Renormalized Dual Basis and continuum limit

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C3NT Workshop QIS-HENP

Wuhan, 17/06/2026

Toward ab-initio continuum limit with quantum methods

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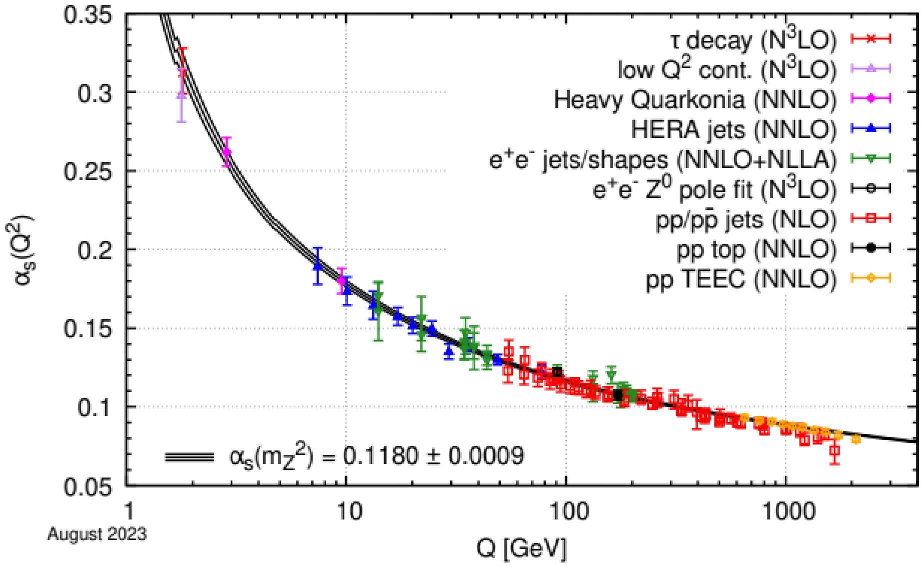
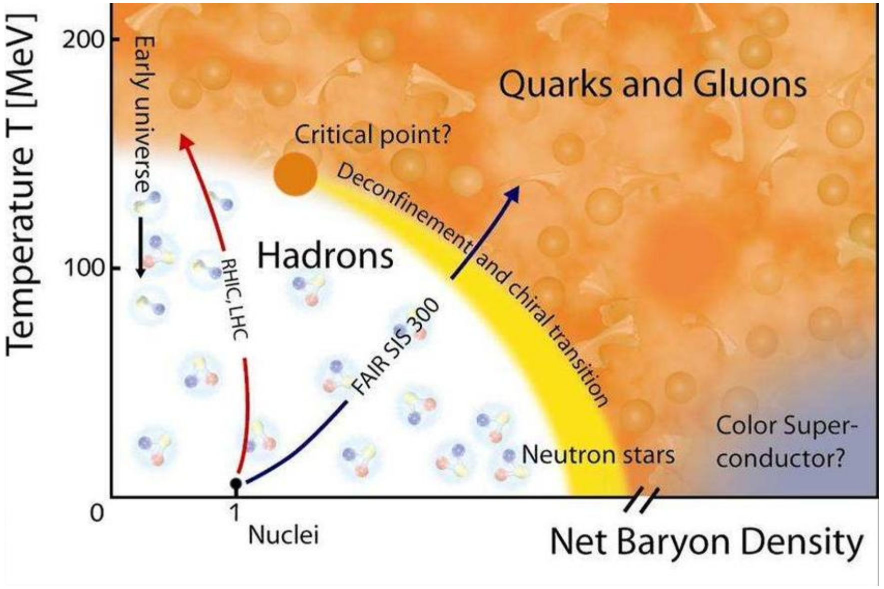
QCD: Lagrangian approach

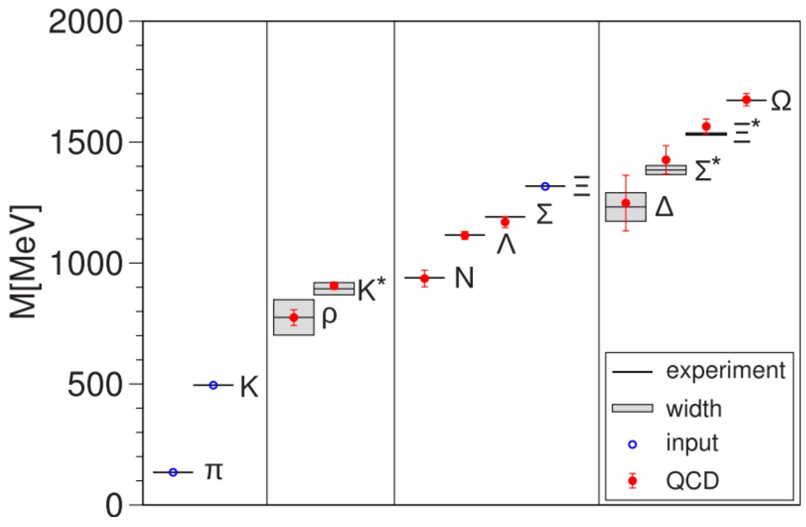


Navas et al., Phys Rev. D **110** (2024)

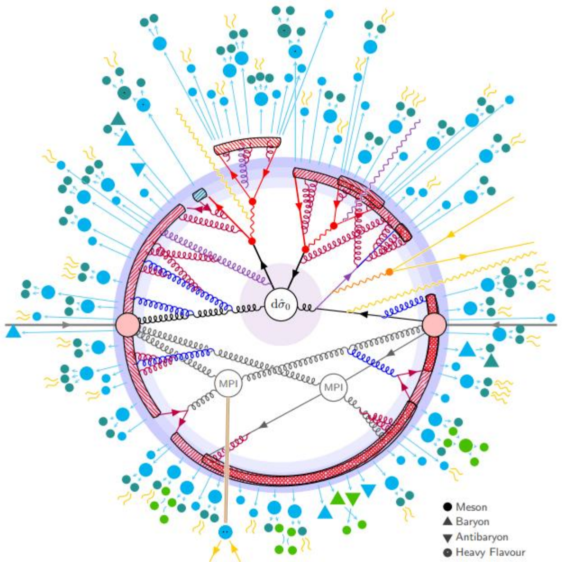
Sign problem



Durante et al. Phys. Scr. **94** (2019)

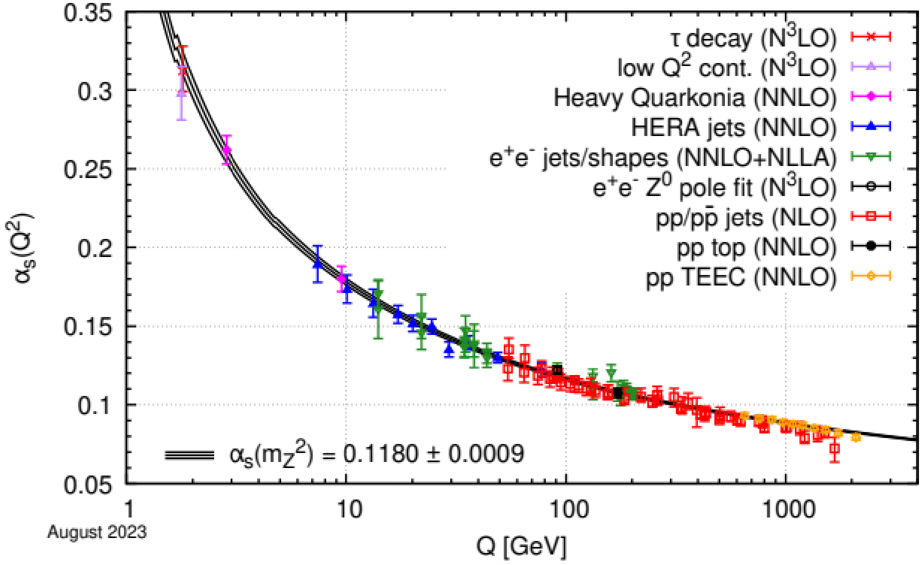


Dürr et al., Science **322** (2008)

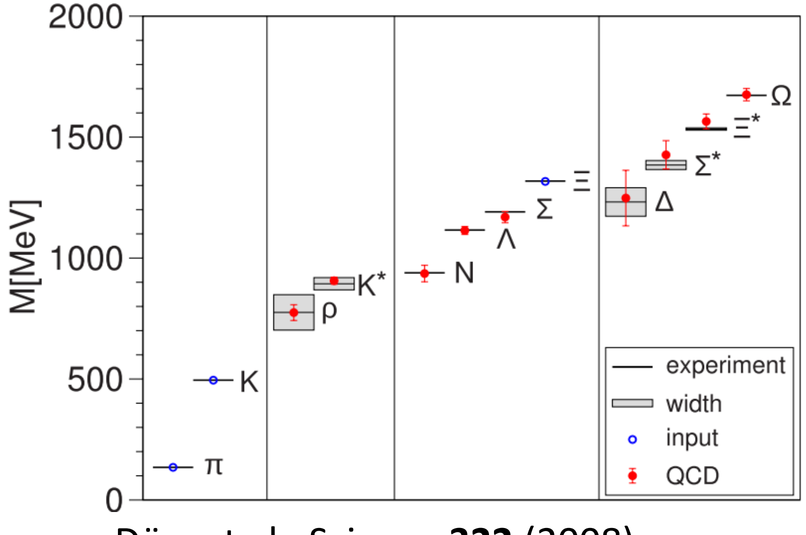


Bierlich, SciPost Phys. Codebases 8 (2022)

QCD: Lagrangian approach



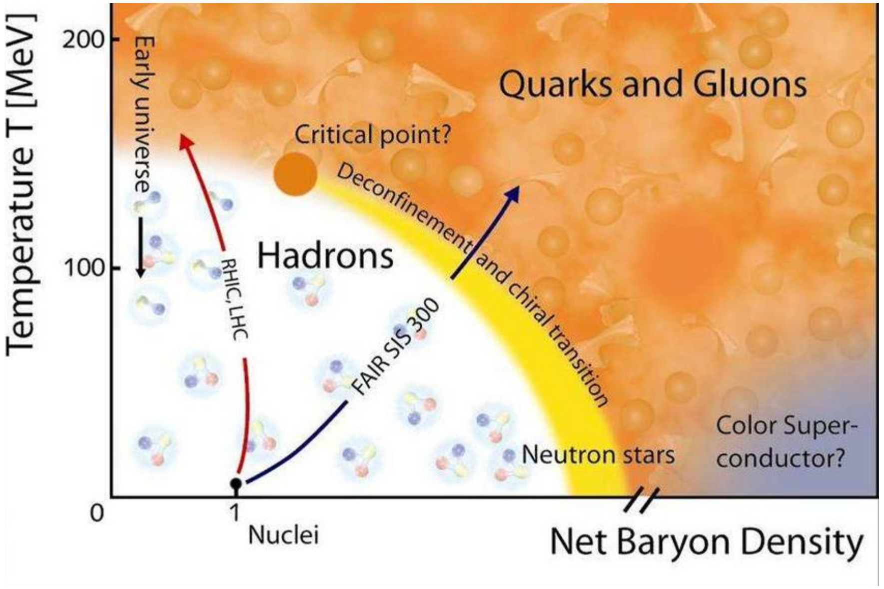
Navas et al., Phys Rev. D **110** (2024)



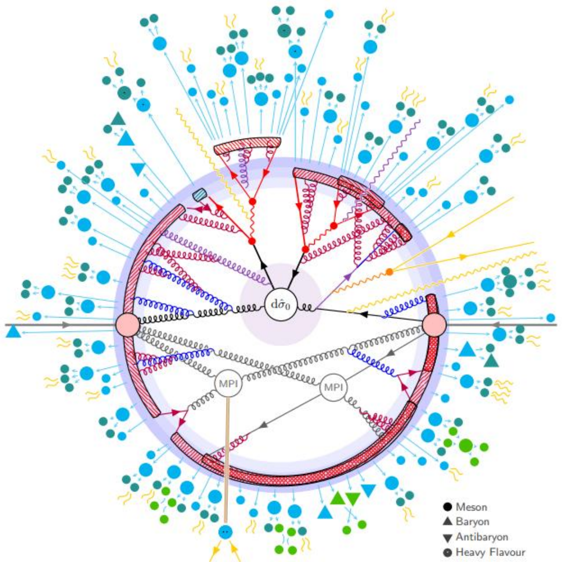
Dürr et al., Science **322** (2008)

Sign problem

Critical slowing down



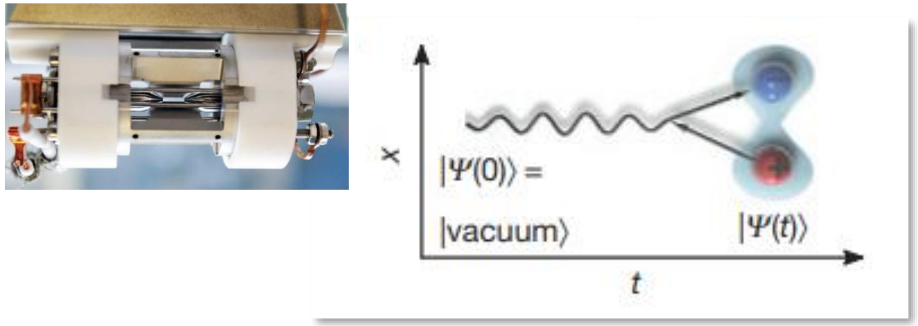
Durante et al. Phys. Scr. **94** (2019)



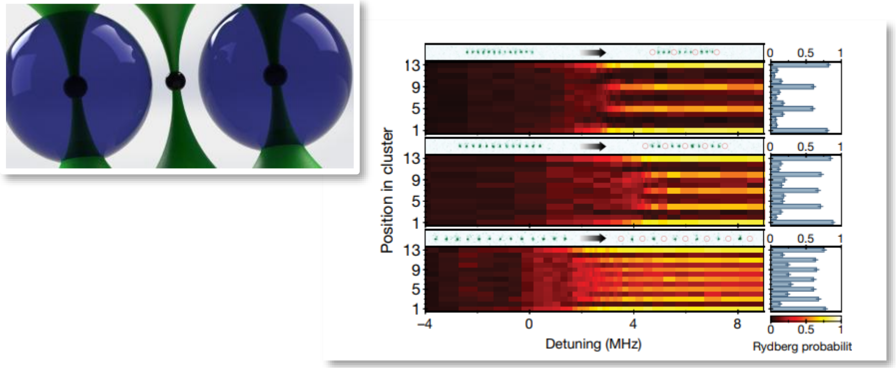
Bierlich, SciPost Phys. Codebases 8 (2022)

The quantum way for gauge theory: experiments

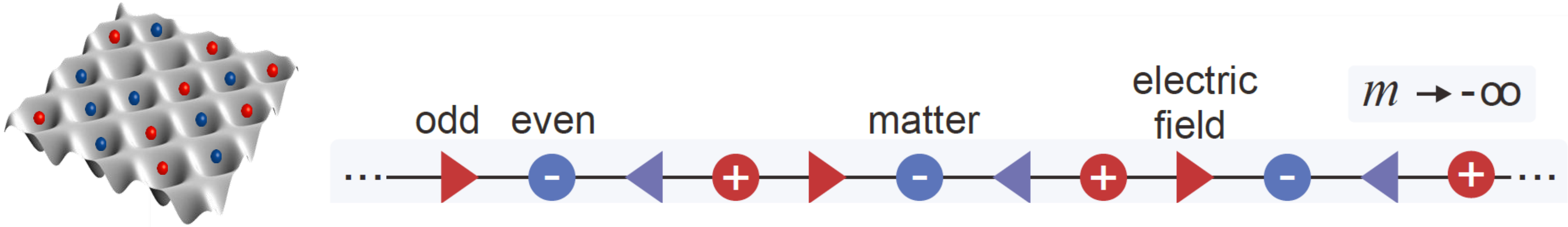
- 1D QED on a lattice: Schwinger model



E. A. Martinez *et al.*, *Nature* **534**, 516 (2016)
C. Kokail *et al.*, *Nature* **569**, 355 (2019)
N. H. Nguyen *et al.*, *PRX Quantum* **3**, 020324 (2022)



H. Bernien *et al.*, *Nature* **551**, 579 (2017)
F. M. Surace *et al.*, *Phys. Rev. X* **10**, 021041 (2020)

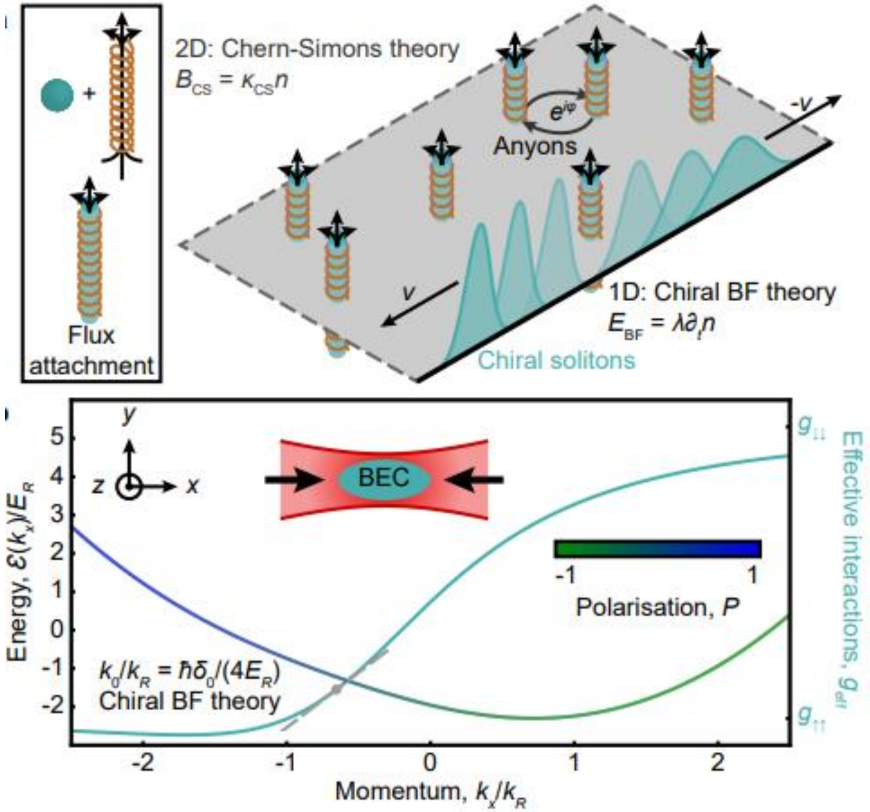


B. Yang *et al.*, *Nature* **587**, 392 (2020); Z.-Y. Zhou *et al.*, *Science* **377**, 311 (2022);
H. Wang *et al.*, Phys. Rev. Lett. **131**, 050401 (2023)

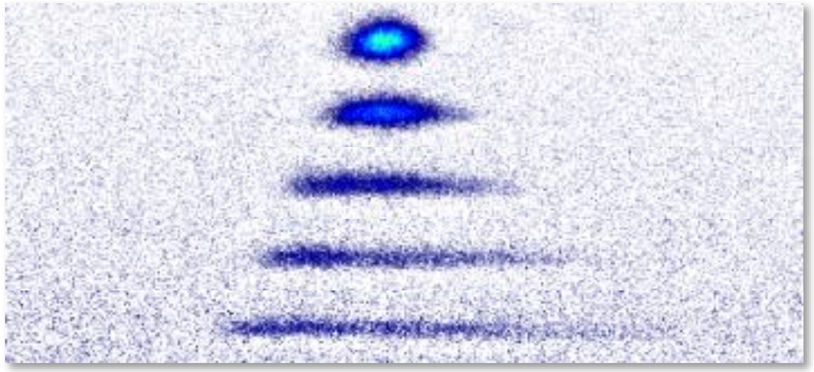
The quantum way for gauge theory: experiments

- 1D QED on a lattice: Schwinger model

- 1D Chiral BF theory in the continuum



Self-generated electric field

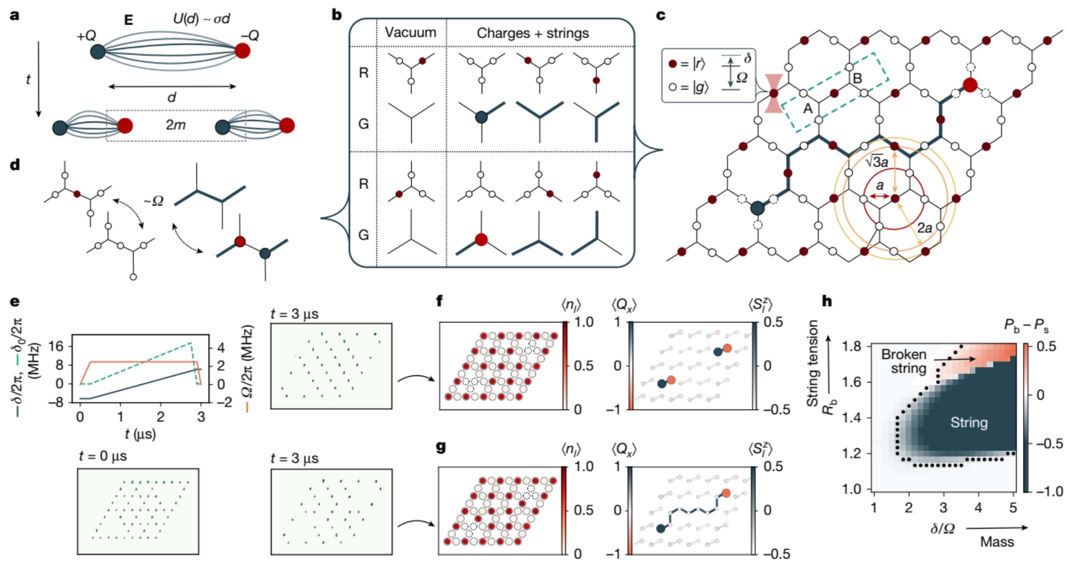


A. Frölian*, C. S. Chisholm*, E. Neri, C. R. Cabrera, R. Ramos, **AC**, and L. Tarruell, Nature **608**, 293 (2022)

The quantum way for gauge theory: experiments

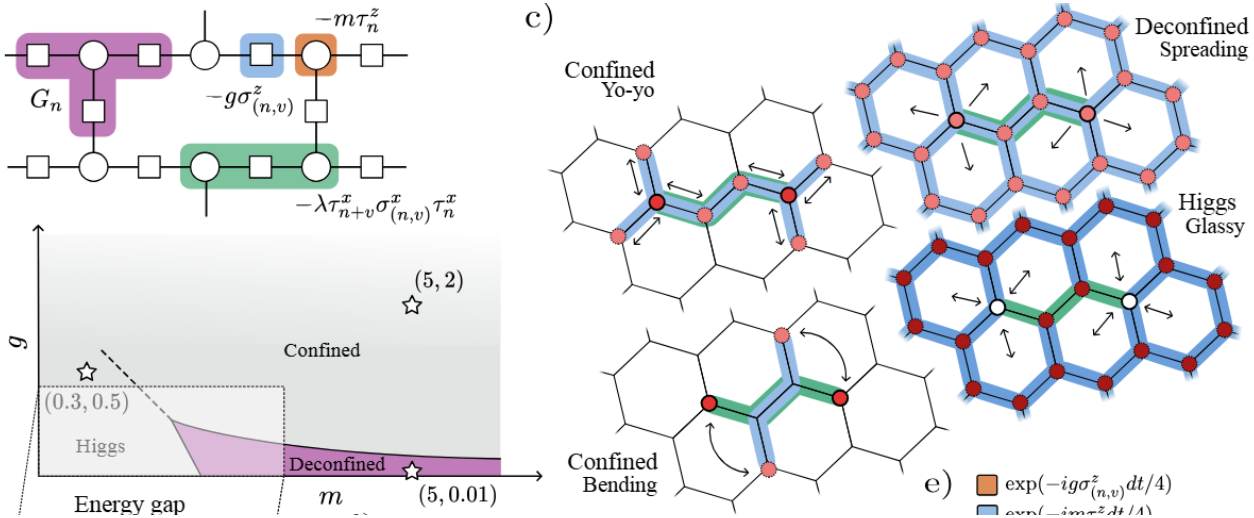
String breaking in small 2D lattice

Analog: Schwinger model with Rydberg



D. González-Cuadra *et al.*, *Nature* **642**, 321–326 (2025)

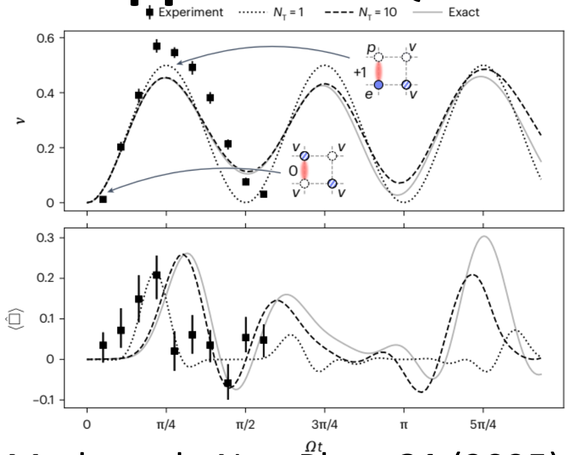
Digital: Superconducting Qubits



Z_2: T.A. Cochran *et al.*, *Nature* **642**, 315–320 (2025)

J. Cobos *et al.*, *arXiv:2507.08088* (see Enrique's talk)

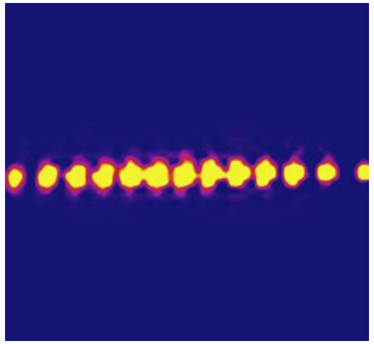
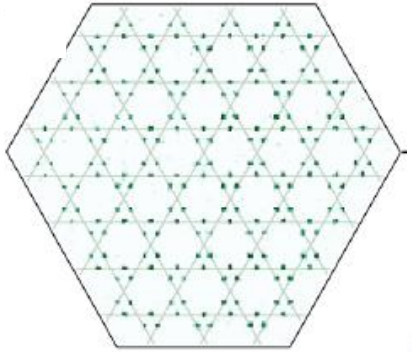
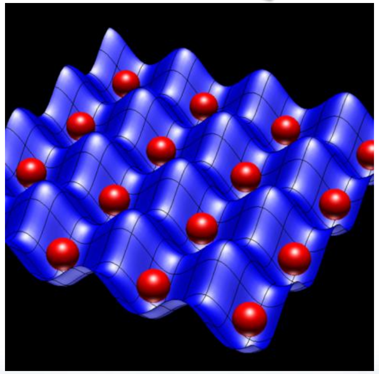
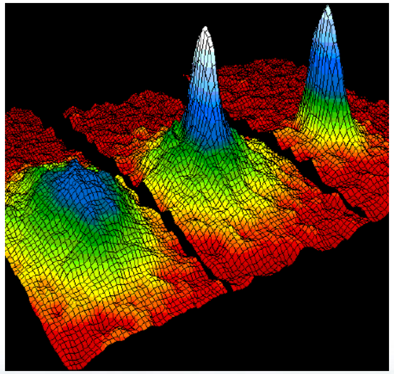
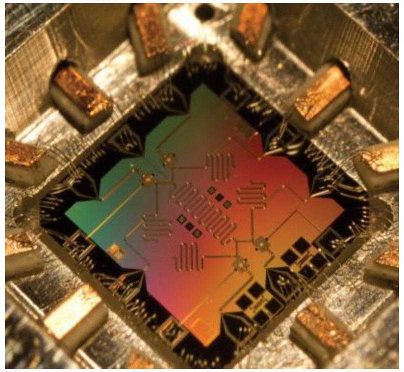
Trapped Ion Qudits



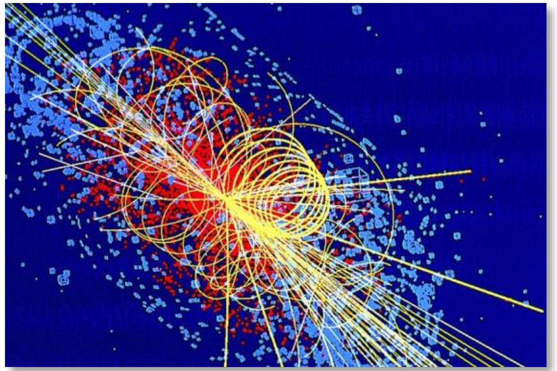
Meth et al., Nat. Phys. **21** (2025)

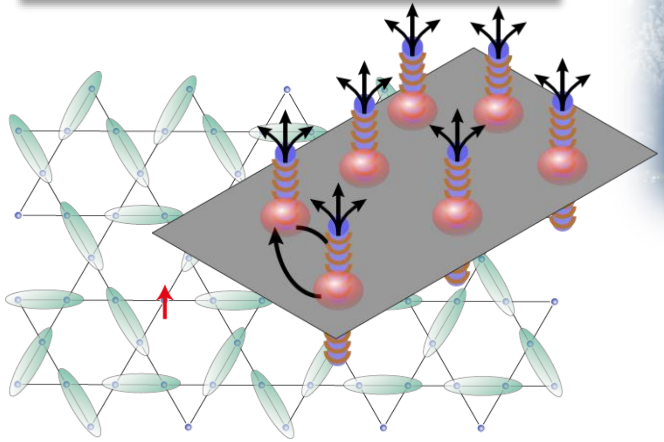
See Simran's and Jad's talk

GAUGE THEORY & QUANTUM SYSTEMS

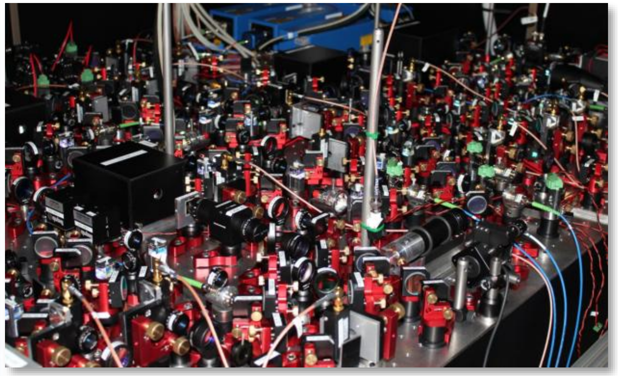
Real world: 2D&3D





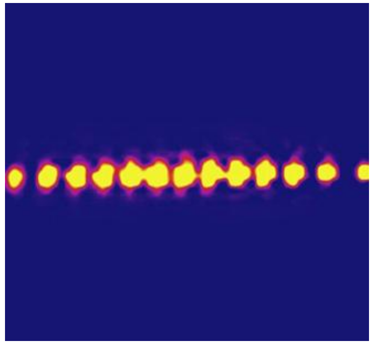
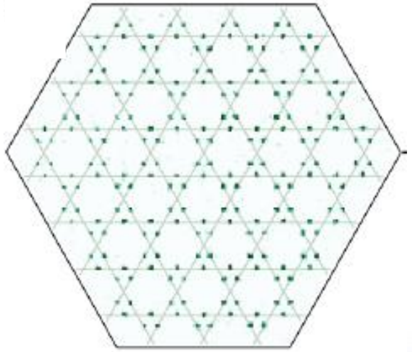
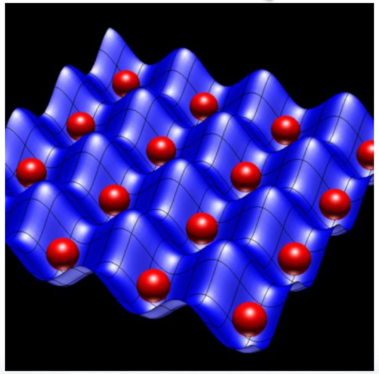
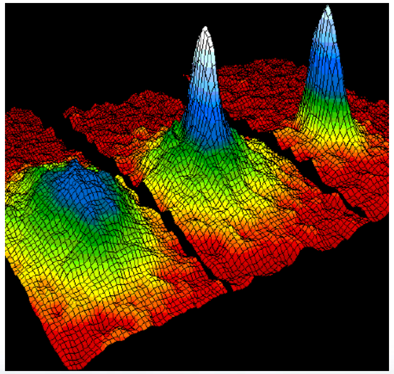
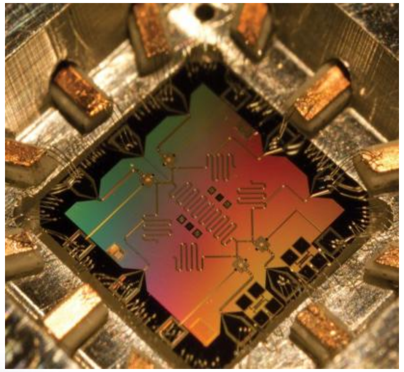
M.C. Bañuls *et al.*, *Eur. Phys. J. D* **74,** 165 (2020)
M. Dalmonte and S. Montangero, *Cont. Phys.* **57** 388 (2016)
E. Zohar, J.I. Cirac, and B Reznik, Rep. Prog. Phys. 79, 014401 (2015)

Experiments

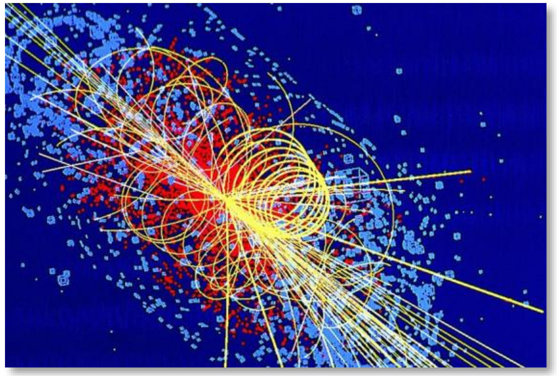


Innsbruck, Heidelberg, Chicago, Munich, Zurich, Paris, Harvard, **Barcelona…**

GAUGE THEORY & QUANTUM SYSTEMS

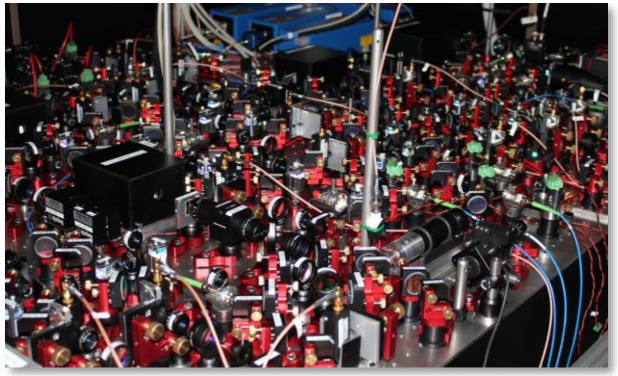






Real world: 2D&3D



Continuum limit

Experiments



Innsbruck, Heidelberg, Chicago, Munich, Zurich, Paris, Harvard, **Barcelona…**

M.C. Bañuls *et al.*, *Eur. Phys. J. D* **74,** 165 (2020)
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E. Zohar, J.I. Cirac, and B Reznik, Rep. Prog. Phys. 79, 014401 (2015)

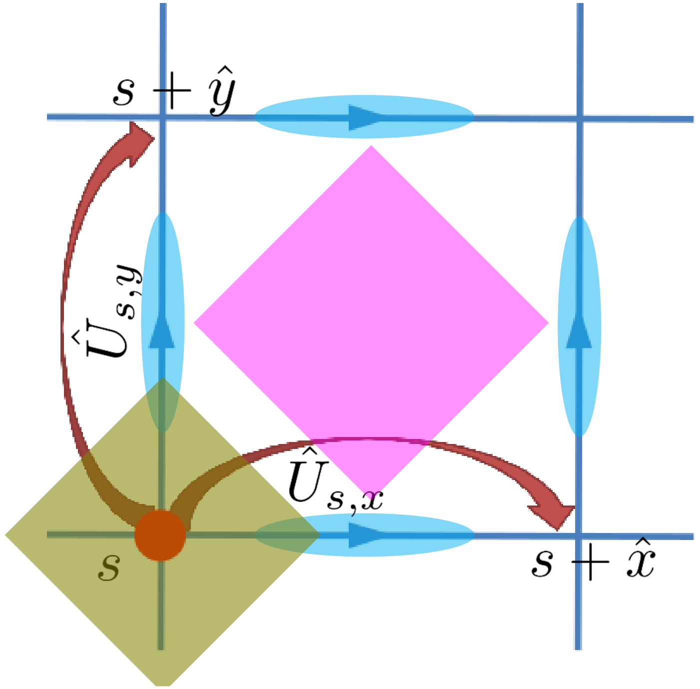
Hamiltonian lattice gauge theory

[Kogut,Susskind '75]

2D Maxwell: Dynamical gauge field = operator on electro-magnetic quanta

$$[\hat{E}_{s,\mu}, \hat{A}_{s,\mu}] = -i \longrightarrow [\hat{E}_{s,\mu}, \hat{U}_{s,\mu}] = -\hat{U}_{s,\mu} \qquad \hat{E}_{s,\mu} = (n+n_0)|n\rangle\langle n|, \ \ n \in Z$$

Hilbert space: U(1) group algebra



Kinetic term preserves Gauss law

$$\hat{G} \equiv \hat{E}_{s,x} + \hat{E}_{s,y} - \hat{E}_{s-\hat{x},x} - \hat{E}_{s-\hat{y},y} - \hat{q}_s$$

$$[\hat{G}, \hat{H}] = 0, \qquad |\psi\rangle \in \{\text{Physical states}\} \iff \hat{G}|\psi\rangle = 0$$

$$e^{i\alpha_s \hat{G}}|\psi\rangle = |\psi\rangle, \forall \alpha_s$$

+ Electro-magnetic energy

$$\hat{H} = -t \sum_{s,\mu=x,y} \hat{c}^\dagger_{s+\mu}\hat{U}_{s,\mu}\hat{c}_s + H.c. + g^2\hat{E}^2_{s,\mu} - \frac{1}{g^2}(\hat{U}\hat{U}\hat{U}^\dagger\hat{U}^\dagger + H.c.)$$

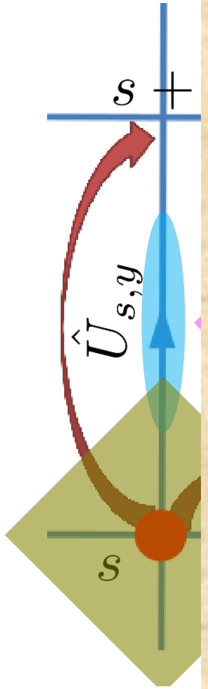
Hamiltonian lattice gauge theory

[Kogut,Susskind '75]

2D Maxwell: Dynamical gauge field = operator on electro-magnetic quanta

$[\hat{E}_{s,\mu}, \hat{A}$ … $\mathbb{Z}$

…ebra



- The anisotropy between time and space reflects in the opposite behavior with respect to the coupling
- *g>>1* Electric term dominates, strong coupling (as 1D)
- *g<<1* Magnetic term dominates, weak coupling, (perturbative) deconfinement
- Competition between the two terms, characteristic feature D>1
- Consequence: Running coupling (dimensional transmutation)

$\ldots_s$

$= 0$

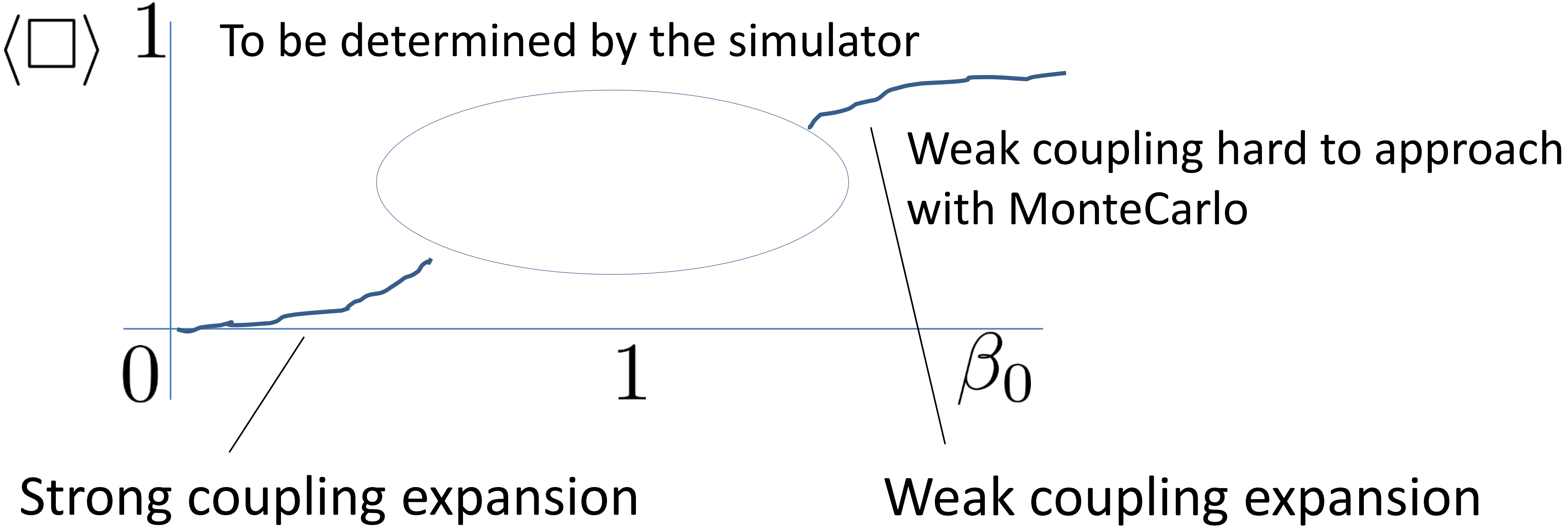
$= |\psi\rangle, \forall \alpha_s$

$$\hat{H} = -t \sum_{s,\mu=x,y} \hat{c}^{\dagger}_{s+\mu}\hat{U}_{s,\mu}\hat{c}_s + H.c. + g^2 \hat{E}^2_{s,\mu} - \frac{1}{g^2}(\hat{U}\hat{U}\hat{U}^{\dagger}\hat{U}^{\dagger} + H.c.)$$

Running coupling on a torus

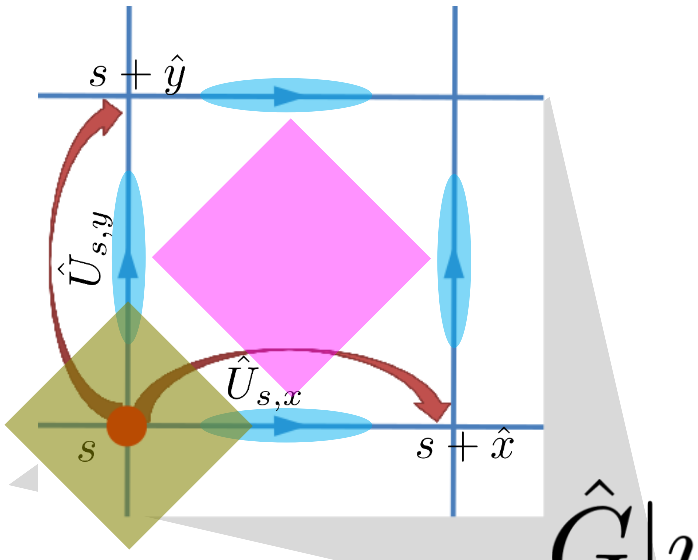
$$\beta_0 = \frac{1}{g^2} \qquad \langle \Box \rangle = \frac{\beta_R}{\beta_0}$$

Renormalization of the coupling [Creutz'80]

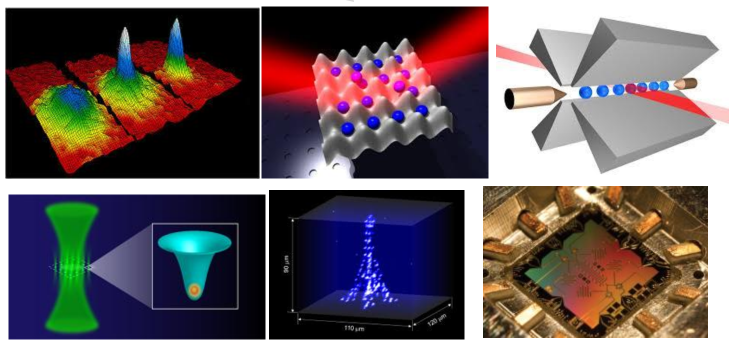


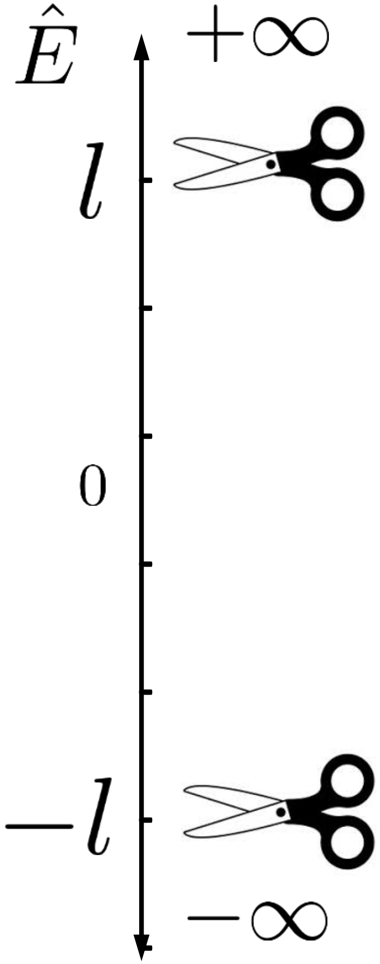
Alternative short-distance observables: G. Clemente, A. Crippa, K. Jansen, Phys. Rev. D **106**, 114511 (2022)

Running coupling with finite resources?



$$\hat{G}|\psi\rangle = 0$$



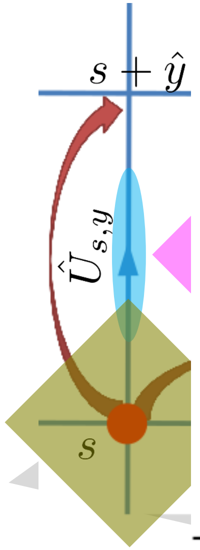


$$g \to 0 \Rightarrow l \to \infty$$

Electric-basis truncation inefficient

Running coupling with finite resources?

ROADMAP FOR QUANTUM(-INSPIRED) LGT SIMULATION



3. arXiv:2407.03058 [pdf, other]

hep-lat cond-mat.str-el hep-th physics.cc

Tensor Networks for Lattice Gauge The Roadmap

Authors: Giuseppe Magnifico, Giovanni Cataldi, M. Pietro Silvi, Simone Montangero

ℓ	d					
	$(2+1)$-dimensions			$(3+1)$-dimensions		
	U(1)	SU(2)	SU(3)	U(1)	SU(2)	SU(3)
1	35	30	164	267	178	3096
2	165	168	752	3437	3670	52476
3	455	600	3738	18487	35280	813438
4	969	1650	19878	64953	214958	17490134
5	1771	3822	43698	177155	967466	69232482
6	2925	7840	82128	408421	3509062	228461186
7	4495	14688	212496	835311	10828494	1245755754
8	6545	25650	333538	1561841	29473038	2782999996

TABLE I. Dressed site Hilbert space dimension d for increasing number ℓ of allowed electric energy density levels in some 2- and 3-dimensional paradigmatic LGTs with dynamical matter and gauge groups U(1), SU(2), and SU(3).

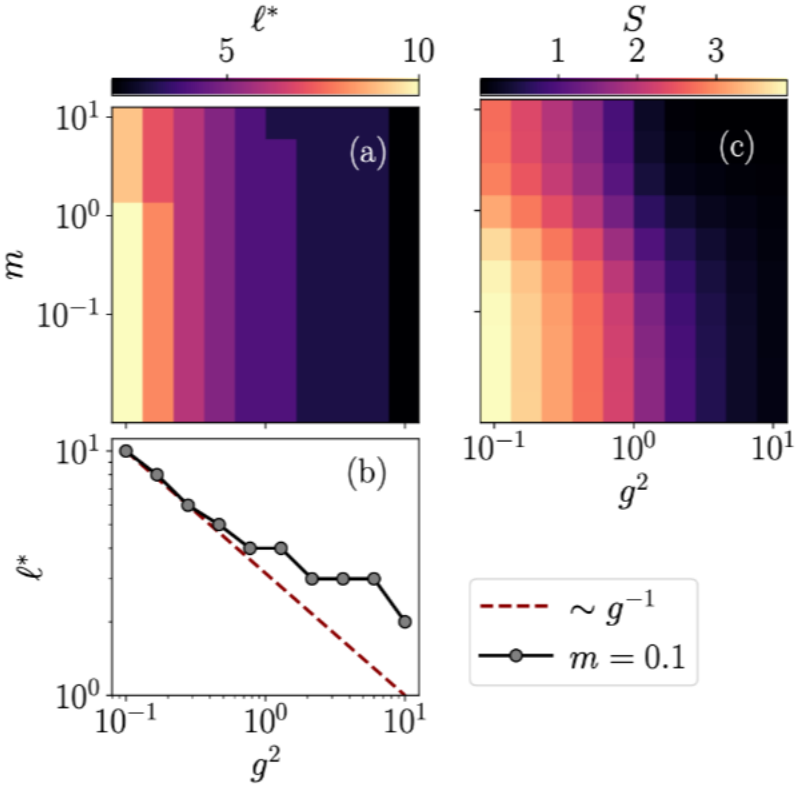
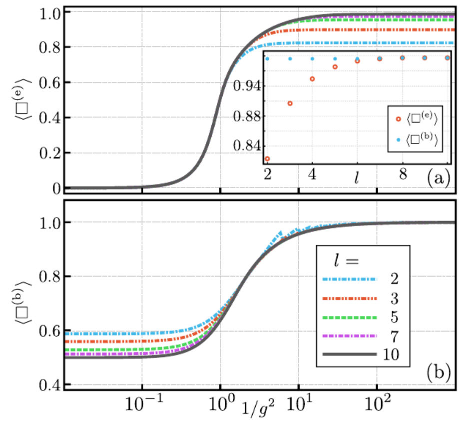


FIG. 4. Exact diagonalization of a QED plaquette for a grid of masses and couplings, $m \in [10^{-2}, 10^1]$ and $g^2 \in [10^{-1}, 10^1]$. (a,b) Minimal gauge truncation ℓ^* required to reach a precision $\epsilon_{\text{trunc}} = 10^{-5}$ in the magnetic energy $\langle \text{Re}\,\hat{U}_\square \rangle$. (c) Corresponding entanglement entropy S associated with a symmetric bipartition of the plaquette.

$$l \to \infty$$

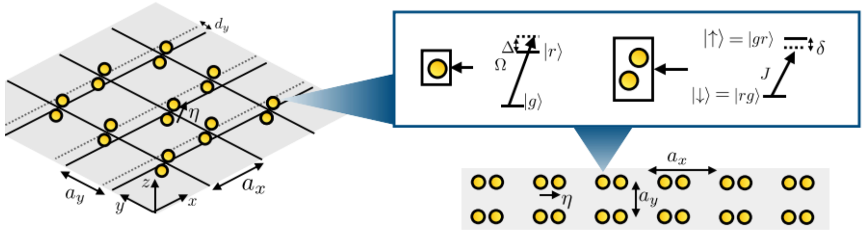
Running of the coupling



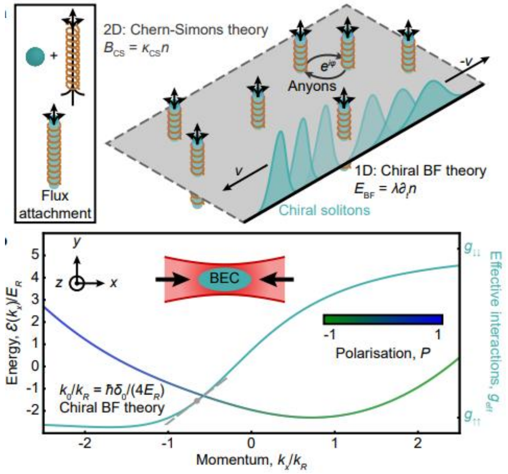
Encoding

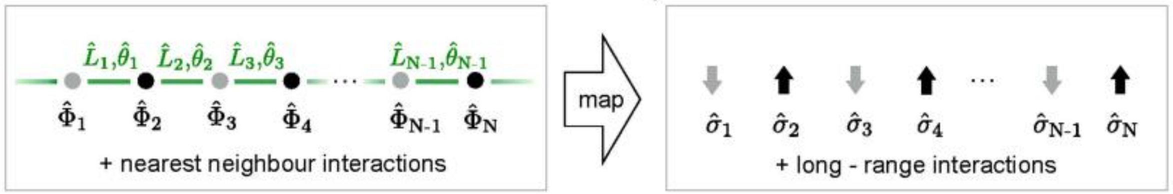
Dual Rokhsar-Kivelson Hamiltonian

AC *et al.*, *PRX* **10**, 021057 (2020)



Experimental demonstration of chiral BF theory

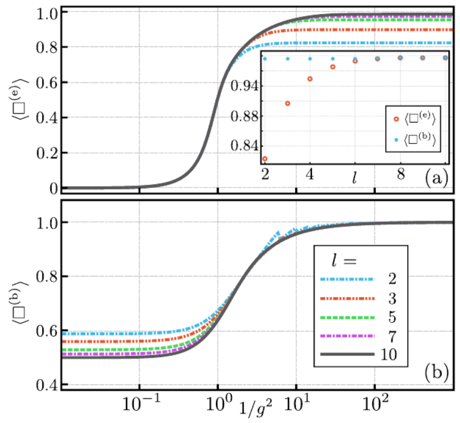




Schwinger model

E.A. Martinez *et al.*, Nature 534 (7608), 516-519 (2016)

C. Muschik *et al.*, New J. Phys. 19 (10), 103020 (2017)



Running of the coupling

Encoding
+
Coupling dependent basis

- 2D QED on small tori

F. Haase*, L. Dellantonio*, **AC***, D. Paulson, A. Kan, K. Jansen, C.A. Muschik, Quantum **5**, 393 (2021)

- 2D pure SU(2) on small tori

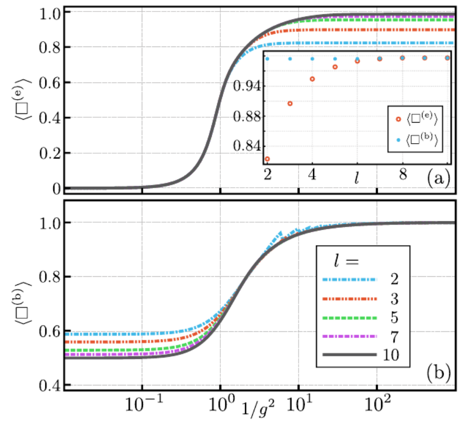
P. Fontana, M. Miranda-Riaza, and **AC**, PRX **15** (2025)

M. Miranda-Riaza

P. Fontana, Ph.D

- 2D QED on large/infinite lattices

M. Miranda-Riaza, P. Fontana and **AC**, PRD **113** (2026)/*in progress*



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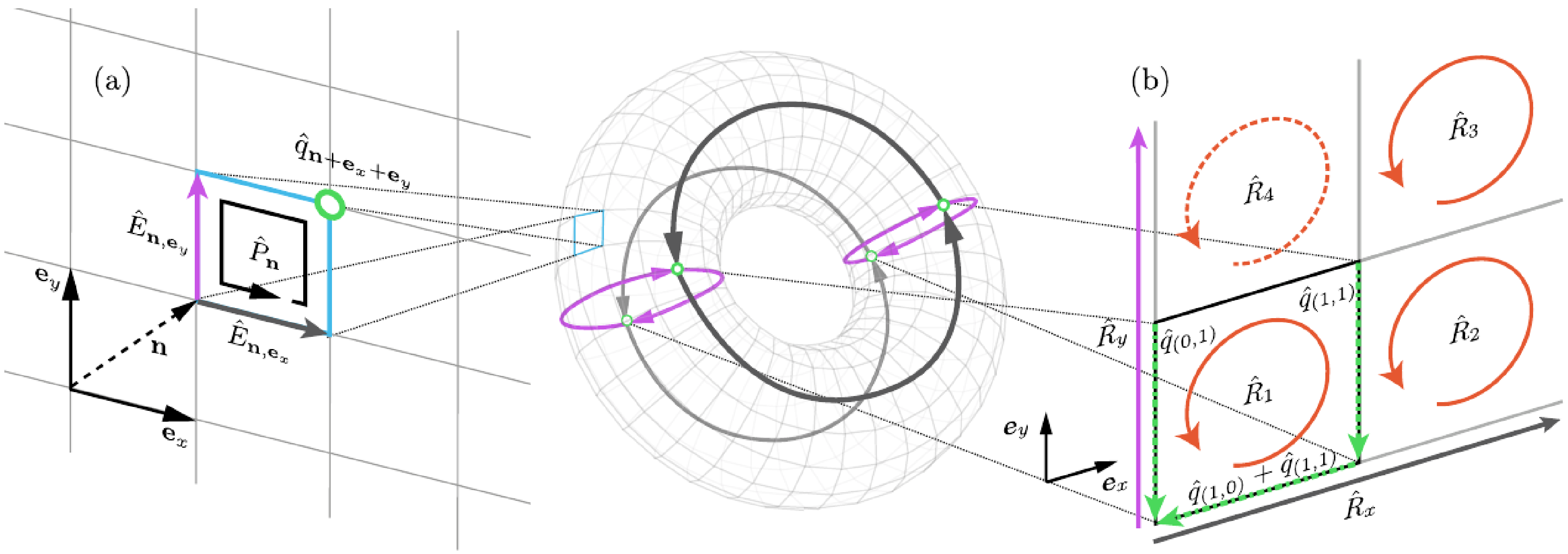
M. Miranda-Riaza

P. Fontana, Ph.D

- 2D QED on large/infinite lattices

M. Miranda-Riaza, P. Fontana and **AC**, PRD **113** (2026)/*in progress*

Running coupling of 2+1 QED on a minimal torus



$$\hat{H}^{(e)} = \hat{H}^{(e)}_E + \hat{H}^{(e)}_B,$$

$$\hat{H}^{(e)}_E = 2g^2\left[\hat{R}_1^2 + \hat{R}_2^2 + \hat{R}_3^2 - \hat{R}_2(\hat{R}_1 + \hat{R}_3)\right],$$

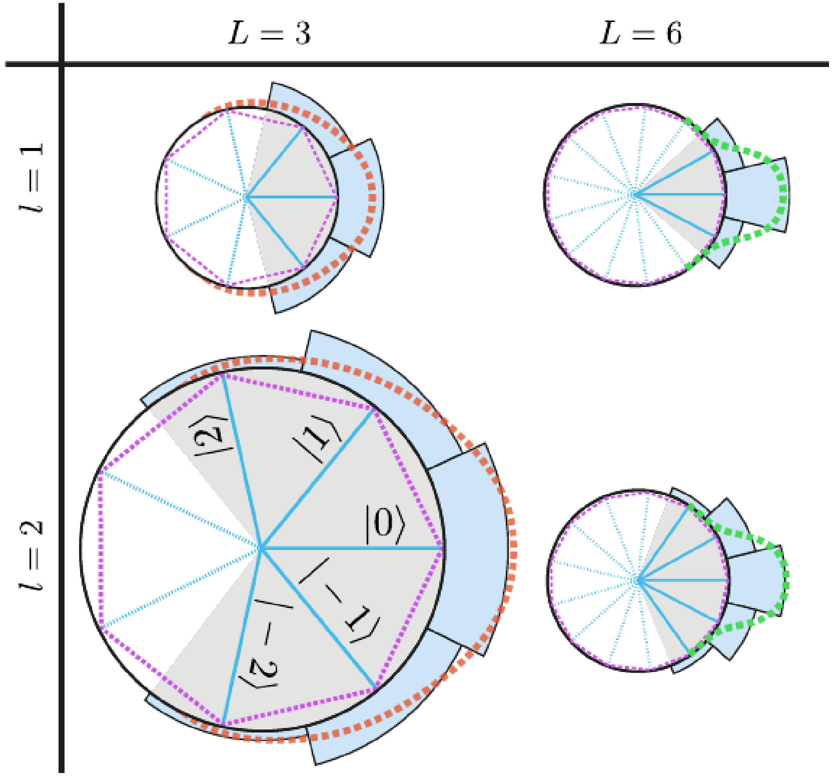
$$\hat{H}^{(e)}_B = -\frac{1}{2g^2a^2}\left[\hat{P}_1 + \hat{P}_2 + \hat{P}_3 + \hat{P}_1\hat{P}_2\hat{P}_3 + \text{H.c.}\right]$$

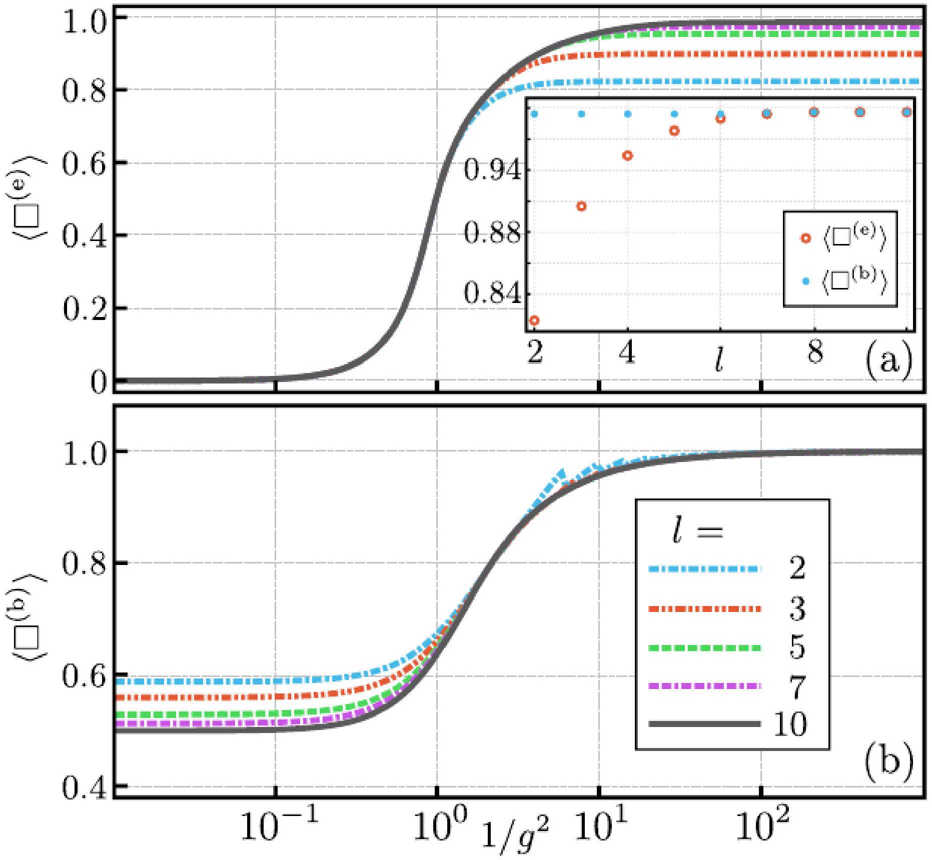
$$\langle\square\rangle = -\frac{g^2a^2}{V}\langle\Psi_0|\hat{H}_B|\Psi_0\rangle$$

J.F. Haase*, L. Dellantonio*, **AC***, D. Paulson, A. Kan, K. Jansen, C.A. Muschik, Quantum **5**, 393 (2021)

Efficient representation at *all couplings*

$L = L(\beta)$ variational parameter for optimizing resources





Varying *L* allows to avoid freezing while minimizing the truncation

J.F. Haase*, L. Dellantonio*, **AC***, D. Paulson, A. Kan, K. Jansen, C.A. Muschik, Quantum 5, 393 (2021)

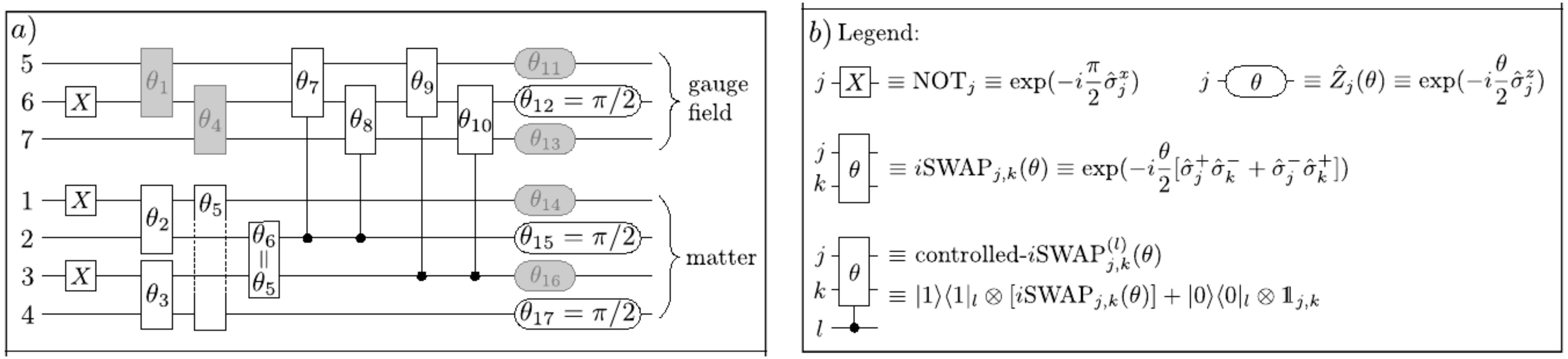
See also C. W. Bauer and D. M. Grabowska, Phys. Rev. D **107**, L031503 (2023)

Variational simulations with trapped ions

Gauge dof: spin l = qudit ➡ $2l+1$ qubits (ions)

$$|-l+j\rangle = |\overbrace{0\ldots0}^{j}\,1\,\overbrace{0\ldots0}^{2l-j-1}\rangle \qquad \hat{E} = \frac{1}{2}\sum_{i=1}^{2l}\prod_{j=1}^{i}\sigma_j^z \qquad \hat{U} = \sum_{i=1}^{2l}\sigma_i^+\sigma_{i+1}^-$$

Variational preparation of the ground state convenient for long range interactions

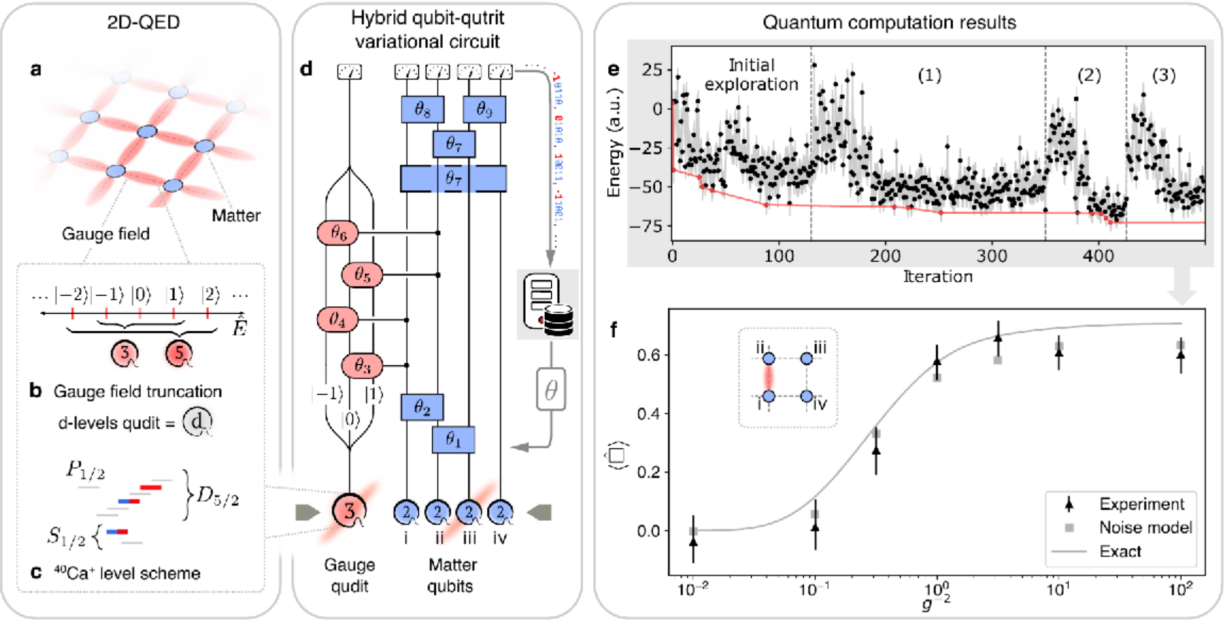


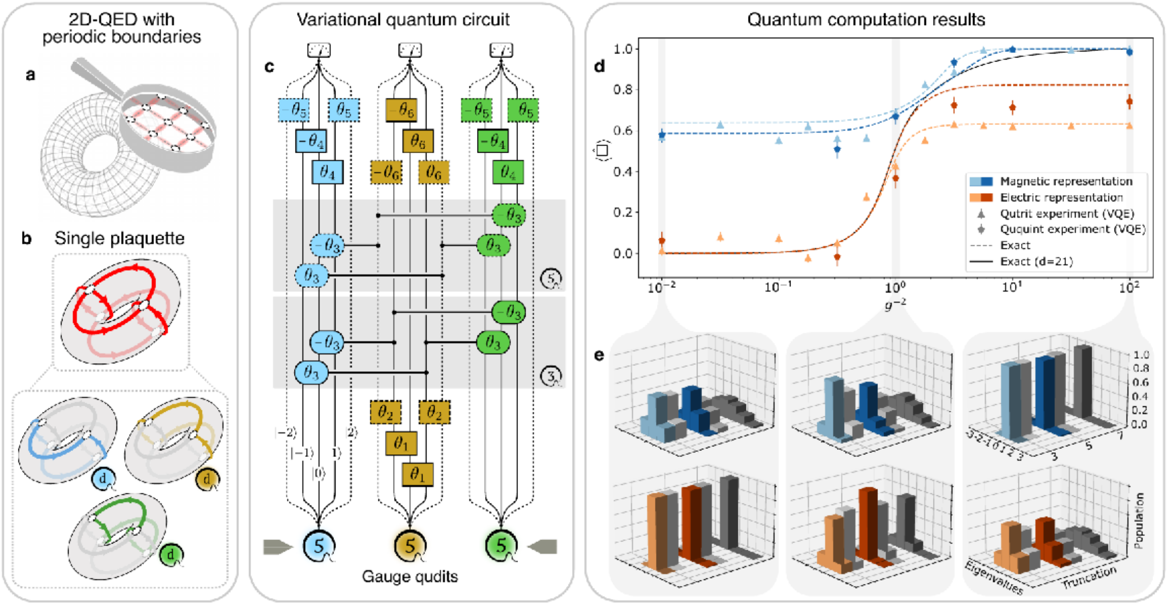
Feasible for moderate truncations in current trapped ions experiments!

D Paulson, L Dellantonio, JF Haase, **AC**,...PRX Quantum 2 (3), 030334 (2021)

Simulating 2D lattice gauge theories on a qudit quantum computer

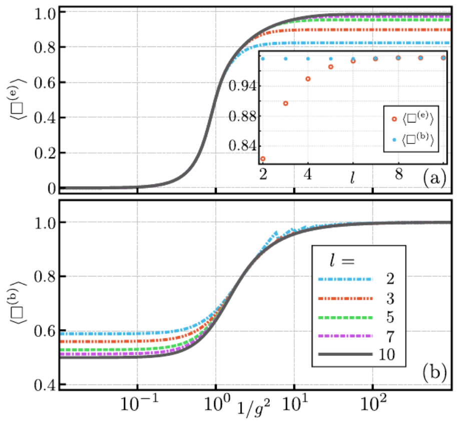
Michael Meth,[1] Jan F. Haase,[2,3,4] Jinglei Zhang,[2,3] Claire Edmunds,[1] Lukas Postler,[1] Andrew J. Jena,[2,3] Alex Steiner,[1] Luca Dellantonio,[2,3,5] Rainer Blatt,[1,6,7] Peter Zoller,[8,6] Thomas Monz,[1,7] Philipp Schindler,[1] Christine Muschik*,[2,3,9] and Martin Ringbauer*[1]





M. Meth *et al.*, *Nature Phys.* **21**, 570–576 (2025)

Running of the coupling



Encoding

+

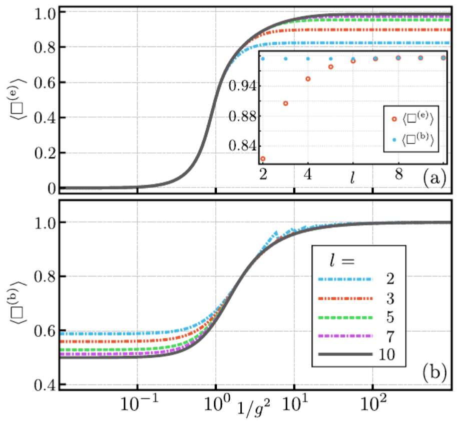
Coupling dependent basis

- 2D QED on small tori

F. Haase*, L. Dellantonio*, **AC***, D. Paulson, A. Kan, K. Jansen, C.A. Muschik, Quantum **5**, 393 (2021)

Open questions:

- Non-Abelian gauge theories? No equivalent of Z_N subgroups…
- Scalability: performance on larger tori/lattices?



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Coupling dependent basis

- 2D QED on small tori

F. Haase*, L. Dellantonio*, **AC***, D. Paulson, A. Kan, K. Jansen, C.A. Muschik, Quantum **5**, 393 (2021)

- 2D pure SU(2) on small tori

P. Fontana, M. Miranda-Riaza, and **AC**, PRX **15** (2025)

M. Miranda-Riaza

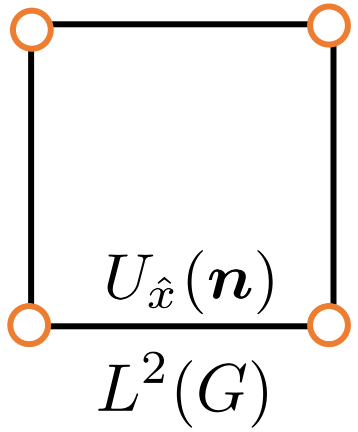
P. Fontana, Ph.D

- 2D QED on large/infinite lattices

M. Miranda-Riaza, P. Fontana and **AC**, PRD **113** (2026)/*in progress*

Renormalized dual basis

Optimal local basis through the single-plaquette Hamiltonian: *divide and conquer*



P. Fontana, M. Miranda-Riaza, and **AC**, PRX **15** (2025), M. Miranda-Riaza, P. Fontana and **AC**, PRD **113** (2026)

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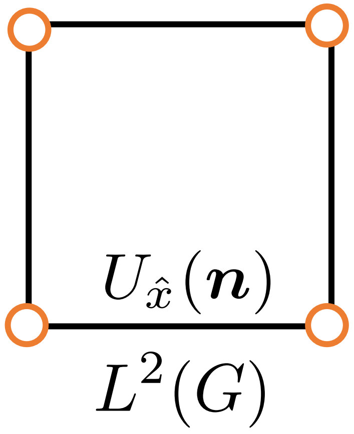

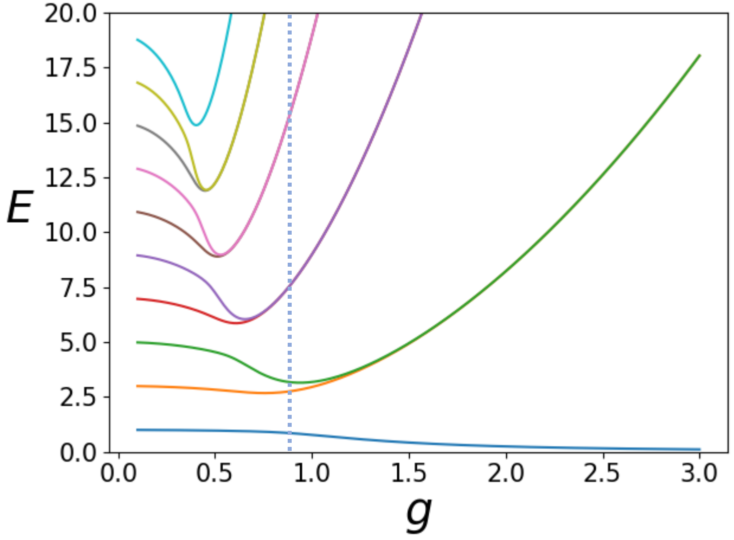

P. Fontana, M. Miranda-Riaza, and **AC**, PRX **15** (2025), M. Miranda-Riaza, P. Fontana and **AC**, PRD **113** (2026)

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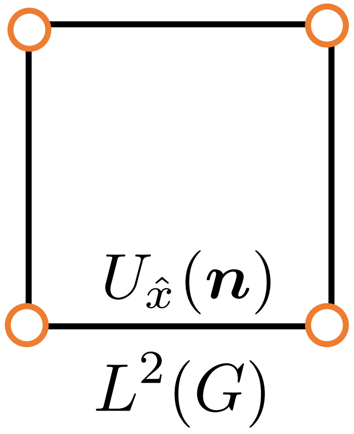

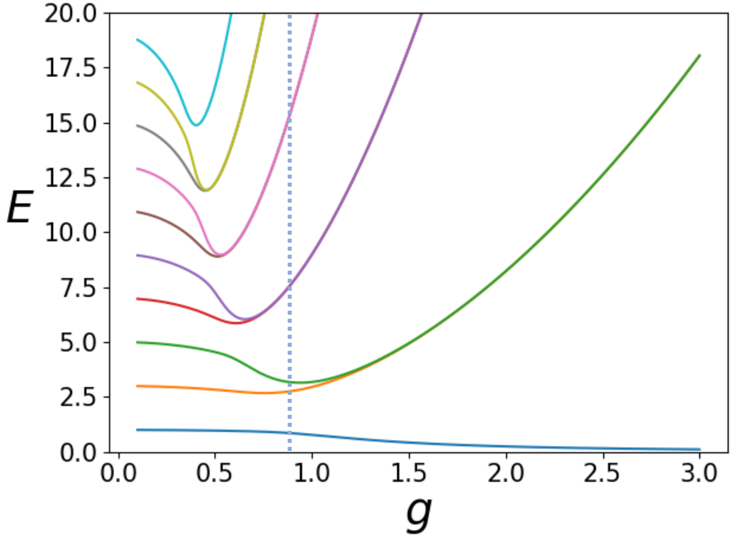

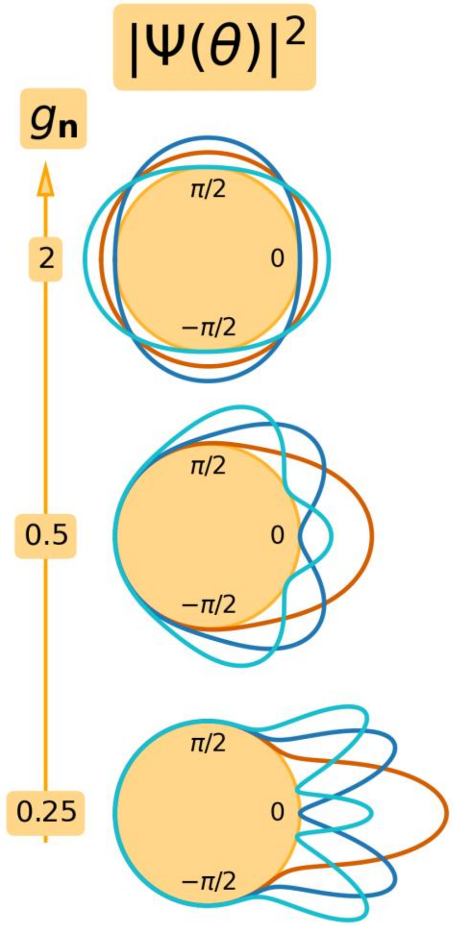

P. Fontana, M. Miranda-Riaza, and **AC**, PRX **15** (2025), M. Miranda-Riaza, P. Fontana and **AC**, PRD **113** (2026)

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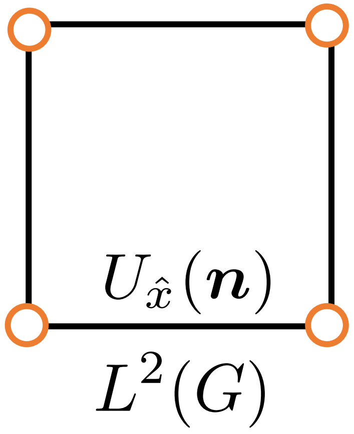
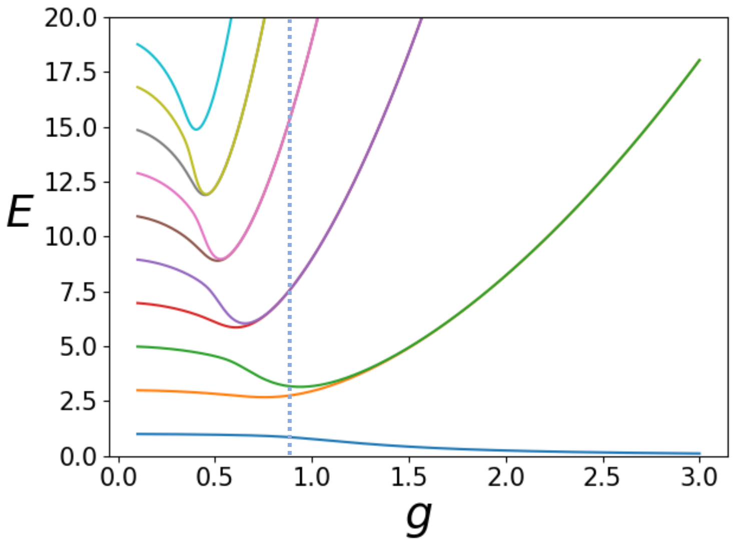
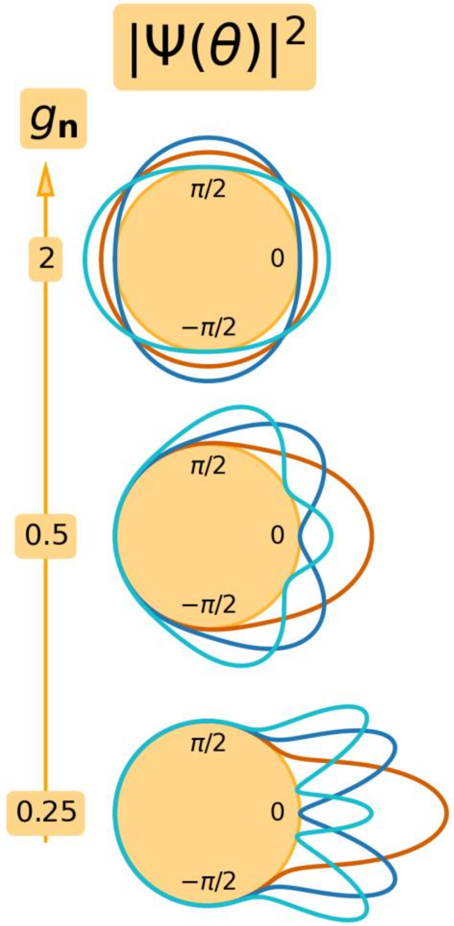
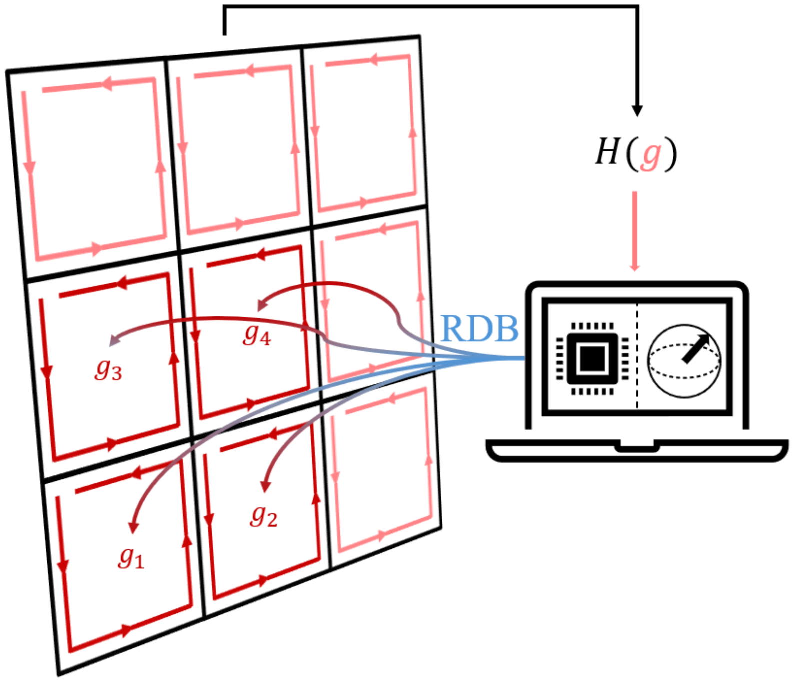


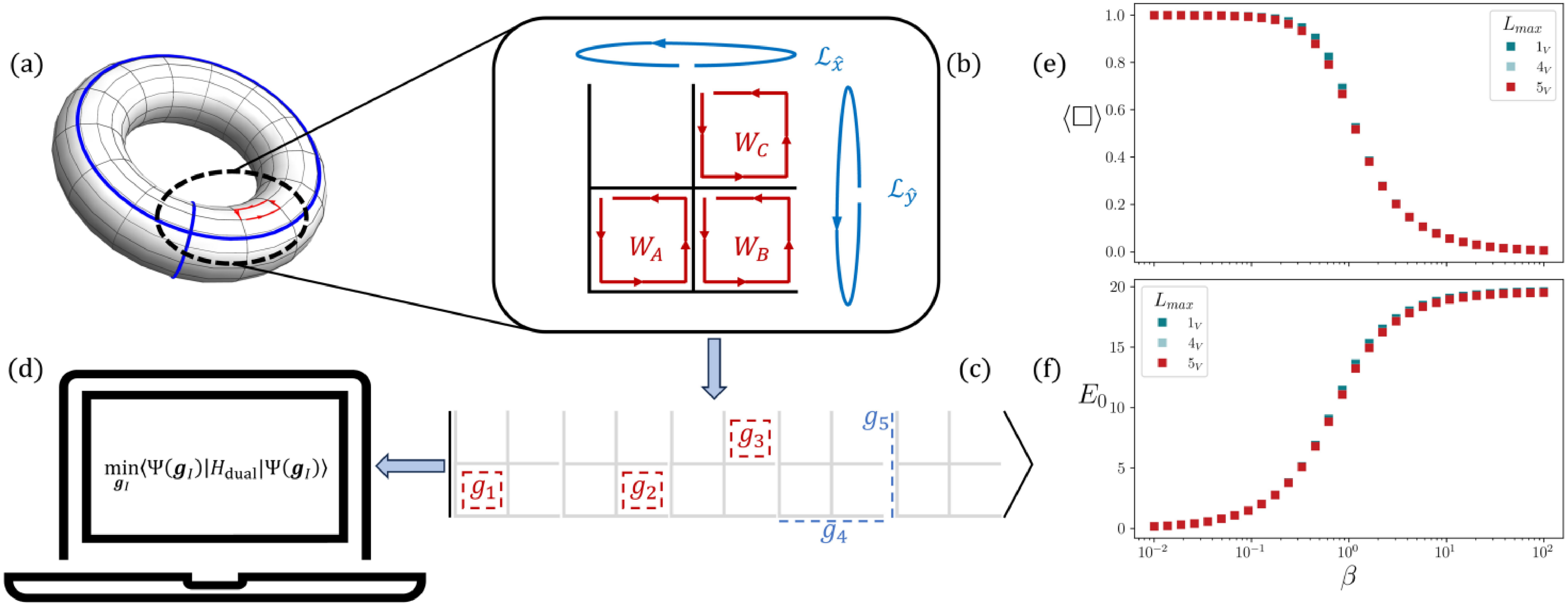






P. Fontana, M. Miranda-Riaza, and **AC**, PRX **15** (2025), M. Miranda-Riaza, P. Fontana and **AC**, PRD **113** (2026)

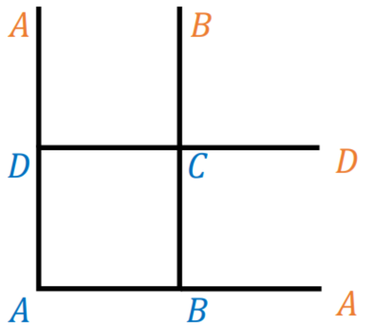
Renormalized dual basis for SU(2)



Variational optimization of states and Hamiltonian representation

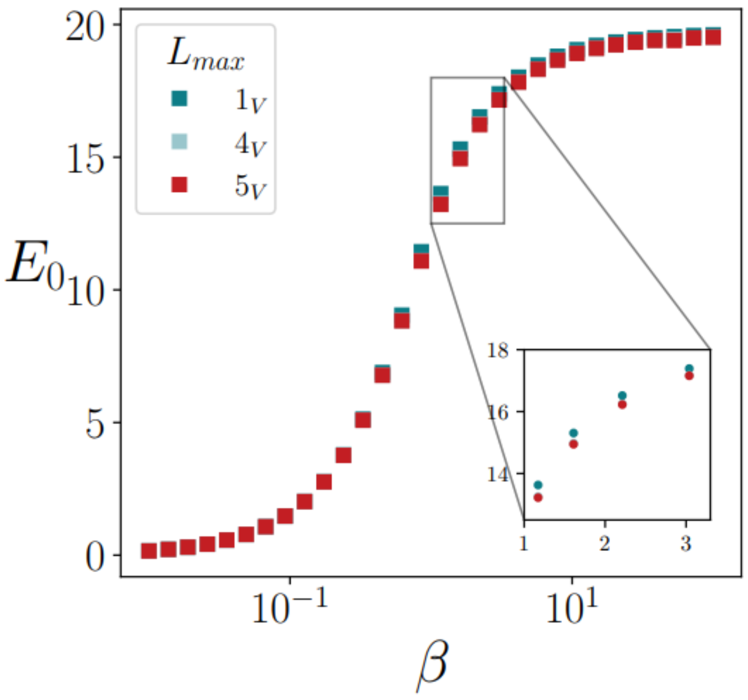
P. Fontana, M. Miranda-Riaza, and **AC**, PRX **15** (2025)

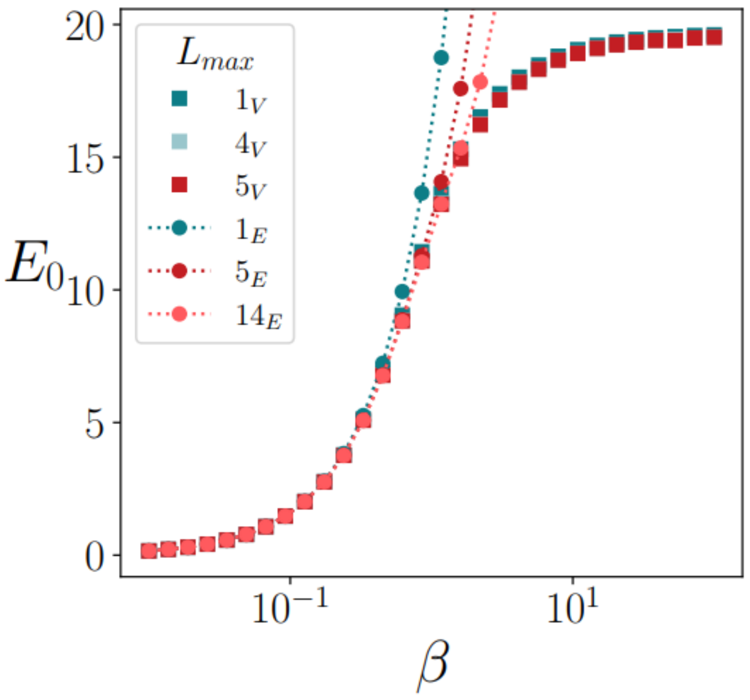
Renormalized dual basis for SU(2)



$$\beta \equiv (2g^2)^{-1}$$

$$\frac{\Delta E_0}{E_0} \equiv \frac{E_0(\boldsymbol{g_O}) - E_0(\boldsymbol{g_V})}{E_0(\boldsymbol{g_V})}$$





P. Fontana, M. Miranda-Riaza, and **AC**, PRX **15** (2025)

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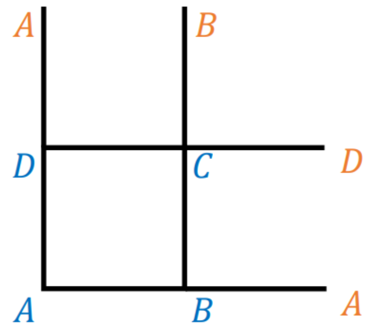
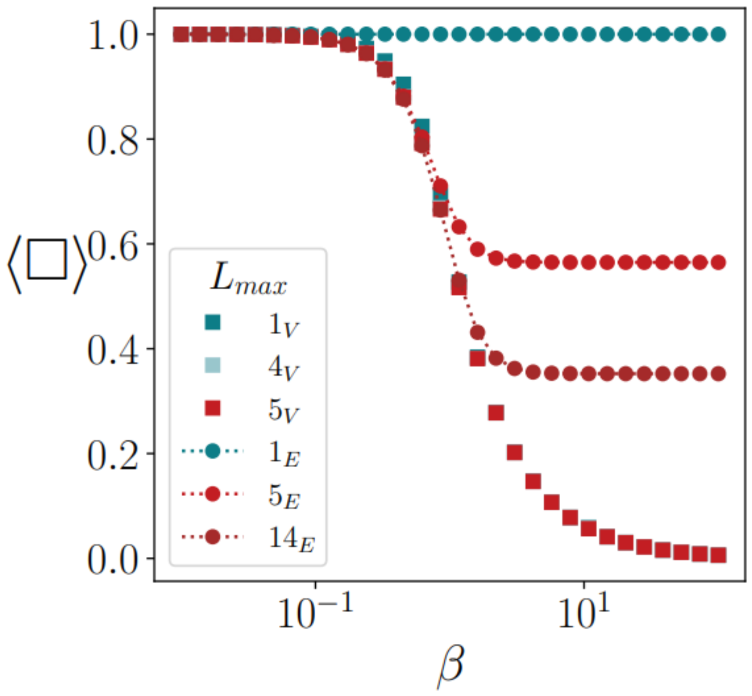


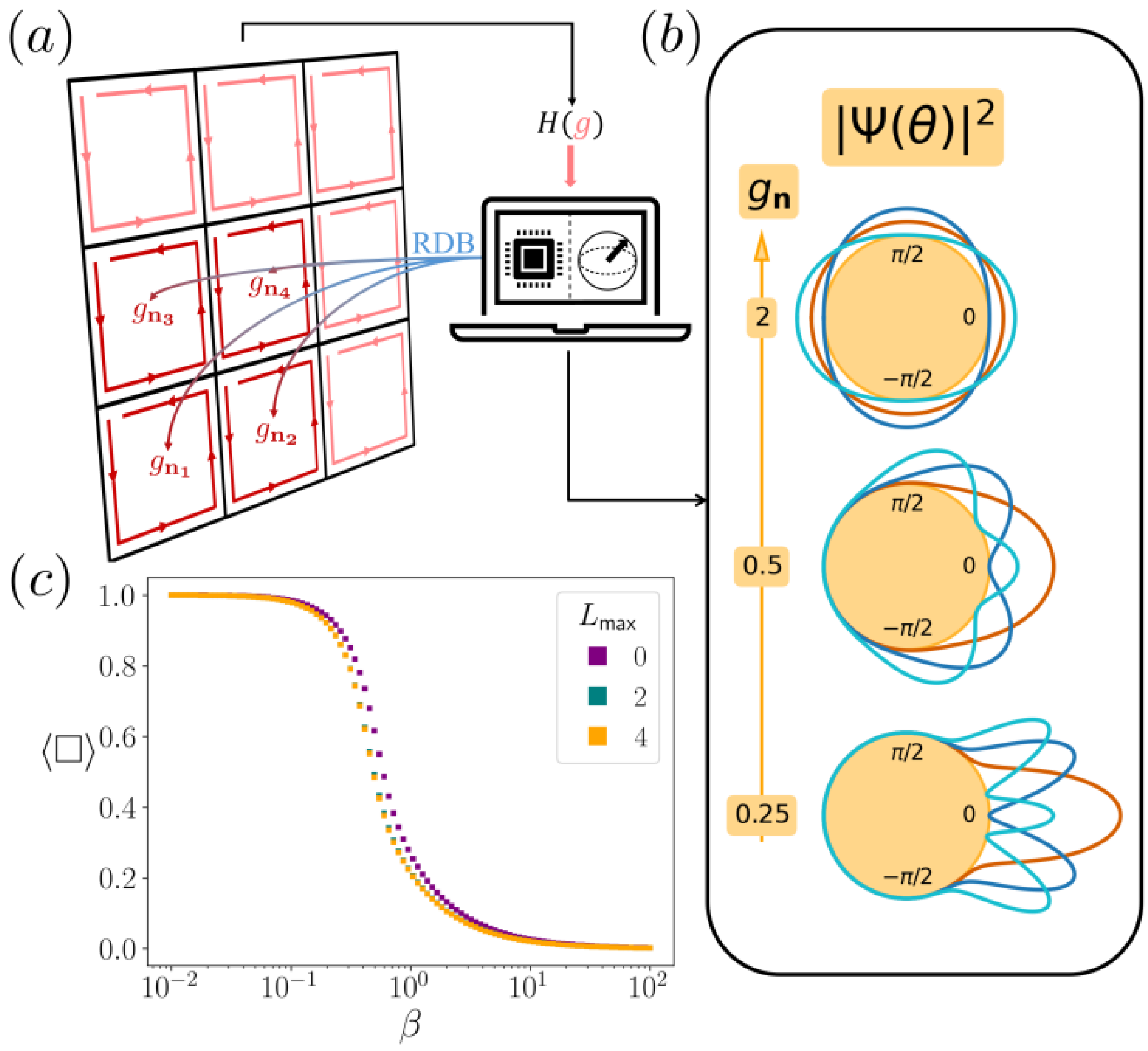
$$\beta \equiv (2g^2)^{-1}$$

$$\langle \square \rangle \equiv \frac{g^2}{2N_{\text{plaq}}} \langle \psi_0 | H_B | \psi_0 \rangle$$



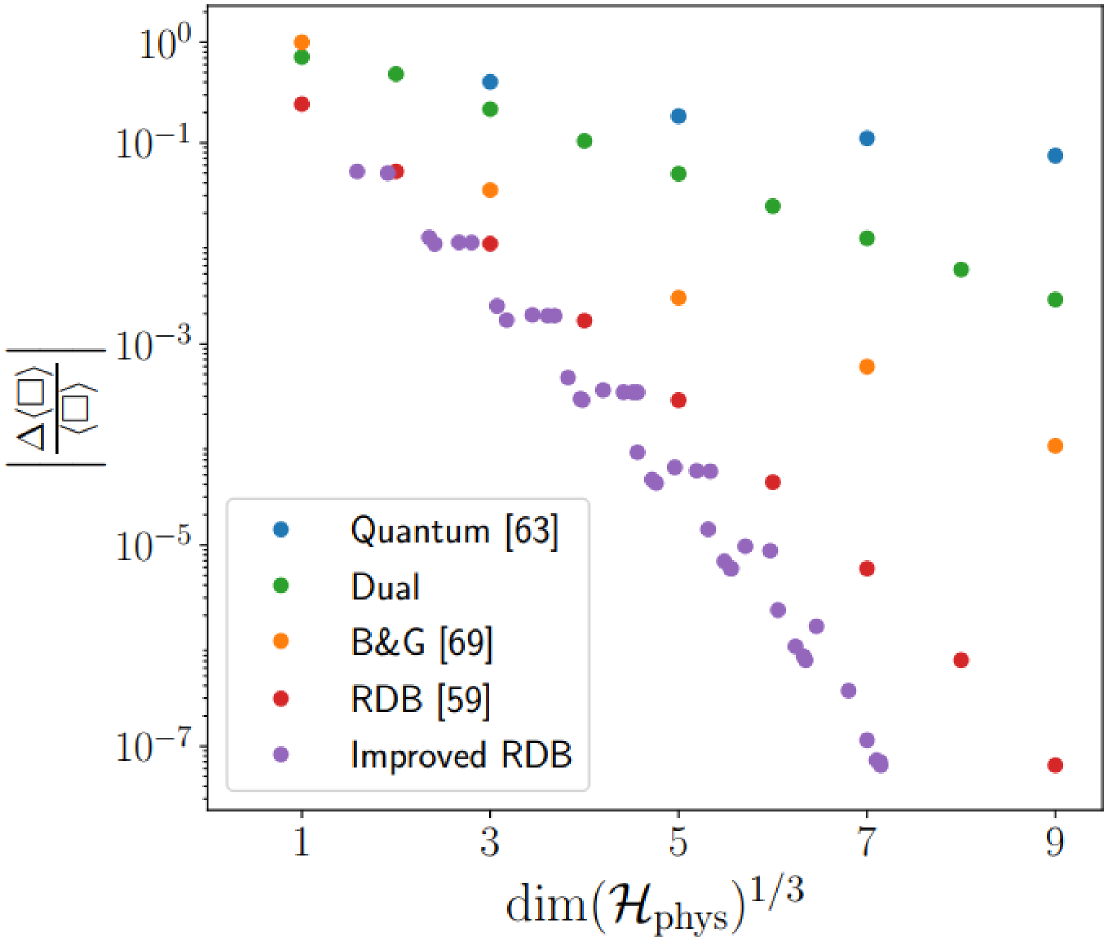
P. Fontana, M. Miranda-Riaza, and **AC**, PRX **15** (2025)

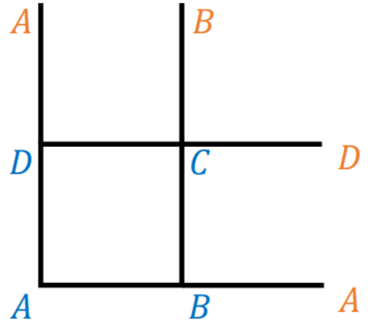
Renormalized dual basis for U(1)



M. Miranda-Riaza, P. Fontana and **AC**, PRD **113** (2026)

RDB for U(1): Comparison





$$\beta = (2g^2)^{-1} = 5 \qquad \langle\Box\rangle \equiv \frac{g^2}{2N_{\text{plaq}}}\langle\psi_0|H_B|\psi_0\rangle$$

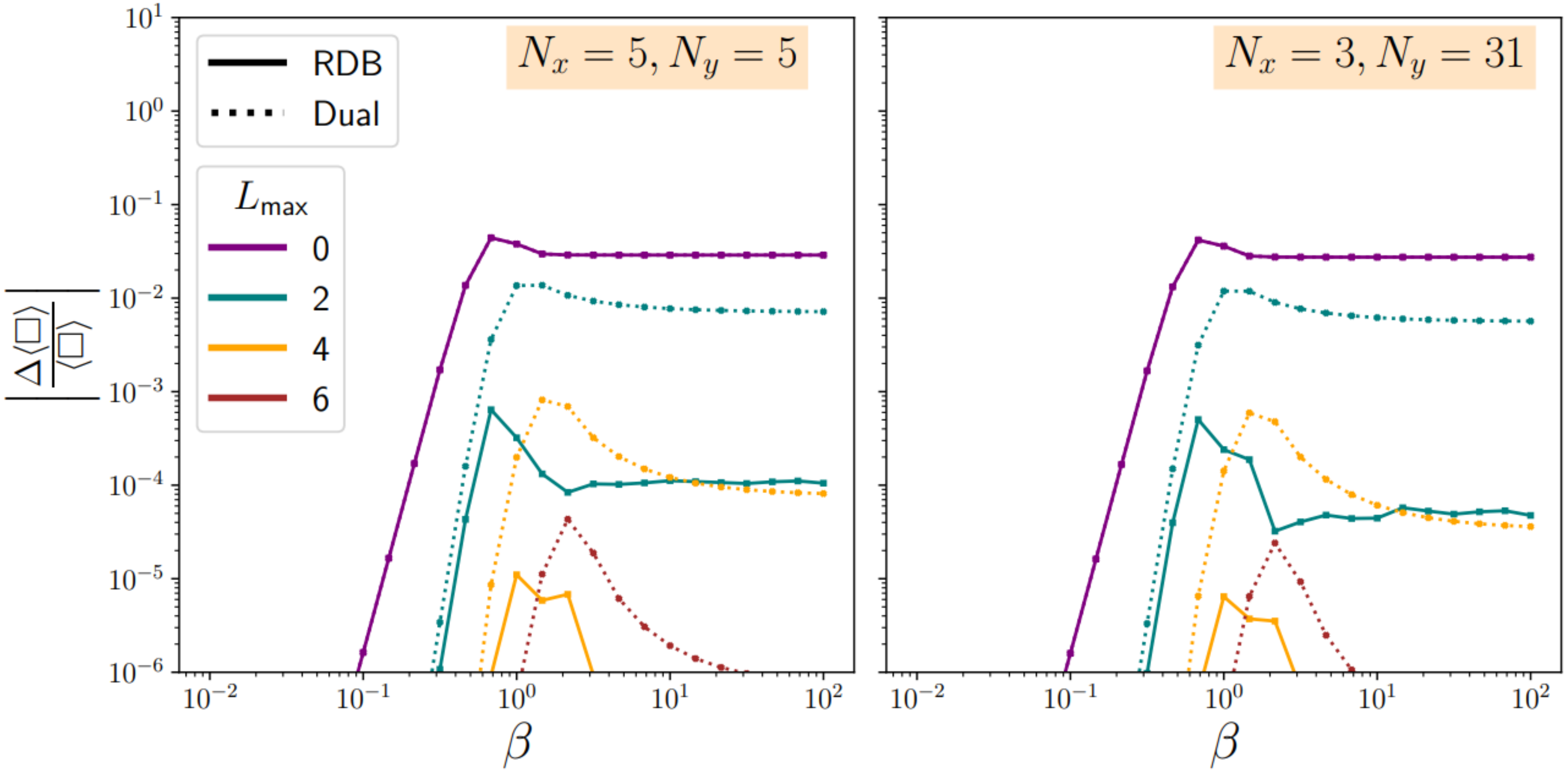
M. Miranda-Riaza, P. Fontana and **AC**, PRD **113** (2026)

RDB for U(1): Scaling to larger lattices

- ➢MPS with DMRG (TeNPy)
- ➢Square/ladder latices in OBC
- ➢Single variational parameter g

$$\langle \square \rangle \equiv \frac{g^2}{2N_{\text{plaq}}} \langle \psi_0 | H_B | \psi_0 \rangle$$

$$\beta \equiv (2g^2)^{-1}$$

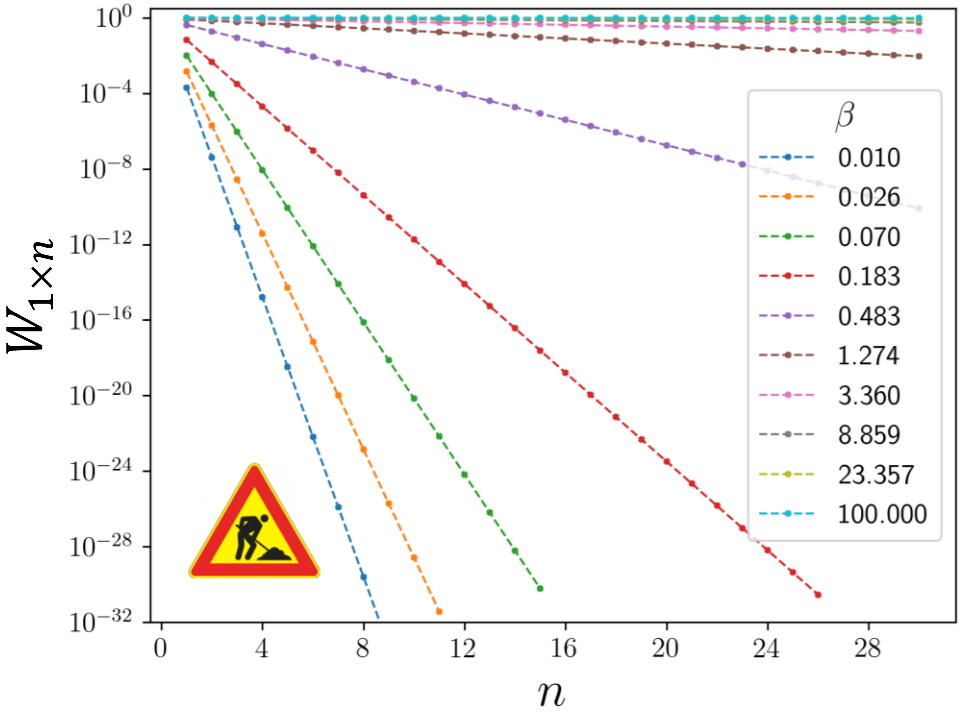


M. Miranda-Riaza, P. Fontana and **AC**, PRD **113** (2026)

RDB for U(1): ∞ lattices

➢VARiPEPS for infinite (2+1)D (E. Weerd and M. Rizzi, PRB **109, L**241117 (2024))

$$\beta \equiv (2g^2)^{-1}$$



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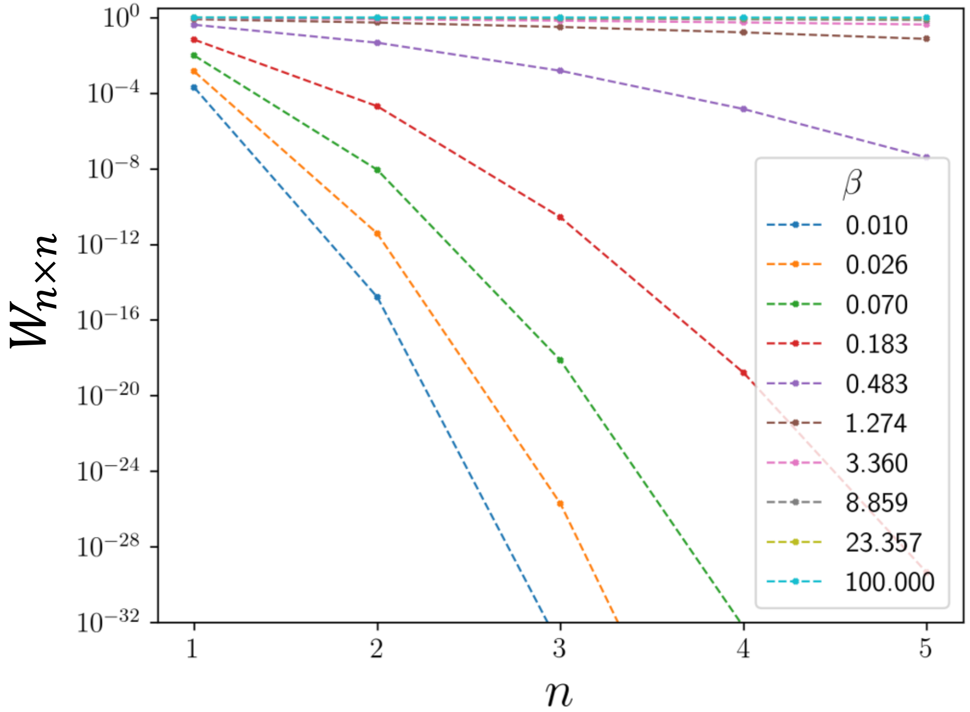
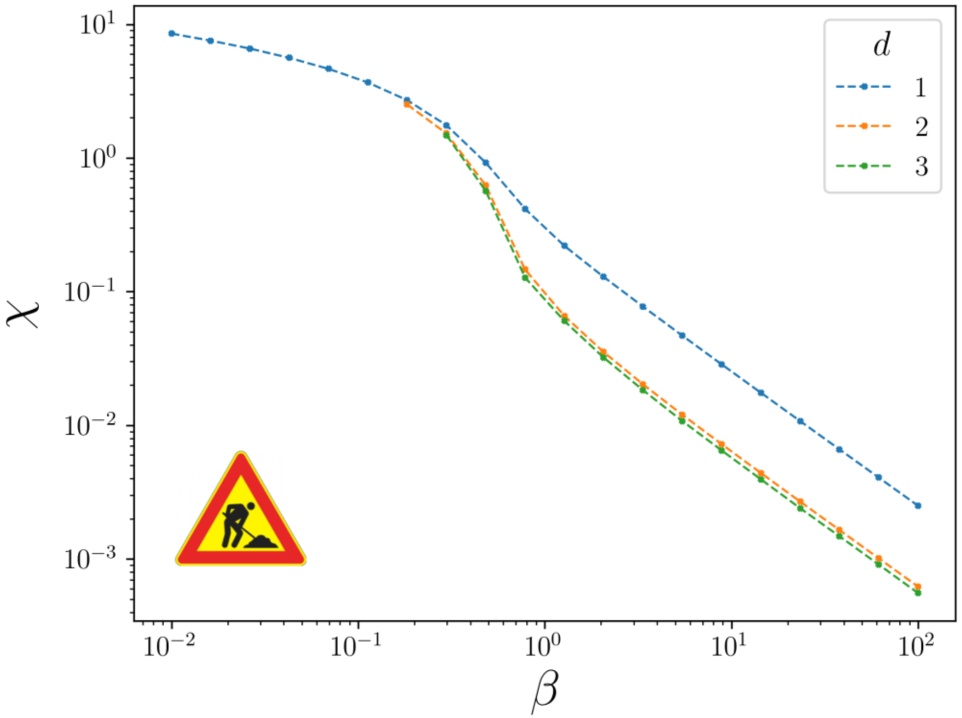




$$W_{n\times n} \simeq \exp[-\sigma IJ - b(I+J) - a]$$

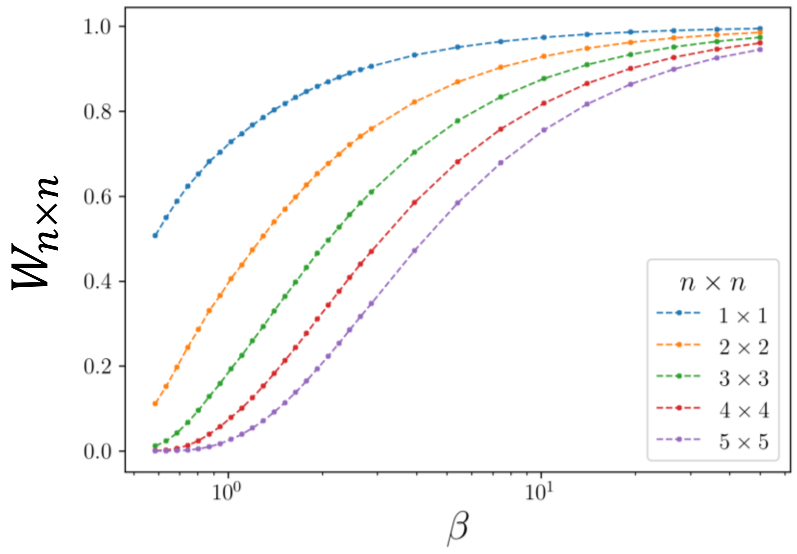
Creutz ratio $\chi_{i\times j} = -\ln \frac{W_{i\times j} W_{i-1\times j-1}}{W_{i\times j-1} W_{i-1\times j}} \to \sigma$

Wilson, Phys. Rev. D **10** (1974)

Creutz, Phys. Rev. Lett. **45** (1980)

Running of the coupling

Renormalization through the Wilson loops: $g = g(a)$



Creutz Phys Rev. D **23** (1981)

Running of the coupling

Renormalization through the Wilson loops: $g = g(a)$

Step scaling

$$F(g_0) = 1 - \frac{W(2,2)}{W(1,3)}$$

$$G(g_0) = 1 - \frac{W(4,4)}{W(2,6)}$$

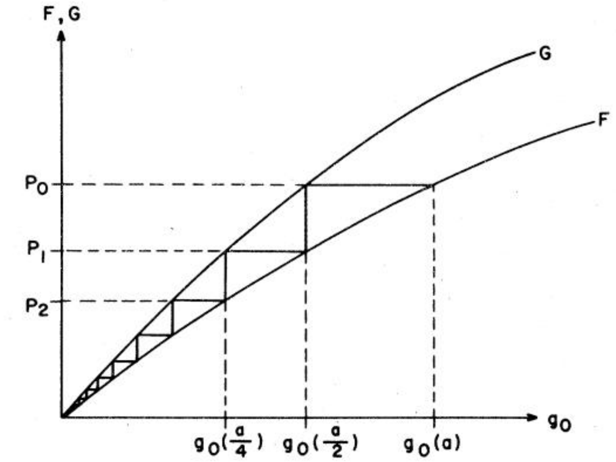


FIG. 1. The "staircase" construction of Sec. II for an asymptotically free theory.

Creutz Phys Rev. D **23** (1981)

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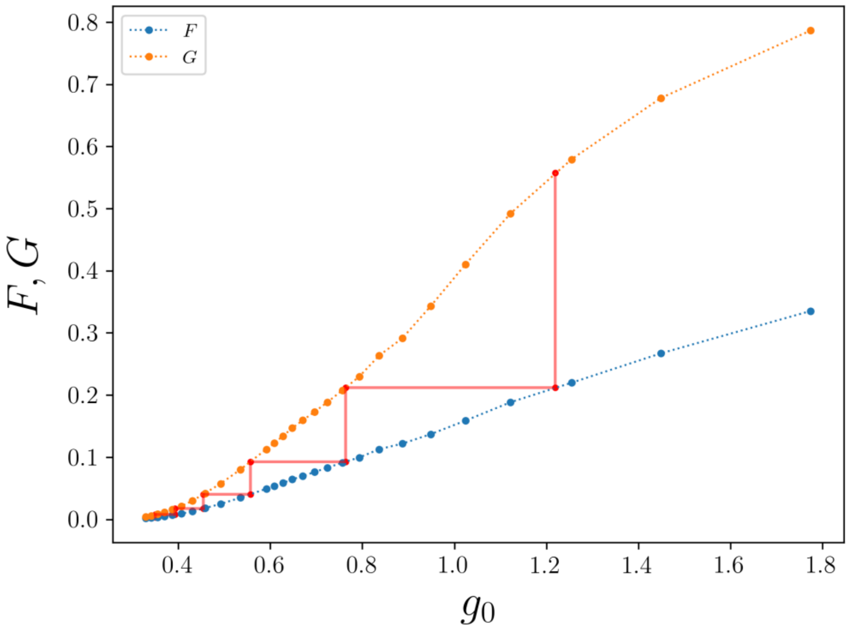

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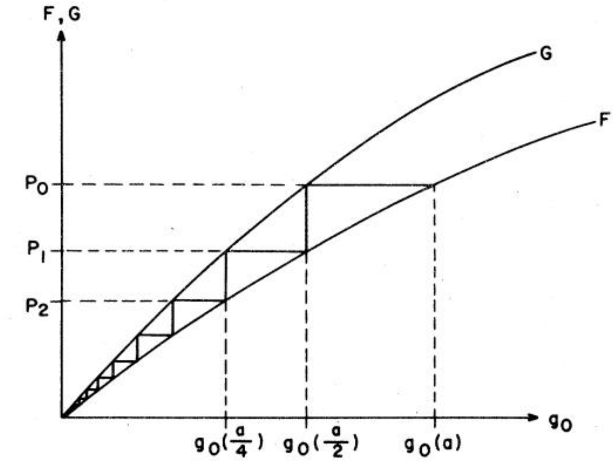

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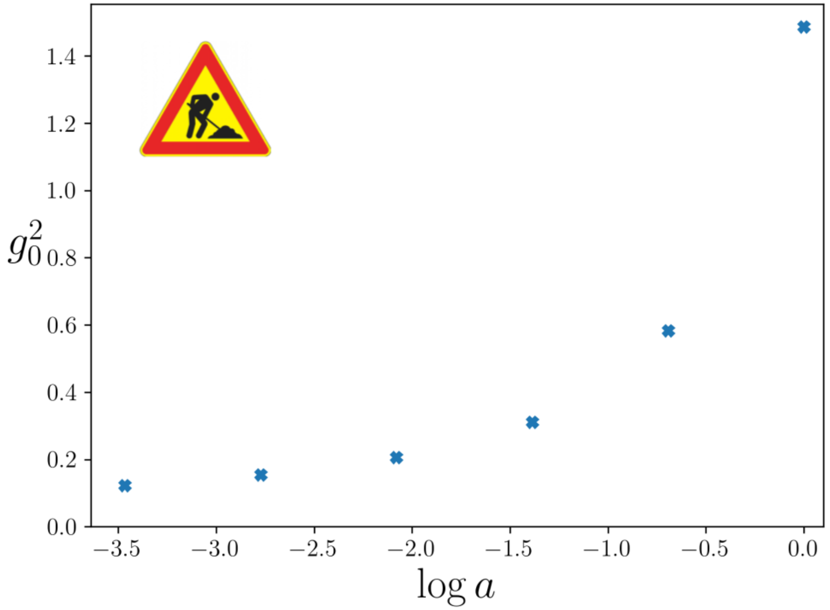
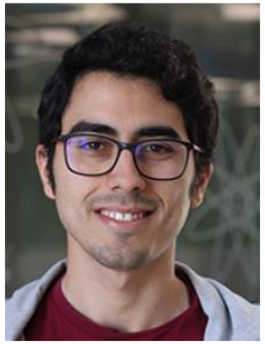


$d = 2 \quad \chi = d^2 \quad l_{max} = 4$

Creutz Phys Rev. D **23** (1981)

RDB Summary: Formulation matters!

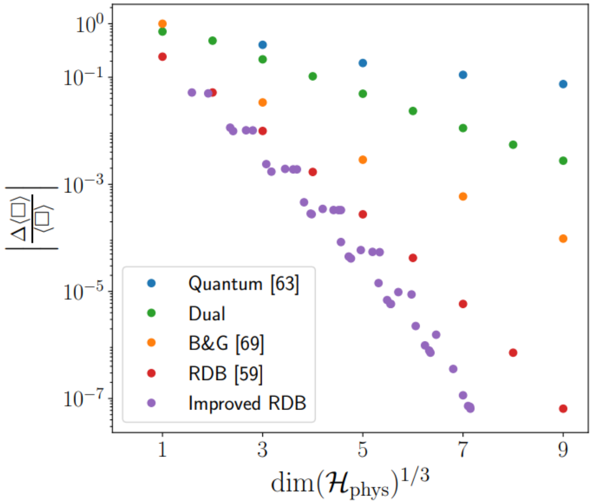


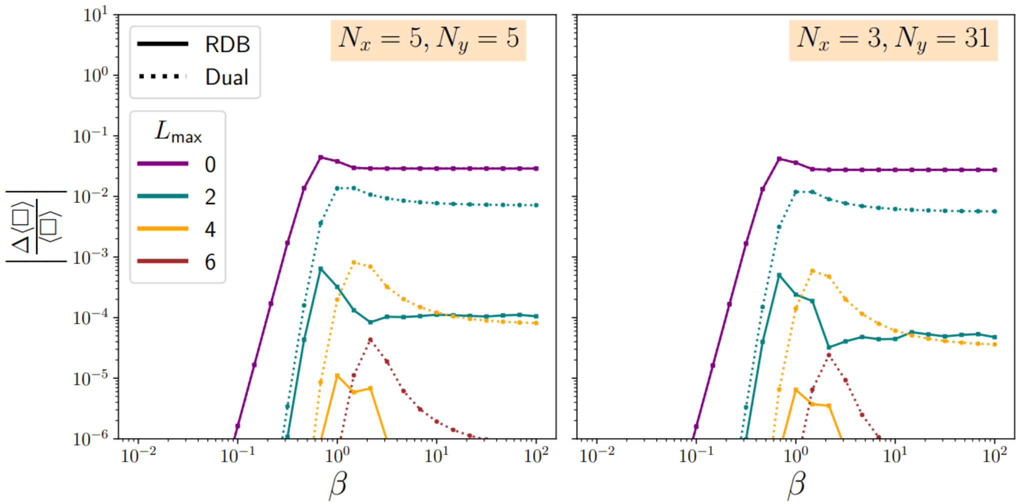
- **SU(2):** P. Fontana, M. Miranda-Riaza, and **AC**, PRX **15** (2025)

β	Standard truncation (electric basis)	Reformulation (interpolating basis)	Variational (interpolating basis)
0.01	1	1	1
1	2744	64	64
100	>2744	125	1

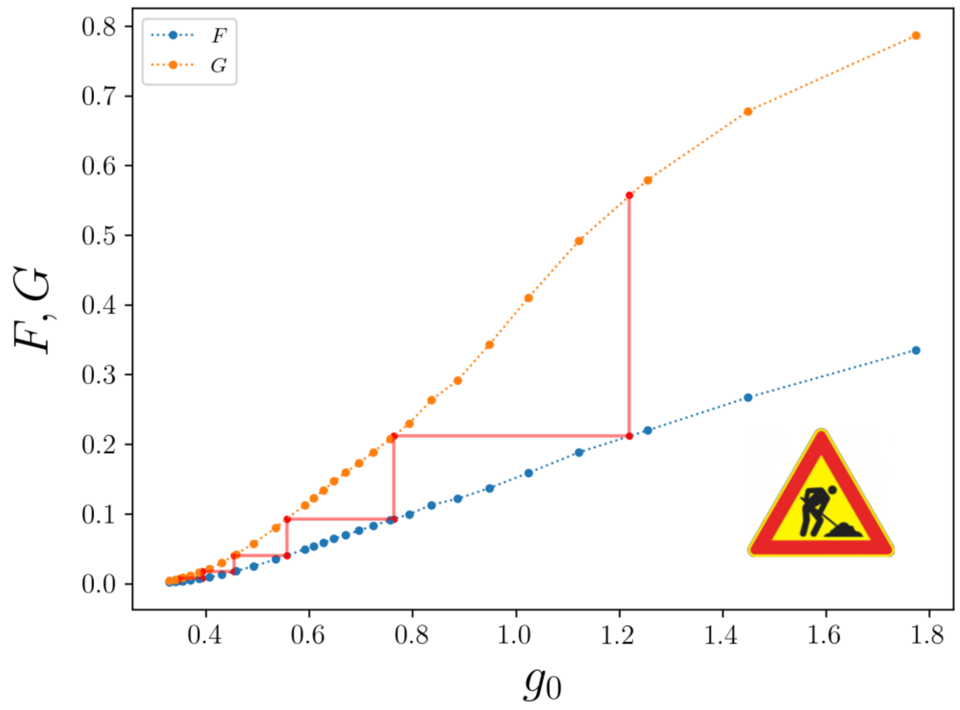
Total # of states to achieve <1% infidelity in the ground state of pure SU(2) on a minimal torus

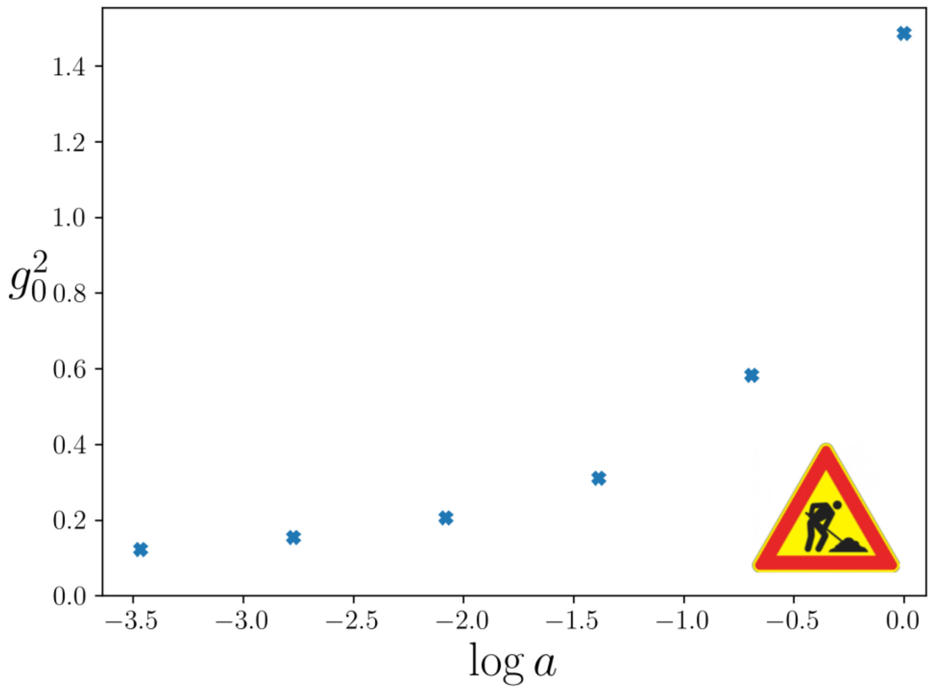
- **U(1):** M. Miranda-Riaza, P. Fontana and **AC**, PRD **113** (2026)





RDB Outlook: *ab-initio* continuum limit





Ongoing: Scaling the infrared cutoff d