Hamiltonian lattice gauge theory

Application to nonequilibrium and dense QCD matter

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References:

Hayata, YH, PRD 103 (2021) , 094502, JHEP 09 (2023) 123; JHEP 09 (2023) 126, Phys. Rev. D 111 (2025) 034513
Hayata, YH, Kikuchi PRD 104 (2021) 7, 074518, 2601.13530
Hayata, YH, Nishimura, JHEP 07 (2024) 106
YH, Tanizaki, Yamamoto, Phys. Rev. D 109 (2024) 114502
YH, Yamamoto, Phys. Rev. D 111 (2025) 014510; PTEP 2025 (2025) 9, 093B04
Hayata, YH, Watanabe, 2601.03843
Fujikura, YH 2605.17183

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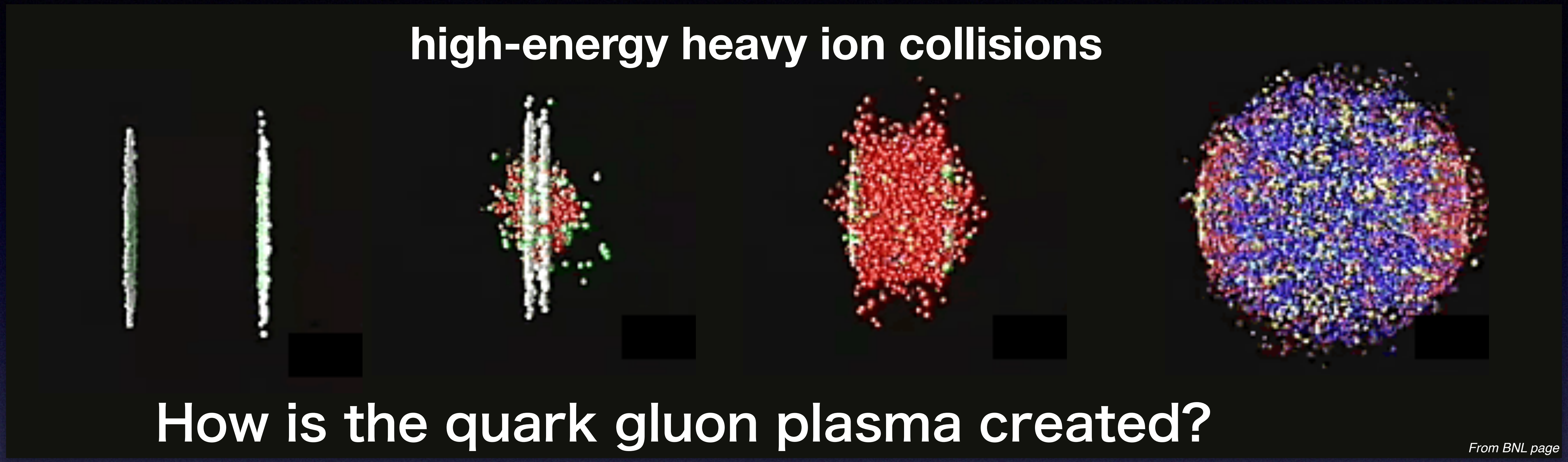
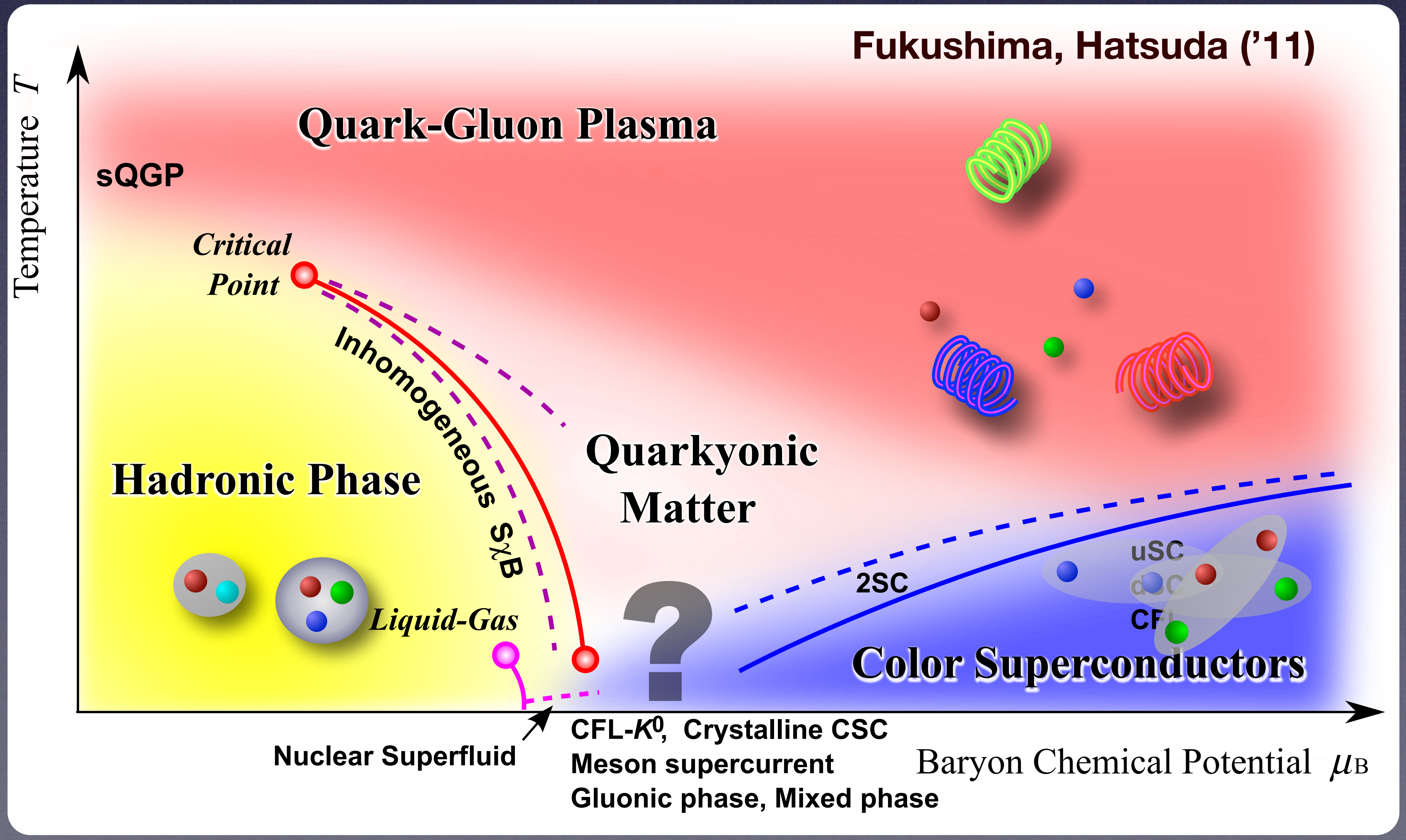
@C3NT workshop: Quantum Information Science in High Energy Nuclear Physics (QIS-HENP) Jun 14 – 19, 2026

Major Challenges in QCD

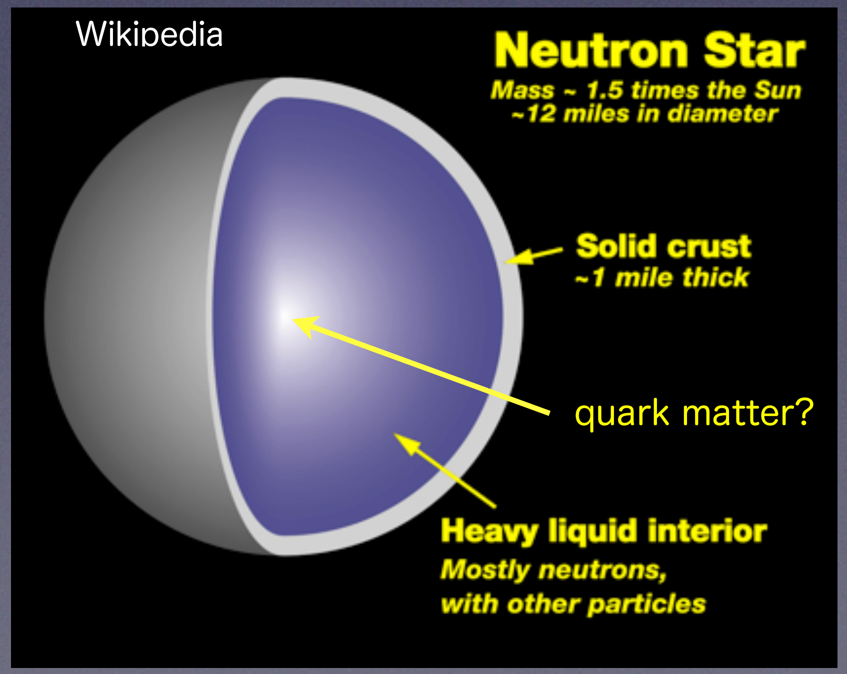
Manybody dynamics of QCD



Dense QCD



What phases are realized in the interior of a neutron star?



Difficulty

Sign problem: Difficulties in first-principles calculations based on importance sampling

$$\langle O \rangle = \int \mathscr{D}A \, \det(D+m) e^{iS} O$$

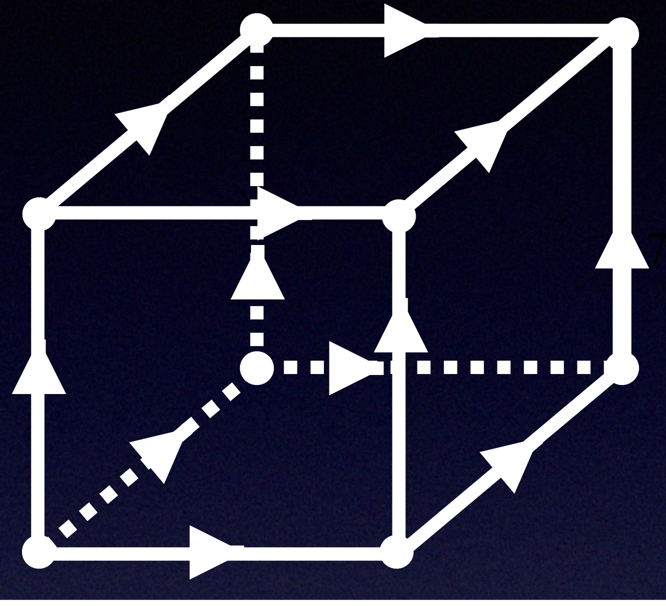
In real-time, finite-density problems, the weight is complex

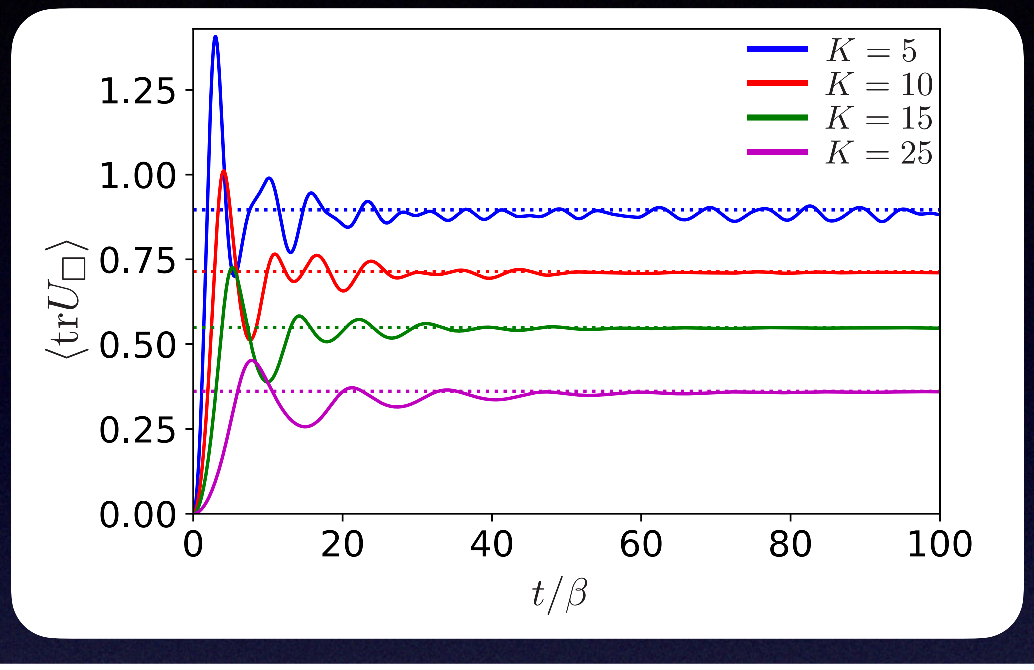
$$\not\approx \frac{1}{N} \sum_j O_j$$

Hamiltonian approach

Directly solve Schrödinger equation!

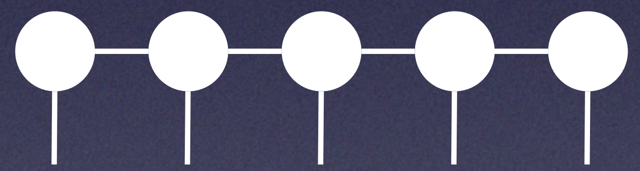
Smaller systems can be simulated directly



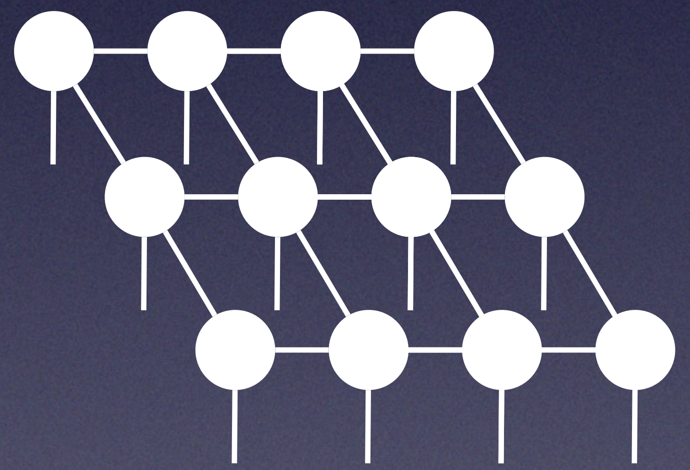


Tensor Networks

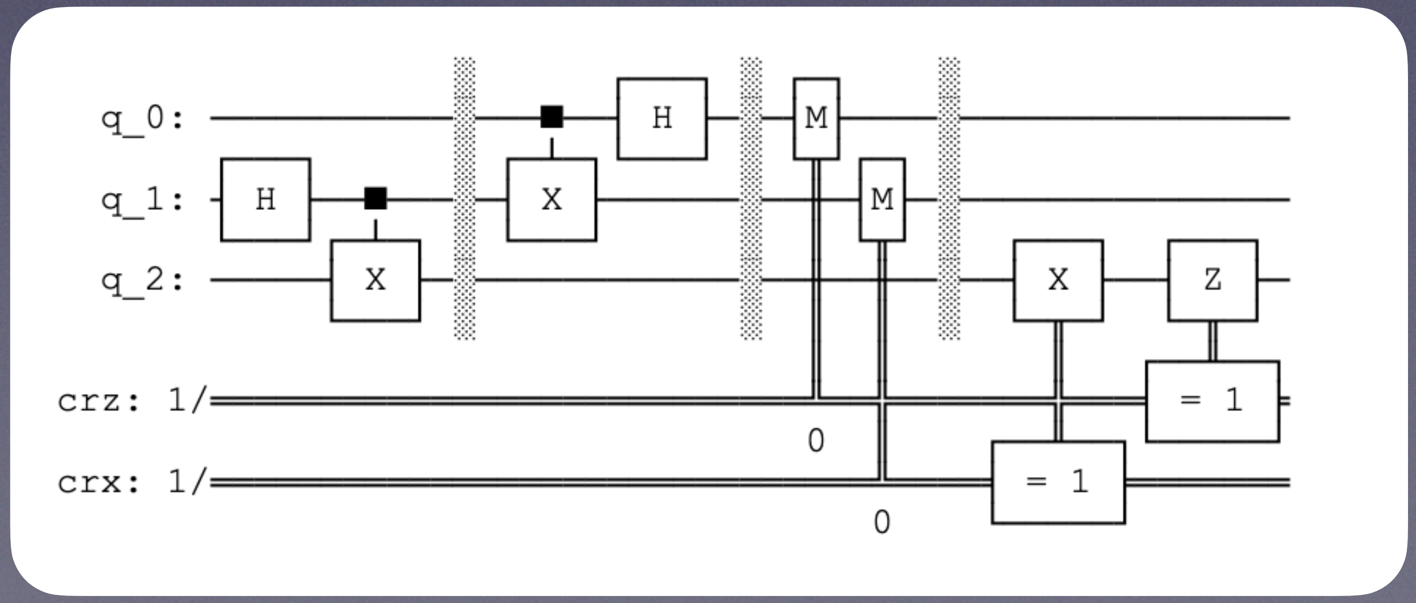
MPS



PEPS



Quantum simulation



Difficulty in Hamiltonian gauge theory

Infinite degrees of freedom

Link variable is continuous (regularization required)

$$U \in \mathrm{SU}(N)$$

continuous

What approximation is compatible with gauge symmetry?

Large gauge redundancy

$$\dim \mathscr{H}_{\mathrm{phys}} \ll \dim \mathscr{H}_{\mathrm{total}}$$

need to solve Gauss law constraint

Outline

- **Formalism**

  - Kogut-Susskind Hamiltonian formalism

- **Application**

  - Confinement-deconfinement phase transition in mean field approximation (JHEP 09 (2023) 123)
  - Thermalization on a small lattice (Phys. Rev. D 103, 094502(2021))
  - QCD_2 at finite density (2605.17183)
  - Quantum computing (2601.13530)

- **Summary**

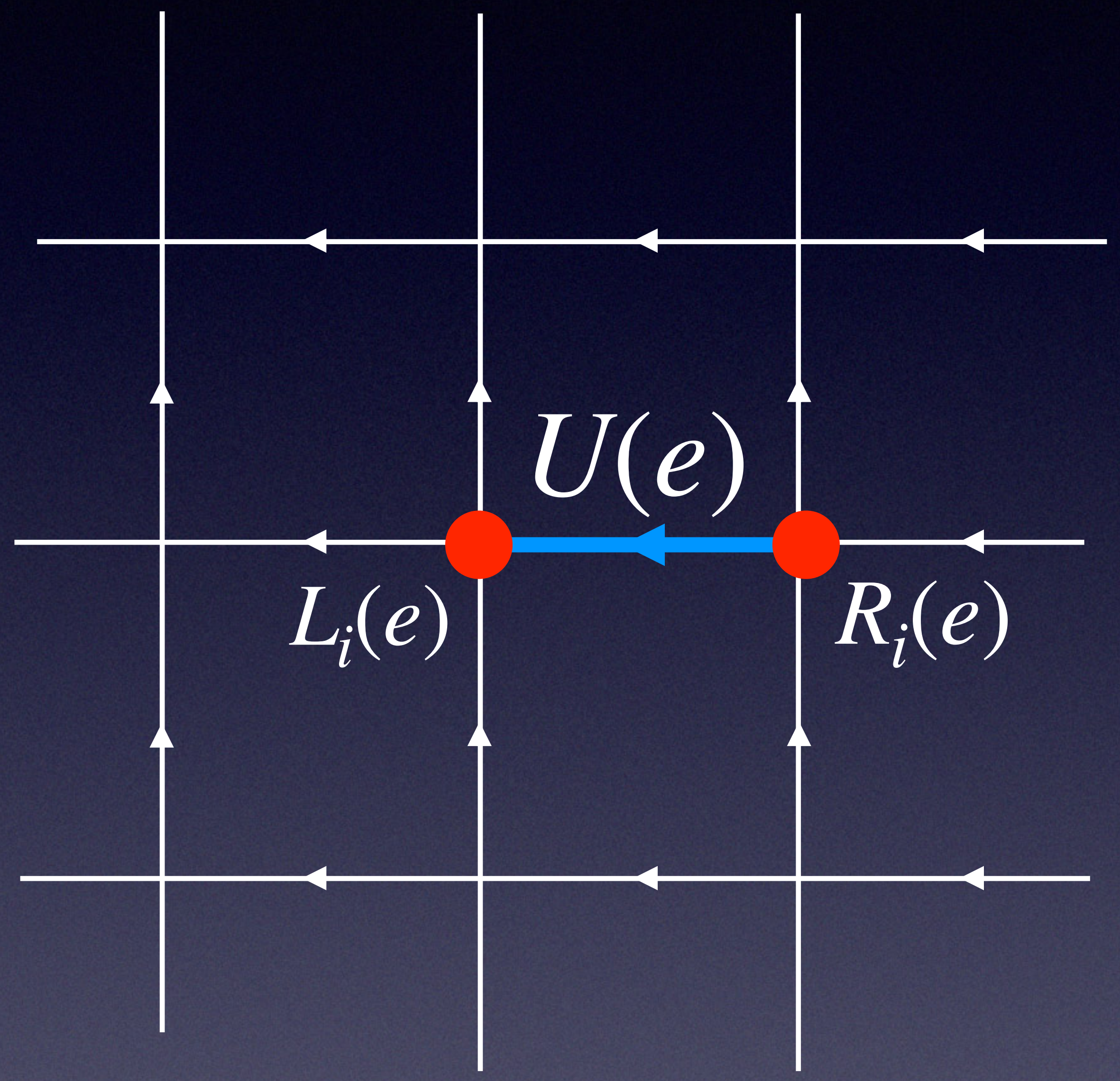
Kogut-Susskind Hamiltonian formalism

Kogut, Susskind ('75)

Kogut-Susskind Hamiltonian formalism

Kogut, Susskind, Phys. Rev. D 11, 395 (1975)

Time is continuous, space is discretized



$e^{i\int A} \to U(e)$:Link variable $\in \mathrm{SU}(N)$ on edge e

$L_i(e), R_i(e)$:Left and right electric fields $\in \mathrm{su}(N)$

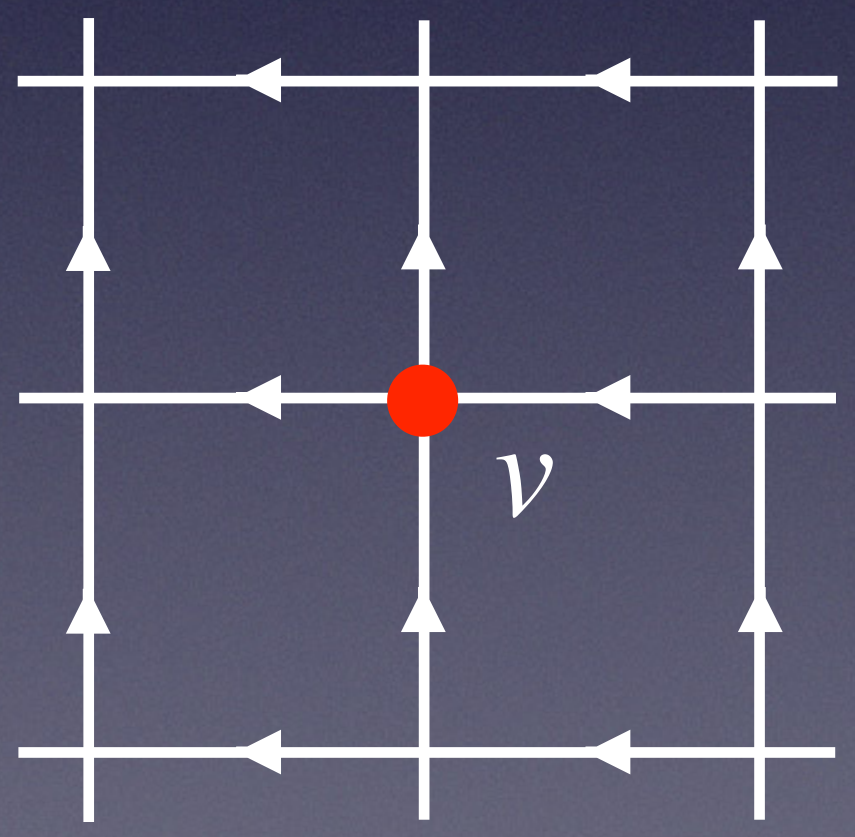
$L_i(e)$ and $R_i(e)$ are not independent

$$[U_{\mathrm{adj}}(e)]_i^{\,j} L_j(e) = R_i(e) \;\Rightarrow\; R_i^2(e) = L_i^2(e) =: E_i^2(e)$$

Commutation relation

$$[A_n^i(x), E_{mj}(x')] = i\delta_{nm}\delta_j^i\delta(x-x')$$

$$[R_i(e), U(e')] = U(e)T_i\delta_{e,e'}$$
$$[L_i(e), U(e')] = T_iU(e)\delta_{e,e'}$$
$$[L_i(e), L_j(e')] = -if^k{}_{ij}L_k(e)\delta_{e,e'}$$
$$[R_i(e), R_j(e')] = if^k{}_{ij}R_k(e)\delta_{e,e'}$$



Gauss law constraint $(D\cdot E)^i|\Psi_{\rm phys}\rangle = 0$

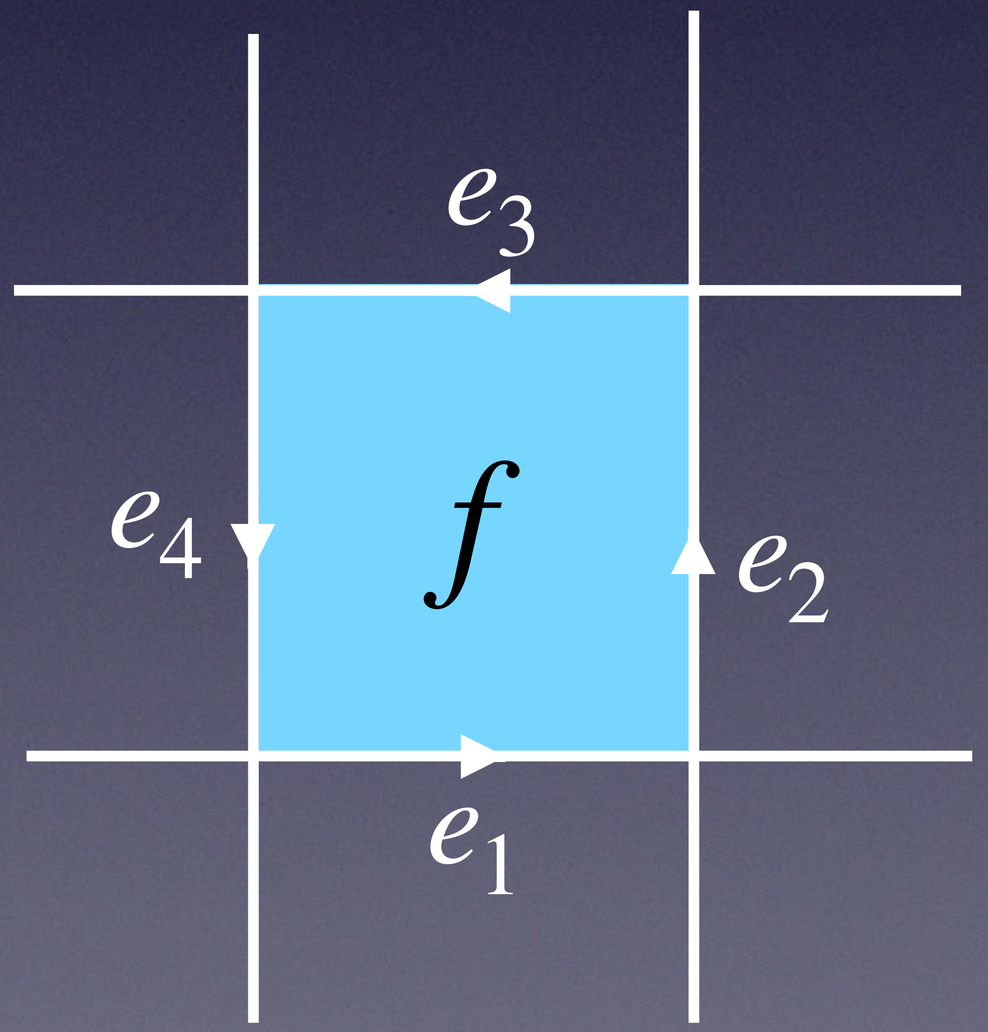
$$\Rightarrow \left(\sum_{e\in C_1|\mathrm{s}(e)=v} R_i(e) - \sum_{e\in C_1|\mathrm{t}(e)=v} L_i(e)\right)|\Psi_{\rm phys}\rangle = 0$$

C_1**:set of edges,** s,t**: source and target functions**

Hamiltonian

$$H = \frac{1}{2}\sum_{e\in C_1}(E(e))^2 - \frac{K}{2}\sum_{f\in C_2}(\mathrm{tr}U(f) + \mathrm{tr}U^\dagger(f))$$

C_2:set of faces



$$U(f) := U(e_4)U(e_3)U(e_2)U(e_1)$$

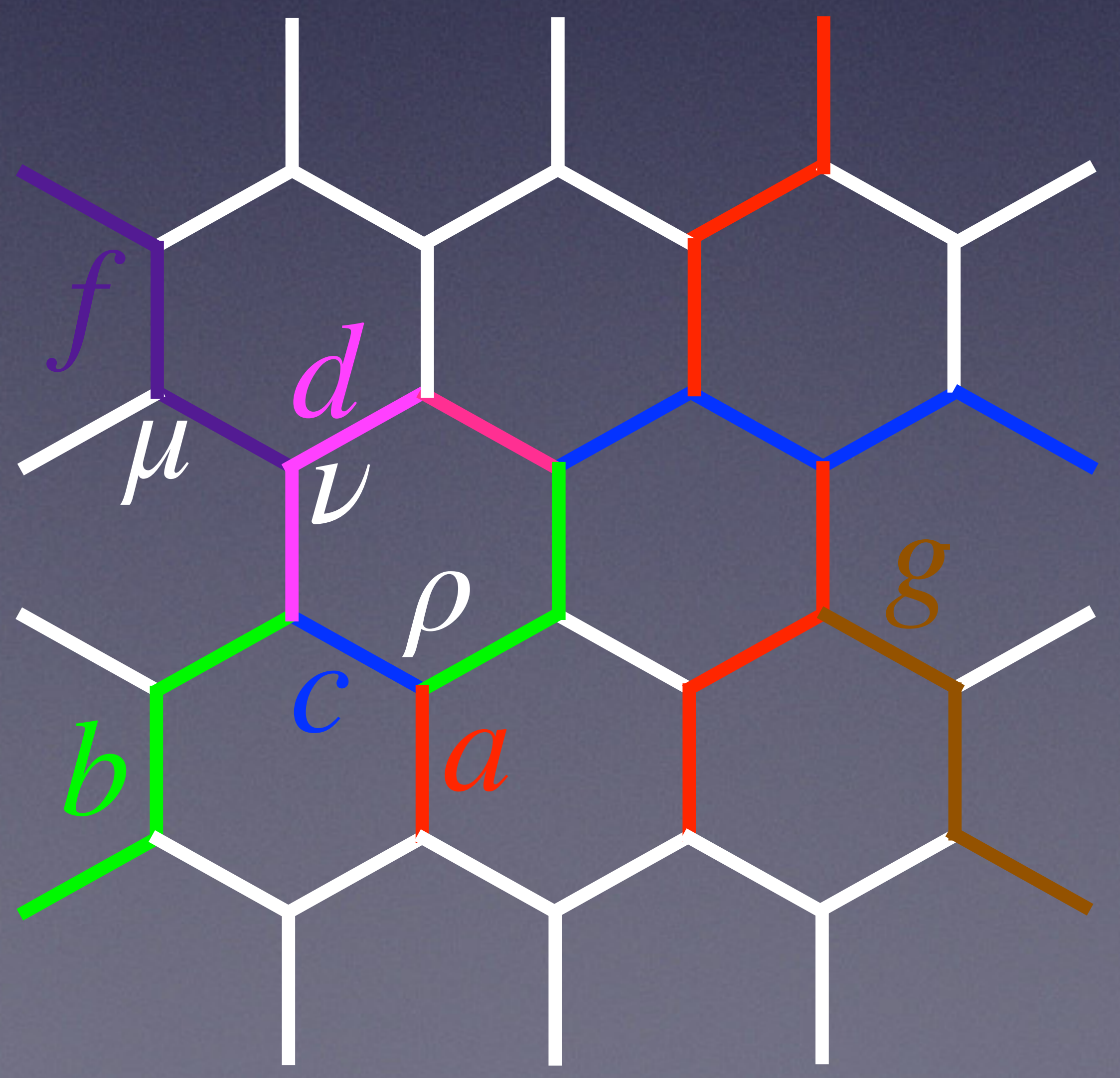
Regularization:

$SU(3) \to SU(3)_k$:Quantum deformation

which preserve properties of gauge symmetry

Solving Gauss law constraint:

Physical states are network of Wilson lines

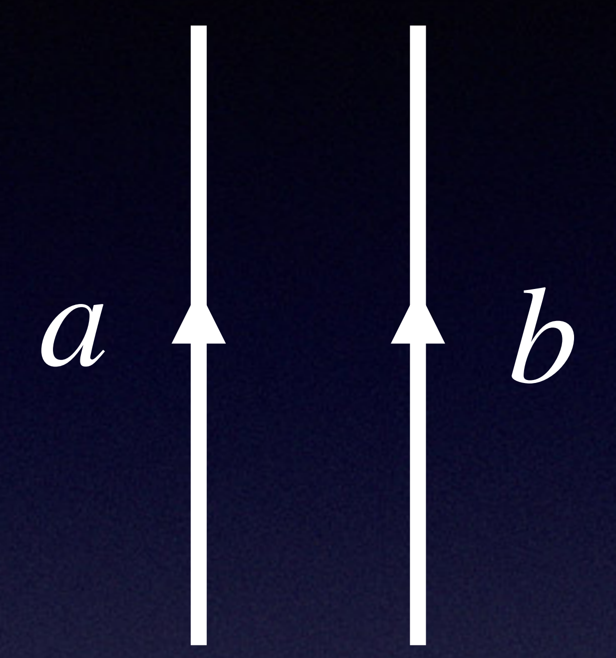


$a, b, c, \cdots$: representation of Wilson lines, e.g., fundamental, adjoint,…

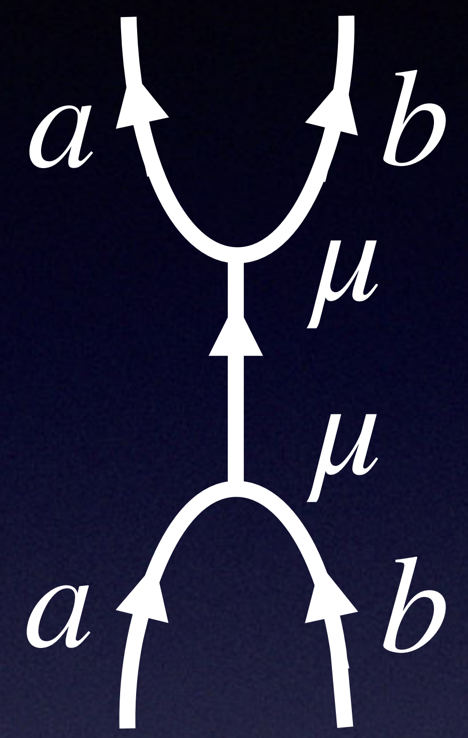
$\mu, \nu, \rho, \cdots \in N^c_{ab}$: Multiplicity index

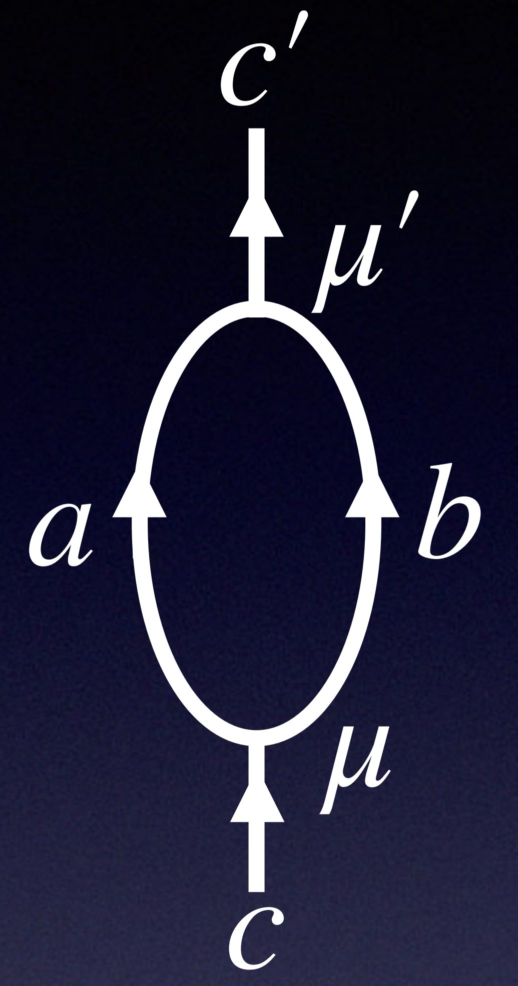
with fusion rule $a \times b = \sum_c N^c_{ab}$

Algebra of Wilson lines for $\mathrm{SU}(3)_k$

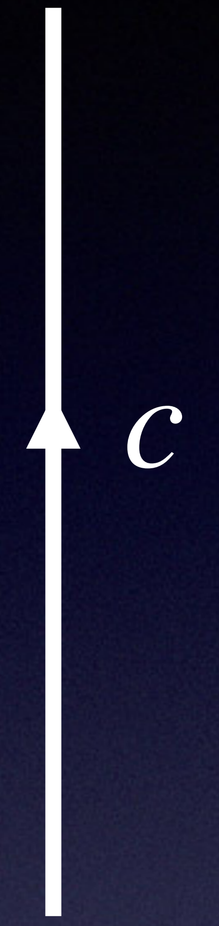


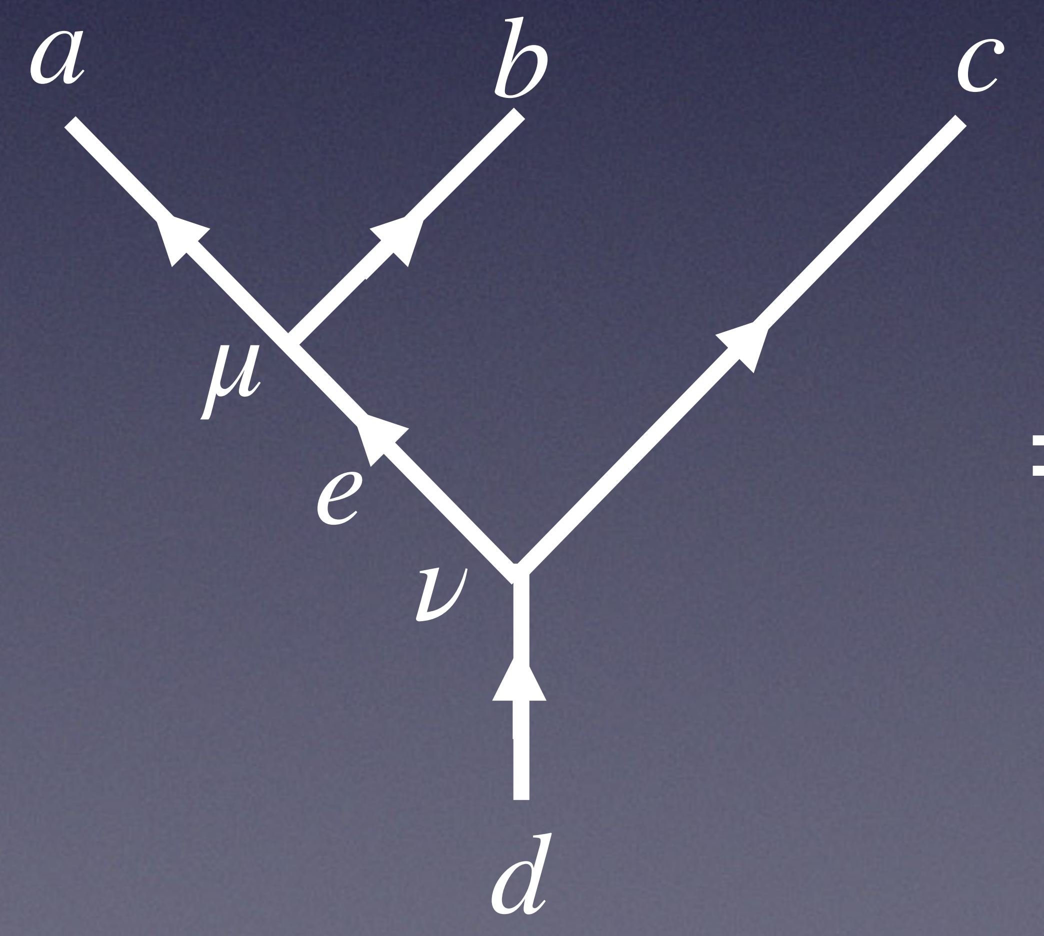
$$= \sum_{c,\mu} \sqrt{\frac{d_c}{d_a d_b}}$$



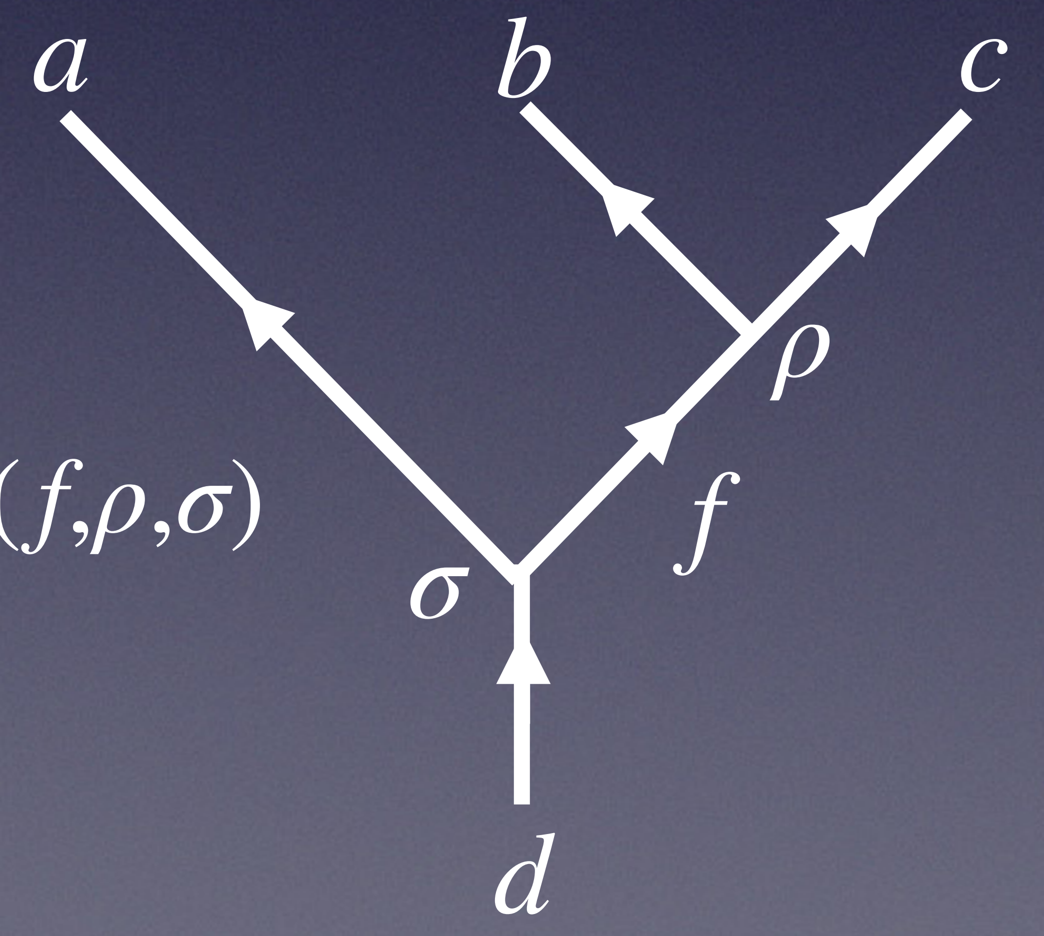


$$= \delta_c^{c'} \delta_\mu^{\mu'} \sqrt{\frac{d_a d_b}{d_c}}$$





$$= \sum_{f,\rho,\sigma} [F_d^{abc}]_{(e,\mu,\nu)(f,\rho,\sigma)}$$



+consistency condition

$\mathrm{SU}(3)_k$ Yang-Mills theory

Hamiltonian $H = \frac{1}{2}\sum_{e \in C_1} (E(e))^2 - \frac{K}{2}\sum_{f \in C_2} (\mathrm{tr}U(f) + \mathrm{tr}U^\dagger(f))$

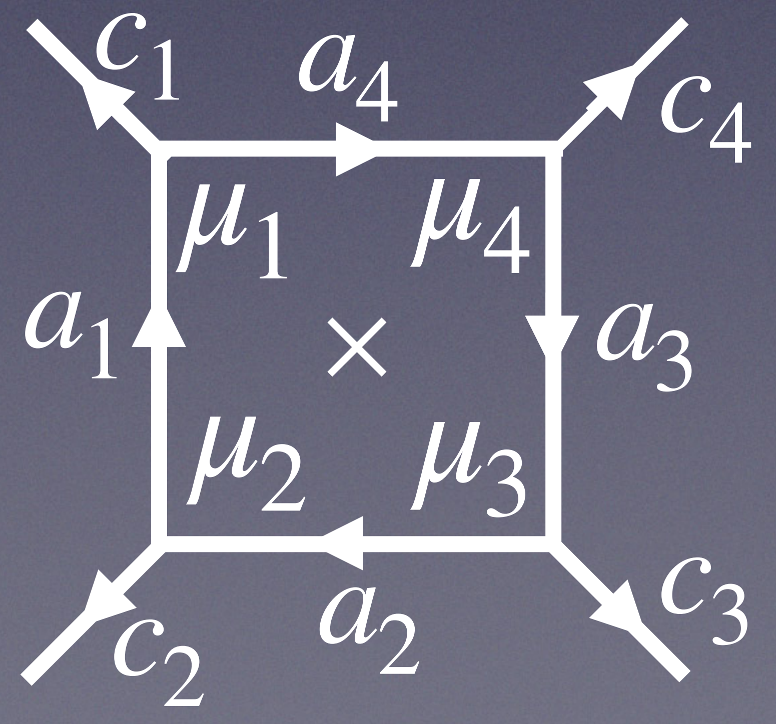
Action on a state

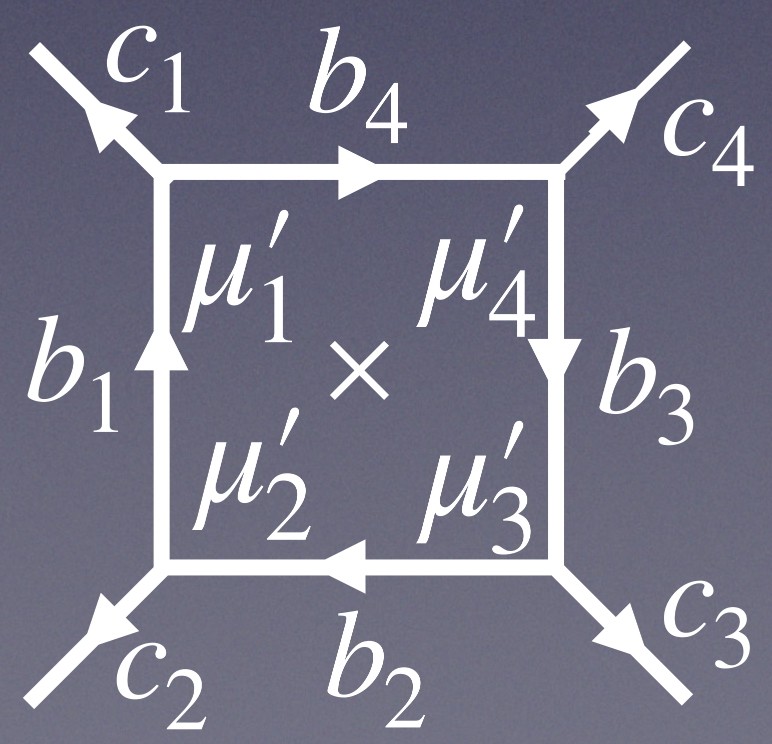
Electric fields E_i^2 

$= C_2(a)$ 

Casimir

Wilson loop

$\mathrm{tr}U$ 

$= \prod_{i=1}^{4}\sum_{b_i} [F_{b_i}^{c_i a_{i-1} \frac{1}{2}}]_{(a_i,\mu_i,\mu'_{i+1})(b_{i-1},\mu_{i-1},\mu'_i)}$ 

Application

Confinement-deconfinement phase transition in mean field approximation for $\mathrm{SU}(3)_k$ in (2+1) dimensions

Variational ansatz for wave function

Dusuel, Vidal, Phys. Rev. B 92 (2015) 12, 125150, Zache, González-Cuadra, Zoller, 2304.02527, Hayata, YH, JHEP 09 (2023) 126

$$|\Psi\rangle = \prod_{f\in\mathcal{F}} \sum_{a_f} \psi(a_f)\,\mathrm{tr}\,U_{a_f}(f)\,|0\rangle$$

We minimize the energy expectation value

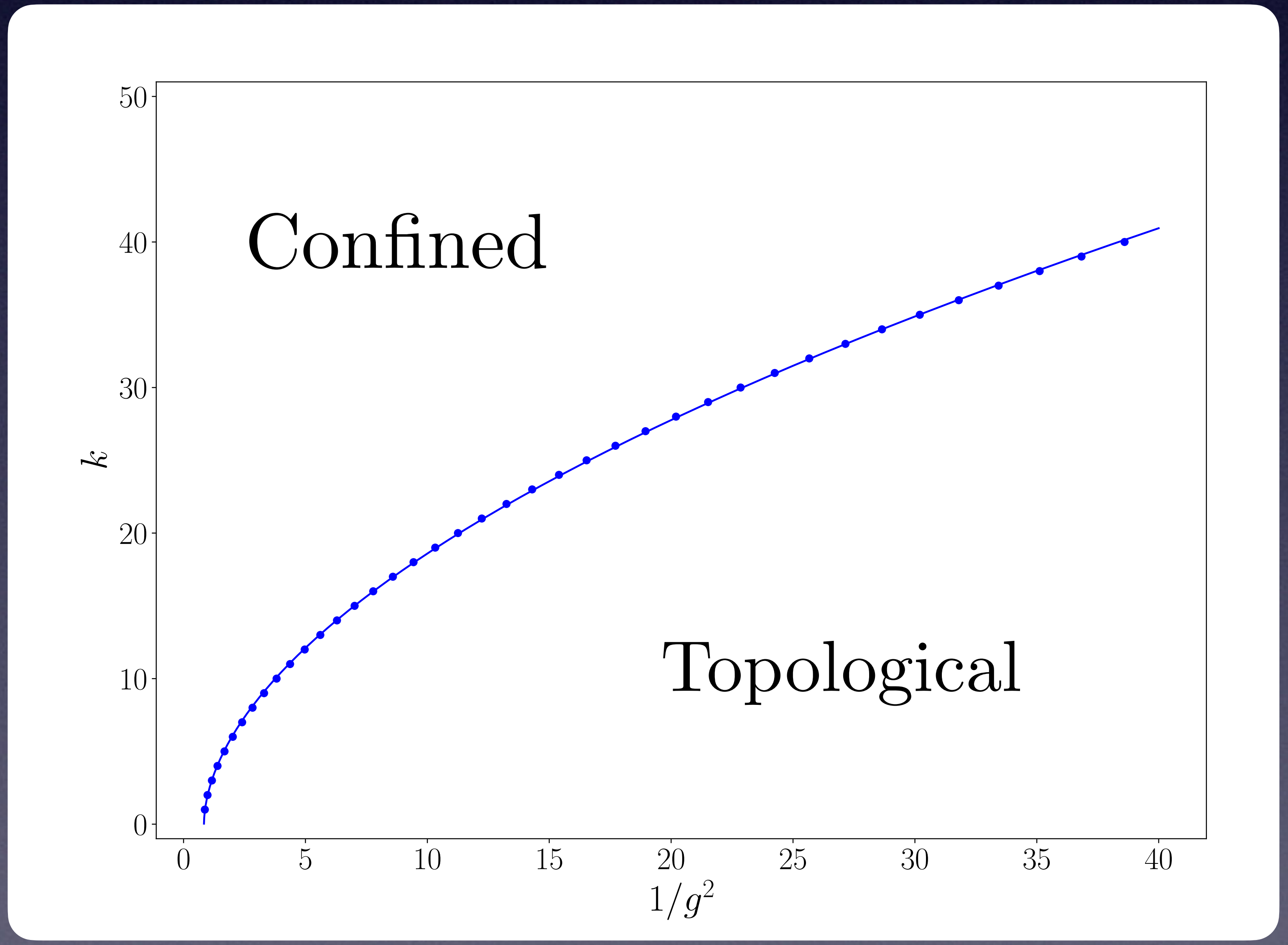
open boundary condition, infinite volume limit

$$E = \min_{\psi} \langle\Psi|\, H \,|\Psi\rangle$$

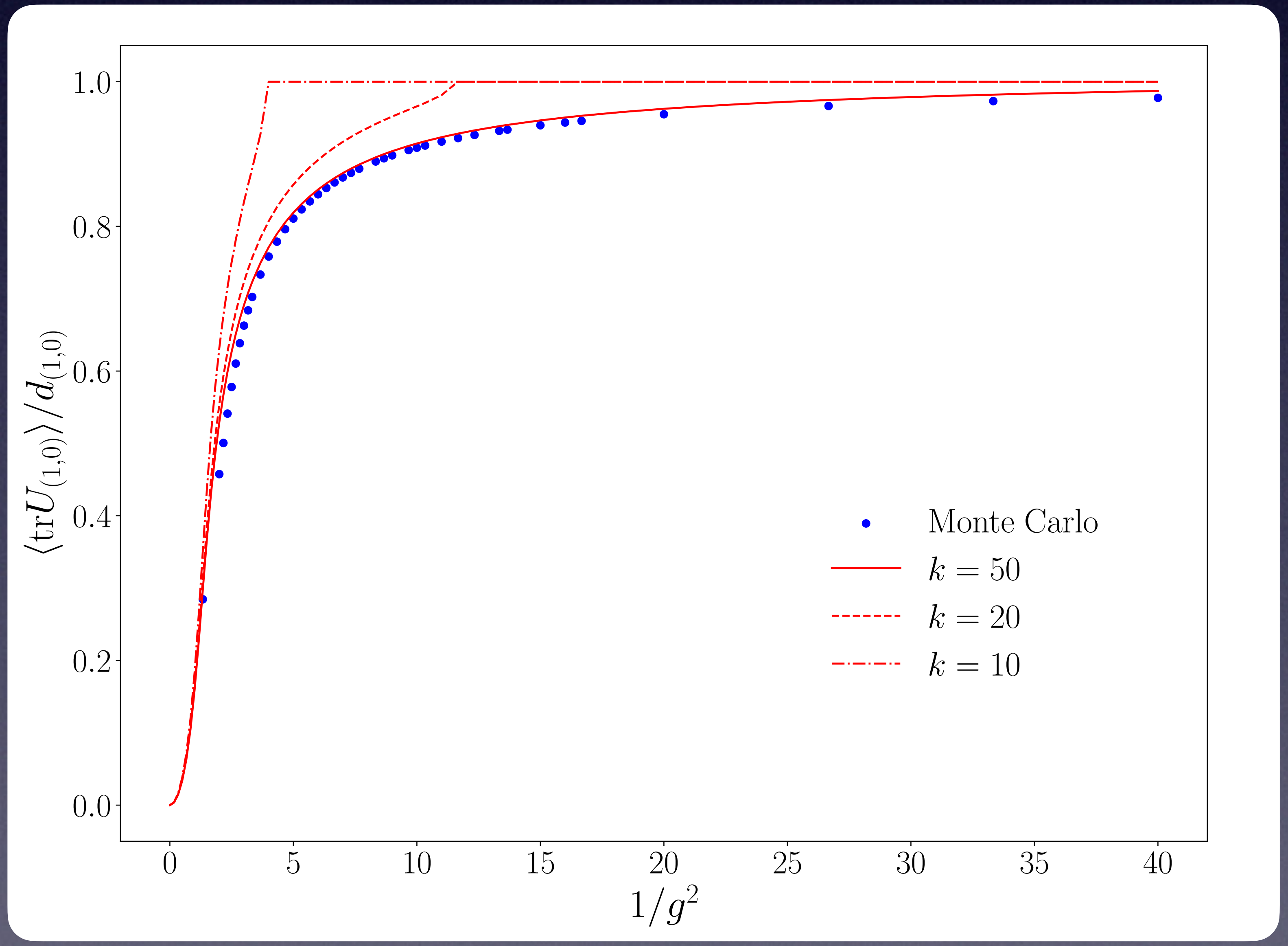
Numerical results

Comparison with Monte-Carlo simulation

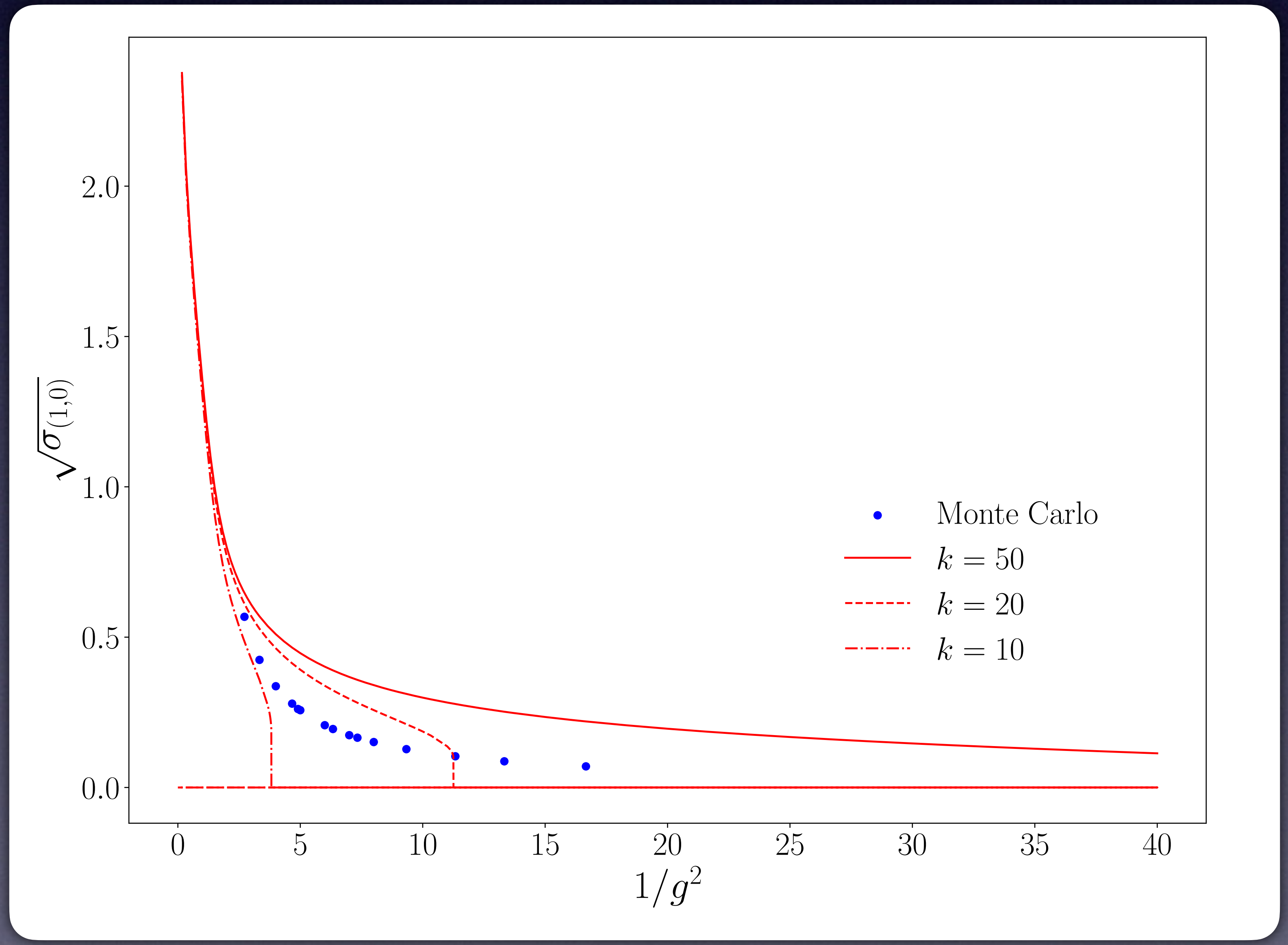
Phase diagram



Plaquette (small Wilson loop)

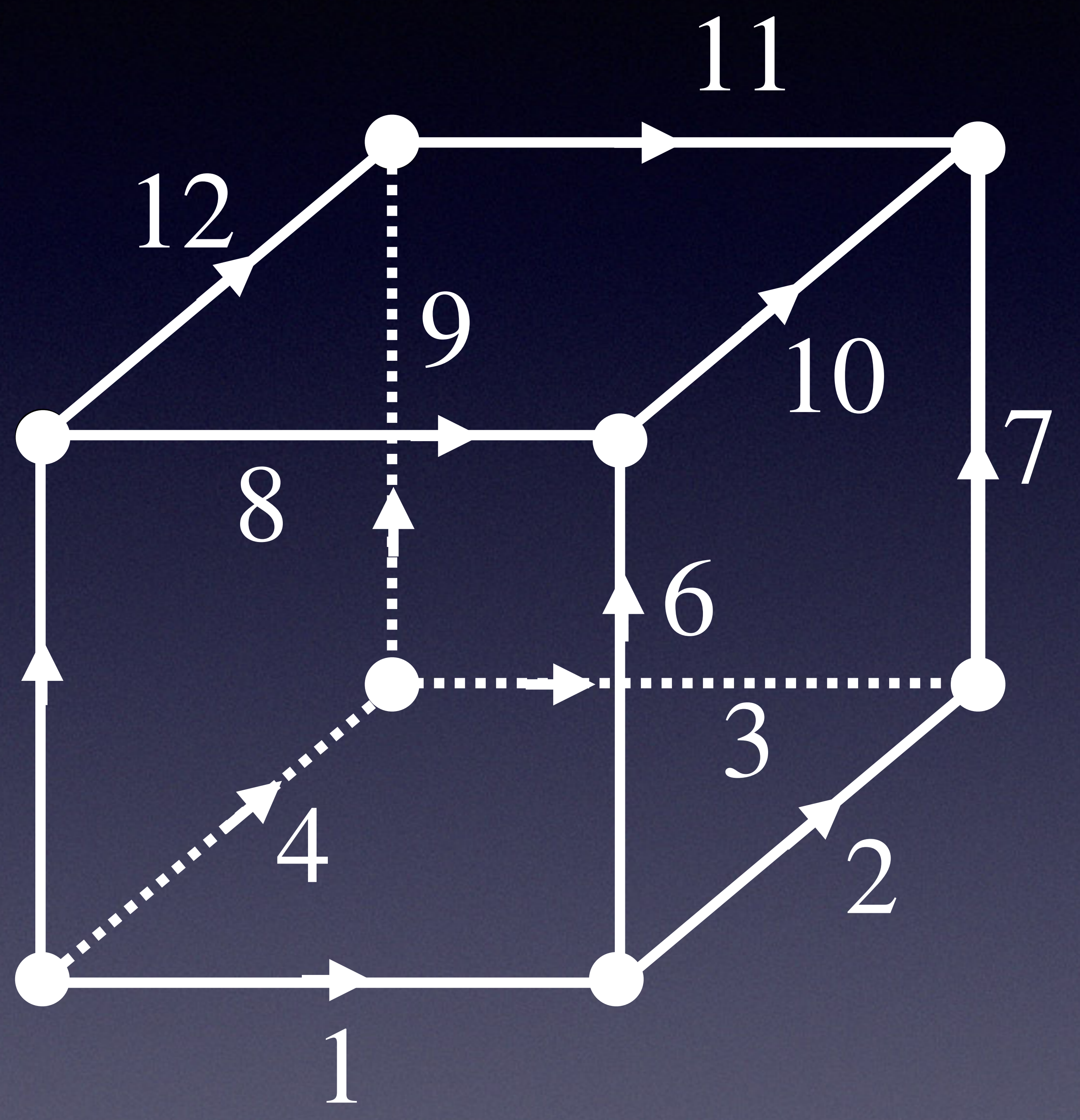


String tension



Good agreement for large k!

Thermalization on a small lattice



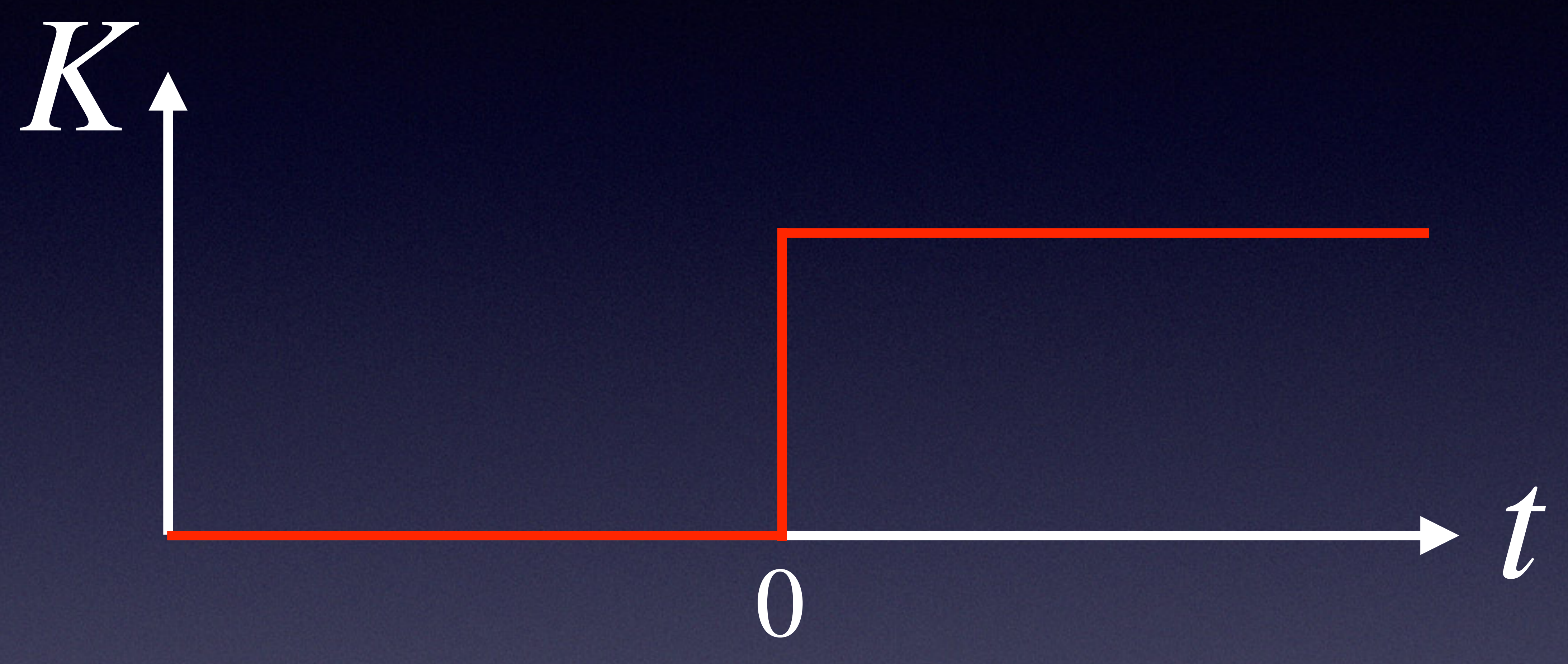
Basis

$$|j_1, \cdots, j_{12}\rangle = |j_1, j_2, j_6\rangle\,|j_2, j_3, j_7\rangle\,|j_3, j_4, j_8\rangle\,|j_1, j_4, j_5\rangle\,|j_6, j_9, j_{10}\rangle\,|j_7, j_{10}, j_{11}\rangle\,|j_8, j_{11}, j_{12}\rangle\,|j_5, j_9, j_{12}\rangle$$

Cutoff $j_i \leq j_{\max} = k/2$

Setup

In order to mimic heavy ion collision experiments, the interaction quenching

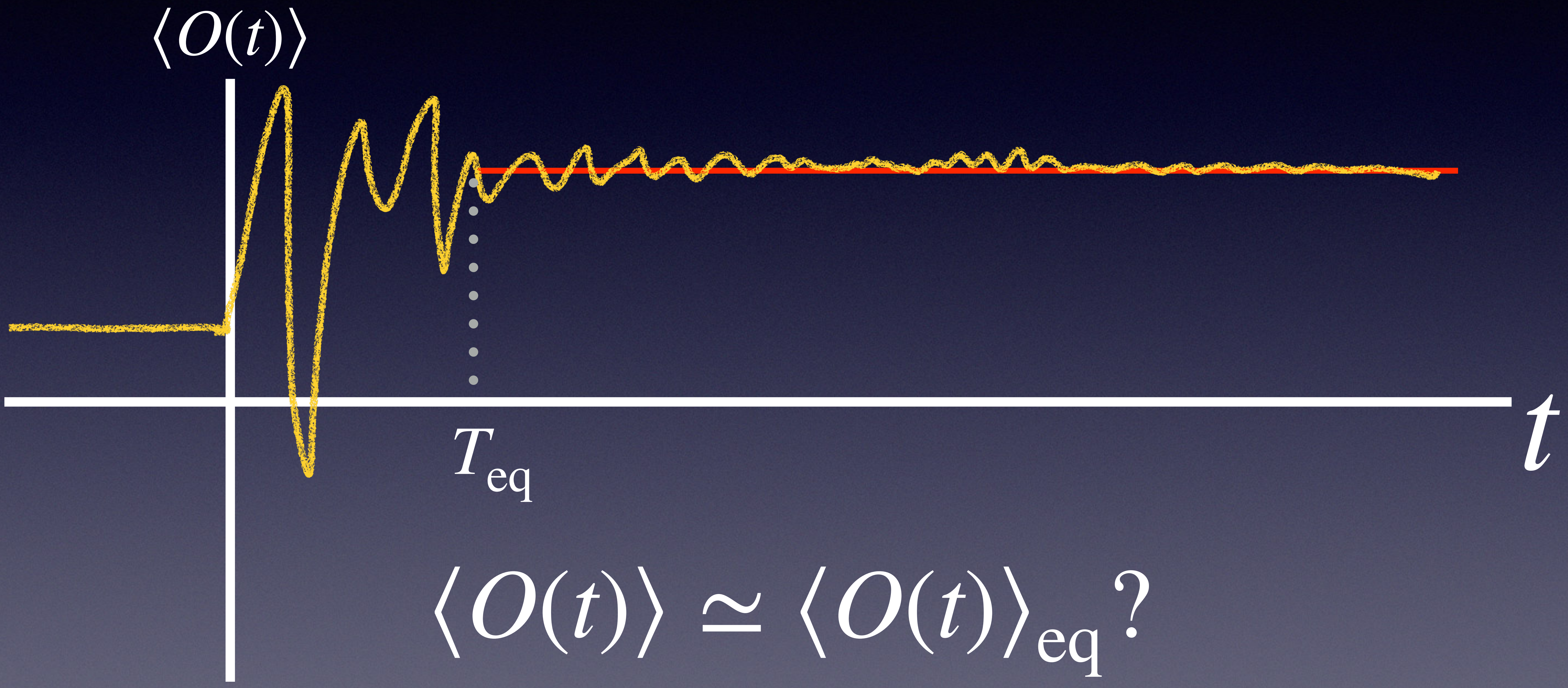


$$t < 0 \ |\mathrm{Vac}\rangle_{K=0}$$

$$t \geq 0 \ |\Psi(t)\rangle = e^{-\mathrm{i}Ht}|\mathrm{Vac}\rangle_{K=0}$$

Expected behavior

for an operator O $\langle O(t) \rangle := \langle \Psi(t) | O | \Psi(t) \rangle$



$$\langle O(t) \rangle \simeq \langle O(t) \rangle_{\mathrm{eq}}?$$

Temperature and Canonical Ensemble

Energy is fixed by an initial condition

$$E = \langle H \rangle = \langle \Psi(t) \,|\, H \,|\, \Psi(t) \rangle$$

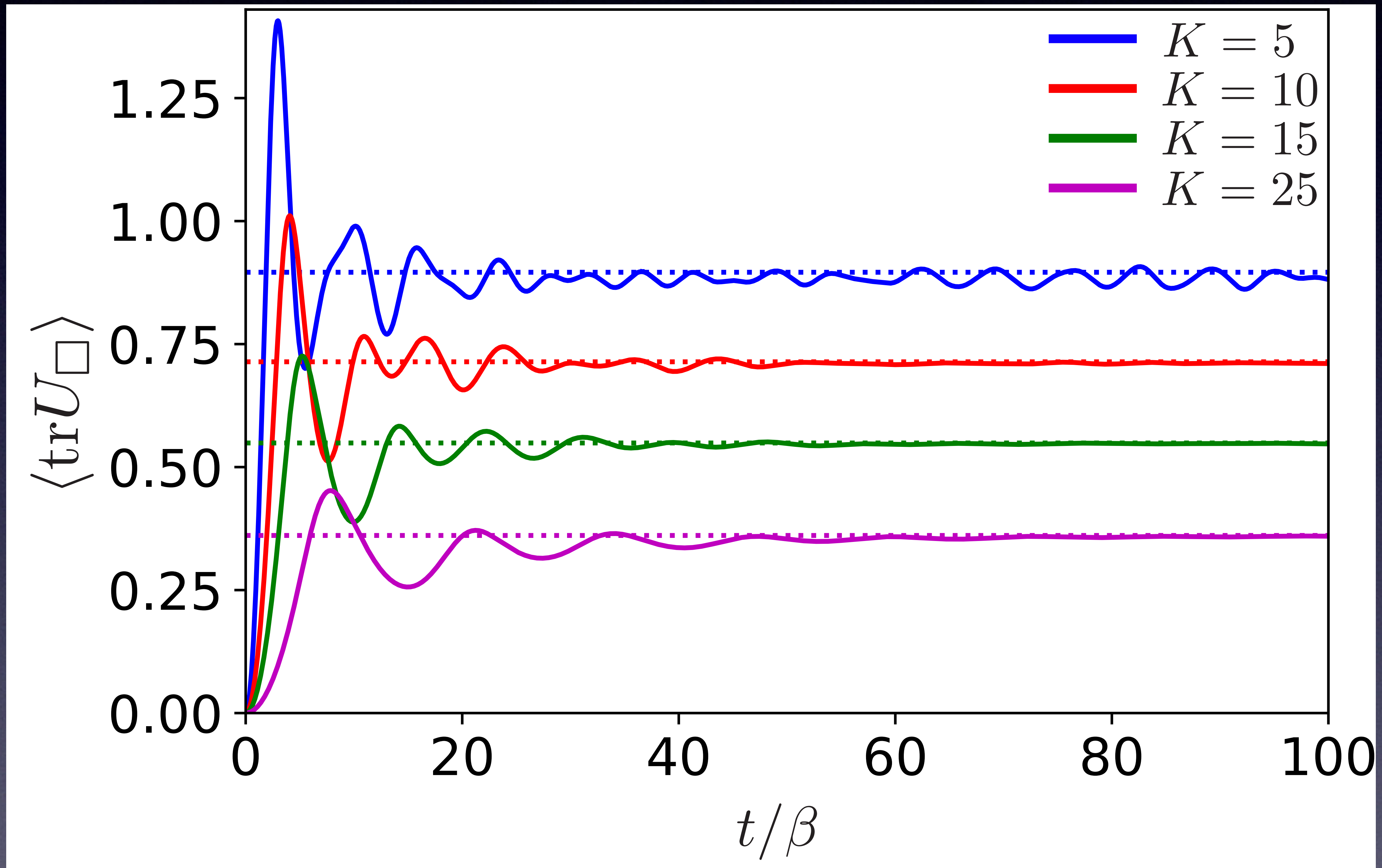
(Independent of time)

For a given energy,
a canonical distribution that reproduces
the expected value can be defined

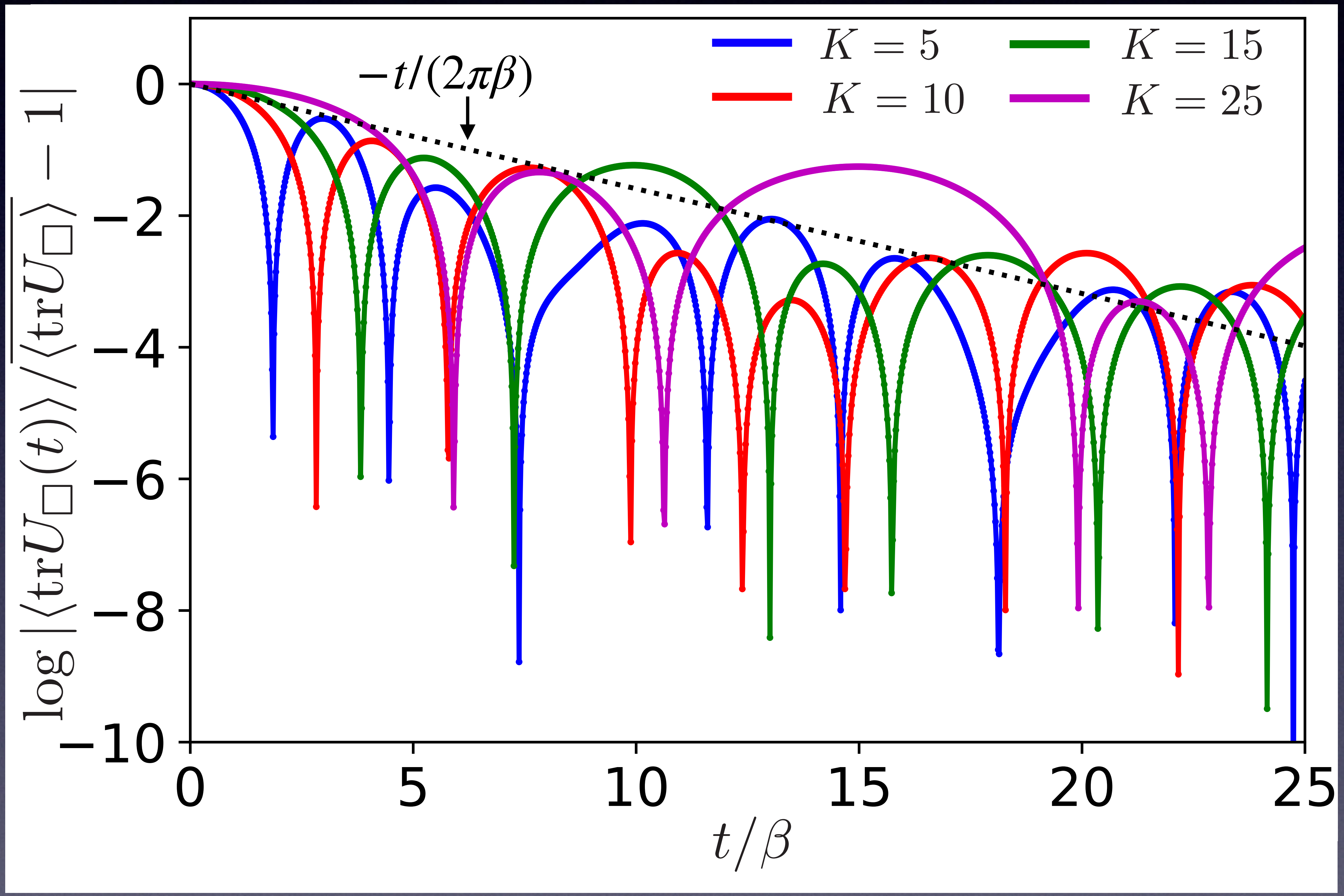
$$E = \langle H \rangle_{\mathrm{eq}} := \mathrm{tr}\rho_{\mathrm{eq}} H \quad \textbf{with} \quad \rho_{\mathrm{eq}} = \frac{e^{-\beta H}}{\mathrm{tr} e^{-\beta H}}$$

Expected value of Wilson loop
Weak coupling (high temperature)

$$\langle \mathrm{tr} U_\square(t) \rangle - \overline{\langle \mathrm{tr} U_\square \rangle} \sim e^{-t/\tau_{\mathrm{eq}}}$$



Steady state observed



Relaxation time

Close to Boltzmann time $2\pi\beta$.

Goldstein, Hara, Tasaki, New J. Phys. 17 (2015) 045002

QCD_2 at finite density

(dimensionless)QCD_2 Hamiltonian

$$J = \frac{ag_0}{2}, w = \frac{1}{2g_0 a}, m = m_0/g_0$$

We use $g_0 = 1$ unit

$$H/g_0 = J\sum_{n=1}^{N-1} E_i^2(n)$$ **Electric field term**

$$+w\sum_{n=1}^{N-1}\left(\chi^\dagger(n+1)U(n)\chi(n) + \chi^\dagger(n)U^\dagger(n)\chi(n+1)\right)$$

Hopping term

$$+m\sum_{n=1}^{N}(-1)^n\chi^\dagger(n)\chi(n)$$ **Mass term**

As a variational ansatz of wave function

- We employ a matrix product state

$$|\psi\rangle = \sum_{\{n_i\}} |n_1\rangle \cdots |n_N\rangle \mathrm{tr} M_1^{n_1} \cdots M_N^{n_N}$$

$[M_i^{n_i}]_{ij}$: $D \times D$ matrix

- Optimize the wave function by density matrix renormalization group technique

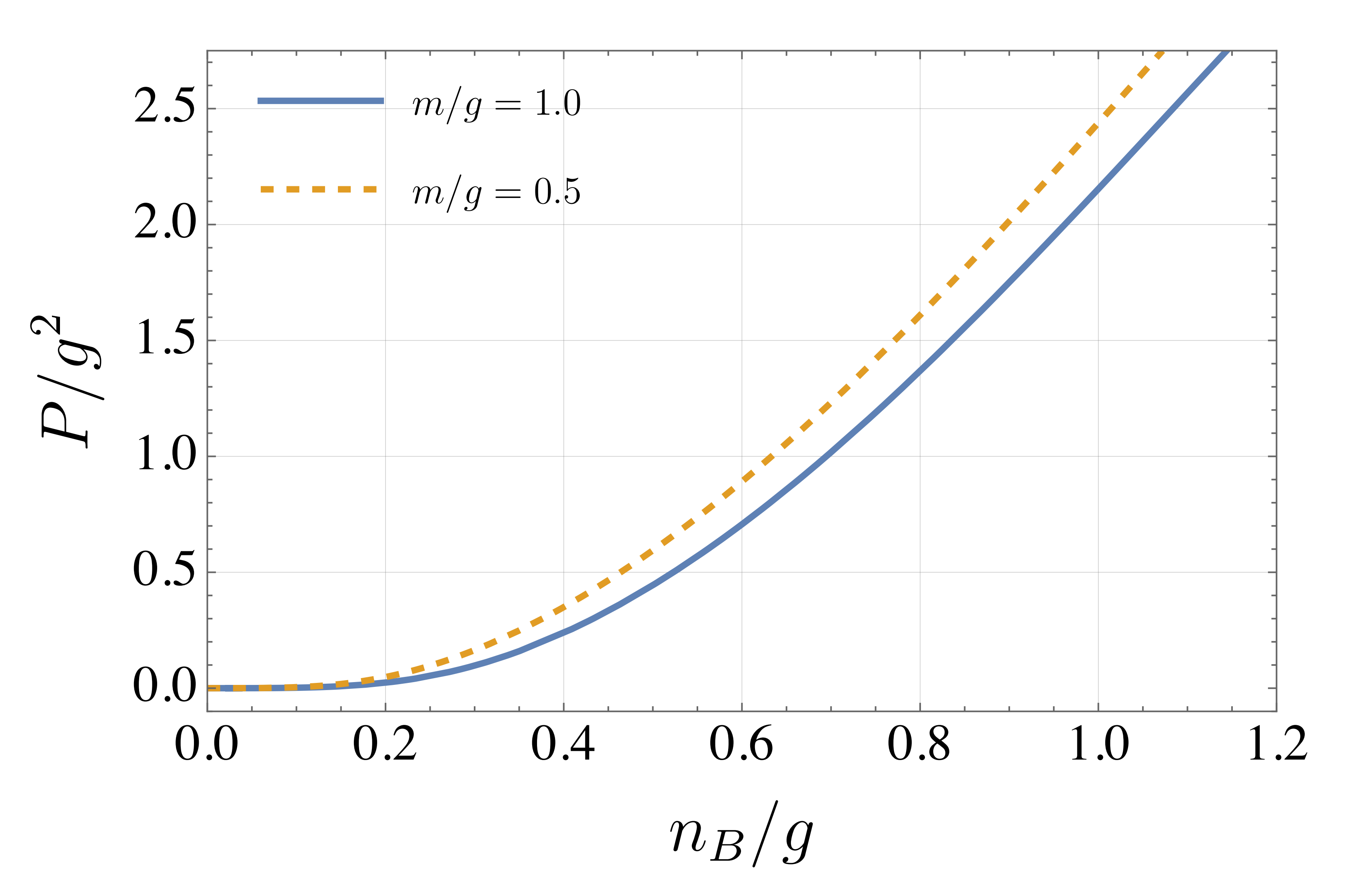
$$E = \min_{\psi} \langle \psi | H | \psi \rangle$$

We employ VUMPS

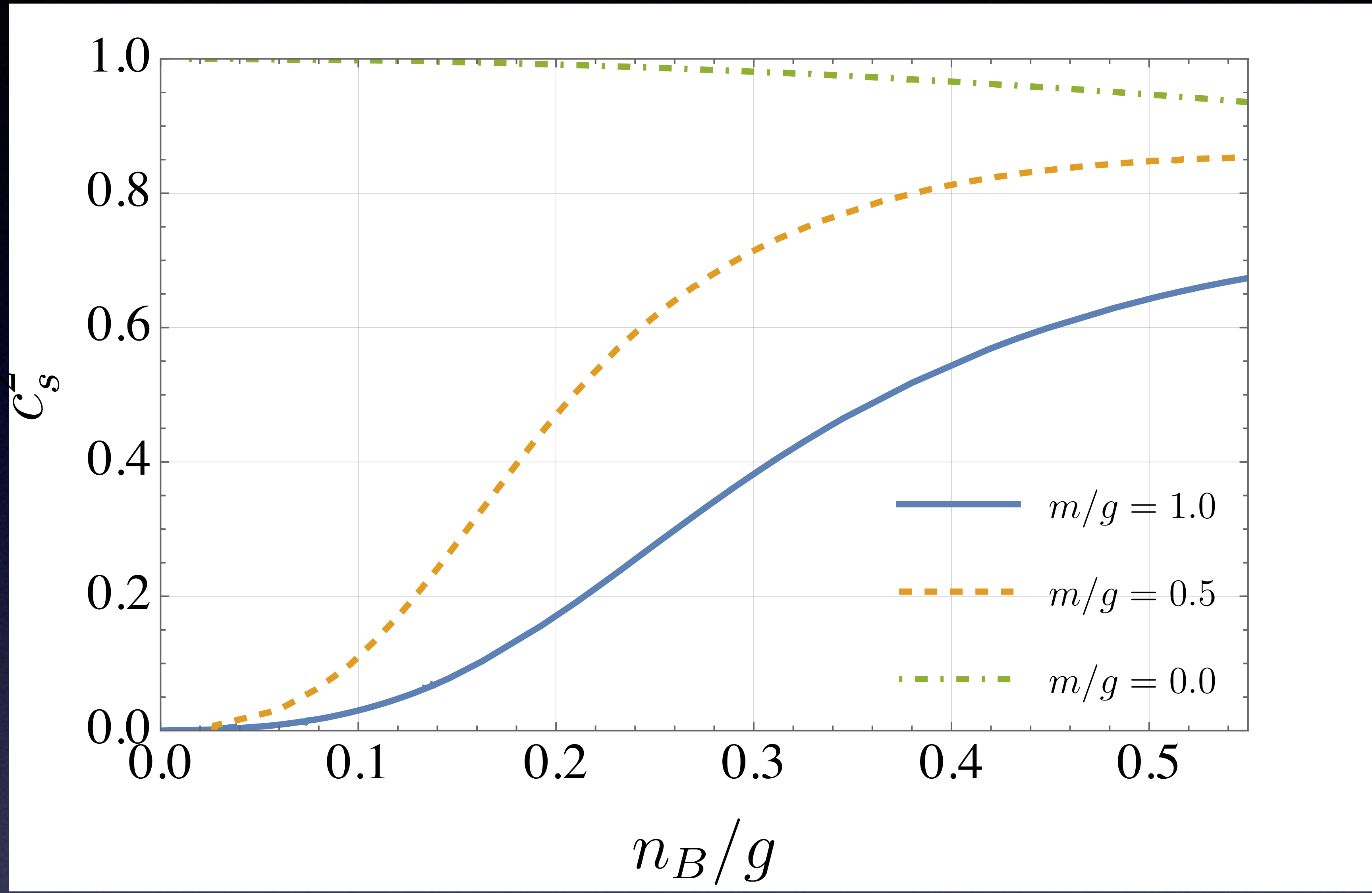
Numerical Results

Fujikura, YH 2605.17183

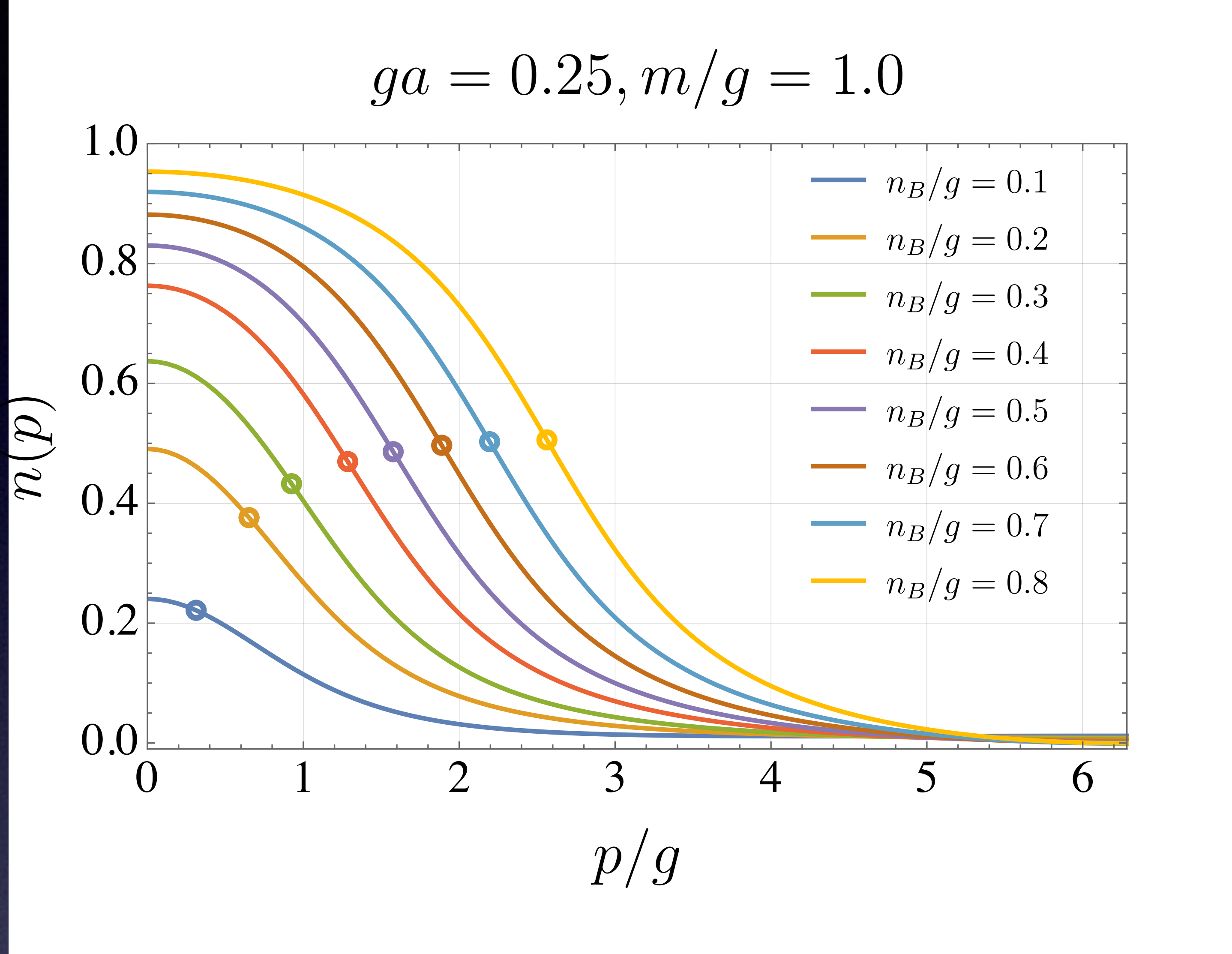
Equation of state

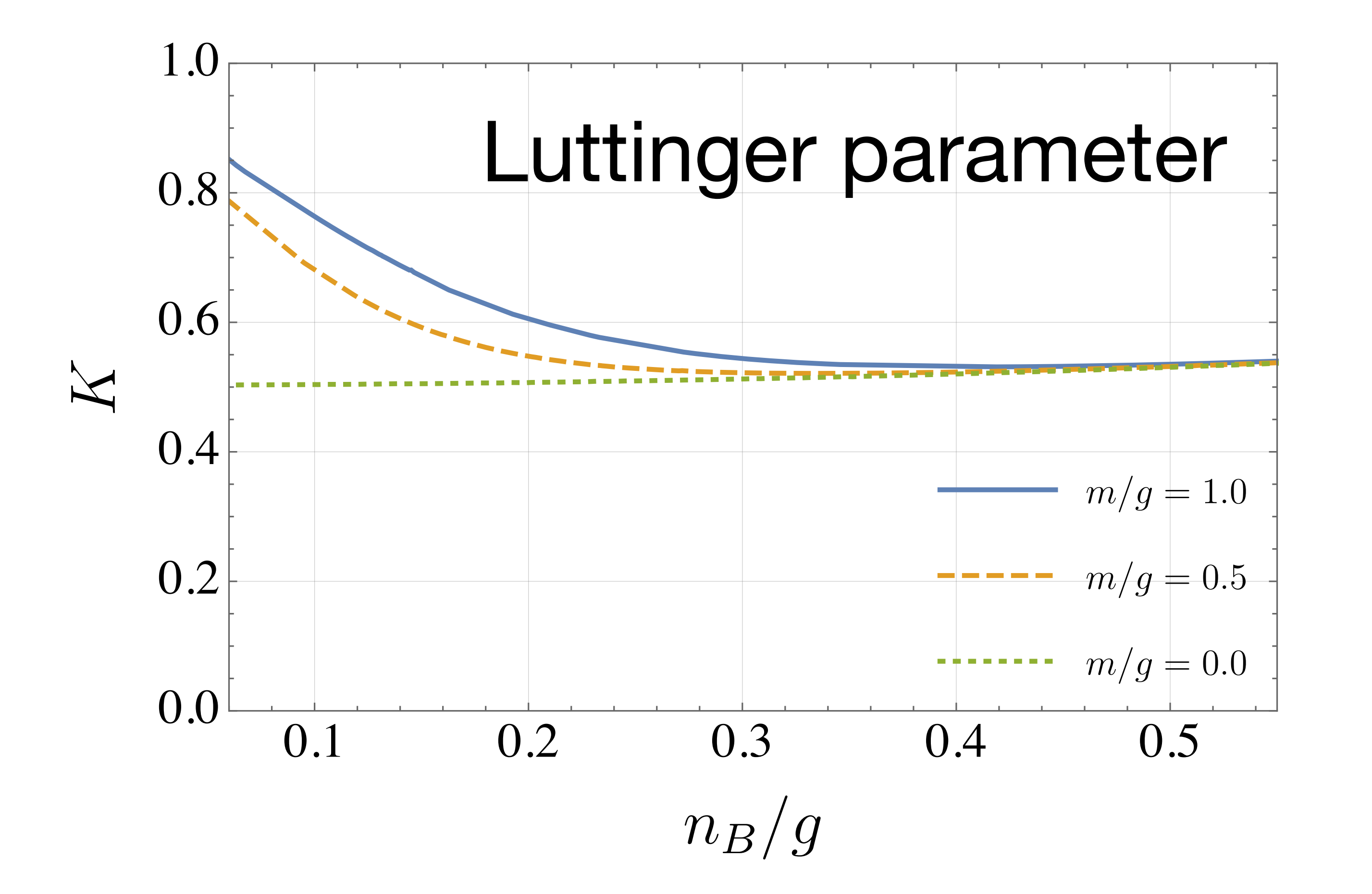


sound velocity



quark distribution



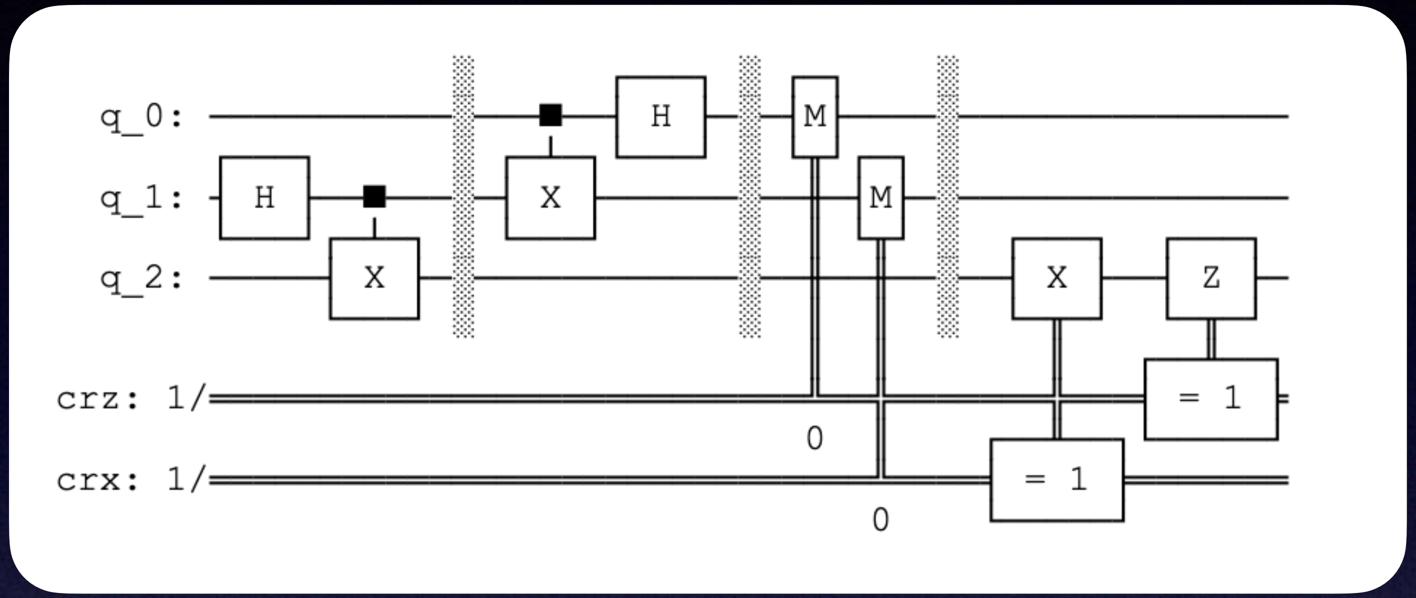


Low-energy behavior: Tomonaga-Luttinger liquid

Baryonic to quark Luttinger liquid crossover

Quantum computing

$$i\partial_t|\psi\rangle = H|\psi\rangle$$



- Natural method to solve quantum systems

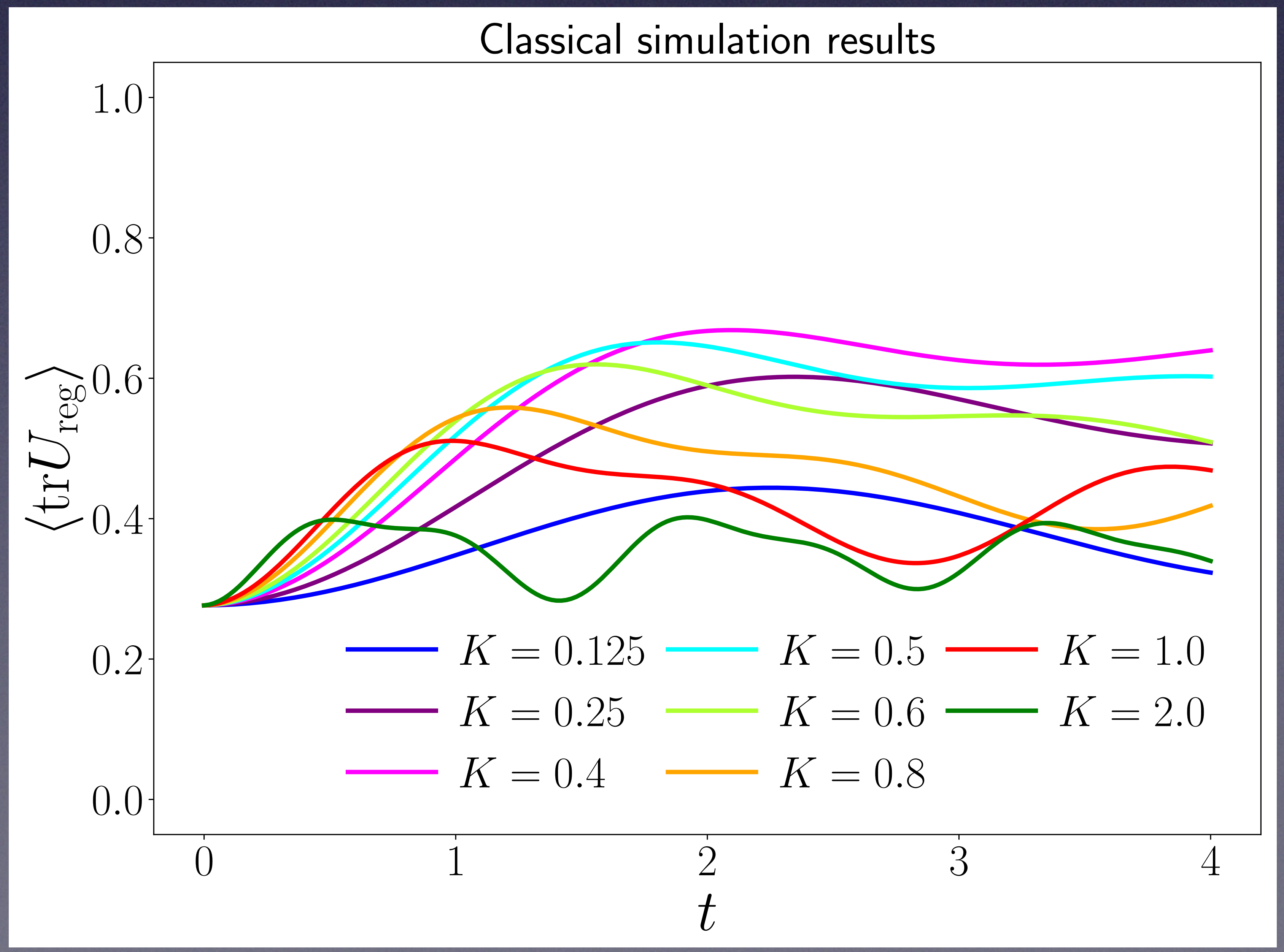
  So far, noise is large,
  the number of qubits are small.

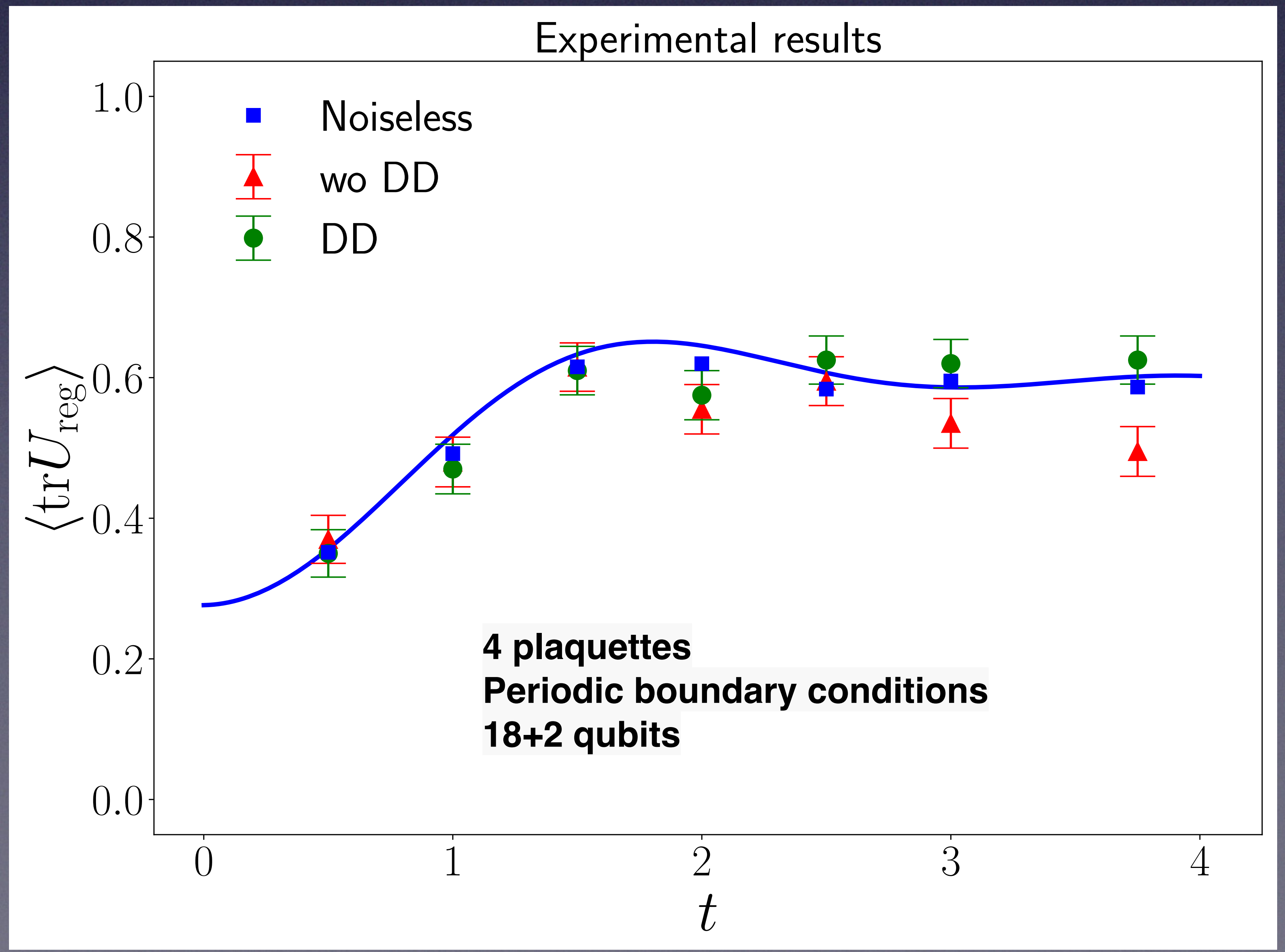
Quantum simulation of (2+1)D non-Abelian gauge theory

Hayata, Hidaka, Kikuchi, 2601.13530

- Analyzing high-dimensional QCD using quantum computation is difficult with current quantum computers
- Non-Abelian gauge theory ~ a theory with non-Abelian anyons
- Simplest model ⇒ Restrict $SU(2)_3$ to integer spins (Fibonacci anyon) so the local states are the two types, $1, \tau$

Simulation in (2+1)d



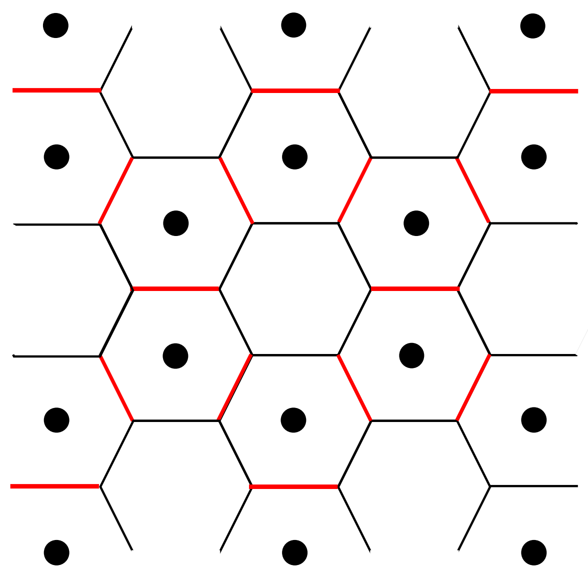


Quantum computer "REIMEI"

Quantinuum H1

Fibonacci anyon model

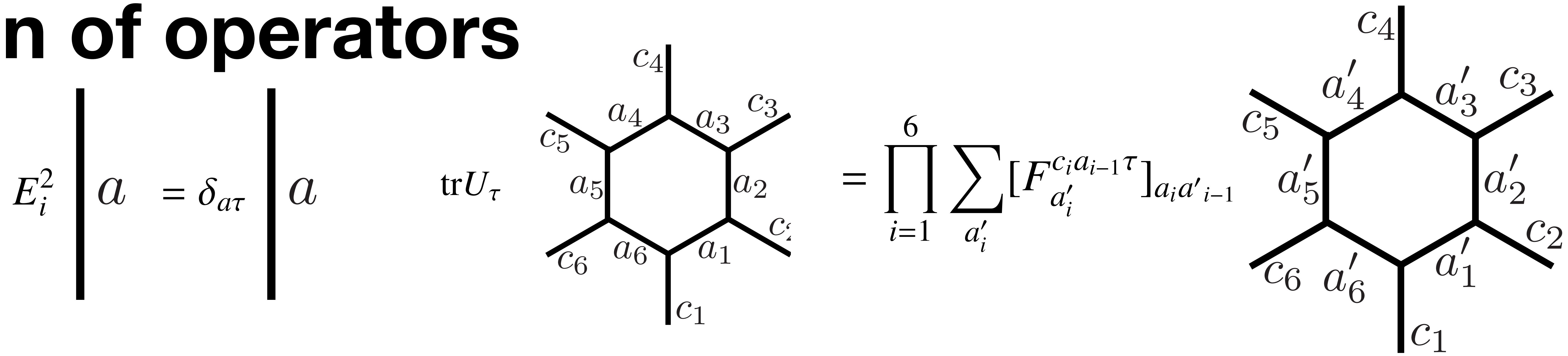
Lattice structure



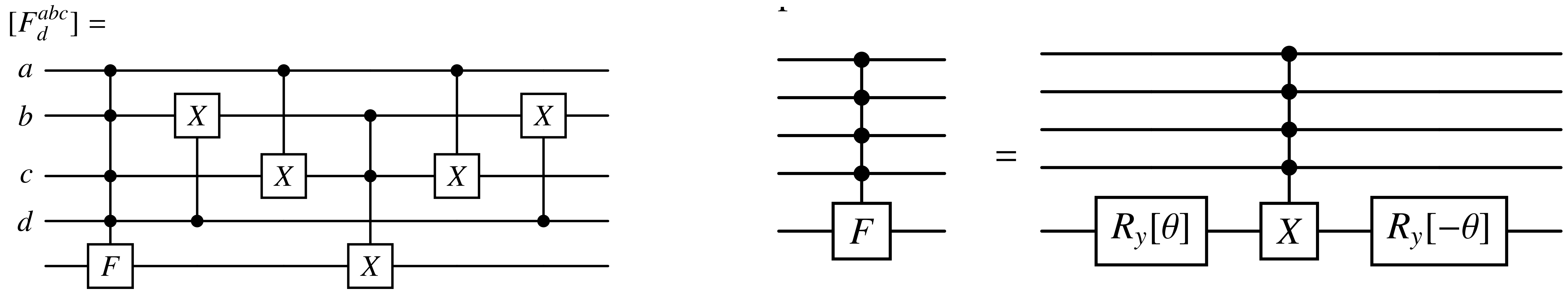
Red: the link acted on by E

Black: the plaquette acted on by B

Action of operators



Implementation of the F-symbols



Summary

Formalism

- **Kogut–Susskind Hamiltonian formalism**
  - Regularization $SU(N) \to SU(N)_k$ (quantum deformation): finite-dim Hilbert space, gauge symmetry preserved; physical states = **networks of Wilson lines**

Application

- $SU(3)_k$ **gauge theory in (2+1) dimensions**: **confinement–topological** phase transition; plaquette & string tension agree with Monte Carlo at large k
- **Thermalization of Yang–Mills in (3+1)-dimensional small systems**: relaxation time ≈ **Boltzmann time τ_eq ~ 2π/T**
- **QCD_2 at finite density** (MPS): **baryon–quark transition, inhomogeneous phase**; Tomonaga–Luttinger liquid
- **Quantum simulation of (2+1)D non-Abelian gauge theory**: Fibonacci anyon on Quantinuum H1 (“REIMEI”), 18+2 qubits