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Approach to thermalization using quantum dynamics

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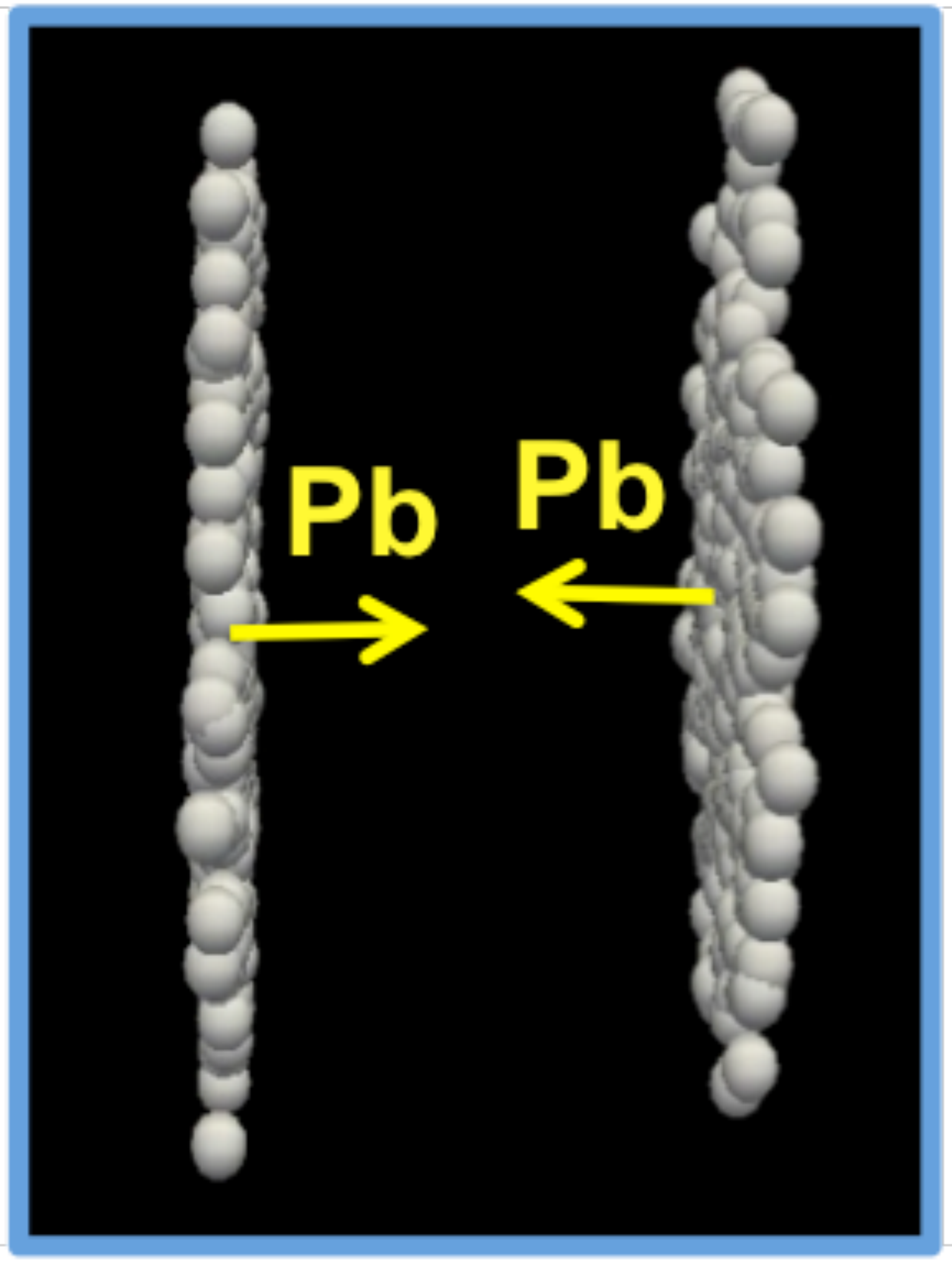
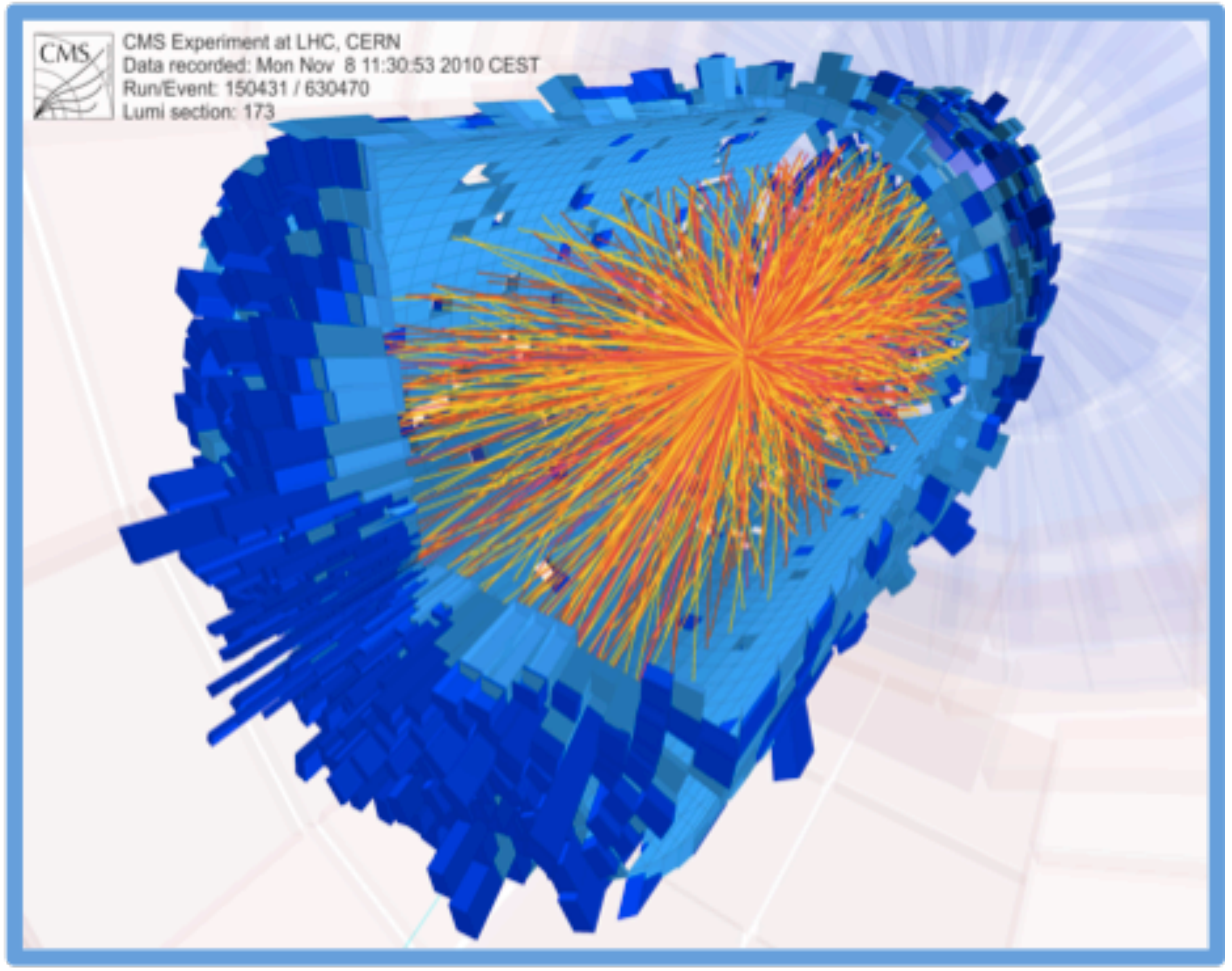
Shile Chen, Pengfei Zhuang, SS, in preparation

Haiyang Shao, Shile Chen, SS, 2509.10835 + 2509.10855

Pre-reaction



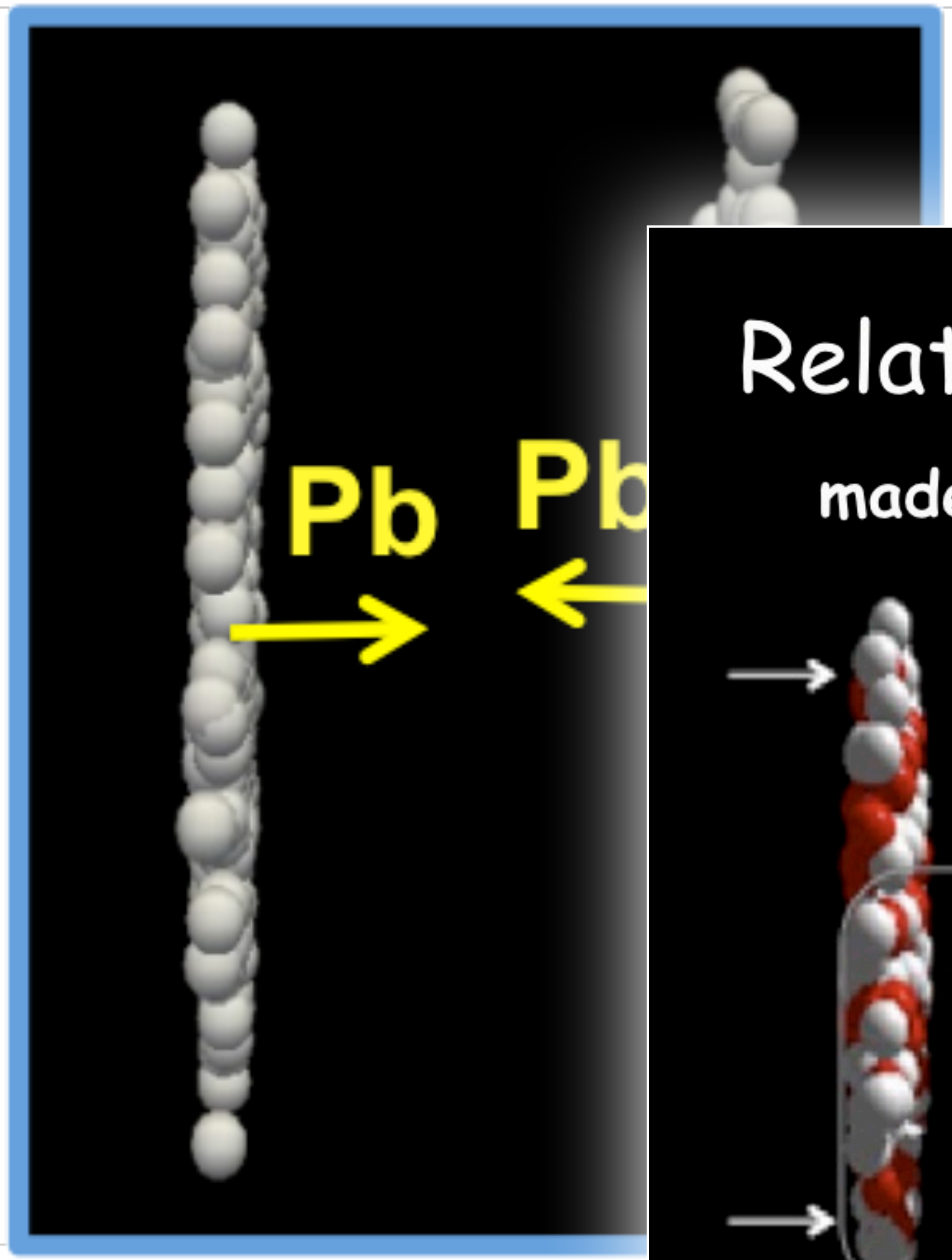
Detection



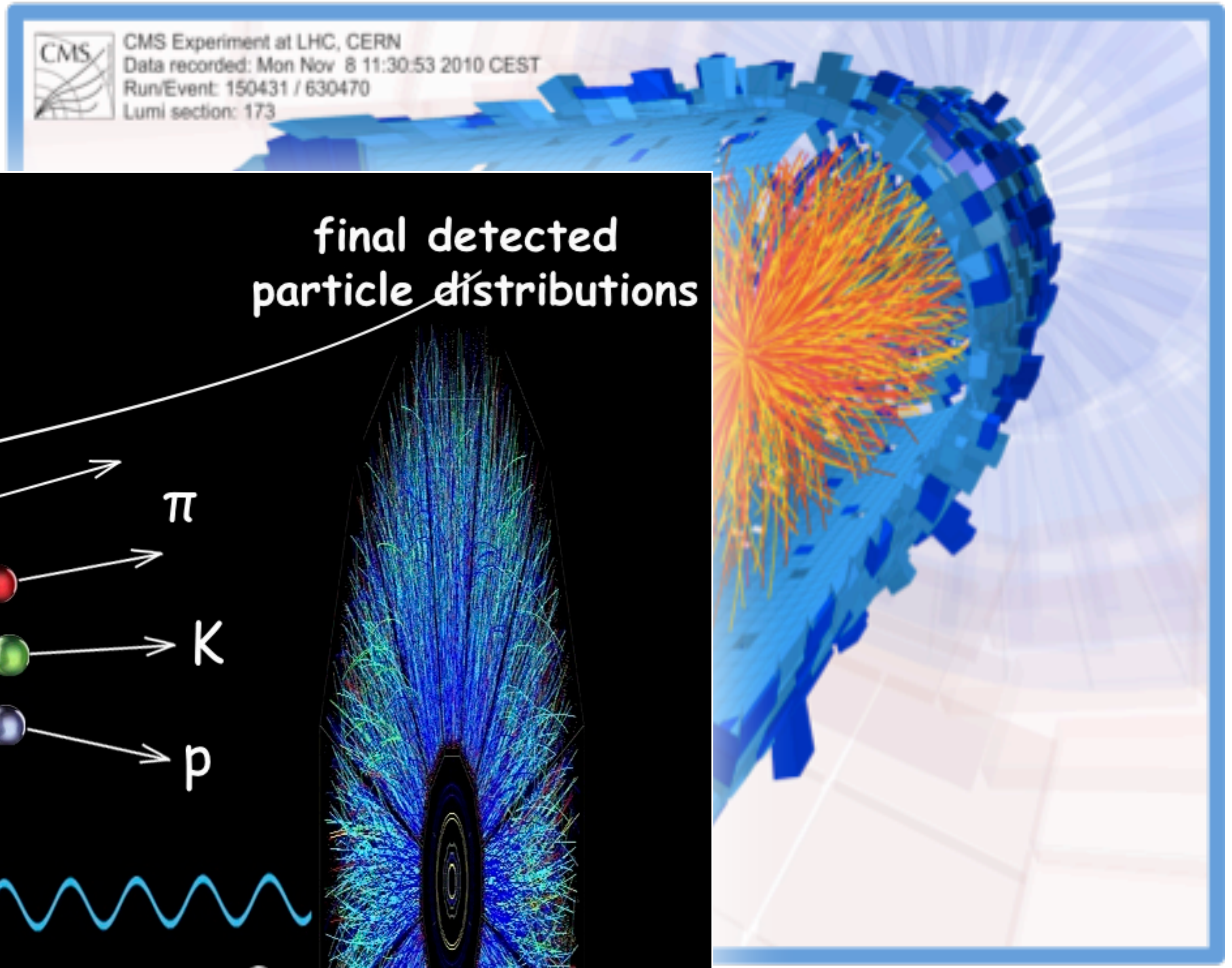
final state particles

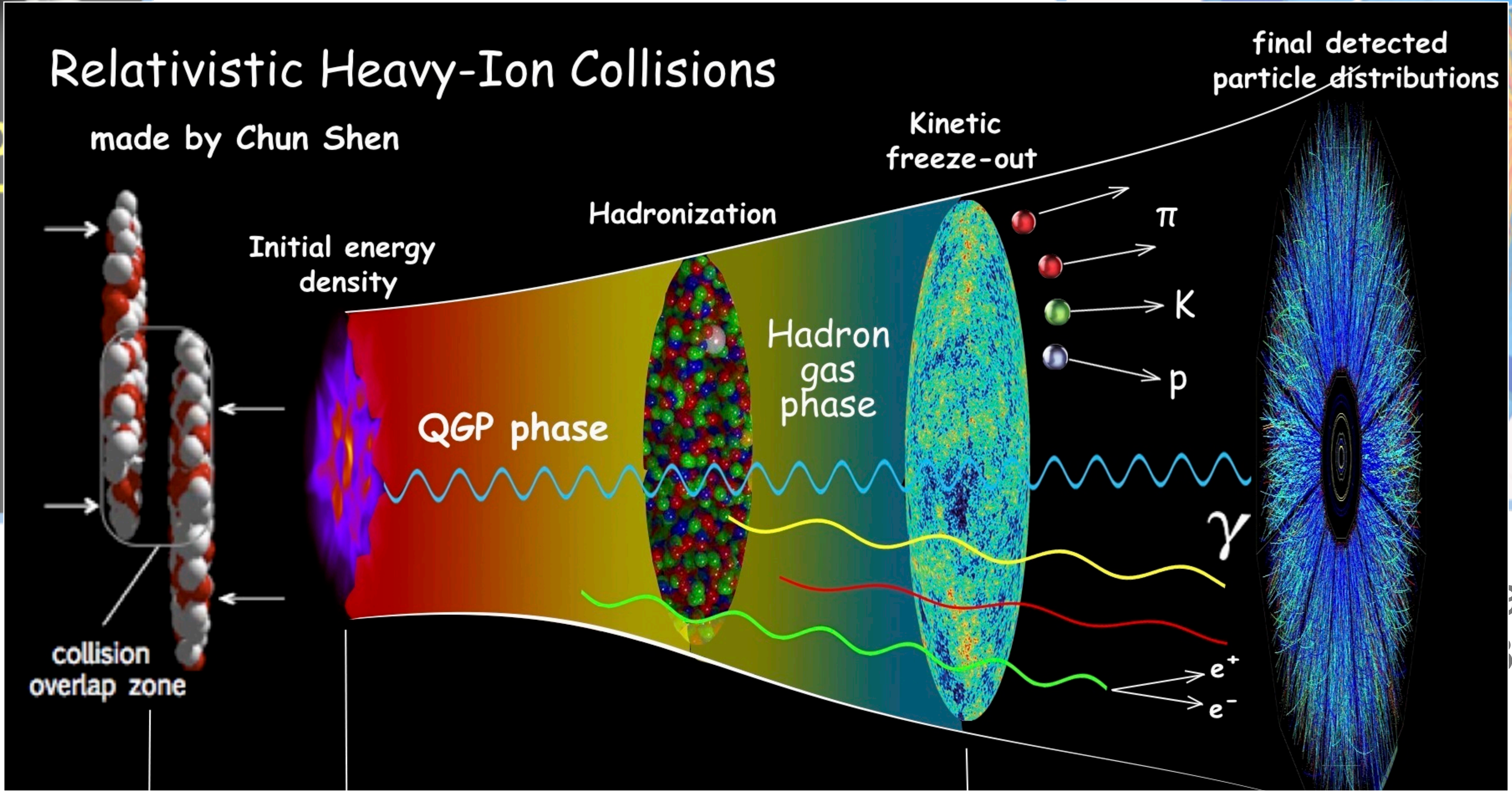
$N \sim 10^{3-4}$

Pre-reaction



Detection



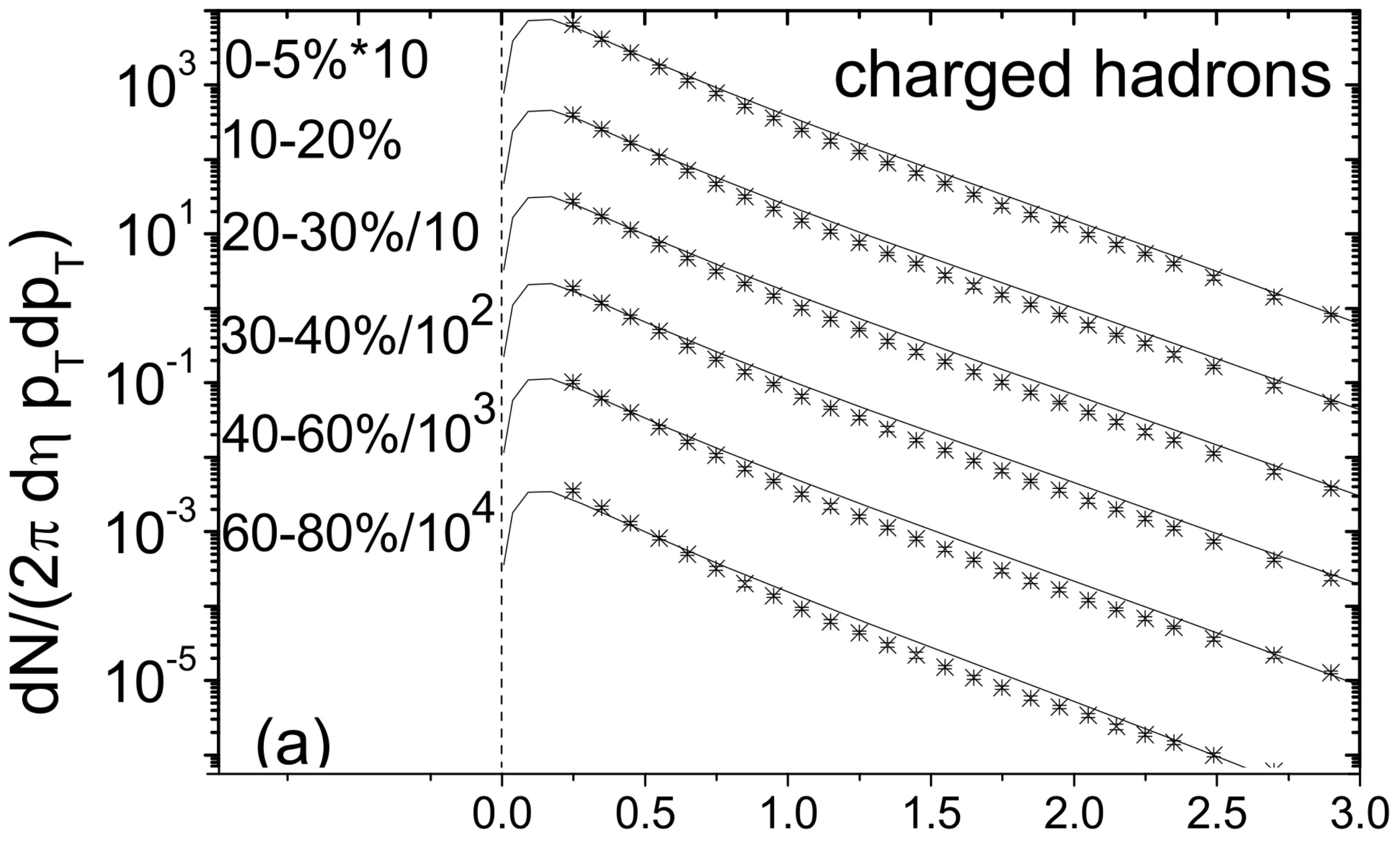


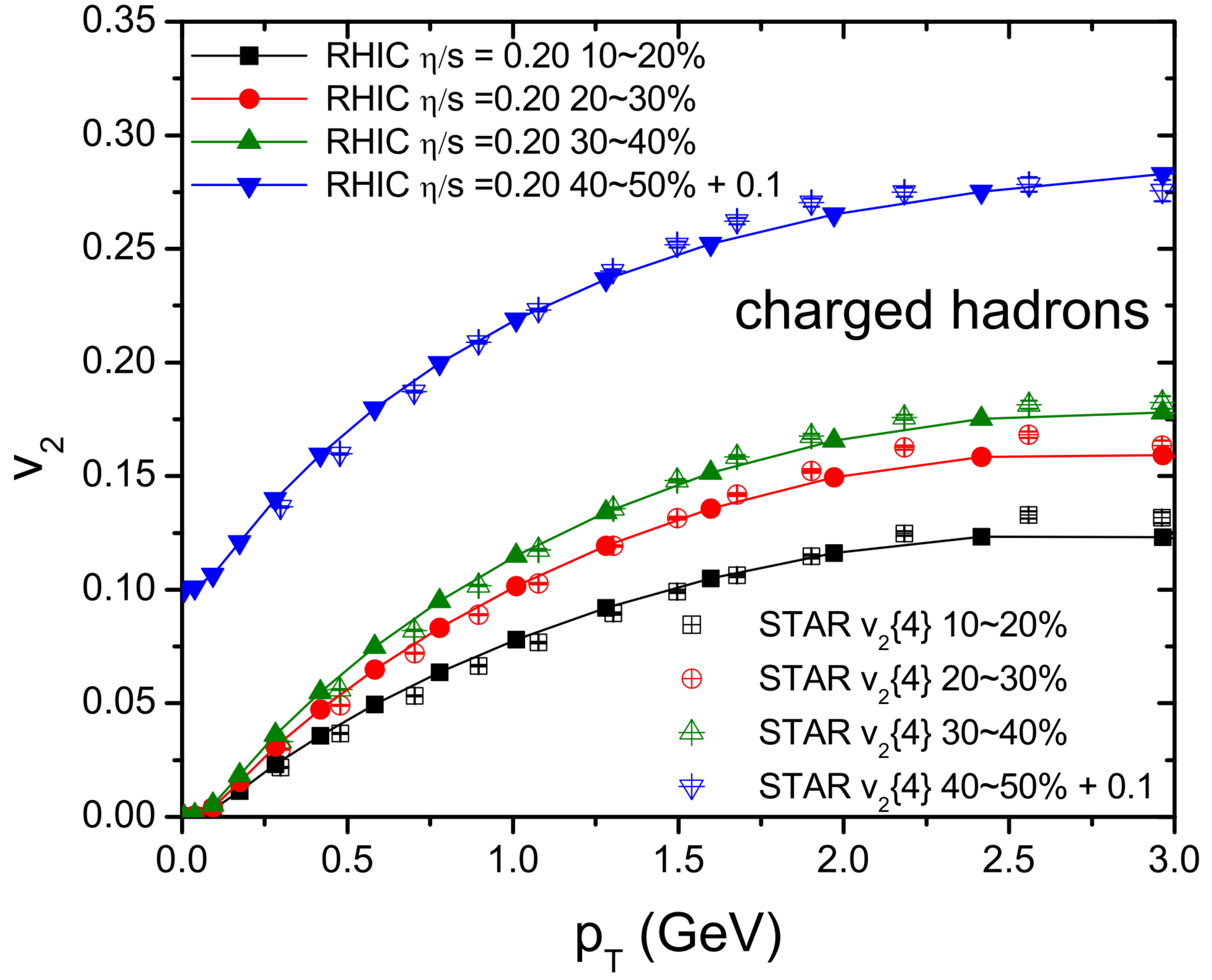
articles

–4

Collectivity in Heavy-Ion Collisions

***Collective Property** of final state particles*:





symbols: data

curves: hydrodynamics theory

(thermal w/ anisotropic blue-shift)

\+ *many **independent probes** of the hot medium*

$$\partial_\mu T^{\mu\nu} = 0\,,$$

$$T^{\mu\nu} = (\varepsilon + P + \Pi)u^\mu u^\nu - (P + \Pi)g^{\mu\nu} + \pi^{\mu\nu}\,.$$

thermodynamic + kinetic quantities (EOS: QCD physics validated)

viscous (non-equilibrium) corrections

Hydrodynamics $\Leftarrow$ near local-equilibrium (many body)

for ~ 10^{3-4} particles?

✓ mean free path (~0.5 fm) $\ll$ system size (~10 fm)

? rapid equilibration? (t ~ 0.5-1 fm/c)

? small system?

Quark-Gluon Plasma:

A strongly coupled, quantum, (quasi)many-body system!

Ideally, full quantum simulation of QCD in 3+1D

Practically, strong coupling QED in 1+1D

– confinement, vacuum chiral condensate

How do the thermal states look like?

1+1D Schwinger model

$$H = \int \left(\frac{E^2}{2} - \bar{\psi}(i\gamma^1 \partial_x - g\gamma^1 A - m)\psi \right) \mathrm{d}x \,.$$

1+1D Schwinger model

$$H = \int \Big(\frac{E^2}{2} - \bar{\psi}(i\gamma^1 \partial_x - g\gamma^1 A - m)\psi \Big) \mathrm{d}x\,.$$

$$\psi(z) = \int \frac{dp}{2\pi\sqrt{2E_p}} \Big(u(p)\hat{a}(p)e^{ipz} + v(p)\hat{c}^\dagger(p)e^{-ipz} \Big)\,,$$

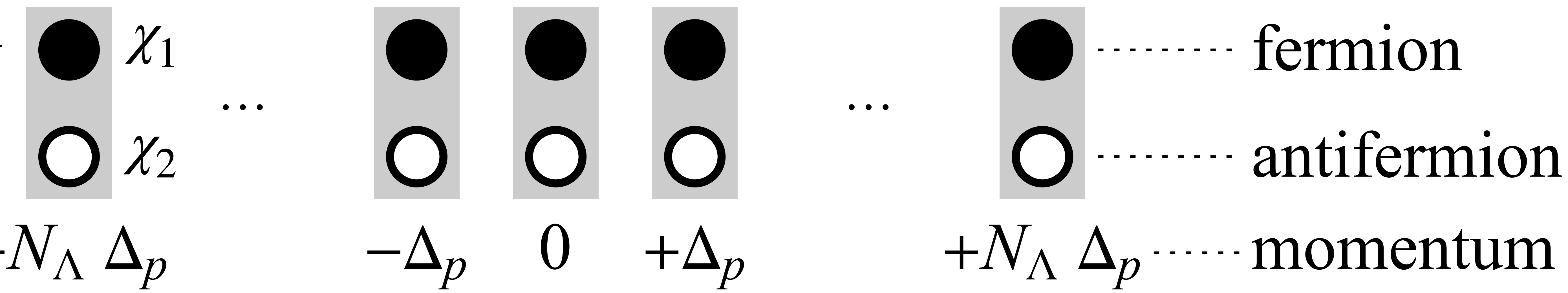
$$= \int \frac{E_p\,\mathrm{d}p}{2\pi} \Big(\hat{a}^\dagger(p)\hat{a}(p) + \hat{c}^\dagger(p)\hat{c}(p) \Big) + g^2_{\mathrm{Sch}} \int \frac{\mathrm{d}q}{4\pi} \hat{e}(q)\hat{e}(-q)\,,$$

$$\hat{e}(q) \equiv \int \frac{\mathrm{d}p}{4\pi q} \Big(S(p+\frac{q}{2}, p-\frac{q}{2}) \big(\hat{a}^\dagger(p-\frac{q}{2})\hat{a}(p+\frac{q}{2}) - \hat{c}^\dagger(p-\frac{q}{2})\hat{c}(p+\frac{q}{2}) \big)$$

$$-A(p+\frac{q}{2}, p-\frac{q}{2}) \big(\hat{a}^\dagger(p-\frac{q}{2})\hat{c}^\dagger(-p-\frac{q}{2}) + \hat{a}(-p+\frac{q}{2})\hat{c}(p+\frac{q}{2}) \big) \Big)\,,$$

1+1D Schwinger model

$|0\rangle$:empty
$|1\rangle$:occupied



$$\{\chi_n^\dagger, \chi_m\} = \delta_{nm}\,, \quad \{\chi_n^\dagger, \chi_m^\dagger\} = \{\chi_n, \chi_m\} = 0\,.$$

Jordan-Wigner

$$\chi_n = \frac{S_n^-}{2}\prod_{m=1}^{n-1}(-iZ_m)$$

$$S_n^\pm \equiv I \otimes \cdots \otimes I \otimes \underbrace{(\sigma_x \pm i\sigma_y)}_{n^{\text{th}}} \otimes I \otimes \cdots \otimes I$$

$$H = \int \frac{E_p\,\mathrm{d}p}{2\pi}\Big(\hat{a}^\dagger(p)\hat{a}(p) + \hat{c}^\dagger(p)\hat{c}(p)\Big) + g_{\text{Sch}}^2 \int \frac{\mathrm{d}q}{4\pi}\hat{e}(q)\hat{e}(-q)\,,$$

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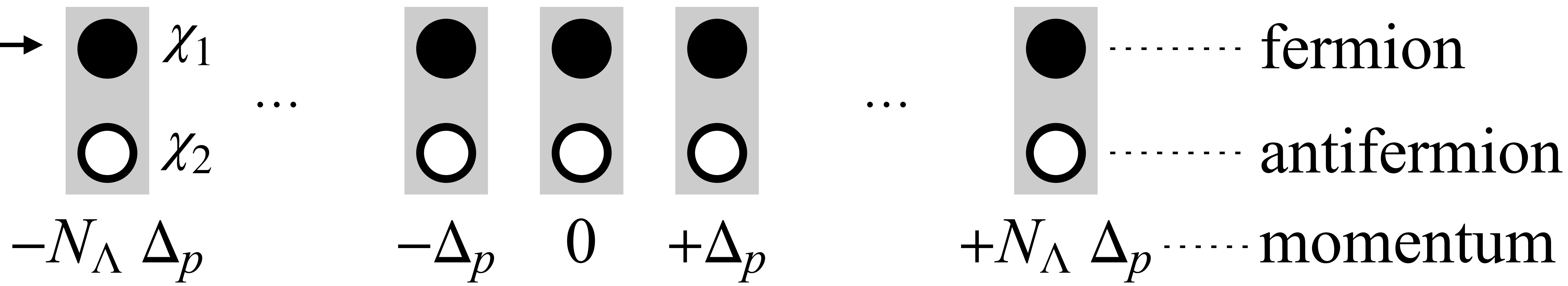

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$H =$

$= \int \frac{E_p\,\mathrm{d}p}{} \left(\hat{c}^\dagger(\ )\hat{c}(\ ) + \hat{c}^\dagger(\ )\hat{c}(\ )\right) + \ ^2 \int \frac{\mathrm{d}q}{} \hat{\ }(\ )\hat{\ }(\ )$

Exact conservation of total momentum:

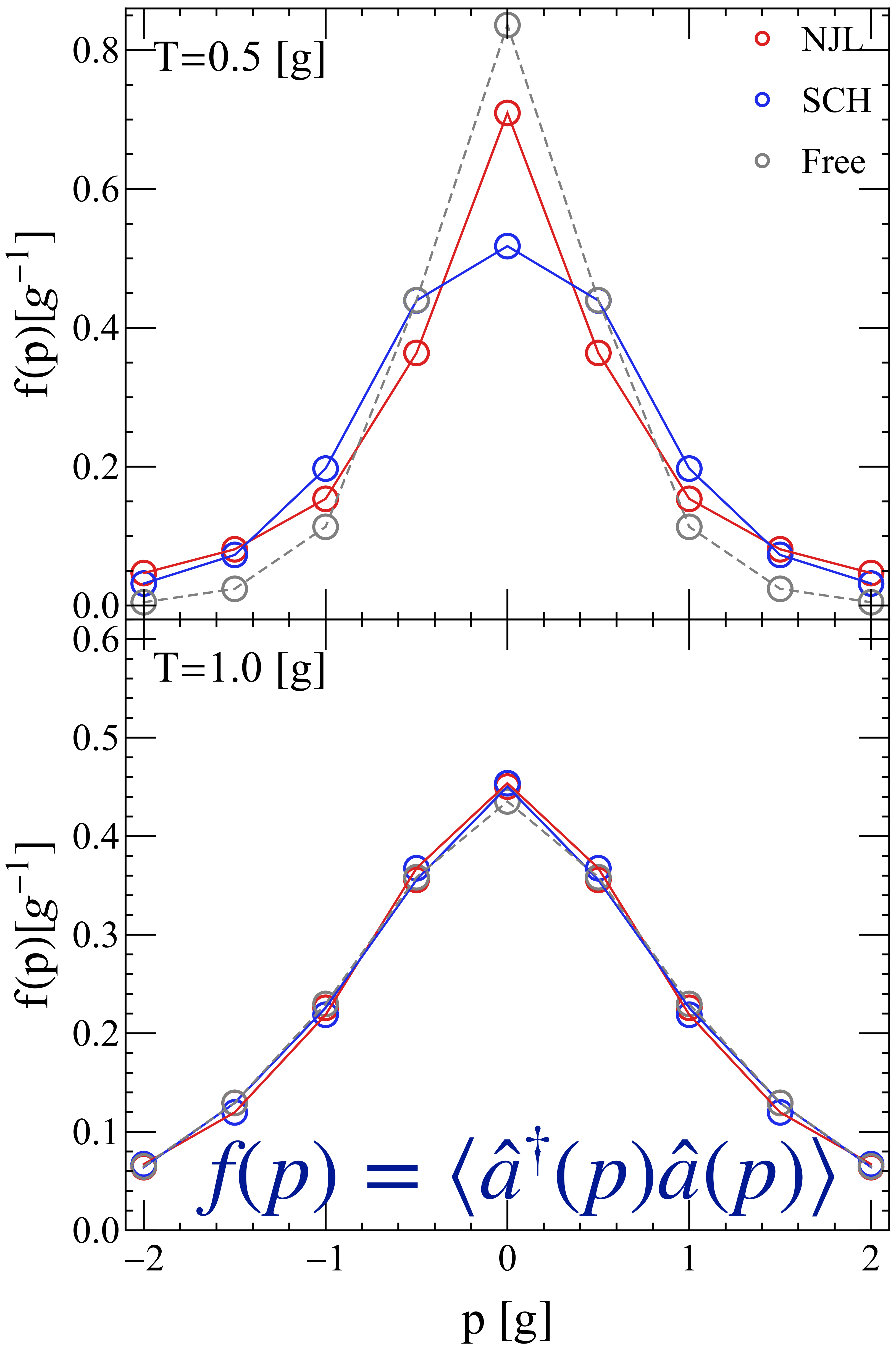
a system with 18 sites:

$$D = 2^{18} = 262144,$$

$$\hat{P} = 0, \hat{Q} = 0: \quad D = 3368.$$

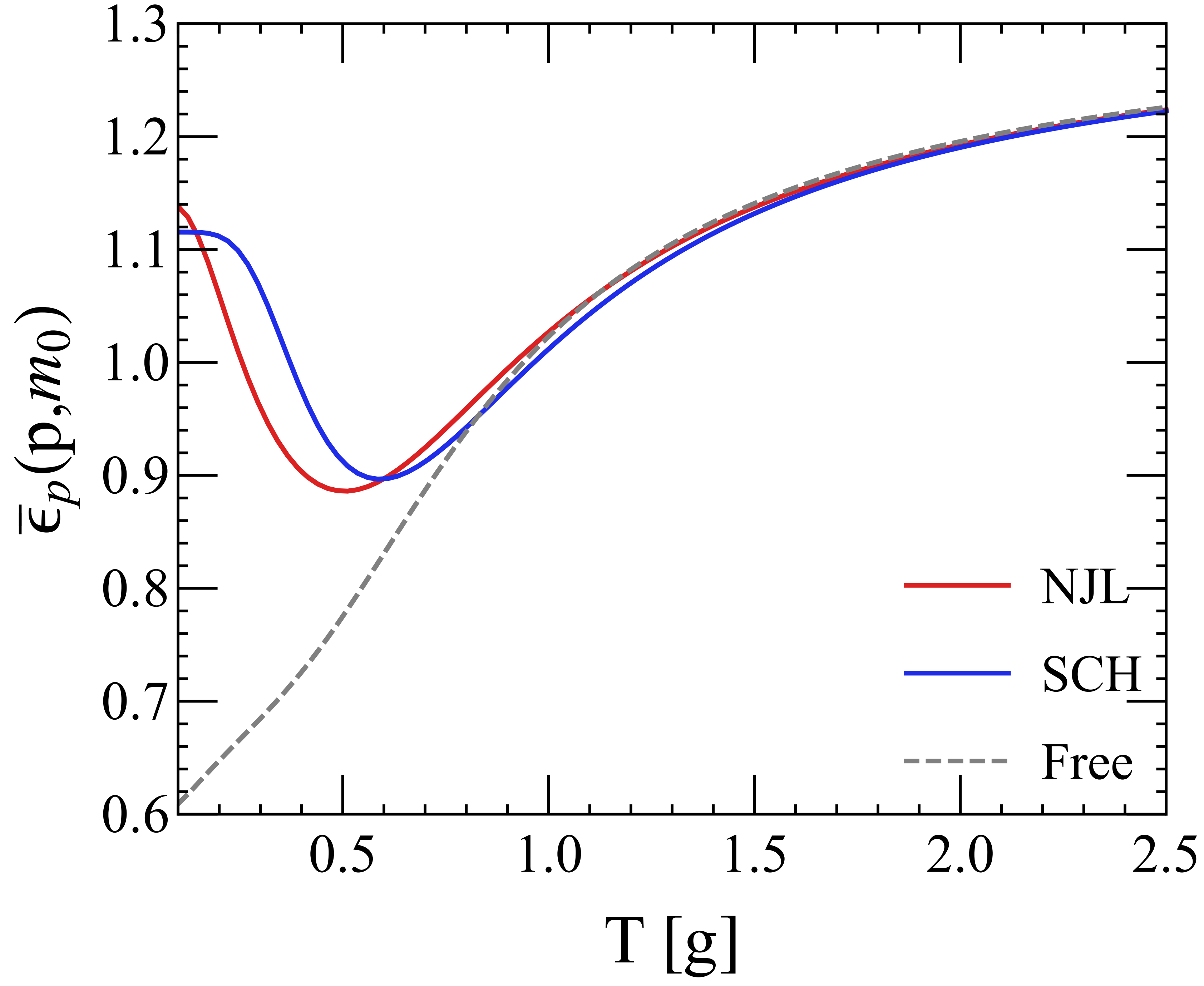
$$-A(p + \frac{}{2}, p - \frac{}{2})(a^\dagger(p - \frac{}{2})c^\dagger(-p - \frac{}{2}) + a(-p + \frac{}{2})c(p + \frac{}{2}))\Big),$$

single-particle distribution

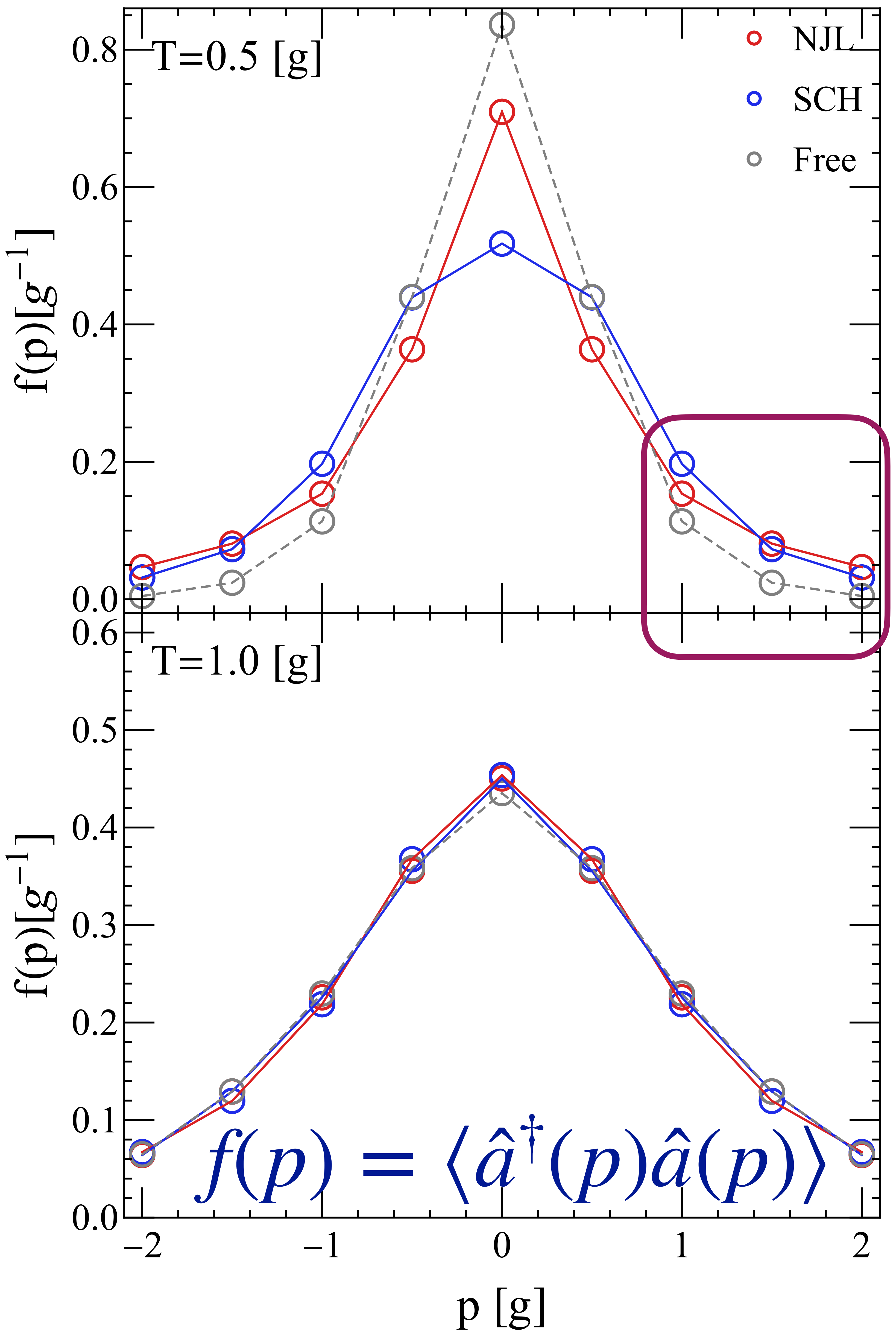


$$f(p) = \langle \hat{a}^{\dagger}(p)\hat{a}(p) \rangle$$

$$\bar{\epsilon} \equiv \left(\int_p \sqrt{m^2 + p^2} f(p)\right) / \left(\int_p f(p)\right)$$



single-particle distribution



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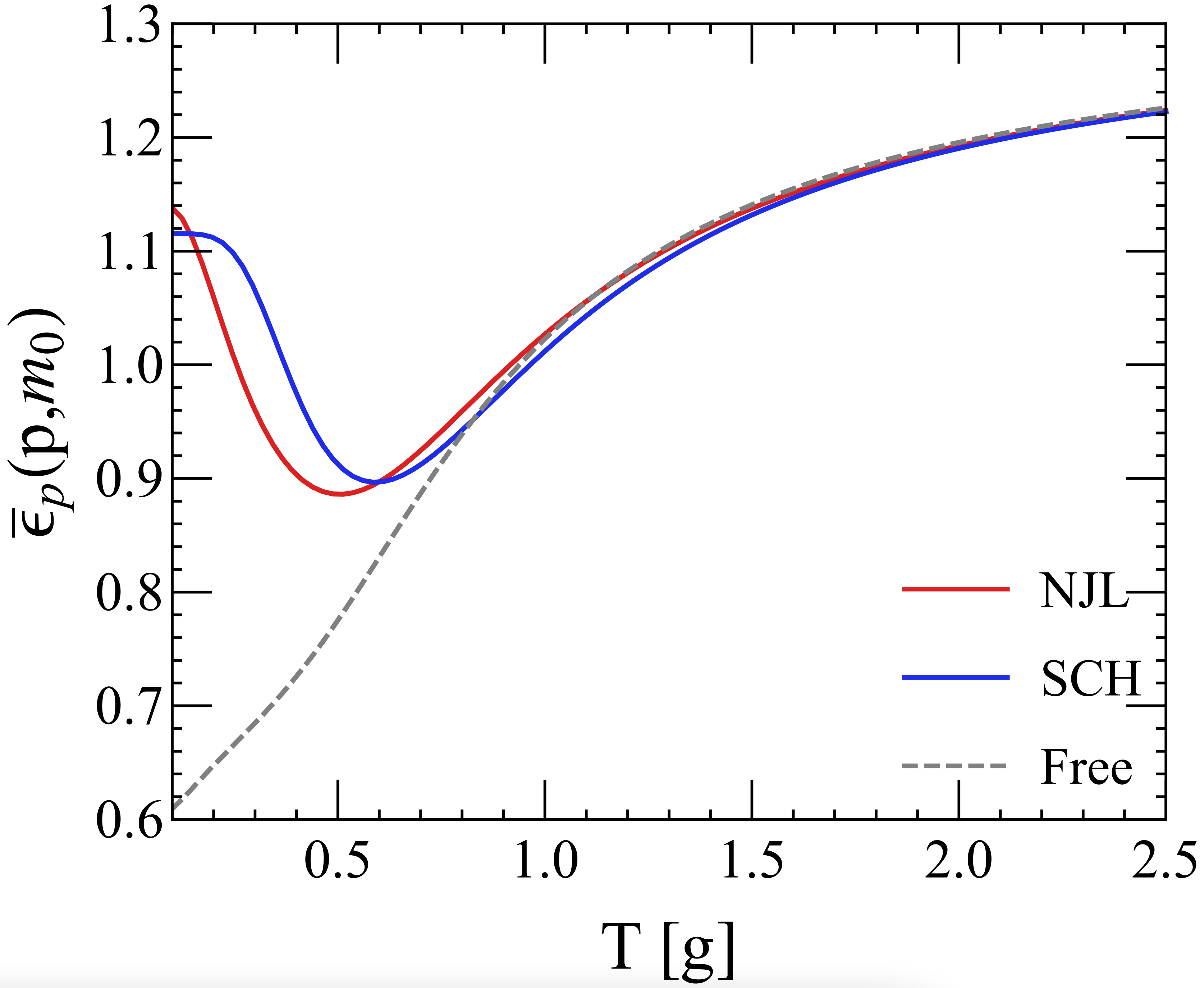
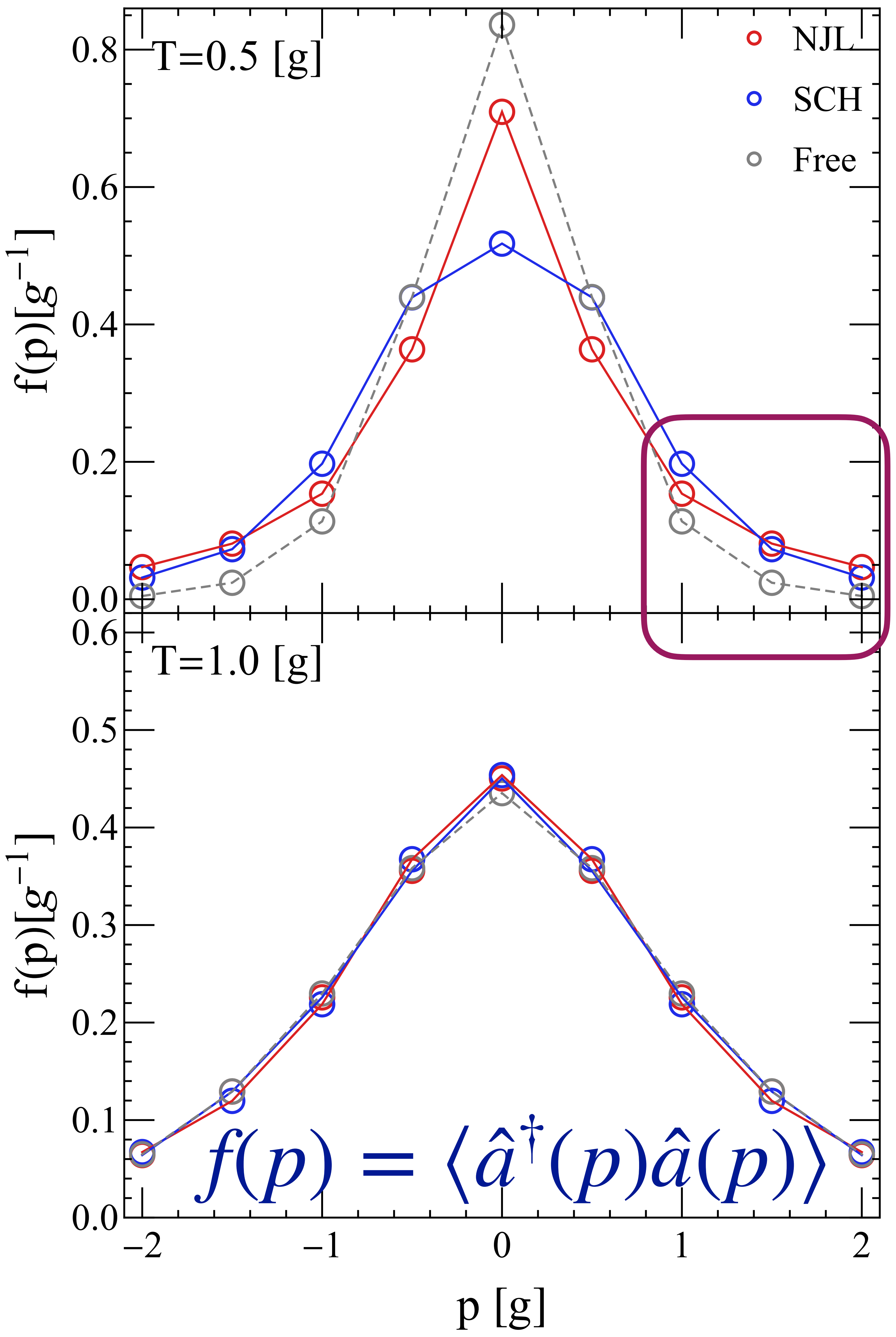


low temperature: relative motion in bound states => high momentum tail

single-particle distribution



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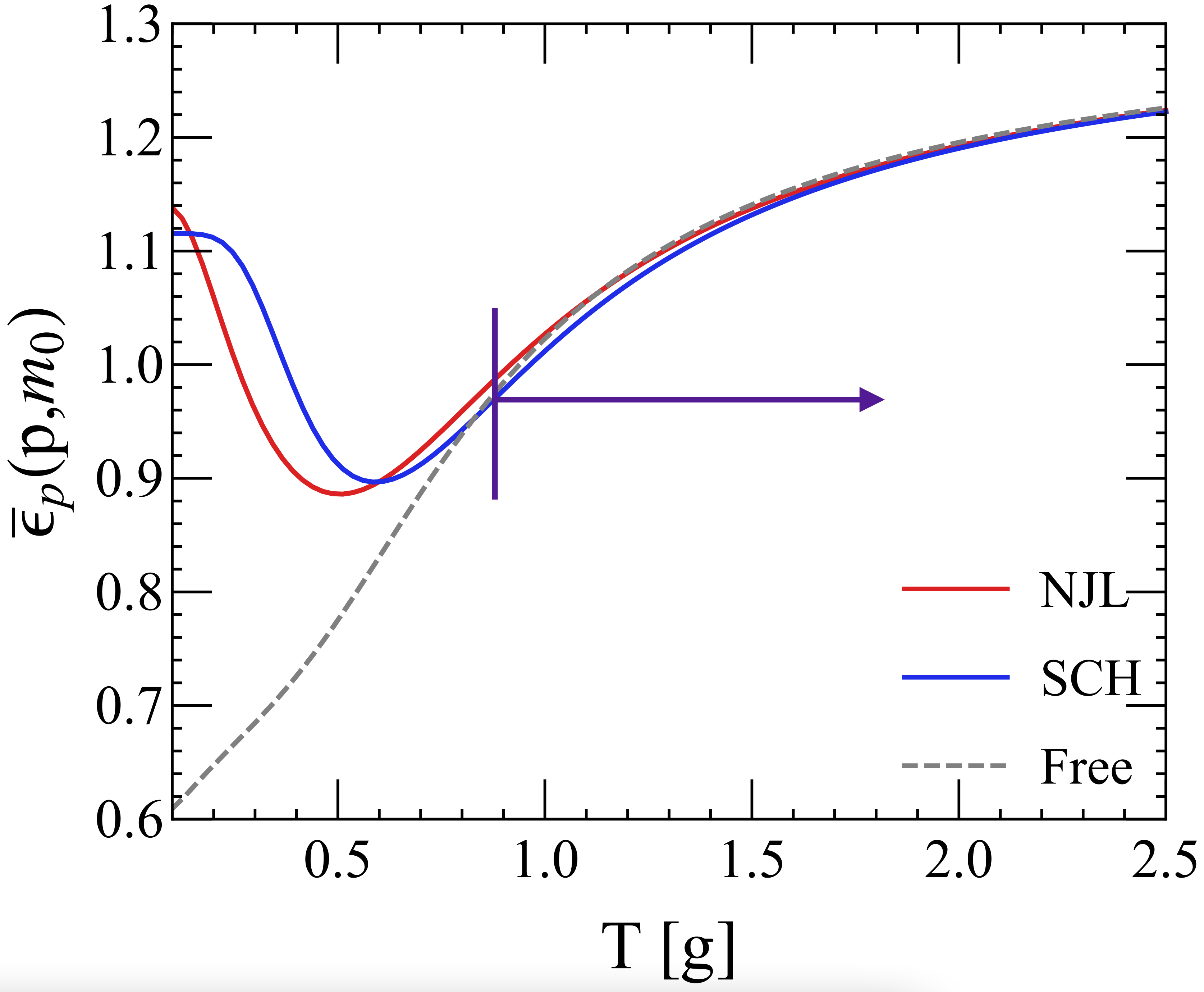
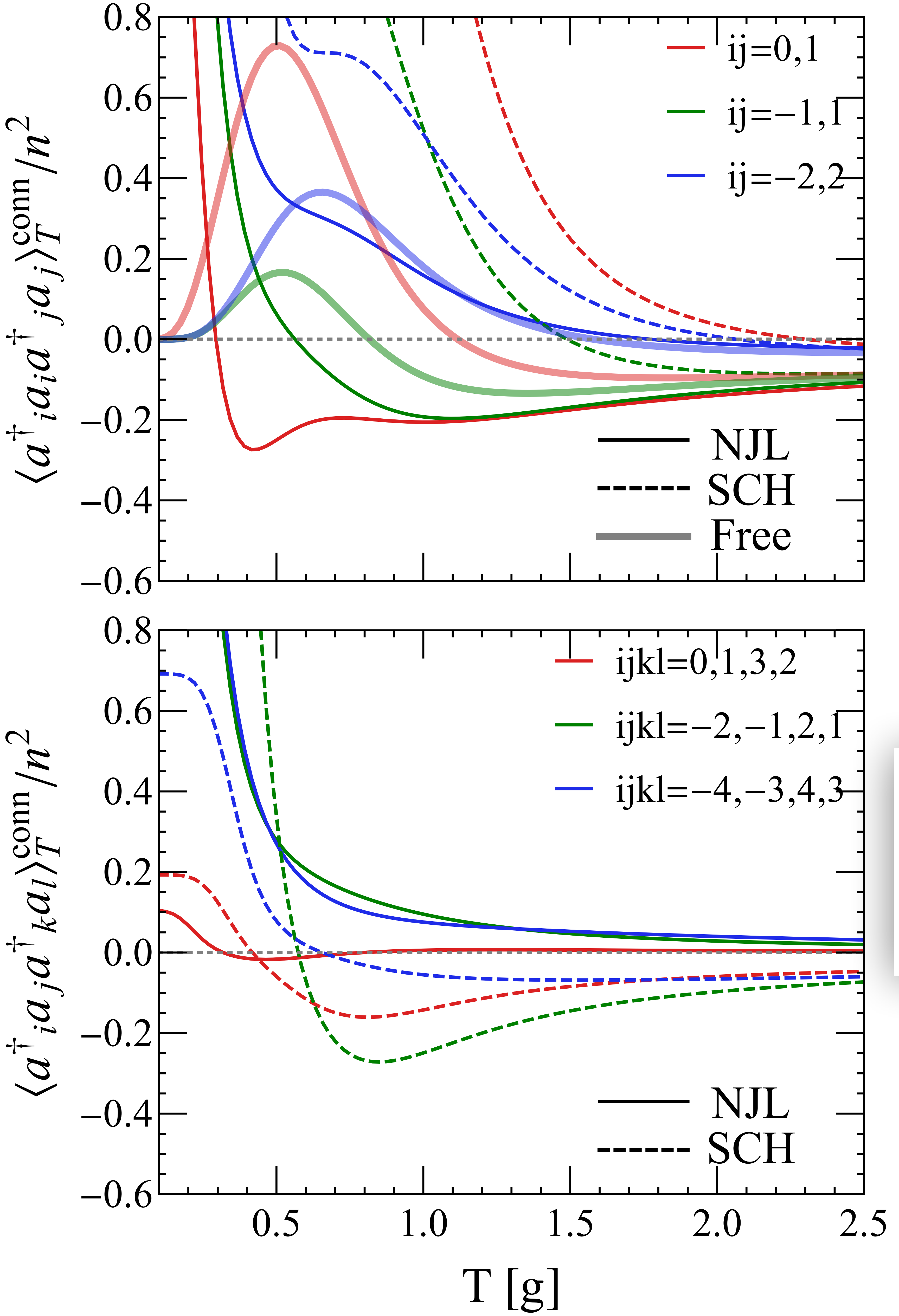


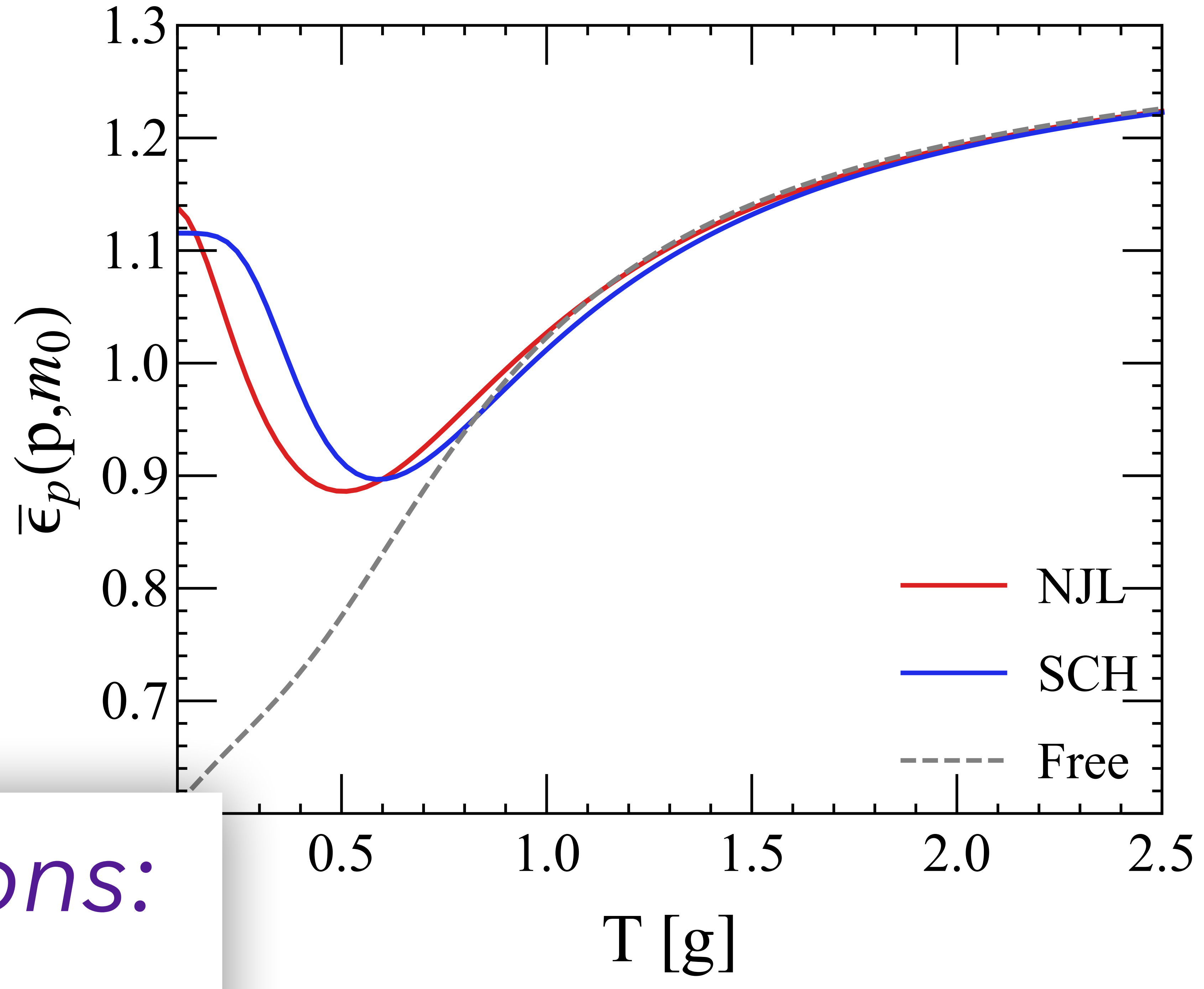
low temperature: relative motion in bound states => high momentum tail

High temperature: free fermion/antifermions

two-particle correlation



$$\bar{\epsilon} \equiv \left(\int_p \sqrt{m^2 + p^2} f(p)\right) / \left(\int_p f(p)\right)$$



two particle correlations: small but non-vanishing

$$\langle \hat{a}^\dagger_i \hat{a}_j \hat{a}^\dagger_k \hat{a}_l \rangle_T^{\mathrm{conn}} \equiv \langle \hat{a}^\dagger_i \hat{a}_j \hat{a}^\dagger_k \hat{a}_l \rangle_T - \langle \hat{a}^\dagger_i \hat{a}_j \rangle_T \langle \hat{a}^\dagger_k \hat{a}_l \rangle_T + \langle \hat{a}^\dagger_i \hat{a}_l \rangle_T \langle \hat{a}^\dagger_k \hat{a}_j \rangle_T .$$

$$\langle \hat{a}^\dagger_i \hat{a}_i \hat{a}^\dagger_k \hat{a}_k \rangle_T^{\mathrm{conn}} \propto f_2(p_i, p_k) - f(p_i) f(p_k) .$$

Can we approach a thermal state?

Hamiltonian

1+1D Schwinger model

$$H = \int \left(\frac{E^2}{2} - \bar{\psi}(i\gamma^1 \partial_x - g\gamma^1 A - m)\psi \right) \mathrm{d}x \, .$$

E: electric field

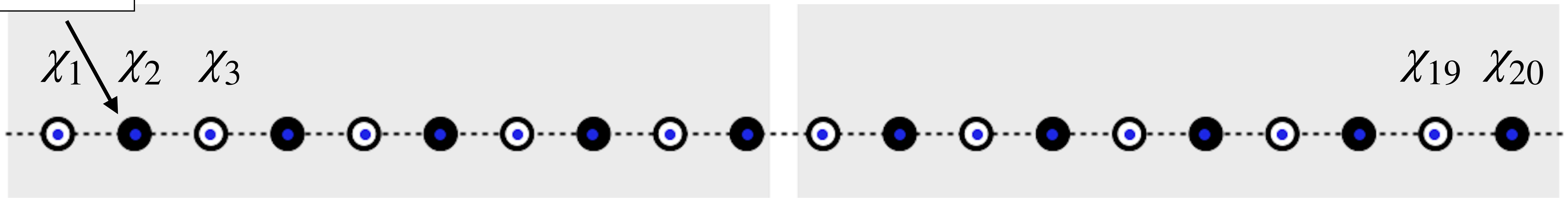
A: electric potential

$\psi, \bar{\psi}$: fermion field

$|0\rangle$:empty

$|1\rangle$:occupied

$$\{\psi_a(x), \psi_b^\dagger(y)\} = \delta_{a,b}\delta(x-y)$$



field operators represented by matrices

$$\{\chi_n^\dagger, \chi_m\} = \delta_{nm} \, , \quad \{\chi_n^\dagger, \chi_m^\dagger\} = \{\chi_n, \chi_m\} = 0 \, .$$

$$H = \frac{1}{4a} \sum_{n=1}^{N-1} (S_n^+ U_n S_{n+1}^- + S_{n+1}^+ U_n^\dagger S_n^-)$$

$$+ \frac{m}{2} \sum_{n=1}^{N} (-1)^n Z_n + \frac{a\,g^2}{2} \sum_{n=1}^{N-1} L_n^2 \, .$$

Jordan-Wigner $\chi_n = \frac{S_n^-}{2} \prod_{m=1}^{n-1} (-iZ_m)$

$$S_n^\pm \equiv I \otimes \cdots \otimes I \otimes \underbrace{(\sigma_x \pm i\sigma_y)}_{n^{\mathrm{th}}} \otimes I \otimes \cdots \otimes I$$

initial state: $|\Psi(t=0)\rangle = e^{i\omega \int_{-w/2}^{+w/2} \bar{\psi}\psi(z)\mathrm{d}z} |\mathrm{vac}\rangle$

vacuum + [excitation @ center]

lattice sites: N = 100

spacing: a = 1/2g

$$|\Psi(t)\rangle = e^{-i\hat{H}_{\mathrm{Sch}}t} |\Psi(t=0)\rangle$$

$$\mathscr{L} \Rightarrow \hat{T}^{\mu\nu}$$

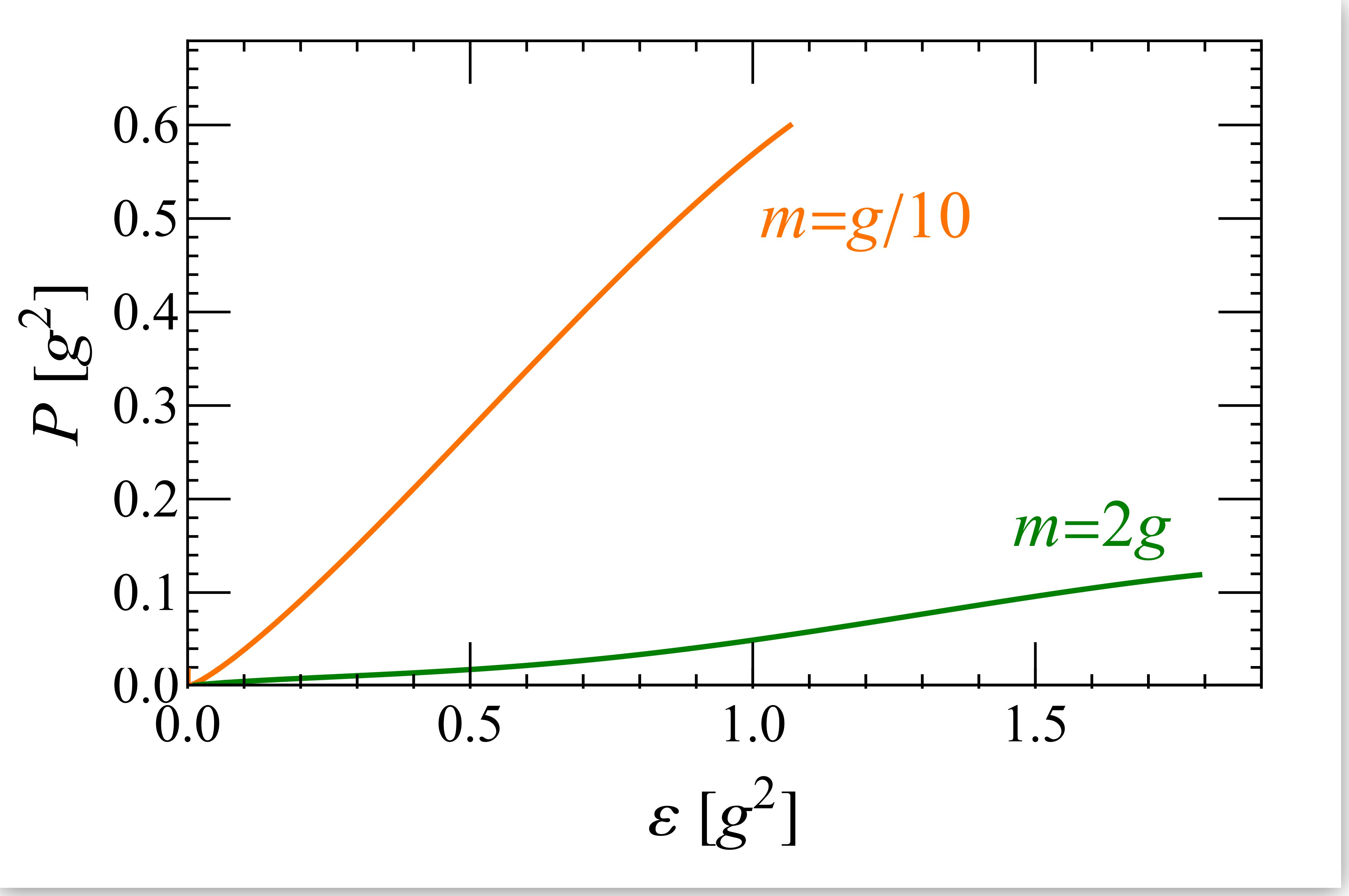
$$\langle\Psi(t)|\hat{T}^{\mu\nu}(x)|\Psi(t)\rangle = T^{\mu\nu}(t,x) = (\varepsilon + P + \Pi)u^\mu u^\nu - (P + \Pi)g^{\mu\nu}$$

bulk pressure: non-equilibrium correction initial state: $|\Psi(t=0)\rangle = e^{i\omega\int_{-w/2}^{+w/2}\bar{\psi}\psi(z)\mathrm{d}z}|\mathrm{vac}\rangle$

lattice sites: N = 100

spacing: a = 1/2g

vacuum + [excitation @ center]



$|\Psi(t)\rangle = e^{-i\hat{H}_{\mathrm{Sch}}t}|\Psi(t=0)\rangle$

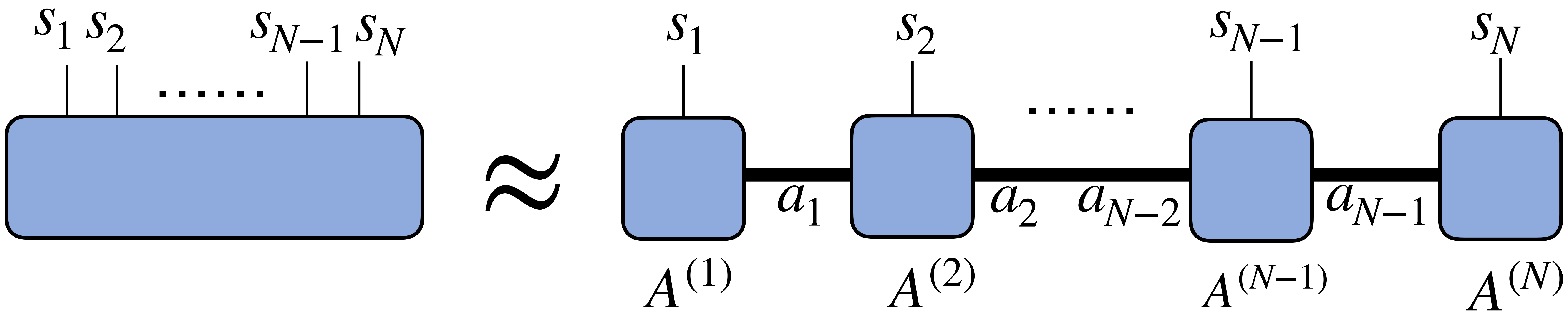
$\mathscr{L} \Rightarrow \hat{T}^{\mu\nu}$

thermal pressure: $P = P_{\mathrm{th}}(\varepsilon)$

$\langle\Psi(t)|\hat{T}^{\mu\nu}(x)|\Psi(t)\rangle = T^{\mu\nu}(t,x) = (\varepsilon + P + \Pi)u^\mu u^\nu - (P + \Pi)g^{\mu\nu}$

bulk pressure: non-equilibrium correction

$$|\Psi\rangle = \sum_{\{s\}} \psi_{s_1 s_2 \ldots s_{N-1} s_N} |s_1\rangle \otimes |s_2\rangle \otimes \ldots |s_{N-1}\rangle \otimes |s_N\rangle$$



$$\psi_{s_1 s_2 \ldots s_{N-1} s_N} \approx \sum_{a_1 a_2 \ldots a_{N-2} a_{N-1}} A^{(1)}_{s_1 a_1} A^{(2)}_{s_2 a_1 a_2} \ldots A^{(N-1)}_{s_{N-1} a_{N-2} a_{N-1}} A^{(N)}_{s_N a_{N-1}}$$

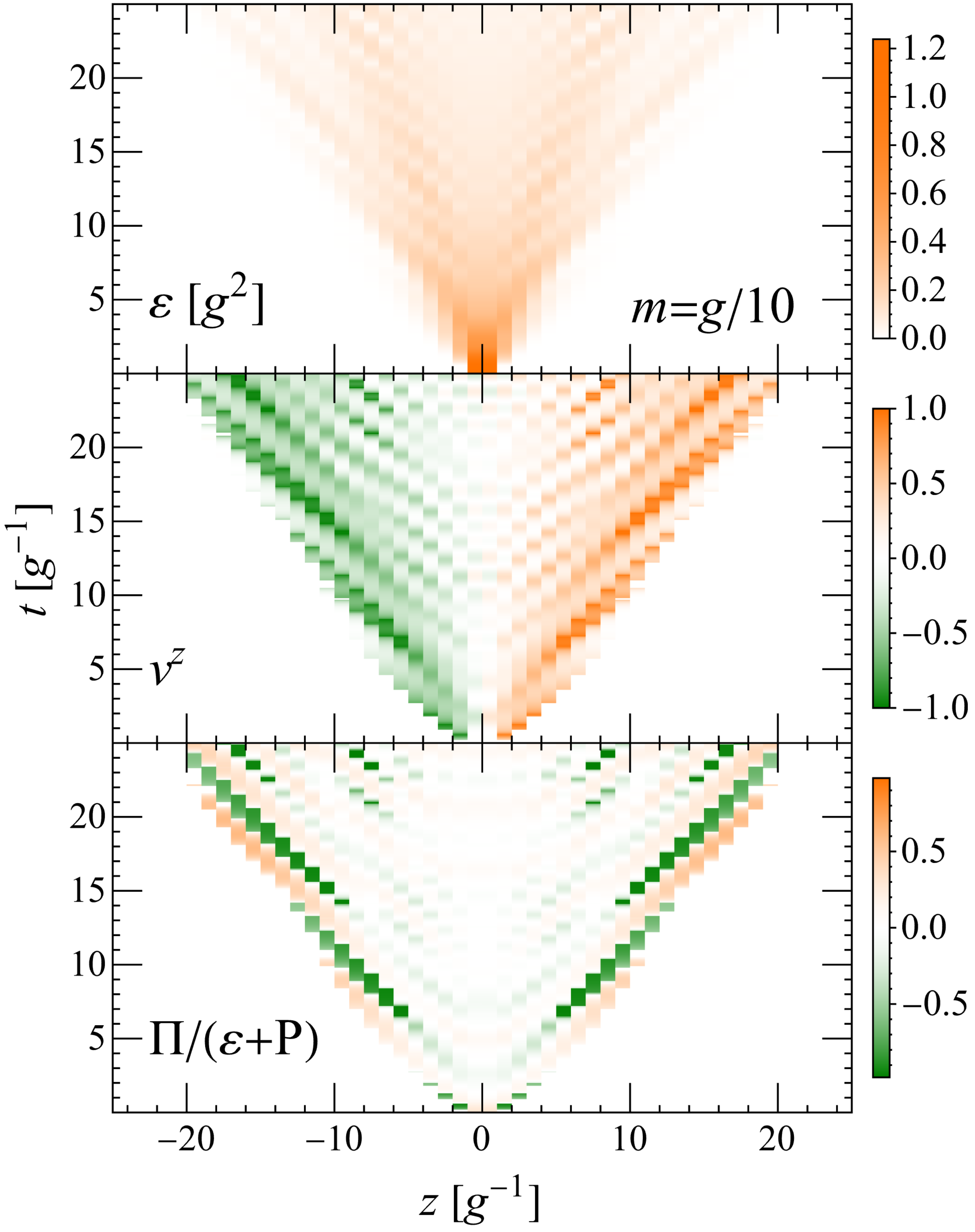
d.o.f.: D^N $N\,D\,d^2$

reproduces the full Hilbert space if *d* sufficiently large,

otherwise, drop states with very high entanglement entropy

Our results converge over d

emergence of hydrodynamics



***Bjorken flow** – an analytical solution to the ideal hydrodynamic equations*

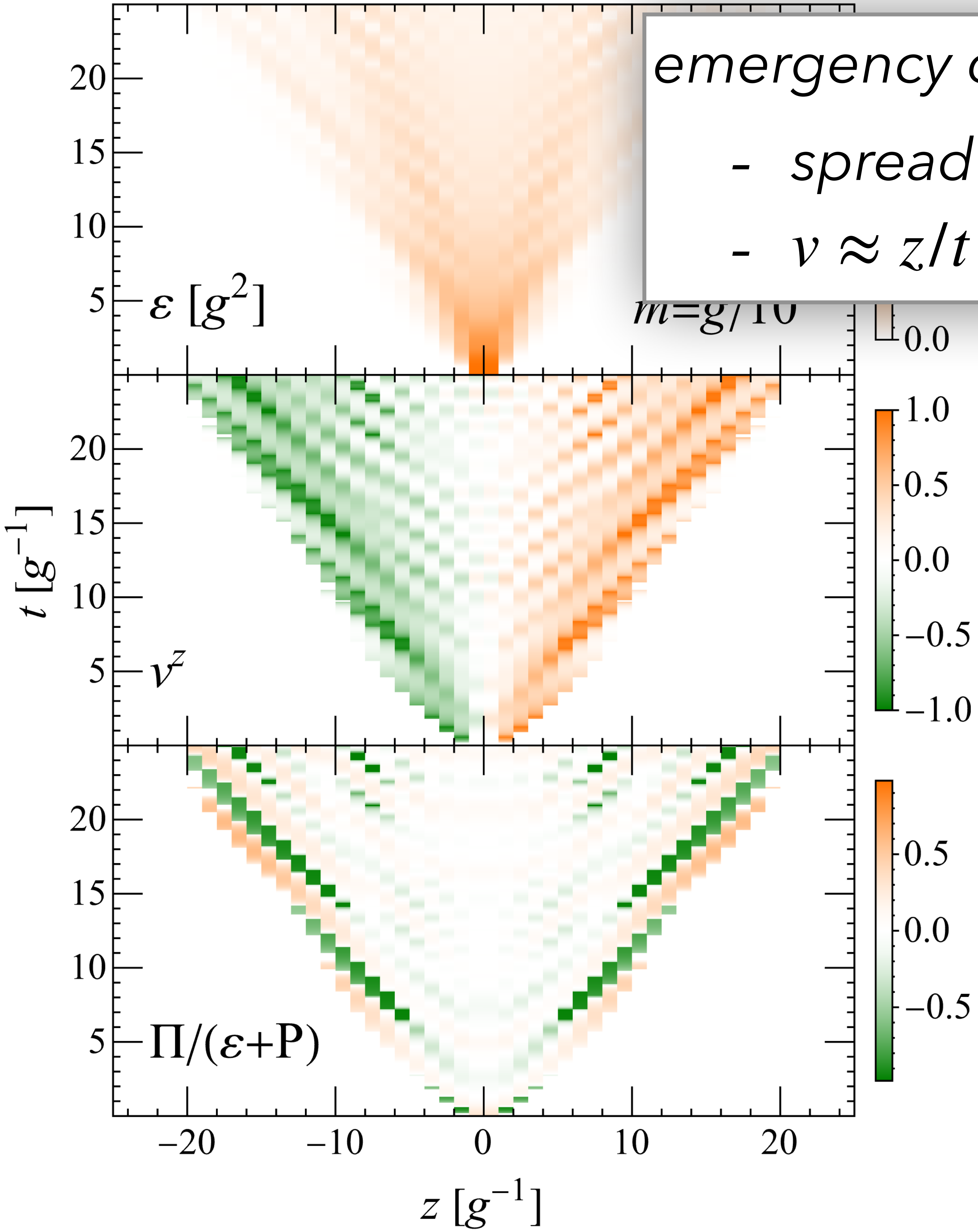
$$\varepsilon \propto \tau^{-1-c_s^2}, \qquad \tau \equiv \sqrt{t^2 - z^2}\,.$$

boost invariant (within a rapidity plateau)

$$v^z = z/t\,.$$

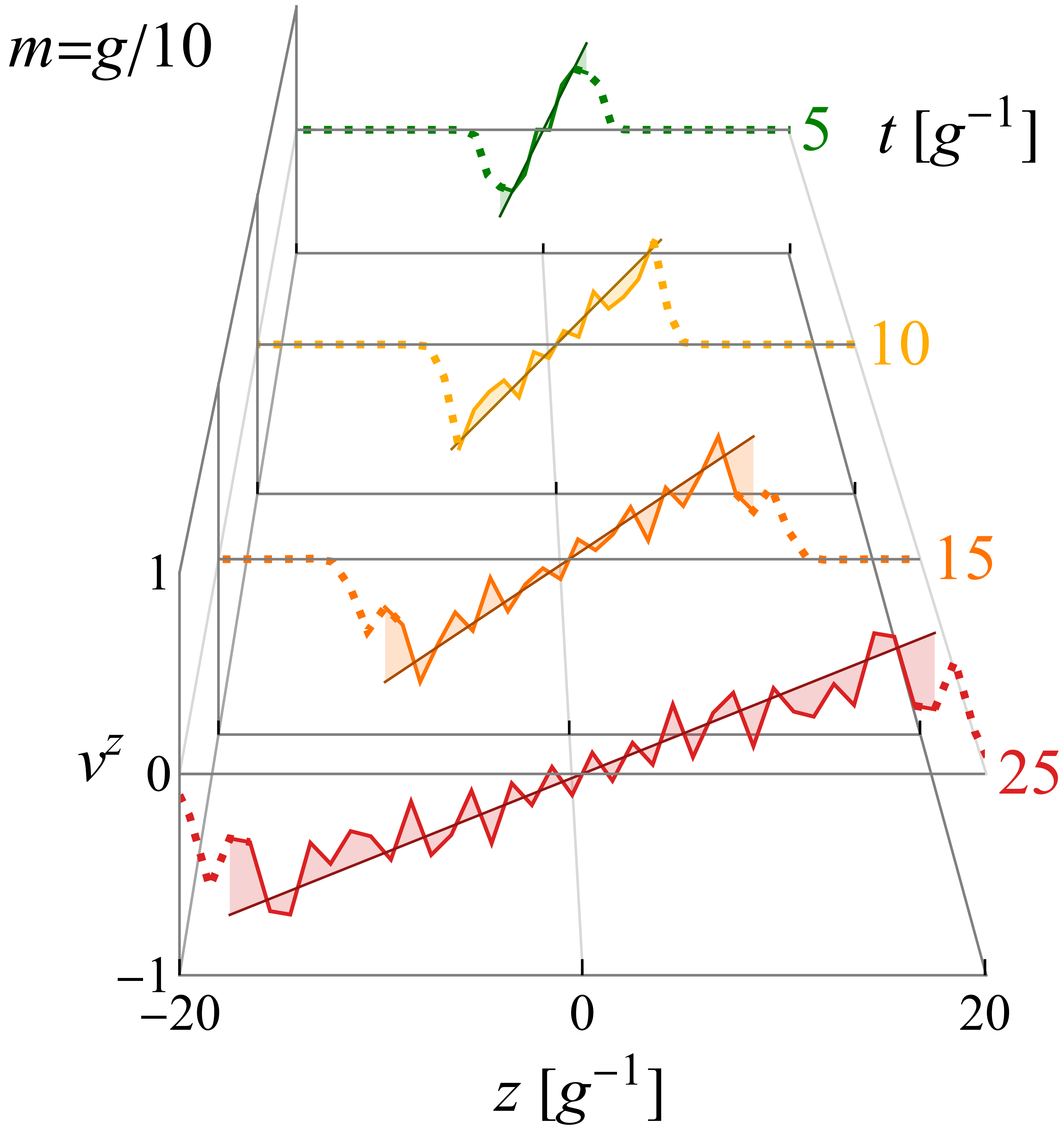
with viscosity: $\Pi \propto \tau^{-2}$

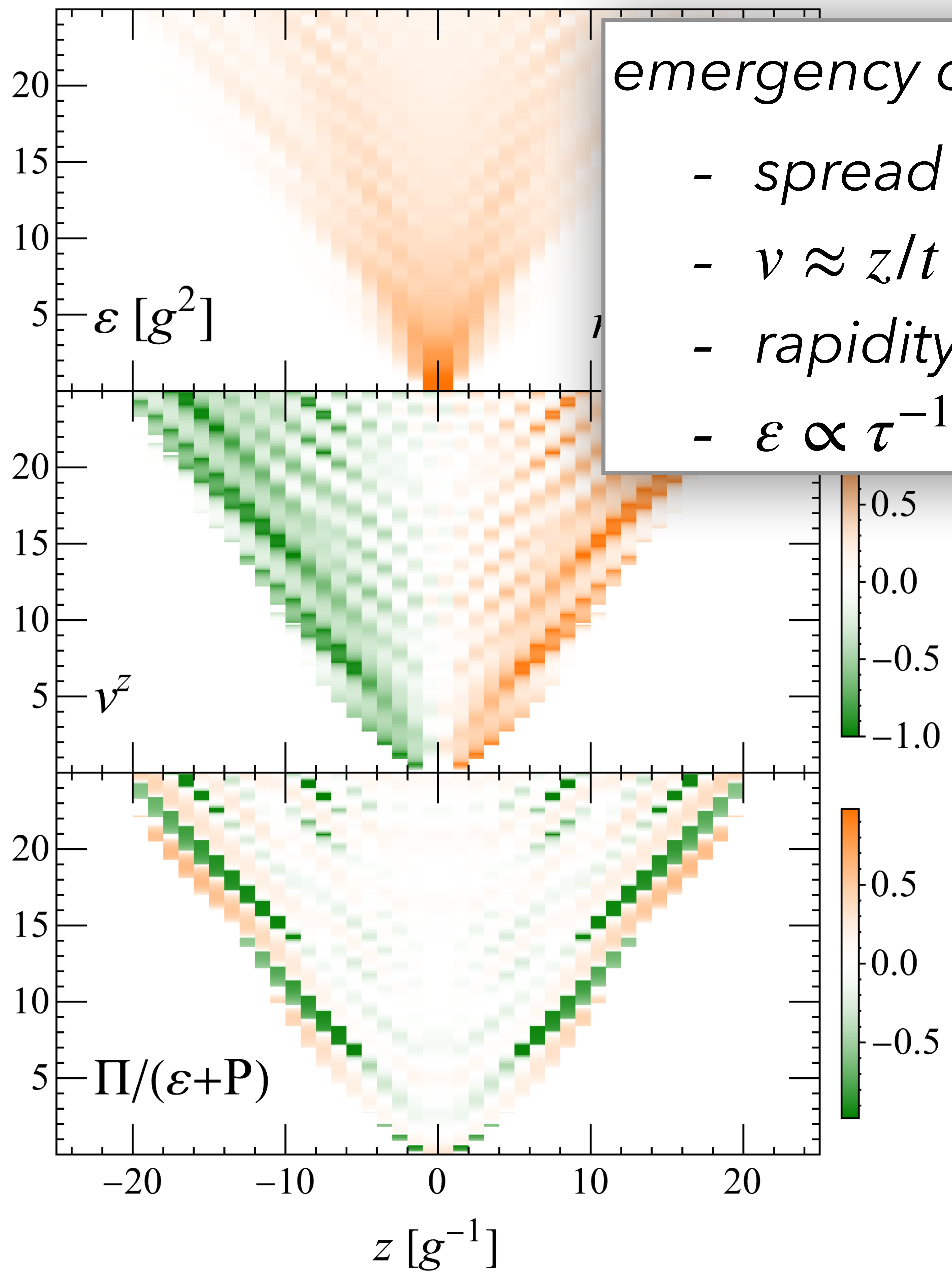
emergence of hydrodynamics



emergency of hydrodynamics

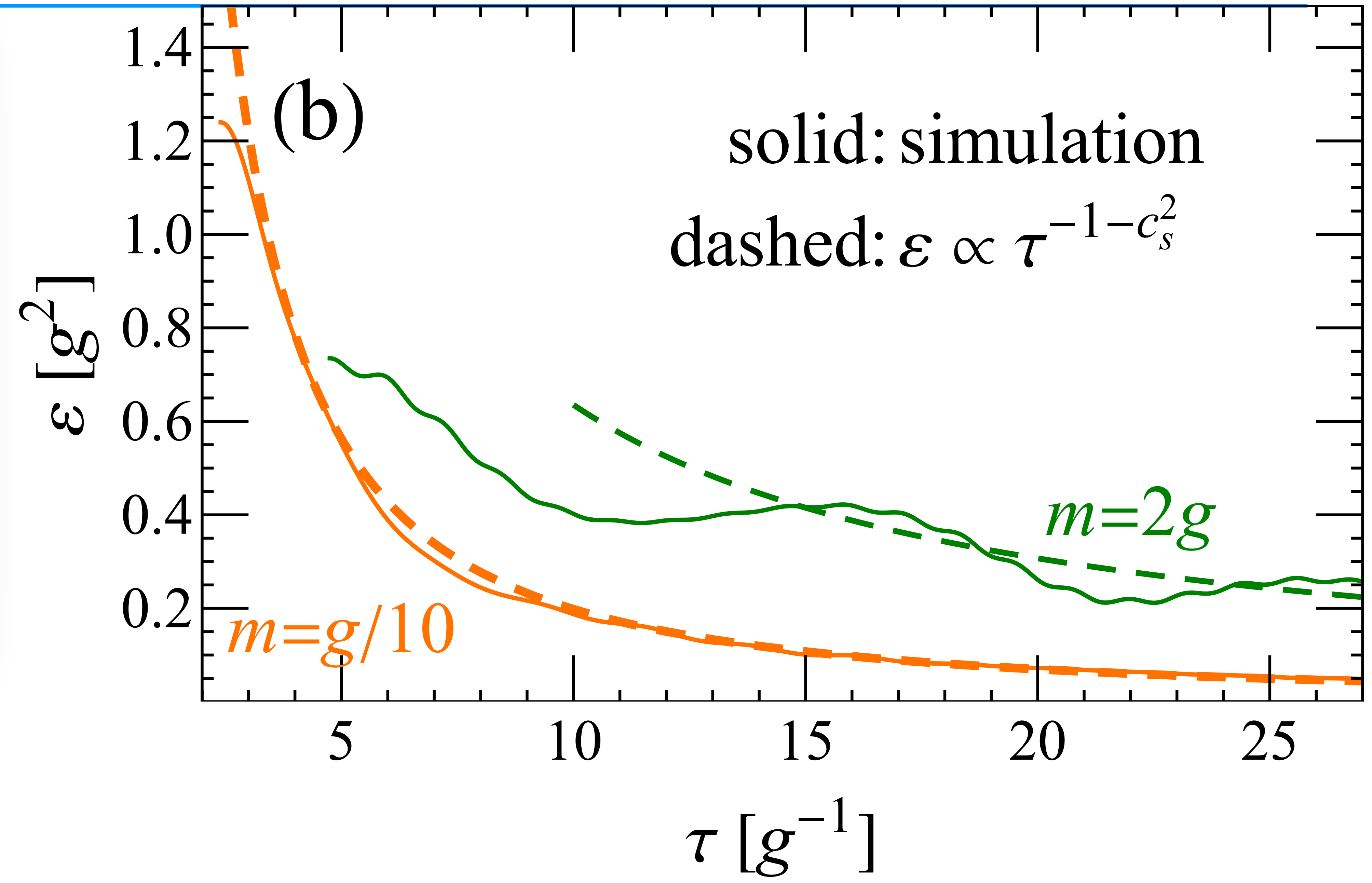
- *spread within light cone*
- $v \approx z/t$

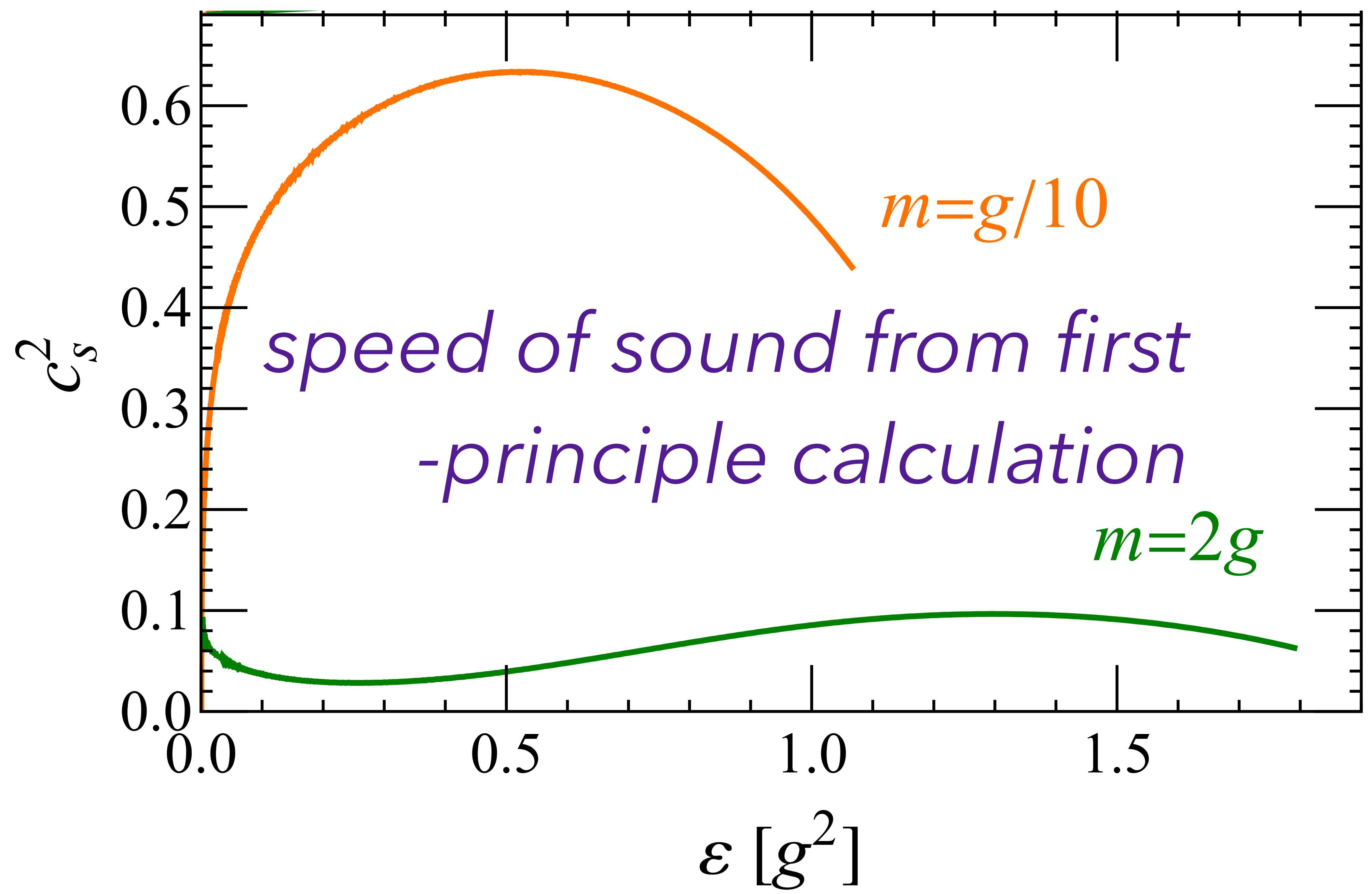




emergency of hydrodynamics

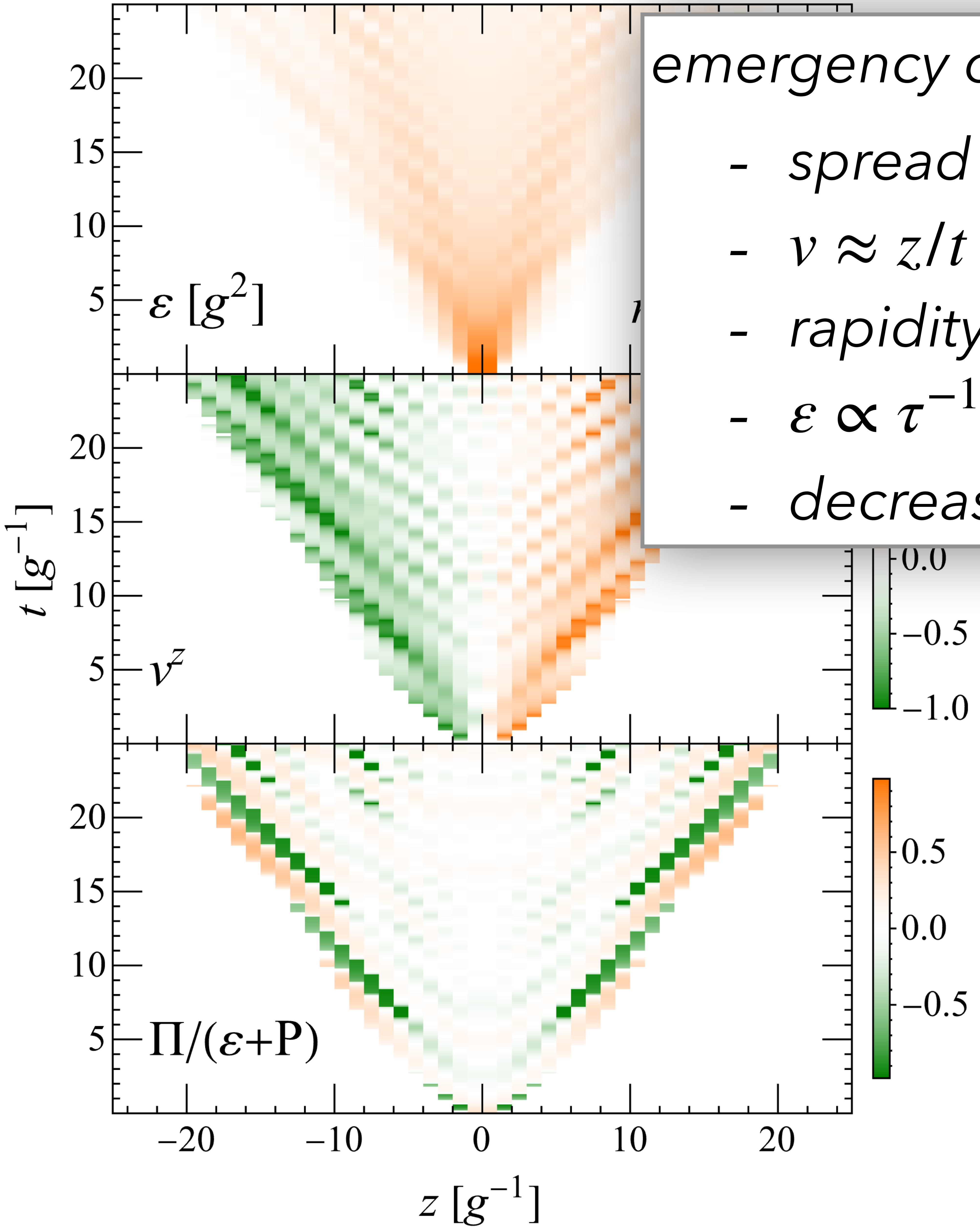
- *spread within light cone*
- $v \approx z/t$
- *rapidity structure*
- $\varepsilon \propto \tau^{-1-c_s^2}$





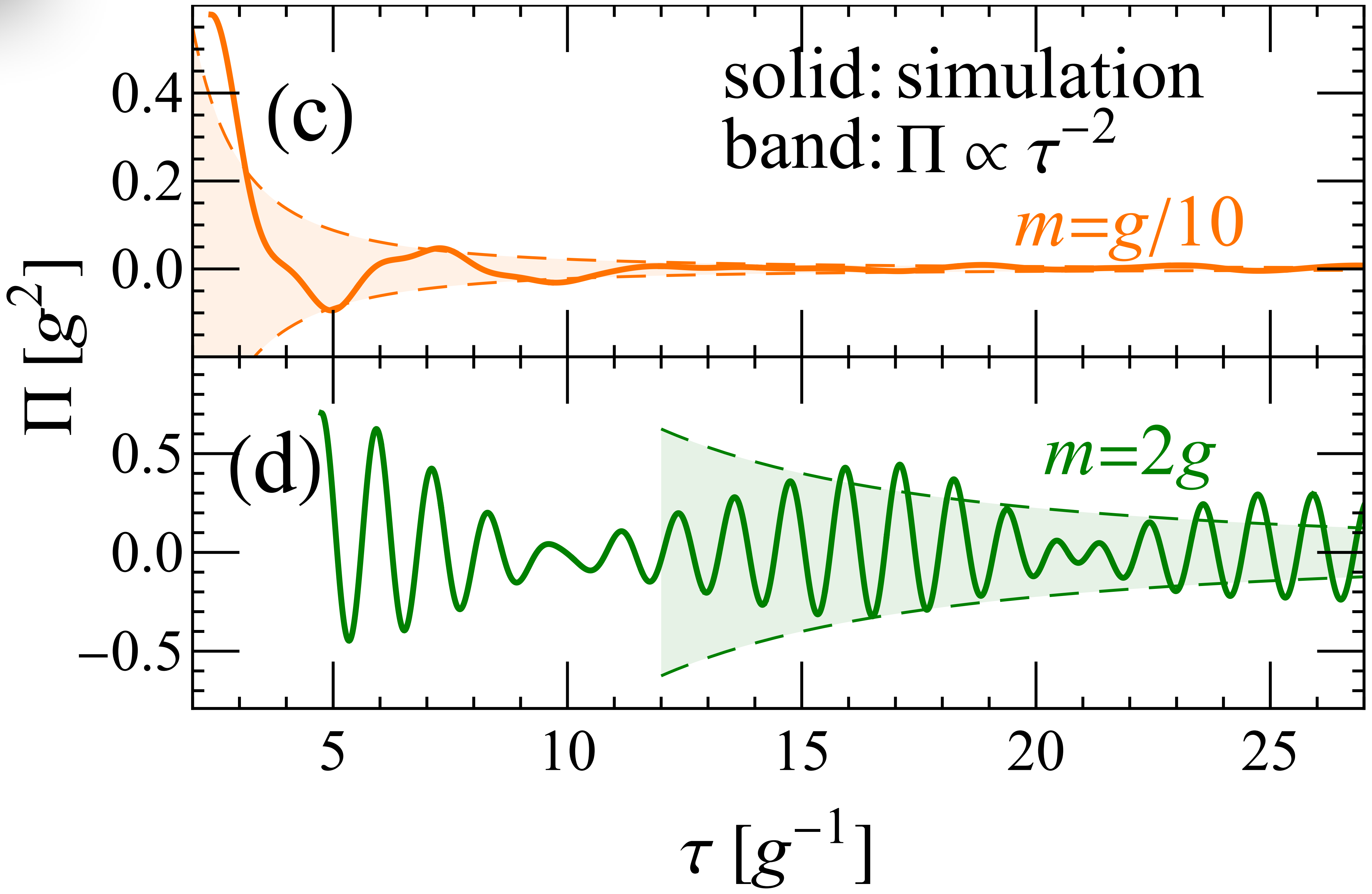
speed of sound from first -principle calculation

emergence of hydrodynamics



emergency of hydrodynamics

- *spread within light cone*
- $v \approx z/t$
- *rapidity structure*
- $\varepsilon \propto \tau^{-1-c_s^2}$
- *decreasing of* Π



summary

Fully exact diagonalized thermal states w/ momentum lattice

- bound state dominated @ low-temperature
- asymptotic free fermions @ high-temperature

Real-time non-perturbative quantum evolution

- emergence of hydrodynamics
  - light-cone structure/rapidity structure
  - energy density, bulk pressure, velocity
- full quantum state accessible, ready to test different microscopic

  derivations of quantum hydrodynamics —— ideals are welcome!