Quantum Simulation of Various Topics in 1+1D Gauge Theory

Xingyu Guo (QUNU Collaboration)
South China Normal University

C3NT Workshop: Quantum Information Science in High Energy Physics

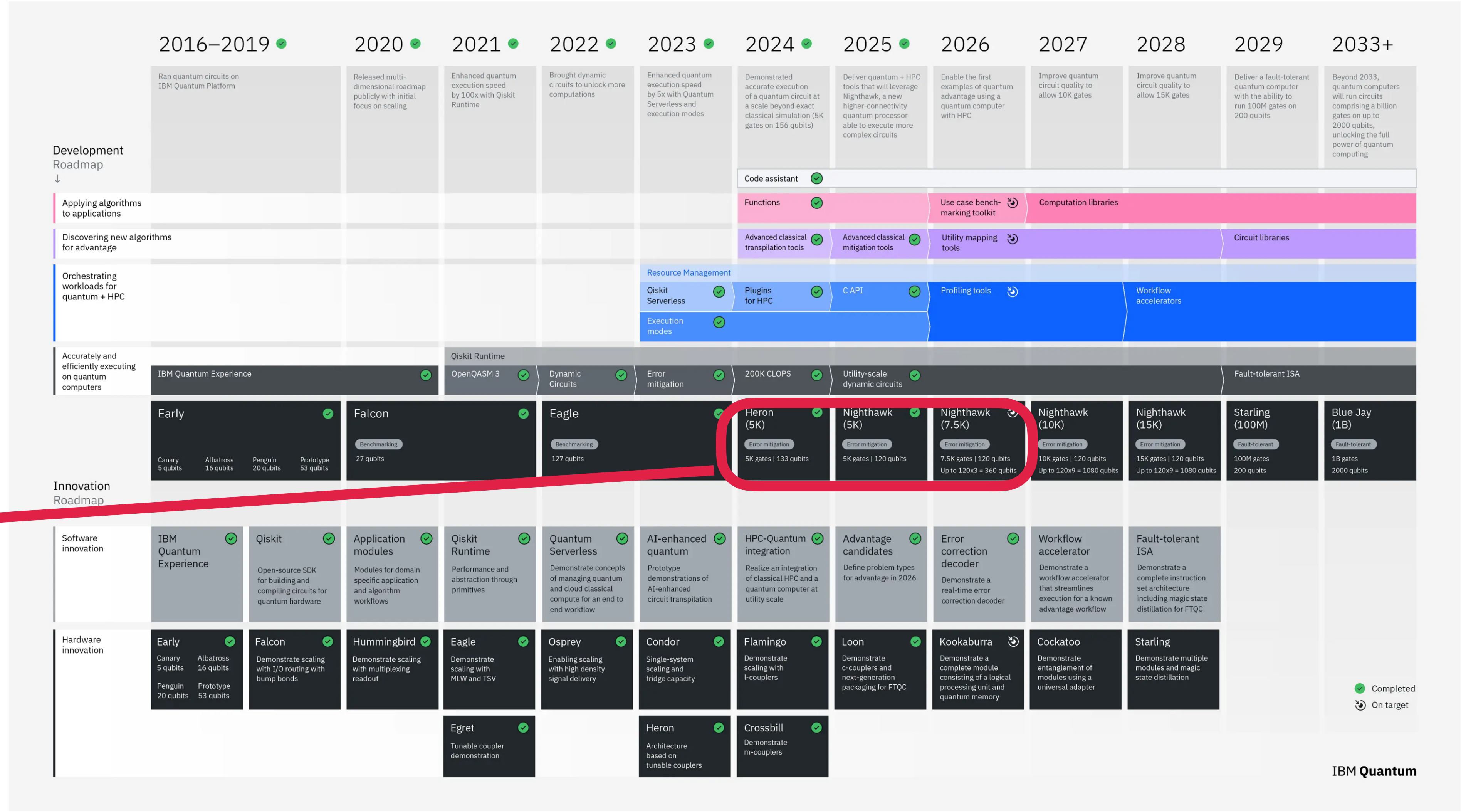
Current State of Quantum Hardware

- Noisy intermediate-scale quantum (NISQ) devices
  - Error rate: 1%~0.1%
  - Number of qubits: 100 ~1000
  - Simulation of 2+1D systems is becoming reasonable

Nighthawk (5K) ✓
Error mitigation
5K gates | 120 qubits

Nighthawk (7.5K)
Error mitigation
7.5K gates | 120 qubits
Up to 120x3 = 360 qubits

Nighthawk (10K)
Error mitigation
10K gates | 120 qubits
Up to 120x9 = 1080 qubits



https://www.ibm.com/quantum/hardware#roadmap

Current State of Quantum Hardware

- Noisy intermediate-scale quantum (NISQ) devices
  - Error rate: 1%~0.1%
  - Number of qubits: 100 ~1000
  - Simulation of 2+1D systems is becoming reasonable
- What meaningful study can we do now?
  - Search for effective, applicable algorithm
  - Meaning topics with small, simple systems

Digital Simulation of Gauge Theory

- Beyond 1+1 D:
  - Many insightful studies
  - Still challenging for current hardwares?
- 1+1D:
  - Gauge field eliminated
  - Simulation is relatively easy
  - Can provide insight to confinement, phase structure, non-Abelian properties, etc.

Gauge Theory in 1+1D

- Schwinger Model:

$$H = -\bar{\psi}(i\gamma^1 \mathrm{D}_1 + m_\alpha)\psi + \frac{1}{2}E^2$$

- Gauss's law:

$$\partial_x E = Q(x) \longrightarrow E(x) = E_0 + \int_0^x \mathrm{d}Q(y)$$

- The remaining gauge link can be eliminated by a gauge transformation.
- Pure fermionic Hamiltonian:

$$H = \frac{1}{2a}\sum_{n=1}^{N-1}\left(\phi_n^\dagger \phi_{n+1} + H.c.\right) + m\sum_{n=1}^{N}(-1)^n \phi_n^\dagger \phi_n + \frac{g^2 a}{2}\sum_{n=1}^{N-1}\left[\varepsilon + \sum_{l=1}^{n}\left(\phi_l^\dagger \phi_l - \frac{1-(-1)^l}{2}\right)\right]^2$$

Schwinger Mechanism

- Pair production under strong electric field.
- Strong field effect: no traditional perturbation method.
- Dynamical evolution: well-established in quantum simulation.
    - Trotterization
- State preparation: VQE + VQD

$$\langle \varphi(\theta) | H | \varphi(\theta) \rangle$$

$$\langle \varphi(\theta) | H | \varphi(\theta) \rangle + \lambda \langle \varphi(\theta) | \varphi_0 \rangle$$

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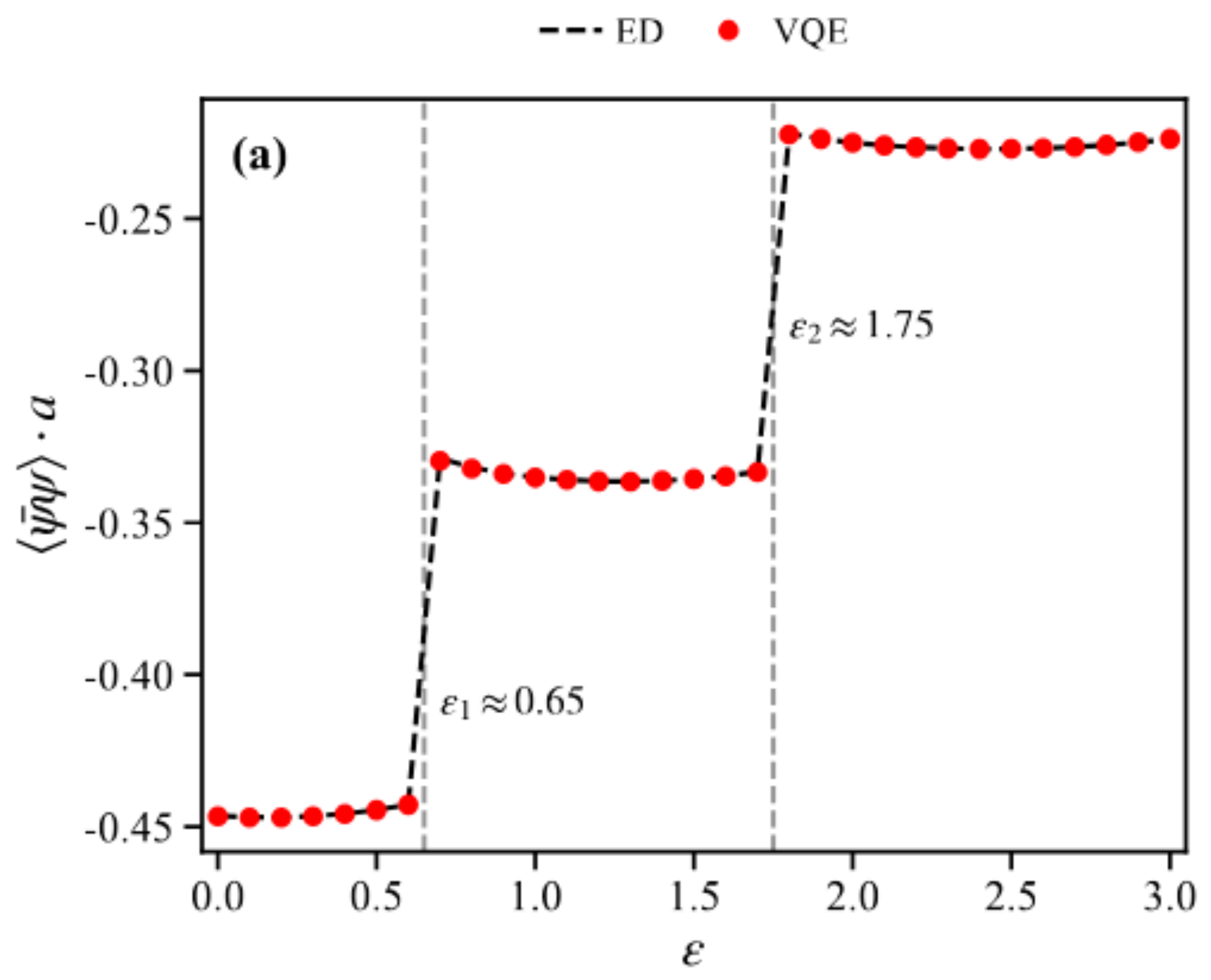
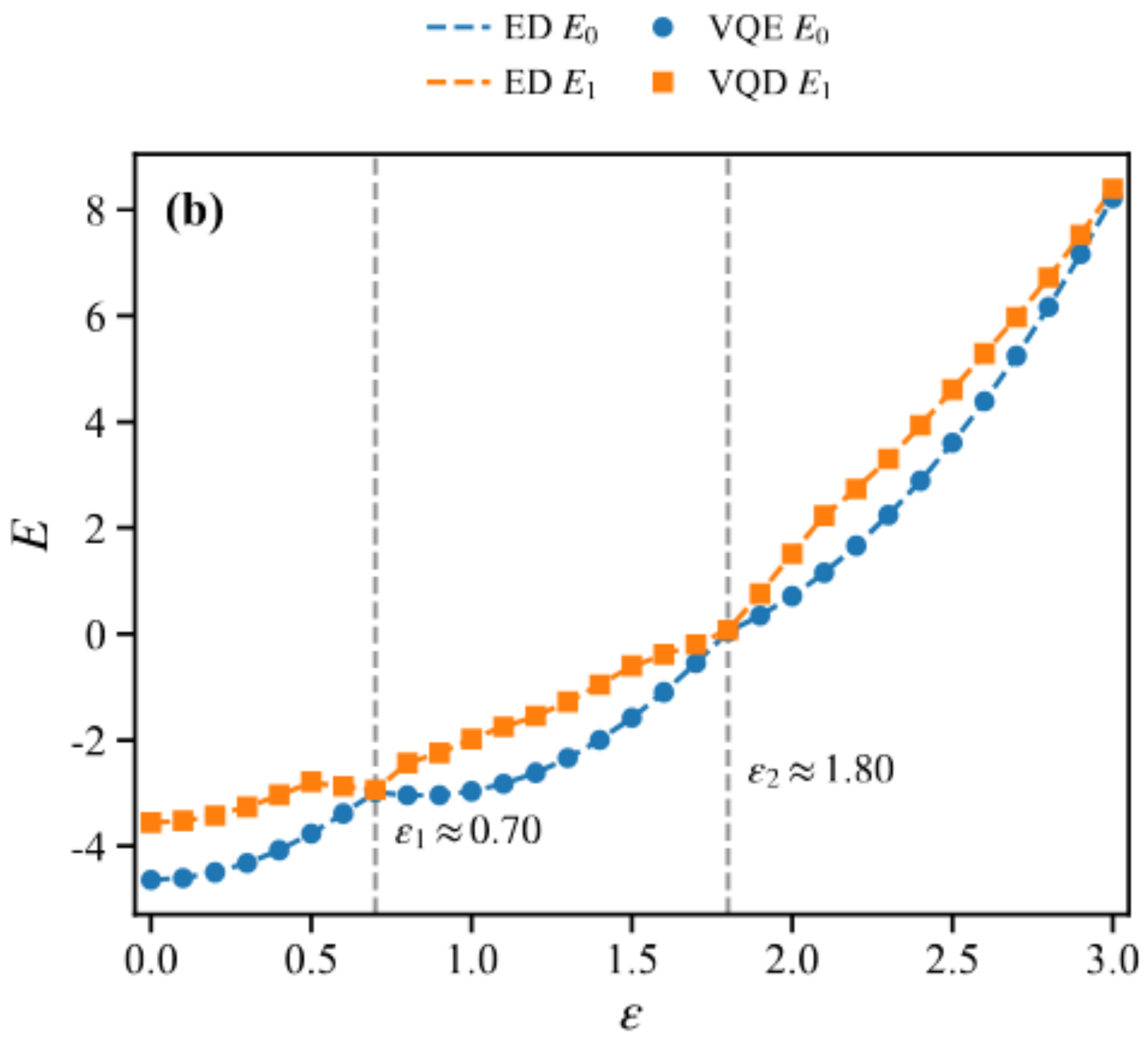
Schwinger Mechanism

- “Jump” of the ground state.
  - First order phase transition at zero temperature.
- But there is no jump in the spectrum.



Chiral condensate of ground state



Energy of ground and first excited state

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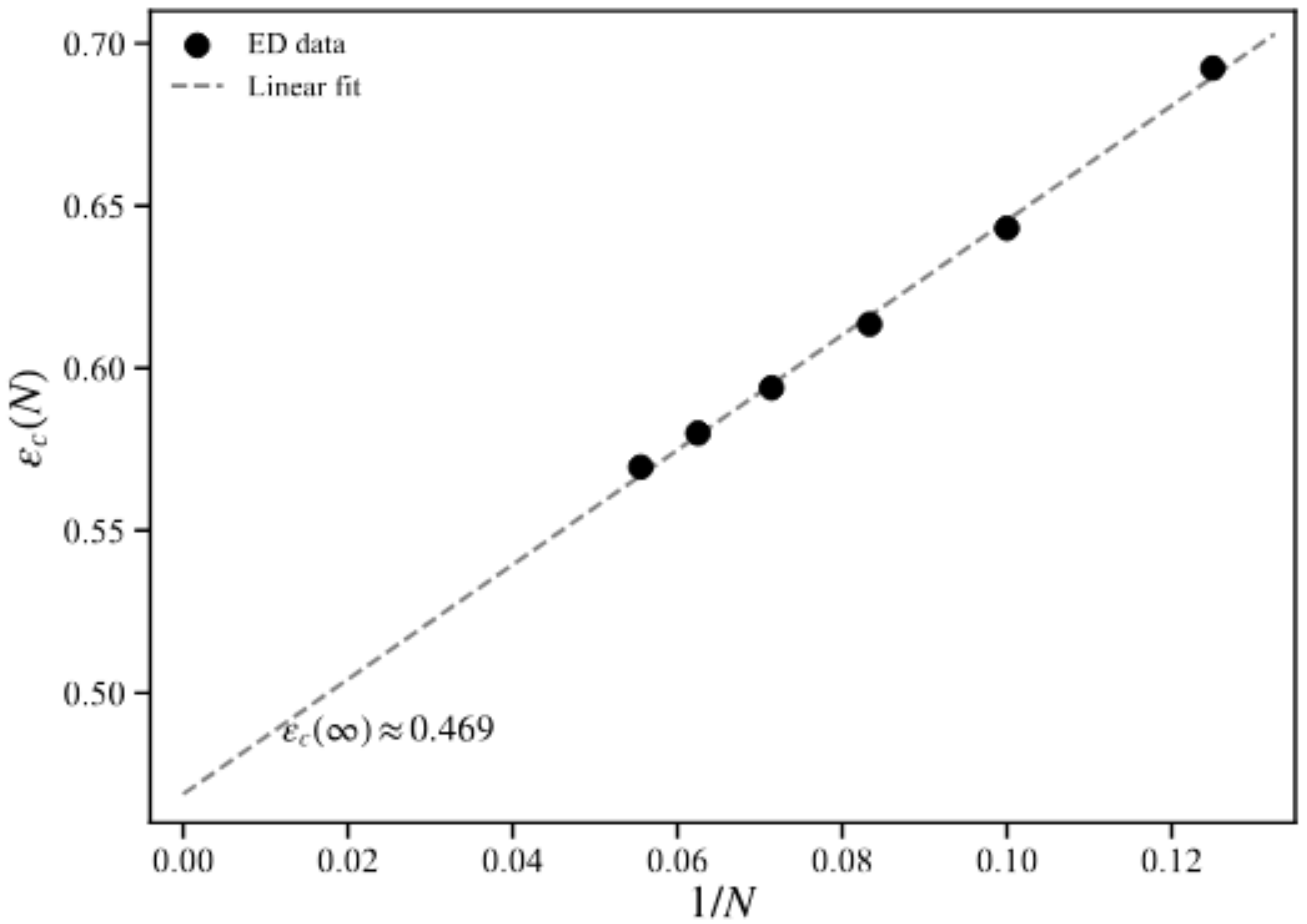
Schwinger Mechanism

- The critical field strength $\epsilon_c \sim 1/N + \epsilon_c(\infty)$.
- $\epsilon_c(\infty)$ is smaller than the theoretical prediction: 0.5.
- Open boundary effect: $\langle H_k \rangle$ gives a negative contribution.



Fitting of the critical field strength

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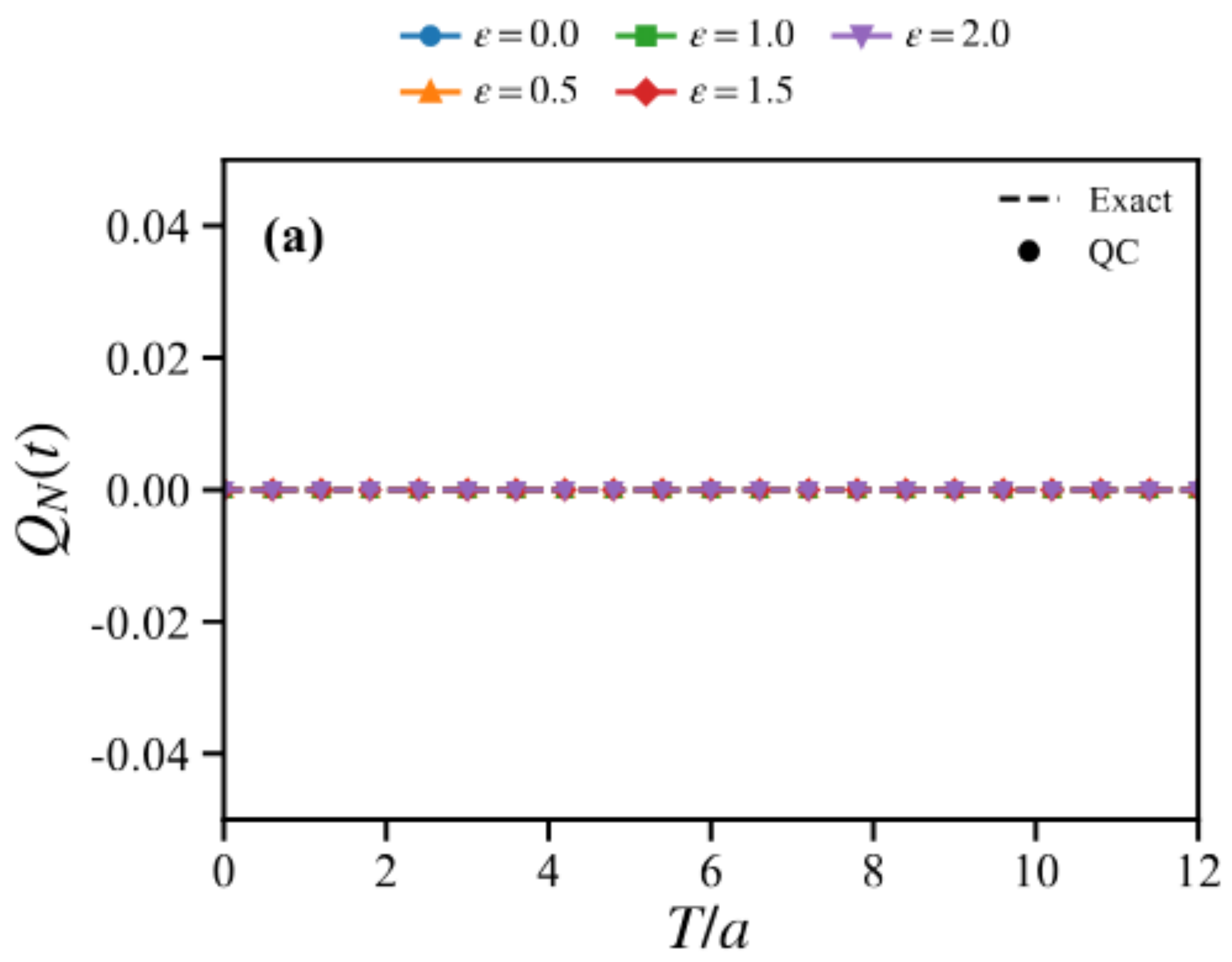
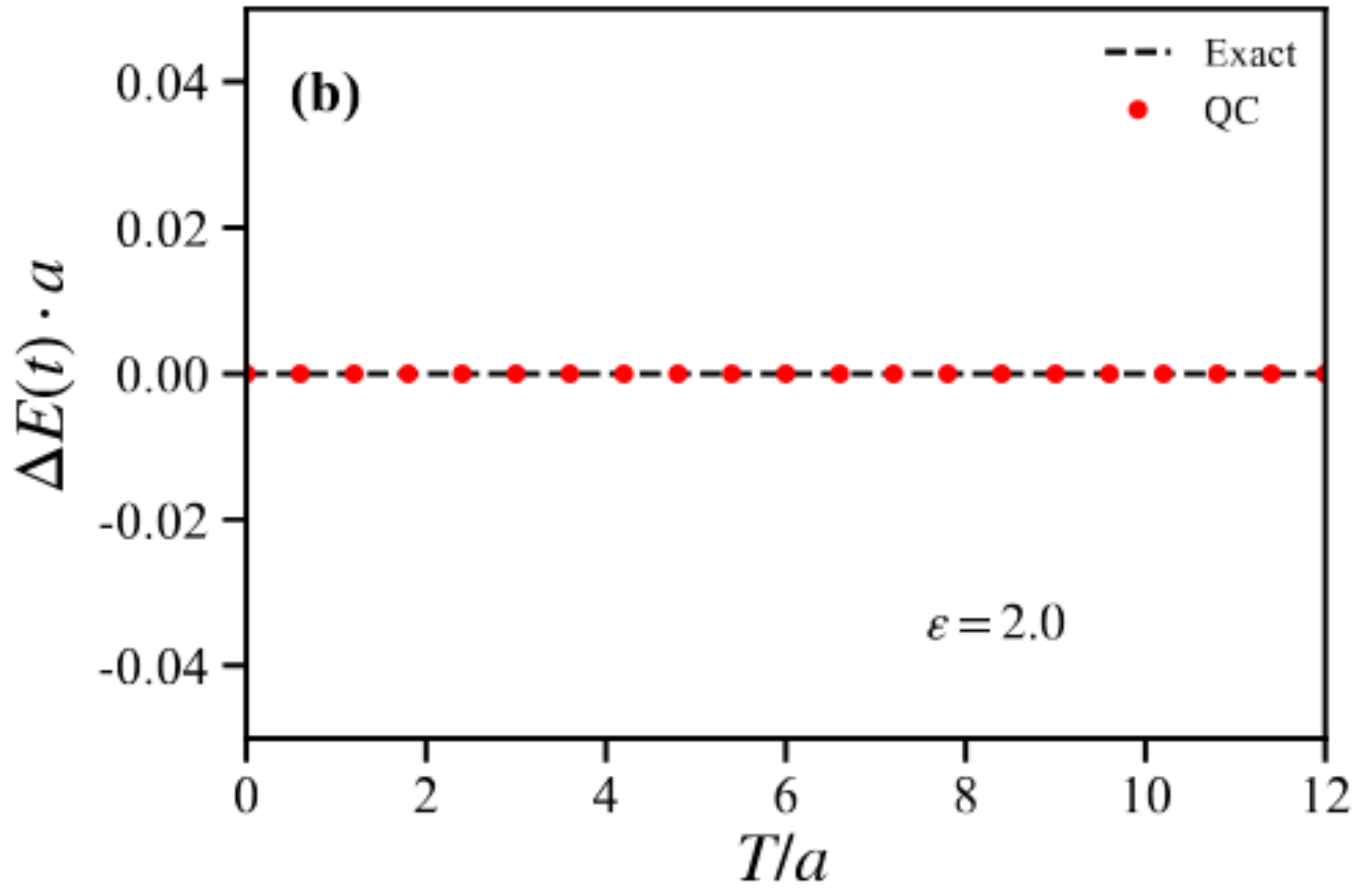
Schwinger Mechanism



Time evolution of total charge



Time evolution of energy shift

- The charge and energy conservation is ensured.

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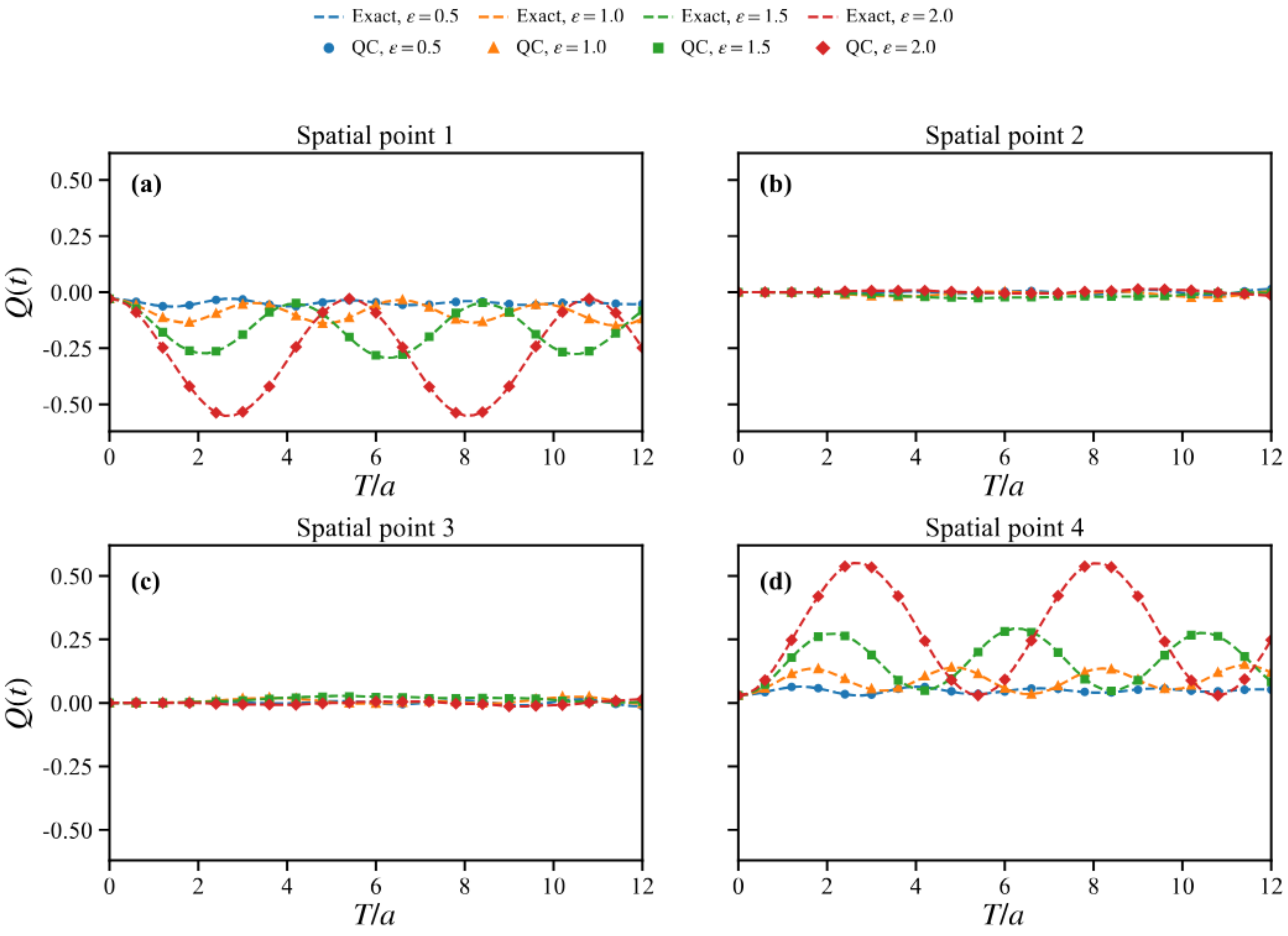
Schwinger Mechanism

- A pair of charge is periodically produced at the boundary.
- No critical field strength observed.
- Consistent with theoretic prediction.



Time evolution of spatial charge density

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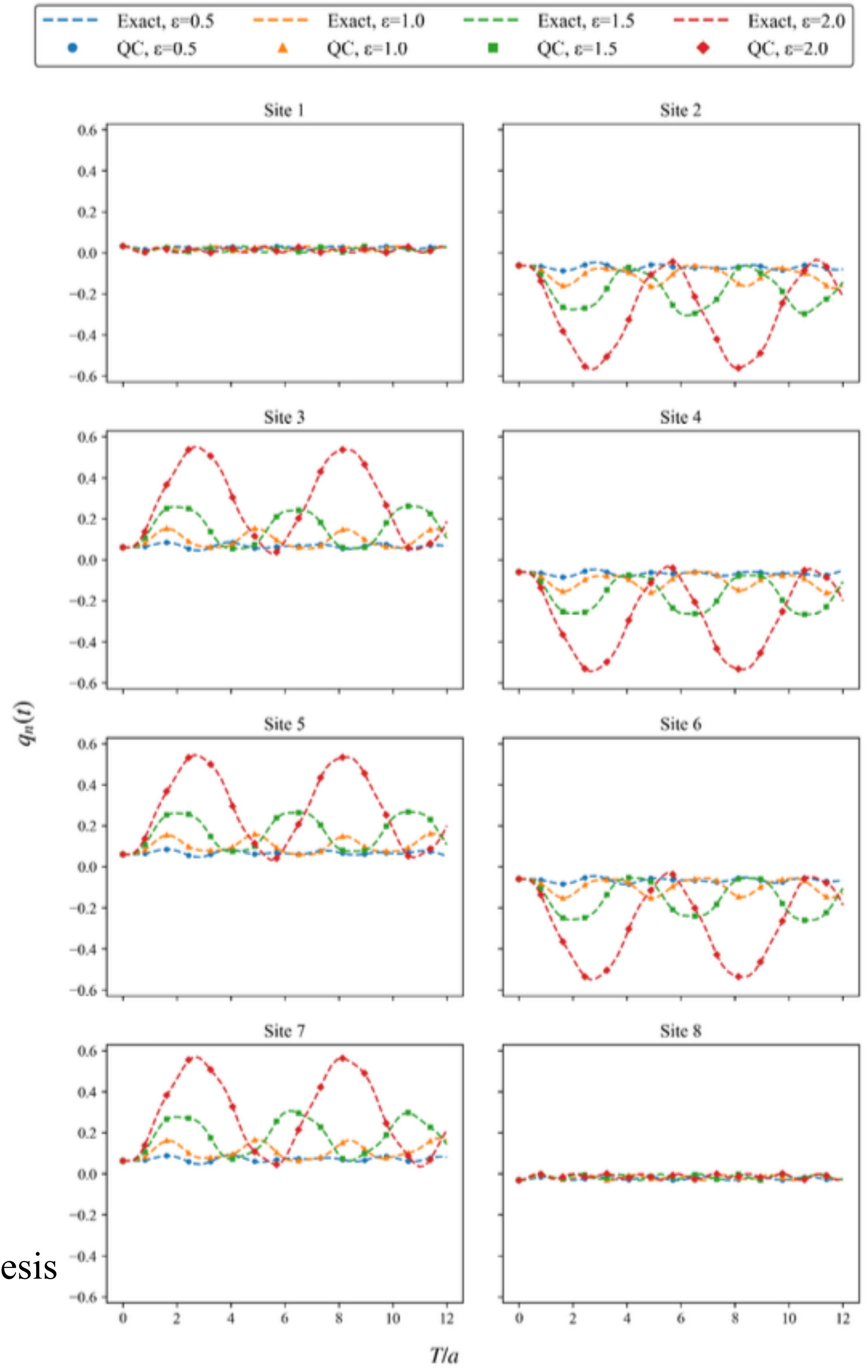
Schwinger Mechanism

- Particle-antiparticle pairs are also produced in the middle.
- Deviation simple periodic behavior.
  - Contributions from higher modes.



Time evolution of charge density on each lattice site

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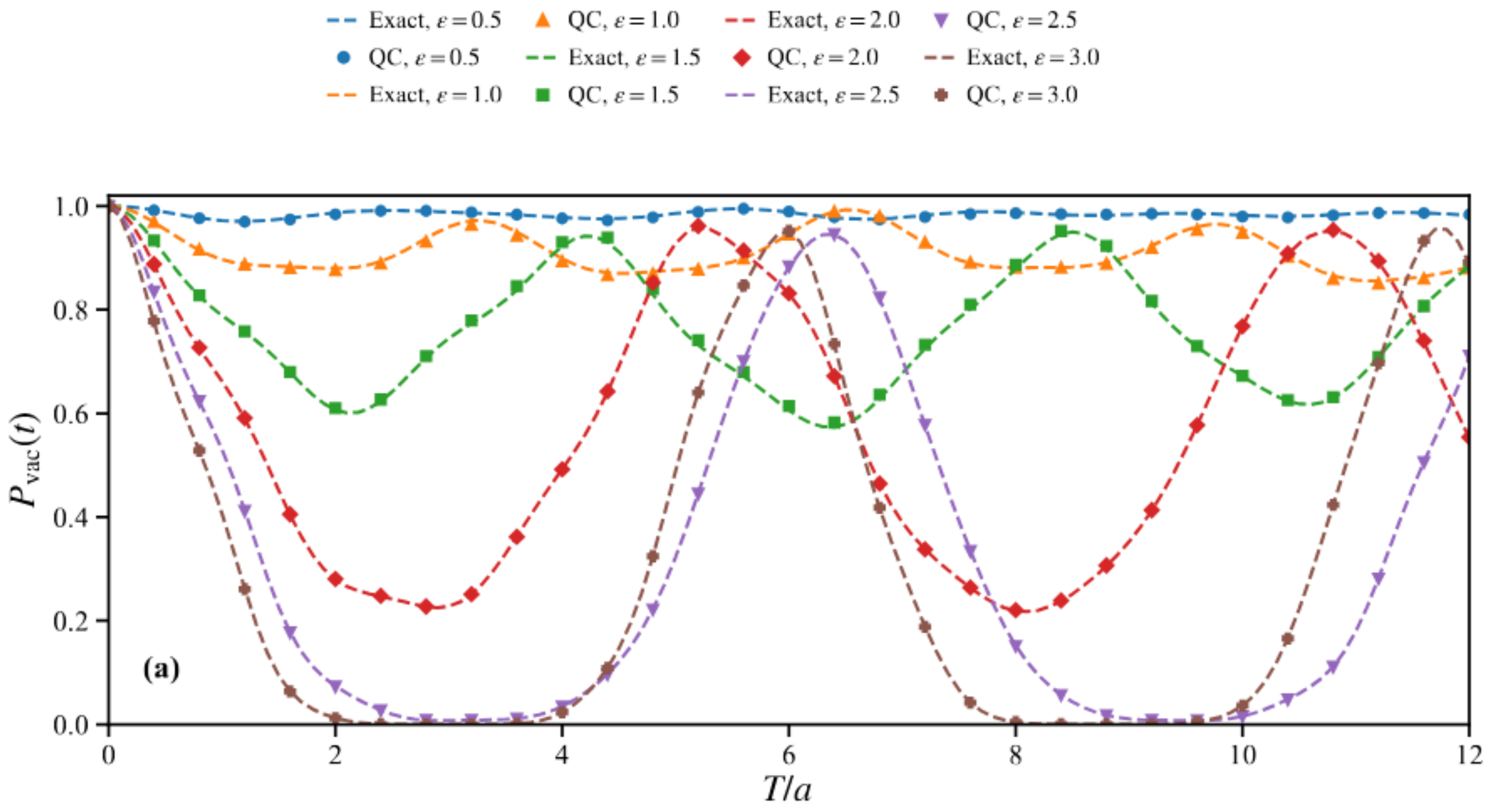
Schwinger Mechanism

- Vacuum state fidelity

$$P_{\mathrm{vac}}(t) = |\langle \psi_0 | \psi(t) \rangle|^2$$

- Describes how far the system deviates from the initial state.



Time evolution of vacuum state fidelity

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Schwinger Mechanism

- Vacuum state fidelity

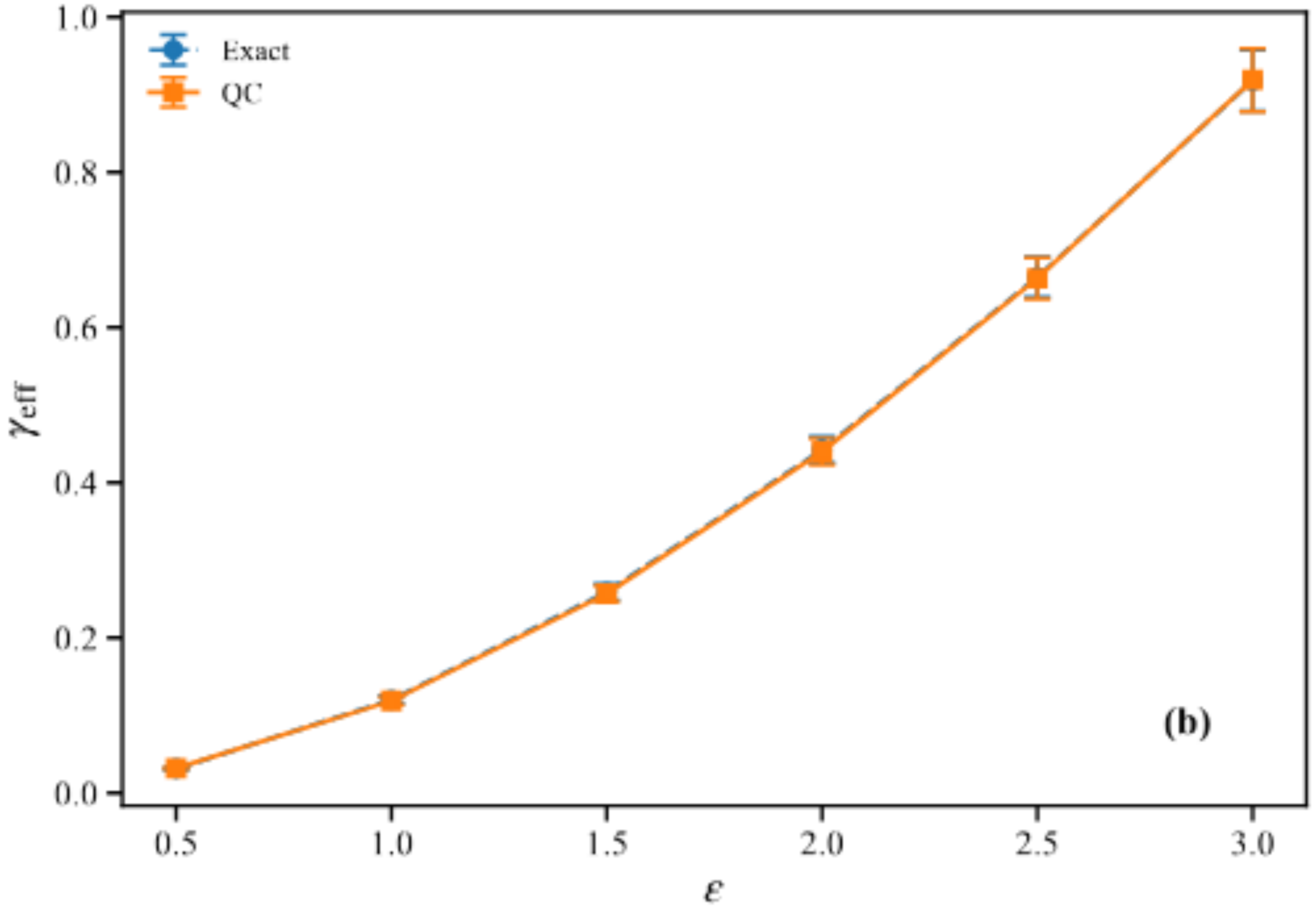
$$P_{\text{vac}}(t) = |\langle \psi_0 | \psi(t) \rangle|^2$$

- Describes how far the system deviates from the initial state.
- Fitting $(0 \leq T/a \leq 1.0)$:

$$P_{\text{vac}}(t;\varepsilon) \simeq A(\varepsilon)\exp\left[-\gamma_{\text{eff}}(\varepsilon)t\right]$$

- In theory, pair production rate:

$$\Gamma \sim E^2 \exp(-a/E)$$



Fitted value of γ_{eff}

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Confinement: String Breaking

- String tension: difference between free energy with and without external field

$$\sigma_\epsilon(\beta) \sim F_\epsilon(\beta) - F_0(\beta) - c$$

$$F = E - TS$$

$$E = \langle H \rangle = \mathrm{tr}(\rho H),\ S = -\mathrm{tr}(\rho \ln \rho)$$

  - Preparation of density matrix ρ
  - Efficient way to calculate entropy S
    - Calculate log of a large matrix is expensive

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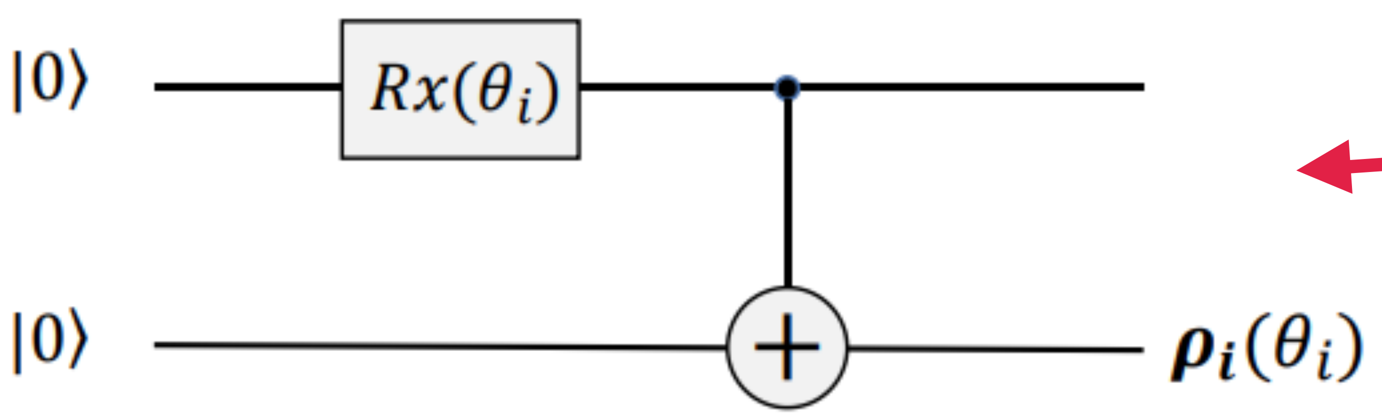
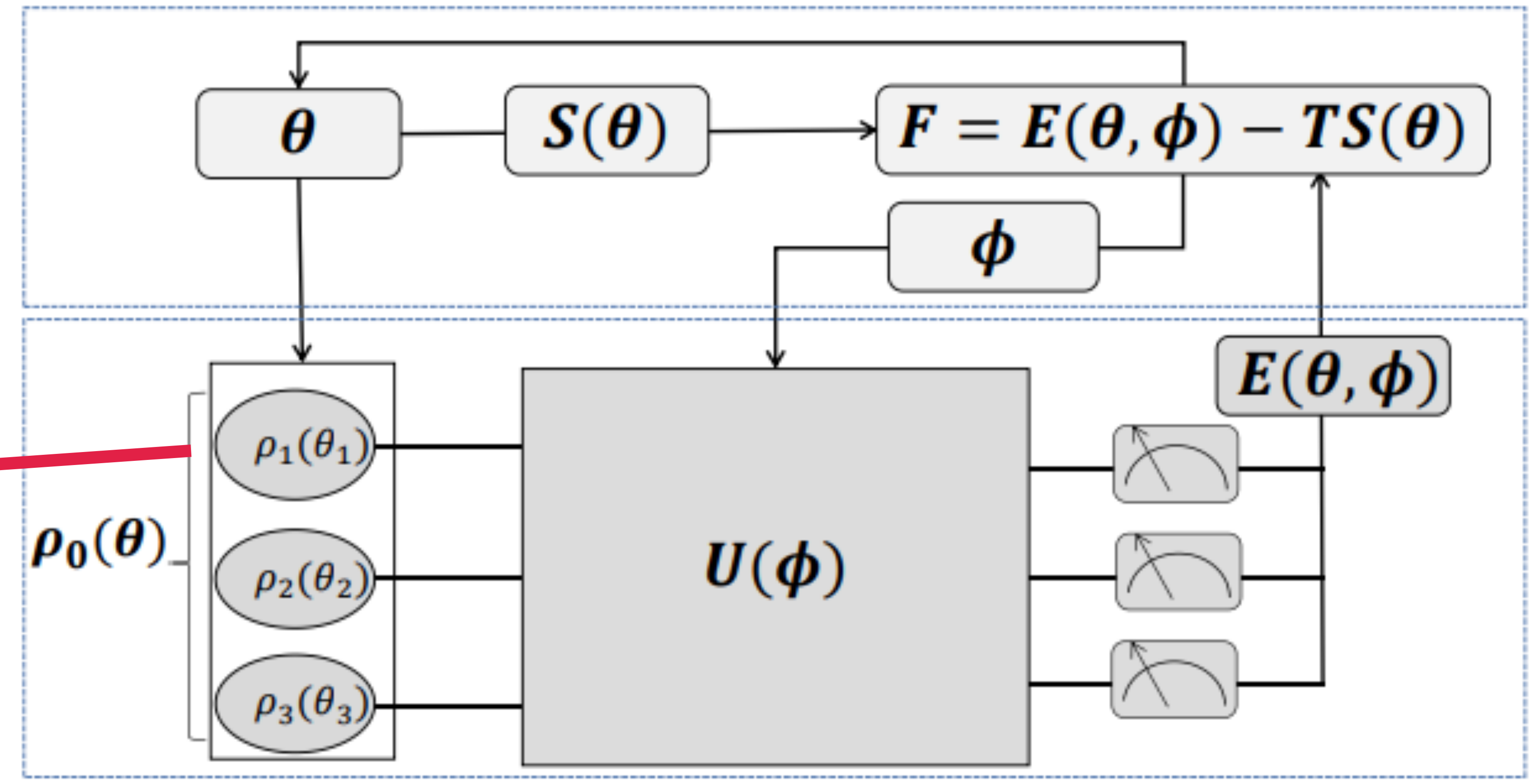
Confinement: String Breaking

- Thermal state preparation: VQA+thermofield double state
  - Assumption: $\rho = U^{\dagger}(\phi)\left[\rho_1(\theta_1) \otimes \rho_2(\theta_2) \otimes \cdots\right] U(\phi)$
    - Require certain pattern in the spectra.
  - $S = -2\sum_i \left(\sin^2\theta_i \log\sin\theta_i + \cos^2\theta_i \log\cos\theta_i\right)$
  - Loss function: the free energy itself.





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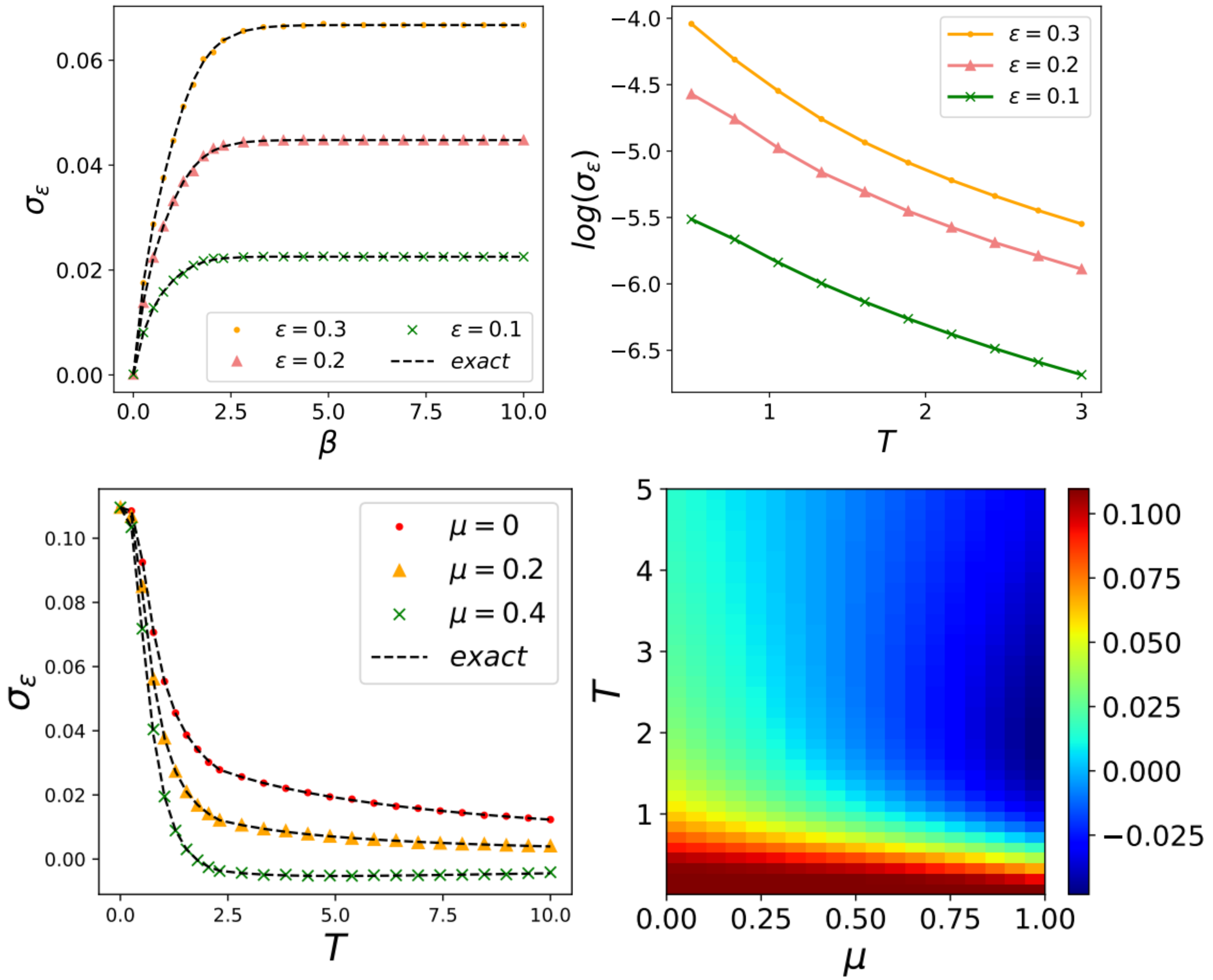
String Breaking in Schwinger Model

- The variational algorithm is accurate.
- The results are qualitatively in agreement with other models.



String tension at different T and μ

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Heavy Quark Potential

- σ can also be related to the potential between a pair of heavy quarks.
  - Heavy quarks are usually treated as statical ($m \to \infty$)
  - Now ϵ is fixed by the charge of the heavy quark.
  - But the distance between heavy quarks can vary.
- Quarkonium dissociation: color screening

Heavy Quark Potential

- Thermal state preparation: a different variational method

$$\rho = e^{-\beta H}/Z = U^{\dagger}\mathrm{diag}(p_1, p_2, \ldots)U$$

$$p_i = \frac{e^{-\beta E_i}}{\sum_j e^{-\beta E_j}}$$

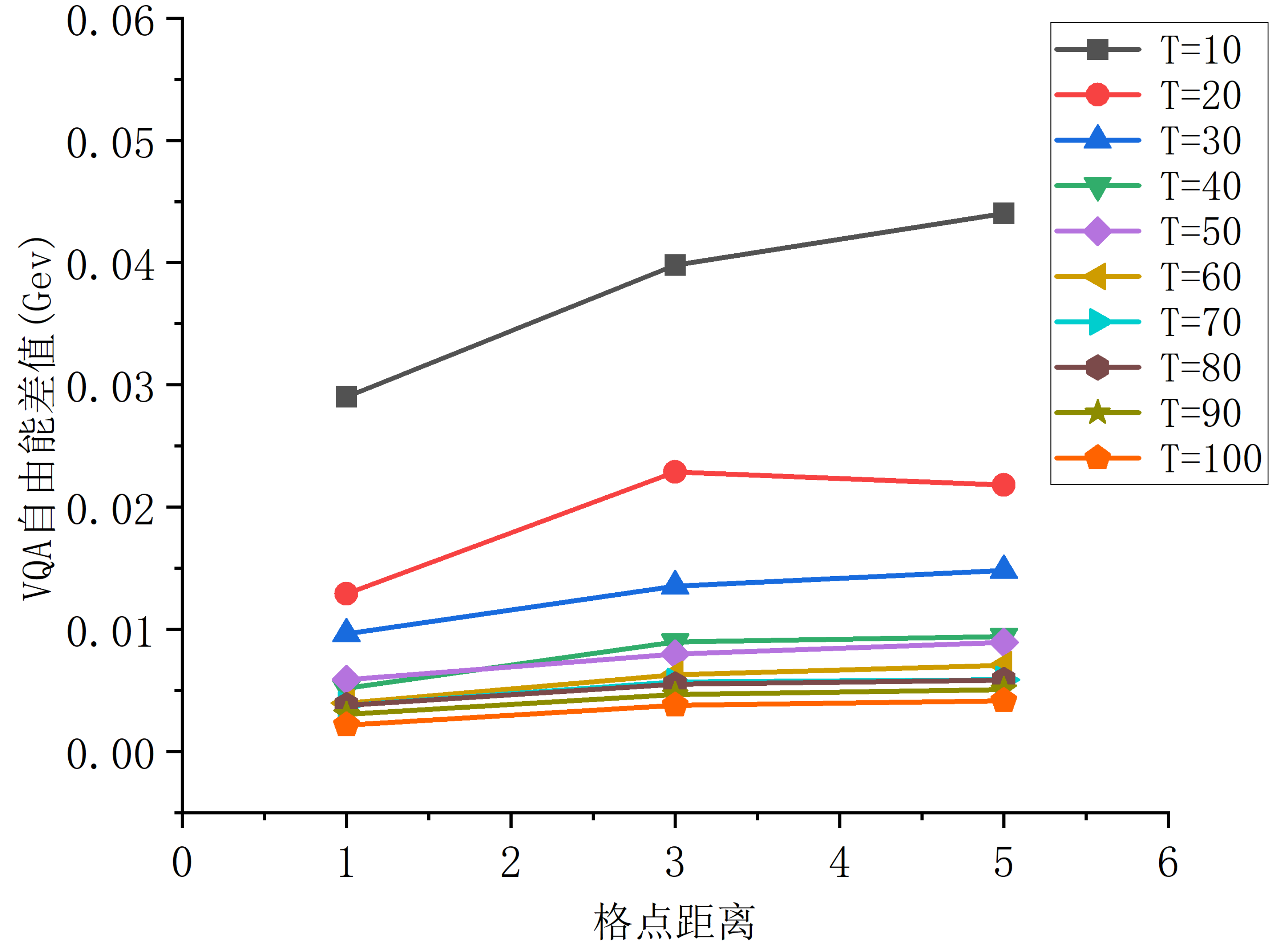
- Variational Ansatz:

$$\rho(\theta,\beta) = U^{\dagger}(\theta)\left(\sum_i p_i(\beta)\,|\varphi_i\rangle\langle\varphi_i|\right)U(\theta)$$

  - $S = -\sum_i \left(p_i \log p_i\right)$
  - The variational parameters are independent of β.
  - No ancillary qubits needed.

Heavy Quark Potential

- The potential drops with temperature
- The slope of potential also seems to drop
  - Need more points to confirm



String tension vs. lattice distance

Chiral Phase Transition

- With quantum simulation, we can explore high chemical potential region.
- We can also consider magnetic field, chiral chemical potential, etc.
- The physics is different at lower dimensions, but the quantum algorithms should be universal.
- Order parameter: chiral condensate $\sigma = \langle \bar{\psi}\psi \rangle$

Chiral Phase Transition in SU(2) Model

- Non-Abelian gauge models:
  - Closer to QCD
  - “Baryon”s and “Meson”s
  - More complicated, but no fundamental difficulties in 1+1D.

Chiral Phase Transition in SU(2) Model

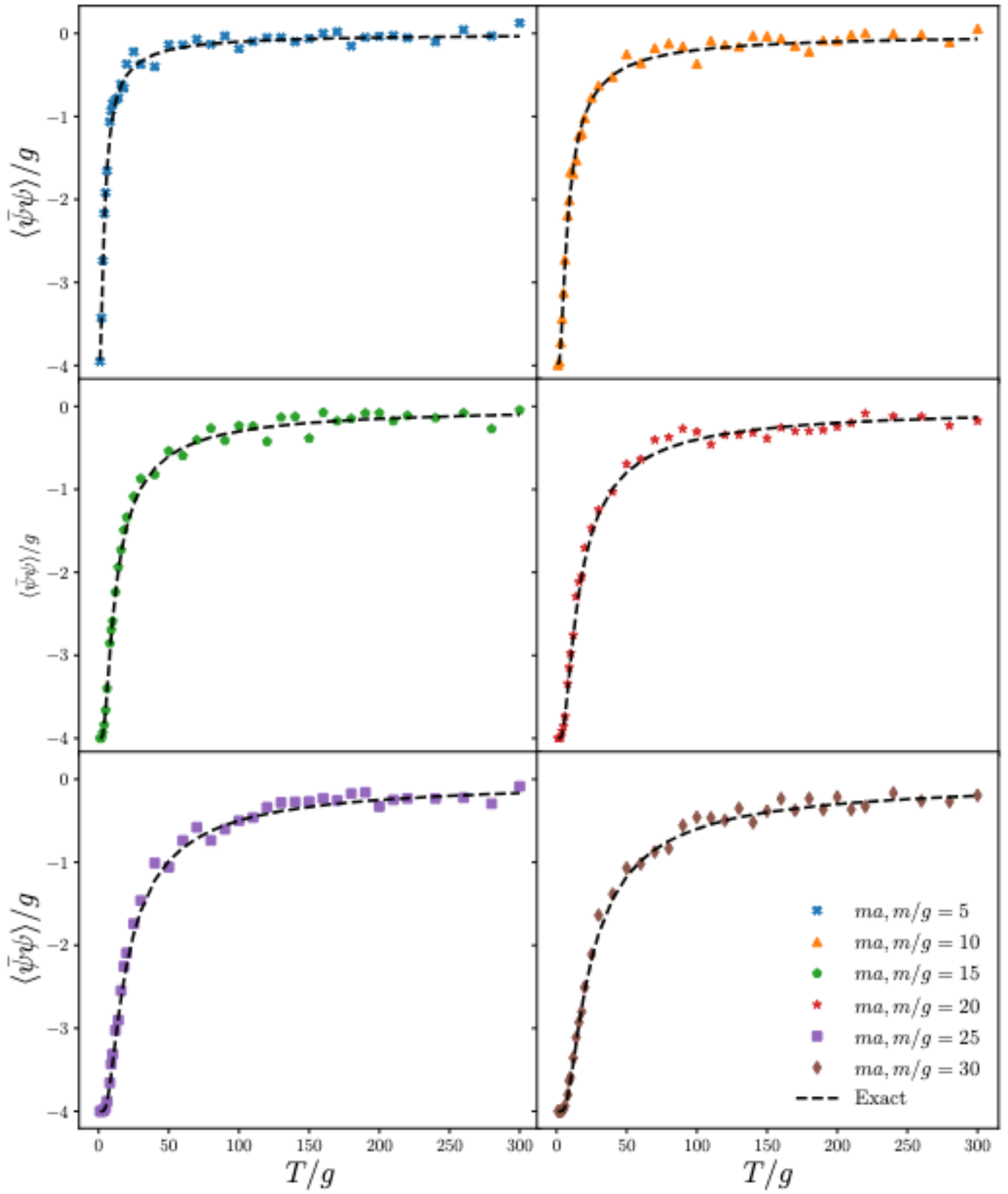
- Variational method with Monte-Carlo:

$$\rho(\theta,\beta) = U^{\dagger}(\alpha)\left(\sum_i p_i(\beta)\,|\,\varphi_i\rangle\langle\varphi_i\,|\right)U(\theta)$$

  - With more qubits, one cannot traverse all φ_i
  - A Monte-Carlo method is used in both optimization and measurements.
    - Optimizaton: a mini-batch of ~20 states per step
    - Measurements: Metropolitan algorithm, flip one qubit in each step.

Chiral Phase Transition in SU(2) Model

- Number of sampled states does not increase exponentially.



8 qubits, 1000 states

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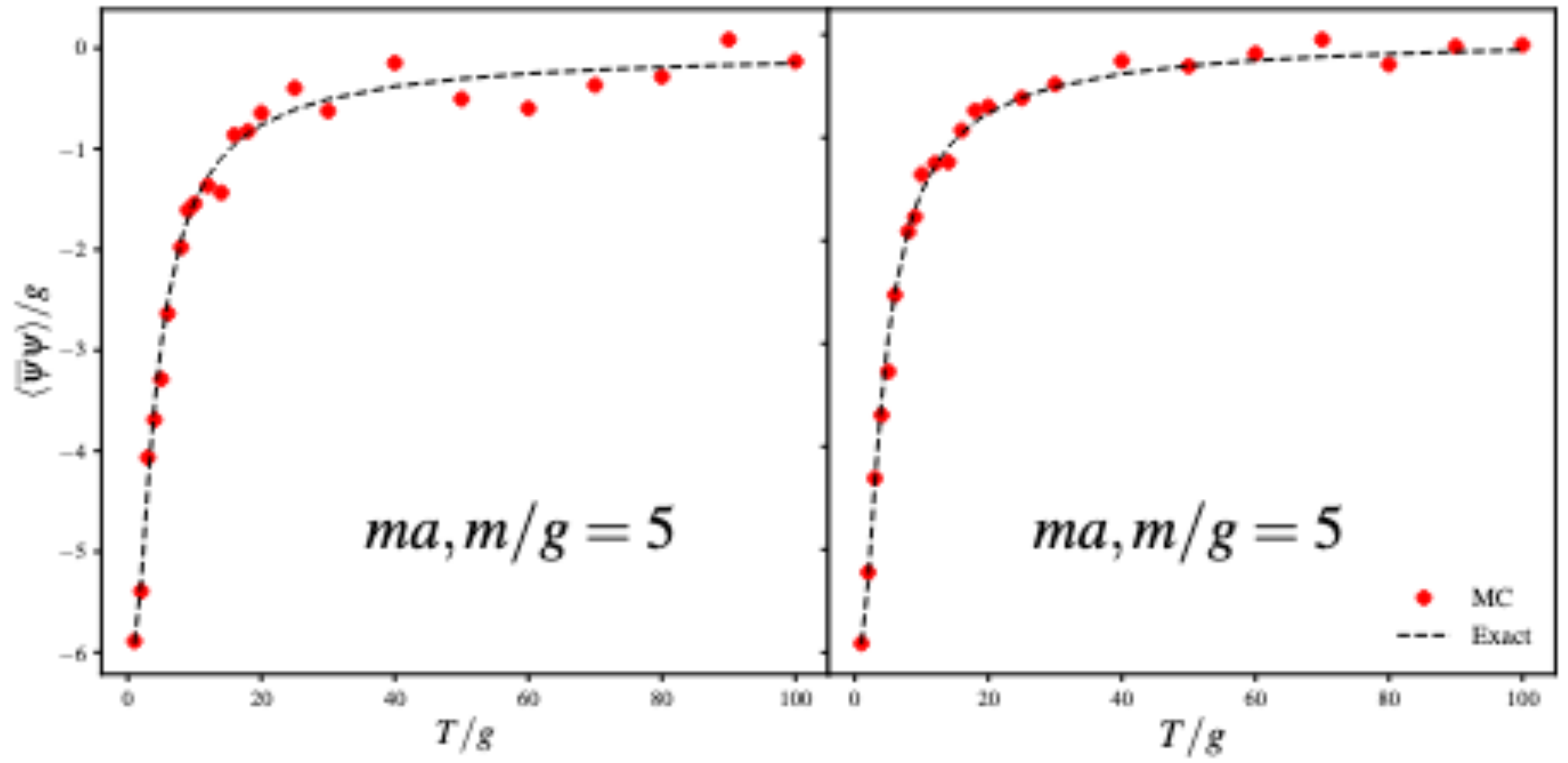
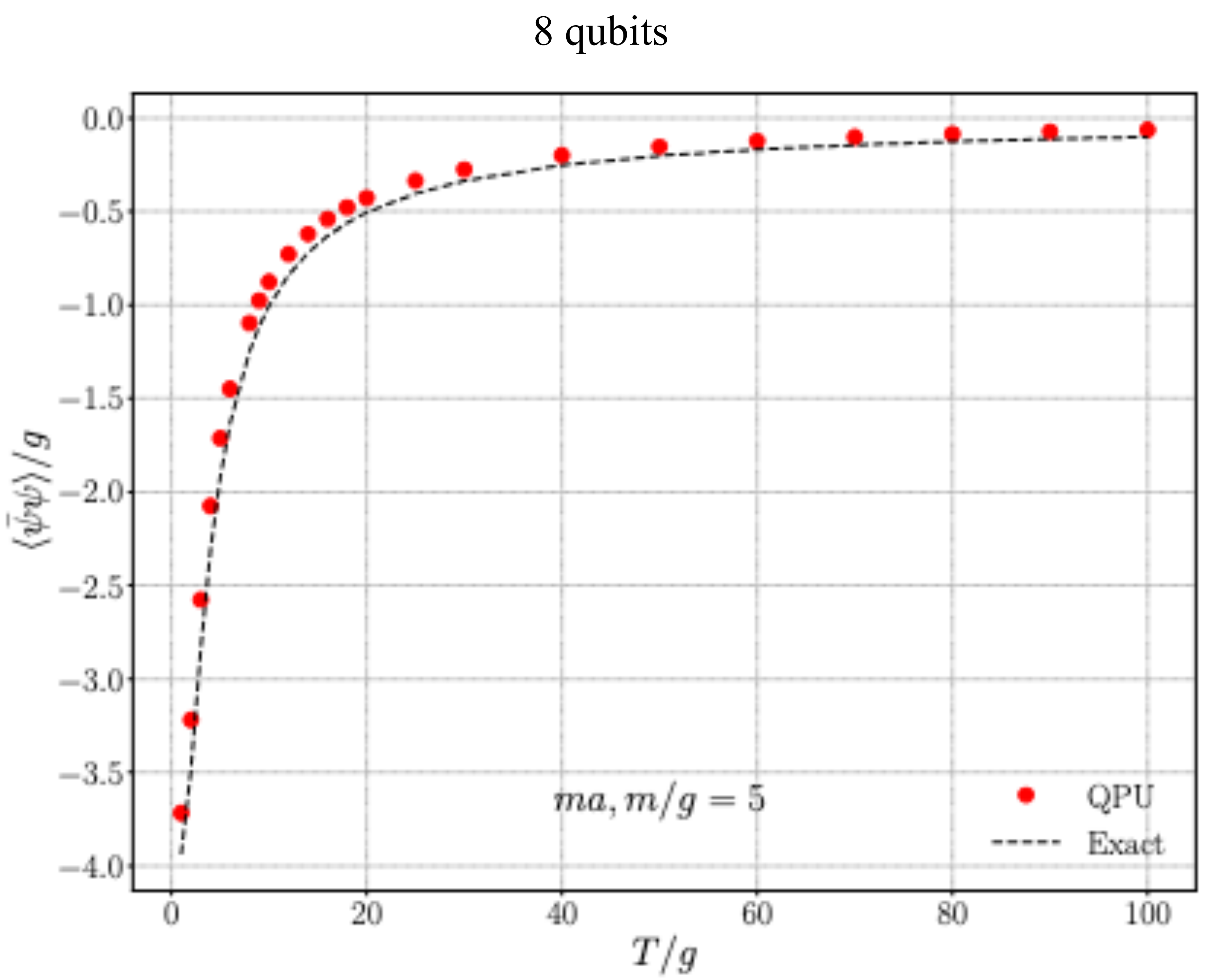


12 qubits, 1000 states

12 qubits, 2000 states

Chiral Phase Transition in SU(2) Model

- Only measurements are done on QC with a simpler ansatz.
- No error mitigation / correction used.
- Our Algorithm runs well on real quantum devices.



Chiral condensate measure on a real QC

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Chiral Phase Transition with B Field

- In principle, no B fields in 1+1D.
  - We can consider this as an "effective" 1D system, like a 1D atom chain.

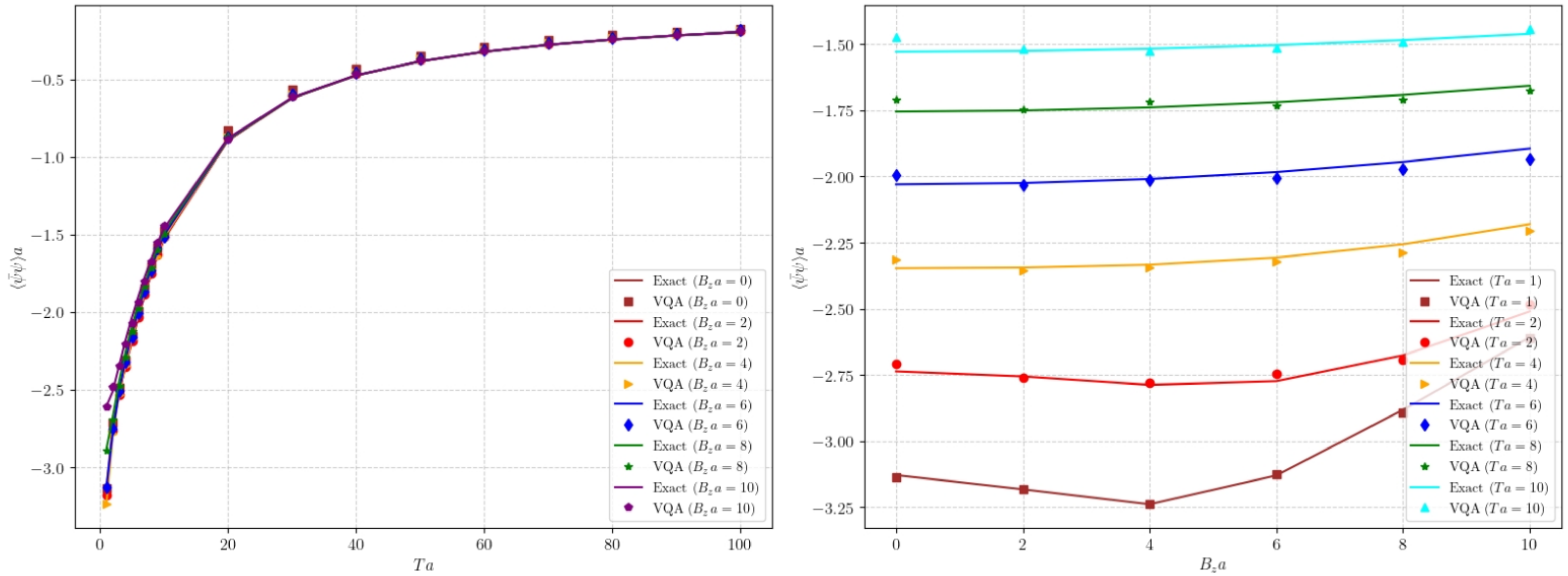
$$H=\bar{\psi}\left(-i\gamma^{1}\partial_{1}+m\right)\psi-g(\bar{\psi}\psi)^{2}-B\bar{\psi}\gamma^{3}\psi$$

- With γ^3, now ψ has 4 components.
- We can still do J-W.

Lvfei Wang's master thesis

Chiral Phase Transition with B Field

- In most region, chiral condensate decreases with magnetic field (inverse magnetic catalysis)
- There is a sign of magnetic catalysis at high μ and low T



String tension vs. temperature (left) and B field (right) at $\mu a = 6$

Lvfei Wang’s master thesis

Summary

- We studied various topics related to real-time dynamics and finite-temperature properties of 1+1D gauge models.
- Our results show that current quantum algorithms can prove useful for studying many topics in HEP.