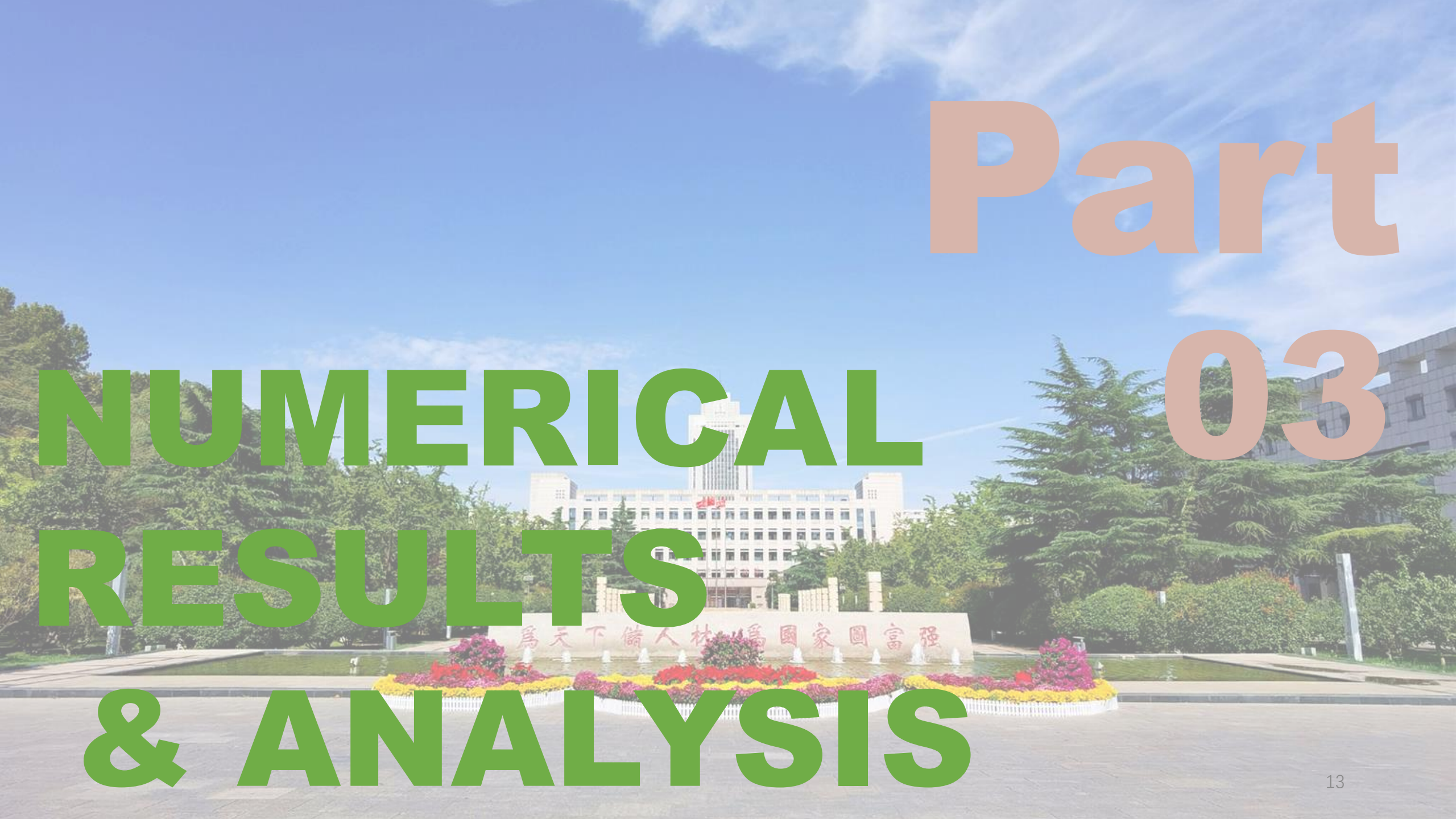
$$C_{ab} = 4\langle(\hat{\mathbf{a}}\cdot\vec{s}_1)\,(\hat{\mathbf{b}}\cdot\vec{s}_2)\rangle$$

$$\vec{s}_1 = \tfrac{1}{2}(\vec{\sigma}\otimes\mathbb{1}) \text{ and } \vec{s}_2 = \tfrac{1}{2}(\mathbb{1}\otimes\vec{\sigma})$$

$$C_{ab} = \frac{\sigma(\uparrow\uparrow) + \sigma(\downarrow\downarrow) - \sigma(\uparrow\downarrow) - \sigma(\downarrow\uparrow)}{\sigma(\uparrow\uparrow) + \sigma(\downarrow\downarrow) + \sigma(\uparrow\downarrow) + \sigma(\downarrow\uparrow)}$$

$\{\hat{r}, \hat{k}, \hat{n}\}$

- $\hat{k}$: the flight direction of the τ^-;
- $\hat{r} = \frac{1}{\sqrt{1-y^2}}(\hat{p} - y\hat{k})$;
- $\hat{n} = \frac{1}{\sqrt{1-y^2}}(\hat{p} \times \hat{k})$



Part 03

NUMERICAL RESULTS & ANALYSIS

Cross section

Pb-Pb UPC

$\sqrt{s_{NN}}$ [TeV]	σ_{LO} [mb]	$\delta\sigma_{\mathrm{NLO}}$ [mb]	σ_{NLO} [mb]	δ[%]
5.02	0.940	8.55×10^{-3}	0.949	0.909
5.36	1.003	9.44×10^{-3}	1.012	0.941
5.52	1.030	9.93×10^{-3}	1.040	0.964

e^+e^-collider

$\sqrt{s}$ [GeV]	σ_{LO} [nb]	$\delta\sigma_{\mathrm{NLO}}$ [nb]	σ_{NLO} [nb]	δ [%]
250	0.5090	4.182×10^{-3}	0.513	0.821
380	0.5350	4.178×10^{-3}	0.539	0.781
1500	1.028	5.613×10^{-3}	1.034	0.546
3000	1.340	3.499×10^{-3}	1.344	0.261

$\mu^+\mu^-$collider

$\sqrt{s}$ [TeV]	σ_{LO} [nb]	$\delta\sigma_{\mathrm{NLO}}$ [nb]	σ_{NLO} [nb]	δ [%]
3	0.4990	1.807×10^{-3}	0.501	0.362
10	0.8290	-3.656×10^{-3}	0.826	-0.441

In Pb-Pb UPC:

- NLO EW contribution **grows** with the increase of $\sqrt{s_{NN}}$;
- Total cross section at NLO about **1 mb** ;

In lepton collider:

- NLO EW contribution **decreases** with the increase of $\sqrt{s}$;
- e^+e^- collider has **superior observability** over $\mu^+\mu^-$ collider at equal $\sqrt{s}$.

Spin correlations

$\sqrt{s_{NN}}$ [TeV]	C_{ab}	C_{pp}	C_{nn}	C_{rr}	C_{kk}
5.02	LO	-0.0213	-0.3281	0.0199	-0.2998
	NLO	-0.0214	-0.3230	0.0195	-0.2956
5.36	LO	-0.0192	-0.3273	0.0214	-0.2968
	NLO	-0.0193	-0.3223	0.0210	-0.2926
5.52	LO	-0.0183	-0.3270	0.0221	-0.2955
	NLO	-0.0184	-0.3220	0.0216	-0.2913

Pb-Pb UPC

$\sqrt{s}$ [GeV]	C_{ab}	C_{pp}	C_{nn}	C_{rr}	C_{kk}
250	LO	0.0335	-0.3090	0.0580	-0.2183
	NLO	0.0334	-0.3070	0.0579	-0.2167
380	LO	0.0514	-0.3030	0.0695	-0.1909
	NLO	0.0513	-0.3012	0.0694	-0.1893
1500	LO	0.0705	-0.2973	0.0796	-0.1604
	NLO	0.0703	-0.2956	0.0795	-0.1590
3000	LO	0.0763	-0.2956	0.0825	-0.1512
	NLO	0.0756	-0.2938	0.0824	-0.1506

e^+e^-collider

$\sqrt{s}$ [TeV]	C_{ab}	C_{pp}	C_{nn}	C_{rr}	C_{kk}
3	LO	0.0743	-0.2905	0.0799	-0.1515
	NLO	0.0737	-0.2944	0.0814	-0.1536
10	LO	0.0821	-0.2938	0.0853	-0.1419
	NLO	0.0799	-0.2921	0.0852	-0.1533

$\mu^+\mu^-$collider

- C_{ab} show an energy-dependent transition from antiparallel spins near threshold to helicity-aligned at high energies;
- The spin correlation is strong in the normal direction;
- Spin correlation along the beam direction is generally suppressed;
- NLO EW corrections produce small numerical shifts and do not alter these qualitative features.

Quantum entanglement

Concurrence:

$$C(\rho) = -\frac{1}{2}(1 + C_{11} + C_{22} + C_{33}) = -\frac{1}{2}(1 + 3D) \qquad D = \frac{C_{11} + C_{22} + C_{33}}{3} = \frac{\mathrm{tr}[C]}{3}$$

entanglement : $-1 \leq D < -\frac{1}{3}$

- No entanglement remains after integrating over the full phase space !

Quantum entanglement

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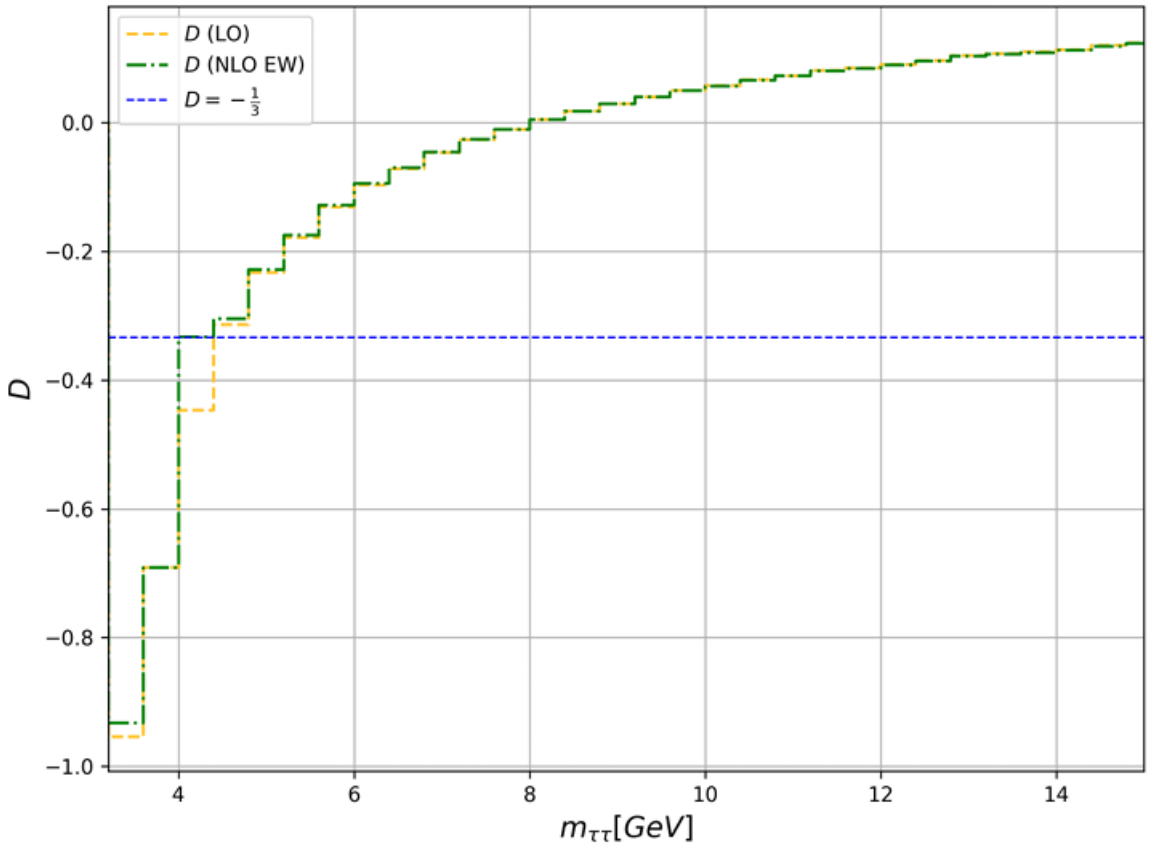

- No entanglement remains after integrating over the full phase space;
- Near threshold, the $^1S_0$ state yields antiparallel spins and an entangled configuration.
- NLO EW corrections slightly reduce the near-threshold entanglement.

Part 04

SUMMARY

Summary

1. We study $\gamma\,\gamma \rightarrow\ \tau^{+}\tau^{-}$ process including $\tau^{+}\tau^{-}$ spin information in Pb-Pb UPC and at lepton collider;

2. We provide the predictions for the corresponding cross section and the spin correlation coefficients;

3. No entanglement remains after integrating over the full phase space, while near threshold the system forms an entangled ${}^{1}S_{0}$ state with antiparallel spins;

4. NLO EW corrections slightly reduce the near-threshold entanglement.

Thanks !