Quantum Simulation of Meson Scattering in Lattice Gauge Theory

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Outline

- **Introduction to quantum simulation of particle scattering**

- **Fermion scattering:**

➢ Quantum circuit for fermion wave packet preparation (Y. Chai et al., Quantum 9, 1638 (2025))

➢ Circuit compression and hardware run with 40~80 qubits (Y. Chai et al., (arXiv:2507.17832, accepted by npj QI))

- **Meson scattering** (Y. Chai, Y. Guo, and S. Kuhn, (arXiv:2505.21240))

- **Ongoing projects:**

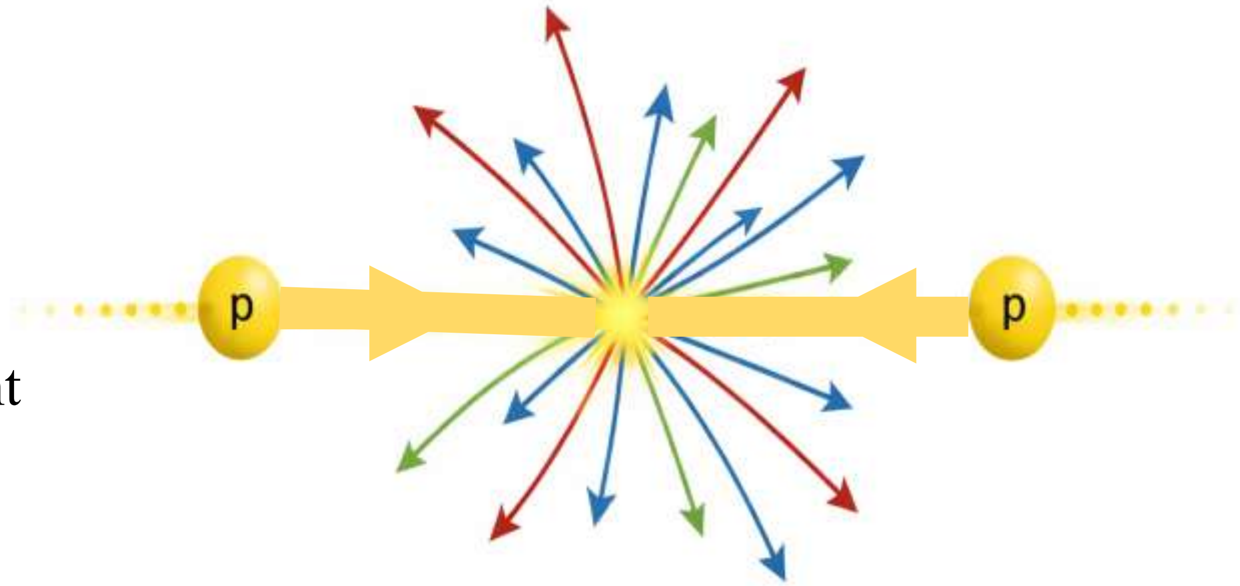
➢ Construct wave packet in OBC

➢ Resonance in scattering

Motivation of Quantum Simulation of Scattering

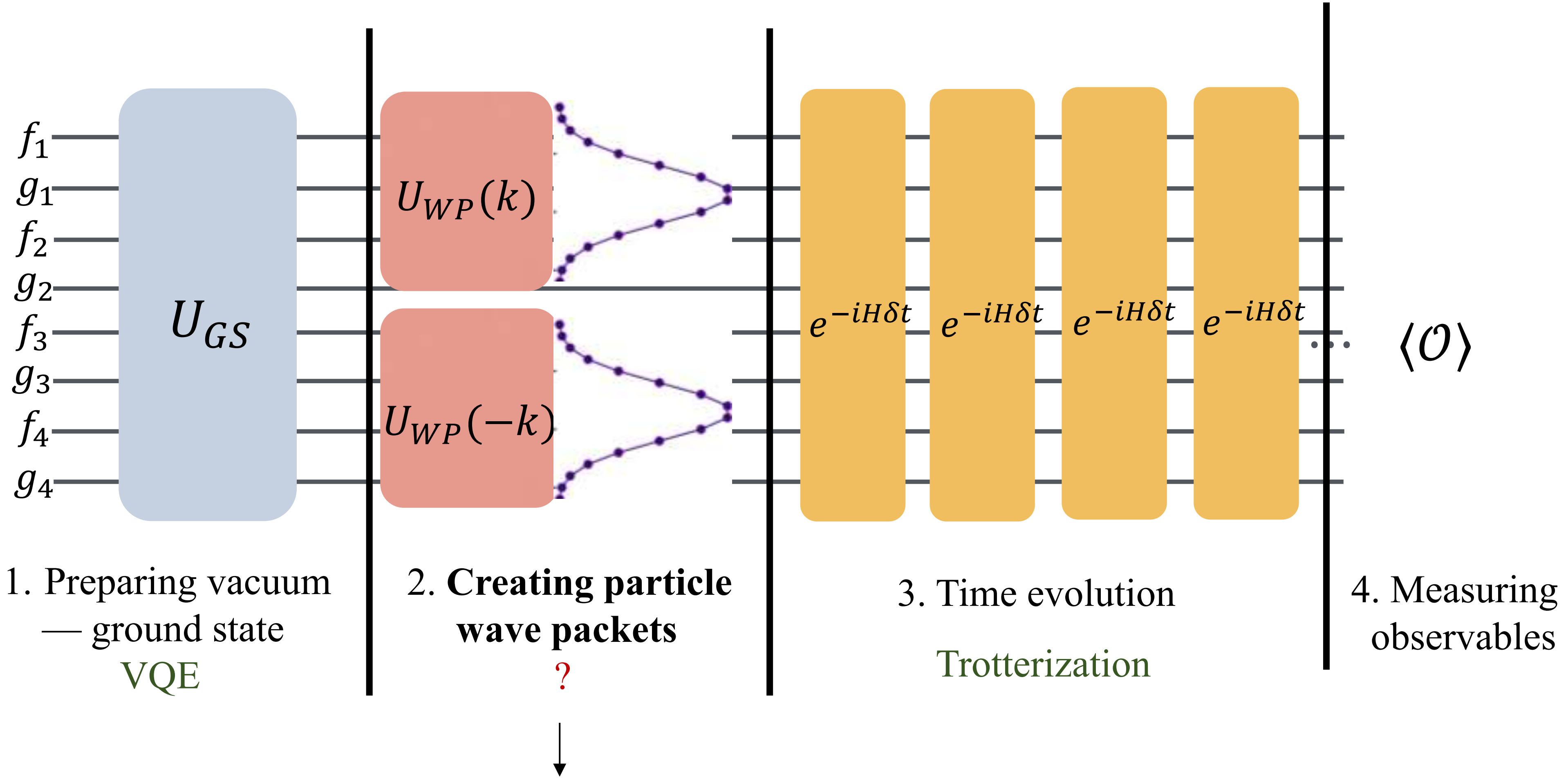
- Collider experiments are essentially particle scattering processes

- Real time dynamics in classical simulation
    - Conventional Monte Carlo: sign problem
    - Tensor Networks : limited by entanglement



- Quantum computers promise to efficiently simulate real-time dynamics

Scattering Process on Quantum Computing



Complicated non-unitary operator

Fermion wave packet preparation is non-unitary

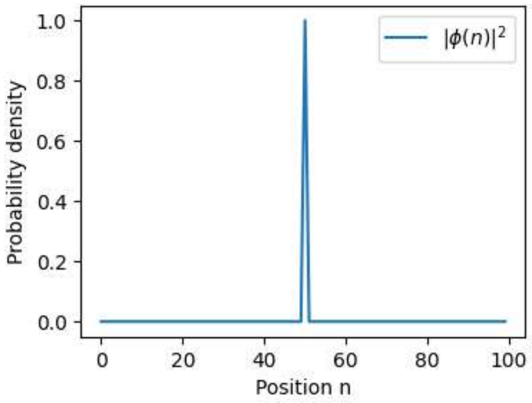
1. Vacuum state $|\Omega\rangle$: ground state of Hamiltonian — VQE or Tensor Networks

2. Initial state $|\psi(0)\rangle = C^\dagger(\phi^c) D^\dagger(\phi^d)|\Omega\rangle$: wave packets of particles

Fermion wave packet: $C^\dagger(\phi^c) = \sum_n \phi^c(n)\, \xi_n^\dagger = \sum_k \phi^c(k)\, \xi_k^\dagger$

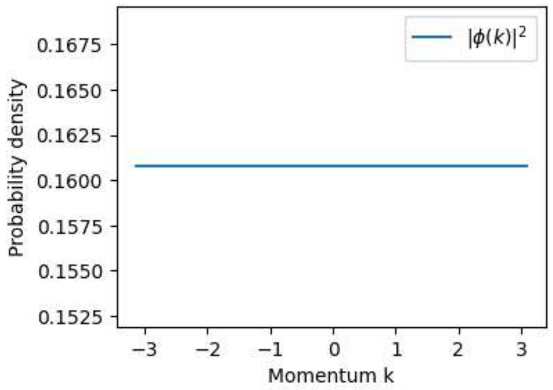
Position space

$$\phi_n = \delta_{n,\bar{n}}$$



Momentum space

$$\phi(k) = 1/\sqrt{2\pi} \cdot e^{-ik\bar{n}}$$



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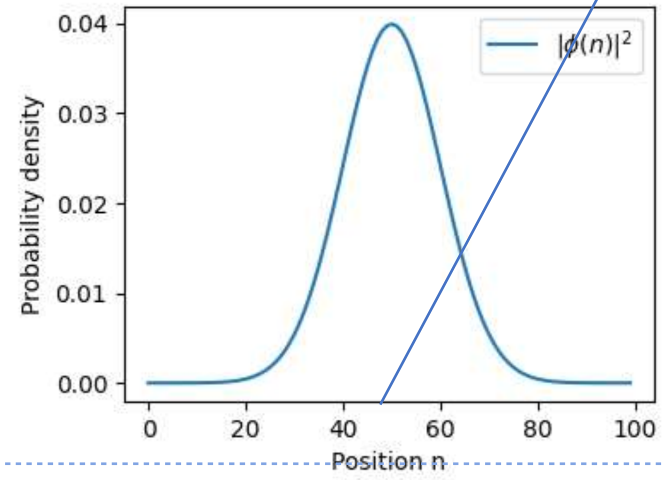
Position space

$$\phi(n) = 1/\mathcal{N}_x \cdot e^{i\bar{k}n} e^{-(n-\bar{n})/\sigma_n^2}$$



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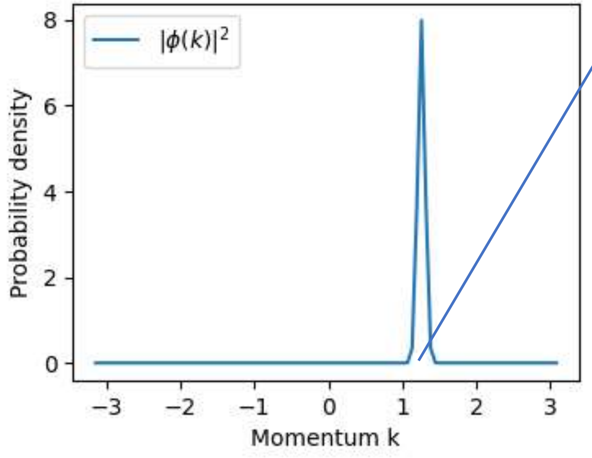
$$\phi(k) = 1/\mathcal{N}_k \cdot e^{-ik\bar{n}} e^{(k-\bar{k})/\sigma_k^2}$$



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Challenges

1. The particle creation operator is non-unitary → No direct quantum circuit

2. A wave packet requires a linear combination of operators → More complicated quantum circuit

Exact circuit decomposition for fermion wave packet

1. Vacuum state $|\Omega\rangle$: ground state of Hamiltonian — VQE or Tensor Networks

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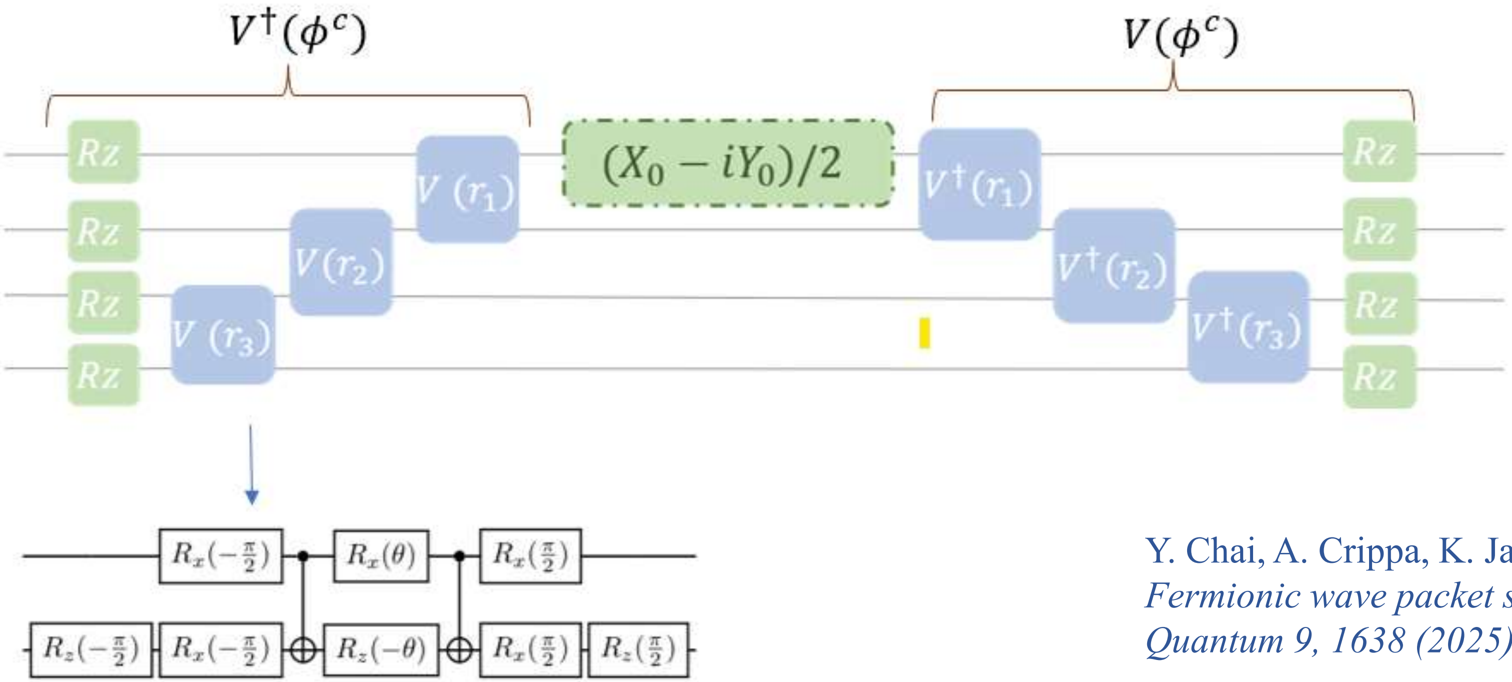
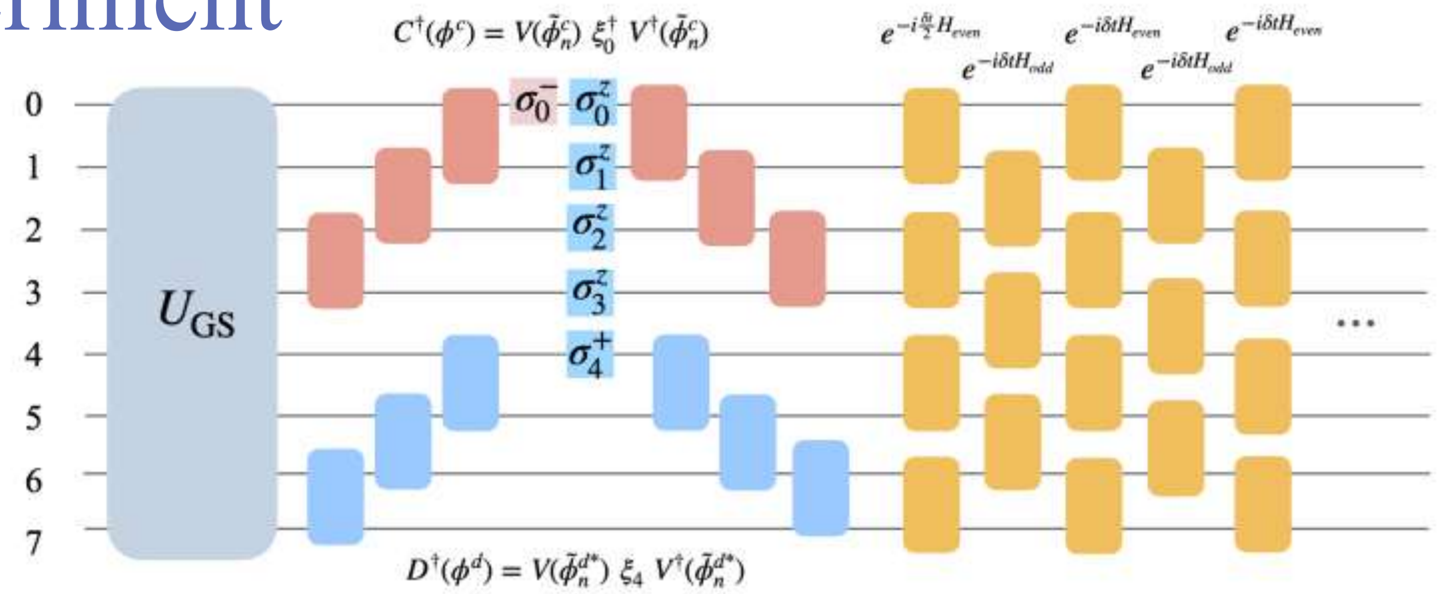


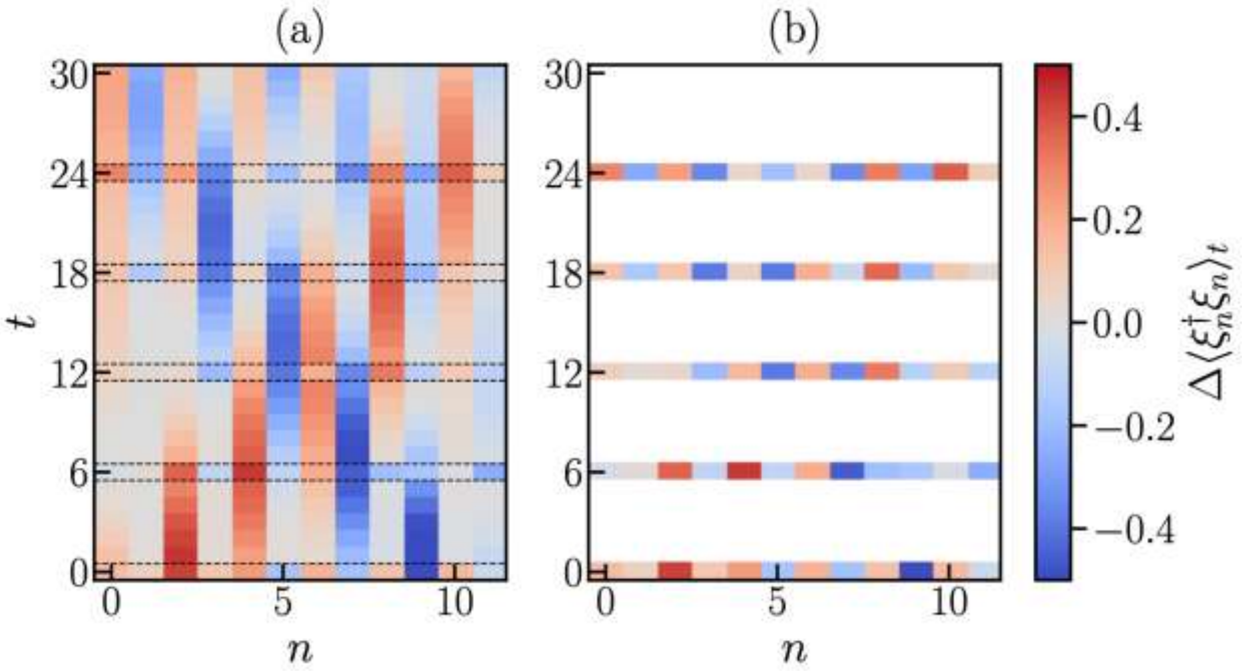
$$C^{\dagger}(\phi^{c}) = V(\phi^{c})\, \xi_{0}^{\dagger}\, V^{\dagger}(\phi^{c})$$

Y. Chai, A. Crippa, K. Jansen, S. Kühn, V. R. Pascuzzi, F. Tacchino, and I. Tavernelli, *Fermionic wave packet scattering: a quantum computing approach*, *Quantum 9, 1638 (2025).*

Fermion scattering experiment

The full quantum circuit for fermion scattering →



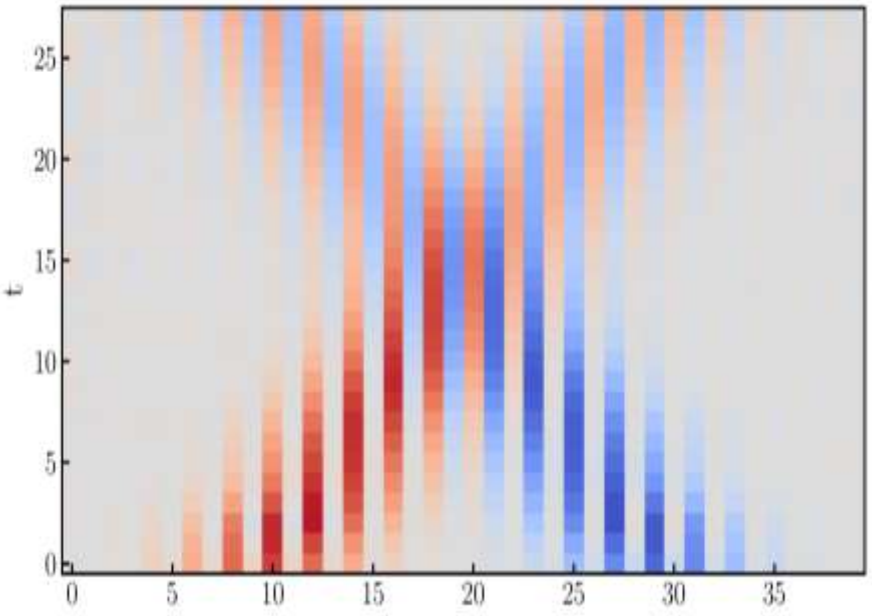


← Hardware run of 12 qubits on ibmq_peekskill

Free fermions:

$$H = -\frac{i}{2a}\sum_n\left(\xi_n^\dagger\,\xi_{n+1} - h.c.\right) + m\sum_n(-1)^n\xi_n^\dagger\xi_n$$

Interacting fermion scattering



Two initial wave packets should be separated

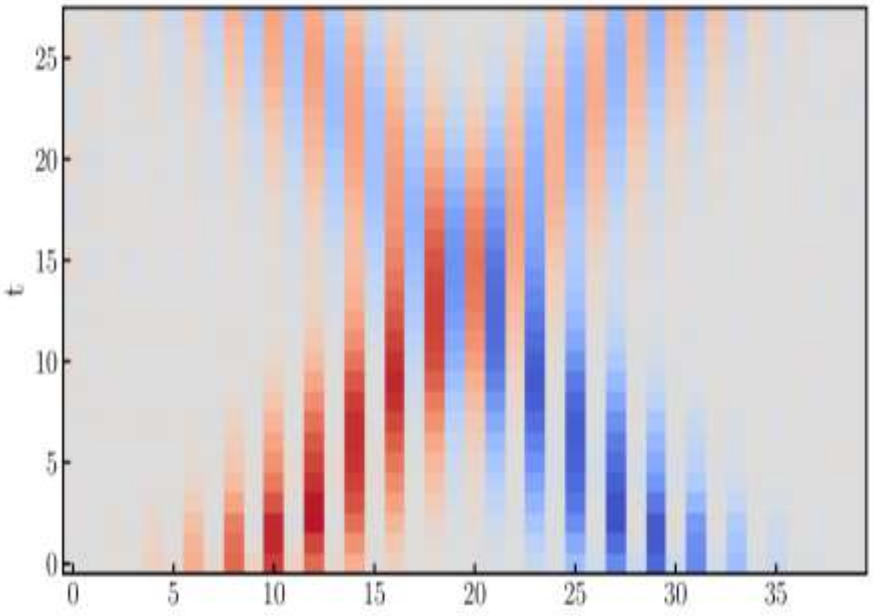
Outgoing particles should be separated

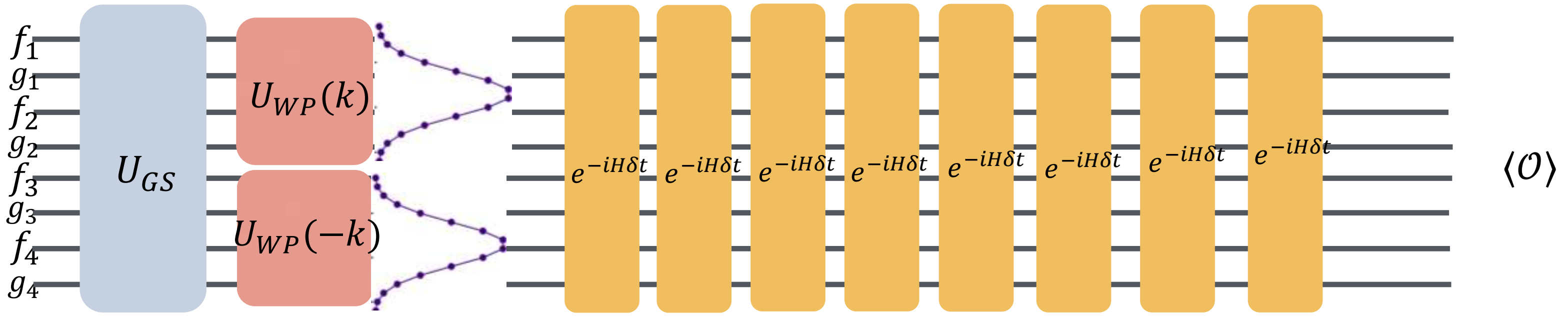
Long time dynamics, deep quantum circuit

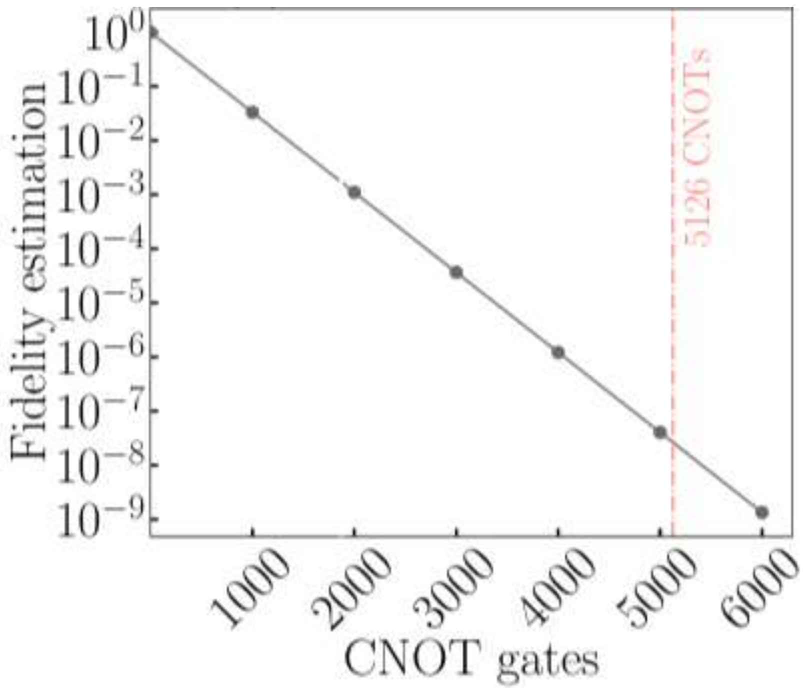
- Interacting fermions: Thirring model

$$H = -\frac{i}{2a}\sum_n \left(\xi_n^\dagger \xi_{n+1} - h.c.\right) + m\sum_n (-1)^n \xi_n^\dagger \xi_n + \frac{g}{a}\sum_n \xi_n^\dagger \xi_n \xi_{n+1}^\dagger \xi_{n+1}$$

Interacting fermion scattering







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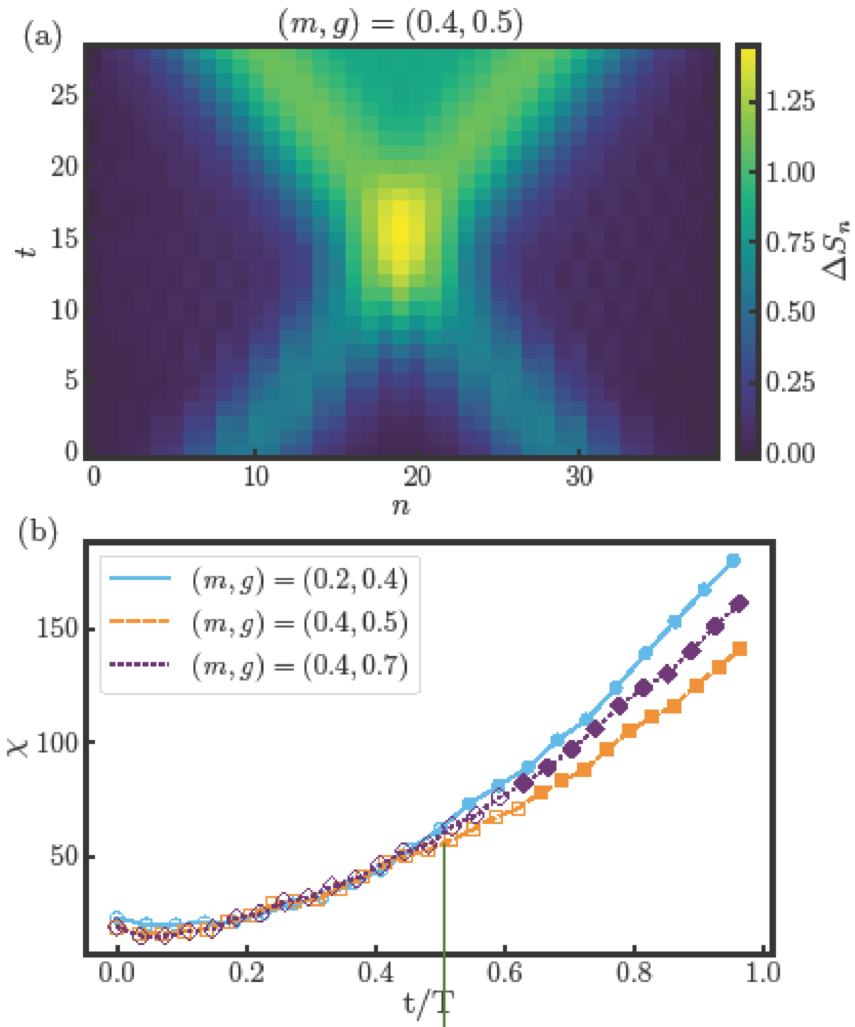
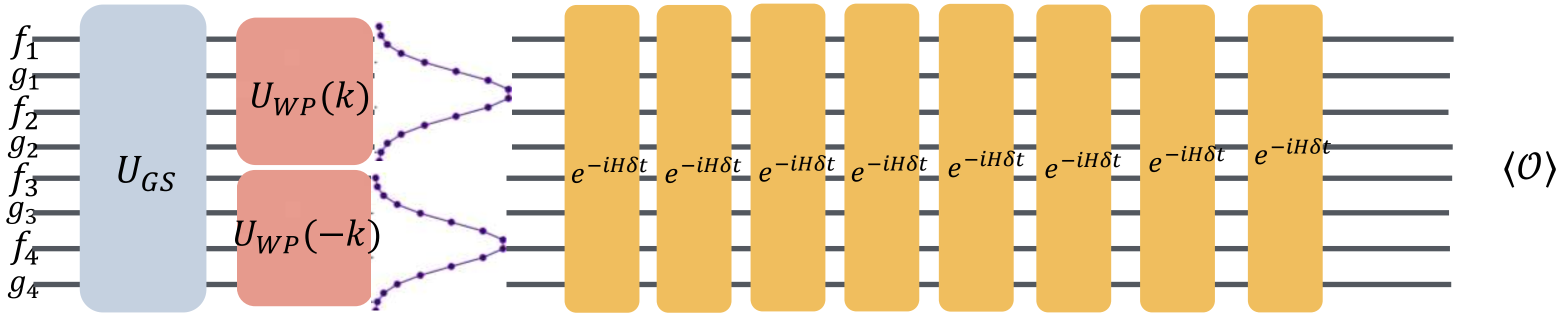
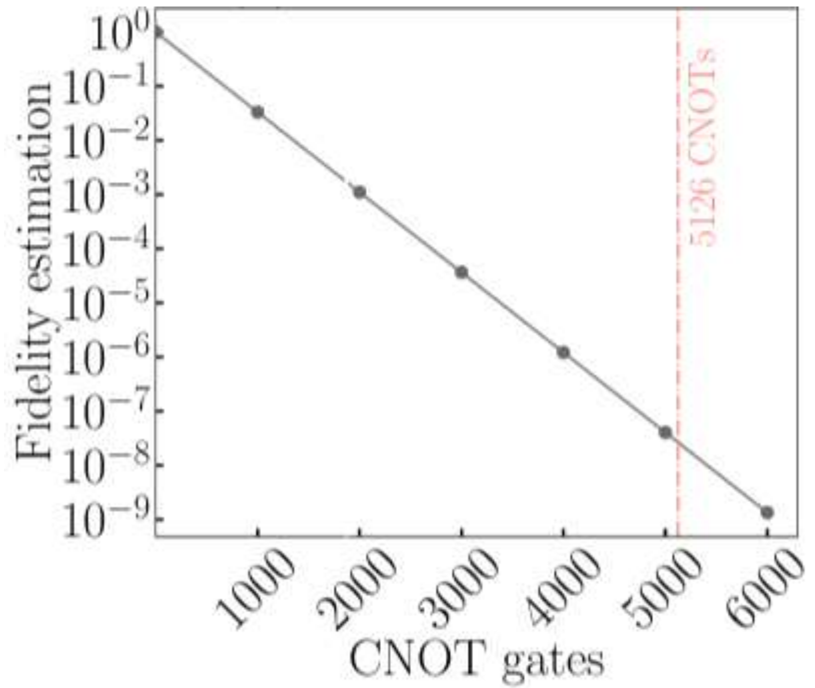
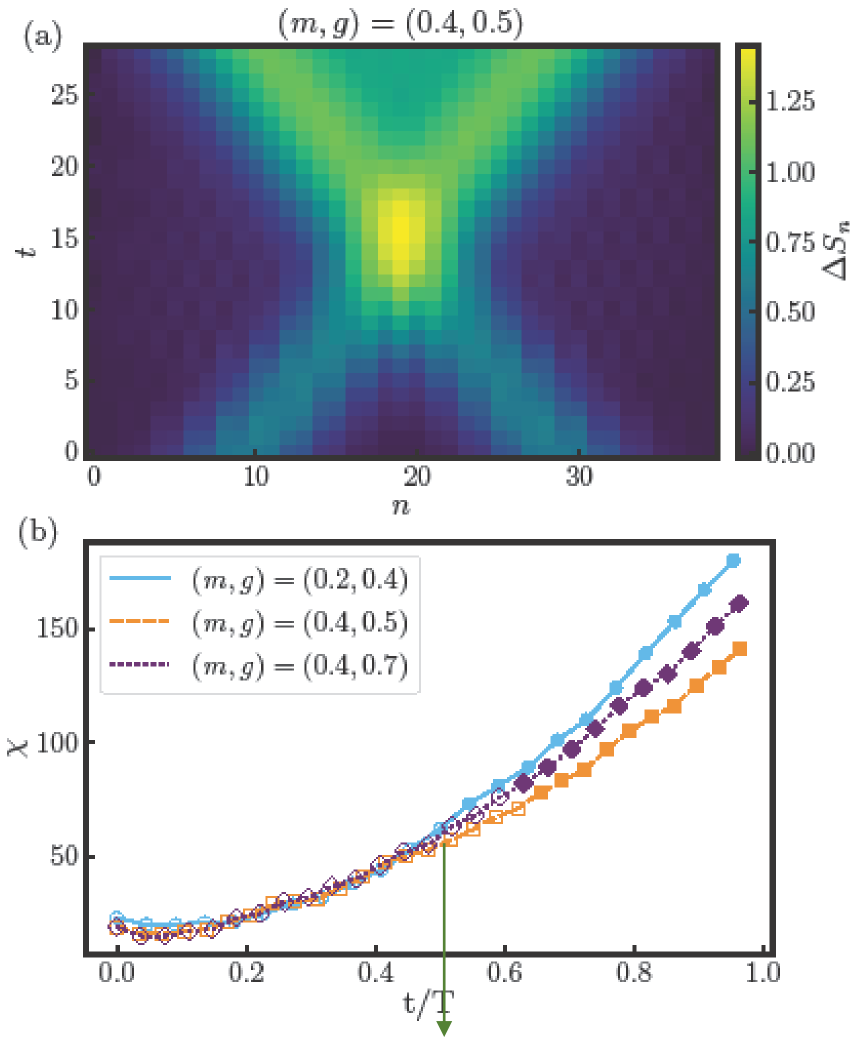


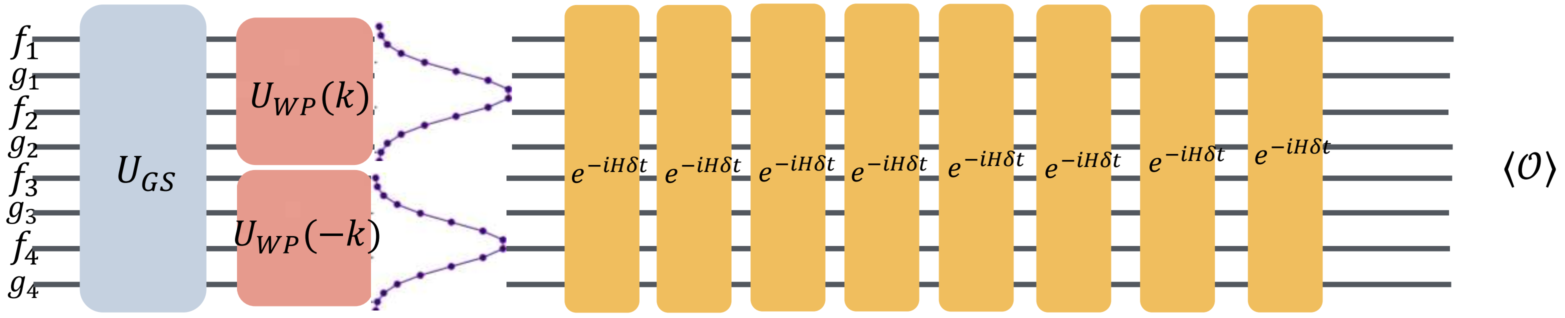


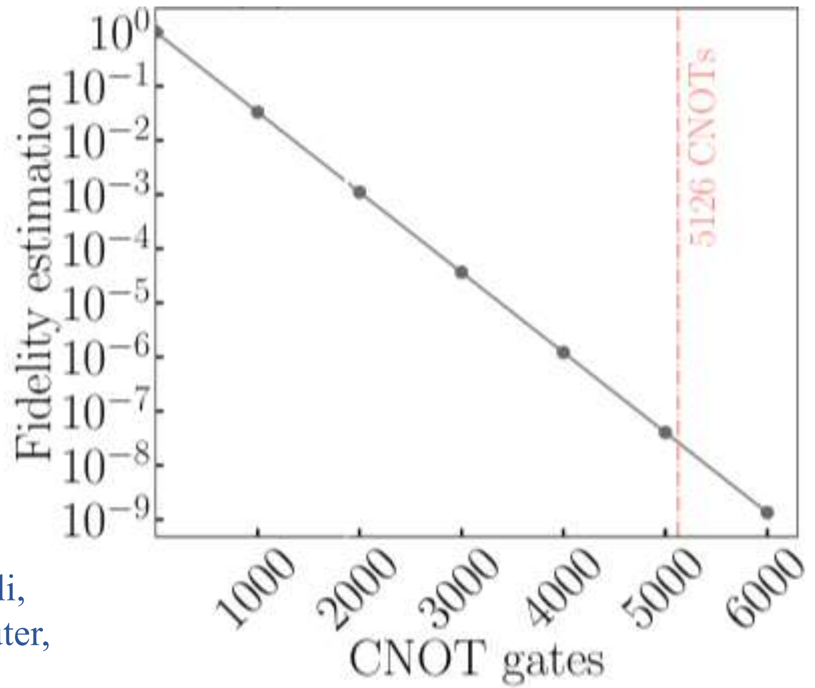


Low bond dimension before collision, easy for MPS

Interacting fermion scattering



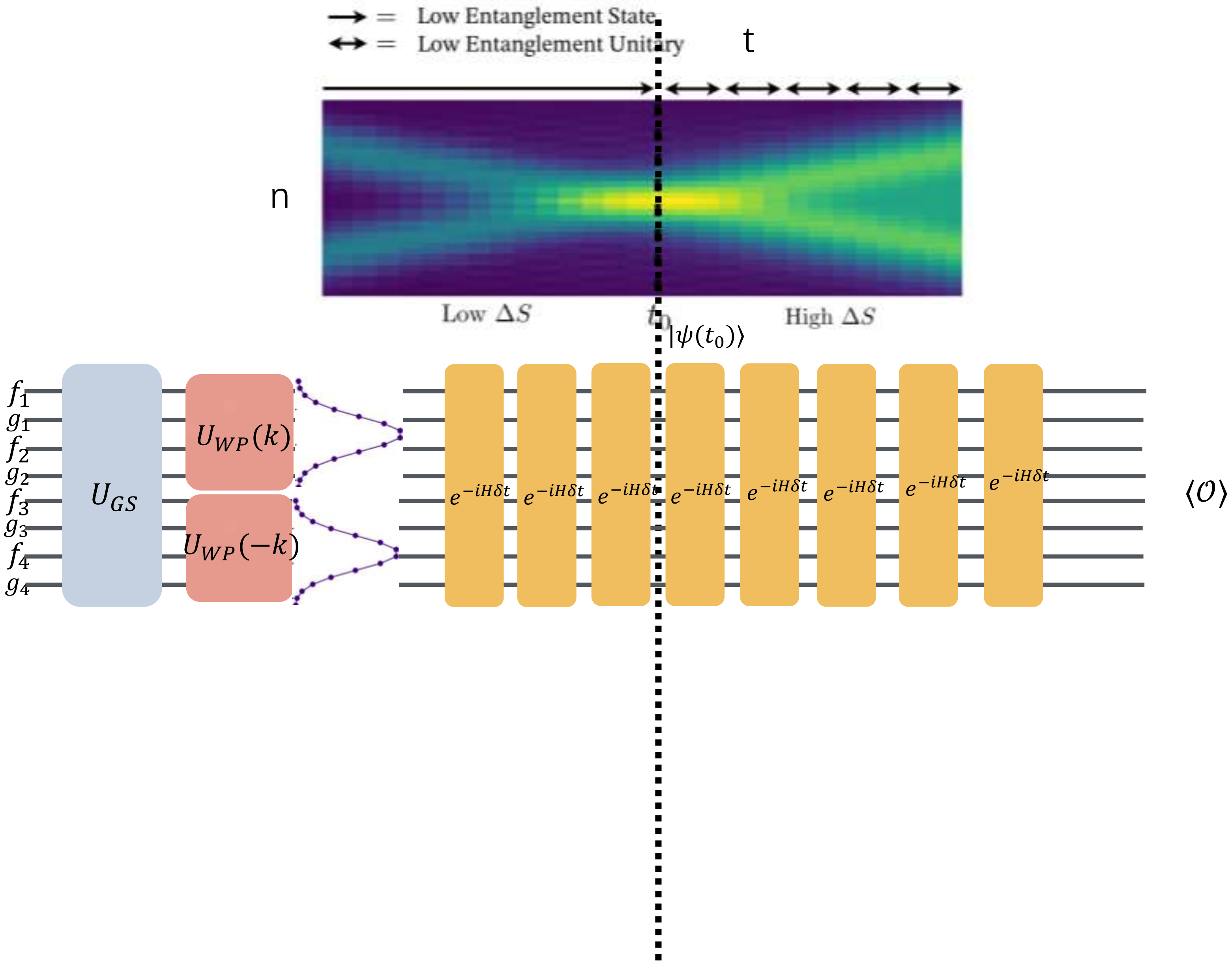




Y. Chai, J. Gibbs, V. R. Pascuzzi, Z. Holmes, S. Kühn, F. Tacchino, and I. Tavernelli, Resource-Efficient Simulations of Particle Scattering on a Digital Quantum Computer, arXiv:2507.17832.

Tensor network circuit compression

Conventional circuit



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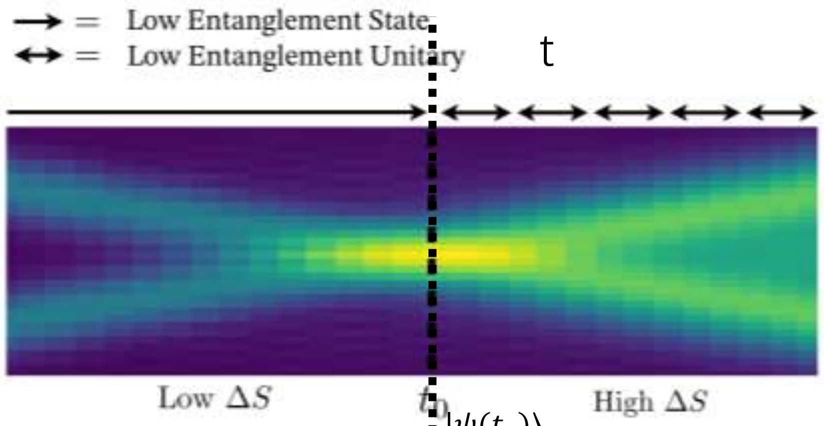

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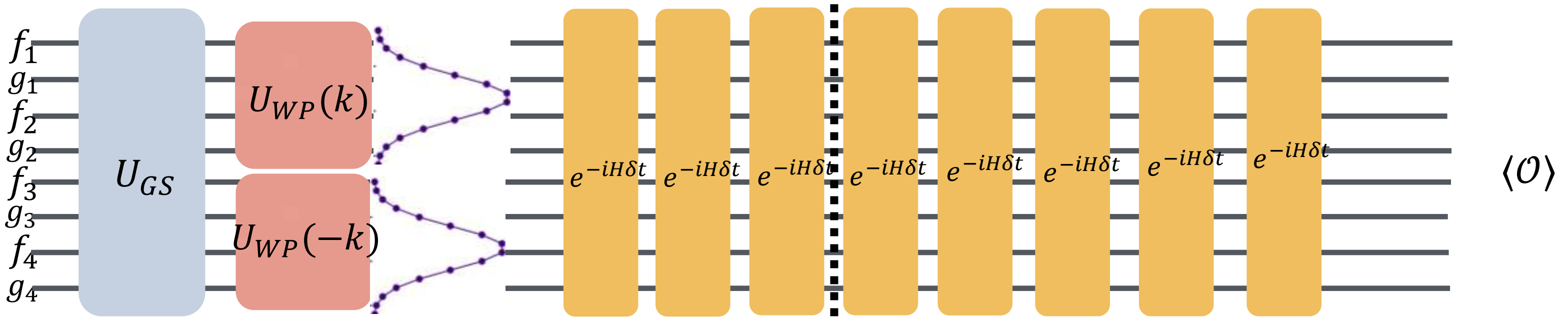
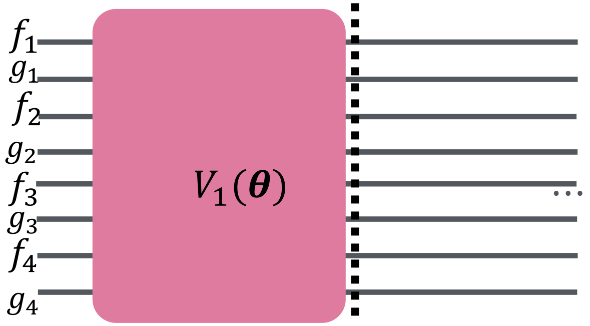


Circuit compression by tensor networks optimization

$$C_{State}(\boldsymbol{\theta}) = |\langle\psi(t_0)|V_1(\boldsymbol{\theta})|0\rangle|^2$$



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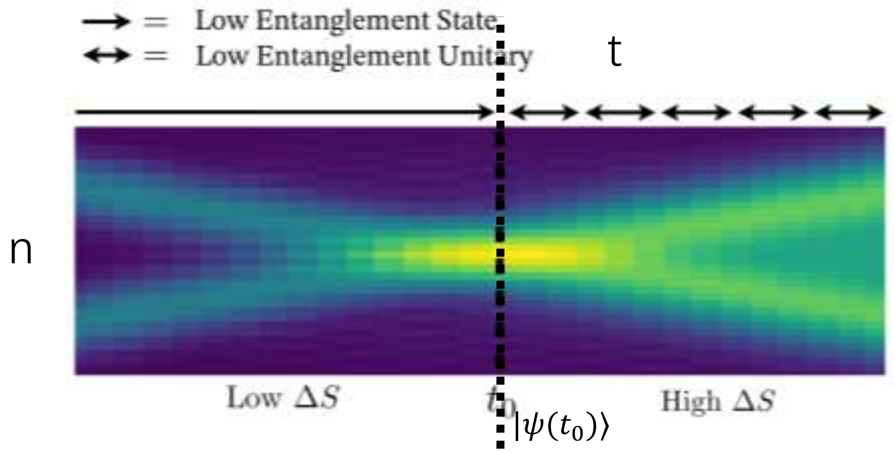

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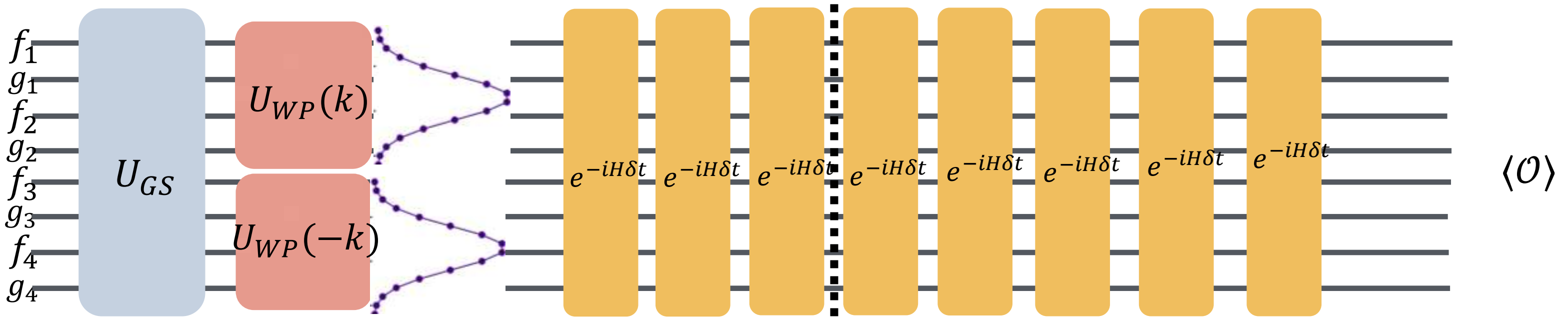

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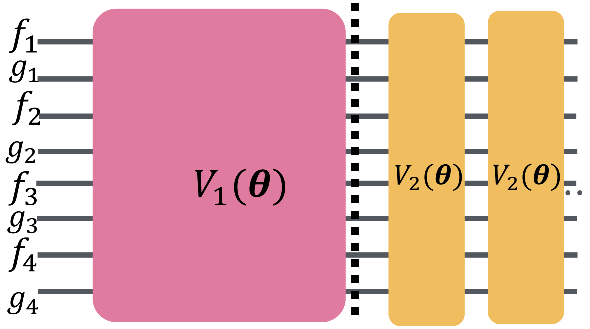


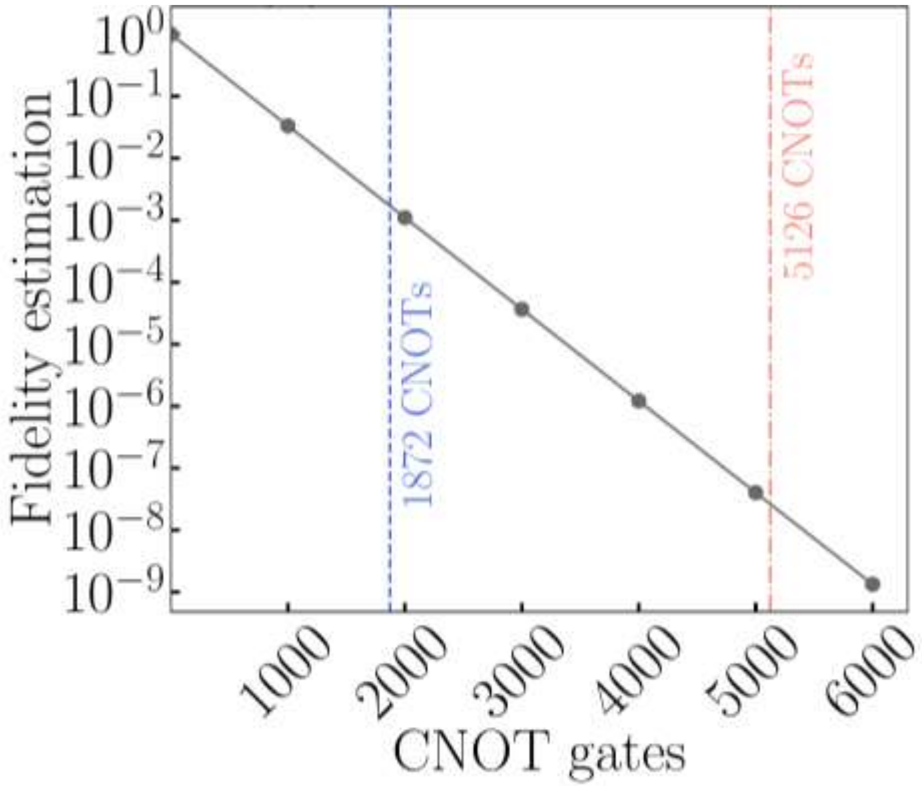
$$C_{Uni}(\boldsymbol{\theta}) = \frac{1}{2^{2N}}\left|Tr(V_2(\boldsymbol{\theta})^\dagger e^{-iH\delta t})\right|^2$$

Hardware run using 40 qubits

$N = 40,\ T = 26,\ \Delta t = 2/3$		
	CNOT layers	CNOT gates
$\lvert\psi(t_0)\rangle$	36 (241)	702 (3371)
e^{-i2H}	12 (18)	234 (351)
In total	96 (331)	1872 (5126)

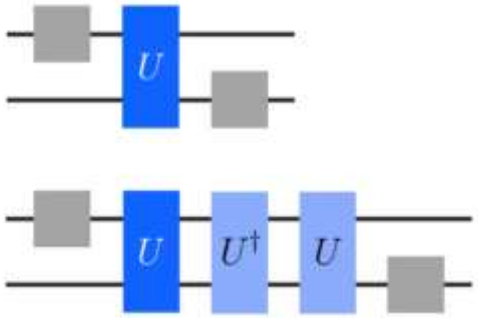
Our optimized circuit → 96, 1872

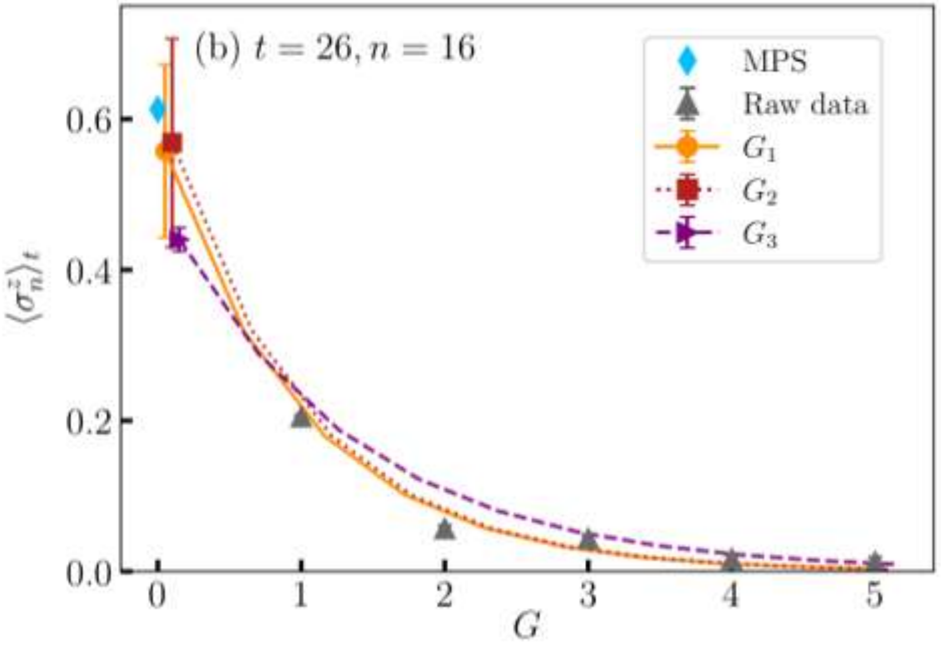
Conventional approach → (331), (5126)

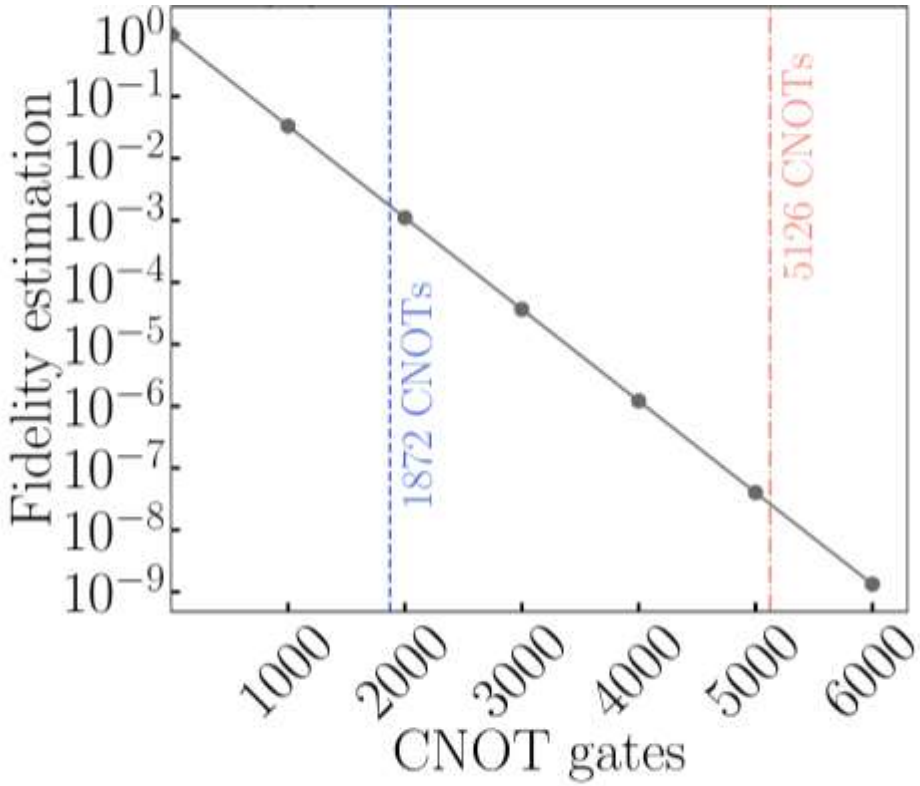


Hardware run using 40 qubits

Error mitigation: zero-noise extrapolation (ZNE)



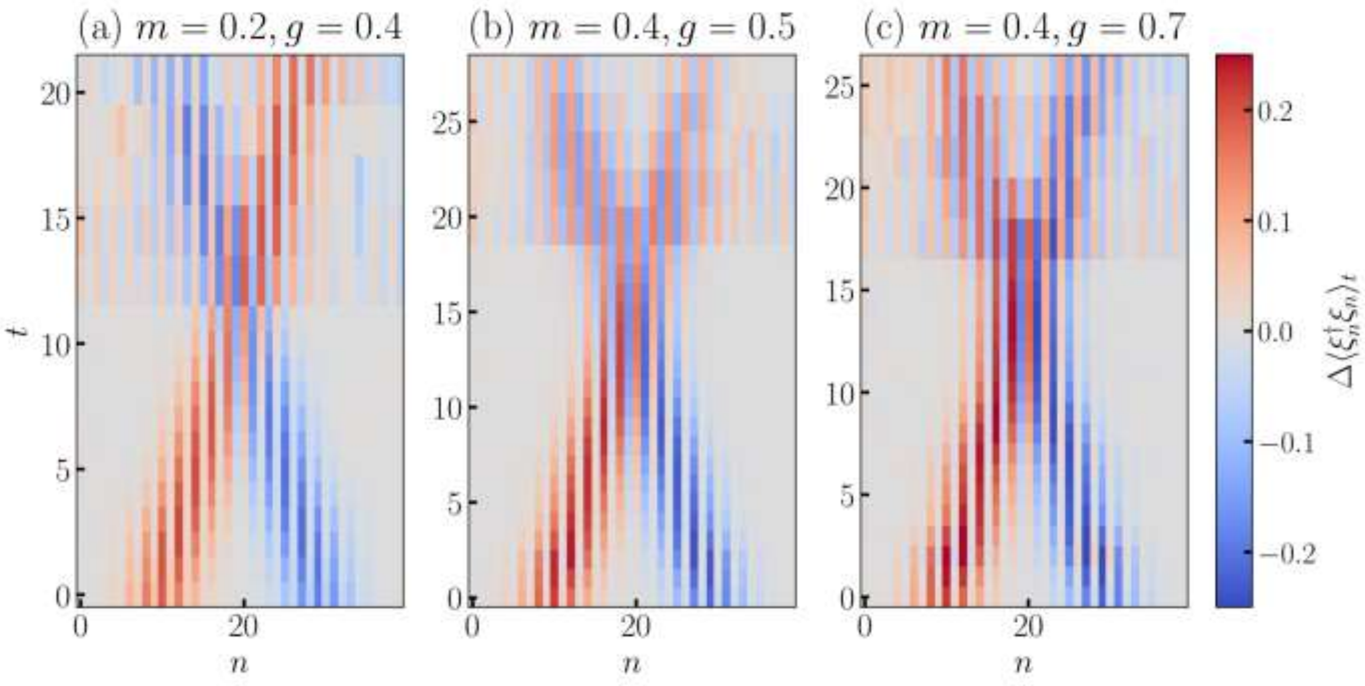




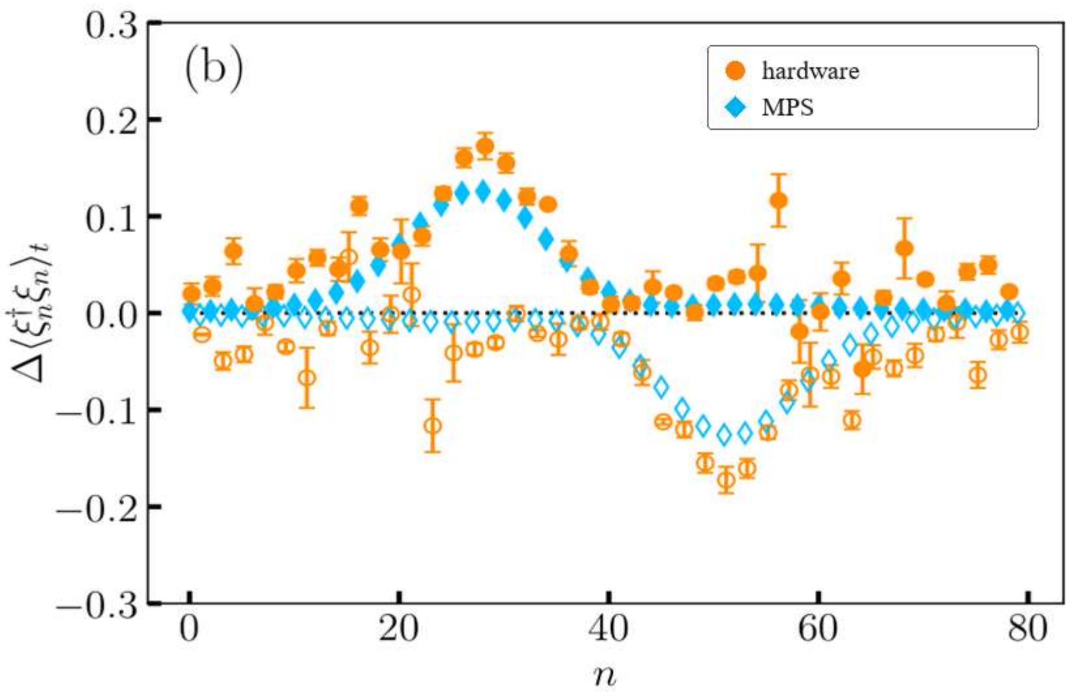
Hardware run for 40 and 80 qubits

IBM device ibm_fez
Error mitigation: zero-noise extrapolation (ZNE)

- Full dynamics with 40 qubits



- State preparation on 80 qubits

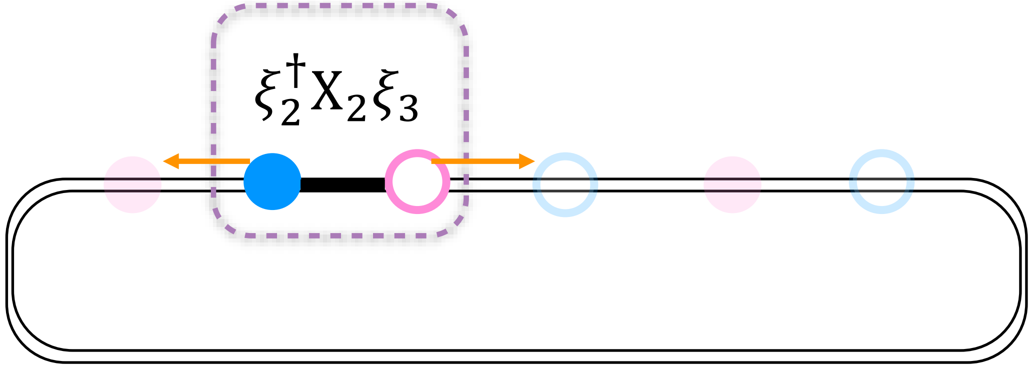


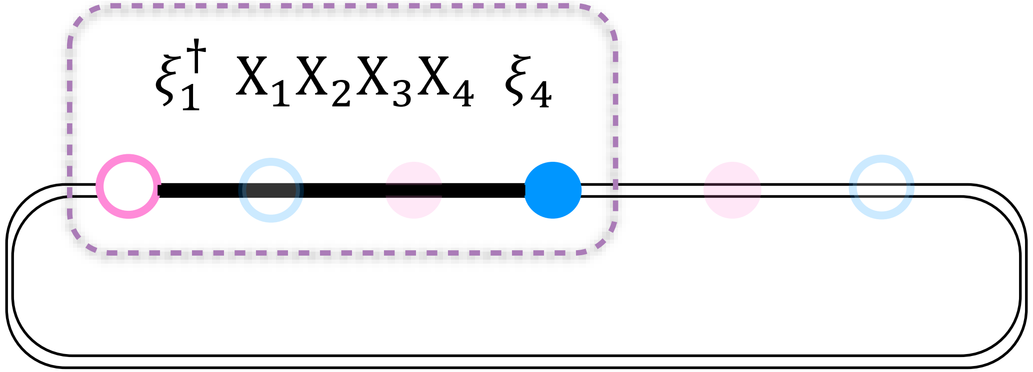
Lattice $\mathbb{Z}_2$ gauge theory in 1+1 D

- Staggered fermion coupled with $\mathbb{Z}_2$ gauge field, PBC

$$H = \frac{1}{2a}\sum_{n=1}^{L}\left(\xi_n^\dagger \mathrm{X}_n \xi_{n+1} + h.c.\right) + m\sum_{n=1}^{L}(-1)^n \xi_n^\dagger \xi_n + \varepsilon \sum_{n=1}^{L} \mathrm{Z}_n$$

- How to create a stable meson state?





Operators with momentum k: $$b_k^\dagger(\mathrm{d}) = \sum_{\mathrm{n}} e^{-ikn}\, \xi_n^\dagger \mathrm{X}_{n,\mathrm{n+d}} \xi_{\mathrm{n+d}}$$

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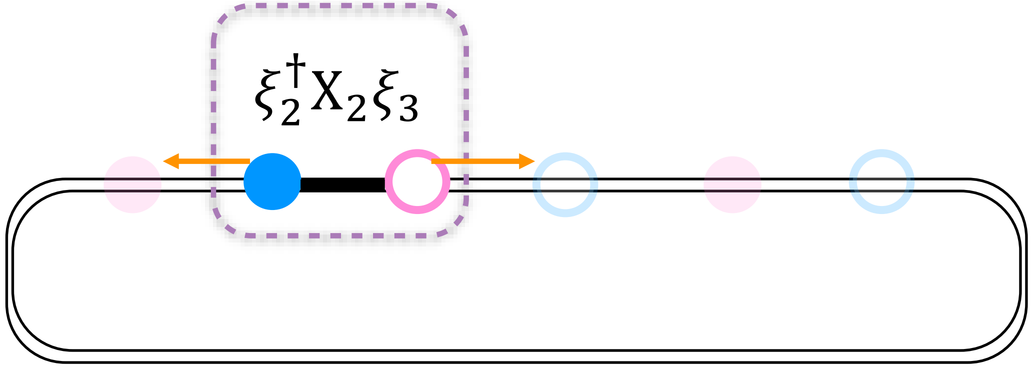

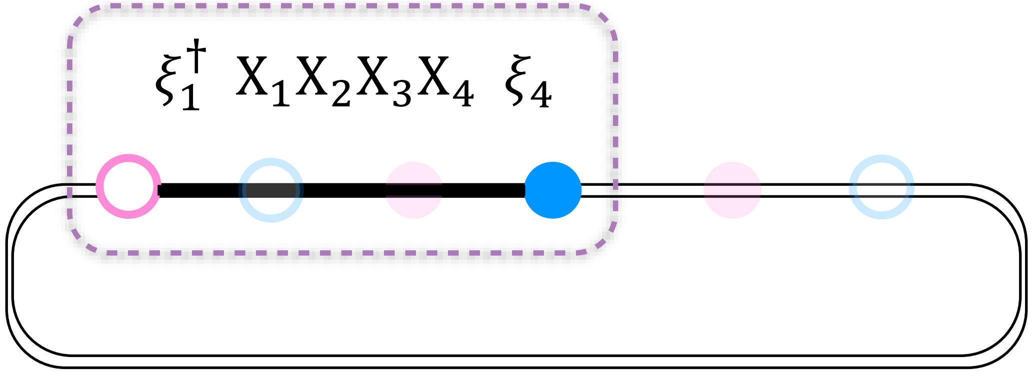

Operators with momentum k: $b_k^\dagger(\mathrm{d}) = \sum_{\mathrm{n}} e^{-ikn}\, \xi_n^\dagger X_{n,\mathrm{n+d}}\xi_{\mathrm{n+d}}$

Operators with charge conjugation c: $b_{k,c}^\dagger(\mathrm{d}) = b_k^\dagger(\mathrm{d}) + \mathrm{c}\ [\mathrm{C}\, b_k^\dagger(\mathrm{d})\mathrm{C}^{-1}],\quad \mathrm{c} = 1, -1$

Quantum Subspace Expansion (QSE)

- General expression for the meson creation operator

$$b_{k,\mathrm{c}}^{\dagger} = \sum_{d} a_d \, b_{k,\mathrm{c}}^{\dagger}(\mathrm{d})$$

Excite eigen state on top of vacuum $|k, \mathrm{c}\rangle_b = b_{k,\mathrm{c}}^{\dagger} |\Omega\rangle$

- Use QSE to determine the coefficients a_{d}

$$\mathcal{H}_{\mathrm{d,l}} = \langle\Omega| \, b_{k,\mathrm{c}}(\mathrm{d}) \, H \, b_{k,\mathrm{c}}^{\dagger}(\mathrm{l}) \, |\Omega\rangle$$

$$\mathcal{S}_{\mathrm{d,l}} = \langle\Omega| \, b_{k,\mathrm{c}}(\mathrm{d}) \; b_{k,\mathrm{c}}^{\dagger}(\mathrm{l}) \, |\Omega\rangle$$

$$\mathcal{H} \, \vec{a} = \mathrm{E} \, \mathcal{S} \vec{a}$$

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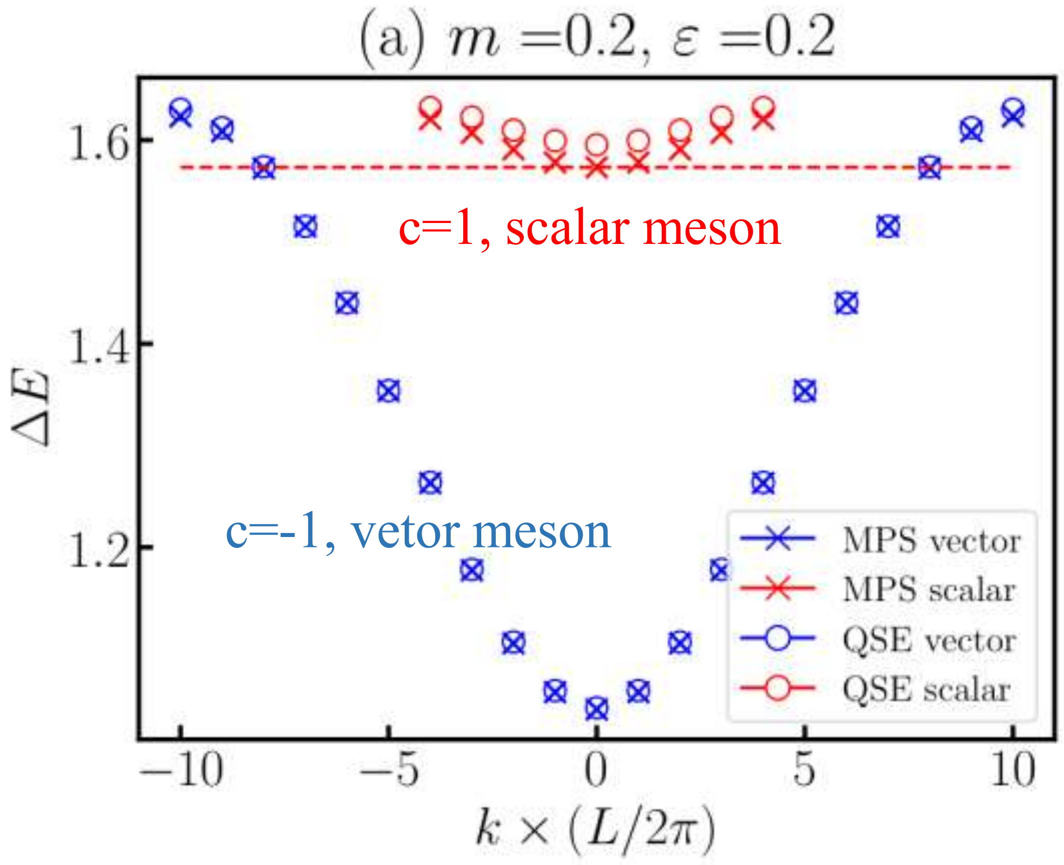
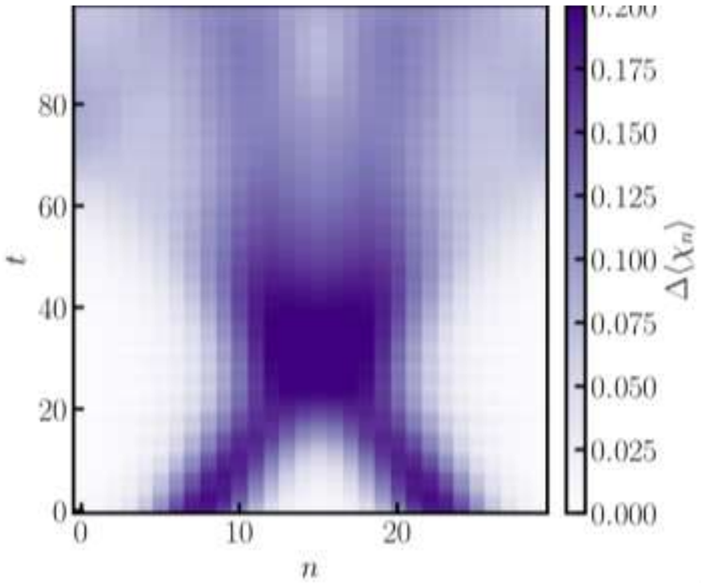
First 30 excited states in a 60 qubits system



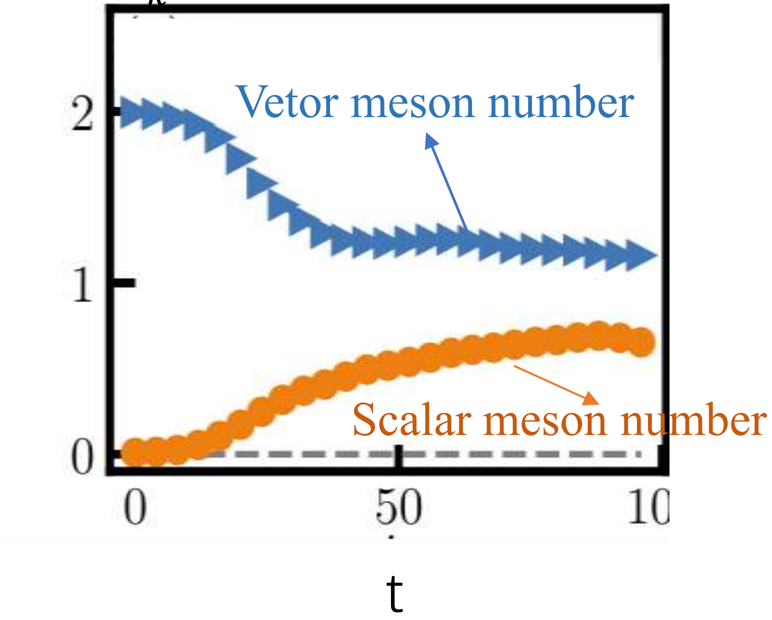
Inelastic meson scattering

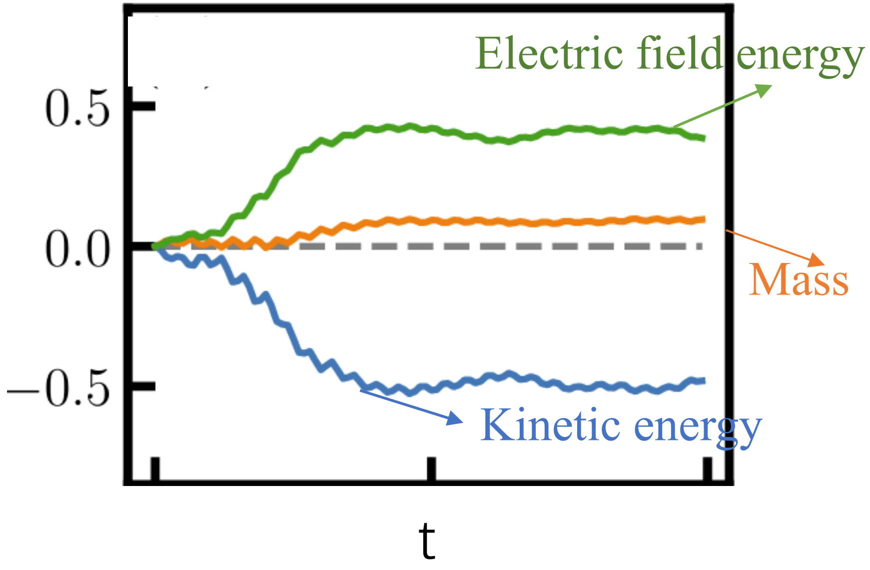
Fermion density



Meson number:

$$\sum_k \langle\psi(t)|\, b^{\dagger}_{k,c}\ b_{k,c}\, |\psi(t)\rangle$$





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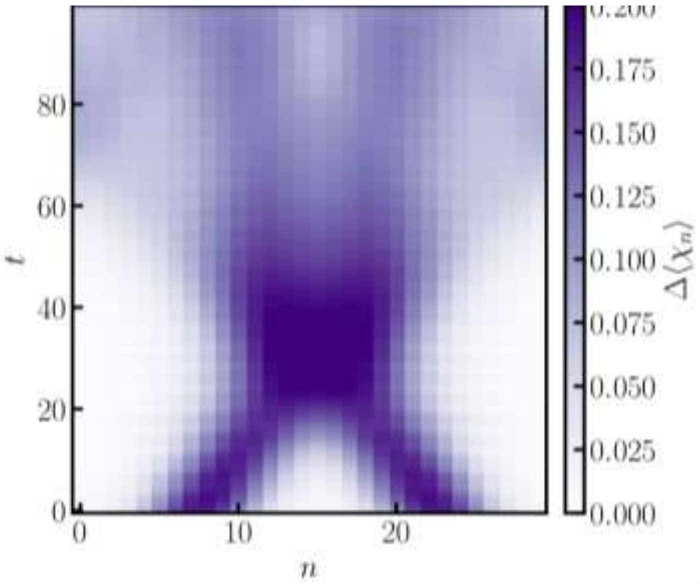
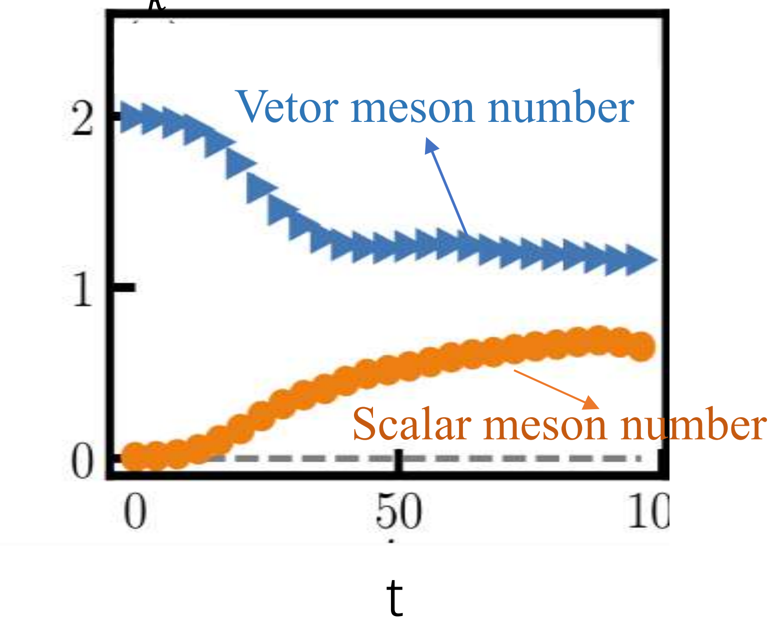
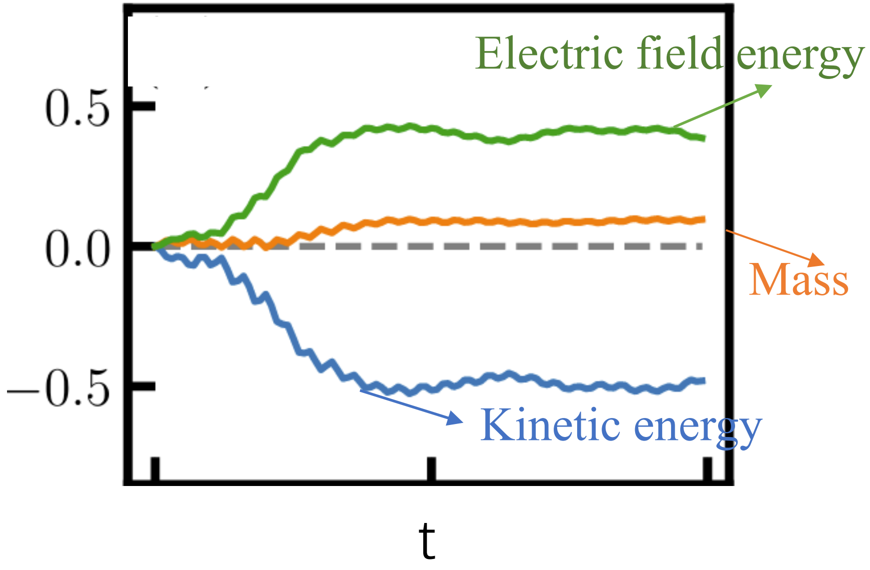
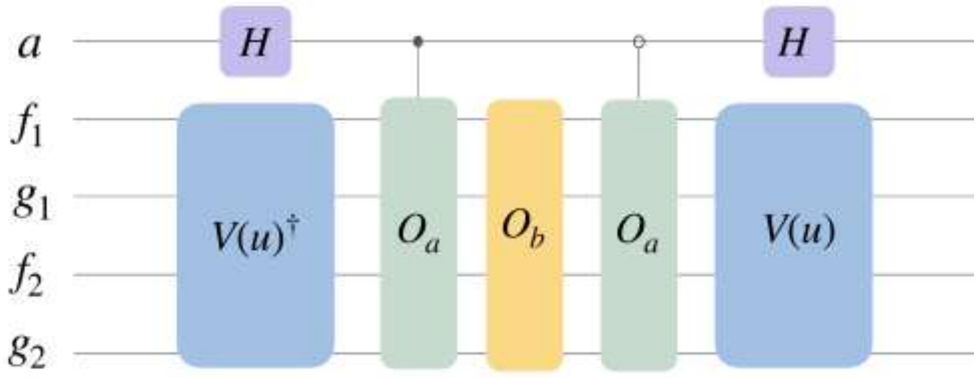


Meson number:

$\sum_k \langle\psi(t)|\, b_{k,c}^{\dagger}\; b_{k,c}\, |\psi(t)\rangle$







	CNOT gates	CNOT depth
$V(u,\tilde{\xi})$ or $V(u,\tilde{\xi})^\dagger$	$2L(L-1)$	$4(2L-3)$
O_a	$4(L-1)$	$4(L-1)$
O_b	$12(L-1)$	$12(L-1)$
Total	$4L^2+16L-20$	$36L-44$

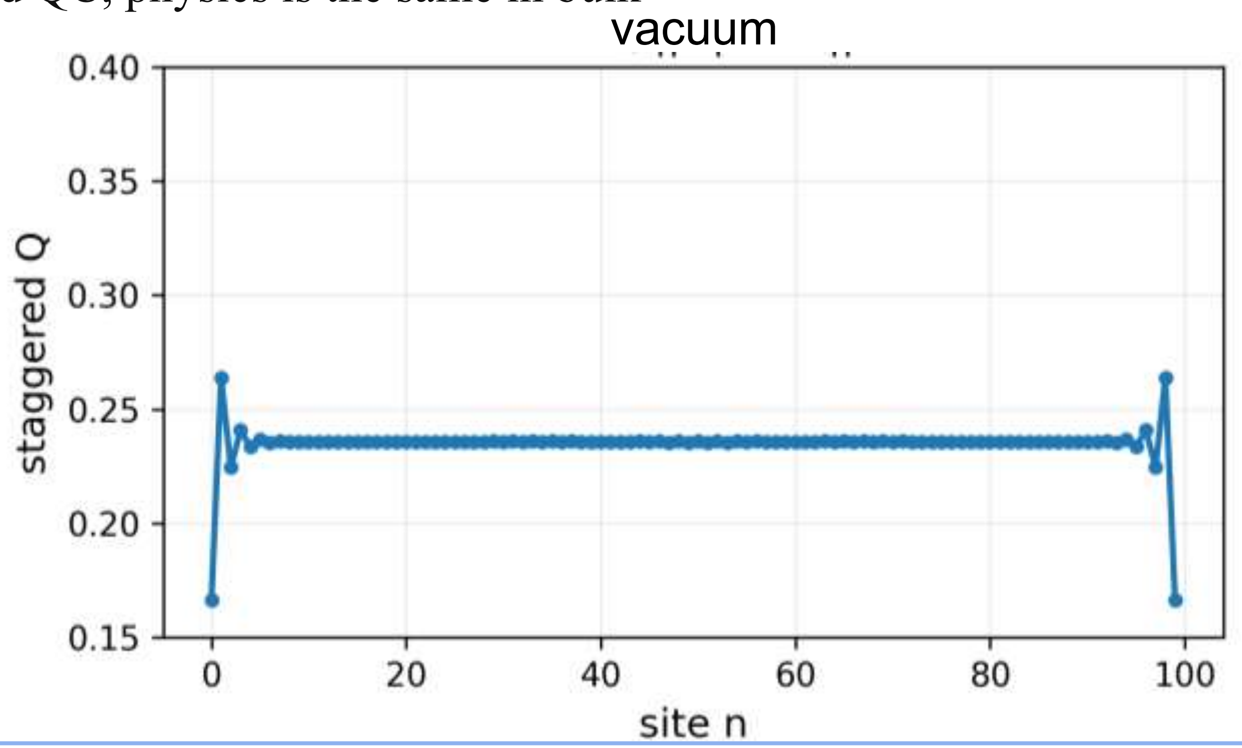
OBC wave packets

Old setup: wave packets in PBC

PBC gives good quantum numbers: momentum k, charge conjugation c ⟶ clear physics.

Boundary term needs more computational resource

OBC is more friendly for TN and QC, physics is the same in bulk



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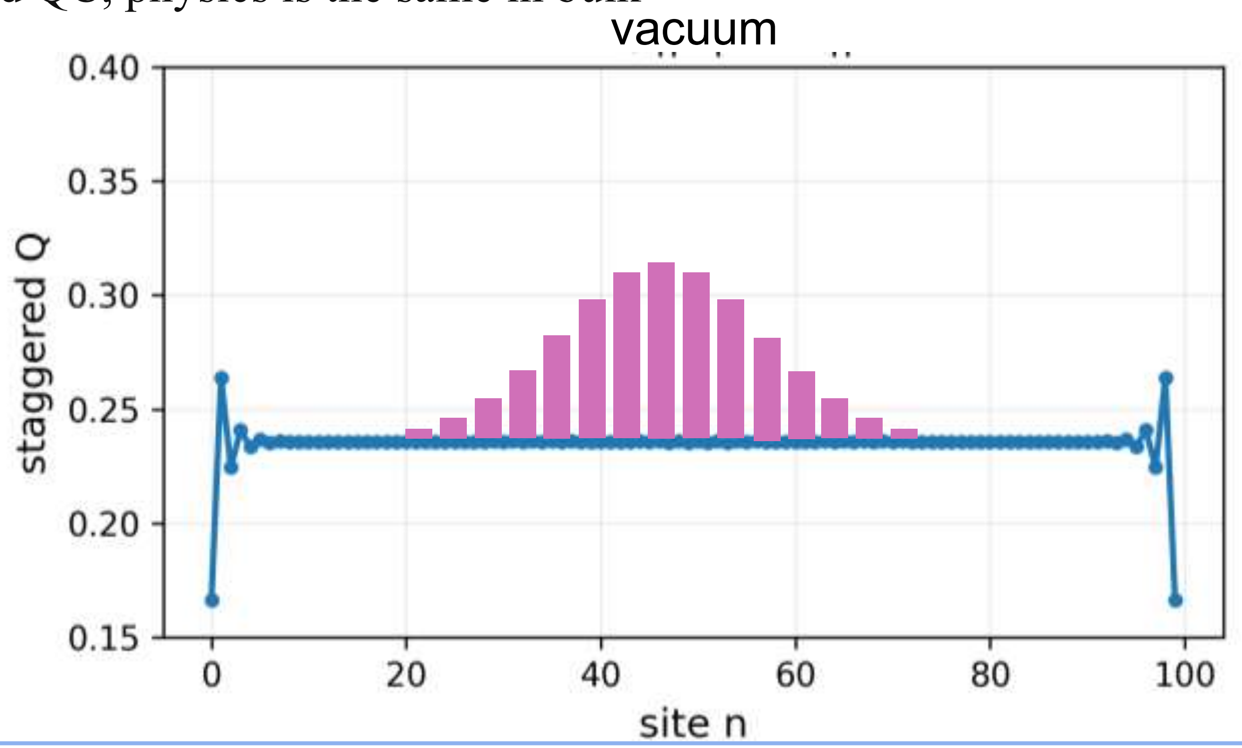

Local meson wave packet in OBC

$$b_k^\dagger = \sum_d a_{k,d}\, b_{k,d}^\dagger \qquad\qquad B_{\bar{k},\bar{n}}^\dagger = \sum_k \phi(k)\ b_k^\dagger$$

Local meson wave packet in OBC

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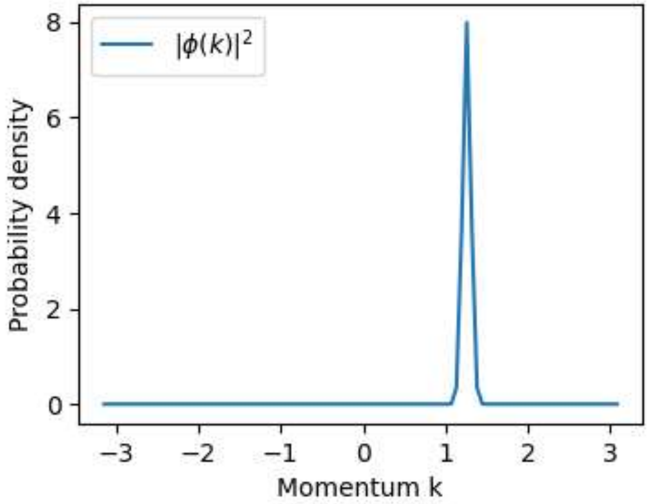
$$B_{\bar{k},\bar{n}}^\dagger = \sum_{\mathrm{d}} a_d \sum_k \phi(k)\ b_{k,d}^\dagger$$

Local meson wave packet in OBC

$$b_k^\dagger = \sum_d a_{k,d}\, b_{k,d}^\dagger$$

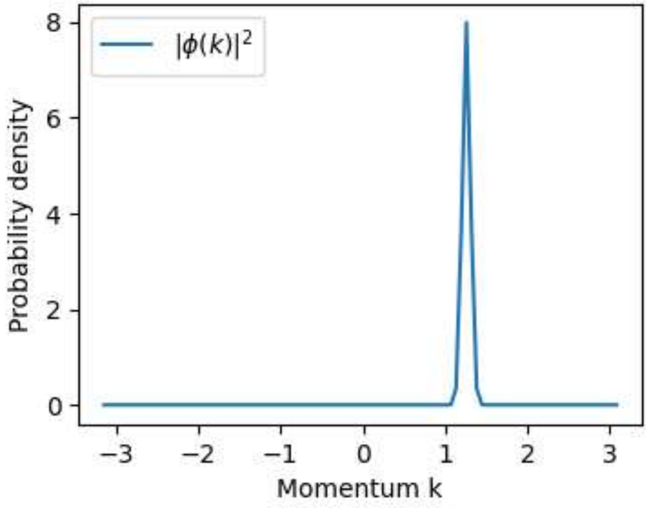
$$B_{\overline{k},\overline{n}}^\dagger = \sum_k \phi(k)\; b_k^\dagger$$

$$B_{\overline{k},\overline{n}}^\dagger = \sum_{\mathrm{d}} a_d \sum_k \phi(k)\; b_{k,d}^\dagger$$

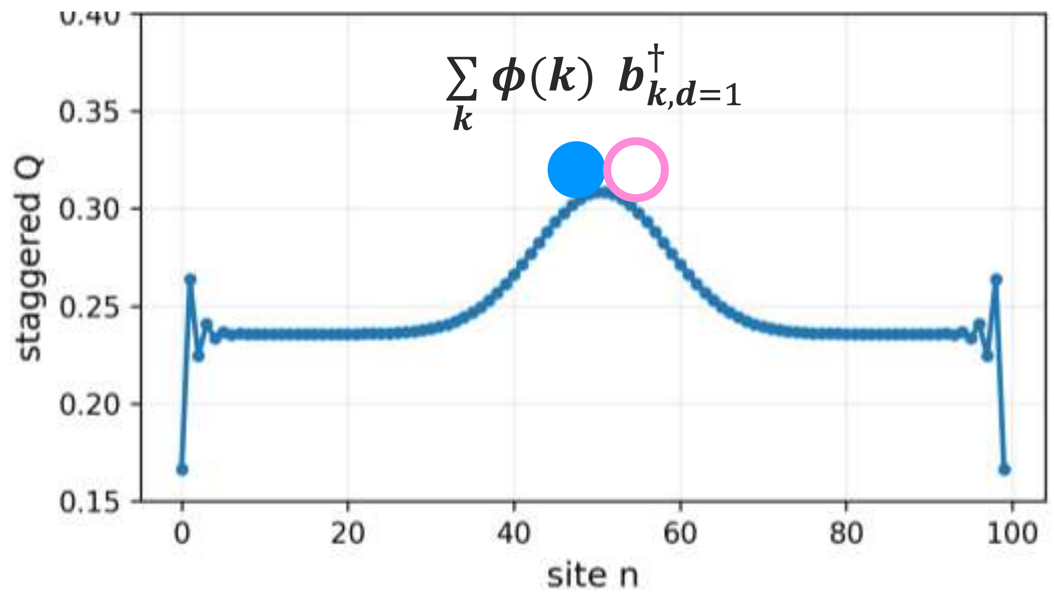


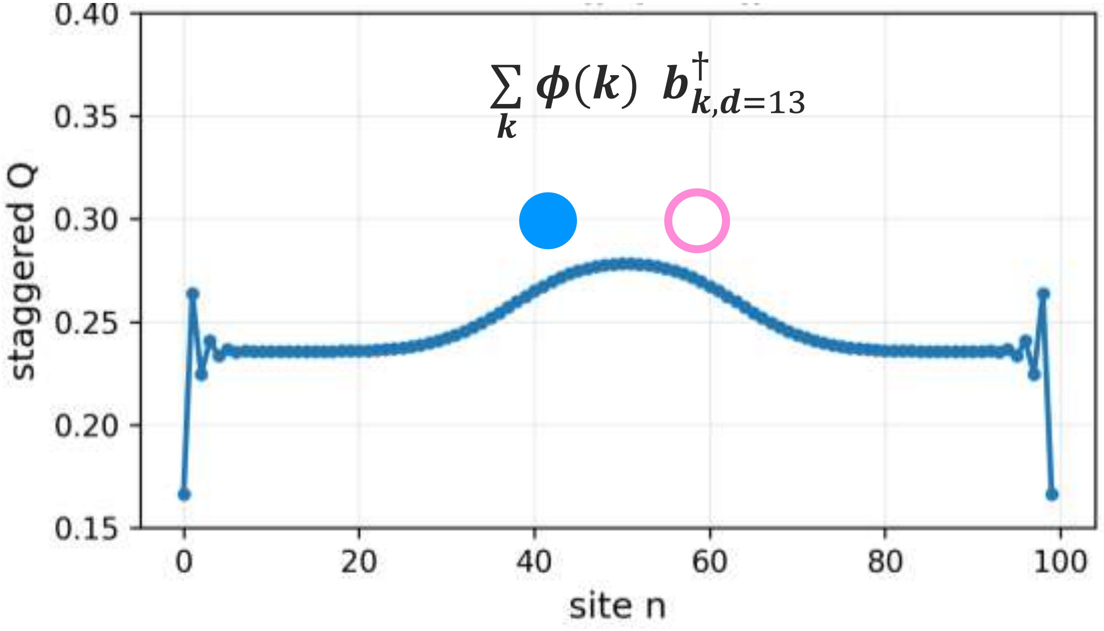
Local meson wave packet in OBC

$$b_k^\dagger = \sum_d a_{k,d}\, b_{k,d}^\dagger \qquad\qquad B_{\bar{k},\bar{n}}^\dagger = \sum_k \phi(k)\; b_k^\dagger$$



$$B_{\bar{k},\bar{n}}^\dagger = \sum_d a_d \sum_k \phi(k)\; b_{k,d}^\dagger$$

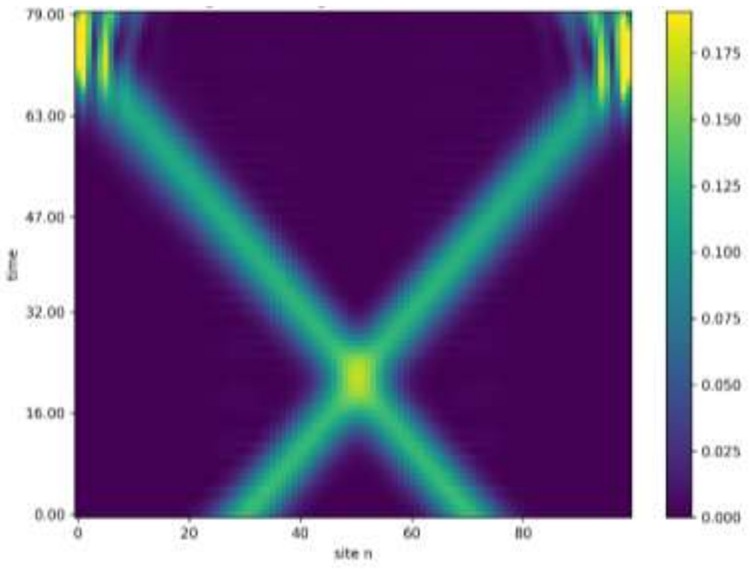




Resonance in scattering of meson in Schwinger model

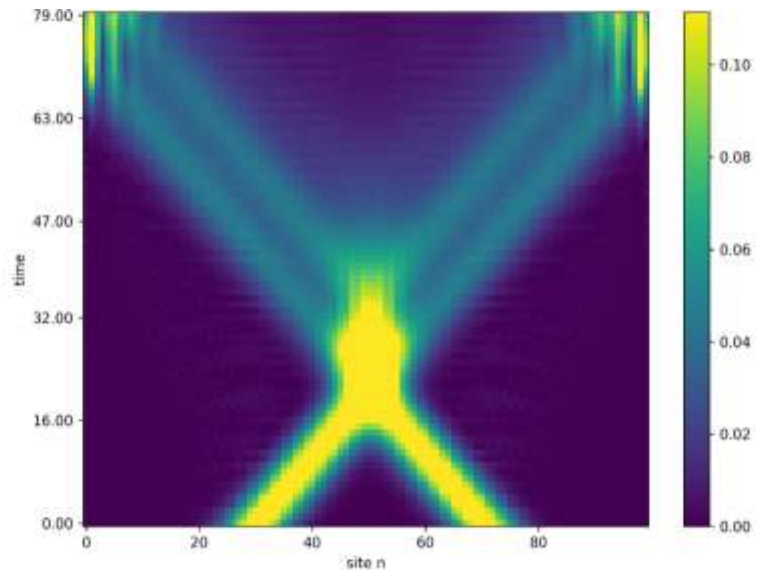
- Same momentum, but boundary electric field induced symmetry broken (C and P), allowing resonance appear

$\theta = 0$



C=P=1

$\theta > 0$



C=1, -1
P=1, 1

TODO:

1. Examine the mass and decay width of the resonance
2. Measure the composition of the resonance state?

Outlook

- Develop method to identify the outgoing particles, and get the S-matrix

- Non-Abelian gauge theories

- Higher dimension, (2+1)D

Thank you!

Arianna Crippa (CQTA)

Karl Jansen (CQTA)

Stefan Kühn (CQTA)

Vincent R. Pascuzzi (IBM, NY)

Francesco Tacchino (IBM Zürich)

Ivano Tavernelli (IBM Zürich)

Yibin Guo (CQTA)

Joe Gibbs
University of Surrey

Zoe Holmes
EPFL

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IBM Quantum

First fermion scattering paper

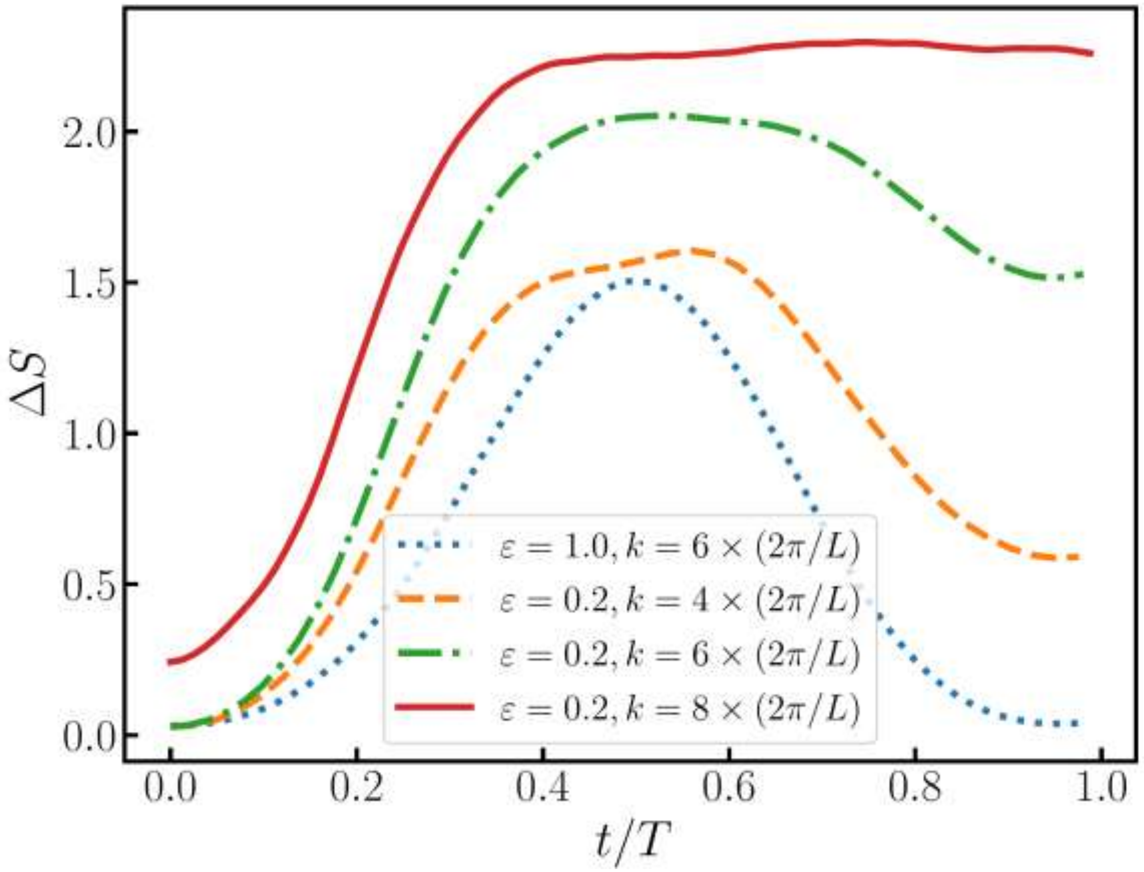
Y. Chai, A. Crippa, K. Jansen, S. Kühn, V. R. Pascuzzi, F. Tacchino, and I. Tavernelli, Quantum **9**, 1638 (2025).

Meson scattering paper

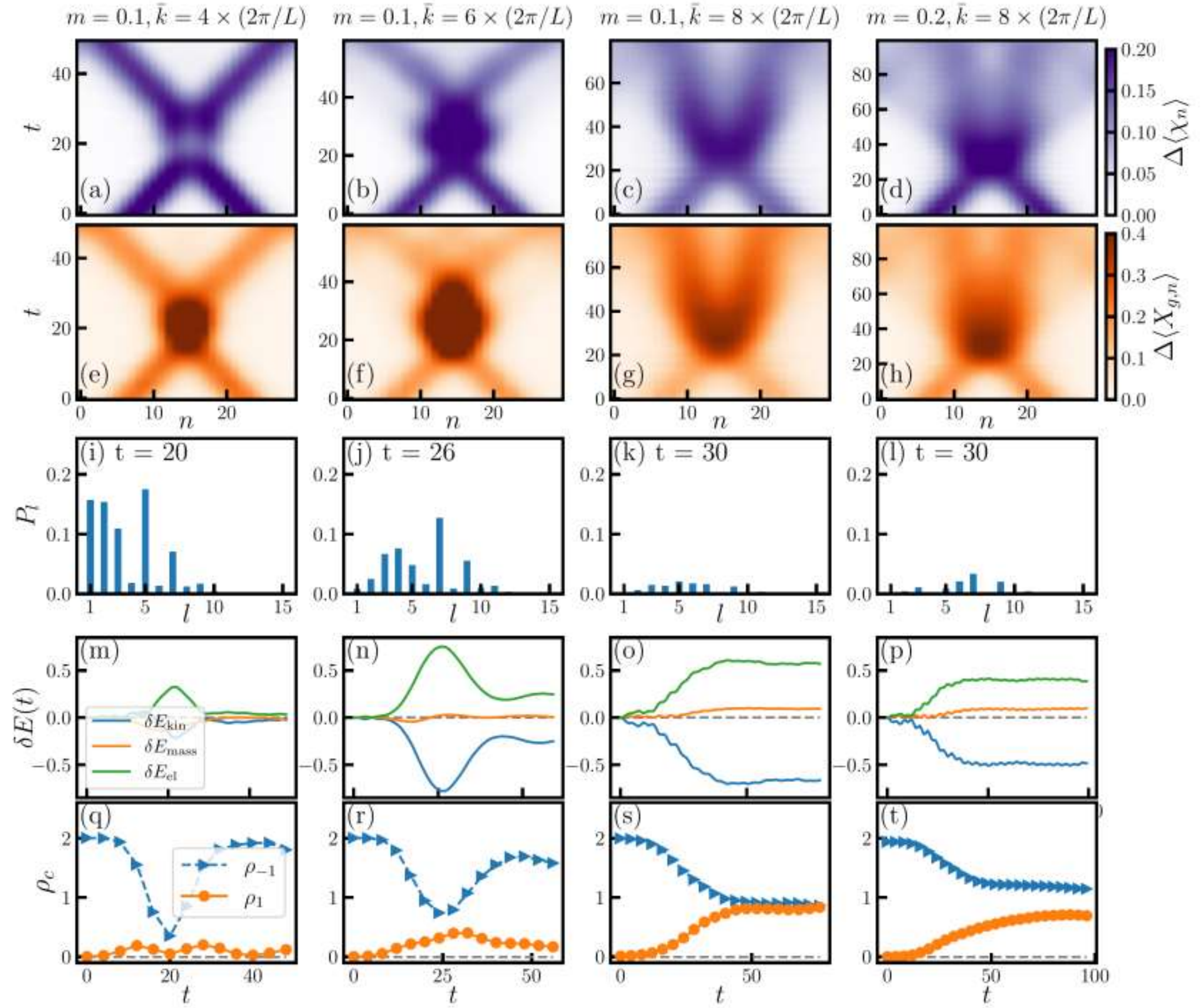
arXiv:2505.21240

Fermion scattering hardware run paper
arXiv:2507.17832,
accepted by npj Quantum Information

Appendix

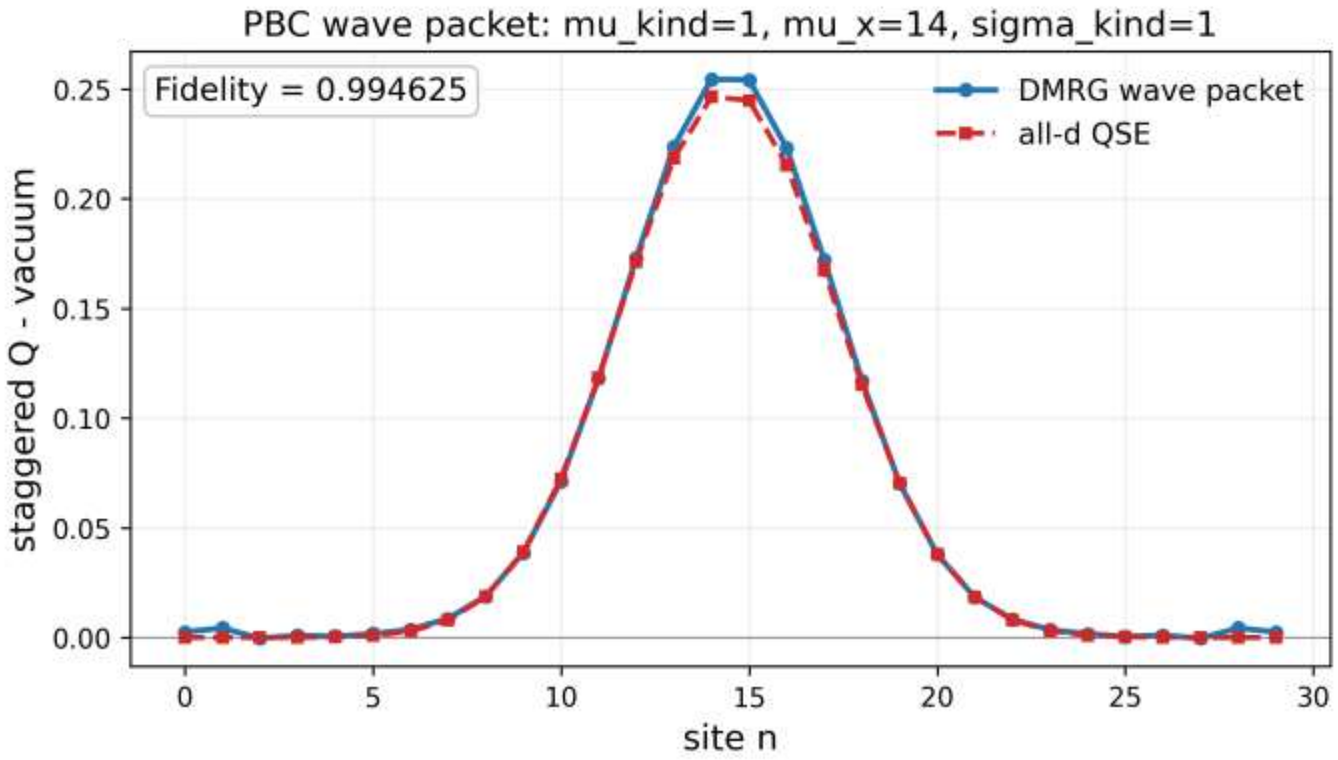


2.0
1.5
1.0
0.5
0.0
ΔS
0.0
0.2
0.4
0.6
0.8
1.0
t/T
$\varepsilon = 1.0, k = 6 \times (2\pi/L)$
$\varepsilon = 0.2, k = 4 \times (2\pi/L)$
$\varepsilon = 0.2, k = 6 \times (2\pi/L)$
$\varepsilon = 0.2, k = 8 \times (2\pi/L)$


$m = 0.1, \bar{k} = 4 \times (2\pi/L)$
$m = 0.1, \bar{k} = 6 \times (2\pi/L)$
$m = 0.1, \bar{k} = 8 \times (2\pi/L)$
$m = 0.2, \bar{k} = 8 \times (2\pi/L)$
$\Delta\langle\chi_n\rangle$
$\Delta\langle X_{g,n}\rangle$
(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
t
n
(i) t = 20
(j) t = 26
(k) t = 30
(l) t = 30
P_l
l
(m)
(n)
(o)
(p)
$\delta E(t)$
δE_{kin}
δE_{mass}
δE_{el}
(q)
(r)
(s)
(t)
ρ_c
ρ_{-1}
ρ_1

Appendix

- Cross check with DMRG in PBC:
  - Wave packet by linear combination of DMRG eigen states
  - Wave packet from QSE



N=100, x=10.0, m/g=1.0

Local meson wave packet in OBC

$$b_k^\dagger = \sum_d a_{k,d}\, b_{k,d}^\dagger$$

$$B_{\overline{k},\overline{n}}^\dagger = \sum_k \phi(k)\;\; b_k^\dagger = \sum_k \phi(k) \sum_{\mathrm{d}} a_{k,d}\, b_{k,d}^\dagger$$

Local meson wave packet in OBC

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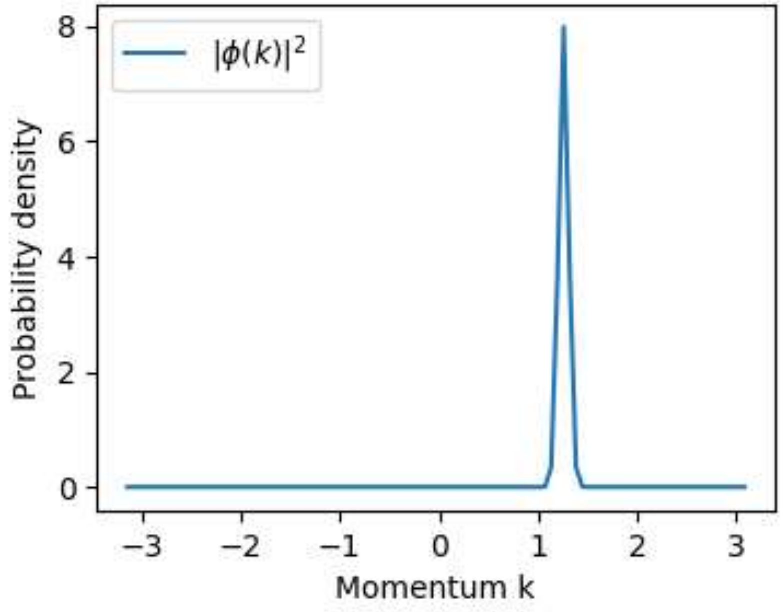

Local meson wave packet in OBC

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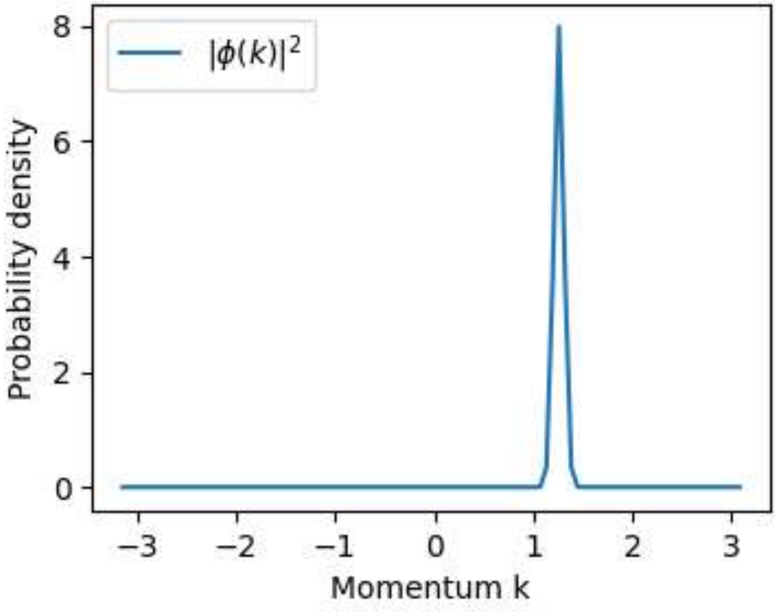
$$B_{\overline{k},\overline{n}}^\dagger = \sum_k \phi(k)\ \ b_k^\dagger = \sum_k \phi(k) \sum_{\mathrm{d}} a_d\, b_{k,d}^\dagger$$



Local meson wave packet in OBC

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$$= \sum_{\mathrm{d}} a_d \sum_k \phi(k)\ \ b_{k,d}^\dagger$$