

水平管两相流压降模型

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利用Friedel correlation公式计算两相流压降:

$$\Delta P_{TP} = \phi_{LO}^2 \cdot \Delta P_{LO}$$

其中:

相关系数公式:
$$\phi_{LO}^2 = \frac{E + 3.24 \cdot F \cdot H}{Fr_{TP}^{0.045} \cdot We_{TP}^{0.035}}$$

单相流压降公式 (Blasius equation):
$$\Delta P_{LO} = \frac{2 \cdot f_{LO} \cdot m'^2 \cdot L}{D \cdot \rho_L}$$

公式以及一些参数、系数还需要进一步确认

计算单相流的摩擦压降, 通常采用Carcy公式⁺:

$$\Delta p_{fric} = \lambda \frac{L}{D_h} \frac{\rho V^2}{2}$$

Friedel相关常数:

$$E = (1-x)^2 + x^2 \cdot \frac{\rho_L \cdot f_{GO}}{\rho_G \cdot f_{LO}}$$

$$F = x^{0.78} \cdot (1-x)^{0.24}$$

$$H = \left(\frac{\rho_L}{\rho_G}\right)^{0.91} \cdot \left(\frac{\eta_G}{\eta_L}\right)^{0.19} \cdot \left(1 - \frac{\eta_G}{\eta_L}\right)^{0.7}$$

Friedel数:
$$Fr = \frac{m'^2}{g \cdot D \cdot \rho^2}$$

Web数:
$$We = \frac{\rho \cdot m'^2 \cdot D}{\sigma}$$

摩擦因子:
$$f_{LO} = \frac{0.079}{Re_{LO}^{0.25}} \longrightarrow \frac{1}{\sqrt{f}} = -1.8 \log \left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{Re} \right]$$

达西摩擦因子: 0.316
范宁摩擦因子: 0.079

雷诺数:
$$Re = \frac{m' \cdot D}{\eta}$$

达西摩擦因子 (Darcy摩擦因子) 和范宁摩擦因子 (Fanning摩擦因子) 之间的关系是: 达西摩擦因子是范宁摩擦因子的四倍。范宁摩擦因子的物理意义是局部剪切应力与局部流动动能密度之间的比率, 而达西摩擦因子则更常用于描述流体在管道中的阻力损失。具体来说, 达西摩擦因子通常用于不可压缩流体的流动分析, 而范宁摩擦因子则在某些情况下更具物理意义。

知乎 +1

讨论后认为需要考虑管壁粗糙度对压降的影响，考虑用Colebrook-White公式（隐式）或者Haaland公式（显式）代替Blasius公式：

Colebrook-White公式是一种工程实用公式，它用于计算流体在光滑和粗糙管道中的摩擦系数。该公式由英国工程师Colebrook和White于1939年提出，在工程领域得到了广泛的应用。

Colebrook-White公式如下：

$$1/\sqrt{f} = -2 * \log_{10}[(\epsilon/D)/3.7 + 2.51/(Re * \sqrt{f})]$$

其中，f是摩擦系数， ϵ 是管道粗糙度，D是管道直径，Re是雷诺数。雷诺数是流体力学中一个重要的无量纲参数，它反映了流体惯性力和粘性力之间的相对重要性。在流体在管道中流动时，雷诺数的大小对流体的速度分布和摩擦系数都有影响。

由于Colebrook-White公式中的摩擦系数f位于方程的左边和右边，因此无法直接求解。为了求解该公式，我们可以使用迭代法。首先，假设一个初始值，然后将该值代入公式右边，计算得到一个新的f值。然后将新的f值代入公式右边，继续迭代，直到得到一个满足精度要求的f值为止。

- 1、水力光滑管：粗糙度对管道阻力几乎无影响，阻力与雷诺数有关；
- 2、水力粗糙管：粗糙度对管道阻力影响很大，阻力与雷诺数和粗糙度有关。
- 3、水力光滑（粗糙）是描述粗糙度对流动影响的，不仅与雷诺数有关，还与粗糙度大小直接关系。若雷诺数较大时，惯性力对流场的影响大于黏滞力，流体流动较不稳定，流速的微小变化容易发展、增强，形成紊乱、不规则的紊流流场。粗糙度越大，对流体阻碍越大。
- 4、流体力学中，雷诺数是流体惯性力与黏滞力比值的量度，它是一个无量纲数。雷诺数较小时，黏滞力对流场的影响大于惯性力，流场中流速的扰动会因黏滞力而衰减，流体流动稳定，为层流。

The Haaland Equation

One way to find the friction factor of flow through a pipe is the approximation by professor S.E. Haaland first proposed in 1983. Like the Colebrook equation, this formula is used to derive the friction factor for a full-flowing circular pipe. It was constructed using experimental results of both laminar and turbulent flow studies. However, Haaland first derived his formula with the intention of making things simple.

The Haaland equation is better suited for hand calculations than any of the other Colebrook derivations. The formula is as follows:

$$\frac{1}{\sqrt{f}} = -1.8 \log \left[\left(\frac{\epsilon/D}{3.7} \right)^{1.11} + \frac{6.9}{Re} \right]$$

```

otk_barrel_3pipe_f_xiuzheng.m x +
1      clc; clear; close all;
2      L = 5.68;
3      P_in = 2e6;
4      rho_l = 1028.4;
5      rho_g = 52.540;
6      mu_l = 1.3819e-4;
7      mu_g = 1.315e-5;
8      sigma = 8.271e-3;
9      g = 9.81;
10     D_range = linspace(2e-3, 10e-3, 50);
11     x_out_list = [0.3, 0.5, 0.75];
12     epsilon = 5e-6; % 定义管道粗糙度为5微米
13     figure('Position', [100, 100, 800, 600]);
14     hold on;

```

定义初始参数

```

51 function delta_P = friedel_pressure_drop(x, rho_l, rho_g, mu_l, mu_g, sigma, D, G, g, dz, epsilon)
52     rho_tp = 1 / (x/rho_g + (1-x)/rho_l);
53     Fr = G^2 / (g * D * rho_tp^2);
54     We = (G^2 * D) / (sigma*rho_tp);
55     Re_LO = G * D / mu_l;
56     Re_GO = G * D / mu_g;
57     f_LO = haaland_friction_factor(Re_LO, epsilon/D);
58     f_GO = haaland_friction_factor(Re_GO, epsilon/D);
59     E = (1 - x)^2 + x^2 * (rho_l * f_GO) / (rho_g * f_LO);
60     F = x^0.78 * (1 - x)^0.24;
61     H = (rho_l/rho_g)^0.91 * (mu_g/mu_l)^0.19 * (1 - mu_g/mu_l)^0.7;
62     phi_LO_sq = E + (3.24 * F * H)/(Fr^0.045 * We^0.035);
63     delta_P_LO = 2 * f_LO * G^2 * dz / (rho_l * D);
64     delta_P = phi_LO_sq * delta_P_LO;
65 end
66 function f = haaland_friction_factor(Re, relative_roughness)
67     term1 = (relative_roughness/3.7)^1.11;
68     term2 = 6.9/Re;
69     f = (-1.8 * log10(term1 + term2))^(-2);
70 end

```

定义公式

```

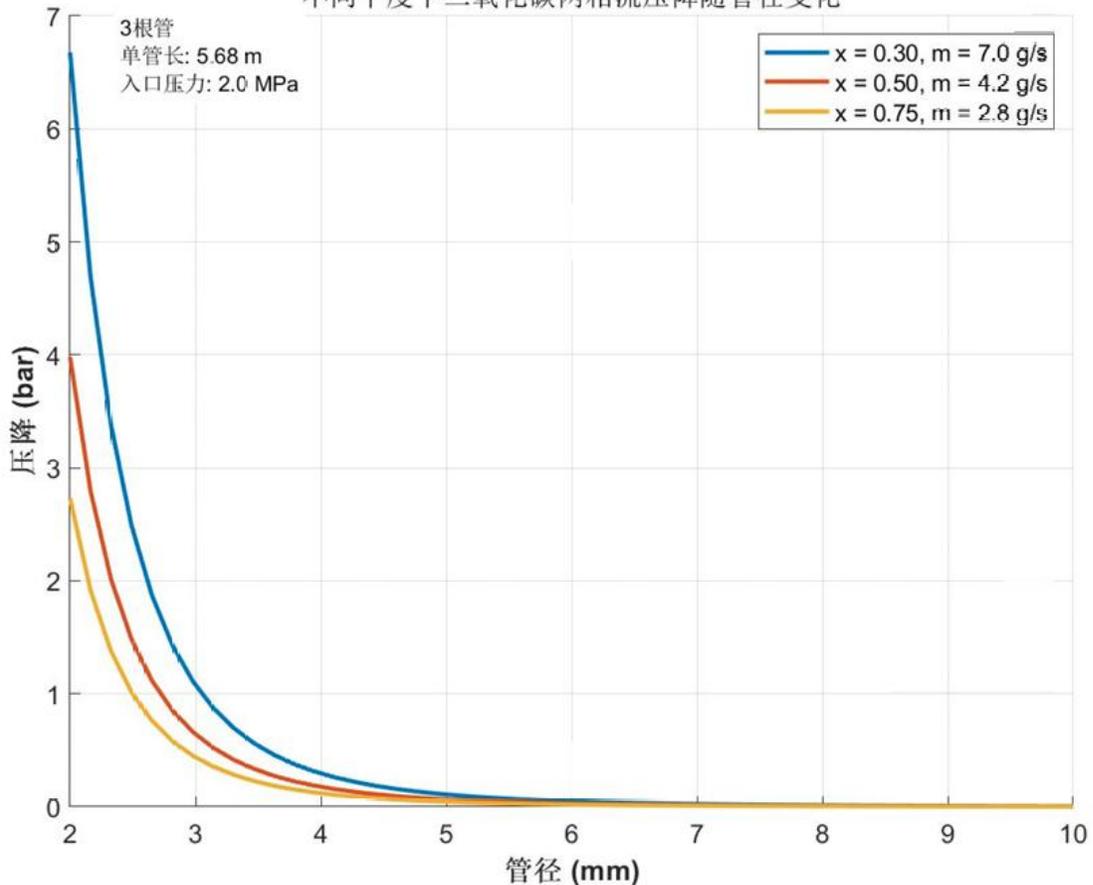
16     colors = lines(length(x_out_list));
17     for k = 1:length(x_out_list)
18         x_out = x_out_list(k);
19         m_dot = (2.1 / x_out) * 1e-3;
20         delta_P_total = zeros(size(D_range));
21         for i = 1:length(D_range)
22             D = D_range(i);
23             A = pi * (D/2)^2;
24             G = m_dot / A;
25             num_segments = 100;
26             delta_z = L / num_segments;
27             delta_P_segment = 0;
28             for j = 1:num_segments
29                 z = (j - 0.5) * delta_z;
30                 x = x_out * (z / L);
31                 delta_P_segment = delta_P_segment + friedel_pressure_drop(...
32                     x, rho_l, rho_g, mu_l, mu_g, sigma, D, G, g, delta_z, epsilon);
33             end
34             delta_P_total(i) = delta_P_segment;
35         end
36         plot(D_range*1000, delta_P_total/1e5, 'Color', colors(k,:), 'LineWidth', 2, ...
37             'DisplayName', sprintf('x = %.2f, m = %.1f g/s', x_out, m_dot*1000));
38     end

```

考虑干度不同对压降积分

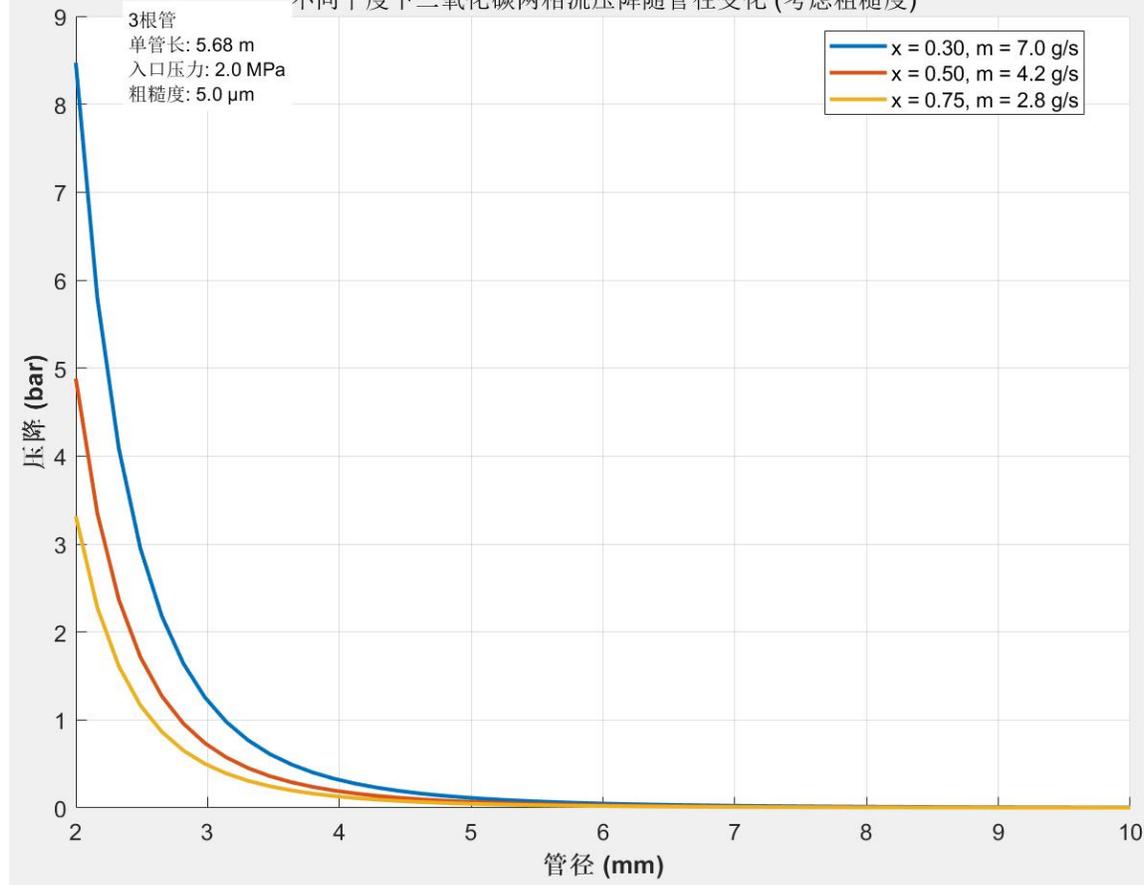
针对OTK桶部冷却：一个OTK桶部stave总热功耗1772.16w；二氧化碳2MPa， -19.503℃：潜热： 281.33kj/kg；通过计算需要6.3g/s蒸发速度带走一条stave的热量， 如果出口干度0.3， 所需质量流量： 6.3g/0.3=21g

不同干度下二氧化碳两相流压降随管径变化



Blasius 公式 (假设水力光滑管)

不同干度下二氧化碳两相流压降随管径变化 (考虑粗糙度)



Haaland公式 (考虑管内壁粗糙度)

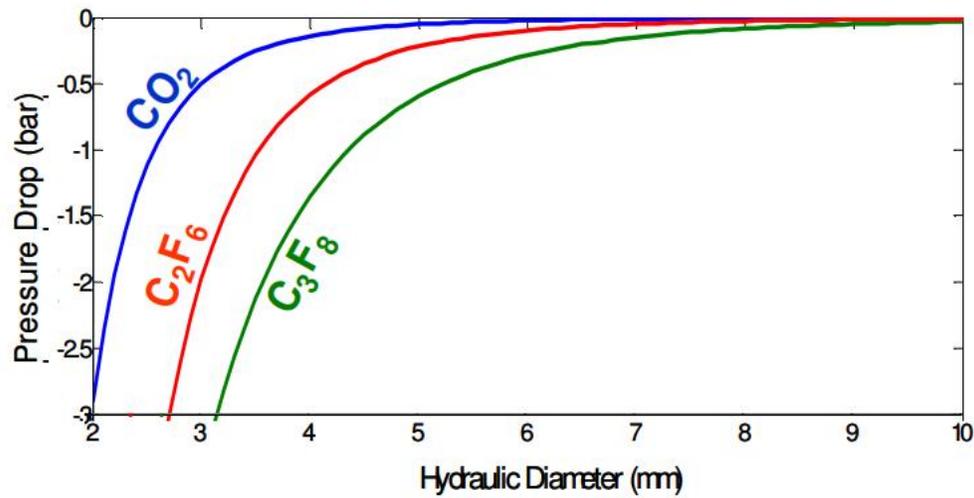
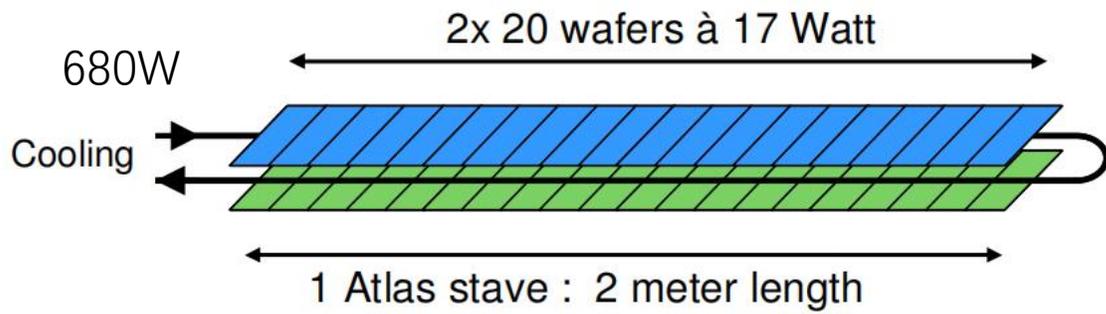


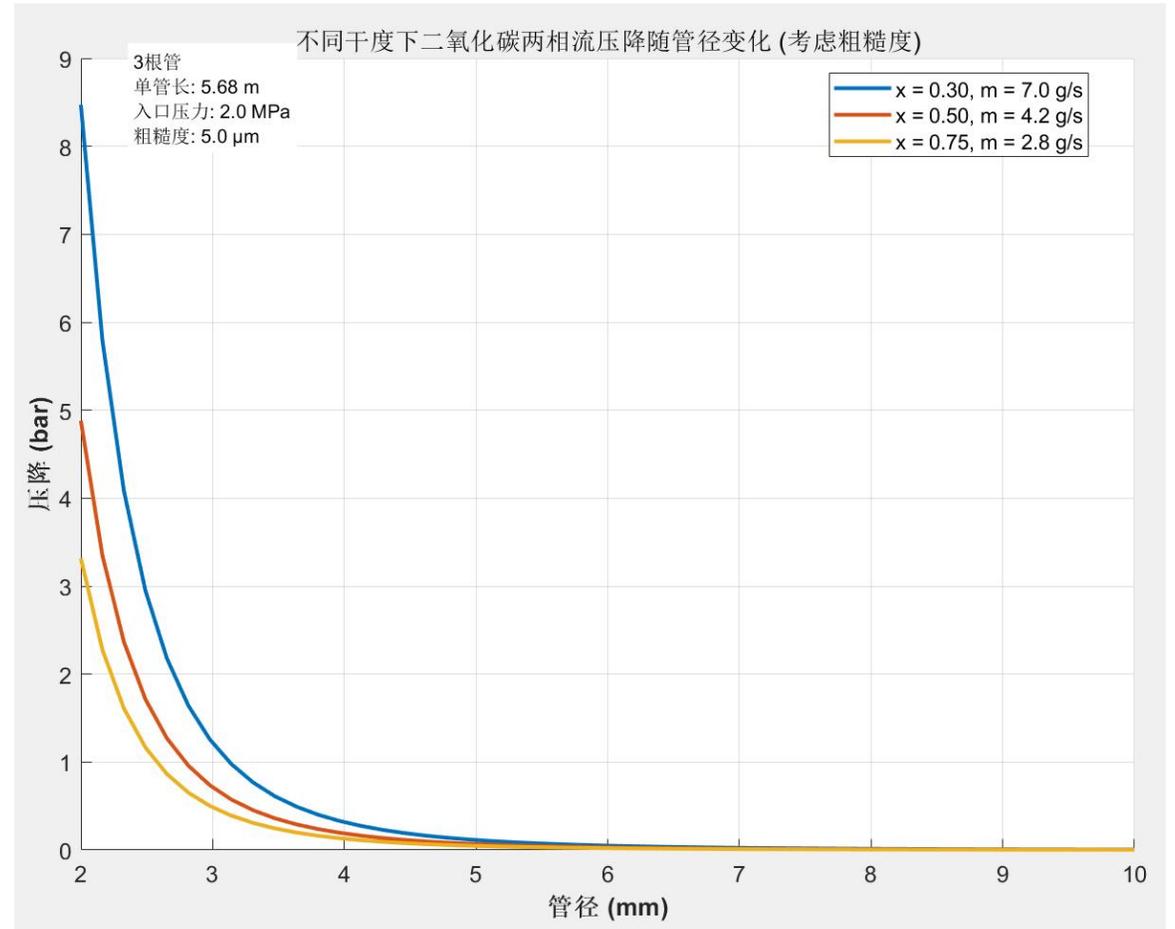
Figure 4: Pressure drop of CO₂, C₃F₈ and C₂F₆ as a function of tube diameter

cern文献:

CO₂: 2.9g/s

C₂F₆: 9.6g/s

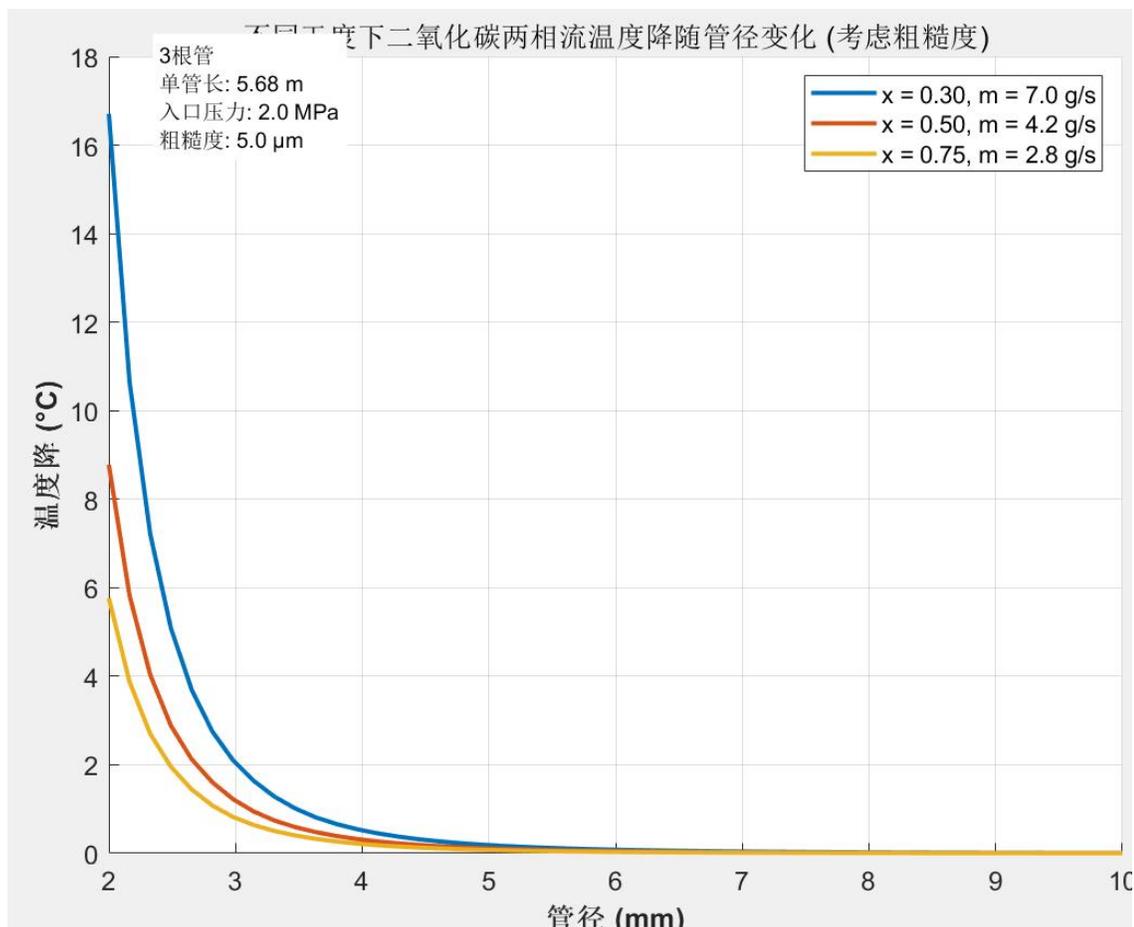
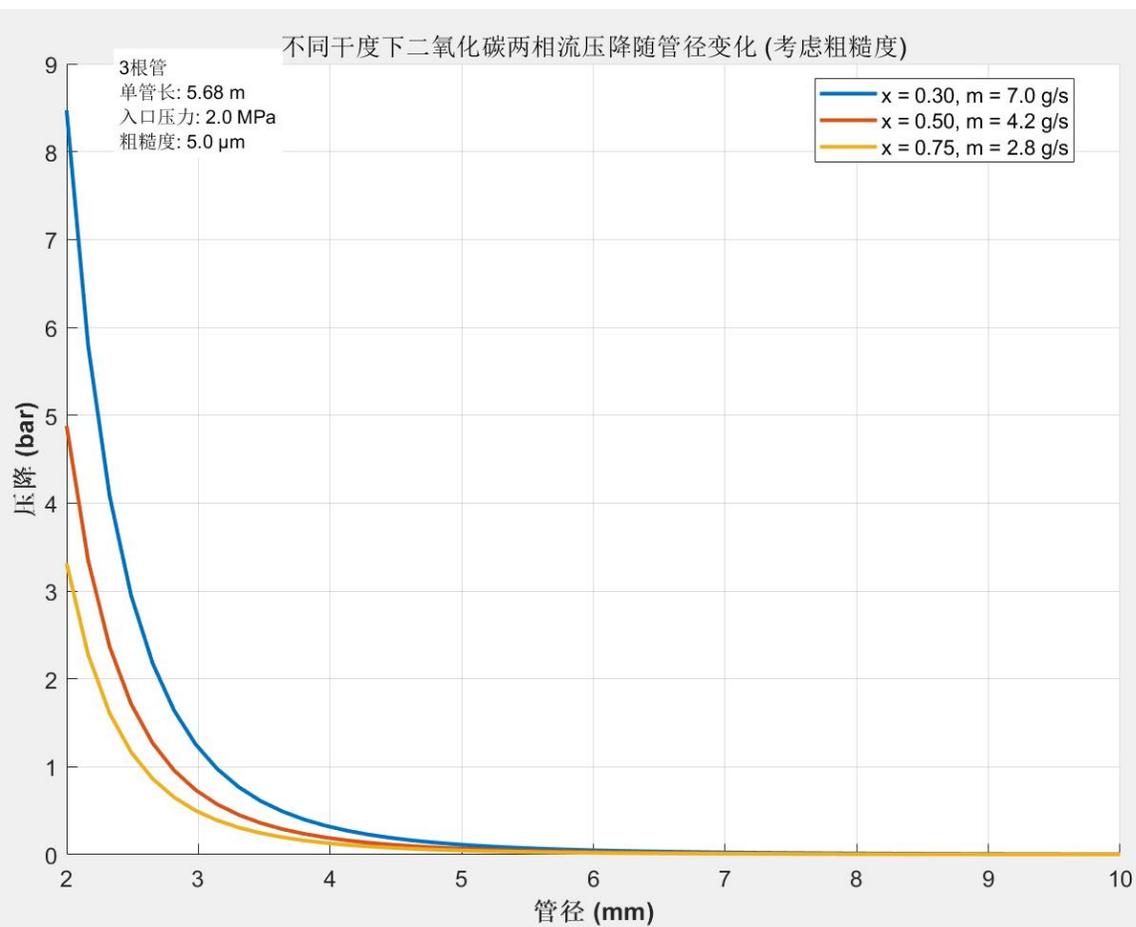
C₃F₈: 8.7g/s



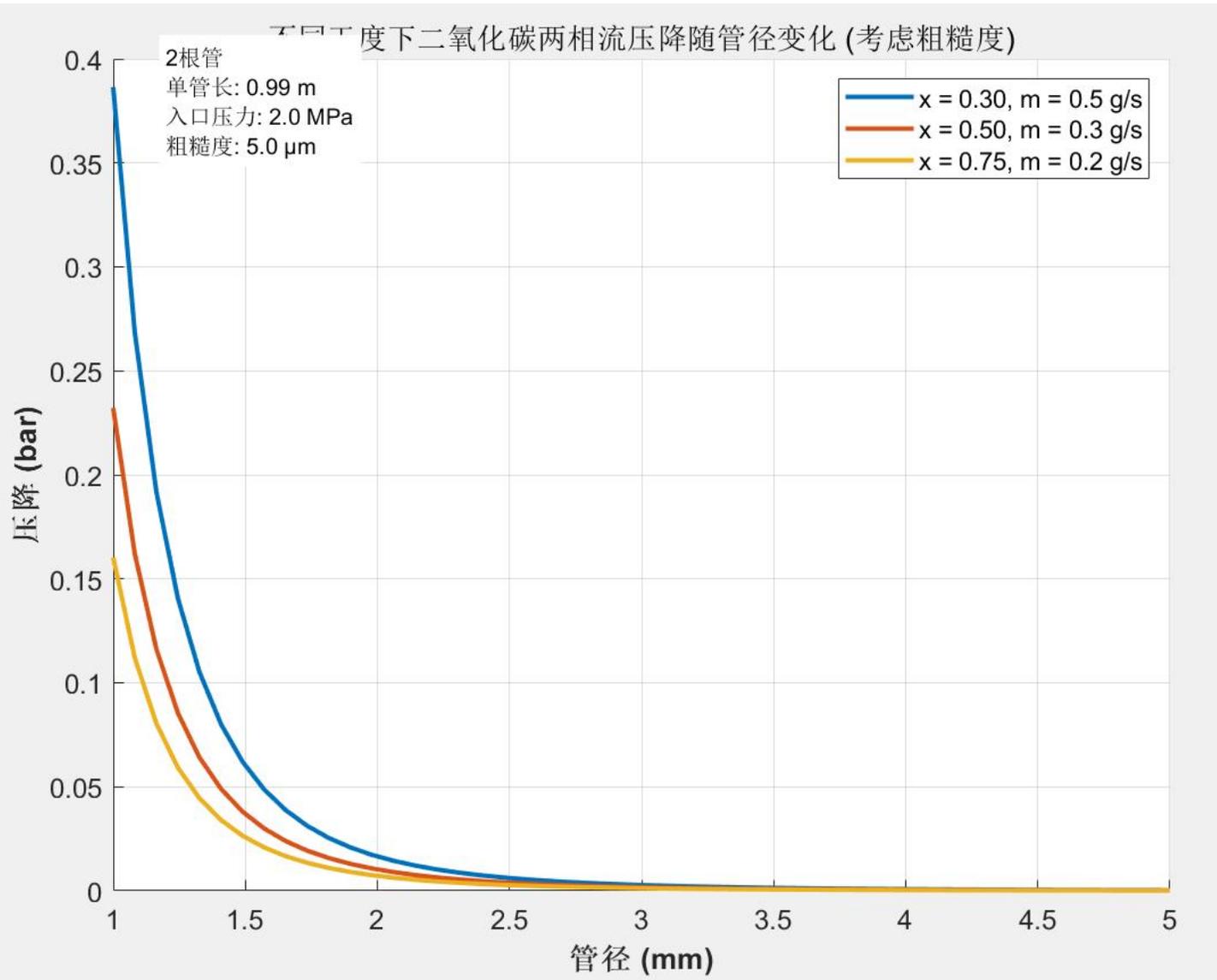
Haaland公式 (考虑管内壁粗糙度)

refprop制冷剂物性参数插值生成温度降和管径的相关曲线

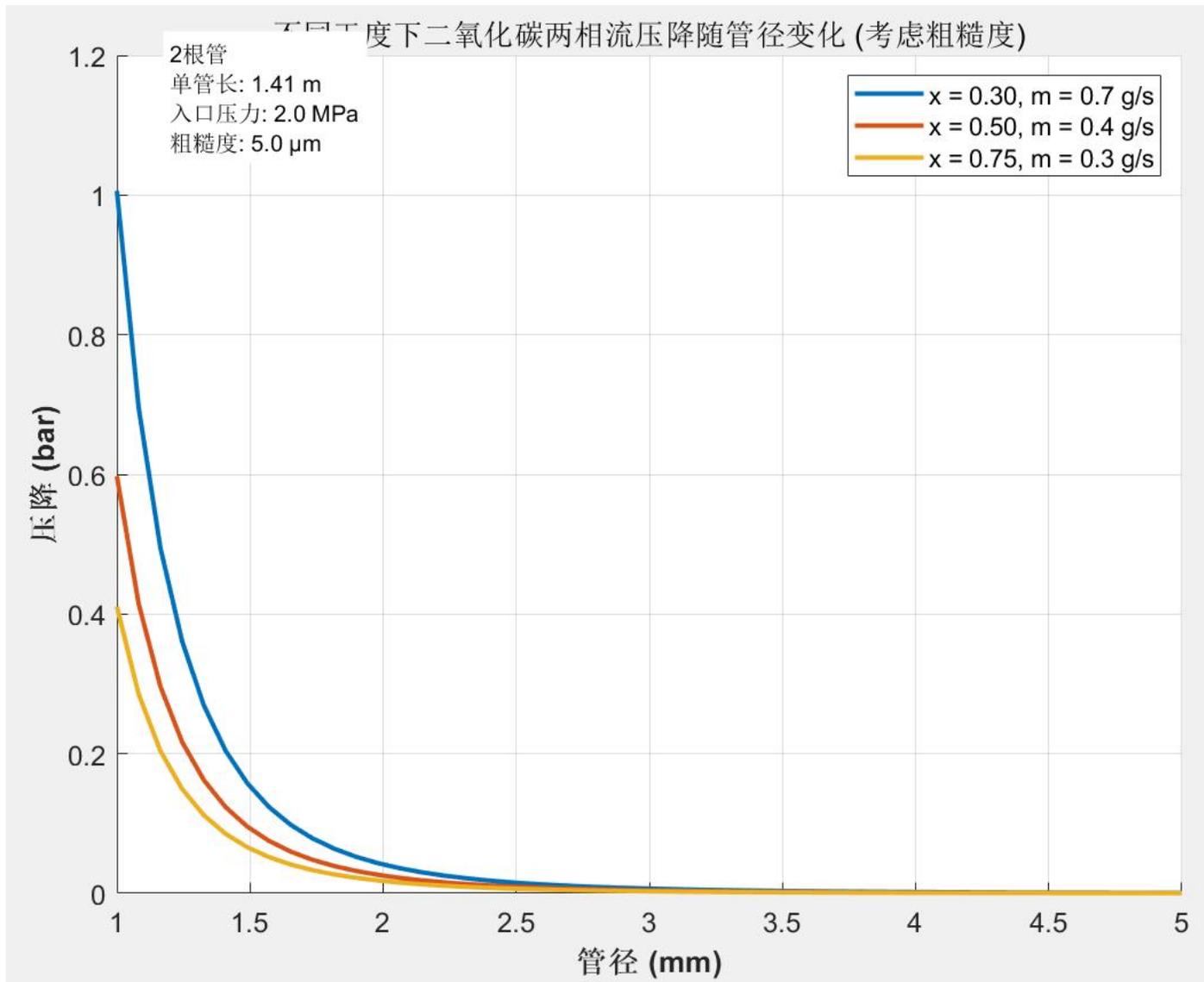
	Temperature (°C)	Pressure (MPa)	Liquid Density (kg/m³)	Vapor Density (kg/m³)	Liquid Enthalpy (kJ/kg)	Vapor Enthalpy (kJ/kg)	Liquid Entropy (kJ/kg-K)	Vapor Entropy (kJ/kg-K)	Liquid Cp (kJ/kg-K)	Vapor Cp (kJ/kg-K)	Liquid Therm. Cond. (W/m-K)	Vapor Therm. Cond. (W/m-K)	Liquid Viscosity (g/cm-s)	Vapor Viscosity (g/cm-s)	Liquid Therm. Diff. (cm²/s)	Vapor Therm. Diff. (cm²/s)	Surf. Tension (N/m)
1	-40.122	1.0000	1116.9	26.006	112.66	435.30	0.66460	2.0491	2.9111	1.0322	0.15945	0.012528	0.0019415	0.00011862	0.00070989	0.0046673	0.012708
2	-38.790	1.0500	1111.7	27.287	115.35	435.57	0.67592	2.0423	2.9178	1.0447	0.15778	0.012668	0.0018989	0.00011939	0.00070339	0.0044438	0.012410
3	-37.504	1.1000	1106.5	28.572	117.95	435.81	0.68681	2.0357	2.9247	1.0572	0.15617	0.012806	0.0018568	0.00012014	0.00069704	0.0042393	0.012124
4	-36.261	1.1500	1101.5	29.861	120.48	436.03	0.69731	2.0294	2.9318	1.0698	0.15461	0.012942	0.0018209	0.00012087	0.00069083	0.0040514	0.011849
5	-35.057	1.2000	1096.7	31.153	122.93	436.22	0.70745	2.0233	2.9389	1.0824	0.15311	0.013077	0.0017850	0.00012158	0.00068476	0.0038781	0.011583
6	-33.890	1.2500	1091.9	32.450	125.32	436.39	0.71727	2.0174	2.9462	1.0950	0.15166	0.013211	0.0017509	0.00012227	0.00067881	0.0037178	0.011327
7	-32.757	1.3000	1087.2	33.752	127.65	436.54	0.72677	2.0117	2.9536	1.1077	0.15026	0.013344	0.0017184	0.00012296	0.00067298	0.0035690	0.011080
8	-31.655	1.3500	1082.7	35.057	129.91	436.66	0.73600	2.0062	2.9611	1.1205	0.14890	0.013476	0.0016875	0.00012362	0.00066725	0.0034305	0.010840
9	-30.583	1.4000	1078.2	36.368	132.13	436.77	0.74496	2.0009	2.9688	1.1334	0.14757	0.013607	0.0016580	0.00012428	0.00066162	0.0033012	0.010608
10	-29.539	1.4500	1073.8	37.684	134.29	436.86	0.75367	1.9957	2.9765	1.1464	0.14629	0.013738	0.0016297	0.00012493	0.00065608	0.0031802	0.010383
11	-28.521	1.5000	1069.5	39.005	136.41	436.93	0.76215	1.9906	2.9844	1.1595	0.14504	0.013869	0.0016027	0.00012556	0.00065062	0.0030667	0.010164
12	-27.528	1.5500	1065.2	40.331	138.48	436.98	0.77042	1.9857	2.9925	1.1727	0.14382	0.013999	0.0015767	0.00012619	0.00064525	0.0029600	0.0099518
13	-26.557	1.6000	1061.0	41.663	140.52	437.02	0.77848	1.9809	2.1006	1.1860	0.14263	0.014129	0.0015518	0.00012680	0.00063995	0.0028594	0.0097453
14	-25.609	1.6500	1056.9	43.001	142.51	437.05	0.78636	1.9762	2.1089	1.1994	0.14147	0.014259	0.0015278	0.00012741	0.00063472	0.0027646	0.0095444
15	-24.682	1.7000	1052.8	44.345	144.46	437.06	0.79405	1.9716	2.1174	1.2131	0.14034	0.014389	0.0015048	0.00012801	0.00062956	0.0026749	0.0093487
16	-23.775	1.7500	1048.8	45.694	146.38	437.05	0.80157	1.9672	2.1260	1.2268	0.13923	0.014519	0.0014825	0.00012861	0.00062446	0.0025899	0.0091579
17	-22.886	1.8000	1044.8	47.050	148.27	437.04	0.80894	1.9628	2.1347	1.2407	0.13815	0.014649	0.0014610	0.00012920	0.00061943	0.0025093	0.0089719
18	-22.015	1.8500	1040.9	48.413	150.13	437.01	0.81615	1.9585	2.1435	1.2549	0.13709	0.014779	0.0014402	0.00012978	0.00061444	0.0024328	0.0087905
19	-21.162	1.9000	1037.0	49.782	151.95	436.97	0.82321	1.9543	2.1525	1.2691	0.13605	0.014910	0.0014202	0.00013036	0.00060951	0.0023599	0.0086133
20	-20.324	1.9500	1033.2	51.158	153.75	436.92	0.83014	1.9501	2.1617	1.2836	0.13504	0.015041	0.0014007	0.00013093	0.00060463	0.0022906	0.0084401
21	-19.503	2.0000	1029.4	52.540	155.52	436.85	0.83694	1.9461	2.1710	1.2983	0.13404	0.015173	0.0013819	0.00013150	0.00059980	0.0022244	0.0082710



针对ITK内桶冷却：一个ITK桶部stave总热功耗81.1w；二氧化碳2MPa， -19.503℃：潜热： 281.33kj/kg；通过计算需要0.2883g/s蒸发速度带走一条stave的热量， 如果出口干度0.3， 所需质量流量： $0.14415\text{g}/0.3=0.4805\text{g}$



针对ITK中桶冷却：一个ITK桶部stave总热功耗81.1w；二氧化碳2MPa， -19.503℃：潜热： 281.33kj/kg； 通过计算需要0.4119g/s蒸发速度带走一条stave的热量， 如果出口干度0.3， 所需质量流量： $0.206\text{g}/0.3=0.6867\text{g}$



针对ITK外桶冷却：一个ITK桶部stave总热功耗162.2w；二氧化碳2MPa， -19.503℃：潜热： 281.33kj/kg； 通过计算需要0.5766g/s蒸发速度带走一条stave的热量， 如果出口干度0.3， 所需质量流量： $0.2883\text{g}/0.3=0.961\text{g}$

