Toward quantum simulations of Maxwell-Chern-Simons theory: constant modes and gauge field truncation

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Based on:

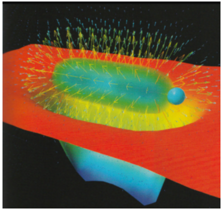
AB, C. Diamantini, N. Dichter, L. Funcke, T. Hartung, K. Jansen, E. Rico, S. Singh, L. Spera

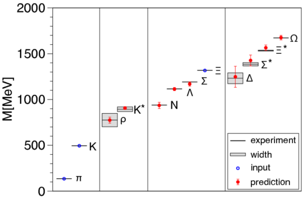
arXiv:2607.XXXXX

Simulating lattice gauge theories

Lagrangian - Monte Carlo

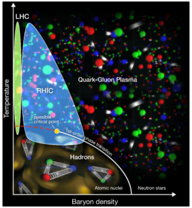
- Hadronic spectrum
- Scattering amplitudes
- Finite temperature
- Confinement
- ...

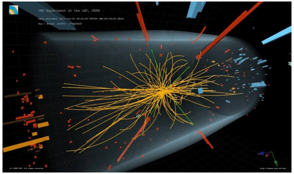




Beyond

- Real time dynamics
- Finite density
- Topological terms
- ...





Topological terms in gauge theories

Topological terms

- **Definition**
  Global action contributions, related to topological invariants
- **Condensed Matter**
  Topological phases of matter, fractional quantum Hall effect ...
- **High-Energy Physics**
  Strong CP problem, axion, quantum gravity ...

Taxonomy

- **Theta term**
  $S_{\text{topo}} \sim \theta \, \mathrm{Tr}(F \wedge F)$, $3+1d$ gauge theories
- **Chern-Simons term**
  $S_{\text{CS}} \sim k \, \mathrm{Tr}(A \wedge \mathrm{d}A)$, $2+1d$ gauge theories
- **Other**
  WZW, BF-theories

Chern-Simons (CS) and Maxwell-Chern-Simons (MCS)

CS action

$$S_{\text{CS}}[A] = -\frac{ik}{4\pi}\int \mathrm{d}^3x\, \epsilon^{\mu\nu\rho}\, A_\mu \partial_\nu A_\rho$$

- **Definition**
  Topological term in $2+1d$ Abelian gauge theories
- **Parameter**
  $k \in \mathbb{Z}$, controlling the ground-state degeneracy on a torus
- **Only global properties**
  No local propagating d.o.f.

Adding a Maxwell term

$$S_{\text{MCS}}[A] = S_{\text{CS}}[A] + \frac{1}{4g^2}\int \mathrm{d}^3x\, F_{\mu\nu}^2$$

- **Role**
  Needed for regularization purposes
- **Physical consequences**
  Massive photon $m_\gamma = ke^2/2\pi$ without Higgs mechanism

[Deser, Jackiw, Templeton; Ann. Phys. 140 (1982)]
[Pisarski; PRD 34 (1986)]
[Eliezer, Semenoff; PLB 286 (1992)]

Non-perturbative phenomena and lattice regularization

Why a lattice regularization?

- Non-perturbative physics
- Microscopic realization of the theory
- Full non-perturbative definition

Euclidean lattice CS

- Naive lattice discretization:

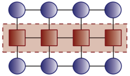
$$S_{\mathrm{CS}}^{\mathrm{lat}}[A] = -\frac{ik}{4\pi} a^3 \sum_x \epsilon^{\mu\nu\rho} A_{x,\mu} \Delta_\nu A_{x+a\hat{\mu},\rho}$$

- Sign problem, Monte Carlo not suited for CS term
- Further challenges: compact vs non-compact, monopoles,...

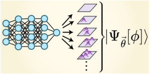
Hamiltonian simulations

Methods

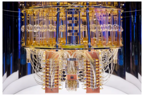
- Tensor Networks



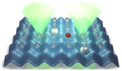
- Variational Wavefunctions



- Quantum Computing



- Quantum Simulators

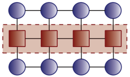
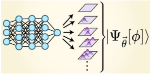


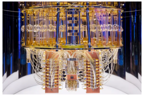
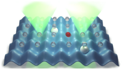
Challenges

- Encoding of the theory
- Initial state preparation
- Time evolution
- Measurement
- ...

Hamiltonian simulations

Methods

- Tensor Networks 
- Variational Wavefunctions 

- Quantum Computing 
- Quantum Simulators 

Challenges

- Encoding of the theory
- Initial state preparation
- Time evolution
- Measurement
- ...

Hamiltonian MCS

Different approaches

- Recent Hamiltonian formulations, mainly for analytical studies

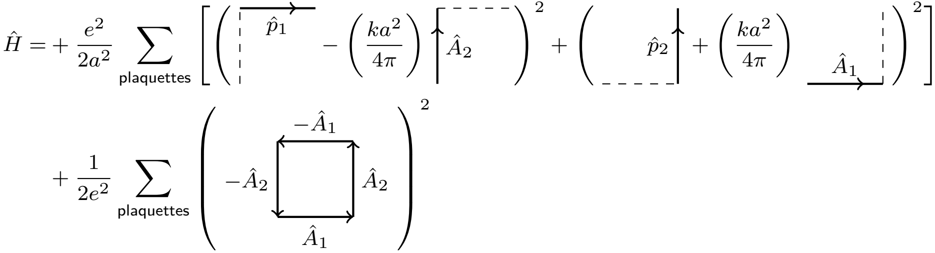
  [Jacobson, Sulejmanpasic; PRD 107 (2023)]
  [Jacobson, Sulejmanpasic; JHEP 11 (2024)]
  [Xu, Chen; JHEP 08 (2025)]

Hamiltonian simulations

- Recent proposal targeting quantum simulations

  [Peng, Diamantini, Funcke, Hassan, Jansen, Kühn, Luo, Naredi; 2407.20225]

- Key dynamical and topological properties recovered

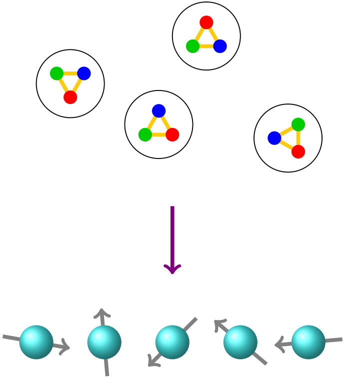


How to fit MCS into a quantum computer?

Infinite dimensional Hilbert space

- $U(1)$ gauge fields: infinite dimensional Hilbert space per link
- Truncate the gauge field so that:
  1. Relevant physical properties are preserved
  2. Correct continuum limit

Truncation



Constant modes

Decomposing the gauge field

- **Fourier decomposition**: spatially constant modes

$$A_i(t, \mathbf{x}) = a_i(t)$$

- **Hamiltonian**

$$H_{\mathrm{CM}} = \frac{Se^2}{2}\left[\left(p_1 - \frac{k}{4\pi}a_2\right)^2 + \left(p_2 + \frac{k}{4\pi}a_1\right)^2\right]$$

- **Configuration space**: torus

$$a_i \sim a_i + 2\pi n_i \qquad n_i \in \mathbb{Z}$$

An exactly solvable model

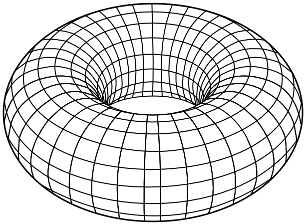
- **Landau problem on a torus**: exact solution

- **Predictions**:
  k-fold ground state degeneracy
  Harmonic-oscillator-like level spacing

- **Strategy**: discretizing the constant modes and check against analytical predictions

Discretizing the constant modes

Our proposal



- Discrete $a_i \equiv$ Discrete torus
- Particle on a periodic lattice in a magnetic background

(An atypical) Hofstadter problem

- **Model**
  Quantum-mechanical charged particle in a square lattice with a magnetic field
- **Parameters**
  Flux-per-plaquette $\Phi_{\text{plaq}} = k/N_x N_y$
- **Continuum limit**
  $\Phi_{\text{plaq}} \to \infty$

Harper equation and chiral symmetry

Harper equation

- Massaging the hopping Hamiltonian one arrives at

$$E\, g_n(x) = -t\Big[g_n(x+1) + g_n(x-1) + 2\cos(2\pi\alpha x + p_n)\, g_n(x)\Big]$$

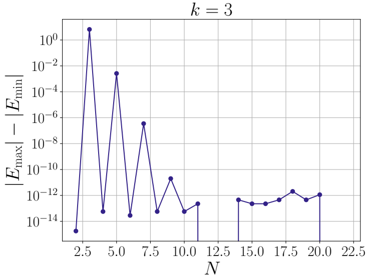
- Twisted bc: $g_n(x+N_x) = g_{n+k}(x)$
- $\dim\mathcal{H} = N_x\, N_y$: ED-friendly!

Rest of the talk $N_x = N_y = N$, $a = 1$

Chiral symmetry

- For even N_x, N_y we can define Γ such that $\{H_{\text{Harper}}, \Gamma\} = 0$

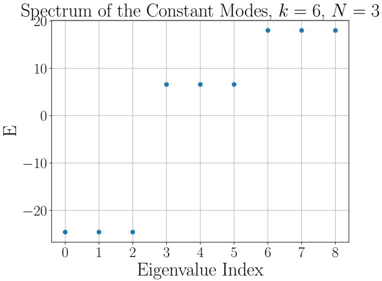
$$\Gamma = \text{diag}_x((-1)^x) \otimes T_{N/2}^{p_y}$$



Regimes of the truncated theory: unphysical regimes ($k > N$)

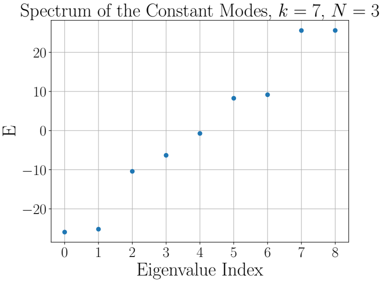
$N|k$

- Standard Hofstadter problem
- $m = k/N$, $q = N/\gcd(m, N)$, q bands



$\gcd(k, N) = 1$

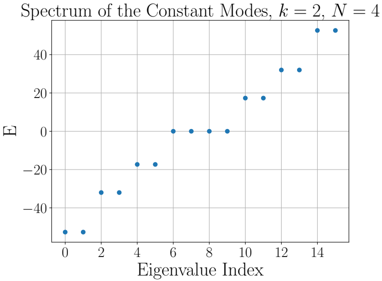
- Continuum k-fold structure lost
- Spectrum determined by the interplay of Φ_{plaq}, twisted bc and $\dim\mathcal{H}$



Regimes of the truncated theory: continuum regimes ($k < N$)

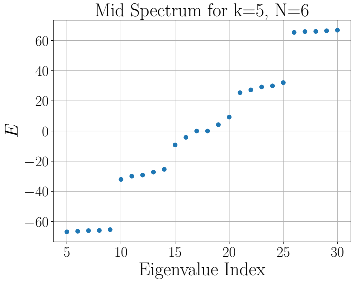
$k|N$

- Exact degeneracy
- Magnetic translations compatible with twisted bc



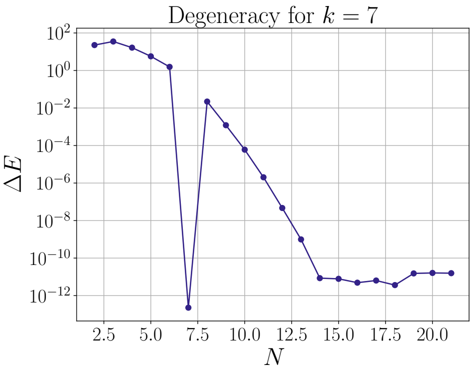
$\gcd(k, N) = 1$

- Dynamical approximate k-fold degeneracy, only exact for $N \to \infty$
- Chiral symmetry controls key features of the spectrum

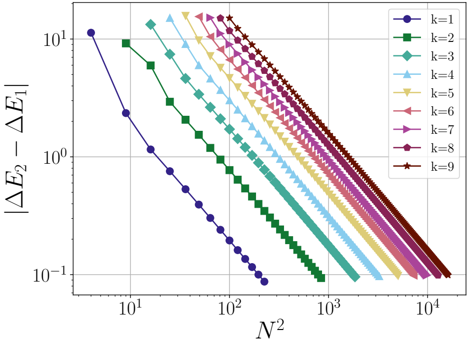


Approaching the continuum limit

Degeneracy



Level spacing



Conclusions and outlook

Conclusions

- Discretization of the constant modes in Hamiltonian MCS theory
- Clear scaling of physical properties in the continuum limit
- Rich physical features also outside of the continuum regime
- Promising continuum limit

Outlook

- Spectrum truncation in the full theory
- Formulation beyond Villain
- Tensor network and quantum computing simulations